New indicators for early detection of critical transitions

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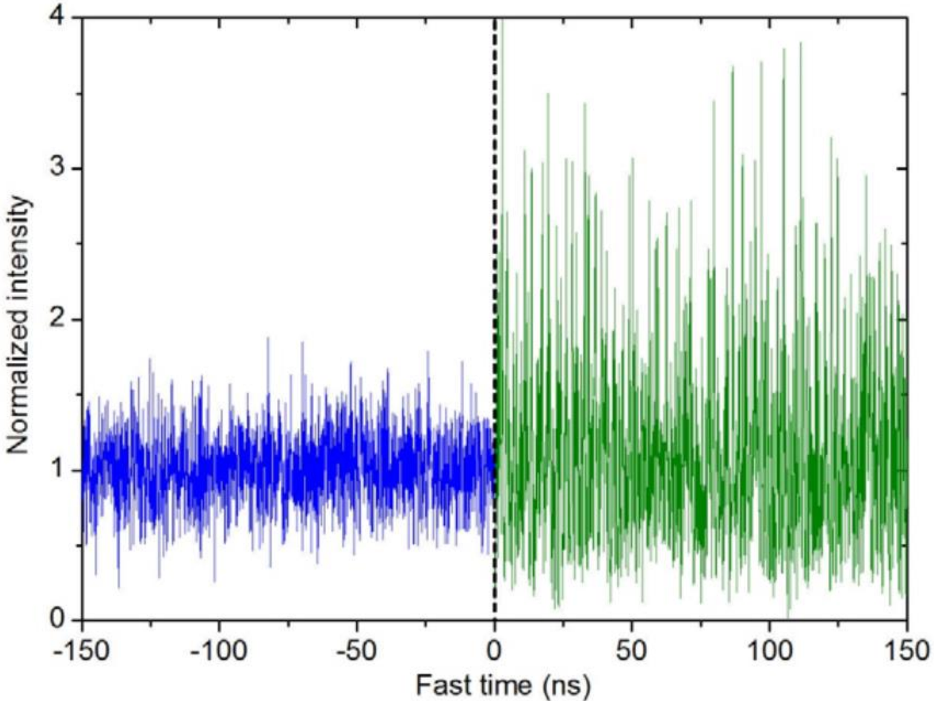
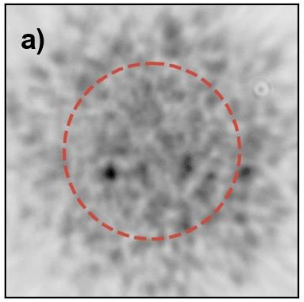
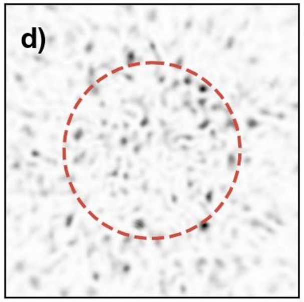
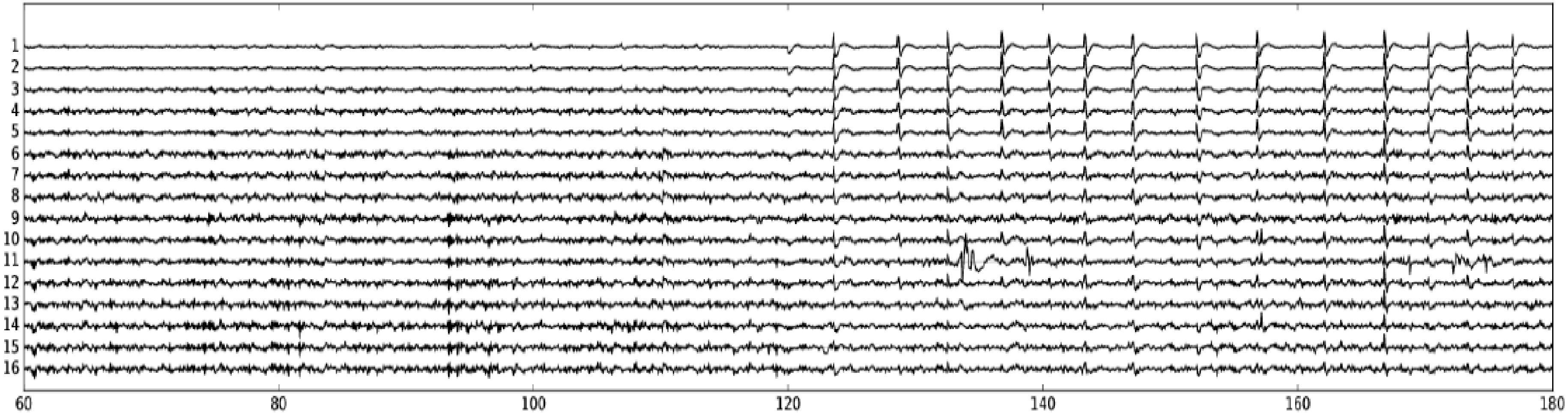
Regime transitions in complex systems









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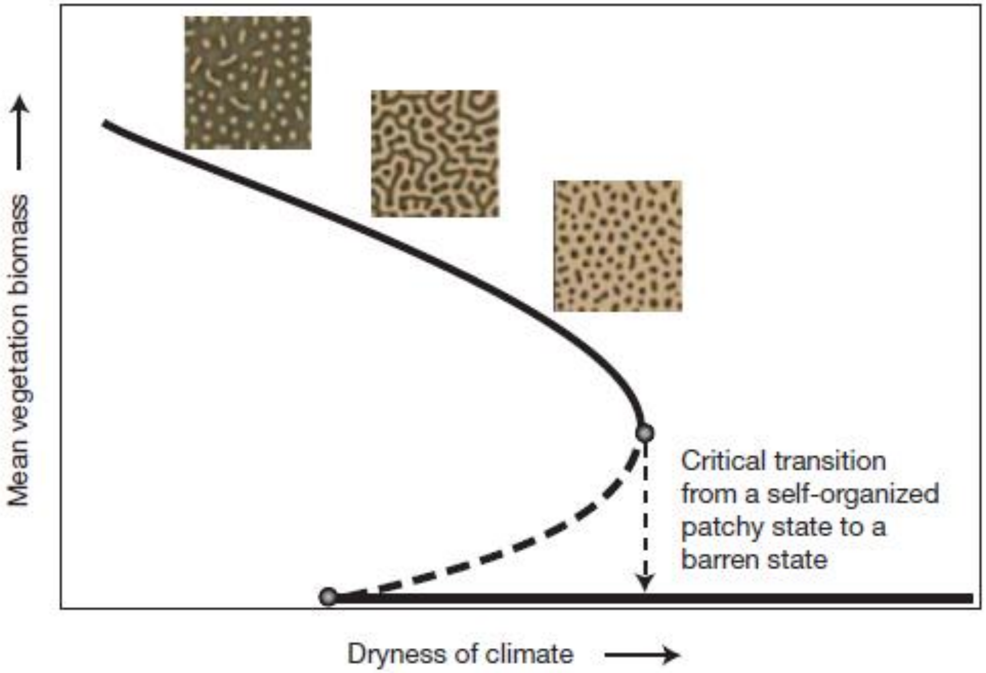
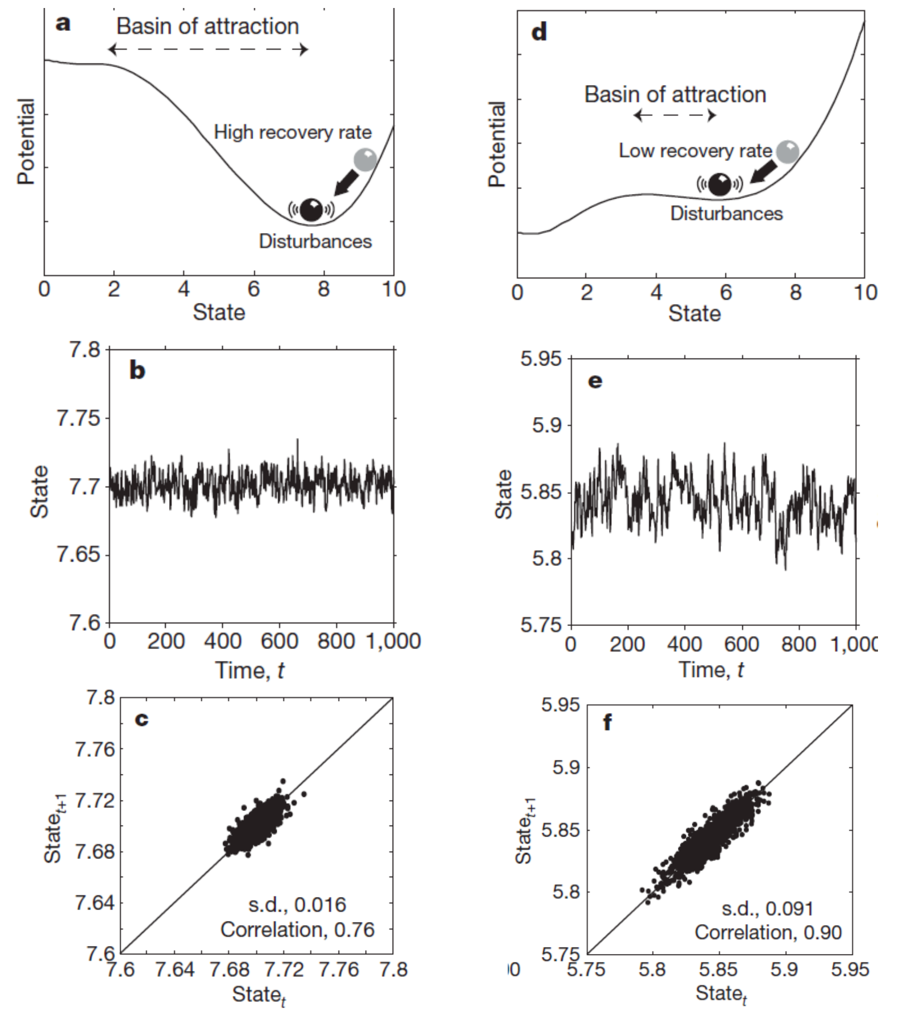
Classical indications of approaching critical transitions



M. Scheffer et al., Nature 461, 53 (2009)

Far from bifurcation | Close to bifurcation



$\Rightarrow$ increase of variance and autocorrelation: *critical slowing down*

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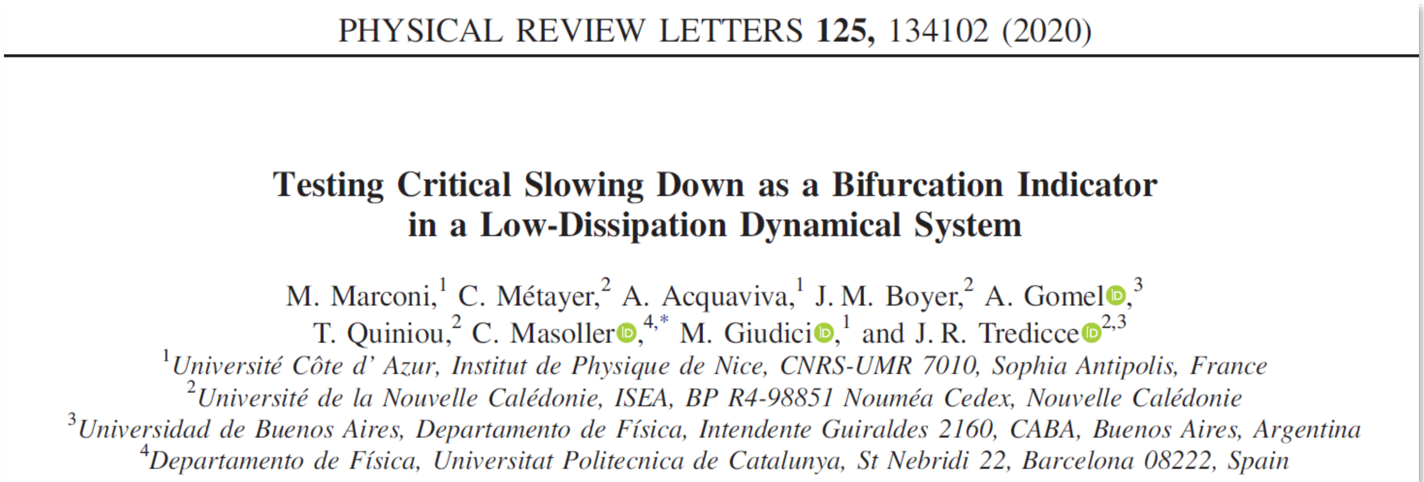
Critical slowing down can occur after the bifurcation


PHYSICAL REVIEW LETTERS **125,** 134102 (2020)

Testing Critical Slowing Down as a Bifurcation Indicator in a Low-Dissipation Dynamical System

M. Marconi,[1] C. Métayer,[2] A. Acquaviva,[1] J. M. Boyer,[2] A. Gomel,[3] T. Quiniou,[2] C. Masoller,[4,*] M. Giudici,[1] and J. R. Tredicce[2,3]
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at a parameter value well above the bifurcation point. We test experimentally the occurrence of critical slowing down by applying a perturbation to the accessible control parameter and we find that this perturbation leaves the system behavior unaltered, thus providing no useful information on the occurrence of critical slowing down. The theoretical analysis reveals the reasons why these tests fail in predicting an

