Enhanced sensitivity to current modulation near dynamic instability in semiconductor lasers with optical feedback and optical injection

Maria S. Torre

Instituto de Física "Arroyo Seco," Universidad Nacional del Centro de la Provincia de Buenos Aires Pinto, 399 (7000) Tandil, Argentina

Cristina Masoller

Instituto de Física, Facultad de Ciencias, Universidad de la República, Igua 4225, Montevideo 11400, Uruguay, and School of Informatics, University of Wales, Bangor, Dean Street, Bangor LL57 1UT, UK

Paul Mandel

Université Libre de Bruxelles, Optique Nonlinéaire Théorique, Campus Plaine C. P. 231, B-1050 Bruxelles, Belgium

K. Alan Shore

School of Informatics, University of Wales, Bangor, Dean Street, Bangor LL57 1UT, UK

We study numerically the response of a semiconductor laser to a small current modulation added to the dc-bias current. The laser is subjected to either optical feedback from an external reflector or optical injection from another laser. We characterize the nonlinear amplification near the Hopf bifurcation leading to the onset of undamped relaxation oscillations. © 2004 Optical Society of America
    OCIS codes: 140.5960, 140.1540, 140.2020, 230.7390.

1. INTRODUCTION

The influence of current modulation on the dynamics of semiconductor lasers with optical feedback or optical injection is of interest for many applications. For example, high-frequency modulation of the injection current is often employed to reduce the relative intensity noise induced by optical feedback or optical injection. It is well known that under current modulation, optical feedback, or optical injection, semiconductor lasers exhibit a rich variety of nonlinear behavior.[1] Both intrinsic interest in nonlinear dynamics and practical applications have spurred a wide range of studies of current-modulated external-cavity lasers[2–7] and of current-modulated optically injected lasers.[8–12]

Sacher *et al.*[2] studied experimentally the dynamics of a current-modulated external-cavity laser diode. The laser operated in a regime such that its dynamics was characterized by two frequencies: current modulation frequency f_{mod} and external cavity frequency f_{ext} ($f_{\text{ext}} = c/2L$, where c is the velocity of light and L is the length of the external cavity). Frequency locking was shown to occur at the rational ratio of $f_{\text{mod}}/f_{\text{ext}} = p/q$, with p, q as integers. It was also shown that the locking behavior has many features in common with the predictions of the one-dimensional standard circle map, thus demonstrating that the current-modulated external-cavity laser belongs to the universal class of systems that exhibit a quasi-periodic route to chaos.

The possibility of controlling the chaotic dynamics of an external-cavity laser diode through a sinusoidal modulation of the injection current was demonstrated numerically and analytically by Liu *et al.*[3] They obtained optimal periodic outputs by calculating the modulation frequency from the mode analysis of the dynamic model. The numerical results predict that chaos can be successfully stabilized to various limit cycles with very small current modulation amplitudes.

Several recent studies have focused on the control of low-frequency fluctuations (LFFs) of external-cavity semiconductor lasers with injection current modulation. In the LFF regime the laser intensity suddenly drops toward zero and then recovers gradually, only to drop again after a certain random delay. Takiguchi *et al.*[4] observed experimentally that, when the frequency of current modulation f_{mod} was close to external cavity frequency f_{ext}, the laser output was synchronized with the current modulation and no dropouts occurred. When f_{mod} was detuned from f_{ext}, the current modulation was not effective to control LFFs and random power dropouts were observed.

Mendez *et al.*[5] studied experimentally the case in which an external-cavity laser without current modulation operates in the excitable regime, exhibiting intensity dropouts only for perturbations larger than a certain threshold. By applying a weak sinusoidal modulation to the injection current, they obtained stable patterns, in which the intensity is periodic with the period of the forc-

ing and presents q dropouts for each p period of the forcing signal. The results were interpreted in terms of solutions of the model of Eguia *et al.*[13]

Buldú *et al.*[6] showed that the power dropouts exhibited by a laser in the LFF regime can be entrained by the joint action of weak periodic current modulation and external noise. The power dropouts, which in the absence of current modulations occur randomly, acquire the periodicity of the current modulation for an optimal amount of external noise, thus demonstrating stochastic resonance. The observed behavior was explained by use of a Lang–Kobayashi model with external colored noise in the modulated injection current.

