ELSEVIER

Volume 386, issue 2 15 December 2007 ISSN 0378-4371

PHYSICA A

STATISTICAL MECHANICS AND ITS APPLICATIONS

Recognized by the European Physical Society

PASI 2006
Disorder and Complexity

Disorder and Complexity

Proceedings of the Pan American Scientific Institute (PASI) Conference "Disorder and Complexity", Mar del Plata (Argentina), 11–20 December 2006

Guest Editor:
Gabriel Cwilich

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Physica A 386 (2007) 681–685
PHYSICA A
www.elsevier.com/locate/physa

Instabilities developed in stratified flows over pronounced obstacles

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Available online 14 September 2007

Abstract

In the present work we study numerical and experimentally the flow of a two-layer stratified fluid over a topographic obstacle. The problem reflects a wide number of oceanographic and meteorological situations, where the stratification plays an important role. We identify the different instabilities developed by studying the pycnocline deformation due to a pronounced obstacle. The numerical simulations were made using the model caffa3D.MB which works with a numerical model of Navier–Stokes equations with finite volume elements in curvilinear meshes. The experimental results are contrasted with numerical simulations. Linear stability analysis predictions are checked with particle image velocimetry (PIV) measurements.

PACS: 47.55.Hd; 47.20.Ft

Keywords: Stratified flows; PIV; Shear flows

Observation of instabilities near density interfaces are important in order to understand different phenomena relevant both for theoretical and practical reasons. It is well known that instabilities develop whenever fluids of different densities are accelerated against the density gradient [1,2]. A most challenging problem is to describe and quantify the characteristic features occurring in the interface of a stratified flow passing over an obstacle. A wide class of phenomena occurs in nature involving the interaction between stable stratified flows and obstacles. In the atmosphere, for example, the flow around buildings or mountains is particularly important because such conditions often go along with high levels of atmospheric pollution due to low wind speeds and suppressed vertical mixing [3]. Furthermore, the interaction of marine currents with topographic features, such as ocean banks and coastlines, results in a complex system of circulation. In this

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0378-4371/$ - see front matter
doi:10.1016/j.physa.2007.08.051

case, observational [4], analytical [5,6], numerical [7], and previous laboratory [8,9] studies have suggested that the combination of streamlines splitting, current intensification, and breaking of internal lee waves, play a significant role as mixing source in the ocean [10]. Also, seamounts are known to enhance biological productivity and to act over the ecological processes that determine the structure of local ocean life [11]. The rich diversity of geo-biophysical scenarios entirely justified the efforts for understanding the physics of the local instability that may appear along the density interface, as well as the conditions for which the generated lee waves break down producing turbulence and vertical mixing. In order to reflect a real oceanographic situation, which is still poorly studied, i.e., the finite-amplitude ocean banks we have chosen a *quasi-prismatic* shape very common in subtropical (see Ref. [12] among others) and tropical seas (see, for example, Refs. [5,13]).

In this work we study numerically and experimentally the instabilities of a two-layer flow over an obstacle. Our experiments were conducted in a $2.0 \times 0.29 \times 0.13\,\mathrm{m}^3$ tank filled with a two-layer stratified water. The upper layer is filled carefully pouring the fluid over a sponge floating on the free surface. Then, the measurements were taken in time scale insignificant compare to the mixing time scales. In fact, after 3 measurements, that takes the order of 30 min, the tank was emptied and new fluid mixture was poured again. For the sake of the simplicity of the experiment, instead of moving the fluid we move the obstacle at different velocities with a calibrated motor. The obstacle is a prism with a height of $h = 0.12\,\mathrm{m}$, a basis of $0.25 \times 0.125\,\mathrm{m}^2$, and a roof of $0.20 \times 0.125\,\mathrm{m}^2$. The two layers had densities $\rho_1 = 1000\,\mathrm{kg/m}^3$ and $\rho_2 = 1002\,\mathrm{kg/m}^3$ due to different salt concentrations, corresponding the index 1 to the upper layer and 2 for the bottom one.

