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Measuring the acoustic response of Helmholtz resonators

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Many experiments have been proposed to investigate acoustic phenomena in college and early undergraduate levels, in particular the speed of sound,[1-9] by means of different methods, such as time of flight, transit time, or resonance in tubes. In this paper we propose to measure the acoustic response curves of a glass beaker filled with different gases, used as an acoustic resonator. We show that these curves expose many interesting peaks and features, one of which matches the resonance peak predicted for a Helmholtz resonator fairly well, and gives a decent estimate for the speed of sound in some cases. The measures are obtained thanks to the capabilities of smartphones.

Execution of the experiment

For the experiment we used a typical beaker found in almost all science laboratories, as shown in Fig. 1. The beaker has height $H = 0.139$ m and radius $a = 0.040$ m. A similar setup is attainable with simple glasses at home (Fig. 2). The beaker was filled with the gas to be measured, side up if the gas is denser than air or side down if the gas is lighter than air.

Two smartphones were employed in this experiment. One smartphone near the beaker was used as a white-noise generator to stimulate the resonant modes in the cavity (black phone just visible to the left behind the white phone in Fig. 2). The other smartphone was used to record the sound (white phone in Fig. 2). Its location is a delicate matter; it must be placed close to the beaker to obtain a good signal but care should be taken to not modify the boundary conditions. We obtained better results when the smartphone protruded slightly into the glass (Fig. 2). This will change (although minimally) the volume and opening area, but the spectrum is measured with much higher accuracy. The measures of the acoustic response must be made with apps that perform a fast Fourier transform in real time. For this purpose we used the app Spektroskop[10] on an iPhone. On Android phones the same can be done with apps like Advanced Spectrum Analyzer[11] that automatically detect the peak frequencies.

The acoustic spectral response of the beaker was obtained for air and three other different gases: oxygen, carbon dioxide, and methane (Figs. 3-6), showing subtle and interesting spectral differences between the four gasses. Different features can be appreciated by means of a little theoretical analysis.

First, the acoustic response of the beaker can be understood by thinking of the beaker as a resonant cavity. The calculus of the normal modes of resonance is beyond the focus of this paper, but can be shown that the wavelength is of the order of $4H$, so the normal frequencies in air for this beaker are in the order of 617 Hz (or 525 Hz in consideration of the end-pipe correction[14]). But in this article we want to see the beaker as a Helmholtz resonator.

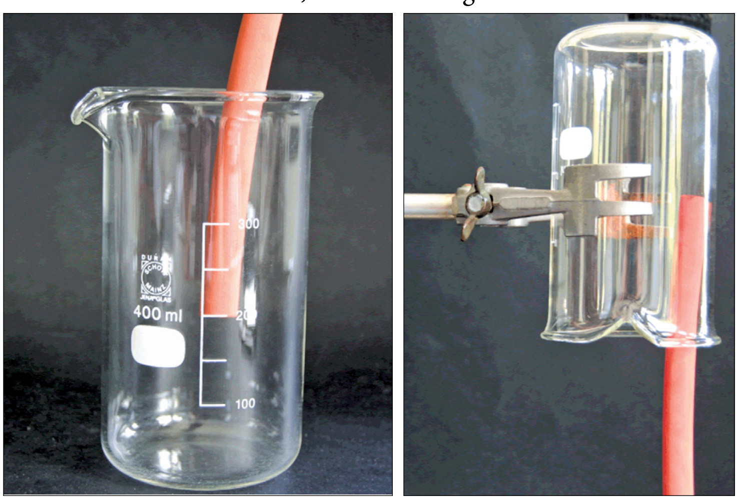

Fig. 1. In physics labs with different gases (left: heavier than air; right: lighter than air).

Fig. 2. Investigation with glasses at home.

Theoretical background and experiment analysis

A rigid cavity with an open neck can be modeled as a mass-spring system, where the cavity is the spring and the neck is the mass, the so-called "Helmholtz resonator" (Fig. 7). The only frequency of this system is given by[12-13]

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Fig. 3. Frequency spectrum with air.

Fig 4. Frequency spectrum with oxygen.

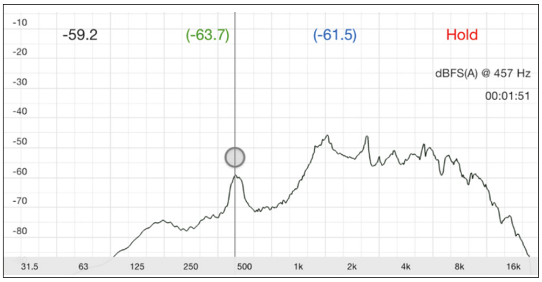

Fig. 5. Frequency spectrum with carbon dioxide.

