School on Applications of Nonlinear Systems to Socio-Economic Complexity

ORGANIZERS

LECTURERS

Hilda Cerdeira
(IFT-UNESP, Brazil)

Pablo Balenzuela
(Universidad de Buenos Aires, Argentina)

Cristina Masoller
(Universitat Politécnica de Catalunya)

Marcelo Kuperman
(Instituto Balseiro-Bariloche, Argentina)

Viktoriya Semeshenko
(Universidad de Buenos Aires, Argentina)

ICTP SAIFR
International Centre for Theoretical Physics
South American Institute for Fundamental Research

Secretary: Humberto Neto, Jandira Oliveira

School on Applications of Nonlinear Systems to Socio-Economic Complexity.xlsx : Program

8:30 - 9:15	**Registration**				
9:15 - 9:30	Welcome				
9:30 - 10:15	Masoller 1	Kuperman 2	Semeshenko 3	Masoller 3	Balenzuela 4
10:15 - 11:00	Balenzuela 1	Masoller 2	Balenzuela 3	Kuperman 4	Semeshenko 4
11:00 - 11:30	BREAK	BREAK	BREAK	BREAK	BREAK
11:30 - 12:15	Semeshenko 1	Semeshenko 2	Kuperman 3	Masoller 4	Hands on Balenzuela Semeshenko
12:15 - 13:00	Kuperman 1	Balenzuela 2	Hands on Balenzuela Semeshenko	Hands on Masoller Kuperman	Hands on Balenzuela Semeshenko
13:00 - 14:30	LUNCH	LUNCH	LUNCH	LUNCH	LUNCH

Masoller: Nonlinear time series analysis
Balenzuela: Opinion formation models
Kuperman: Evolutionary game theory
Semeshenko: Economic and financial networks

13:00 - 14:30	LUNCH	LUNCH	LUNCH	LUNCH	LUNCH
14:30 - 15:15	Presentation posters	Hands on Masoller Kuperman	IFT-Colloquium: Marcelo Kuperman **(14:00)**	Hands on Masoller Kuperman	Hands on Balenzuela Semeshenko
15:15 - 17:00	Hands on Masoller Kuperman	Hands on Masoller Kuperman	Hands on Balenzuela Semeshenko **(15:30)**	Hands on Masoller Kuperman	Presentation projects
16:15 - 18:00			Hands on Balenzuela Semeshenko		

The destructive effect of human stupidity:
a revision of Cipolla's fundamental laws

School on Applications of Nonlinear Systems to Socio-Economic Complexity, Oct. 17 – Oct. 22 2022

Nonlinear time series analysis

Cristina Masoller

Departamento de Fisica

Universitat Politecnica de Catalunya

Class 1: From dynamical systems to complex systems

Class 2: Univariate time series analysis

Class 3: Univariate time series analysis

Class 4: Bivariate and multivariate analysis

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Campus d'Excel·lència Internacional

International Centre for Theoretical Physics South American Institute for Fundamental Research

cristina.masoller@upc.edu @cristinamasoll1

Outline

Class 1: From dynamical systems to complex systems

- Dynamical systems
- Bifurcations
- Logistic Map
- Chaotic attractors
- Synchronization
- Kuramoto Model
- Networks

Class 2: Univariate time series analysis

Class 3: Univariate time series analysis

Class 4: Bivariate and Multivariate analysis

cristina.masoller@upc.edu @cristinamasoll1

The beginning of dynamical systems theory

- Mid-1600s: Newtonian mechanics $$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

- **Isaac Newton**: studied planetary orbits and solved analytically the "two-body" problem (earth-sun).

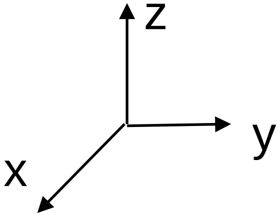
- Since then: a lot of effort for solving analytically the "three-body" problem (earth-sun-moon) – Impossible.

cristina.masoller@upc.edu @cristinamasoll1

Late 1800s: Henri Poincare (French mathematician)

- Instead of asking "*which are the exact positions of the planets (trajectories)*?"

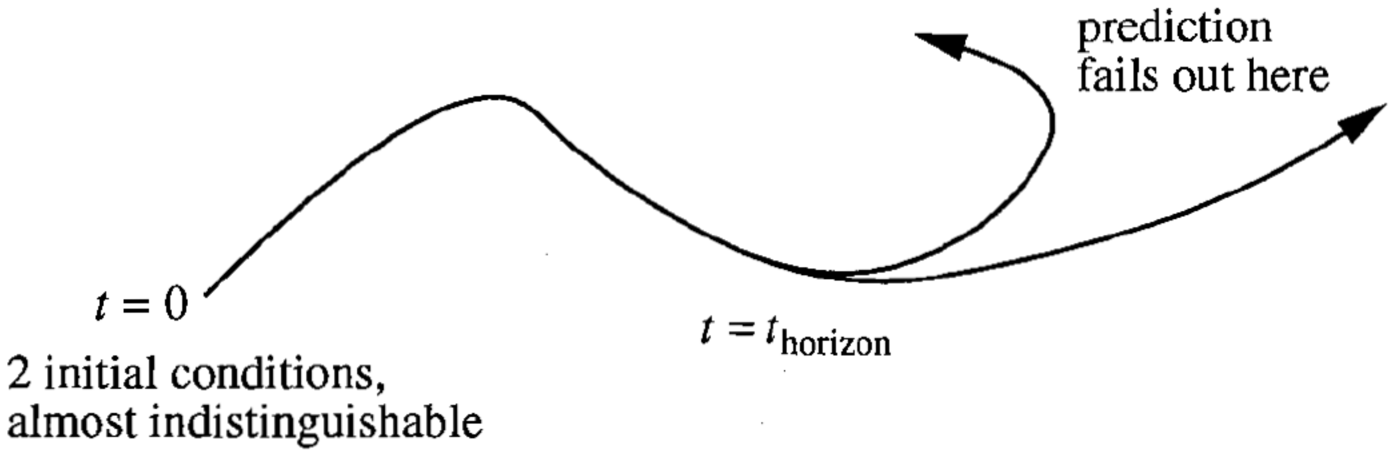
  he asked: "*is the solar system* **stable** *for ever, or will planets eventually run away*?"
- He developed a **geometrical** approach to solve the problem.
- Introduced the concept of "**phase space**".
- Search for structures that divide the phase space into regions where "trajectories" have quantitatively different behavior.



- *Poincaré recurrence theorem*: certain systems will, after a sufficiently long but finite time, return to a state very close to the initial state.
- He also had the intuition of the possibility of chaos.

cristina.masoller@upc.edu @cristinamasoll1

***Poincare**: “The evolution of a **deterministic** system can be aperiodic, unpredictable, and strongly depends on the initial conditions”.*



Deterministic system: the initial conditions fully determine the future state.

Deterministic **chaotic** system: there is no randomness but the system can be, in the long term, unpredictable.

A problem in time series analysis: How to determine the prediction horizon? How to estimate the uncertainty?

cristina.masoller@upc.edu @cristinamasoll1

1950s: First computer simulations

- Computes allowed to experiment with equations.
- Huge advance in the field of "*Dynamical Systems*".
- 1960s: **Eduard Lorenz** (American mathematician and meteorologist at MIT): simple model of convection rolls in the atmosphere.

$$\frac{dx}{dt} = -\sigma x + \sigma y,$$
$$\frac{dy}{dt} = -xz + rx - y,$$
$$\frac{dz}{dt} = xy - bz.$$

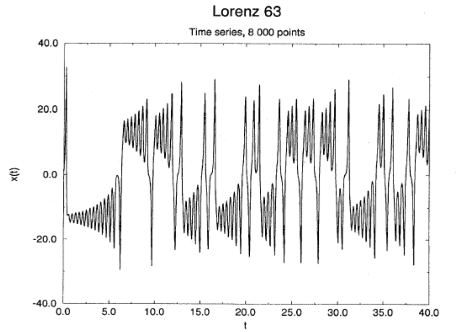
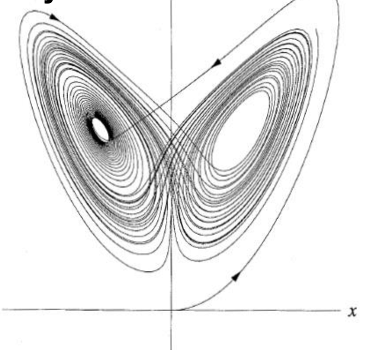


FIG. 1. Chaotic time series $x(t)$ produced by Lorenz (1963) equations (11) with parameter values $r=45.92$, $b=4.0$, $\sigma=16.0$.

2D projection of 3D attractor



- Most famous **chaotic** attractor.

cristina.masoller@upc.edu @cristinamasoll1

Lorentz describing deterministic chaos:

The present determines the future.

But

The approximate present does not approximately determine the future.

Which system may be chaotic?

Continuous dynamical systems described by 3 or more ordinary differential equations.

Problems in time series analysis: How to quantify chaos? How to distinguish chaos from noise?

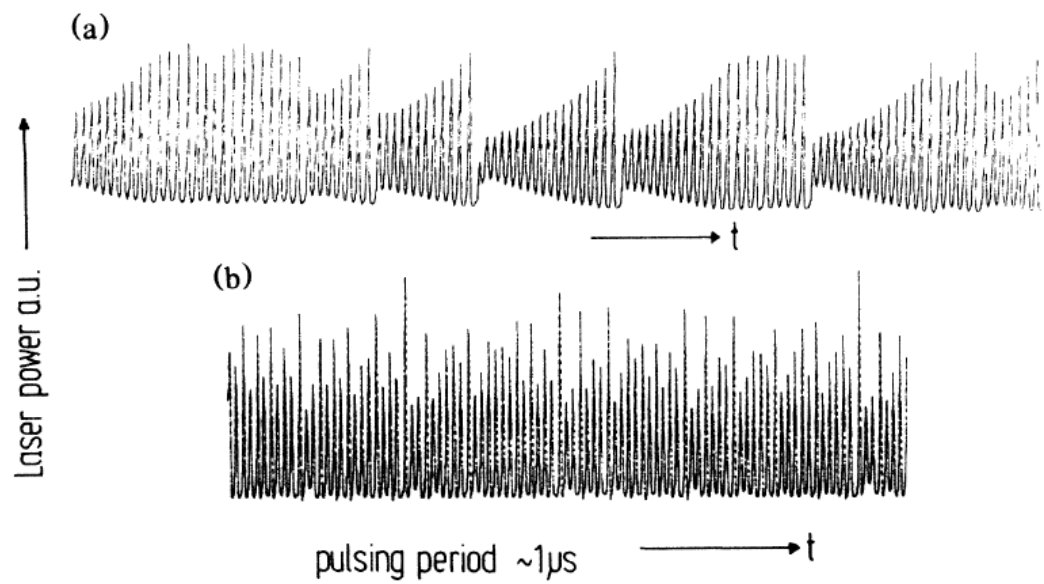
cristina.masoller@upc.edu @cristinamasoll1

Can we observe chaos experimentally?

