A simple mechanism for controlling vortex breakdown in a closed flow.

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This work is focused to study the development and control of the laminar vortex breakdown of a flow enclosed in a cylinder. We show that vortex breakdown can be controlled by the introduction of a small fixed rod in the axis of the cylinder. Our method to control the onset of vortex breakdown is simpler than those previously proposed, since it does not require any auxiliary device system. The effect of the fixed rods may be understood using a simple model based on the failure of the quasi-cylindrical approximation. We report experimental results of the critical Reynolds number for the appearance of vortex breakdown for different radius of the fixed rods and different aspect ratios of the system. Good agreement is found between the theoretical and experimental results.

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INTRODUCTION

The development of structural changes in vortical flows and, particularly, vortex breakdown [1, 2, 3, 4, 5] has been intensively investigated during the last years [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. The characteristic and fundamental signature of vortex breakdown is the appearance of a stagnation point followed by regions of reversed axial flows with a bubble structure when the swirl is sufficiently large. This structural change is also accompanied by a sudden change of the size of the core and the appearance of disturbances downstream the enlargement of the core. Vortex breakdown is very important in several applications of Fluid Mechanics such as aerodynamics, combustion, nuclear fusion reactors or bioreactors. The presence of vortex breakdown may be beneficial or detrimental, depending on each particular application [11, 13].

In order to explain the origin of vortex breakdown many proposals have been given. Some of them are based on instability mechanisms, while in other cases the proposed theories consider that hydrodynamical instabilities do not play a significant role. In any case, it is widely accepted that instabilities arise after the occurrence of vortex breakdown.

Vortex breakdown (VB) has been observed not only in open flows, but also in experiments performed in confined flows, for example, in closed cylinders [3]. It is worth noting that the characteristics of VB in both cases are strongly similar, suggesting the possibility that the basic mechanism of VB is the same in both situations [10].

While considerable theoretical effort has been done in the study of VB emergence in open flows, less results have been obtained for closed flows. In a way, this may be explained by the fact that in the later case, the basic flow to be studied is much more complex than those typically observed in open ducts. On one hand, experimental measurements show that flows in open channels can be accurately described with the relatively simple q-vortex model [5]. On the other hand, an analytical description for closed flows which are considerable more complex than closed flows is not known [3, 6, 8]. However, from an experimental point of view, experiments performed with flows confined in cylinders are very attractive because they are simpler to control. Due to the great number of practical implications of VB, the development of mechanisms for controlling its emergence is of considerable interest.

Recently, different methods for controlling VB in closed flows have been proposed using different techniques such as co–rotation and contour–rotation of the end-walls [13], the addition of near-axis or at the end wall swirl [11], or temperature gradients [19]. Experiments show that such methods may increase or decrease the critical Reynolds number to develop VB. In all the mentioned cases, it is necessary to use auxiliary devices, such motors or heat sources. The aim of this work is to introduce a method simpler, from a practical point of view, than those previously mentioned for controlling vortex breakdown in closed flows.

This paper is organized as follows. In Sec. we analyze a simple model to predict the onset of vortex breakdown. In Sec. we present the experimental observations. These results show that control of VB may be performed with the presence of rods *at rest*, located along the axis of the cylinder, without the necessity of transferring swirl. The comparison between the experiments and the theoretical model is given in Sec. . Finally, in Sec. we present a summary and the conclusions.

THE MODEL

As already mentioned in the introduction, the similarity between vortex breakdown in open and closed flows is striking. In particular, experiments and simulations show that, the velocity fields around the point where VB develops, have the same structure in open and closed flows. The similarity of both situations suggests that the formation of VB depends mainly on the local velocity field, and the details of the flow far from the location of VB can be neglected. For example, in figure 1, we can observe (a) the experimental streamlines just before the appearance of VB from Ref. [3] and (b) a sketch of an open flow with divergent section. In subsection we present