The dynamics of optically injected semiconductor lasers under current modulation has also been studied extensively. A recent report was made of the experimental observation of bistability when a laser is subjected to a weak current modulation and a high optical injection rate.[11] In these experiments, cw optical injection induces a subcritical Hopf bifurcation in the optical output. This means that two distinct amplitudes of the oscillations are available for certain values of the control parameters. Nizette *et al.*[10–12] analyzed analytically and numerically the laser response, taking as control parameters the optical injection rate and optical detuning. As a function of the injection rate several branches of periodic and quasiperiodic intensity oscillations were found but no signature of bistability.[10] However, analysis of the laser equations with optical detuning as the control parameter lead to cases of bistability.[11] When modulation of the current is nearly resonant with the intrinsic relaxation oscillation frequency of the laser, under strong enough optical injection, a double locking occurs: the intensity oscillations are locked to the injection current modulation and the optical frequency is locked to the frequency of the injected field.[8,9]

Here we study numerically the enhancement of the laser response for particular values of current modulation frequency. We consider both a laser with optical feedback and a laser with optical injection. In a previous analysis we studied the response of an external-cavity vertical-cavity surface-emitting laser to a small modulation added to the dc-bias current.[7] The simulations were based on a multimode model that included carrier diffusion and spatial profiles for transverse modes. It was found that the effect of the injection current modulation depends strongly on the modulation frequency: if f_{mod} is close to the frequency of the relaxation oscillations of free-running laser f_{RO}, a small amplitude modulation induces a large amplitude spiking behavior. We studied this response by plotting the maximum, minimum, and average values of the intensity oscillations as a function of the period of the modulation. For small modulation amplitude and a weak feedback level we found the symmetric, Lorenzian-shaped resonance peak that is characteristic of a nonlinear oscillator under external periodic forcing. For a larger modulation amplitude or feedback level we found that the peak was distorted in a complex and unexpected way.

Here we have performed numerical calculations within the framework of a single-mode rate-equation model with the aim of clarifying the origin of the enhanced sensitivity

of the laser to the external periodic perturbation of the injection current. We show that enhancement and distortion of the response are related to the occurrence of a Hopf bifurcation in which the relaxation oscillations are no longer damped and the laser exhibits limited-cycle intensity oscillations. In the case of an external-cavity laser, the Hopf bifurcation occurs when the feedback level increases. In the case of an optically injected laser, the Hopf bifurcation occurs for a fixed injection strength when the frequency of the injected field is detuned away from the region of stable injection locking.

In Section 2 we present the model used to simulate the dynamics of current-modulated diode lasers with optical feedback and optical injection. In Section 3 we present the results of numerical simulations for external-cavity lasers. In Section 4 we present the results of the numerical simulations for optically injected lasers, and in Section V we present a summary and the conclusions.

2. MODEL

The rate equations for a single-mode semiconductor laser with optical feedback, optical injection, current modulation, and white noise are[2,14]

$$\dot{E} = k(1 + i\alpha)(N - 1)E(t) + \gamma E(t - \tau)\exp(-i\omega\tau)$$
$$+ i\Delta w E(t) + E_{inj} + \sqrt{D}\xi(t), \tag{1}$$

$$\dot{N} = \{j[1 + \delta\sin(2\pi t/T_{mod})] - N - N|E|^2\}/\tau_n. \tag{2}$$

Here, E is the slowly varying optical field envelope; N is the normalized carrier density; k is the cavity loss; α is the linewidth enhancement factor; ω is the optical frequency of the solitary laser; γ is the feedback level; τ is the delay time; $E_{inj} = k_{inj}\sqrt{P_{inj}}$ is the amplitude of the injected field; $k_{inj} = \tau_l^{-1}\sqrt{\eta_{inj}(1/r_2 - r_2)}$ is the injection parameter (where r_2 is the laser output mirror reflectivity, τ_l is the laser cavity round-trip time, and η_{inj} is the coupling efficiency of the injected light to the circulating field in the laser cavity); P_{inj} is the injected power (in units of photon number); Δw is the detuning between the free-running oscillation frequency and the frequency of the injected signal; $\xi(t)$ is a complex Gaussian white noise; D is the noise intensity, j is the normalized injection current; δ is the modulation amplitude; T_{mod} is the modulation period; and τ_n is the carrier lifetime. The model includes a single reflection in the external cavity and therefore is valid only for weak feedback levels. In the numerical simulations the laser internal parameters are $k = 300\,ns^{-1}$, $\alpha = 3$, $\tau_n = 1\,ns$, $D = 1 \times 10^{-5}\,ns^{-1}$, and $j = 2$.