VOLUME 57, NUMBER 22 PHYSICAL REVIEW LETTERS 1 DECEMBER 1986

Evidence for Lorenz-Type Chaos in a Laser

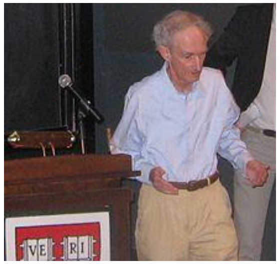
C. O. Weiss and J. Brock[a]
Physikalisch-Technische Bundesanstalt, D-3300 Braunschweig, Federal Republic of Germany
(Received 18 April 1986)



optically pumped NH_3 laser

cristina.masoller@upc.edu @cristinamasoll1

The 1970s



- **Robert May** (Australian, 1936): population biology
- "Simple mathematical models with very complicated dynamics", *Nature* (1976).

$$x_{t+1} = f(x_t)$$

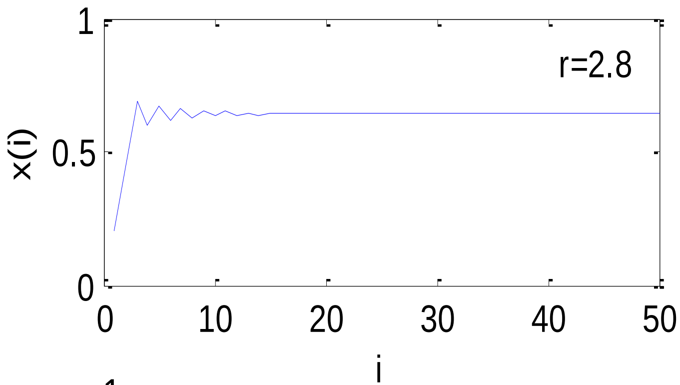
A classical example: **The Logistic map** $f(x) = r\,x(1-x)$

$x \in (0,1)$, $r \in (0,4)$

- Difference equations ("iterated maps"), in spite of being simple and deterministic, can exhibit: **stable points**, **stable cycles**, and **apparently random fluctuations**.

cristina.masoller@upc.edu @cristinamasoll1

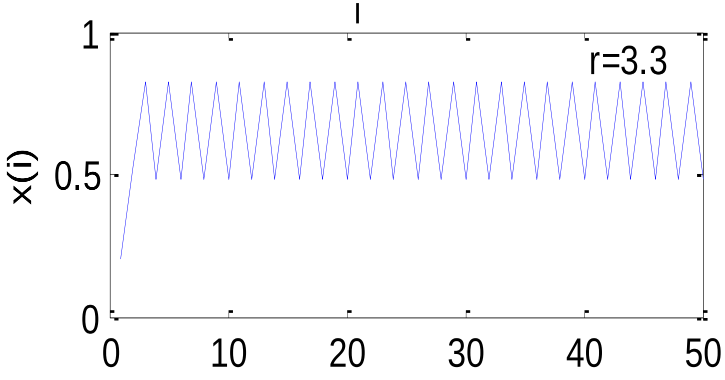
The logistic map: $x(i+1) = r\, x(i)[1 - x(i)]$ $x \in (0,1)$, $r \in (0,4)$



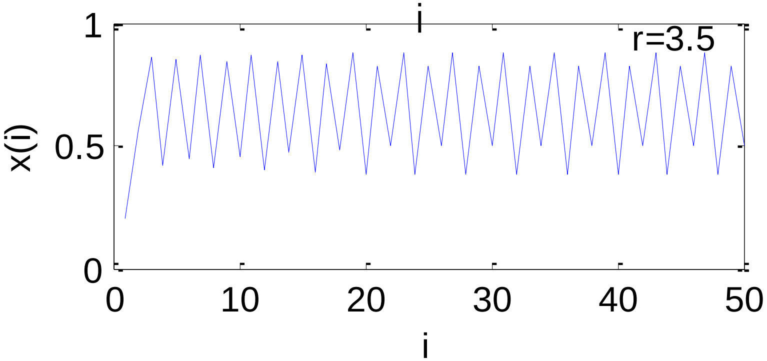
r=2.8, Initial condition: x(1) = 0.2

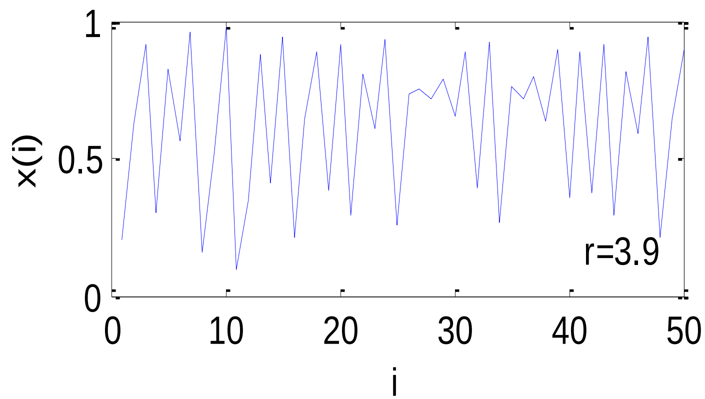
Transient relaxation → long-term stability

The fixed point is the solution of: $x = r\, x\, (1-x) \Rightarrow x = 1 - 1/r$



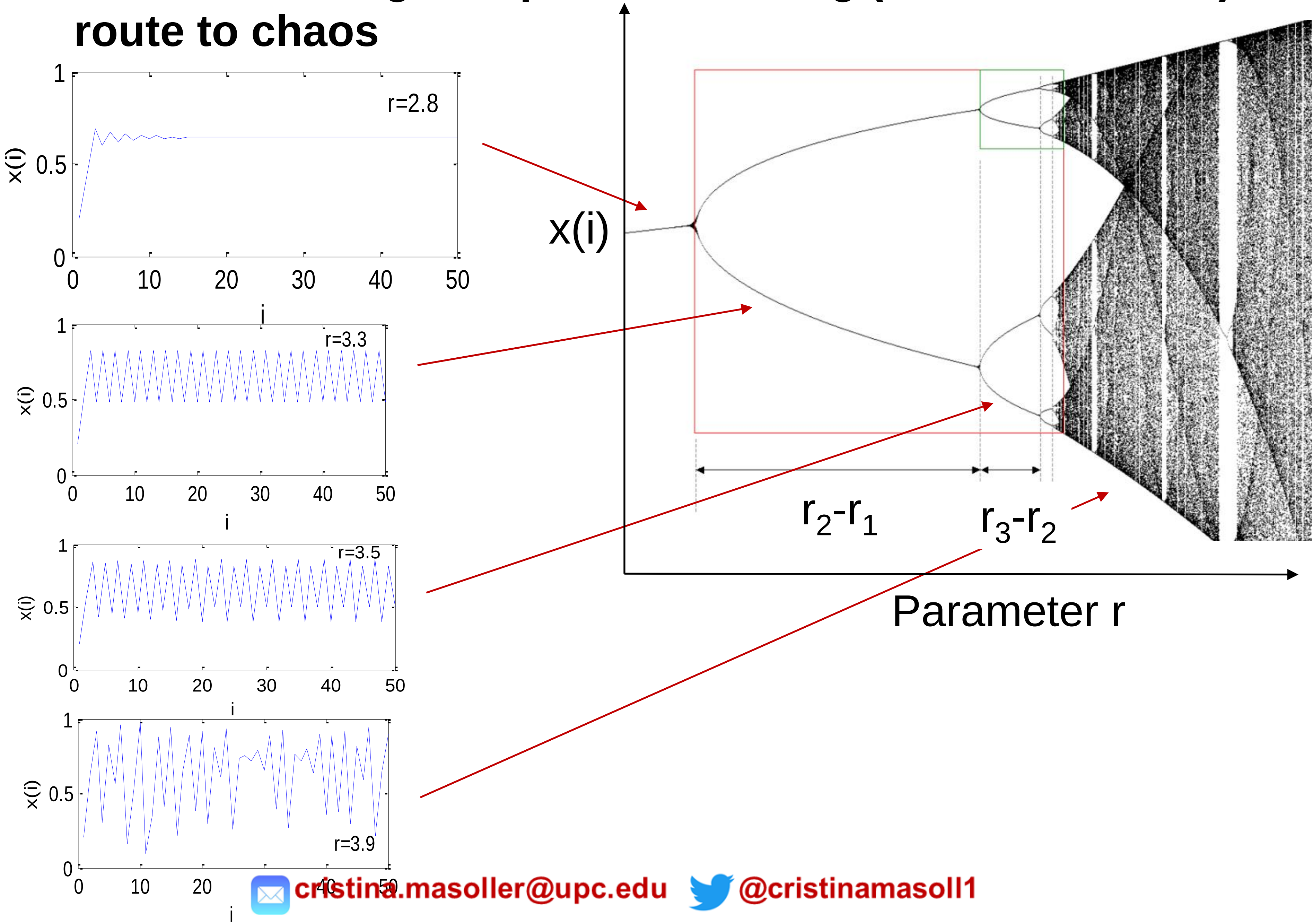
Transient dynamics → oscillations (regular or irregular)





cristina.masoller@upc.edu @cristinamasoll1

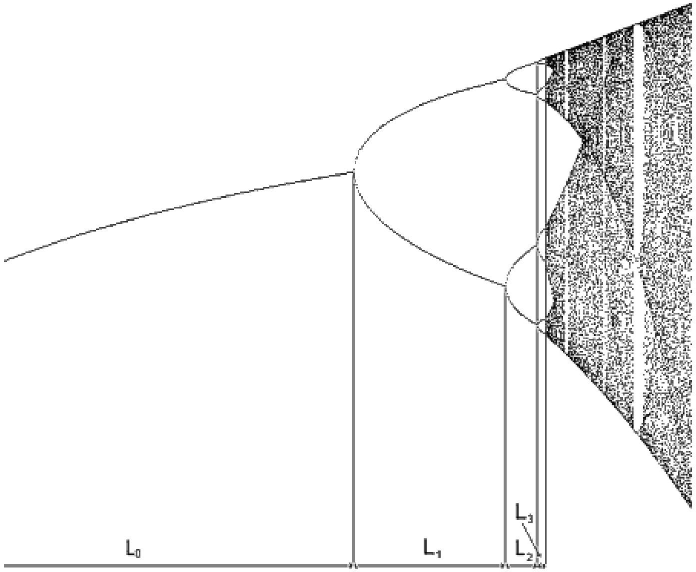
Bifurcation diagram: period-doubling (or subharmonic) route to chaos



cristina.masoller@upc.edu @cristinamasoll1

Order within chaos (1975)