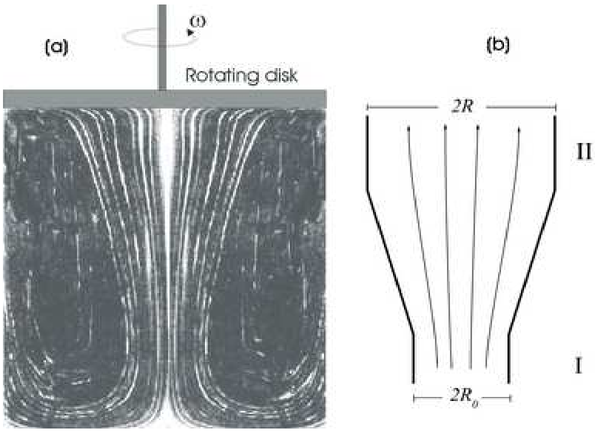

Figure 1: (a) Streamlines of the enclosed flow prior to the development of VB (taken from Ref. [Escudier 1981]). The streamlines are divergent near the point where VB develops, suggesting that locally the flow may be described as the flow inside a duct that experiment an expansion as it is shown in (b).

a simple model to predict the onset of VB in open flows. We shall employ this model to describe locally the enclosed flow and to study the influence of local geometrical changes on VB. In particular, the effect on VB development of a fixed rod located at the axis of the cylinder is discussed in subsection . The results will be compared with the experimental observations in subsequent sections.

Failure of quasi–cylindrical approximation and VB

In the past, it has been shown that simple theoretical vortex models of open flows may predict behaviors very similar to those occurring in vortex breakdown. Various authors associated the emergence of VB with the failure of the quasi-cylindrical approximation in inviscid theories [5]. This approach gives a prediction about the position of VB that is in good agreement with experiments [4]. A classical approach based on an axisymmetric inviscid analysis was given by Batchelor [20]. In this model, that can be applied outside the boundary layer, it is assumed that the fluid in steady motion passes through a transition from a duct of radius R_0 to another duct of different radius R as it is shown in Fig. 1(b).

Based on Batchelor's model, we assume that upstream, region I in Fig. 1(b), the fluid moves inside a duct of radius R_0 following a Rankine vortex with uniform axial velocity. This vortex has a core of radius c_0 which rotates like a solid body with constant angular velocity σ and an outer irrotational region. The whole vortex has an axial motion with velocity U_0. In a cylindrical system of coordinates (r,θ,z) with the z-axis in the direction of the duct the velocity field is written as:

$$\begin{aligned} v_r &= 0, \qquad (1)\\ v_\theta &= \begin{cases} \sigma r & 0<r<c_0, \\ \Gamma/r & c_0<r<R_0, \end{cases} \\ v_z &= U_0. \end{aligned}$$

where Γ is the circulation in the outer region and in order to assure the continuity of the velocity at $r=c_0$ it must satisfy $\Gamma=\sigma c_0^2$.

Now, in order to solve the fluid motion downstream, region II in Fig. 1, we introduce the axisymmetric streamfunction Ψ related to the velocity field by means of

$$v_z = \frac{1}{r}\frac{\partial\Psi}{\partial r}, \qquad (2)$$

$$v_r = -\frac{1}{r}\frac{\partial\Psi}{\partial z}. \qquad (3)$$

From Euler's equation the following equation for the streamfunction is obtained [20]:

$$r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Psi}{\partial r}\right)+\frac{\partial^2\Psi}{\partial z^2} = r^2\frac{dH}{d\Psi}-K\frac{dK}{d\Psi} \qquad (4)$$

where $H=\frac{1}{2}v^2+\frac{p}{\rho}$, $K=rv_\theta$, ρ is the density, p is the pressure and v^2 is the magnitude of the velocity. For the flow given by Eqs. (1), we have that

$$H=\frac{2\sigma^2}{U_0}\Psi+\frac{1}{2}U_0^2,\ K=\frac{2\sigma}{U_0}\Psi,\ for\, 0<r<c_0 \qquad (5)$$

