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Anticipation in the synchronization of chaotic time-delay systems

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Abstract

We study numerically the synchronization of two time-delay chaotic systems, in a unidirectional coupling configuration. The coupling is delayed in time to represent the finite speed at which the information is transmitted from one system (master system) to the other (slave system). We simulate coupled Mackey–Glass and Ikeda systems. We show that, when the delay time of the systems, τ, is greater than the delay time of the coupling, τ_2, for adequate parameters a regime of anticipated synchronization occurs. In this regime, the slave system at time t, synchronizes to the future state of the master system, at time $t + \tau - \tau_2$, anticipating its chaotic evolution. Anticipation in the synchronization is not destroyed by small parameter differences between the systems, but in this case the systems are not perfectly synchronized. © 2001 Elsevier Science B.V. All rights reserved.

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In recent years, the synchronization of two unidirectionally coupled chaotic systems has become an area of active research, in part due to its potential application to secure communication [1–5]. The chaotic output of a transmitter (master system) is used as a carrier in which a message is encoded. The signal is transmitted to an identical chaotic system (slave system), which synchronizes with the master system, and the message can be recovered from the output of the slave system.

Of special interest is the synchronization of chaotic time-delay systems. Time-delay systems are described by delay differential equations of the form $\mathrm{d}x/\mathrm{d}t = f(x(t), x(t-\tau))$, where τ is a delay time. To calculate $x(t)$ for times greater than t, one must know the value of $x(t)$ over the interval $(t, t-\tau)$. Therefore, time-delay systems have

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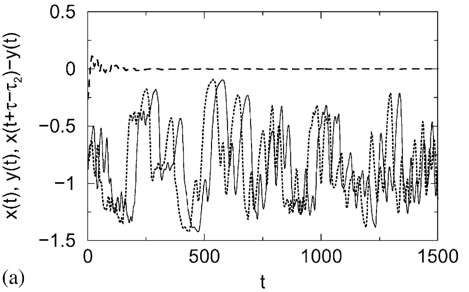

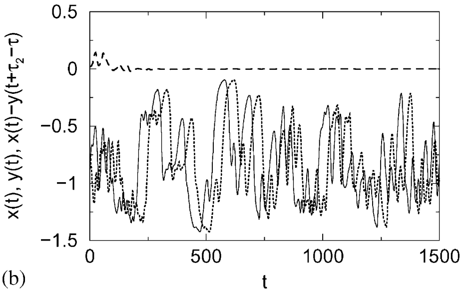

Fig. 1. Synchronization of identical Mackey–Glass equations. The parameters are $a = 0.2$, $b = 0.1$, $\gamma_1 = 1$, $\gamma_2 = 0.2$ and $\eta = 0.8$. (a) Anticipated synchronization for $\tau = 100$, $\tau_2 = 60$. $x(t)$ (solid line), $y(t)$ (dotted line), and $x(t + \tau - \tau_2) - y(t)$ (dashed line) are plotted as a function of time. (b) Retarded synchronization for $\tau = 100$, $\tau_2 = 140$. $x(t)$ (solid line), $y(t)$ (dotted line), and $x(t) - y(t + \tau_2 - \tau)$ (dashed line) are plotted as a function of time.

an infinite-dimensional phase space. It has been shown that the number of positive Lyapunov exponents, and the dimension of the attractor, increases linearly with the delay time [6]. Time-delay systems are good candidates for secure communication, since to improve security high-dimensional systems with a large number of positive Lyapunov exponents are preferable.

In this paper, we study the synchronization of two unidirectionally coupled time-delay systems, which have the same parameters and the same delay time, τ. The information is transmitted from one system to the other at finite speed, and this introduces an additional delay time, τ_2, in the coupling. We consider a coupling configuration of the form

$$\mathrm{d}x/\mathrm{d}t = f(x) + \gamma_1 g(x_\tau)\,, \tag{1}$$

$$\mathrm{d}y/\mathrm{d}t = f(y) + \gamma_2 g(y_\tau) + \eta g(x_{\tau 2})\,. \tag{2}$$

In (1) and (2), $x_\tau = x(t-\tau)$, $y_\tau = y(t-\tau)$, $x_{\tau_2} = x(t-\tau_2)$, γ_1, γ_2 and η are the feedback and coupling coefficients respectively. Synchronized solutions will exist only if the functions f and g in (1) and (2) are identical (with identical parameters) and the coefficients γ_1, γ_2 and η are related by $\gamma_1 = \gamma_2 + \eta$. In this case, for solutions that satisfy $x_{\tau_2} = y_\tau$, Eqs. (1) and (2) become identical. Notice that $x_{\tau_2} = y_\tau$ implies that $x(t) = y(t - \tau + \tau_2)$. Depending on the difference $\tau - \tau_2$ there is anticipated or retarded synchronization.