Five different set of heights of the layers were chosen, always keeping the ratio between the lower and the upper in 1.5. The experiment was repeated for each set of heights with a wide range of velocities analyzing the resulting different behaviors. In the upper layer, a dash of $KMnO_4$ was added in order to obtain a good visual contrast between both layers. Experimental velocity fields were obtained with the PIV technique, using 50 μm diameter polyamide seeding particles. Particles were illuminated with a light plane from a 100 mW green laser, and a digital camera captured the motion of the particles at 20 frames per second. In Fig. 1 we show three snapshots of the bank moving along the whole channel. It can clearly seen that no perturbation coming from the far boundary affects the core of the experiment. We have focused on the central region where the structures are persistent and the effects of the boundaries can be neglected. We have also made thoughtful PIV measurements and taken detailed movies, showing compatibility with our working hypothesis. This hypothesis was also tested numerically by using constant upstream and downstream velocity at large distances from the bank.

The numerical simulations considered here were obtained with the in-house flow solver *caffa3d.MB* developed by Usera et al. [14] as a join work of the team at the *Rovira i Virgili Universitat* (Spain) and the *Universidad de la República* (Uruguay) team. It is an original Fortran95 implementation of a fully implicit finite volume method for solving the 3D incompressible Navier–Stokes equations in complex geometry. Further description of this model can be found in Ref. [14]. The numerical results are in good agreement with the experimental results, as can be seen in Fig. 2.

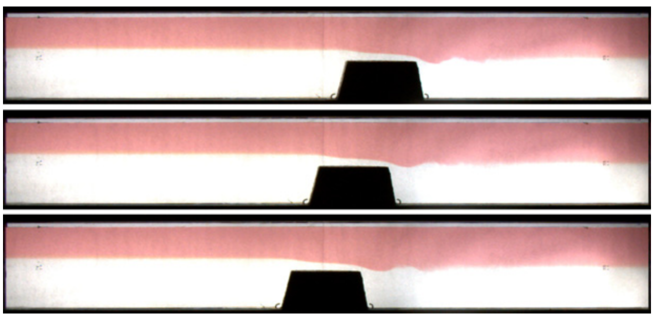

Fig. 1. Flow over the leftward moving obstacle for $d_2 = 15\,\mathrm{cm}$ (bottom layer), $d_1 = 10\,\mathrm{cm}$ (top layer) and $U = 0.30\,\mathrm{cm/s}$. Three different snapshots covering the whole channel are shown.