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Outline

- Analysis method: ordinal analysis

- Application to regime transitions
  - Desertification: High $\rightarrow$ low biomass (vegetation 2D data – observational and simulated)

  - Laser turn-on: Low $\rightarrow$ high coherence (speckle images)

  - Eyes closed $\rightarrow$ eyes open (multichannel EEG data)

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Data analysis method: ordinal analysis

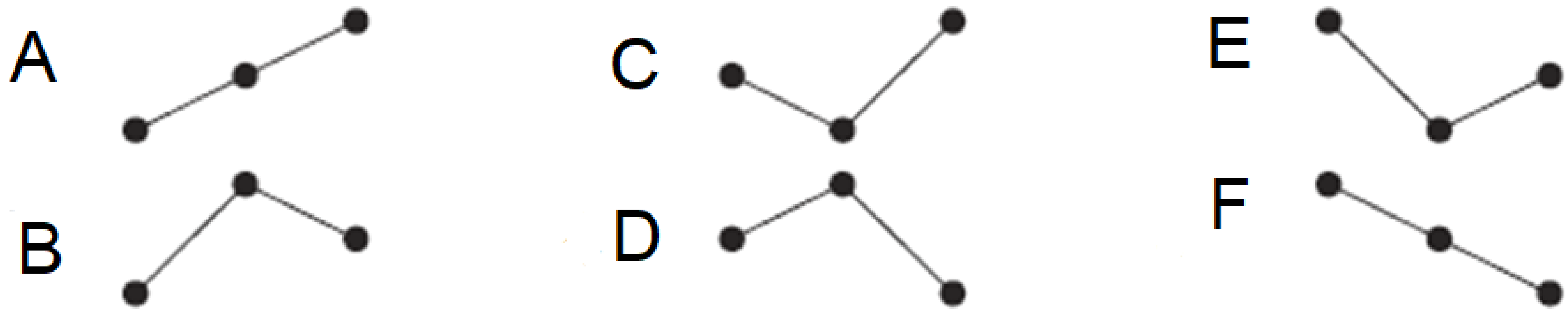
$$\{\ldots x_i,\ x_{i+1},\ x_{i+2},\ \ldots\}$$

Possible order relations among three numbers (e.g., 2, 5, 7)

{…2, 5, 7…}

{…5, 2, 7…}

{…7, 2, 5 …}



{…2, 7, 5…}

{…5, 7, 2…}

{…7, 5, 2…}

Bandt and Pompe: Phys. Rev. Lett. 2002

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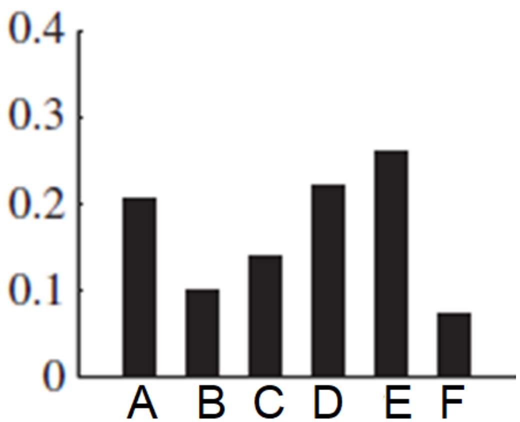
From the frequency of occurrence of the patterns, we calculate the "ordinal probabilities"



?

A. Analyze the probability values; use them as features for ML algorithms
B. Analyze "information theory measures" (e.g. entropy)— a form of nonlinear dimensionality reduction.

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

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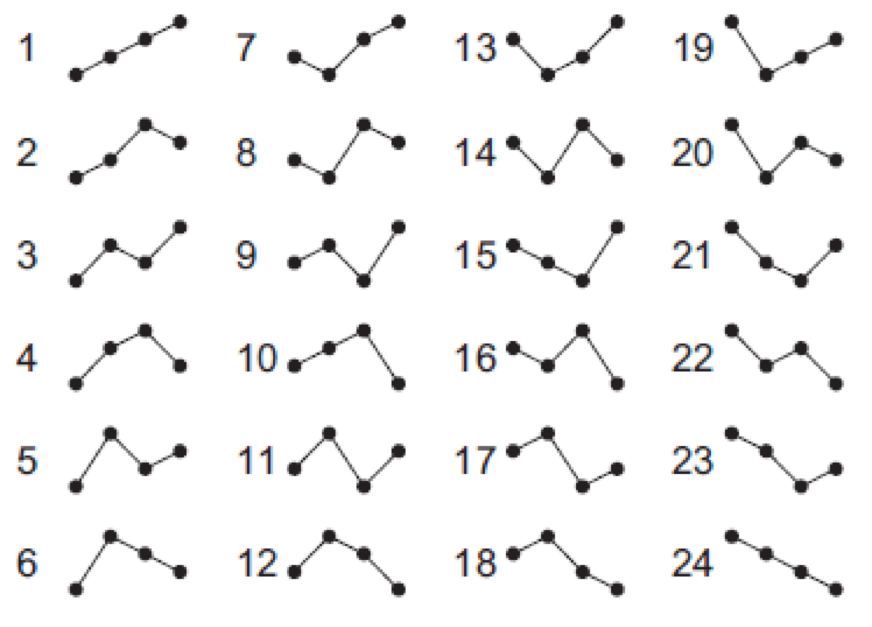
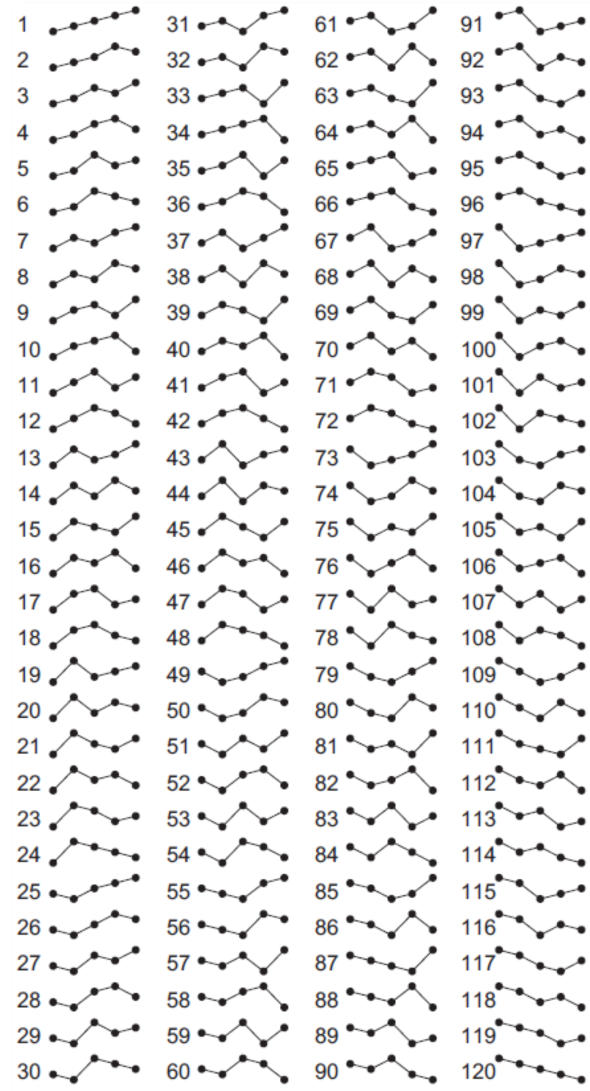
The number of ordinal patterns increases as D!



A problem for short datasets.

U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327



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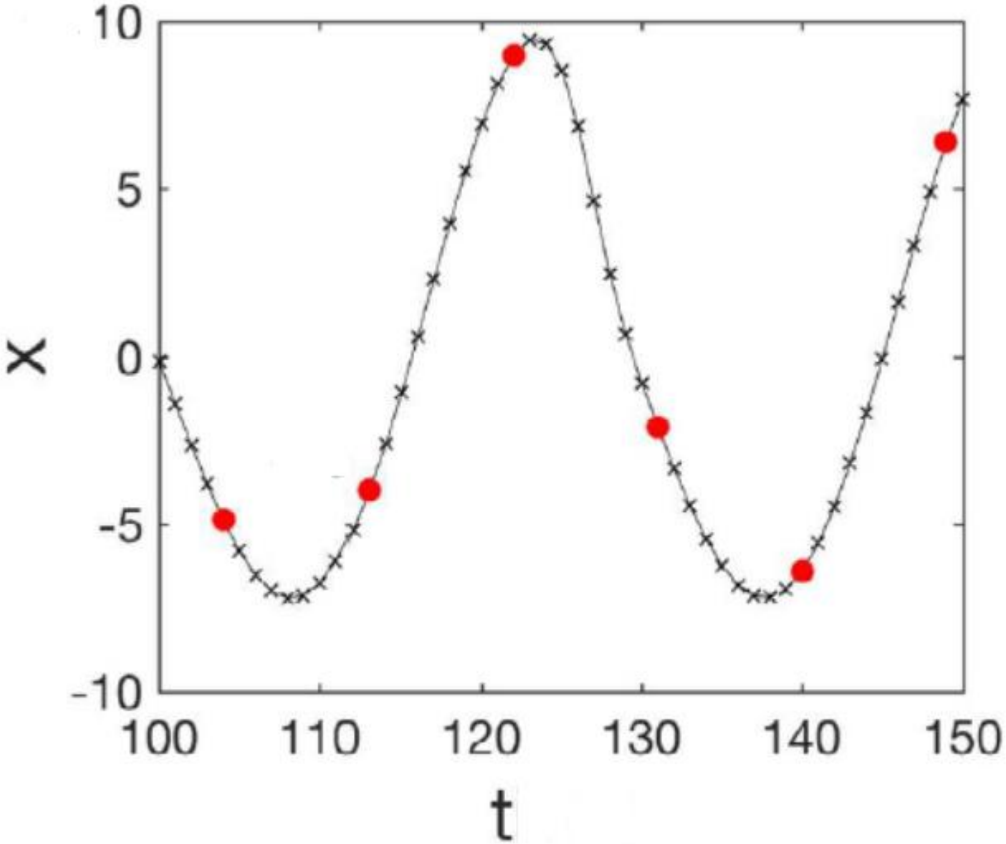
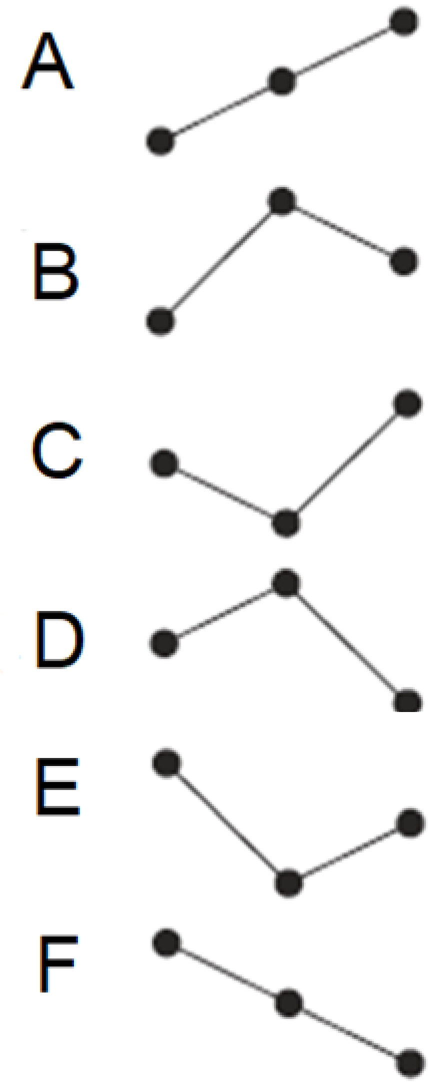
Using the “ordinal code”, which is the message?

A B F C





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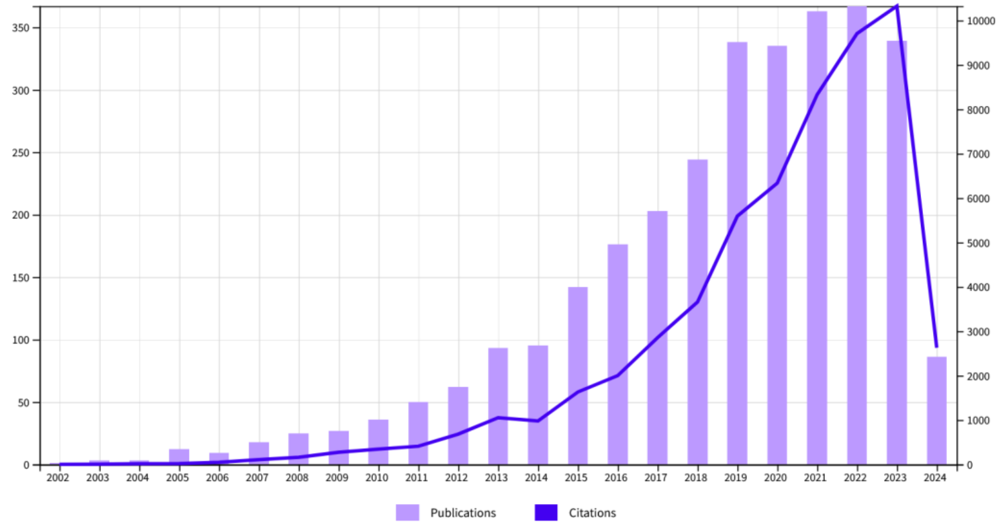
Ordinal analysis has been extensively used



Citations of the original work by Bandt and Pompe (PRL 2002)

- to test if a model is good for the data,
- to fit the model's parameters,
- to classify different types of data based on similarities of probabilities of ordinal patterns.