M. Feigenbaum (American mathematician and physicist 1944-2019), using a small HP-65 calculator, discovered "hidden" order in the route to chaos: the <u>scaling of the bifurcation points of the Logistic map.</u>



$$\delta = \lim \frac{L_i}{L_{i+1}} = 4.669201...$$

HP-65 calculator: the first magnetic card-programmable handheld calculator

cristina.masoller@upc.edu @cristinamasoll1

A universal law

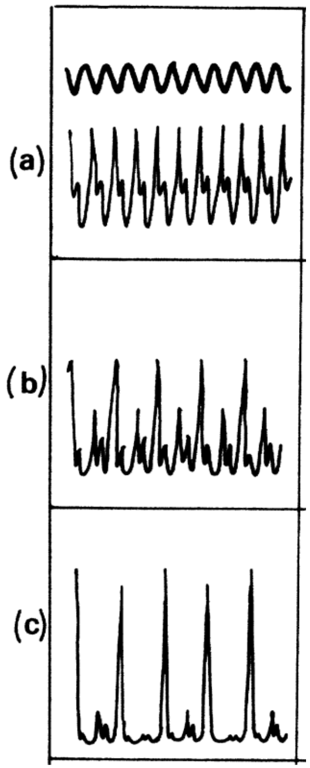
Feigenbaum demonstrated that the same behavior, with the same mathematical constant (δ=4.6692…), occurs for a wide class of functions. $x_{t+1} = f(x_t)$

$\Rightarrow$ Very different systems (in chemistry, biology, physics, etc.) go to chaos in the same way, quantitatively.

Can we observe the period doubling route experimentally?

(about 10 years later) With a modulated laser, keeping constant the modulation frequency and increasing modulation amplitude.

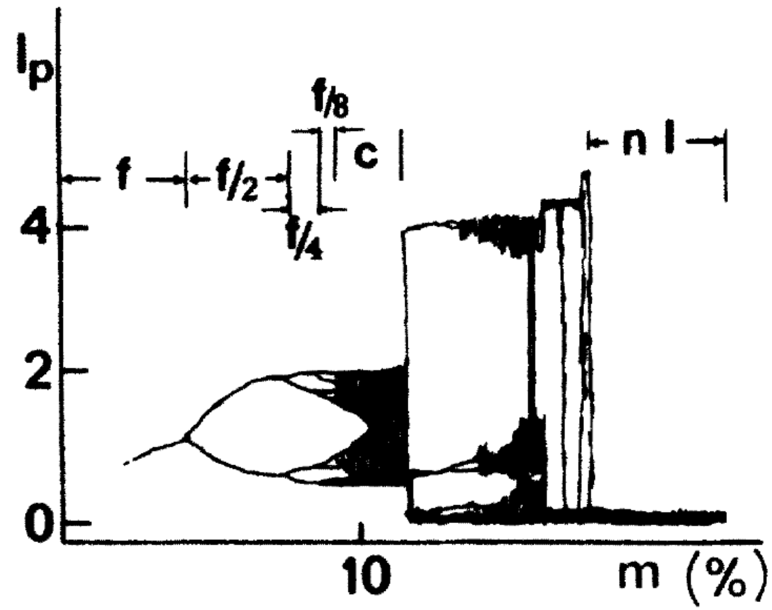
Laser output intensity



m (%)

Time

J. R. Tredicce et al, Phys. Rev. A 34, 2073 (1986).



Problems in time series analysis:

- *How to identify an approaching bifurcation point (tipping point)?*
- *How to distinguish transient from non-transient behavior?*

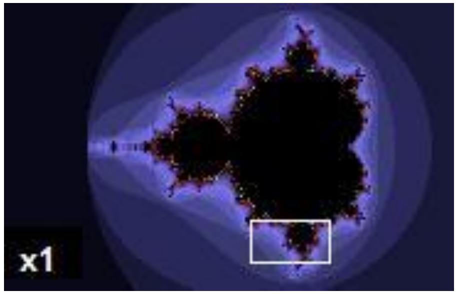
cristina.masoller@upc.edu @cristinamasoll1

The late 1970s

- **Benoit B. Mandelbrot** (Polish-born, French and American mathematician 1924-2010): “self-similarity” and **fractal objects:**

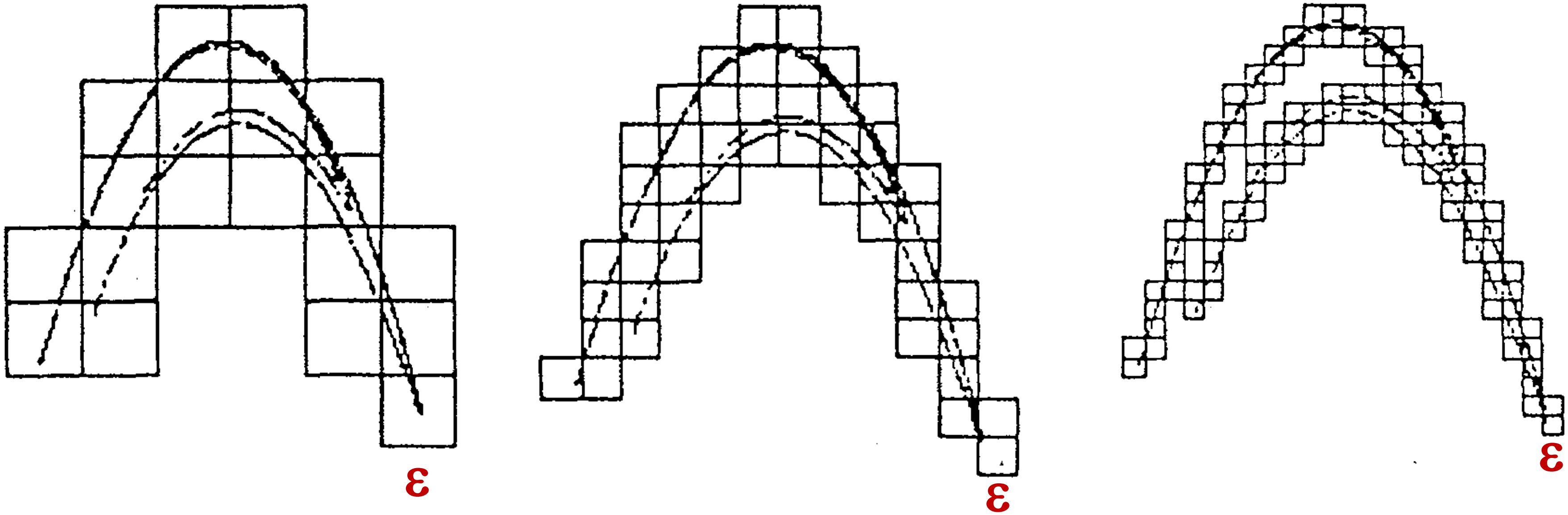
  each part of the object is like the whole object but smaller.

- Because of his access to IBM's computers, Mandelbrot was one of the first to use **computer graphics** to create and display fractal geometric images.



cristina.masoller@upc.edu @cristinamasoll1

How to estimate the dimension of a fractal?



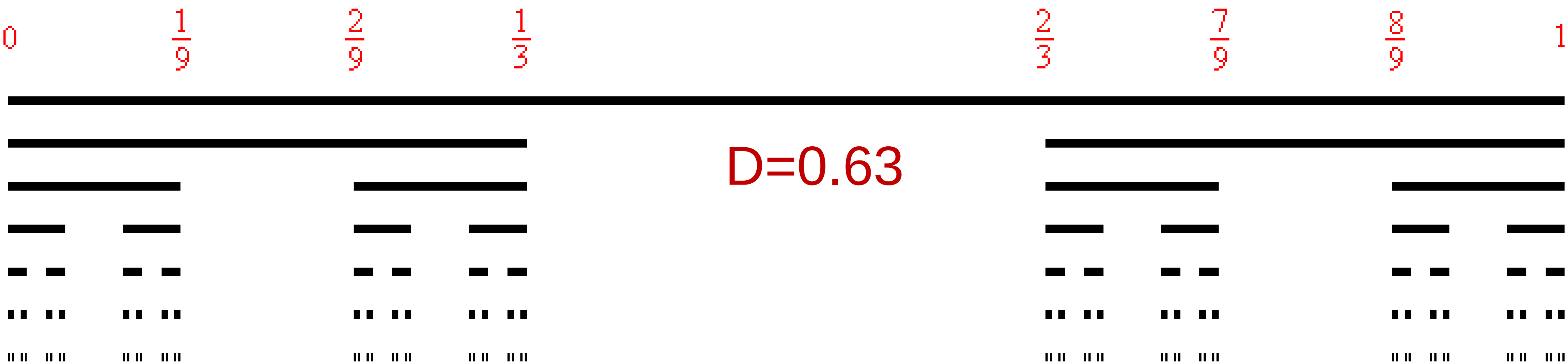
Box counting: number of occupied boxes scales as $(1/\varepsilon)^D$

Abarbanel et al, Reviews of Modern Physics 65, 1331 (1993).

cristina.masoller@upc.edu @cristinamasoll1

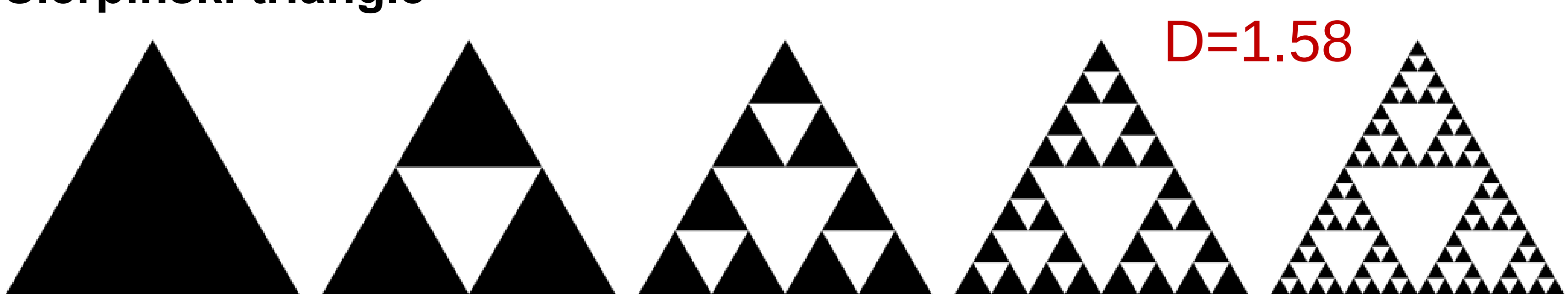
Examples

1. **Cantor set** (introduced by German mathematician Georg Cantor in 1883)



Fractal structure: each part of the object resembles the hole object.