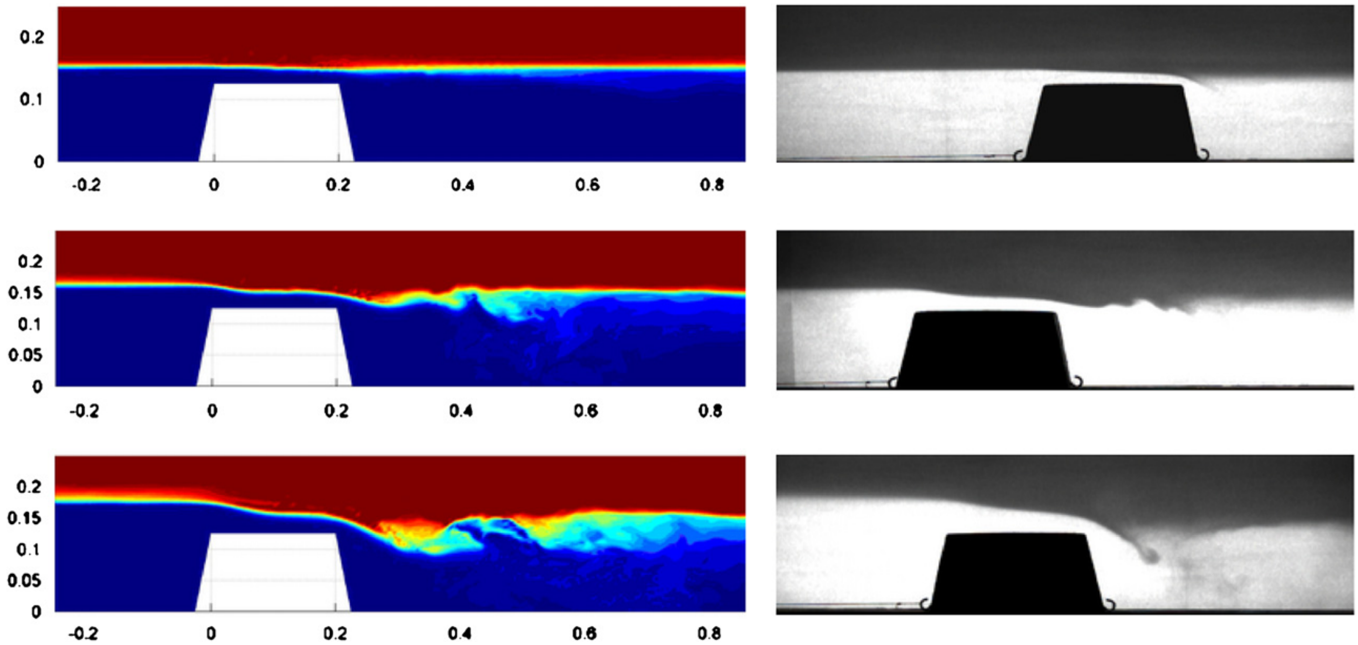

Fig. 2. Numerical simulations of a flow, from left to right, past an obstacle (left) and experiments of a leftward moving obstacle (right) corresponding to $d_1 = 0.10\,\mathrm{m}$ (upper layer), $d_2 = 0.15\,\mathrm{m}$ (bottom layer) and three different velocities: (a) $U = 0.16\,\mathrm{cm/s}$ (top), (b) $U = 0.38\,\mathrm{cm/s}$ (middle) and (c) $U = 0.64\,\mathrm{cm/s}$ (bottom).

The different regimes of a two-layer stratified flow are described by the Froude number $F = \sqrt{F_1^2 + F_2^2}$, where $F_i^2 = u_i^2/(g'd_i)$ $(i = 1, 2)$, d_i and u_i are the fluid height and velocity in each layer and $g' = g((\rho_2 - \rho_1)/\rho_2)$ is the so-called reduced gravity [3]. For $F > 1$ the flow is termed supercritical (characterized by the fact that disturbances at the interface cannot propagate upstream against the background flow), while for $F < 1$ the flow is subcritical (disturbances may propagate in both directions). We shall denote F_0 the value of the Froude number in the region upstream the obstacle. Since in the present experiments the two layers have the same velocity in this region, the expression for F_0 becomes

$$F_0 = \sqrt{\frac{U^2}{g'}\frac{(d_1 + d_2)}{d_1 d_2}}.$$

When F_0 reaches a critical value, F_{0t}, the structure of the flow changes past the obstacle, leading to the formation of a dissipative internal hydraulic jump [3]. In this critical situation, $F = 1$ above the obstacle. This means that there is a transition from subcritical to supercritical flow over the body. The dependence of F_{0t} with $H = h/d_2$ is in good qualitative agreement with the dependence obtained from hydraulic theories [3]. For instance, the critical value F_{0t} obtained from the experimental data for $d_1/d_2 = \frac{2}{3}$ and $H = \frac{5}{6}$ is $F_{0t} = 0.053$ in good agreement with the theoretical prediction, $F_{0t} = 0.0583$.

We also observed that as F_0 exceeds a second critical value, F_{0c}, Kelvin–Helmholtz (KH) instability takes place at the interface in the lee side of the obstacle (see Fig. 2 middle). The jet formed between the obstacle and the interface is the responsible for this instability (see Fig. 3(a)).