Thus, Eq. (4) takes the form

$$\Psi_{rr}-\frac{1}{r}\Psi_r+\Psi_{zz}=-\frac{4\sigma^2}{U_0^2}\Psi+\frac{2\sigma^2}{U_0}r^2 \qquad (6)$$

where the subscripts, r and z, denote derivatives. The relations (5) hold also downstream for the steady flow, so that Eq. (6) determines the cylindrical flow downstream. The general solution for the downstream rotational region of the flow for any kind of cylindrical ducts is

$$\Psi(r)=\frac{1}{2}U_0r^2+AF_1(\gamma r)+BY_1(\gamma r) \qquad (7)$$

where F_1 and Y_1 are the Bessel functions of the first and second kind respectively and $\gamma=2\sigma/U_0$ [20]. The downstream flow in the irrotational region is given by

$$v_r = 0, \qquad (8)$$

$$v_\theta = \Gamma/r, \qquad (9)$$

$$v_z = U, \qquad (10)$$

where U is a constant that can be determined using conservation laws.

Now, using the fact that downstream the fluid is constrained to move inside a cylindrical duct of radius R, in order to avoid the divergence of Ψ at $r=0$ it follows that $B=0$. The no mass flow condition at the solid boundary is automatically satisfied because the flow has no radial component. Therefore, the downstream cylindrical flow in the rotational region ($r<c$, where c is the radius of the rotational core) is given by

$$\begin{aligned} v_r &= 0, \\ v_\theta &= \sigma r + \frac{AS}{R} J_1\left(\frac{S}{R} r\right), \\ v_z &= U_0 + \frac{AS}{R} J_0\left(\frac{S}{R} r\right), \end{aligned} \tag{11}$$

whereas in the irrotational region ($c<r<R$), the flow is written as

$$\begin{aligned} v_r &= 0, \\ v_\theta &= \Gamma/r, \\ v_z &= \frac{R_0^2-c_0^2}{R^2-c^2} U_0, \end{aligned} \tag{12}$$

where, using mass conservation,

$$A = \frac{(c_0^2-c^2)U_0}{2cJ_1(\frac{S}{R}c)}, \tag{13}$$

and S is the swirl parameter, defined as $S = R\gamma = \frac{2\sigma R}{U_0}$. Using the continuity of the pressure, we obtain the following implicit equation that gives the value of the core radius c,

$$\frac{(R_0^2-c_0^2)}{(R^2-c^2)} - \frac{\frac{S}{R}(c_0^2-c^2)J_0(\frac{S}{R}c)}{2cJ_1(\frac{S}{R}c)} = 1. \tag{14}$$

We solved this equation for different values of the parameters. When the value of S is increased, it may happen that two branches of solutions for c collide and disappear. In Fig. 2 the curve labelled (a) represents the ratio c/R as a function of the swirl S. We observe in this figure that for each curve (corresponding to the different values of d, parameter to be introduced in the following subsection) there exists a critical value, above which the solution does not exist.

In Batchelor approach [20], the disappearance of these solutions is interpreted as the signal of VB emergence. We discuss now a way to support this interpretation. As already mentioned, the typical signature of VB is the formation of a stagnation point, followed by regions with reversion of the axial velocity. In the neighborhood of the stagnation point, a quasi-cylindrical description of the flow is doomed to failure, because in this place the flow is strongly dependent on the axial coordinate z [4]. This is revealed by the disappearance of cylindrical solutions. In the absence of VB a quasi-cylindrical description is suitable and, probably, there exists a cylindrical solution to describe the flow. Then, it is reasonable to assume that the critical value S_{VB} for the emergence of VB and the critical value S_c for the disappearance of cylindrical solutions are very close to one another, i.e. $S_{VB} \approx S_c$, and the identification of both may be a practical criteria for VB. From now on,