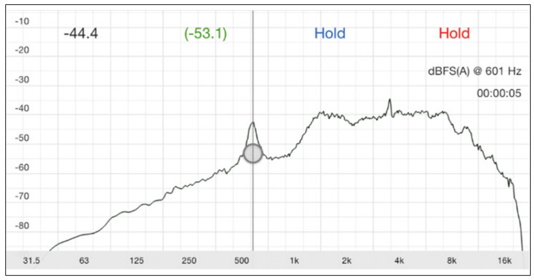

Fig. 6. Frequency spectrum with methane.

$$f=\frac{c}{2\pi}\sqrt{\frac{A}{V\cdot L'}}, \tag{1}$$

where L' is the effective length of the neck, A is the area of the neck, V is the volume of the cavity, and c is the speed of sound in the inner gas.

Because a little amount of mass of gas is moving outside the edges of the neck dragged by the gas inside the neck, the effective length of the neck L' is slightly greater than the physical length of the neck L. This end correction depends on the boundary conditions[12-16]

with an outer end flanged: $L' = L + 1.7a$, (2a)

with an outer end unflanged: $L' = L + 1.4\,a$, (2b)

where a is the radius of the opening. In our case, the resonator is a cylindrical glass, then the neck has a real null length $L = 0$; thus, the effective length must be expressed completely by the end correction of a flanged border. Moreover, by means of this particular geometry, the volume of the cavity is $V = AH$, where H is the height of the cylinder. Then, with all that in mind, the resonant frequency is

$$f=\frac{c}{2\pi}\sqrt{\frac{1}{H\,1.7\,a}}. \tag{3}$$

We then assume that the best defined peak in the middle region of the spectrum corresponds to the Helmholtz frequency (Eq. 3). So, from this peak, we can determine the speed of sound, which can be expressed as a function of this frequency of resonance:

$$c = 2\pi f\sqrt{1.7\,a\,H}. \tag{4}$$

Let us call this the measured speed of sound.

In order to compare, remember that assuming that the gas is an ideal gas, the speed of sound is given by the well-known thermodynamical relation[17]

$$c=\sqrt{\frac{\gamma RT}{M}}, \tag{5}$$

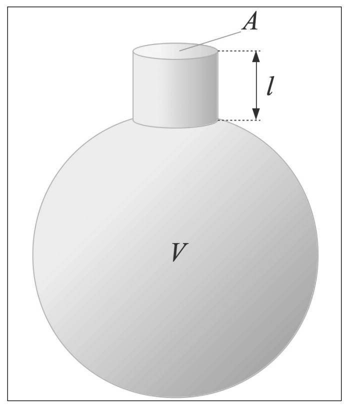

Fig. 7. Helmholtz resonator.

where γ is the adiabatic index of the gas, R is the universal gas constant (8.31 $\mathrm{J{\cdot}mol^{-1}{\cdot}K^{-1}}$), T is the absolute temperature in kelvin, and M is the molar mass of the gas.

The resonance frequencies of the best defined peaks in the middle region of the spectra and the calculated results from Eqs. (4) and (5) are shown in Table I for the aforementioned gases. As can be seen, if we assume that the best defined peak in the middle frequency regime is centered at the frequency predicted by the theoretical model for an ideal Helmholtz resonator, then we get reasonable estimates for the speed of sound in each gas, with the methane case showing the largest discrepancy with the accepted values.

An interesting extension could consist of employing other gases such as helium or sulfur hexafluoride, with densities and speed of sound considerably different from those of air.

Table I. Experimental results for a temperature of 17.5 °C

Gas	Measured resonance frequency (Hz)[18]	Speed of sound: measured (m/s)	Speed of sound: reference values (m/s)	Deviation in %
Air (γ = 1.4, M = 0.029 kg/mol)	557 ± 3	340 ± 2	341	0.3
Oxygen (γ = 1.4, M = 0.032 kg/mol)	533 ± 3	325 ± 2	325	0.0
Carbon dioxide (γ = 1.4, M = 0.044 kg/mol)	457 ± 3	279 ± 2	277	0.7
Methane (γ = 1.3, M = 0.016 kg/mol)	598 ± 3	365 ± 2	443	17.6

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18. The frequencies were not read from the display, but from the exported data.

Fermi Questions

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Question 1: "Up" – A Fermi question about buoyancy

Is it possible for a house to fly by the uplift of balloons, like in the movie "Up"?

(written by R. Allart, A. Cazenave, and A. Müller, University of Geneva)

Question 2: Traffic jams

How much total time do Americans spend waiting in traffic each year? What is the total cost of this time?

Look for the answers online at *tpt.aapt.org*. Question suggestions are always welcome! For more Fermi questions and answers, see the now available *Guesstimation 2.0: Solving Today's Problems on the Back of a Napkin,* by Lawrence Weinstein (Princeton University Press, 2012).

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