2. **Sierpiński triangle**



cristina.masoller@upc.edu @cristinamasoll1

Examples of fractal objects in nature

Broccoli D=2.66

Human lung D=2.97

Coastline of Ireland D=1.22

cristina.masoller@upc.edu @cristinamasoll1

An in finance?

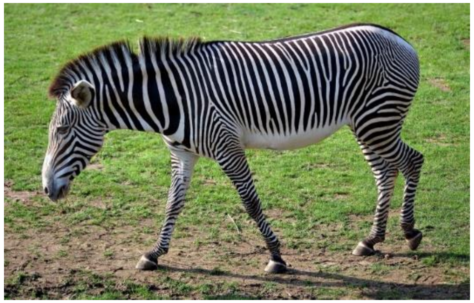
- The fractal concept is not an abstraction but a mathematical formulation of a well-known fact: movements of a stock or currency all look alike when a market chart is enlarged or reduced.
- An observer cannot tell which of the data concern prices that change from week to week, day to day or hour to hour.

How Fractals Can Explain What's Wrong with Wall Street,
B. B. Mandelbrot, Scientific American Sept. 2008

cristina.masoller@upc.edu @cristinamasoll1

Spatial patterns: how “self-organization” emerges?



- **Ilya Prigogine** (Belgium, born in Moscow, Nobel Prize in Chemistry 1977).
- Studied chemical systems far from equilibrium.
- Discovered that the interplay of (external) **input of energy** and **dissipation** can lead to “self-organized” patterns.


cristina.masoller@upc.edu @cristinamasoll1

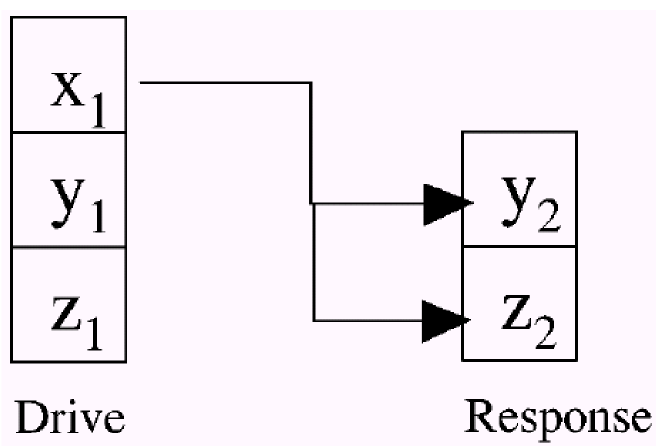
The 1990s: can two chaotic systems synchronize?

VOLUME 64, NUMBER 8 PHYSICAL REVIEW LETTERS 19 FEBRUARY 1990

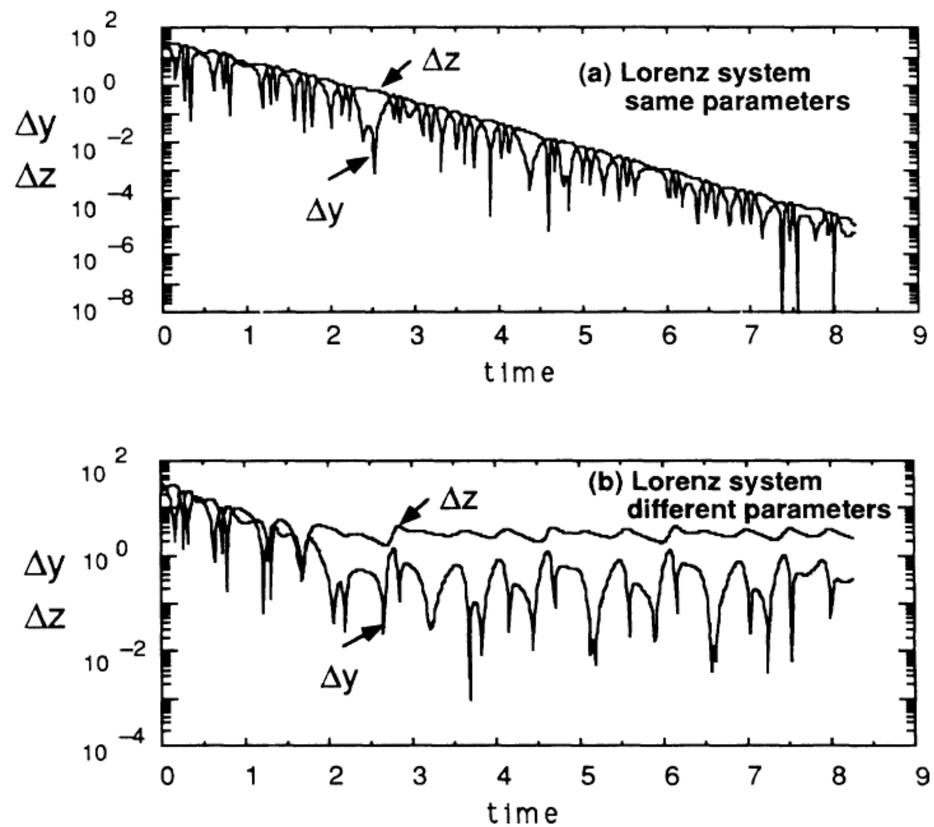
Synchronization in Chaotic Systems

Louis M. Pecora and Thomas L. Carroll
Code 6341, Naval Research Laboratory, Washington, D.C. 20375
(Received 20 December 1989)

Coupled Lorenz systems



$$t \to \infty \quad |y_2 - y_1| \to 0, \ |z_2 - z_1| \to 0$$



cristina.masoller@upc.edu @cristinamasoll1

In fact, the first observation of synchronization was done much earlier (mutual *entrainment* of two pendulum clocks)

In mid-1600s **Christiaan Huygens** (Dutch mathematician) noticed that two pendulum clocks mounted on a common board synchronized and swayed in opposite directions (in-phase also possible).

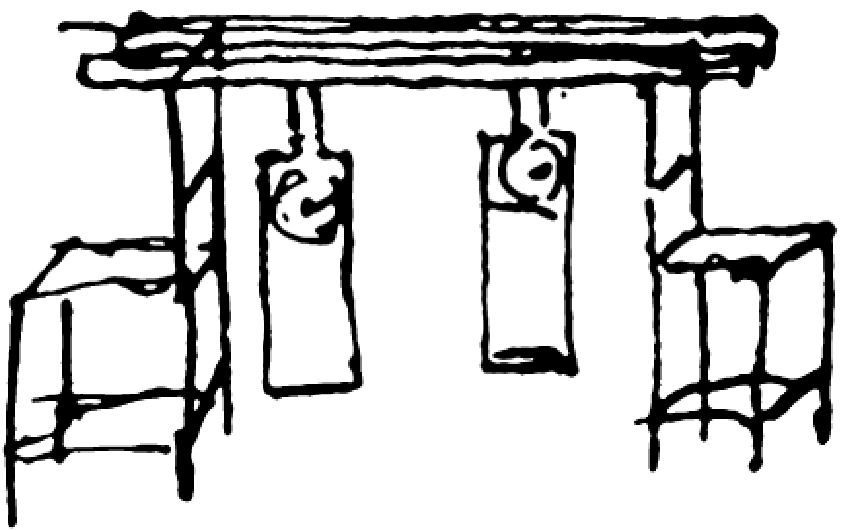
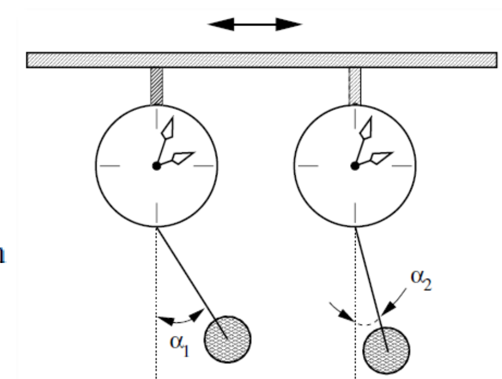


Figure 1.2. Original drawing of Christiaan Huygens illustrating his experiments with two pendulum clocks placed on a common support.



(lots of videos in internet)

cristina.masoller@upc.edu @cristinamasoll1

Can we observe the synchronization of two chaotic systems?