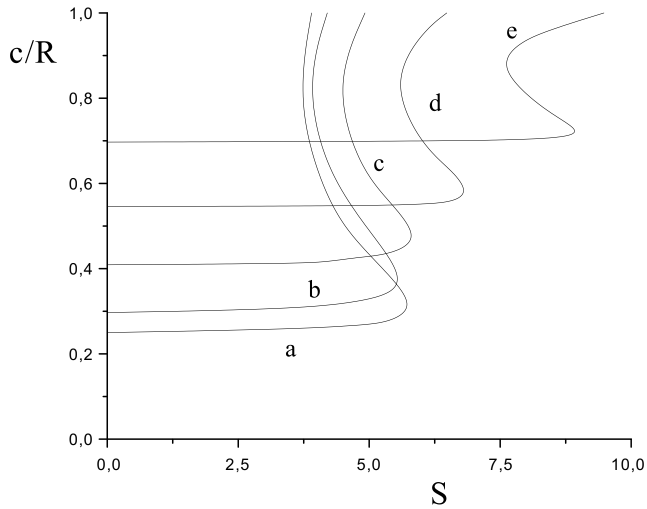

Figure 2: Theoretical results for the dimensionless core size, c/R, as a function of the swirl, S, and different radius of the fixed rod, (a) $d=0$, (b) $d=0.2$, (c) $d=0.4$, (d) $d=0.6$ and (e) , $d=0.8$. Other parameter values: $R_0=1$, $c_0=0.25$ and $R=1.2$

we use this criterion to *estimate* the critical conditions for the emergence of VB. We shall employ the above model of open flows to study qualitatively the flow in the closed cylinder. We shall assume that, if the open and closed flows are similar around the localization of VB, then the phenomena that takes place in both cases are similar. The flow inside the cylinder is analogous to the open flow near a transition between two cylindrical ducts (regions I and II of Fig. 1). We assume that in region I the flow is given by Eq. (1) and the cylindrical flow in region II is given by the Eqs. (11). Thus the radius of the core c is to be determined with the Eq. (14). As already mentioned, for some range of values of R_0, c_0 and U_0, cylindrical solutions do not exist in region II if S is above a critical value S_c (see Fig. 2). According to Batchelor's criterion, this means that VB takes place inside region II, when the swirl parameter S is larger than a critical value S_c. This behavior is in qualitative agreement with VB inside the closed cylinder.

Effect of rods presence at the axis

We now estimate how the changes in the geometry of the region II affect the development of VB. More specifically, we consider the effect produced by the presence of cylinders along the duct axis. Taking into account the local similarity between open and closed flows, we extend the model introduced in the previous sub-section to this case. Using the formalism described above, we obtain the following equation that determines the radius of the rotational core c in presence

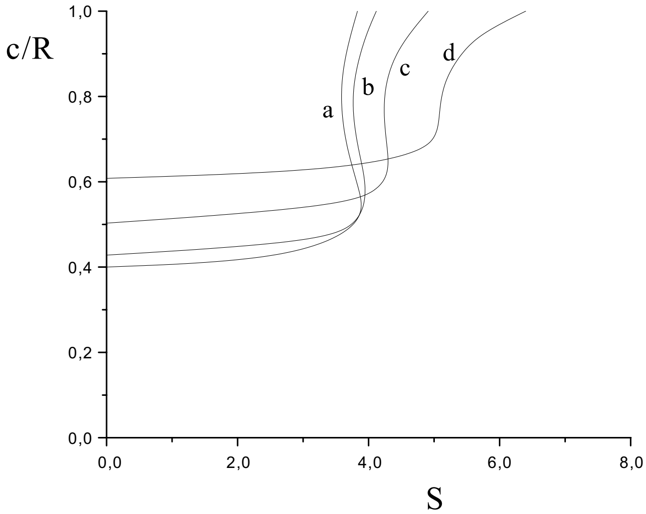

Figure 3: As figure 2 but with $R_0 = 1$, $c_0 = 0.4$ and $R = 1.2$, for (a) $d = 0$, (b) $d = 0.2$, (c) $d = 0.4$, (d) $d = 0.6$.