3. RESULTS FOR EXTERNAL-CAVITY LASERS

First we study the case in which the current-modulated laser is subjected only to optical feedback ($E_{inj} = 0$). Modulation amplitude δ, modulation period T_{mod}, and feedback level γ are the free parameters of our study. We consider a fixed delay time of $\tau = 0.7\,ns$, which corresponds to an external cavity of approximately 10 cm, and
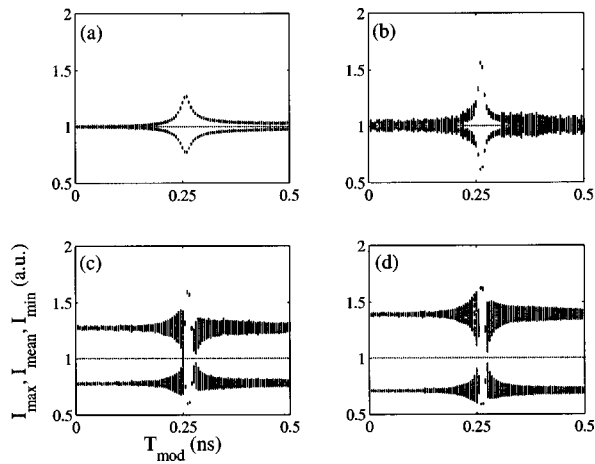
Fig. 1.   External-cavity laser under current modulation.   Effect of the optical feedback strength.   We plot the maximum, minimum, and average values of the intensity oscillations versus the modulation period for weak modulation ($\delta = 0.01$):   (a) without feedback and (b)–(d) with feedback.   The feedback strength is (b) 0.8 ns^{-1}, (c) 0.88 ns^{-1}, (d) 0.96 ns^{-1}.
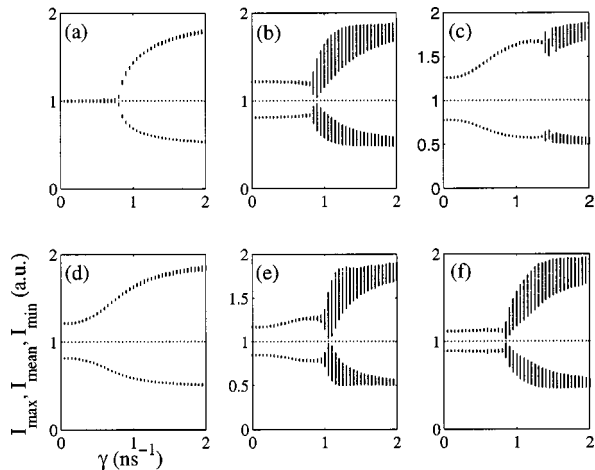

Fig. 2.   External-cavity laser under current modulation.   Effect of the modulation period.   We plot the local maxima and minima, and the average value of the intensity oscillations versus the optical feedback strength: (a) without injection current modulation and (b)–(f) with weak injection current modulation. The modulation amplitude is $\delta = 0.01$ and the period of T_{mod} is (b) 0.25 ns, (c) 0.26 ns, (d) 0.265 ns, (e) 0.27 ns, (f) 0.28 ns.

feedback levels weak enough so that, in the whole range of parameters considered, the laser operates in a single external cavity mode, which is the solitary laser mode, slightly perturbed by the external cavity.

To characterize the laser response to current modulation, in Fig. 1 we show the maximum, the average, and the minimum of the intensity oscillations ($I_{\max}$, I_{mean}, and $I_{\min}$, respectively) as a function of modulation period. To distinguish clearly the regions in which the dynamics is entrained to the modulation from those in which no entrainment occurs, we plot the maxima and minima for each oscillation period.   When the dynamics is not locked to the modulation there are several (local) maxima and minima.   In contrast, when the dynamics is entrained to the modulation, there is a single (global) maximum and minimum (however, spontaneous emission noise leads to

a spread in the values of the global extremes, in particular close to the Hopf bifurcation).