Important advantage: robust to noise & outliers.

I. Leyva, J. M. Martinez, C. Masoller, O. A. Rosso, M. Zanin, "20 Years of Ordinal Patterns: Perspectives and Challenges", EPL 138, 31001 (2022).

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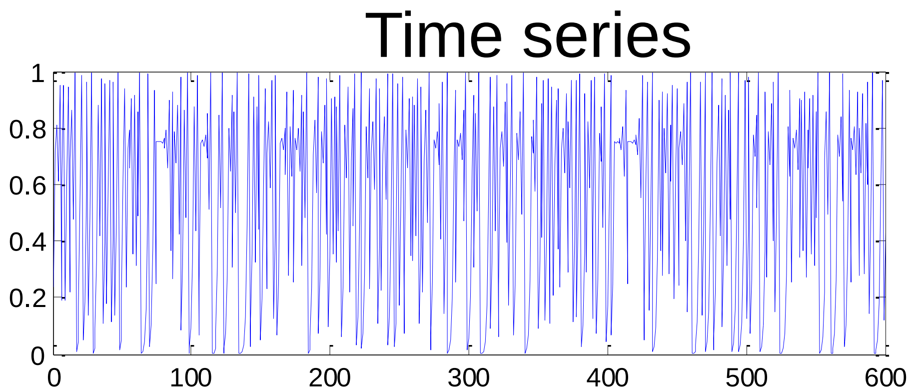
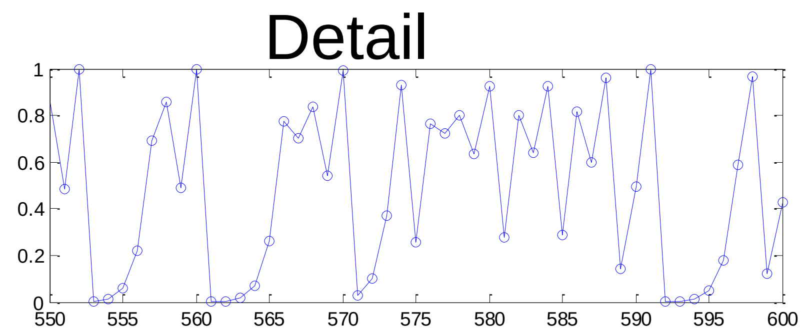
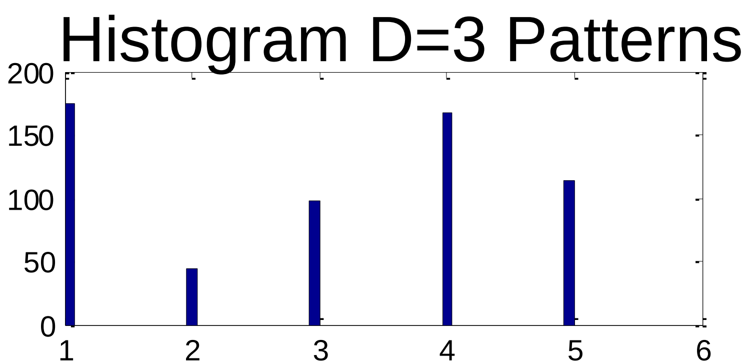
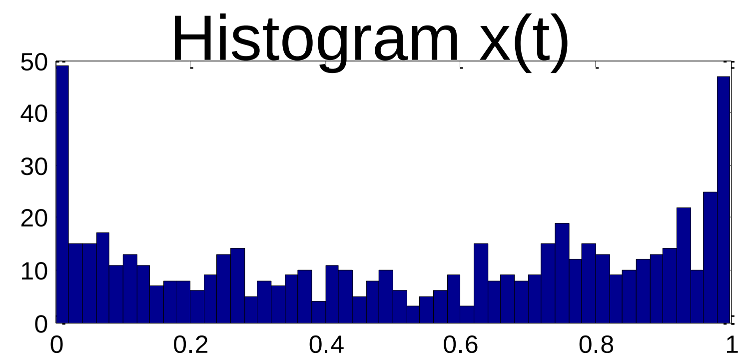
Example: chaotic time series generated with the Logistic map

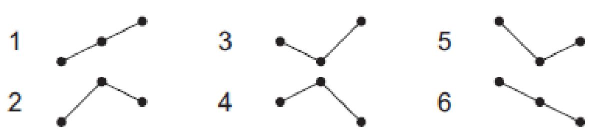
$$x(i+1) = r\, x(i)[1 - x(i)] \quad r=3.99$$











↑
forbidden

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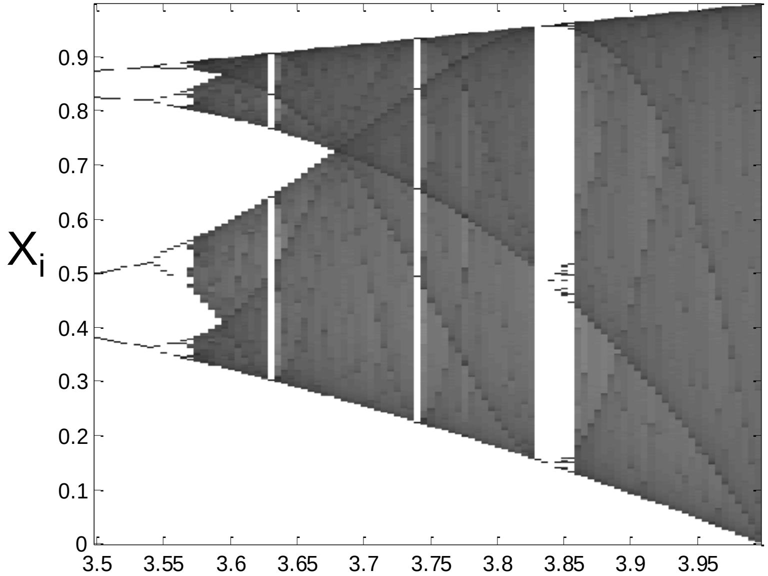
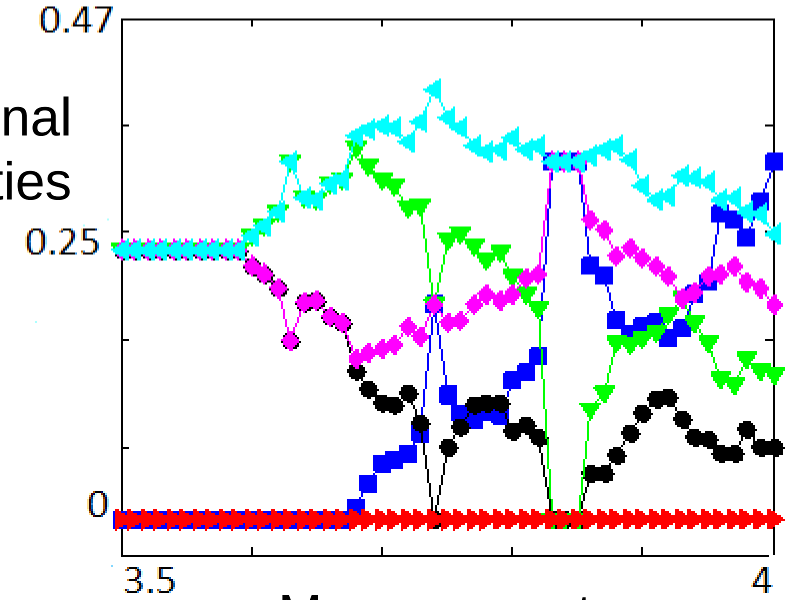
“Normal” and “Ordinal” bifurcation diagrams of the Logistic map

Normal bifurcation diagram



Map parameter

Ordinal diagram with D=3



Ordinal Probabilities

Map parameter, r

012 021 102 120 201 210

Pattern **210** is always forbidden; pattern **012** is more probable as r increases

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Example of application: analysis of ECG signals

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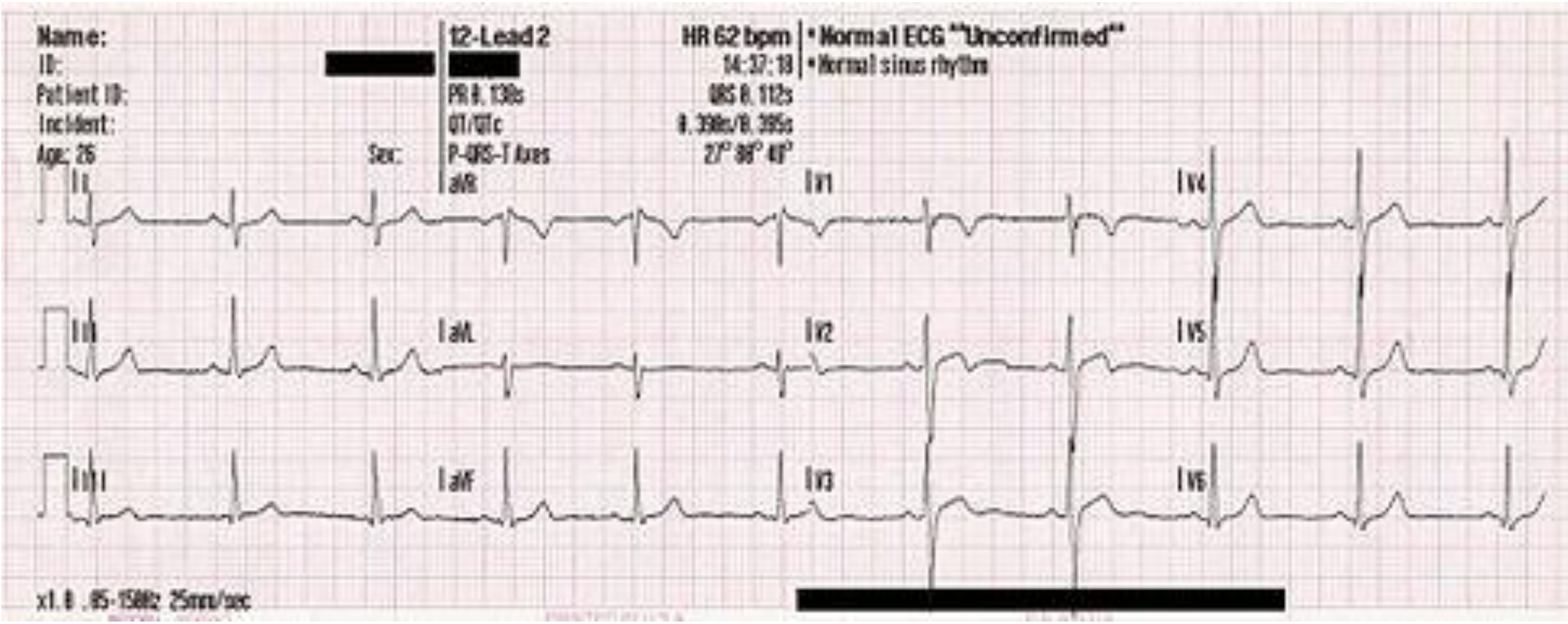
Analysis of sequences of inter-beat intervals



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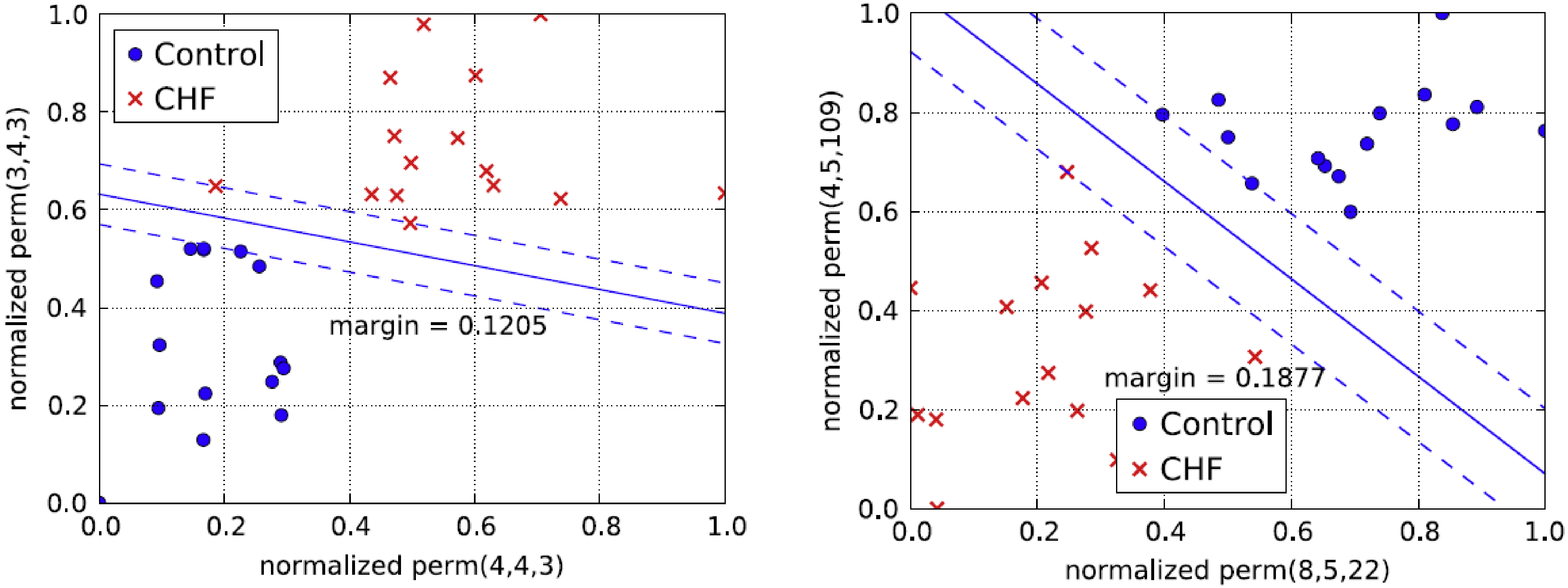

- Analysis of raw data (statistics of ordinal patterns is almost unaffected by anomalies - outliers)
- The probabilities are normalized with respect to the smallest and largest values occurring in the data set.