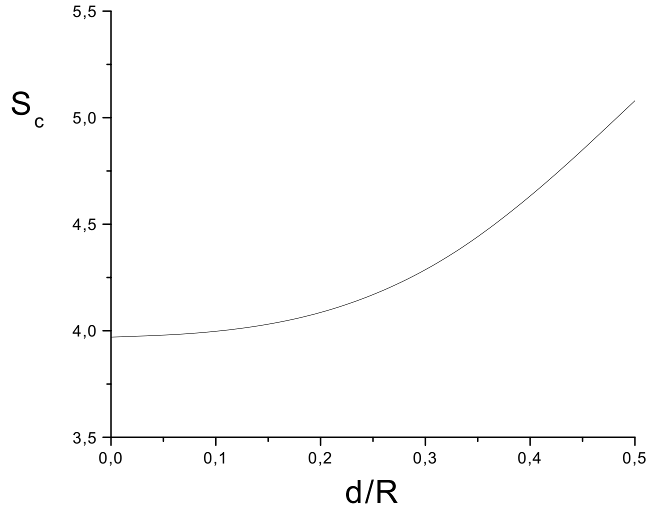

Figure 4: Critical value of the swirl parameter, S_c, for the disappearance of cylindrical solutions as a function of d/R, for the case $R_0 = 1$, $c_0 = 0.4$ and $R = 1.2$

of the inner cylinder of radius d

$$\frac{(R_0^2 - c_0^2)}{(R^2 - c^2)} - \frac{A_{II}}{U_0}\frac{S}{R}J_0(\frac{S}{R}c) - \frac{B_{II}}{U_0}\frac{S}{R}Y_0(\frac{S}{R}c) = 1, \qquad (15)$$

with

$$A_{II} = \frac{U_0}{2cd}[\frac{d(c_0^2 - c^2)Y_1(\frac{S}{R}d) + cd^2Y_1(\frac{S}{R}c)}{J_1(\frac{S}{R}c)Y_1(\frac{S}{R}d) - J_1(\frac{S}{R}d)Y_1(\frac{S}{R}c)}], \qquad (16)$$

$$B_{II} = \frac{U_0}{2cd}[\frac{d(c_0^2 - c^2)J_1(\frac{S}{R}d) + cd^2J_1(\frac{S}{R}c)}{Y_1(\frac{S}{R}c)J_1(\frac{S}{R}d) - Y_1(\frac{S}{R}d)J_1(\frac{S}{R}c)}]. \qquad (17)$$

In order to appreciate the influence of the inner cylinder on the disappearance of cylindrical solutions, figures 2 and 3 show the ratio c/R as a function of the swirl S. The effect of a very slender inner cylinder may be to increase (Fig. 2) or decrease (Fig. 3) slightly the critical value S_c. However, for all the situations considered, the critical value S_c increases as long as d is above a threshold. In this case, the emergence of VB was transferred to larger values of the swirl parameter, and VB was suppressed in the range between the old and new critical values of S (see Fig. 4). These results show that the model predicts the suppressing effect of the slender cylinders. In the next section, we present the experimental results of that were performed to study the effect of the rods in confined flows.

EXPERIMENTAL SETUP AND RESULTS

The experimental setup consists of an acrylic cylindrical container of inner radius $R = 40$ mm and a rotating top disk at a variable height H rotating with angular velocity Ω (figure 5).

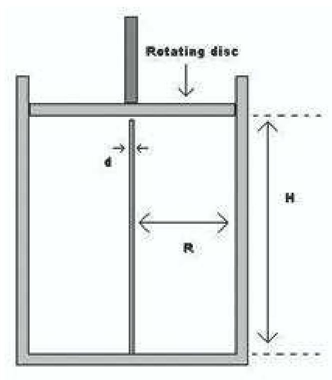

Figure 5: Experimental setup. Closed cylinder, radius R and height H, with a rotating top wall. Along the axis of the cylinder a fixed rod of radius d is located.