Different feedback levels are considered.   In Fig. 1(a) there is no feedback, and the resonance peak characteristic of a forced nonlinear oscillator can be seen.   The peak occurs for $T_{\mathrm{mod}} = T_{\mathrm{RO}}$, where T_{RO} is the period of the relaxation oscillations of the free-running laser.   A single maximum and a single minimum are observed for all values of T_{mod}, indicating entrainment of the laser oscillations to the current modulation, as is well known for free-running lasers.   As the feedback level increases, the peak is enhanced [Fig. 1(b)].   A further increase of the feedback level induces a strongly nonlinear distortion of the peak [Figs. 1(c) and 1(d)].   It can also be observed that
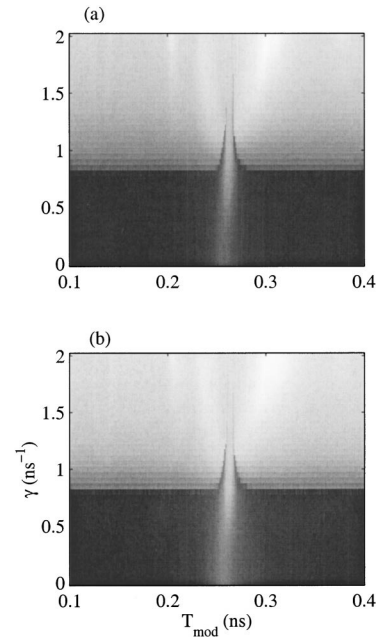

Fig. 3.   External-cavity laser under current modulation as a function of the modulation period and the optical feedback strength: (a) $I_{\max} - I_{\mathrm{mean}}$ and (b) $I_{\mathrm{mean}} - I_{\min}$.   The values are represented by gray tones: a dark tone represents a small value, and a light tone represents a large value.
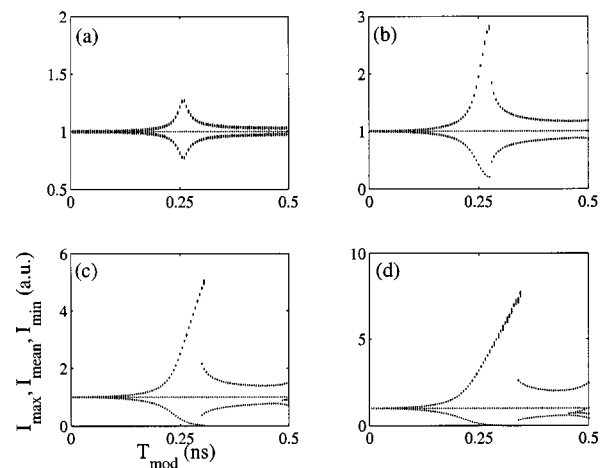

Fig. 4.   Effect of the modulation amplitude.   We plot the maximum, minimum, and average values of the intensity oscillations versus the modulation period.   The modulation amplitude is δ equal to (a) 0.01, (b) 0.05, (c) 0.1, (d) 0.2.

the dynamics is entrained to the current modulation only in a small region of T_{mod} (where the peak occurs). Outside this region the dynamics is quasi-periodic.

A different characterization of the interplay between weak optical feedback and small current modulation can be made by plotting I_{max}, I_{mean}, and I_{min} as a function of the feedback level for fixed δ and T_{mod}. Figure 2(a) displays I_{max}, I_{mean}, and I_{min} versus γ in the absence of current modulation ($\delta = 0$). A Hopf bifurcation occurs at $\gamma \sim 0.75 \, \text{ns}^{-1}$. Figures 2(b)–2(f) display I_{max}, I_{mean}, and I_{min} versus γ for $\delta = 0.01$ and different values of T_{mod}. While bifurcation diagrams with a sharp bifurcation are found for $T_{\text{mod}} < T_{\text{RO}}$ and $T_{\text{mod}} > T_{\text{RO}}$ [Figs. 2(b) and 2(f)], smooth transitions reminiscent of bifurcations in the presence of noise are observed for $T_{\text{mod}} \sim T_{\text{RO}}$ [Figs. 2(c)–2(e)]. It can be observed that the dynamics is entrained to the current modulation in Figs. 2(b), 2(c), 2(e), and 2(f) only for feedback levels below the Hopf bifurcation; above the Hopf bifurcation the dynamics is quasi-periodic. The exception is for modulation period $T_{\text{mod}} = 0.265 \, \text{ns}$ [Fig. 2(d)], where a smooth transition occurs and the dynamics is entrained to the current modulation in all the region of feedback levels studied ($\gamma < 2 \, \text{ns}^{-1}$).