Ulrich Parlitz & coworkers, Computers in Biology and Medicine 42, 319 (2012).

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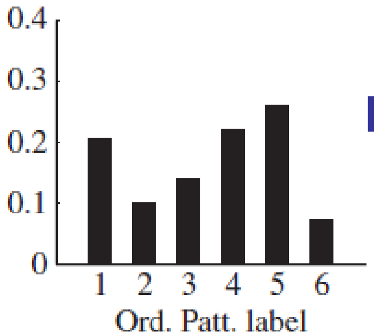
Permutation entropy: Shannon's entropy computed from ordinal probabilities

Time series

Ordinal probabilities



Permutation entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

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Ordinal analysis of two-dimensional patterns

1	2	1
8	3	4
6	7	5

1	2
8	3

1283 $\Rightarrow$0132

2	1
3	4

2134 $\Rightarrow$1023

8	3
6	7

8367 $\Rightarrow$3012

3	4
7	5

3475 $\Rightarrow$0132

Spatial permutation entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

2x2 pixels: 24 possible patterns

Haroldo V. Ribeiro and coworkers, PLoS ONE 7, e40689 (2012)

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Example of application: analysis of liquid crystal textures

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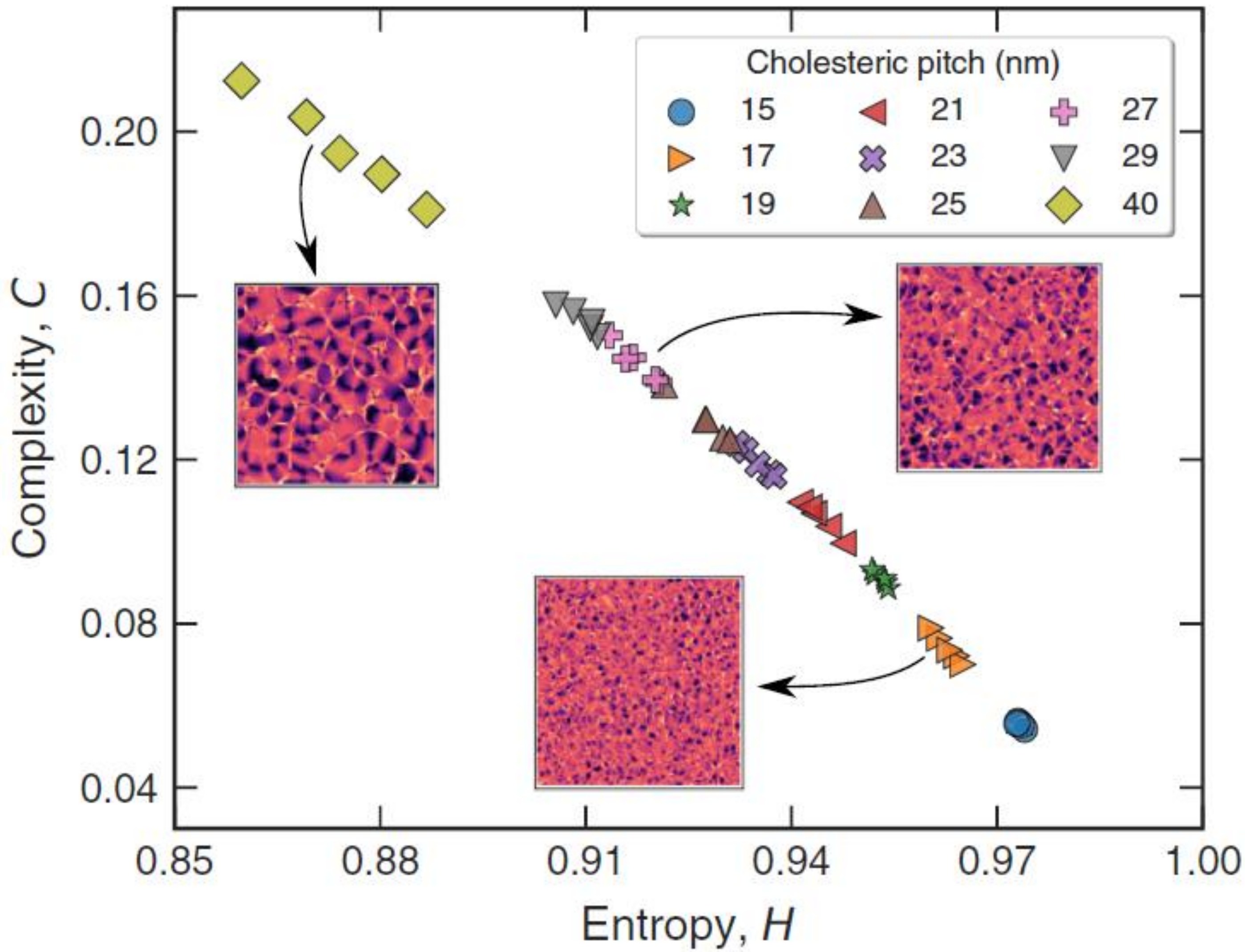

Haroldo V. Ribeiro and coworkers, PRE 99, 013311 (2019)

Can the variation of the permutation entropy give an early indication of an approaching transition?

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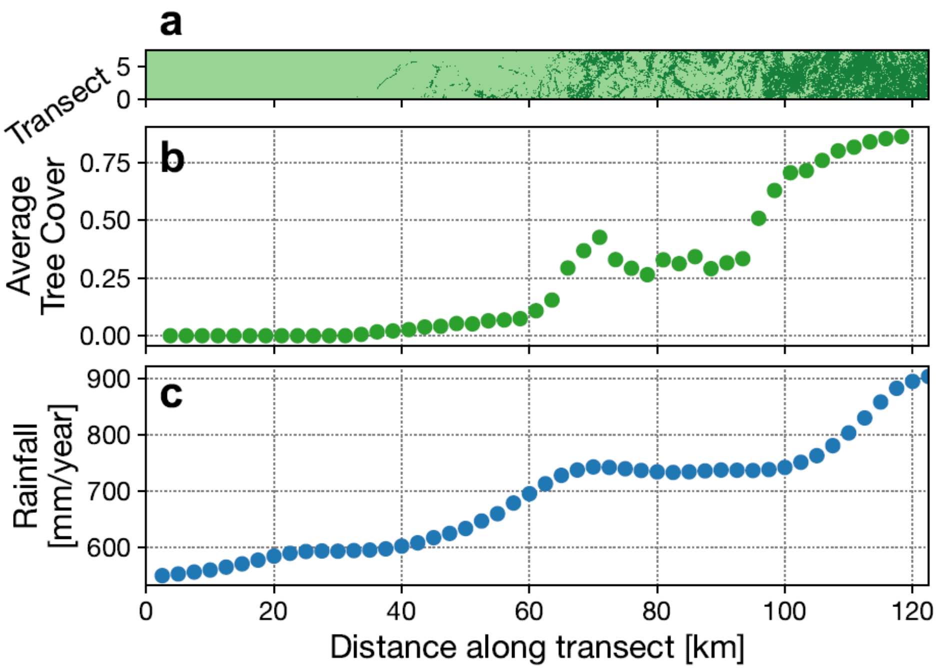
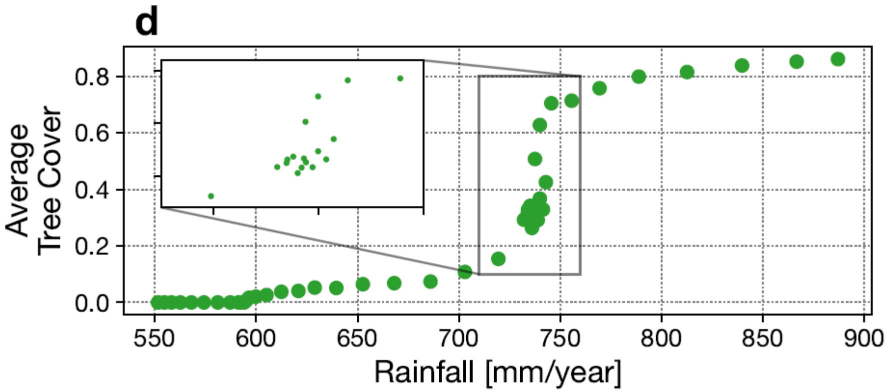
Analysis of high-resolution vegetation data





Data from the Serengeti–Mara ecosystem in northern Tanzania and southern Kenya.

G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2022).

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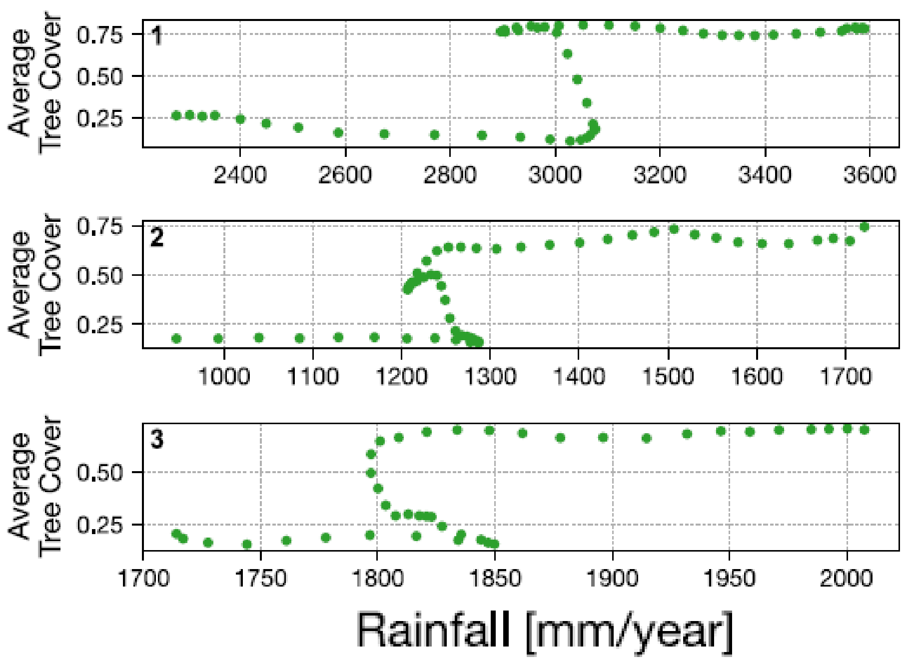
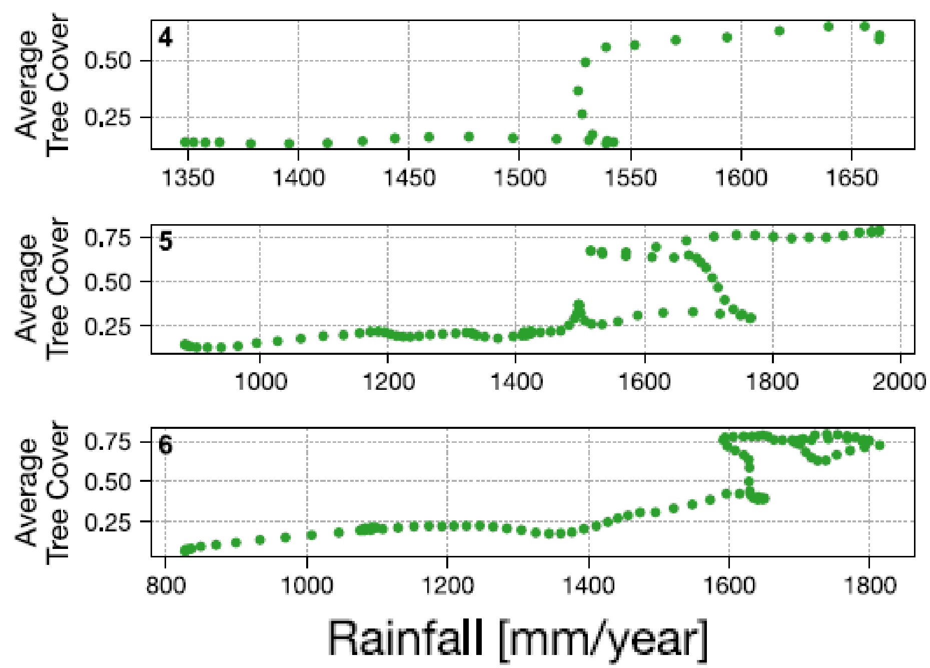
Analysis of low-resolution satellite vegetation data