VOLUME 72, NUMBER 13 PHYSICAL REVIEW LETTERS 28 MARCH 1994

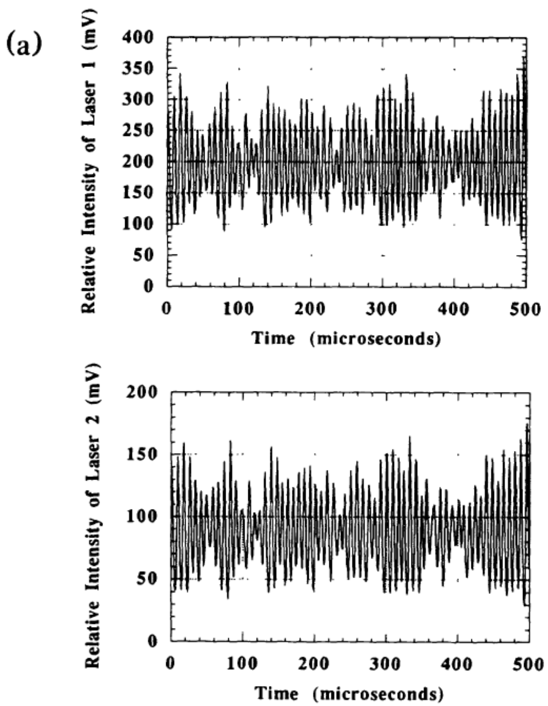
Experimental Synchronization of Chaotic Lasers

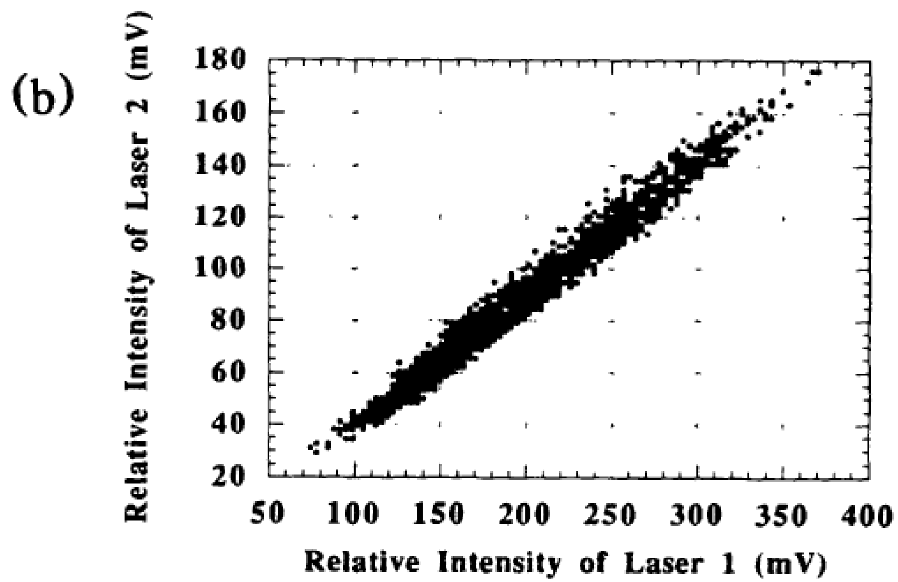
Rajarshi Roy and K. Scott Thornburg, Jr.

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332

(Received 30 August 1993)

We report the observation of synchronization of the chaotic intensity fluctuations of two Nd:YAG lasers when one or both the lasers are driven chaotic by periodic modulation of their pump beams.





cristina.masoller@upc.edu @cristinamasoll1

Different types of synchronization

- Complete: $y(t) = x(t)$ (identical systems)
- Phase: the phases of the oscillations are synchronized, but the amplitudes are not.
- Lag: $y(t+\tau) = x(t)$
- Generalized: $y(t) = F(x(t-\tau))$ (*F and* τ can depend on the coupling strength)

More problems of time series analysis:

How to detect coupling, how to detect delay in the coupling, and how to quantify synchronization?

cristina.masoller@upc.edu @cristinamasoll1

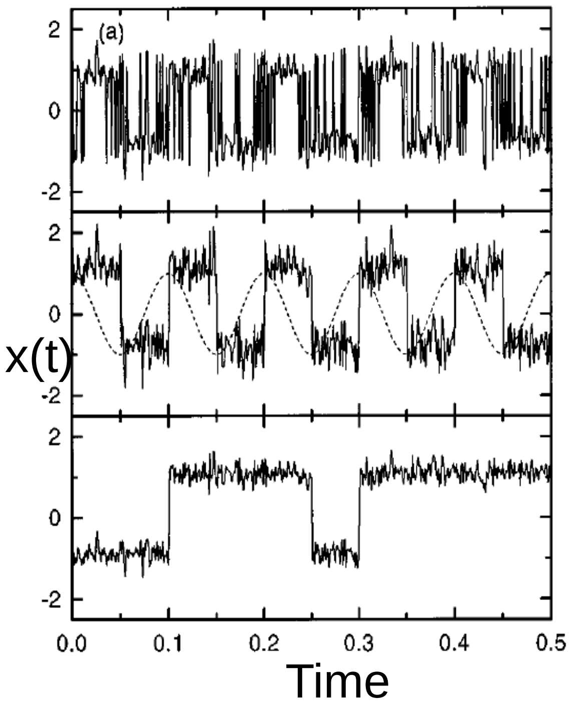
Effect of noise in nonlinear systems? (late 80' and 90')

Stochastic resonance: an optimal level of noise can, in some **bistable** systems, enhance the detection of a weak signal, improving the performance of the system.

Bistable system — Periodic signal — Noise

$$\dot{x}(t) = -V'(x) + A_0 \cos(\Omega t + \varphi) + \xi(t)$$

$$V(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$$



Gammaitoni, Hanggi et al, Rev. Mod. Phys. 70, 223 (1998).

cristina.masoller@upc.edu @cristinamasoll1

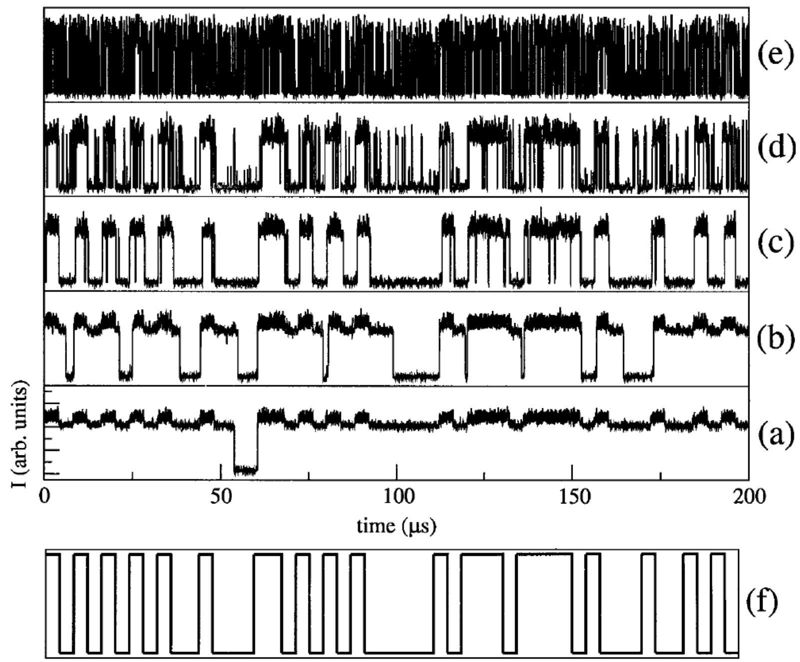
Can we observe the stochastic resonance phenomenon?

VOLUME 85, NUMBER 22 PHYSICAL REVIEW LETTERS 27 NOVEMBER 2000

Experimental Evidence of Binary Aperiodic Stochastic Resonance

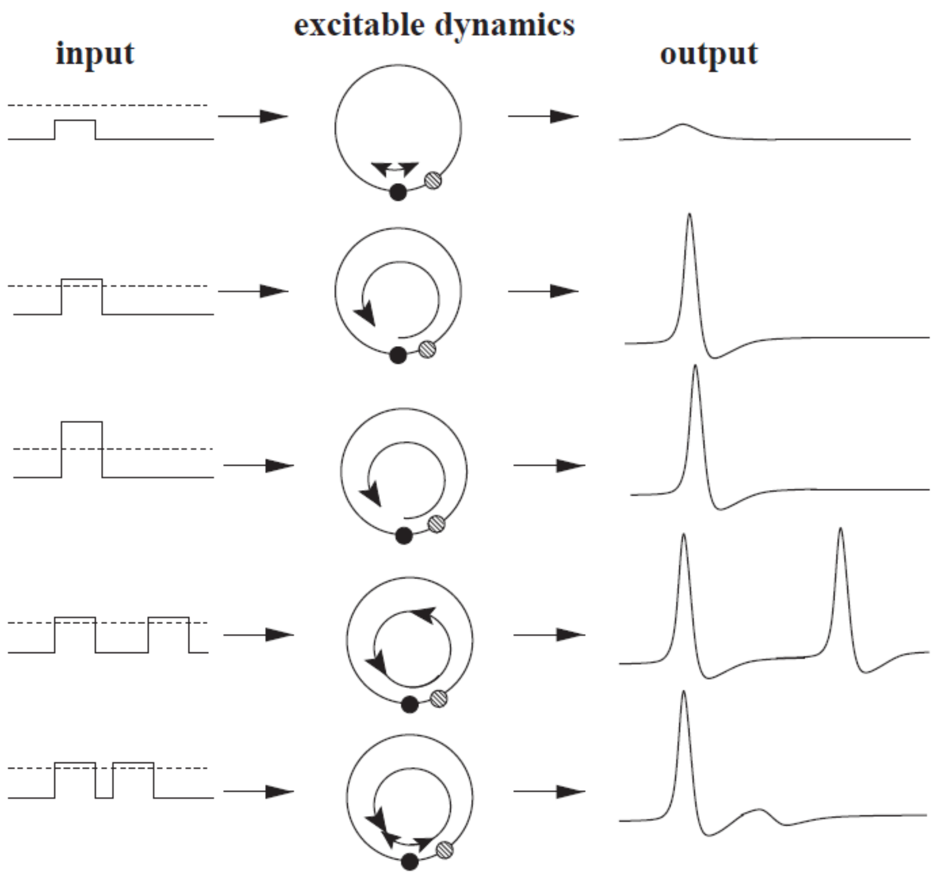
Sylvain Barbay,[1] Giovanni Giacomelli,[1,3,*] and Francesco Marin[2,3]
[1]*Istituto Nazionale di Ottica Applicata, Largo E. Fermi 6, 50125 Firenze, Italy*
[2]*Dipartimento di Fisica, Università di Firenze, and Laboratorio Europeo di Spettroscopia Nonlineare, Largo E. Fermi 2, 50125 Firenze, Italy*
[3]*Istituto Nazionale di Fisica della Materia, unità di Firenze, Italy*
(Received 14 March 2000)

(using a bistable laser that emits in two orthogonal polarizations)



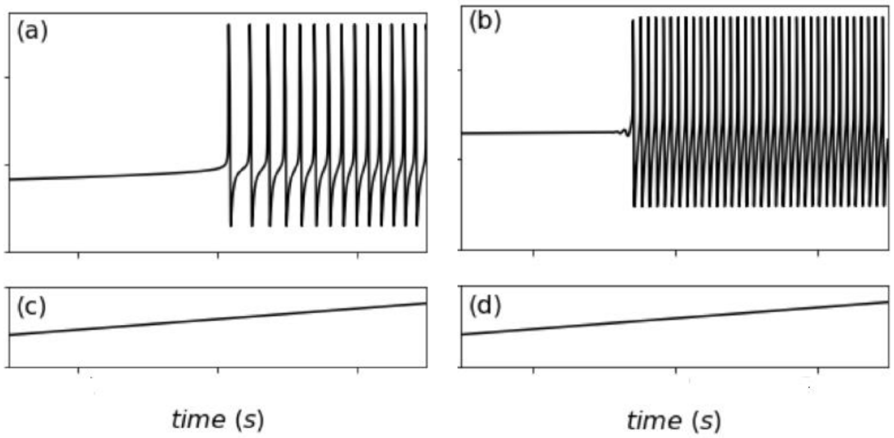
cristina.masoller@upc.edu @cristinamasoll1

An excitable system: a peculiar type of dynamical system



B. Lindner et al., Phys. Rep. 392, 321 (2004)

Response when a control parameter increases in time



M. Masoliver PhD thesis (2020)

cristina.masoller@upc.edu @cristinamasoll1

Role of noise in excitable systems?