The amplitude of the intensity oscillations in parameter space (T_{mod}, γ) is displayed in Fig. 3. The horizontal axis is the modulation period, the vertical axis is the feedback level, and gray tones represent the value of $I_{\text{max}} - I_{\text{mean}}$ [Fig. 3(a)] and $I_{\text{mean}} - I_{\text{min}}$ [Fig. 3(b)]: a dark tone represents a small value, whereas a light tone represents a large value (here, only the global maximum and minimum values are represented). It can be observed that as the feedback level increases the height of the peak increases (it becomes lighter) and the peak moves slightly to the right, because the relaxation oscillation period increases slightly with γ. The breaking of the resonance peak symmetry for feedback levels above the Hopf bifurcation point is clearly visible.

Next we study the effect of modulation amplitude δ. To distinguish from optical feedback-induced effects, we consider the free-running laser ($\gamma = 0$). In Fig. 4 we plot I_{max}, I_{mean}, and I_{min} versus T_{mod} for selected values of δ. For weak modulation we see a symmetric resonance peak [Fig. 4(a)], but as the modulation amplitude increases we observe a distortion of the peak that gradually becomes asymmetric [Figs. 4(b) and 4(c)]. For large enough δ ($\delta > 0.14$) the distortion of the peak leads to bistability [Fig. 4(d)]. Similar results have previously been found in the literature for a ring laser under periodic loss modulation[15,16] and for a solitary laser diode under periodic modulation of the pump or loss.[17]

4. RESULTS FOR OPTICALLY INJECTED LASERS

Next we consider the case in which the current-modulated laser is subjected only to optical injection ($\gamma = 0$). We consider an injection strength of $E_{\text{inj}} = 5 \, \text{ns}^{-1}$. Taking into account that for vertical-cavity lasers typical values are $r_2 = 0.99$, $\tau_l = 0.045 \, \text{ps}$, $\eta_{\text{inj}} \sim 1$, and the injection parameter is $k_{\text{inj}} = 110 \, \text{ns}^{-1}$,[18] thus the rate of injected power $P_{\text{inj}} = (E_{\text{inj}}/k)^2$ to solitary laser power $P_{\text{sol}} = j - 1$ is $P_{\text{inj}}/P_{\text{sol}} \sim 2 \times 10^{-3}$.
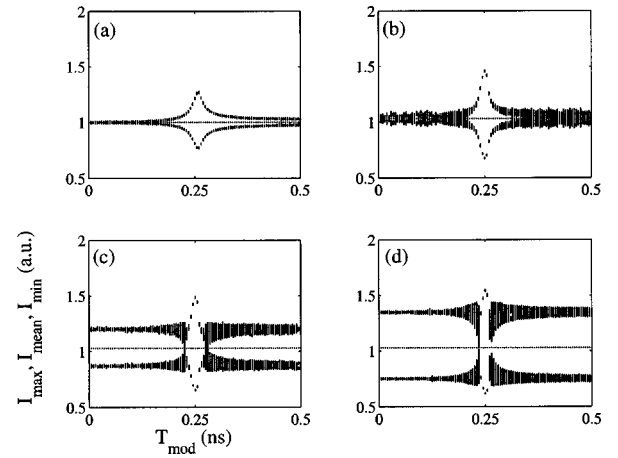