The fluid used was water dissolutions of glycerin at 60% in mass, with $\nu = 1 \times 10^{-5}$ m^2/s. Temperature was kept constant at 20^oC. The Reynolds number corresponding to the rotating top wall, $Re = \Omega R^2/\nu$, varies between 600 and 2600, with a 1% error. Four different aspect ratios H/R were used: 1, 1.5, 2 and 2.5. The visualization system consists of a vertical sheet of light with 2 mm of thickness, generated by two slide projectors. Small quantities of fluorescein were injected into the flow through a small hole in the bottom disk. Pictures were taken using a 5 megapixel Canon digital camera. To investigate the effect of the inner cylinders on VB, we have used three axial fixed rod of radius $d = 1$ mm, $d = 2.5$ mm and $d = 5$ mm.

For convenient comparison with previous works [3] , we first studied the dynamical behavior of usual vortex break-

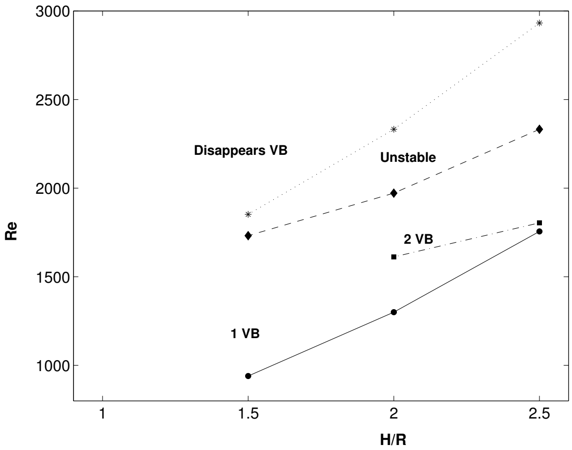

Figure 6: Critical Reynolds number corresponding to the appearance of a single VB (circles), double VB (squares), oscillatory bubbles (diamond) and disappearance of VB (stars) for the case without fixed rod.

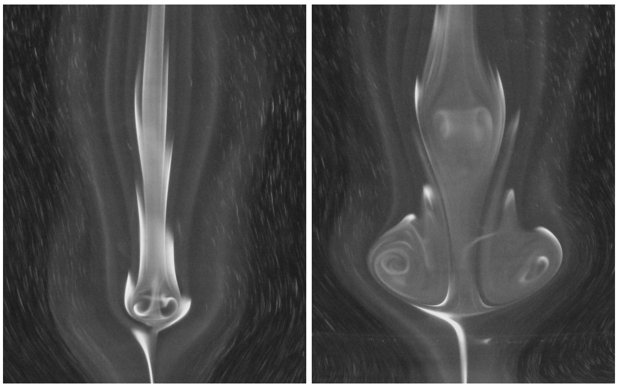

Figure 7: Flow visualization showing one and two vortex breakdown, without axial rod. (a) Re = 1300, H/R = 2. (b) Re = 1756, H/R=2.5.

down without axis rod. In Fig. 6 the experimental results for different aspect ratios can be observed. For $H/R = 1$, vortex breakdown does not appear. For $H/R = 1.5$, vortex breakdown take place for Re = 940. With further increment of Reynolds number, the recirculation bubble becomes oscillatory (Re = 1732), and for Re = 1852, it disappears. These results agree with classical work of Escudier [3]. For $H/R = 2$ and 2.5, first one VB, and subsequently two VB, are generated when the Reynolds number is increased. It is possible to see in figure 7 the interior detailed structure of the recirculation bubble.

We consider the situation in which an axial fixed rod of radius d is introduced. For $d = 1$ mm, the changes in the flow are very small in comparison with the situation without rod. However, for $d = 2.5$ mm and $d = 5$ mm, we observe that the critical Reynolds number to produce vortex breakdown is increased. Experimental results are summarized in Table I. In Figs. 8 and 9 the critical Reynolds number for the appearance of the first bubble as a function of the aspect ratio and the radius of the rod respectively is shown. We note that for values

Table I: Reynolds numbers corresponding to the appearance of the first VB for different aspect ratios and radius of the rod.