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Two possible indicators of the desertification transition

Spatial Permutation Entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

Ordinal patterns defined by the values of 2x2 pixels

Spatial correlation

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

w_{ij}=1 if pixels I and j first neighbors, else w_{ij}=0

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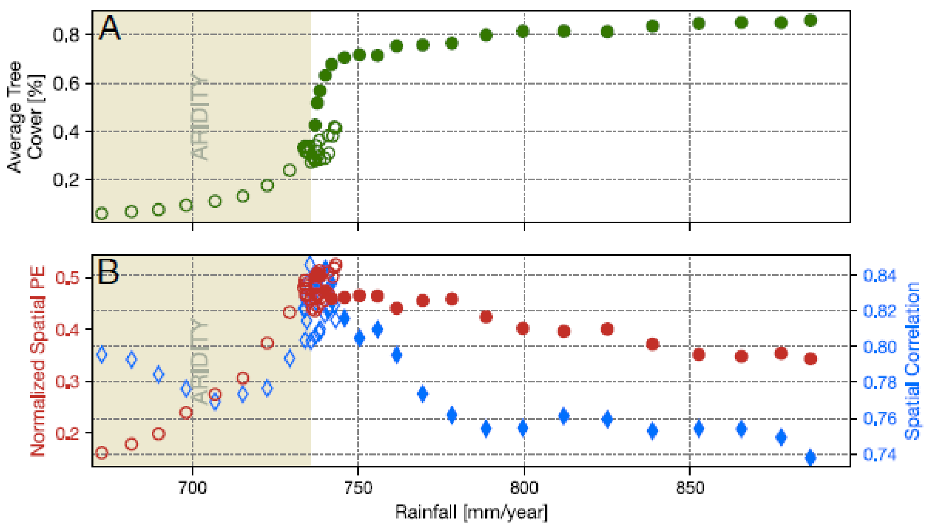
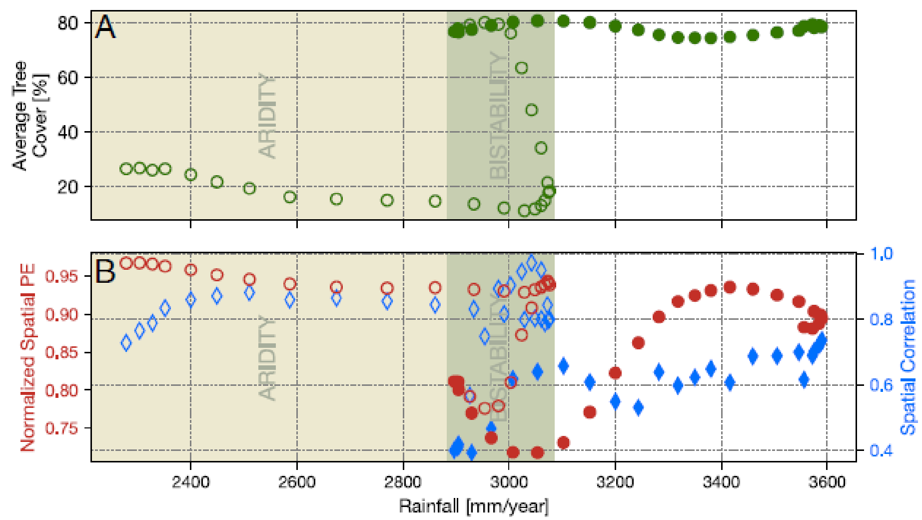
Results

High-resolution data



Low-resolution data

(transect 1; large variability across transects)



G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2022).

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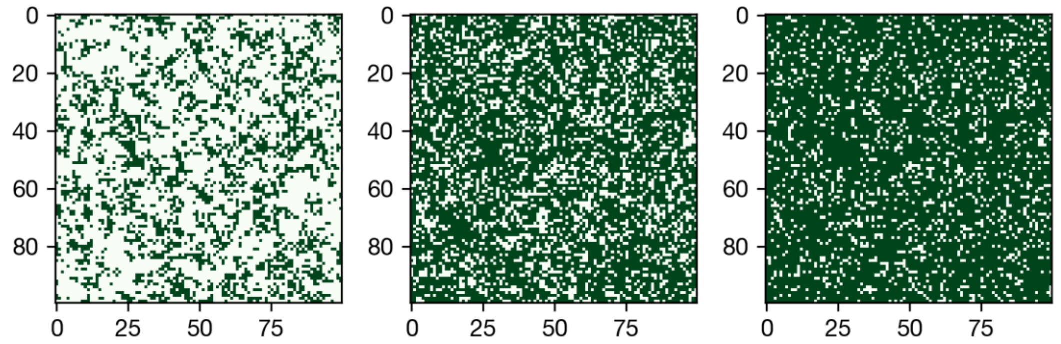
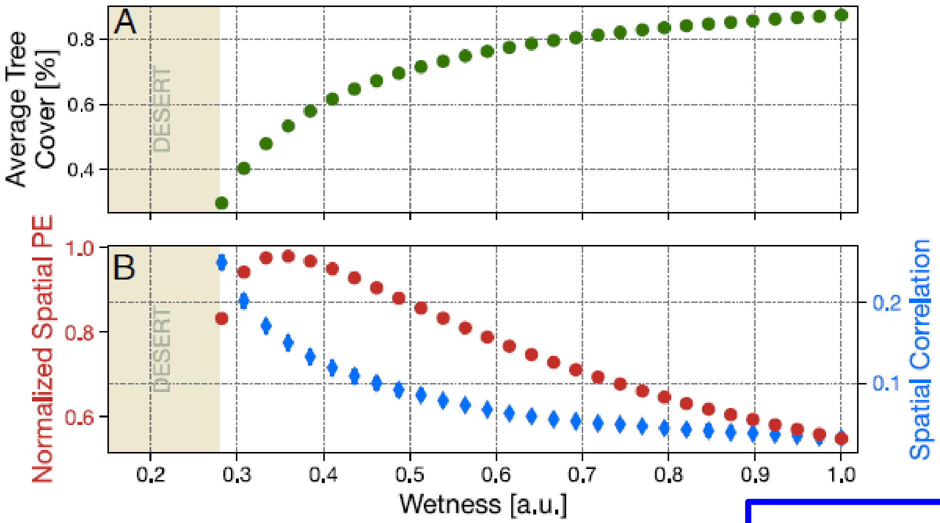
To gain insight: simulations of vegetation models

A) Cellular automata model





$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

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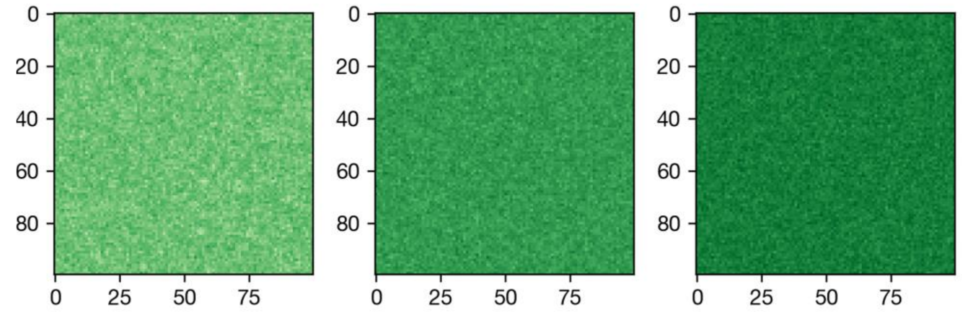
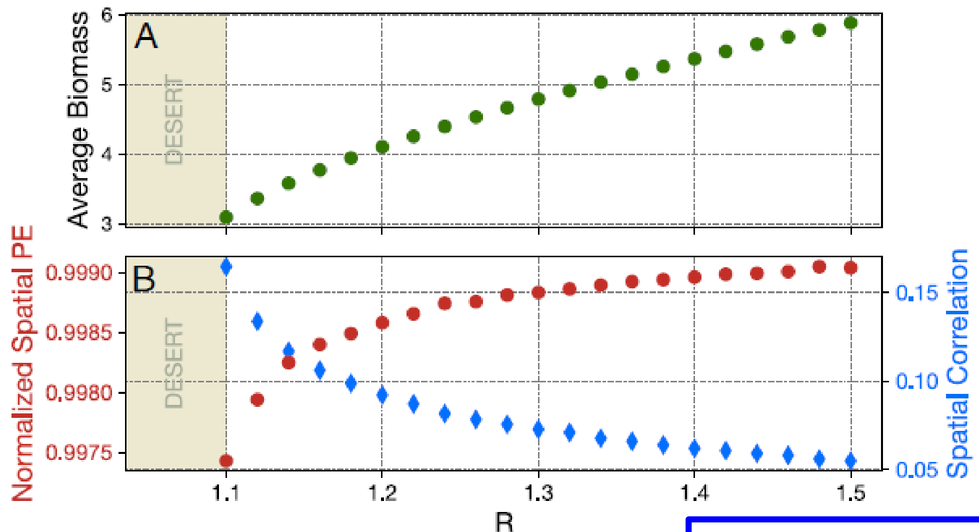
To gain insight: simulations of vegetation models

B) Local Positive Feedback model
(two partial differential equations)





$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

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Can we test the indicator in controlled experimental data?

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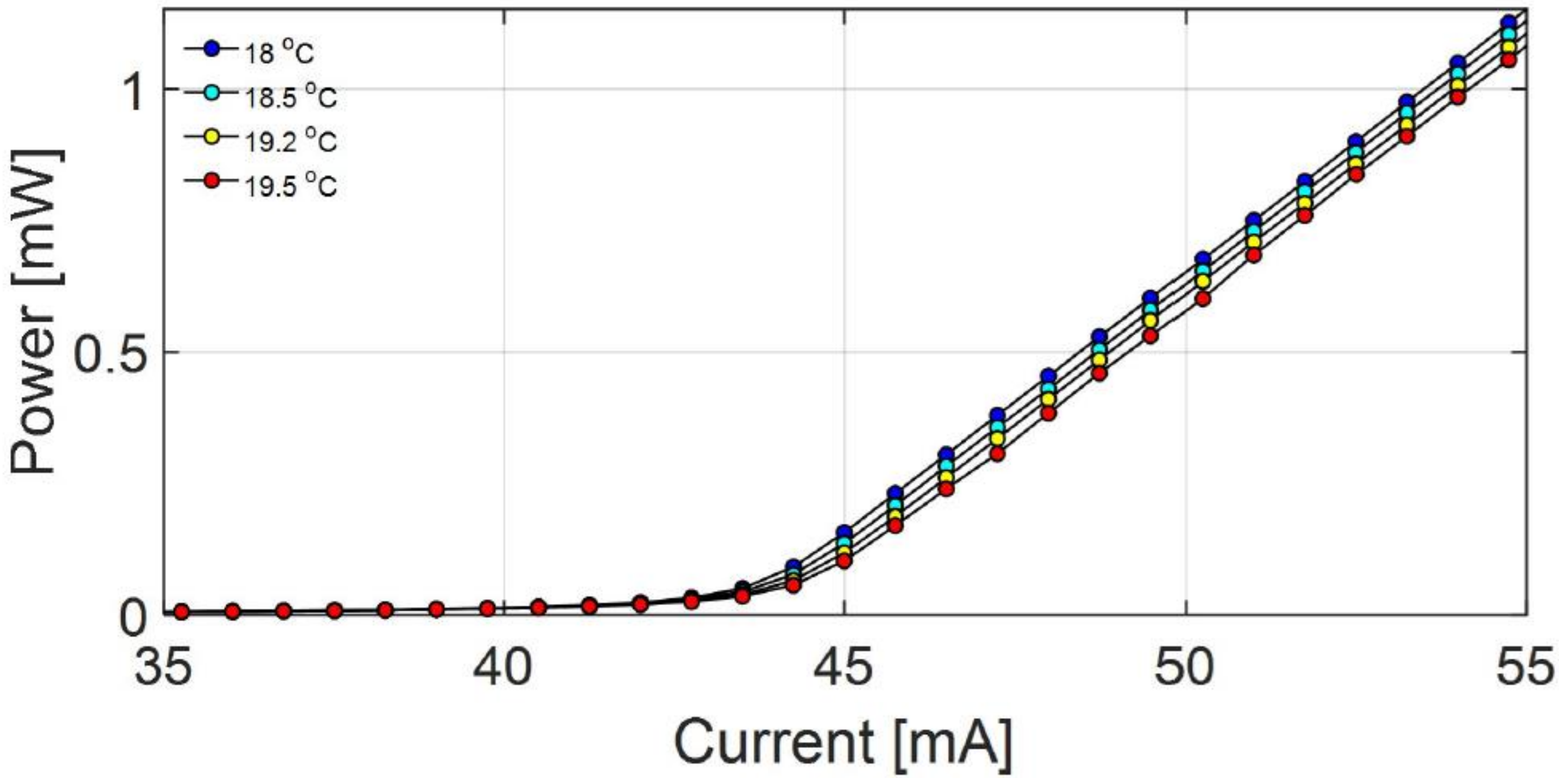
Analysis of the turn-on transition of a diode laser