The Fig. 3(b) shows the second critical value F_{0c} as a function of H for which KH instability develops at the interface. When the amplitude of the waves arising from this instability is sufficiently large, they become unstable against three-dimensional perturbations and secondary instability develops near the interface, which in turn, causes turbulence downstream (Fig. 2 middle). We notice that before the emergence of KH instability, turbulence is already present due to the hydraulic jump (i.e., $F_{0t} < F_{0c}$), but this turbulence is located mainly below the interlayer surface for $F_0 \lesssim F_{0c}$.

The PIV measurements allows us to perform a detailed study of the instabilities near the interface. Fig. 3(a) shows a typical profile of the horizontal component of the velocity as a function of the vertical coordinate z. In

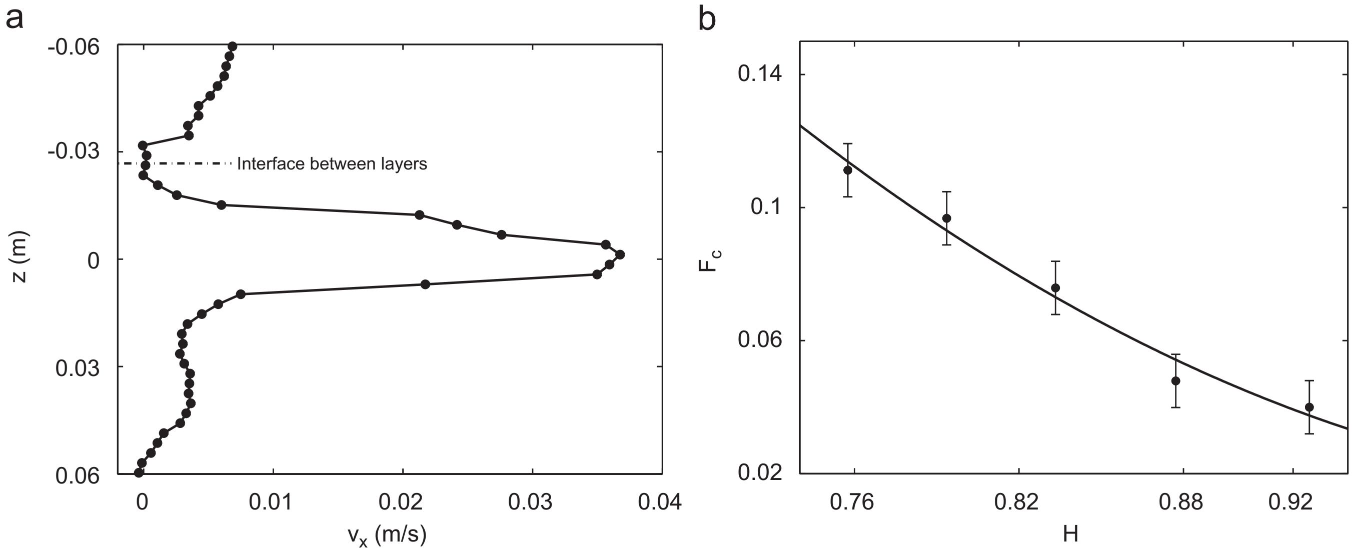

Fig. 3. (a) Experimental profile of the horizontal velocity near the interface at the end of the obstacle as a function of the vertical coordinate z obtained from PIV for $F_0 = 0.076$ and $H = \frac{5}{6}$. (b) Critical Froude number for the emergence of shear instability as a function of H. The points and error bars correspond to the experimental values.

order to study the stability of this jet, we use a triangular jet model in which the velocity in x-axis is defined as

$$U = \begin{cases} U_1 & \text{for } |z| \geqslant b, \\ U_2 + (U_1 - U_2)|z|/b & \text{for } |z| < b, \end{cases} \tag{1}$$

where b is the thickness of the jet and $z = 0$ is defined in order to obtain $z = b$ at the interface between layers, and we consider that the two layers have the same velocity U in this region. The normal mode stability analysis for this flow yields that the more unstable mode corresponds to the wavenumber $k = 1.225/b$. For $b = 1.4\,\text{cm}$, the corresponding wavelength is $\lambda = 7.2\,\text{cm}$ which is in good agreement with the experimental data of $\lambda = 6\,\text{cm}$. From this analysis we obtain that this jet is partially stabilized in the proximity of a wall, as occur over the obstacle, in the sense that instabilities grow slowly in comparison with the unbounded case. This result is in agreement with similar results obtained previously by Hazel [15]. This argument explains why the instabilities appear downstream, but not over the obstacle.