H/R	d=1 mm	d=2.5 mm	d=5 mm
1	No VB	892	1012
1.5	940	1108	1132
2	1300	1420	1636
2.5	1756	1876	2260

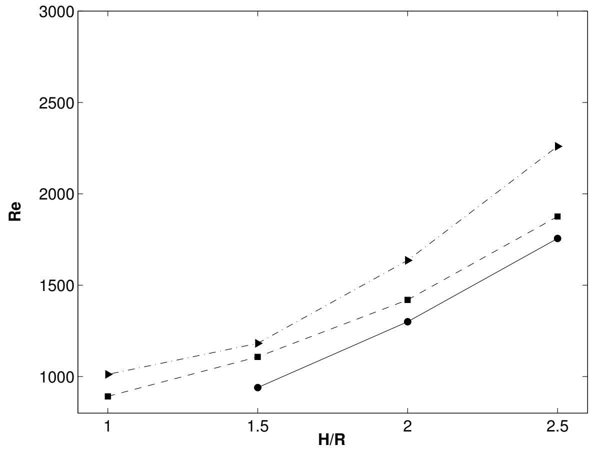

Figure 8: Critical Reynolds numbers corresponding to the appearance of the first bubble as a function of the aspect ratio H/R. Without axial rod (circles) and with axial rod; $d = 2.5$ mm (squares) and $d = 5$ mm (triangles).

of Re larger that Re_c, the size of the bubble and its dynamics is still affected by the presence of the inner cylinder. For example, Figure 10, shows vortex breakdown for $H/R = 2.5$ and Re = 2260. We can observe that without an axial rod, there are two oscillating bubbles (Fig. 10a). For $d = 2.5$ mm, there are two VB, but, in this case, they are steady (Fig. 10b) and for $d = 5$ mm, only one VB appears. In addition, the size of the bubble corresponding to the first VB was clearly decreased with the presence of the rods.

DISCUSSION

From the tables and figures shown above it can be concluded that the usage of the inner rods at rest, may be employed to control the vortex onset. Comparing Figs. 4 and 9, we observe that the experimental results about the effect of the cylinders on the first bubble formation are in agreement with the prediction of the simple model, assuming the hypothesis that the swirl parameter increases as Re increases. This dependence of S on Re is suggested by the fact that both quantities are proportional to the angular momentum of the flow. Recently, Husain *et al* proposed arguments that support this hypothesis [11].

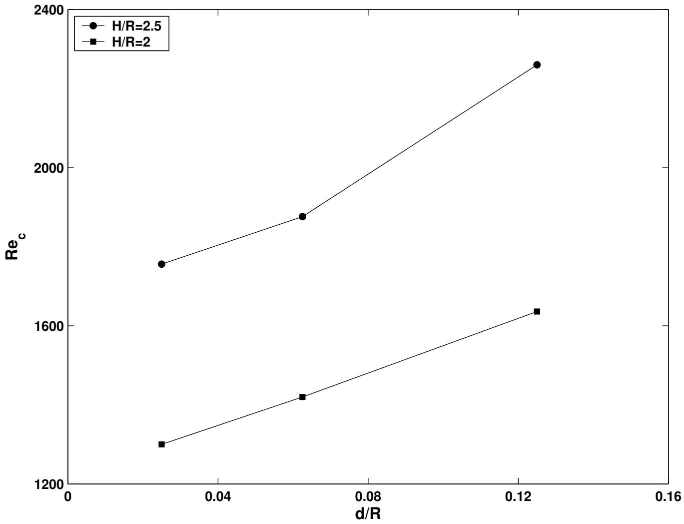

Figure 9: Critical Reynolds number as a function d/R for the appearance of single VB and different aspect ratios, $H/R = 2$ (squares) and $H/R = 2.5$ (circles).