Fig. 5. Optically injected laser under weak current modulation. Effect of the frequency detuning. We plot the maximum, minimum, and average values of the intensity oscillations versus the modulation period. The modulation amplitude is $\delta = 0.01$: (a) without optical injection and (b)–(d) with weak optical injection. The optical injection strength is $E_{\text{inj}} = 5 \, \text{ns}^{-1}$ and the detuning between the free-running oscillation frequency and the frequency of the injected field is (b) $-10.8 \, \text{GHz}$, (c) $-10.6 \, \text{GHz}$, (d) $-10.2 \, \text{GHz}$. The Hopf bifurcation occurs for $\Delta \omega \sim -10.9 \, \text{GHz}$.
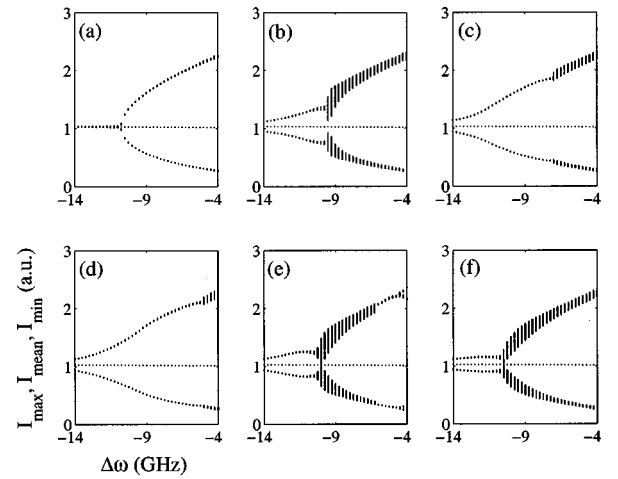

Fig. 6. Optically injected laser under weak current modulation. Effect of the modulation period. We plot the maximum, minimum, and average values of the intensity oscillations versus detuning between the free-running oscillation frequency and the frequency of the injected signal. The optical injection strength $E_{\text{inj}} = 5 \, \text{ns}^{-1}$: (a) without current modulation ($\delta = 0$) and (b)–(f) with weak injection current modulation of amplitude $\delta = 0.01$ and the period of I_{mod} is (b) 0.24 ns, (c) 0.25 ns, (d) 0.255 ns, (e) 0.26 ns, (f) 0.27 ns.

Figure 5 displays the average and the local extrema (maxima and minima) of the intensity oscillations and the global average intensity as a function of the modulation period. Figures 5(b)–5(d) show results for the optically injected laser and, for comparison, Fig. 5(a) shows results for the solitary laser [same as Fig. 1(a)]. Frequency detuning $\Delta \omega$ is varied near the Hopf bifurcation that is the boundary of stable injection locking in the absence of current modulation. Under weak current modulation, in narrow regions of parameter space the laser might be doubly locked: its optical frequency is locked to that of the injected field, and its intensity oscillations are phase

locked to the injection current modulation, as reported previously by Nizette *et al.*[11] In Fig. 5 results similar to those shown in Fig. 1 can be observed. Here parameter $\Delta\omega$ plays the role of feedback level γ. As $\Delta\omega$ increases a Hopf bifurcation occurs, which in the absence of current modulation determines the boundary of static (locking) and dynamic (nonlocking) behavior.

Plotting the local maxima, minima, and global average (I_{max}, I_{min} and I_{mean}, respectively) as a function of frequency detuning for fixed T_{mod} (see Fig. 6) gives results similar to those shown in Fig. 2. Although bifurcation diagrams with a sharp bifurcation are found for $T_{mod} < T_{RO}$ and $T_{mod} > T_{RO}$, smooth transitions reminiscent of bifurcations in the presence of noise are observed for $T_{mod} \sim T_{RO}$.

5. SUMMARY AND CONCLUSIONS

To summarize, we have studied the dynamics of external-cavity semiconductor lasers and optically injected semiconductor lasers when a small sinusoidal modulation is added to the dc-bias current. We characterized the laser dynamics by the extrema and average of the intensity oscillations. For optically injected lasers we found nonlinear amplification for weak injection strengths and frequency detunings close to the Hopf bifurcation that limits the stable locking region. For external-cavity lasers we found strong nonlinear amplification for weak feedback levels and modulation amplitudes, close to the Hopf bifurcation instability that gives rise to sustained relaxation oscillations. For larger modulation amplitudes there is a distortion of the resonance peak that signs bistability. This bistable behavior differs from the bistability in optically injected diode lasers reported in Refs. 8, 9, and 11, which occurs for weak modulation amplitude and strong optical injection.

In our simulations the external-cavity laser and the optically injected laser have a similar response to an external periodic forcing, because the parameters we consider correspond to the weak feedback limit (the laser operates in only one cavity mode) and to the weak injection limit. Otherwise the two systems should respond quite differently. A study of the response of an external-cavity laser operating near the second Hopf bifurcation where the laser intensity exhibits quasi-periodic oscillations (of frequencies f_{RO} and f_{ext} for a larger feedback level) is the object of future work. The enhanced sensitivity to current modulation reported in this paper might be of interest for practical applications to provide access to enhanced modulation amplitude by simple frequency detuning.