VOLUME 78, NUMBER 5 PHYSICAL REVIEW LETTERS 3 FEBRUARY 1997

Coherence Resonance in a Noise-Driven Excitable System

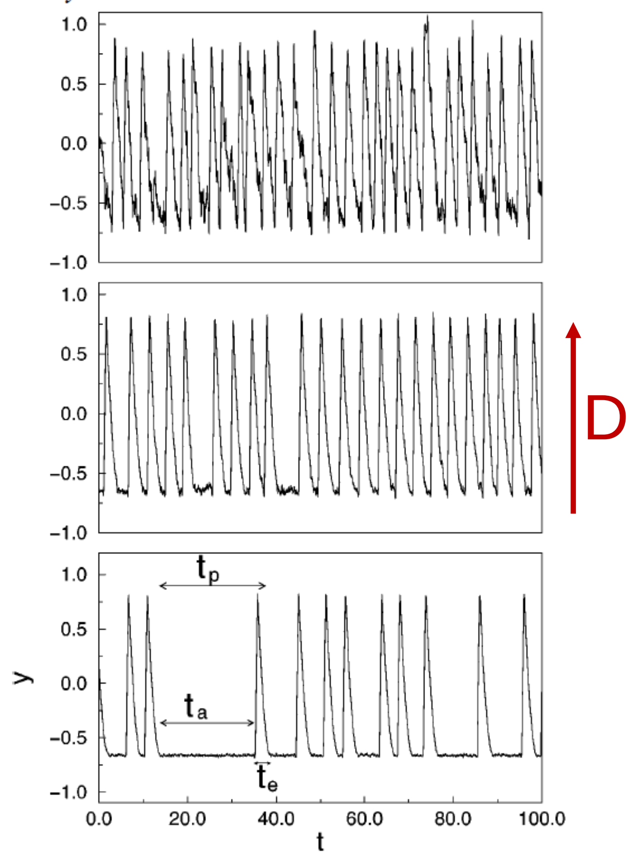
Arkady S. Pikovsky* and Jürgen Kurths*
Max–Planck–Arbeitsgruppe "Nichtlineare Dynamik" an der Universität Potsdam Am Neuen Palais 19, PF 601553, D-14415,

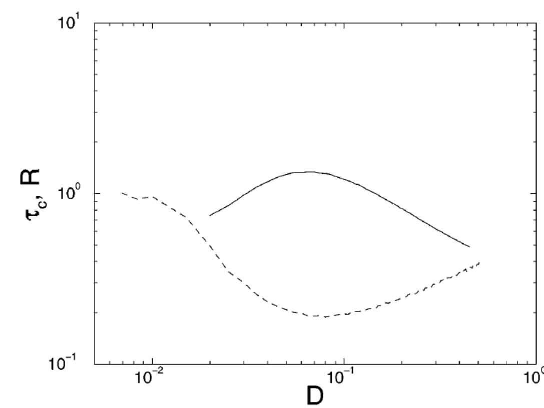
Fitz Hugh–Nagumo model

$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y\,,$$

$$\frac{dy}{dt} = x + a + D\xi(t)$$

D=0: stable behavior





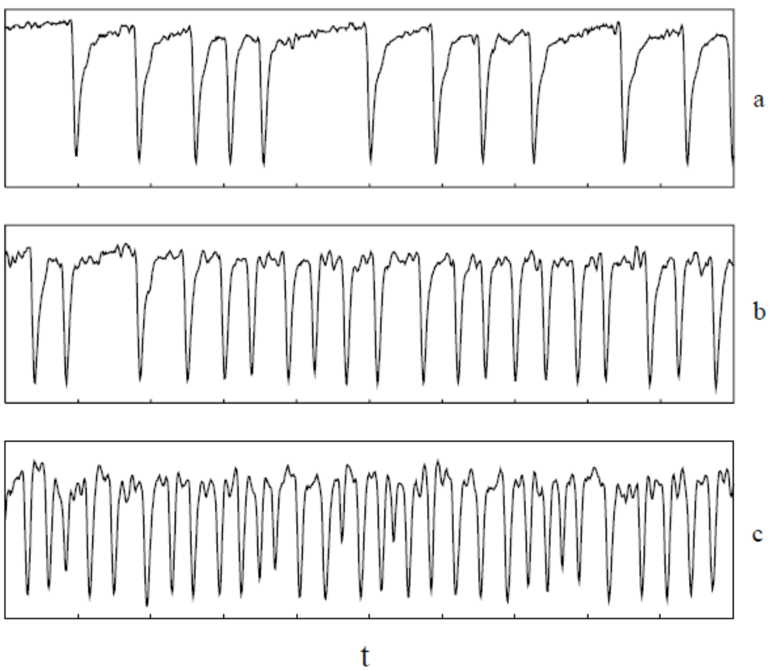
cristina.masoller@upc.edu @cristinamasoll1

Coherence and stochastic resonance have been observed in excitable lasers

VOLUME 84, NUMBER 15 PHYSICAL REVIEW LETTERS 10 APRIL 2000

Experimental Evidence of Coherence Resonance in an Optical System

Giovanni Giacomelli
Istituto Nazionale di Ottica, Largo E. Fermi 6, 50125 Firenze, Italy
Massimo Giudici and Salvador Balle
Departamento de Física Interdisciplinar, Instituto Mediterraneo de Estudios Avanzados (CSIC-UIB), 07071 Palma de Mallorca, Spain
Jorge R. Tredicce
Institut Non-Linéaire de Nice, UMR 6618 Centre National de la Recherche Scientifique-Université de Nice Sophia-Antipolis, 06560 Valbonne, France

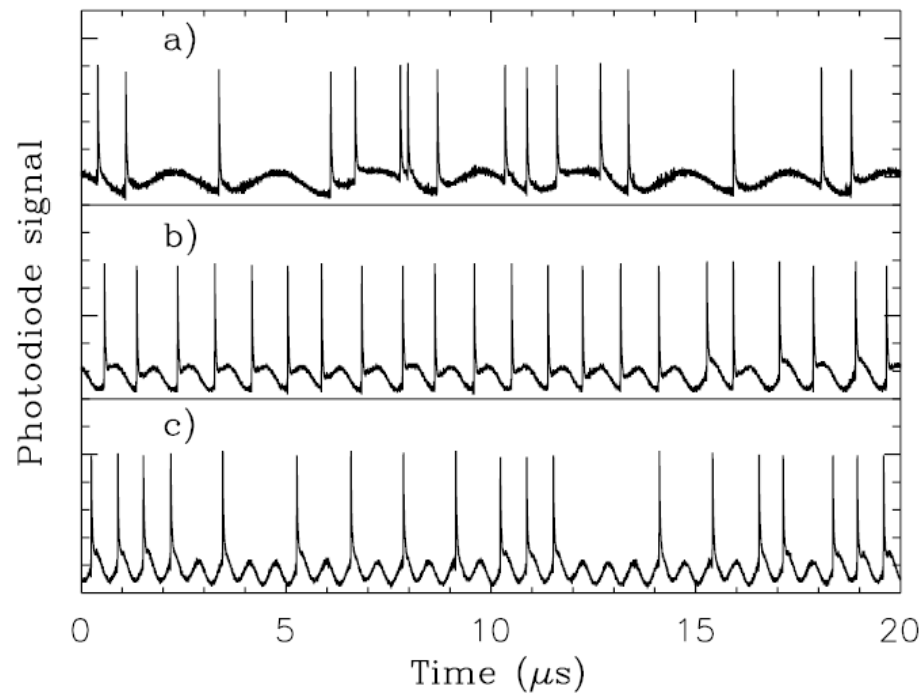


(varying the level of noise)

VOLUME 88, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JANUARY 2002

Experimental Evidence of Stochastic Resonance in an Excitable Optical System

Francesco Marino, Massimo Giudici,* Stéphane Barland,† and Salvador Balle
Department de Física Interdisciplinar, Instituto Mediterráneo de Estudios Avanzados (CSIC-UIB), C/ Miquel Marqués 21, E-07190 Esporles, Spain
(Received 1 August 2001; published 10 January 2002)



(varying the frequency of the signal)

cristina.masoller@upc.edu @cristinamasoll1

But what is “noise”?

Someone's noise is another one's signal

(example: for a climatologist “weather” is noise).

A problem in time series analysis: How to “find the signal”?

(example: filter out noise, compress data).

A two-dimensional ***random walk*** *or drunkard’s walk*

(The Viking Press, New York,1955)

In social systems, a **Brownian agent** generalizes the concept of a Brownian particle: is an active particle that has internal states, can store energy, information, assets, and interacts with other agents and with the environment.

cristina.masoller@upc.edu @cristinamasoll1

Stochastic resonance in social systems?

- In a model of opinion formation (Kuperman and Zanette, 2002), opinions are affected by:
  - social imitation, occurring via majority rule;
  - fashion, expressed by an external modulation acting on all agents;
  - individual uncertainty, expressed by random noise.

  Stochastic resonance was observed because a optimal amount of noise leads to a strong amplification of the system's response to the external modulation (fashion).
- The phenomenon also occurs if one varies the system's size keeping fixed amount of noise (Tessone and Toral, 2005): an optimal response is achieved for an optimal population size ("system size stochastic resonance").