In summary, the existence of a concentrated jet in the lee side of the obstacle is demonstrated both, through laboratory experiments and numerical simulation. The normal mode analysis of the jet shows that the KH instability induces a secondary instability and the appearance of turbulence near the interface between the two layers. Furthermore, the results of this stability analysis explains the existence of the instabilities past the obstacle, but not over it. Critical values of the Froude number for the emergence of the internal hydraulic jumps are experimentally obtained in agreement with theoretical predictions. Although the internal hydraulic jumps develops for values of F less than those necessary for KH, the turbulence generated by it remain below the layers interface. Thus, the KH instability at the surface constitutes a more efficient source of two layer mixing than the internal hydraulic jump.

We acknowledge financial support from the Latin American Center for Physics (CLAF), PEDECIBA, and Grants PDT 54/037 and FCE 9028 (Uruguay). R.M. acknowledge financial support from the Brazilian agencies, CNPq and FAPERN.

References

[1] P. Drazin, W. Reid, Hydrodynamic Stability, second ed., Cambridge University Press, Cambridge, 2004.
[2] S. Abarzhi, K. Nishihara, R. Rosner, Multiscale character of the nonlinear coherent dynamics in the Rayleigh–Taylor instability, Phys. Rev. E 73 (2006) 036310.

[3] P.G. Baines, Topographic Effects in Stratified Flows, Cambridge University Press, Cambridge, 1988.
[4] D. Boyer, P. Davies, Laboratory studies of orographic effects in rotating and stratified flows, Ann. Rev. Fluid Mech. 32 (2000) 165.
[5] P. Travassos, F. Hazin, J. Zagaglia, R. Rocha, J. Schober, Thermohaline structure around seamounts and islands off North-Eastern Brazil, Arch. Fish. Mar. Res. 47 (1999) 211.
[6] D. Farmer, L. Armi, Stratified flow over topography: the role of small-scale entrainment and mixing in flow establishment, Phil. Trans. R. Soc. London A 455 (1999) 3221.
[7] J. Verron, C. LeProvost, A numerical study of quasigeostrophic flow over isolated topography, J. Fluid Mech. 154 (1985) 231.
[8] D. Boyer, P. Davies, W. Holland, F. Biolley, H. Honji, Stratified rotating flow over and around isolated three-dimensional topography, Phil. Trans. R. Soc. London A 322 (1987) 213.
[9] O.S. Eiff, P. Bonneton, Lee-wave breaking over obstacles in stratified flow, Phys. Fluids 12 (2000) 1073.
[10] Q. Schiermeier, Oceanography: churn, churn, churn, Nature 447 (2007) 522.
[11] A. Genin, Bio-physical coupling in the formation of zooplancton and fish aggregations over abrupt topographies, J. Mar. Syst. 50 (2004) 3.
[12] C. Chen, R. Beardsley, R. Limeburner, A numerical study of stratified tidal rectification over finite-amplitude banks. Part II: George Bank, J. Phys. Oceanogr. 25 (1995) 2111.
[13] G. Boehlert, A. Genin, A review of the effects of seamounts on biological processes, in: B. Keaton, P. Fryer, R. Batiza, G. Boehlert (Eds.), Seamounts, Islands and Atols, Geophysical Monograph, vol. 43, American Geophysical Union, Washington, DC, 1987, p. 319.
[14] G. Usera, A. Vernet, J. Ferré, Use of time resolved PIV for validating LES/DNS of the turbulent flow within a PCB enclosure model, Flow Turbulence Combust. 77 (2006) 77.
[15] P. Hazel, Numerical studies of the stability of inviscid stratified flows, J. Fluid Mech. 51 (1972) 39.