ACKNOWLEDGMENTS

REFERENCES

1.   J. Ohtsubo, "Chaotic dynamics in semiconductor lasers with optical feedback," in *Progress in Optics*, E. Wolf, ed., Vol. 44 (Elsevier, North Holland, Amsterdam, 2002), pp. 1–84.
2.   J. Sacher, D. Baums, P. Pankin, W. Elsässer, and E. O. Göbel, "Intensity instabilities of semiconductor lasers under current modulation, external light injection, and delayed feedback," Phys. Rev. A **45**, 1893–1905 (1992).
3.   Y. Liu, N. Kikuchi, and J. Ohtsubo, "Controlling dynamical behavior of a semiconductor laser with external optical feedback," Phys. Rev. E **51**, R2697–R2700 (1995).
4.   Y. Takiguchi, Y. Liu, and J. Ohtsubo, "Low-frequency fluctuation induced by injection-current modulation in semiconductor lasers with optical feedback," Opt. Lett. **23**, 1369–1371 (1998).
5.   J. M. Mendez, R. Laje, M. Giudici, J. Aliaga, and G. B. Mindlin, "Dynamics of periodically forced semiconductor laser with optical feedback," Phys. Rev. E **63**, 066218 (2001).
6.   J. M. Buldú, J. Garcia-Ojalvo, C. R. Mirasso, and M. C. Torrent, "Stochastic entrainment of optical power dropouts," Phys. Rev. E **66**, 021106 (2002).
7.   M. S. Torre, C. Masoller, P. Mandel, and K. A. Shore, "Transverse-mode dynamics in directly modulated vertical-cavity surface-emitting lasers with optical feedback," submitted to IEEE J. Quantum Electron.
8.   T. B. Simpson, "Phase-locked microwave-frequency modulations in optically-injected laser diodes," Opt. Commun. **170**, 93–98 (1999).
9.   T. B. Simpson and F. Doft, "Double-locked laser diode for miscrowave photonics applications," IEEE Photon. Technol. Lett. **11**, 1476–1478 (1999).
10. M. Nizette, T. Erneux, A. Gavrielides, and V. Kovanis, "Stability and bifurcations of periodically modulated, optically injected laser diodes," Phys. Rev. E **63**, 026212 (2001).
11. M. Nizette, T. Erneux, A. Gavrielides, V. Kovanis, and T. B. Simpson, "Bistability of pulsating intensities for double-locked laser diodes," Phys. Rev. E **65**, 056610 (2002).
12. M. Nizette, "Temporal dynamics of driven nonlinear optical systems," Ph.D. dissertation (Universite Libre de Bruxelles, Bruxelles, Belgium, 2003).
13. M. C. Eguia, G. B. Mindlin, and M. Giudici, "Low-frequency fluctuations in semiconductor lasers with optical feedback are induced with noise," Phys. Rev. E **58**, 2636–2639 (1998).
14. G. H. M. van Tartwijk and D. Lenstra, "Semiconductor lasers with optical injection and feedback," Quantum Semiclass. Opt. **7**, 87–143 (1995).
15. T. Erneux, S. M. Baer, and P. Mandel, "Subharmonic bifurcation and bistability of periodic solutions in a periodically modulated laser," Phys. Rev. A **35**, 1165–1171 (1987).
16. P. Mandel, P. Nardone, and T. Erneux, "Periodic loss modulation in a ring laser: influence of inhomogeneous broadening and detuning," J. Opt. Soc. Am. B **5**, 1113–1120 (1988).
17. C. Mayol, R. Toral, C. R. Mirasso, S. I. Turovets, and L. Pesquera, "Theory of main resonances in directly modulated diode lasers," IEEE J. Quantum Electron. **38**, 260–269 (2002).
18. J. Y. Law, G. H. M. van Tartwijk, and G. P. Agrawal, "Efects of tranverse-mode competition on the injection dynamics of vertical-cavity surface-emitting lasers," Quantum Semiclass. Opt. **9**, 737–747 (1997).