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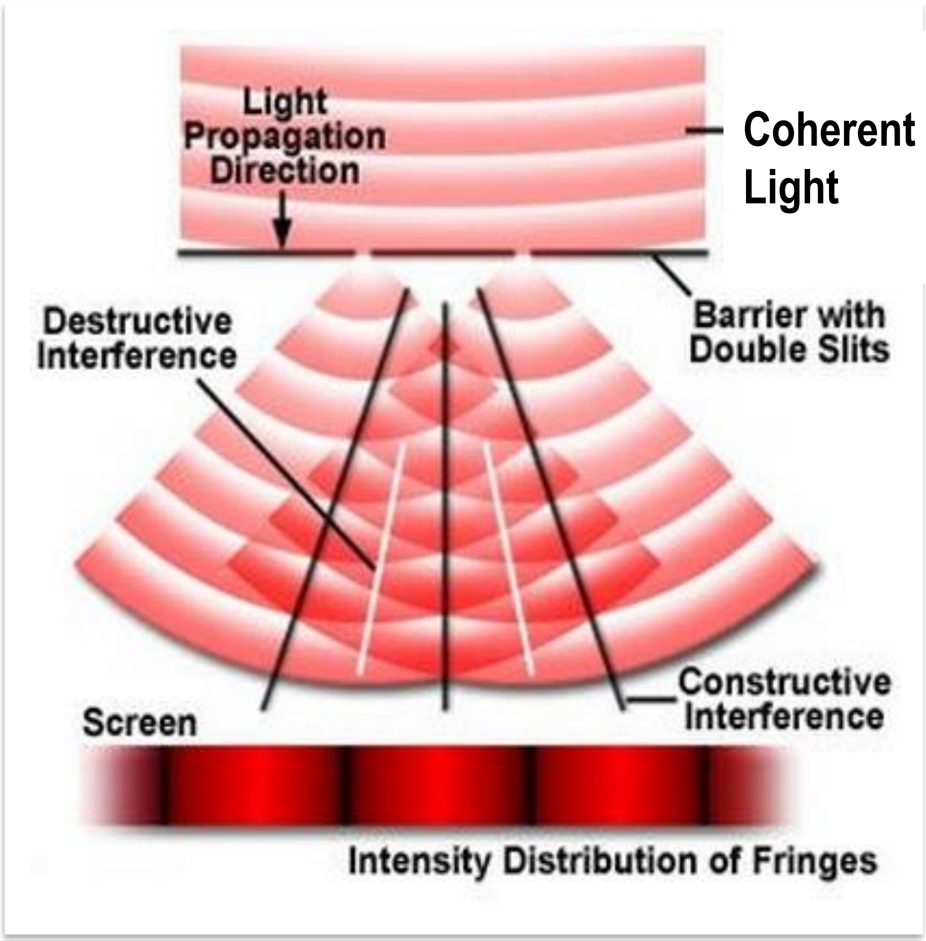
Quick reminder of the interference of coherent waves



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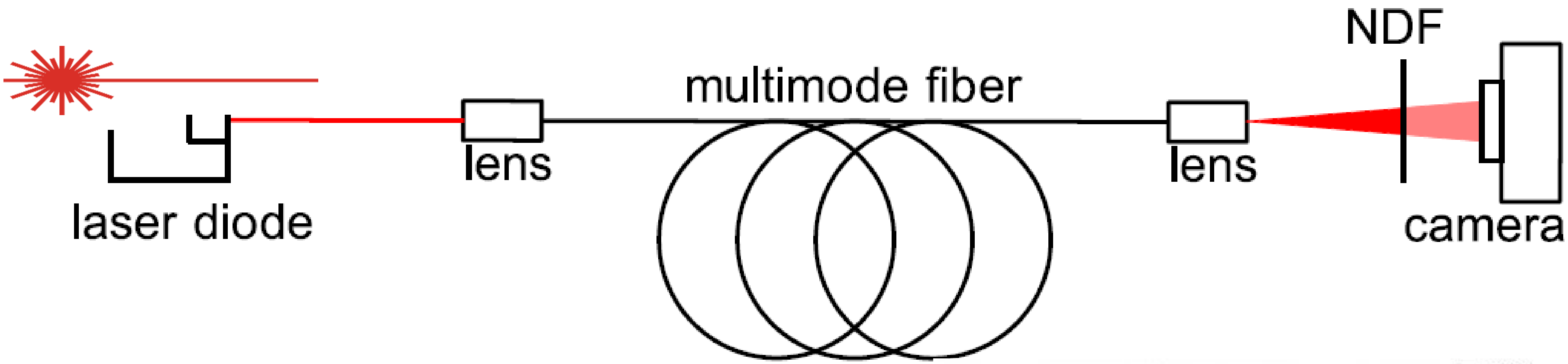
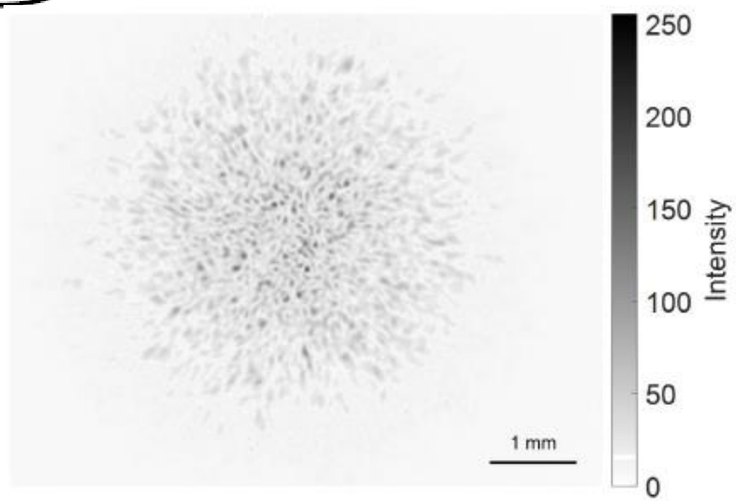
Speckle pattern: generated by the interference / scattering of coherent waves





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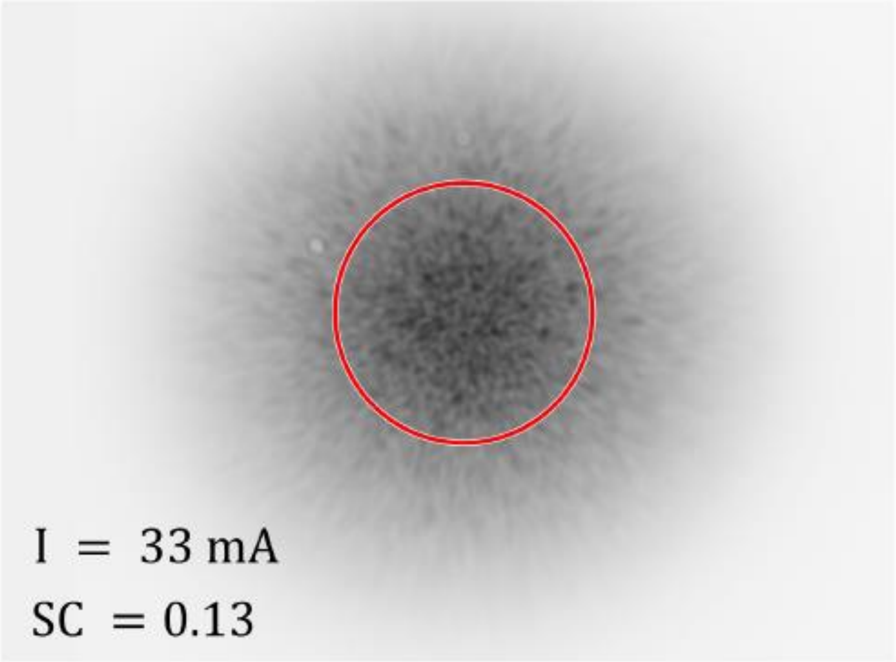
The speckle pattern reveals the level of coherence of the laser light

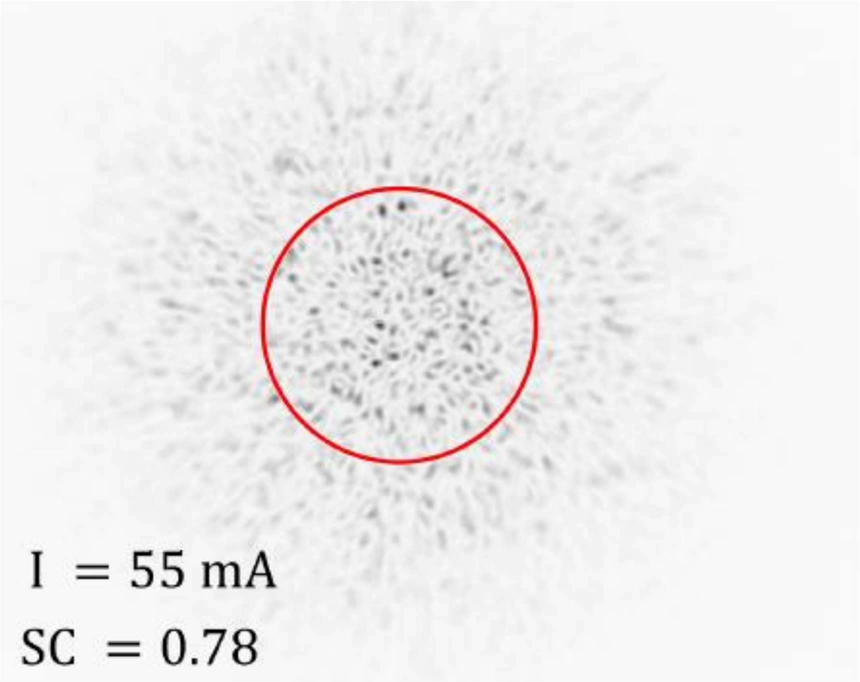
Below threshold

Low light coherence → low speckle contrast



Above threshold

High light coherence→ high speckle contrast



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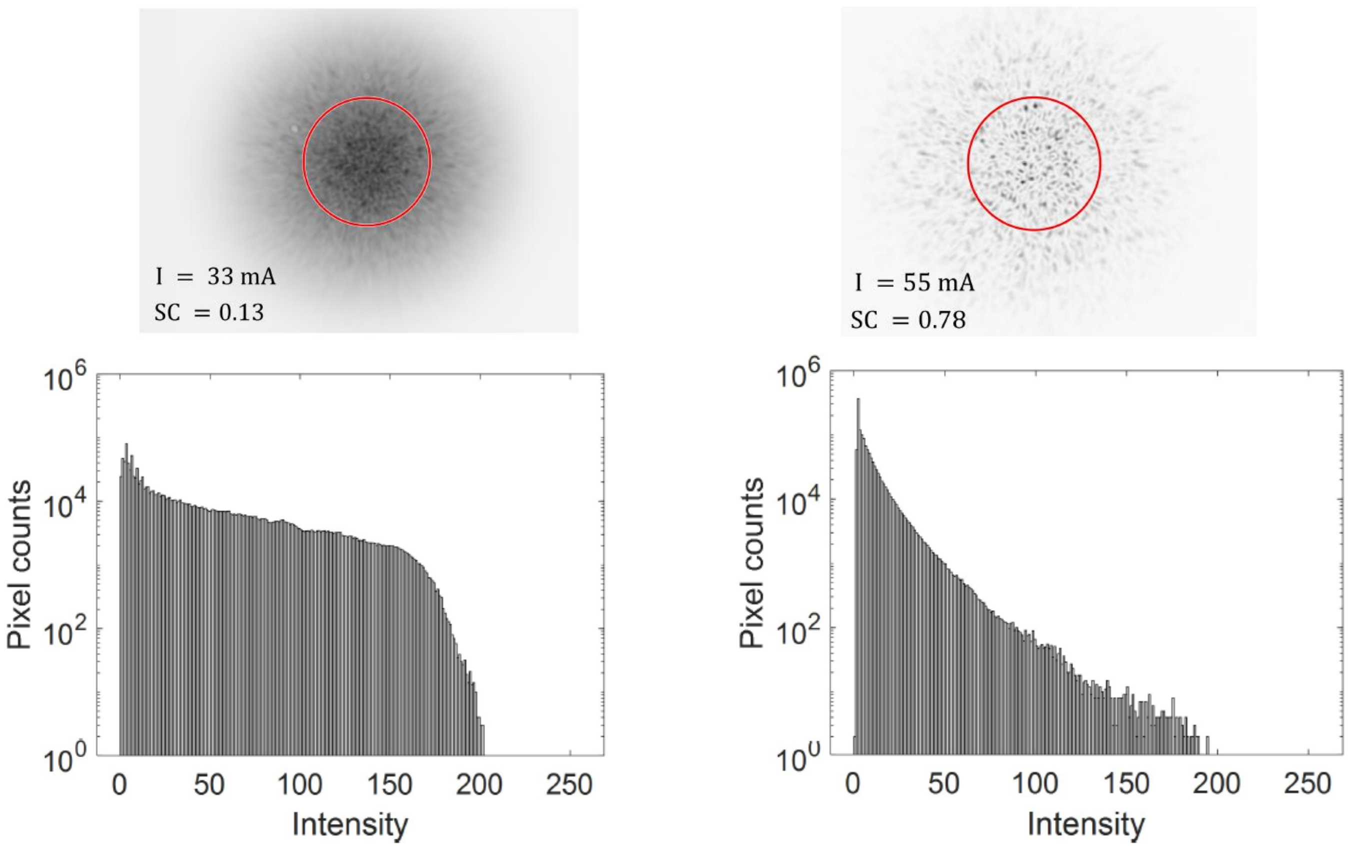
Quantification of speckle contrast



$$\boxed{SC = \sigma/\langle I \rangle}$$

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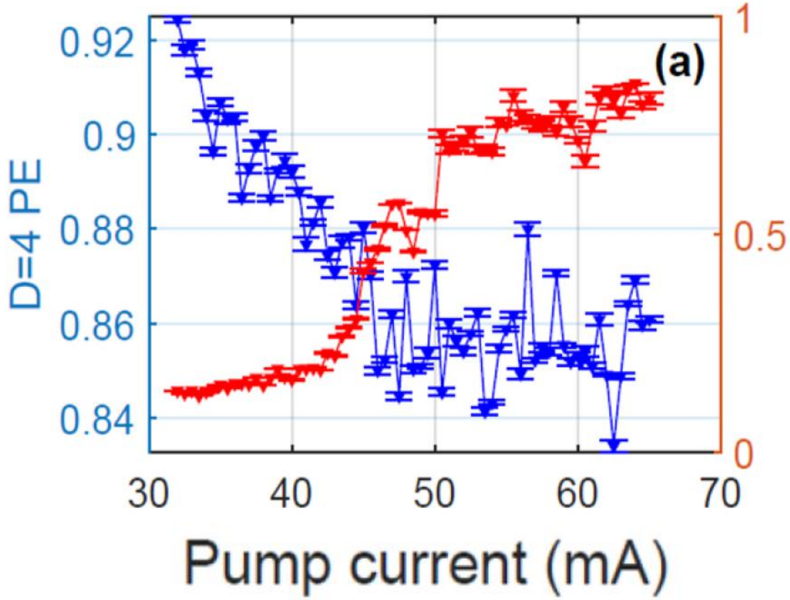
Results