Kuperman and Zanette, Eur. Phys. J. B **26**, 387 (2002).
Tessone and Toral, Physica A 351, 106 (2005).
Castellano et al, Rev. Mod. Phys. 81, 591 (2009).

cristina.masoller@upc.edu @cristinamasoll1

Late 90s, early 2000s: synchronization of a large number of dynamical systems

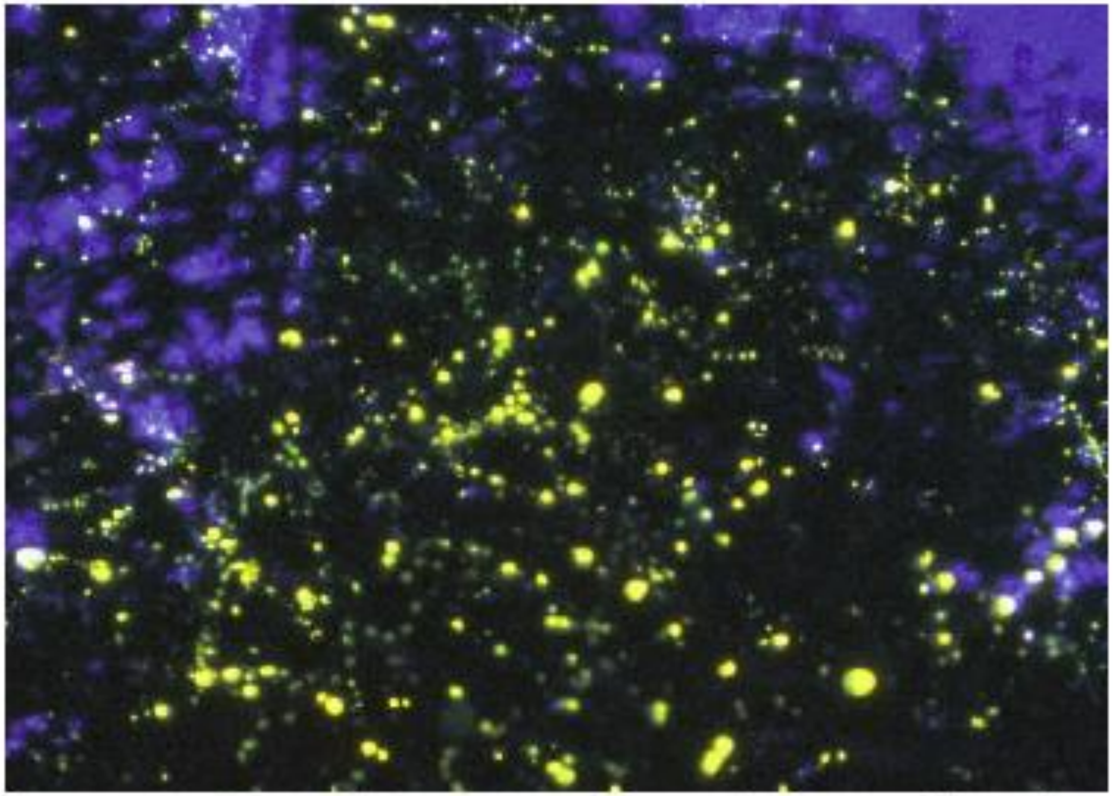


Figure 1 | Fireflies, fireflies burning bright. In the forests of the night, certain species of firefly flash in perfect synchrony — here *Pteroptyx malaccae* in a mangrove apple tree in Malaysia. Kaka *et al.*[2] and Mancoff *et al.*[3] show that the same principle can be applied to oscillators at the nanoscale.

cristina.masoller@upc.edu @cristinamasoll1

Another example of synchronization: the opening of the London Millennium Bridge, June 10, 2000

Source: BBC

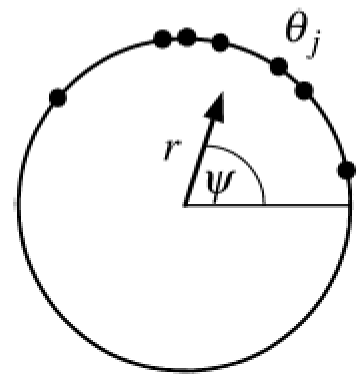
Crowd synchrony on the Millennium Bridge, Strogatz et al, Nature 438, 43 (2005)

cristina.masoller@upc.edu @cristinamasoll1

The Kuramoto model (Japanese physicist, 1975)

Model of **all-to-all** coupled **phase oscillators**.

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) + \xi_i, \qquad i = 1 \ldots N$$



K = coupling strength, ξ_i = stochastic term (noise)

Describes the emergence of collective behavior

How to quantify?

With the **order parameter**:

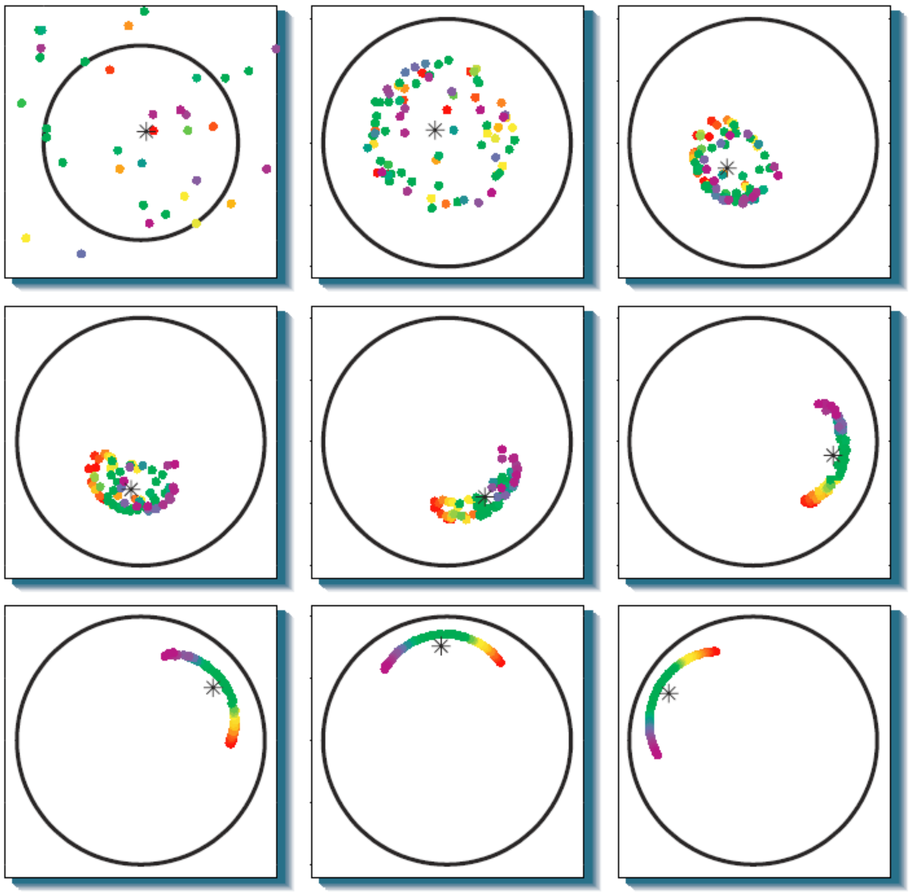
$$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

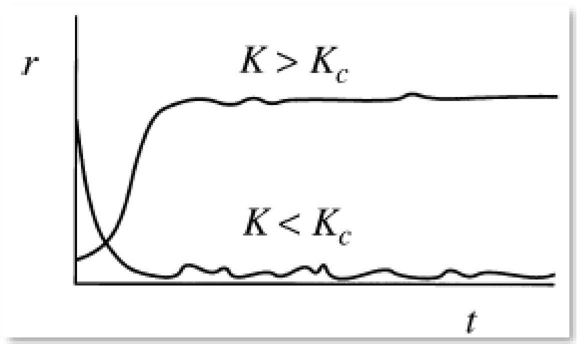
r =0 incoherent state (oscillators scattered in the unit circle)

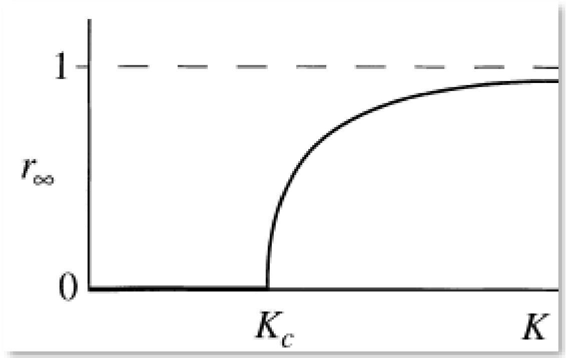
r =1 all oscillators are in phase ($\theta_i=\theta_j\ \forall$ i,j)

cristina.masoller@upc.edu @cristinamasoll1

Synchronization transition as the coupling strength increases







Strogatz, Nature 2001

Video: https://www.ted.com/talks/steven_strogatz_on_sync

cristina.masoller@upc.edu @cristinamasoll1

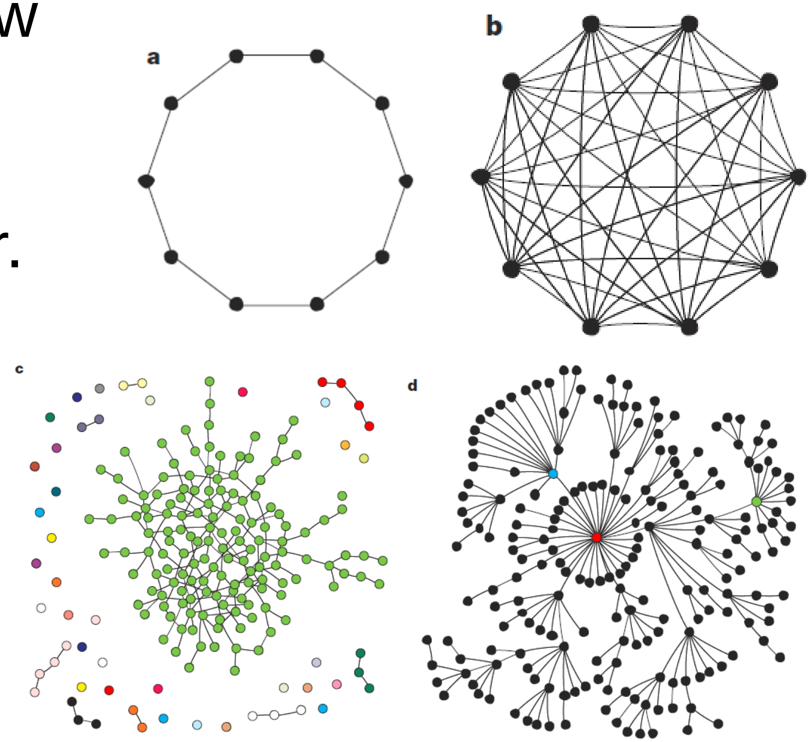
2000s to present: from chaotic systems to complex systems

- Complicated systems (large sets of linear elements with linear interactions) are not complex.
- Complex systems: large number of elements, where the elements and/or their interactions are **nonlinear**.
- Main difference: in a complex system a “reductionist” approach does not work.
- The “emergent behavior” in a complex system can not be predicted studying the behavior of the individual units.

cristina.masoller@upc.edu @cristinamasoll1

Complexity science

- **Networks** (or **graphs**) are used for mathematical modelling of complex systems.
- Emergent properties, not present in the individual elements.
- The challenge: to understand how the **structure** of the network and the **dynamics** of individual units determine the collective behavior.
- Applications
  - Epidemics
  - Rumor spreading
  - Transport networks
  - Financial, Economics
  - Brain, physiology, etc.

