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Rotational energy in a physical pendulum

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Smartphone usage has expanded dramatically in recent years worldwide. This revolution also has impact in undergraduate laboratories where different experiences are facilitated by the use of the sensors usually included in these devices. Recently, in several articles published in the literature,[1-2] the use of *smartphones* has been proposed for several physics experiments. Although most previous articles focused on mechanical experiments, an aspect that has received less attention is the use of *rotation sensors* or *gyroscopes*. Indeed, the use of these sensors paves the way for new experiments enabling the measurement of angular velocities. In a very recent paper the conservation of the angular momentum is considered using rotation sensors.[3] In this paper we present an analysis of the rotational energy of a physical pendulum.

Fig. 1. Smartphone mounted on a bike wheel and a scheme indicating the axes orientation.

Experimental setup

The experimental setup consists of a smartphone on the periphery of a bicycle wheel that can rotate freely in a vertical plane as shown in Fig. 1. The smartphone is an LG Optimus P990 2X (three-axis gyroscope MPU3050 Invensense, accuracy 0.0001 rad/s). The moment of inertia of the wheel, easily obtained by means of small oscillations, is 0.040 kg·m^2. The distance from the center of mass of the wheel to the center of mass of the smartphone is $R = 0.30$ m and the mass of the smartphone $m = 0.14$ kg.

The application AndroSensor[4] running under Android was used to record the values measured. The magnitudes relevant in this work are those reported by the rotation sensor according to the x-axis. Once recorded, data can be exported and analyzed using appropriate software.

Analysis of the motion

In our experiment the bike wheel is set in motion performing full rotations in one direction. During a single rotation the energy is very nearly conserved, and upon using the bottom of the bicycle wheel as the zero of potential energy and measuring the angle from this lowest potential energy position, we can write the total energy of the system during the rotation as

$$E = \frac{1}{2} I \omega_{\text{bottom}}^2 = \frac{1}{2} I \omega_{\text{top}}^2 + 2mgR,$$

where I is the moment of inertia of the wheel-phone combination about the pivot point and ω_{bottom} and ω_{top} are the angular velocities when the smartphone is at the bottom or top position, respectively. Thus, energy conservation implies that

$$\omega_{\text{bottom}}^2 - \omega_{\text{top}}^2 = \frac{4mgR}{I} = \text{constant}.$$

In our setup the constant was determined from the parameters to be 31.3 (rad/s)^2.

As time goes by, due to the effect of a weak dissipation, the energy decreases and, at a given point, the angular velocity first vanishes. Then, the wheel reverses the direction of spinning and starts to oscillate around the stable equilibrium point. In Fig. 2, the square of the angular velocity, proportional to the kinetic energy, is plotted as a function of time. At the beginning of the graph, the wheel is rotating in one direction; however, at $t \sim 38$ s, indicated by the green circle, the angular velocity vanishes for the first time, and the wheel starts oscillating. Before the green circle, when the wheel is rotating, relative maxima and minima are achieved when the smartphone is passing through the lowest or highest points, respectively. After the green circle, when the wheel is oscillating, relative minima coincide with turning points or zeros of the angular velocity, while relative maxima correspond to the smartphone passing through the lowest point in one direction or the other according to the sign of the angular velocity.

In this figure, two exponential curves, fitting the relative maxima and minima, reveal that the energy decreases roughly exponentially. The vertical difference between

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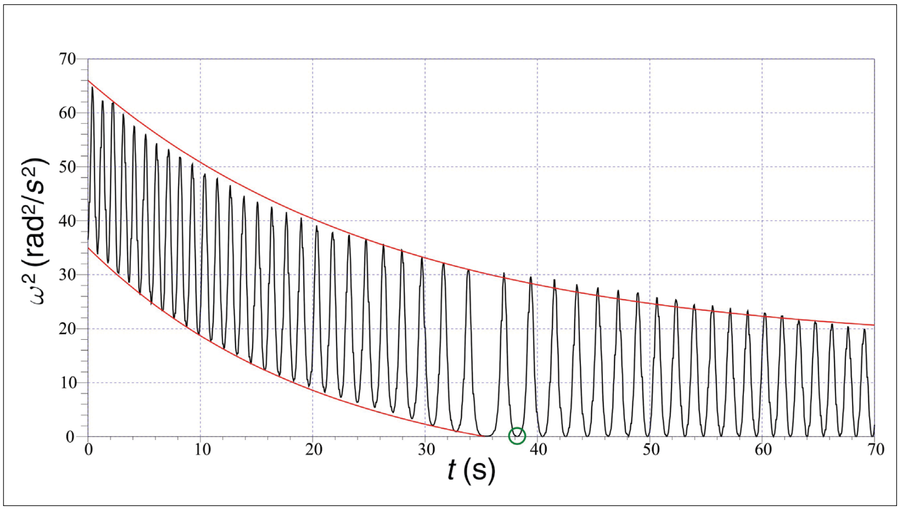

Fig. 2. Angular velocity squared (black), proportional to the kinetic energy, and two exponential curves fitting relative maxima and minima (red). The green circle indicates the transition from rotations to oscillations.

these curves is about 31 (rad/s)^2. This value shows excellent agreement with the value of the constant $4mgR/I$ previously obtained from direct measures of the parameters.

Final remarks

To conclude, we remark that the use of rotation sensors allows a broad spectrum of measures applicable in different mechanical experiments, for example, spring, double, or torsion pendula, or coupled oscillators. Other possibilities, to be considered in a future work, are given by the simultaneous use of acceleration and rotation sensors that allows, among other things, to obtain a full characterization of the motion of simple systems.

References

1. See for example, Jochen Kuhn and Patrik Vogt, "Smartphones as experimental tools: Different methods to determine the gravitational acceleration in classroom physics by using every Figure 1(a). Photographs of timer assembly. day devices," *Eur. J. Phys. Educ.* **4**, 16 (2013).
2. Patrik Vogt, Jochen Kuhn, and Sebastian Müller, "Experiments using cell phones in physics classroom education: The computer-aided *g* determination," *Phys. Teach.* **49**, 383 (Sept. 2011).
3. Asif Shakur and Taylor Sinatra, "Angular momentum," *Phys. Teach.* **51**, 564 (Dec. 2013).
4. Several applications available at http://play.google.com allow recording the values measured by the sensors.

Fermi Questions

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Question 1: Horse exhaust

If we still rode horses for transportation, our pollution problems would be quite different. How much exhaust does one horse produce annually? How much if we replaced all of our cars and trucks with horses?

Question 2: Feeding transportation

How much more land would we need for crops if we still used horses for transportation? Assume that we replace every car with a horse.

Look for the answers online at *tpt.aapt.org*. Question suggestions are always welcome! For more Fermi questions and answers, see the now available *Guesstimation 2.0: Solving Today's Problems on the Back of a Napkin*, by Lawrence Weinstein (Princeton University Press, 2012).

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