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

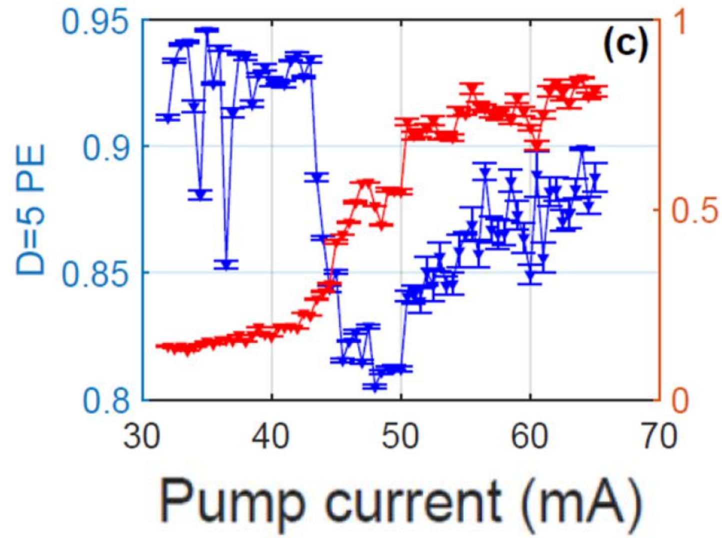
Speckle Contrast

$$SC = \sigma/\langle I \rangle$$

Pattern: x x
x x



Pattern: x
x x x
x



G. Tirabassi et al., "Permutation entropy-based characterization of speckle patterns generated by semiconductor laser light", APL Photonics 8, 126112 (2023).

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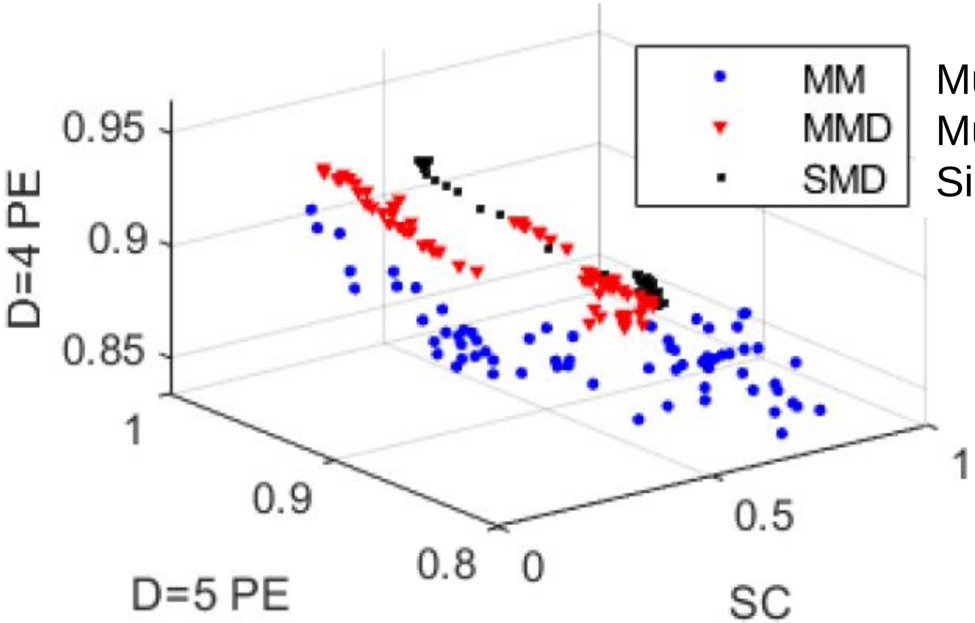
Three features allow to differentiate the speckle patterns according to the type of medium that generated the speckles

$$SC = \sigma/\langle I \rangle$$

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

Pattern: x x
x x

Pattern: x
x x x
x



Multimode fiber
Multimode fiber and diffuser
Single mode fiber and diffuser

Accuracy of a random forest classifier

Solitary laser: 99.4 % ± 0.4 %

Laser with optical feedback: 97.1 % ± 1.3 %

G. Tirabassi et al., "Permutation entropy-based characterization of speckle patterns generated by semiconductor laser light", APL Photonics 8, 126112 (2023).

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Useful for the analysis of more complex, temporal data?

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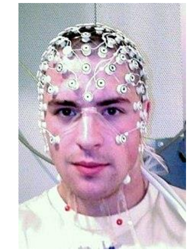
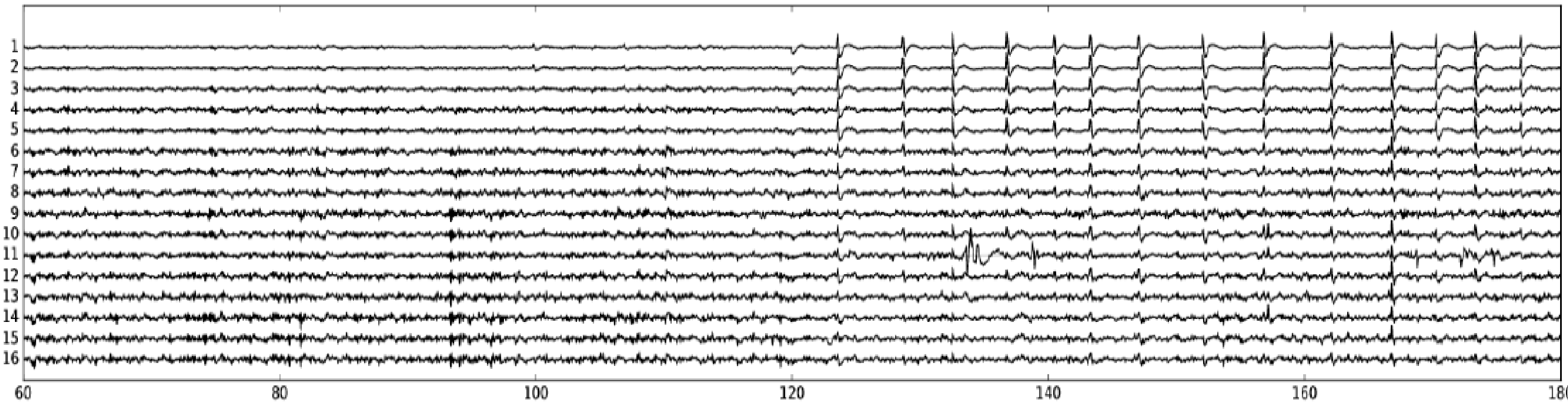
Analysis of EEG signals recorded from healthy subjects.



Eyes closed | Eyes open



Time (seconds)

TABLE I. Description of the datasets used.

	DTS1	DTS2
Sampling rate (Hz)	256	160
Time task (seg)	120	60
Total points	30 720	9600
Number of electrodes	16	64
Number of subjects	71	109

DTS1: Britbrain (Zaragoza)
DTS2: Physionet

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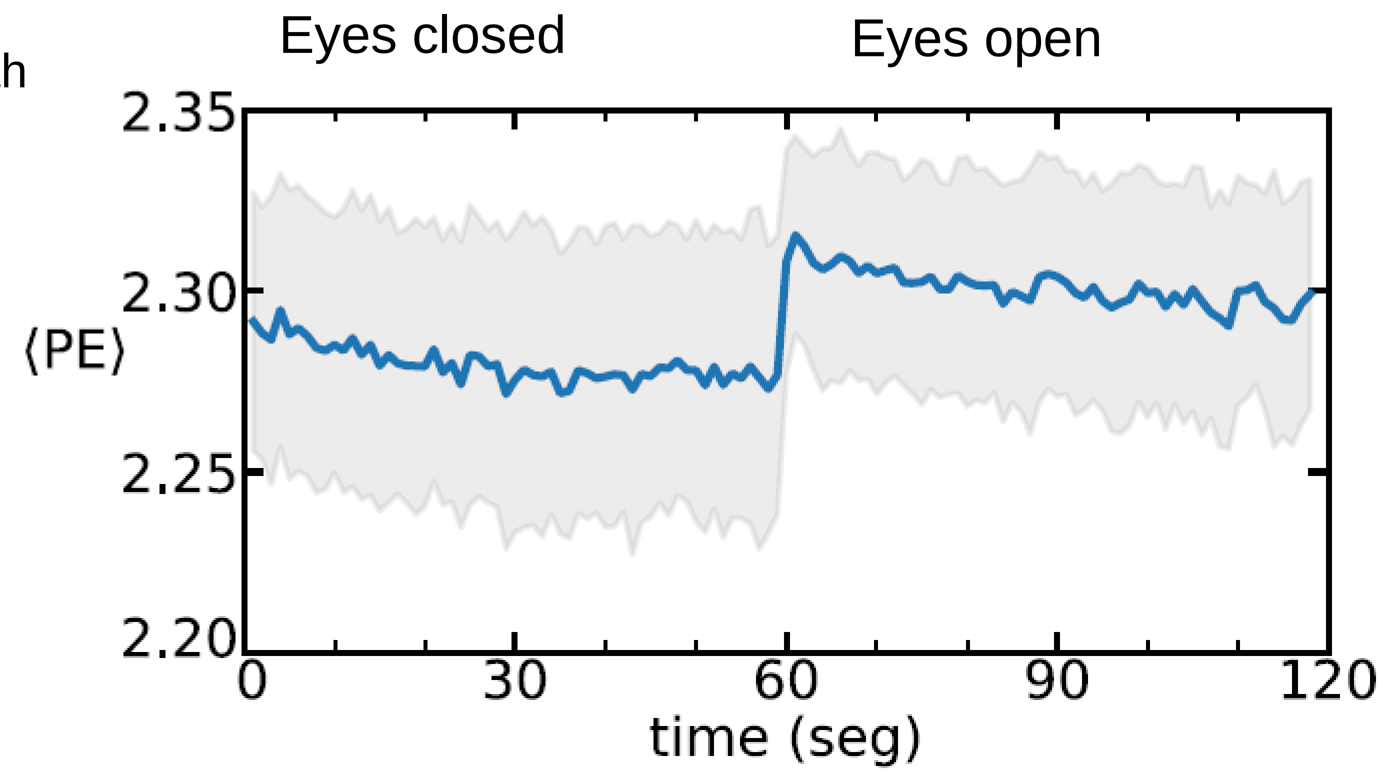
The Permutation Entropy increases in the "eyes open" state

$$\langle \mathrm{PE} \rangle = \frac{1}{N[\text{electrodes}]} \sum_i \mathrm{PE}^i$$

PE was calculated with patterns of length 4 (# of possible patterns 24) in time windows containing >4000 patterns

Gray region: σ of ⟨PE⟩ values across subjects



C. Quintero-Quiroz et al., "Differentiating resting brain states using ordinal symbolic analysis", Chaos 28, 106307 (2018).