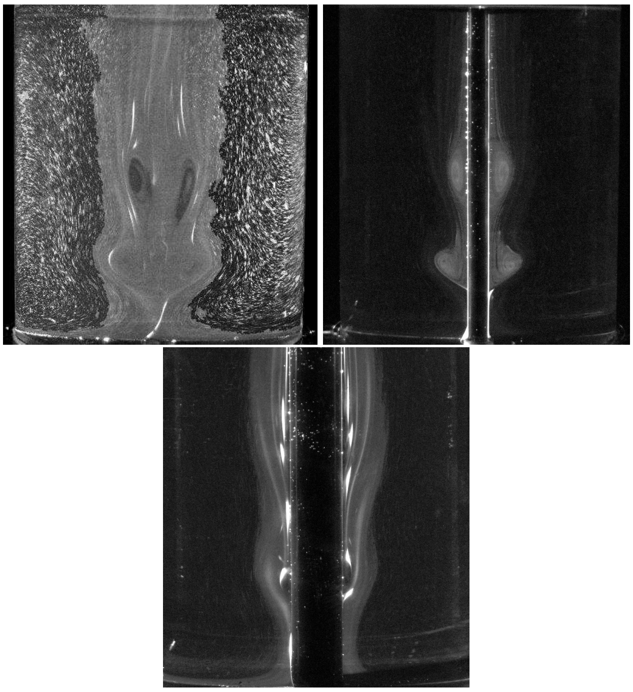

Figure 10: Vortex breakdown for $H/R = 2.5$ and Re=2260. (a) without axial rod; (b) for $d = 5$ mm; (c) for $d = 10$ mm.

The changes of the critical Reynolds number due to the rods presence are clearly noticeable, as follows from table 1. We also observe that in the presence of two bubbles, larger values of the Reynolds number are needed for the development of the second bubble in comparison with the situation without rod. It is worth noting that our results are in agreement with those of Hussain *et al* [11], in so far as that the very slender rod at rest does not introduce significant changes in the flow. The radius ratio used by these authors was $d/R = 0.04$, which is similar to the small radius ratio value we considered, that is $d/R = 0.025$. Significant results, in our experiments, are obtained above $d/R = 0.0625$.

It is also interesting to compare our results with those obtained by Mullin *et al* [12]. These authors reported that they did not obtain noticeable changes in the emergence of VB with the addition of a inner cylinder with ratio $d/R = 0.1$. However, in our experiments we observed appreciable changes in the flow, for values of the ratio $d/R = 0.0625$ and $d/R = .125$. Looking carefully at Figures 2 and 3 of [12], we observe slight differences which, according to the authors, are within the experimental errors. For this reason, we took specially careful measurements in these cases.

SUMMARY AND CONCLUSIONS

In this work, we present a method for controlling the onset of VB. It consists basically of the addition of a small cylinder at rest in the axis of the cylindrical container. The experiments we performed show that this procedure increases the critical Reynolds number for the emergence of VB, and consequently suppresses the onset of VB in a certain range of values of Re.

The control technique proposed here is simpler than other previously proposed in the literature, since it does not require additional auxiliary devices. It is worth noting that our method, unlike the approach proposed by Husain et al [11], does not imply the addition of swirl near the axis. The simplicity of a method is in general an interesting feature, becoming more feasible to be used in engineering devices. Moreover, the required modification of the duct is relatively small. The volume ratio (cylinder to row) is $V_1/V_2 \sim 4 \times 10^{-3}$, while the decrease of the critical Reynolds number is about 10% . So that the shift of the critical Reynolds number is 20 times larger that the percent modification of the volume of the cylinder, showing the effectiveness of the method.

The effect of the rods is consistent with the results of a simple theoretical model of VB based on the failure of the quasi-cylindrical approximation, which assumes that S increases with increasing Re. This agreement supports the idea of that to explain VB a local description of the flow is adequate, and the details of the flows far for the localization of VB do not play an important role. This point is very appealing from a theoretical point of view, since it implies that to study VB in closed flows we must not consider all the rather complex velocity fields, but that it is enough to consider only the local field near the axis of the cylinder.

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