Fig. 1 shows numerical solutions of coupled Mackey–Glass equations [7] $[f(x) = -bx,\ g(x_\tau) = ax_\tau/(1 + x_\tau^{10})]$. The values of the parameters a and b are the same for the master and the slave system, and the coefficients γ_1, γ_2, and η verify $\gamma_1 = \gamma_2 + \eta$. Fig. 1(a) exhibits anticipated synchronization (for $\tau > \tau_2$), and Fig. 1(b) retarded synchronization (for $\tau < \tau_2$). In Fig. 1(a), it is clearly observed that the trajectory of the slave system, $y(t)$ (dotted line), foresees the chaotic trajectory of the master system, $x(t)$ (solid line). The dashed line indicates the value of $x(t + \tau - \tau_2) - y(t)$, and demonstrates that after a transient time, $y(t) = x(t + \tau - \tau_2)$. Fig. 2 shows similar results for coupled

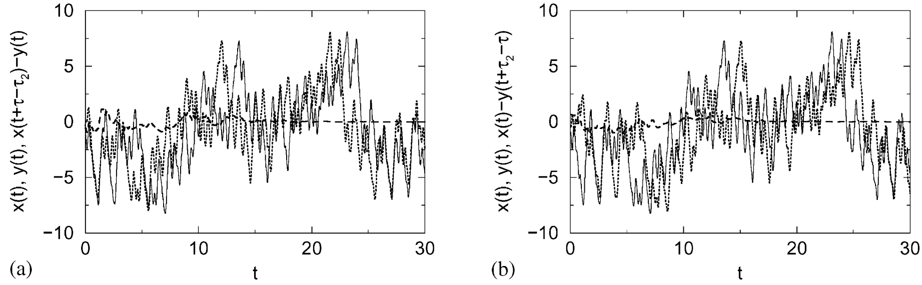

Fig. 2. Synchronization of identical Ikeda equations. The parameters are $a = 20$, $b = 1$, $\gamma_1 = 1$, $\gamma_2 = 0.2$ and $\eta = 0.8$. (a) Anticipated synchronization for $\tau = 2$, $\tau_2 = 0.5$. (b) Retarded synchronization for $\tau = 2$, $\tau_2 = 3.5$. The solid, dotted and dashed lines have the same meaning as in Fig. 1.

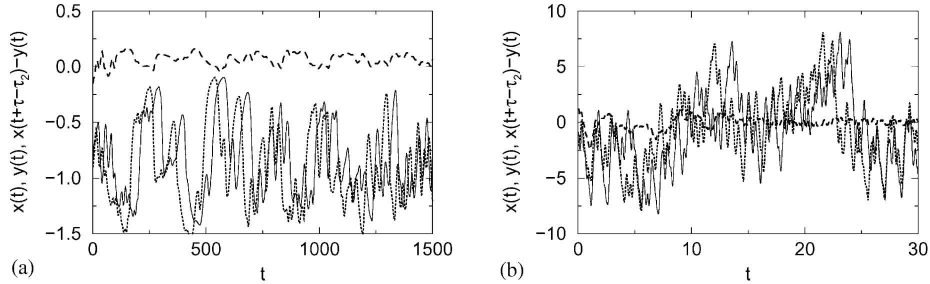

Fig. 3. Synchronization of nonidentical systems. (a) Coupled Mackey–Glass equations. The parameters of the master system are as in Fig. 1(a), and the parameters of the slave system are $a = 0.2$ and $b = 0.09$. (b) Coupled Ikeda equations. The time delay of the master system is as in Fig. 2(a), and the time delay of the slave system is $\tau = 1.95$.

Ikeda equations [8] $[f(x) = -bx,\ g(x_\tau) = a\sin(x_\tau)]$ with the same parameter values for the master and the slave systems. Fig. 3 exhibits anticipated synchronization when the parameters of the master and the slave system slightly differ. Fig. 3(a) shows solutions of coupled Mackey–Glass equations, with different values of the parameter a, and Fig. 3(b) shows solutions of coupled Ikeda equations with the master delay time $\tau = 2$, and the slave delay time $\tau = 1.95$. Comparing with Figs. 1(a) and 2(a) (where the master and the slave systems are identical), we can notice that the difference of $x(t + \tau - \tau_2) - y(t)$ does not decay to zero, but shows small oscillations that indicate that the synchronization is not complete. The synchronization improves when the value of η is increased (and simultaneously the value of γ_2 is decreased such that $\gamma_2 = \gamma_1 - \eta$).

In summary, we have studied the synchronization of two unidirectionally coupled time-delay systems, when the coupling involves an additional delay time, τ_2 and have found a regime of synchronization where the states of the two systems are related by $y(t) = x(t + \tau - \tau_2)$. This regime of synchronization occurs when the coupling η is large enough, and the feedback and coupling coefficients are related by $\gamma_1 = \gamma_2 + \eta$. When $\tau_2 < \tau$, anticipated synchronization occurs, and the slave system foresees the chaotic evolution of the master system. When $\tau < \tau_2$, retarded synchronization occurs, and the state of the slave system is retarded in time with respect to the state of the master system. Synchronization is a result of the memory of the systems, which is stored in the delay terms of (1) and (2). Synchronization is robust with respect to small parameter differences between the systems, but if the systems are not identical the synchronization is not complete.

Anticipated synchronization in unidirectionally coupled time-delay systems was recently discovered by Voss [9]. In the coupling schemes considered in [9], the delay time of the coupling was equal to the delay time of the system. It was found that the slave system anticipates in time the master system by a time interval of length τ. In our case, the coupling delay time, τ_2, is generally different from the system delay time, τ. It could be expected that an additional time delay in the coupling would defy synchronization because it changes the structures of the coupled systems, and it is hard to synchronize non-identical systems. However, we find synchronization such that one system lags in time to the other, with the lag time determined only by the difference between τ and τ_2.

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