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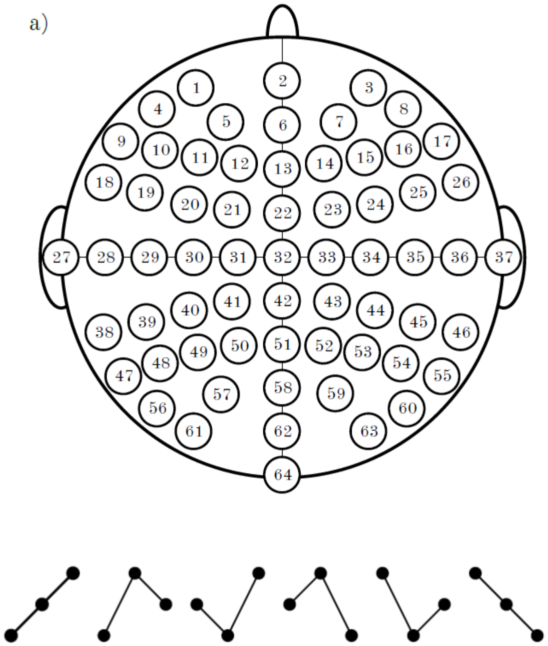
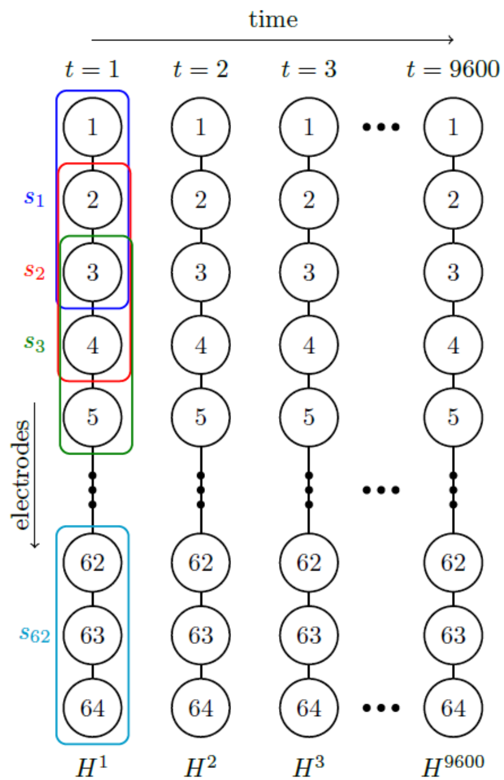
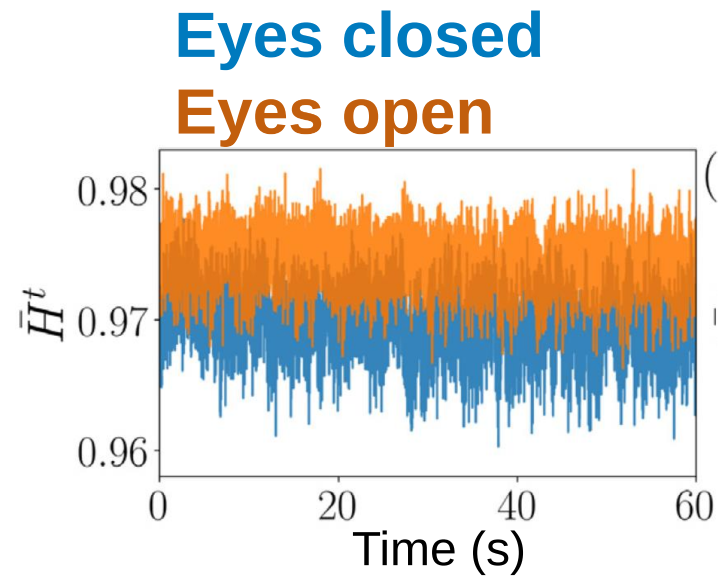
Spatial approach to compute the Permutation Entropy







(H_t is averaged over subjects)

At each time: data values of 64 channels $\Rightarrow$ 62 ordinal patterns to calculate 6 probabilities.

Bruno Boaretto and coworkers, "Spatial permutation entropy distinguishes resting brain states", Chaos, Solitons & Fractals 171, 113453 (2023).

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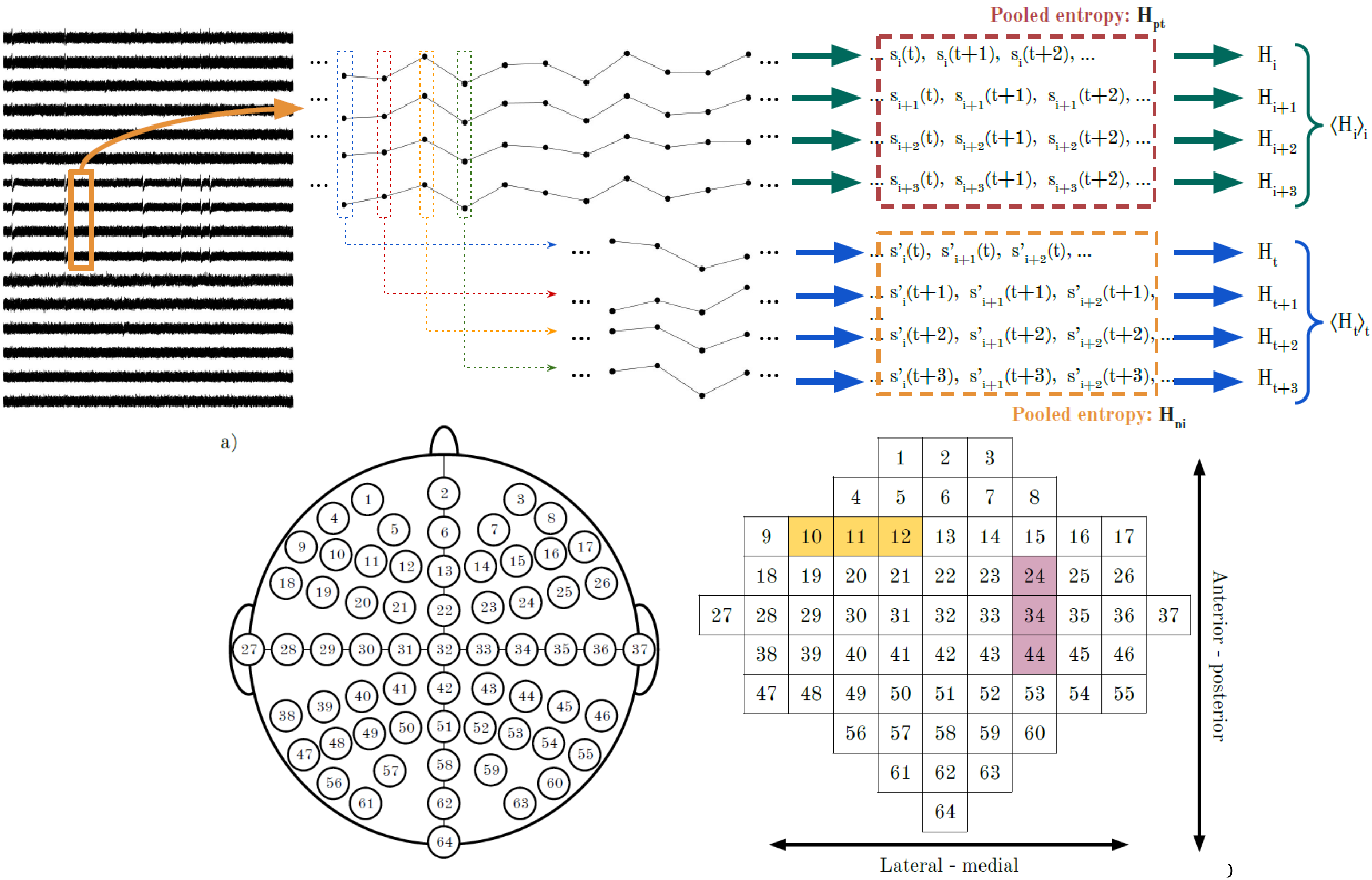
Four approaches to calculate the permutation entropy



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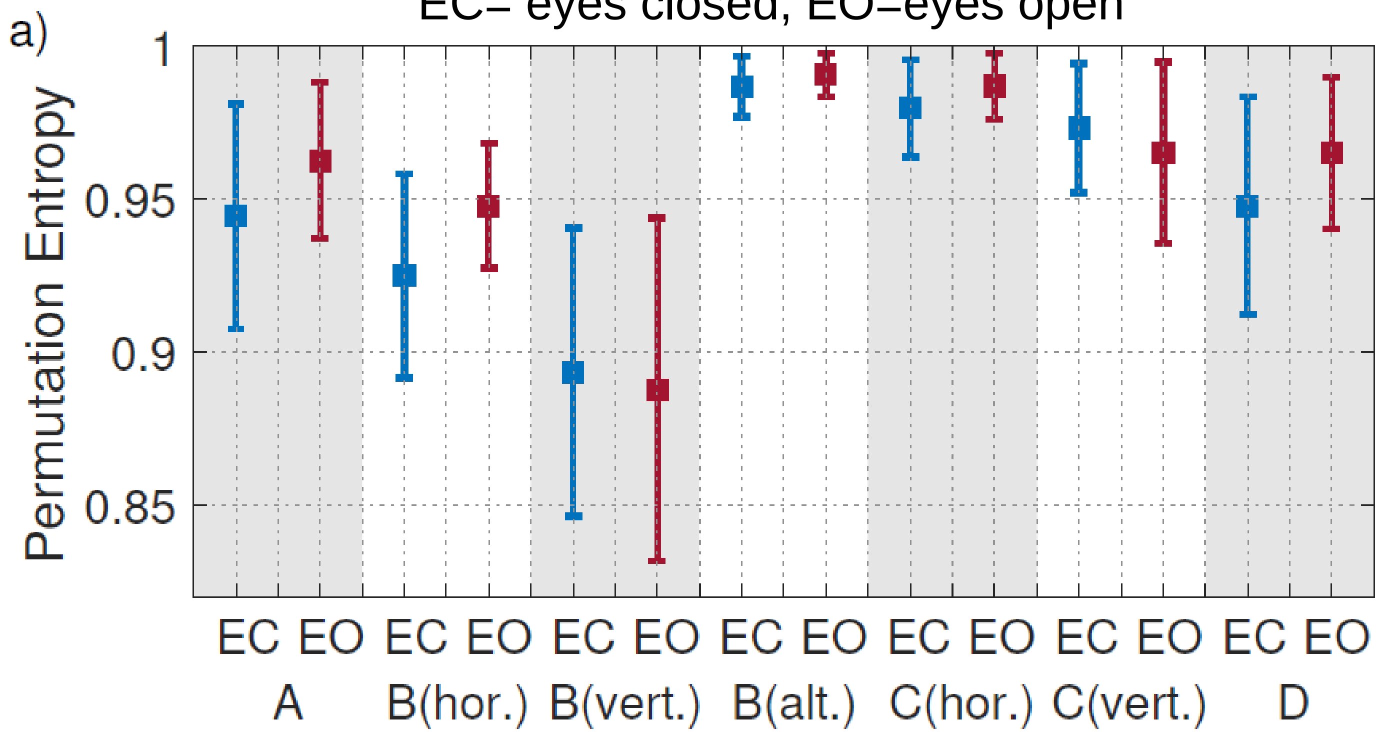
Results

EC= eyes closed, EO=eyes open



error bars:
σ over 107
subjects

A: temporal coding
B: spatial coding (horizontal, vertical, and alt. symbols)
C: spatial pooling (horizontal and vertical symbols)
D: temporal pooling

J. Gancio, C. Masoller, G. Tirabassi, "Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches", Chaos 34, 043130 (2024).

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Random forest classification of eyes open-eyes closed states

		Accuracy	F1 score	Precision	Recall	Specificity
Horizontal	$\langle H_t^s \rangle_t$	61 ± 7	59 ± 10	63 ± 10	57 ± 16	65 ± 16
	$\sigma\left(H_t^s\right)$	66 ± 7	65 ± 9	66 ± 9	$\mathbf{67 \pm 15}$	65 ± 15
	H_{pi}^s	58 ± 8	54 ± 12	61 ± 11	50 ± 16	66 ± 15
Vertical	$\langle H_t^s \rangle_t$	54 ± 9	55 ± 12	54 ± 10	59 ± 17	50 ± 15
	$\sigma\left(H_t^s\right)$	56 ± 9	59 ± 10	56 ± 9	64 ± 15	48 ± 16
	H_{pi}^s	55 ± 9	56 ± 11	55 ± 10	59 ± 17	51 ± 16
Temporal	$\langle H_i^s \rangle_i$	63 ± 8	56 ± 13	70 ± 15	49 ± 16	77 ± 15
	$\sigma\left(H_i^s\right)$	$\mathbf{69 \pm 8}$	$\mathbf{66 \pm 10}$	$\mathbf{73 \pm 12}$	62 ± 14	76 ± 13
	H_{pt}^s	64 ± 8	58 ± 13	72 ± 14	51 ± 16	$\mathbf{78 \pm 14}$

Using filtered data tends to improve the performance.

Performance is as good as that of other statistical measures.

J. Gancio, C. Masoller, G. Tirabassi, "Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches", Chaos 34, 043130 (2024).

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Take home messages and outlook

Ordinal analysis methods allow us to uncover patterns and relationships in data, which can characterize (and sometimes predict) the behavior of complex systems.

1. We have shown, in empirical, experimental and synthetic data, that the permutation entropy may give an indication of an approaching transition.

2. It can be used to find differences in high-dimensional datasets.

3. Ongoing work: synchronization transition, climate data

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- G. Tirabassi and C. Masoller, "*Entropy-based early detection of critical transitions in spatial vegetation fields*", PNAS 120, e2215667120 (2022).
- G. Tirabassi et. Al,*"Permutation entropy-based characterization of speckle patterns generated by semiconductor laser light*", APL Photonics 8, 126112 (2023).
- J. Gancio et. al, "*Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches*", Chaos 34, 043130 (2024).

Thank you for your attention!

AGAUR

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