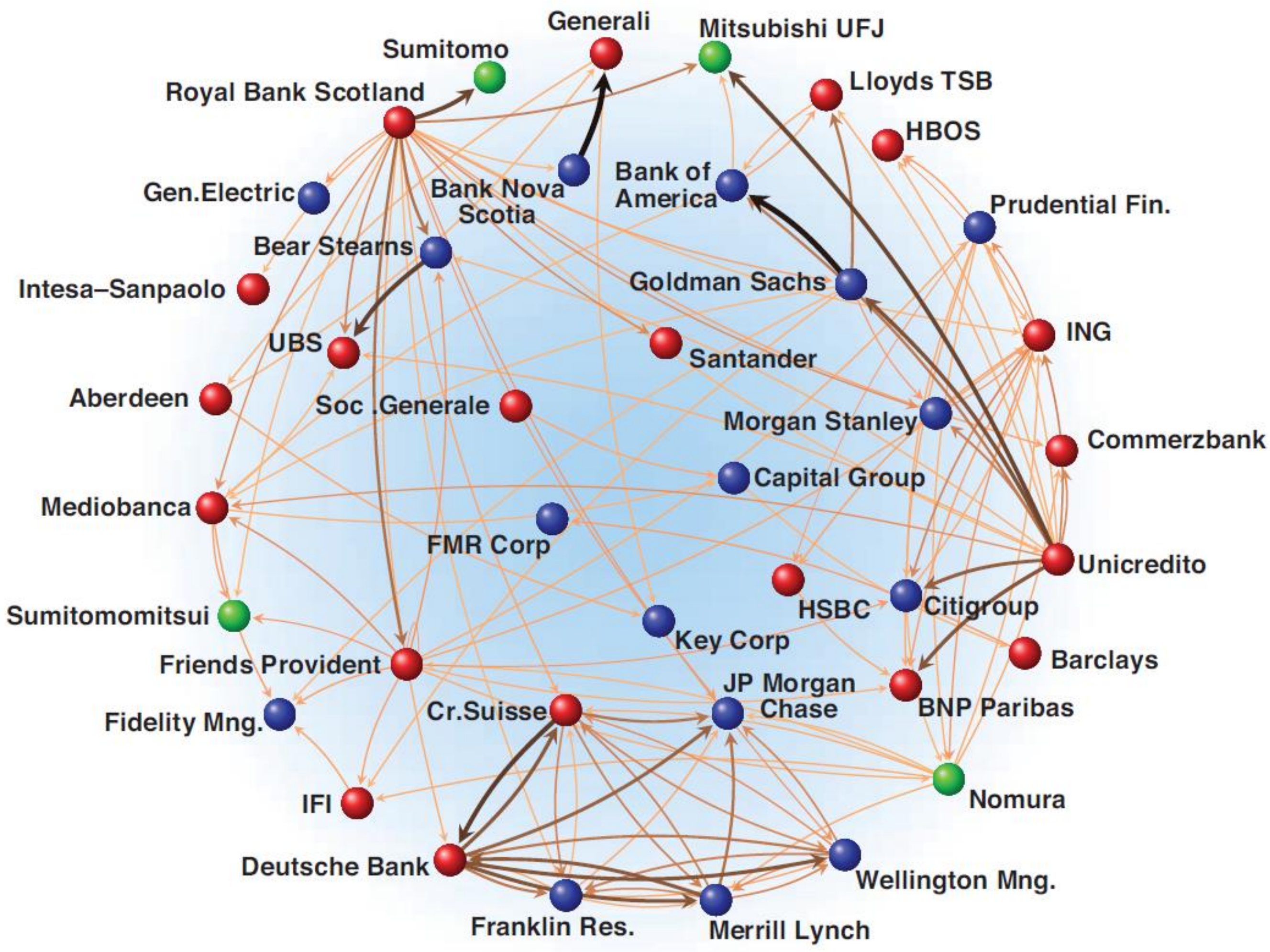


S. Strogatz, Nature 2001

cristina.masoller@upc.edu @cristinamasoll1

Real-world example: international financial network

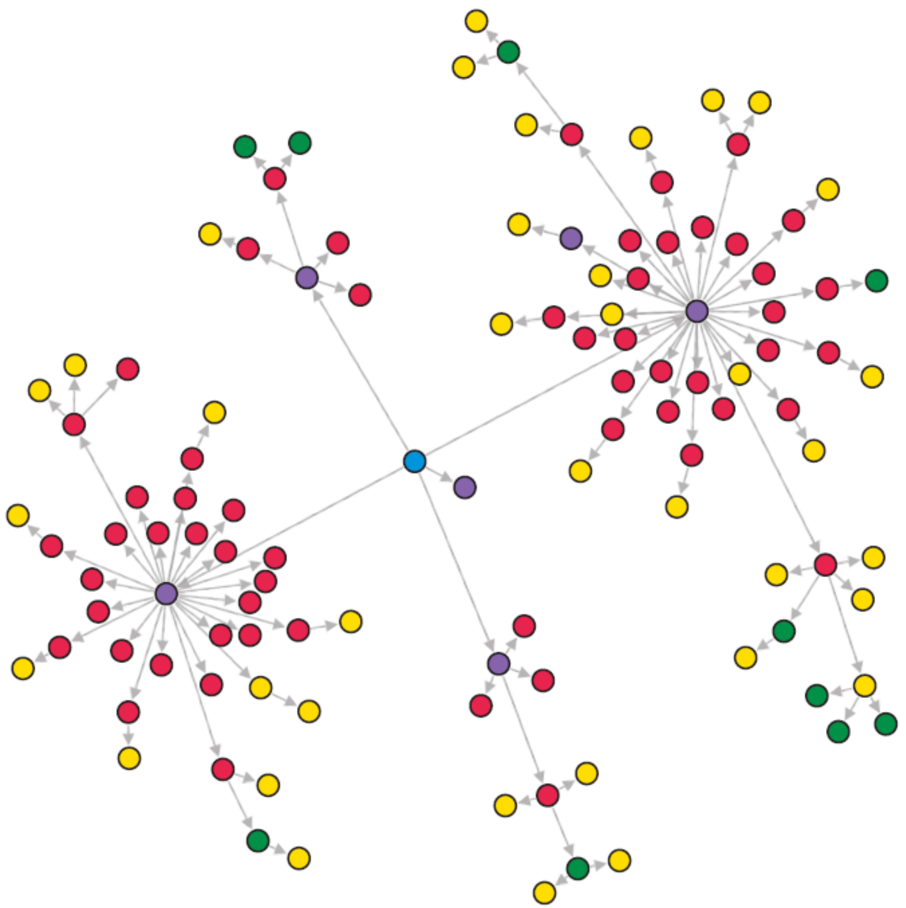
- The nodes represent major financial institutions
- The links (directed and weighted) represent the strongest relations among them.
- Node colors indicate different geographical areas: EU (red), North America (blue), other (green).



F. Schweitzer et al., Science 325, 422 (2009).

cristina.masoller@upc.edu @cristinamasoll1

Real-world example: transmission of Covid-19



Source: Alison Hill, The math behind epidemics, https://physicstoday.scitation.org/doi/10.1063/PT.3.4614

- Transmission network seeded by an unknown infected individual (**blue**) who attended a training course with other fitness instructors (**purple**).
- The fitness instructors spread the infection to students in their classes (**red**), to family (yellow), and to coworkers (**green**).

Time series analysis problems:
- how to "reconstruct" the network from observed data? -
- how to predict the existence or the absence of a link?

cristina.masoller@upc.edu @cristinamasoll1

Kuramoto model in a complex network

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j=1}^{N} A_{ij} \sin(\theta_j - \theta_i)$$

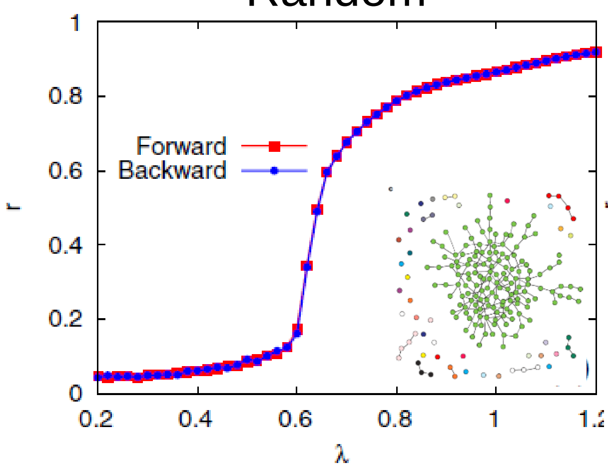
Adjacency matrix: A_{ij} = 1 if i and j are connected, else A_{ij} = 0.

Order parameter $r = \frac{1}{N}\left|\sum_{j=1}^{N} e^{i\theta_j}\right|$

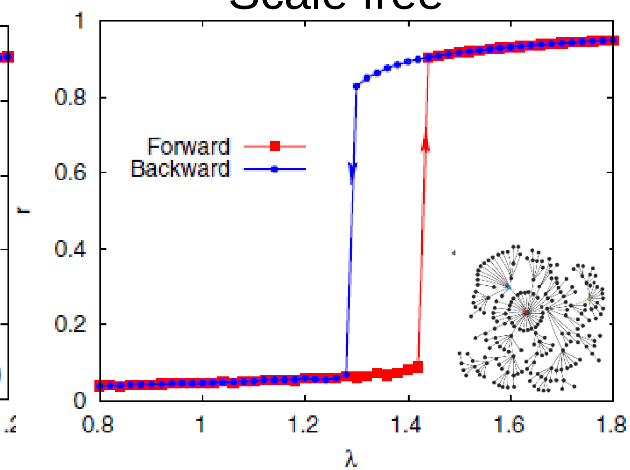
$$\omega_i = k_i = \sum_{j=1}^{N} A_{ij}$$

Fast oscillators have many links, slow oscillators only few.

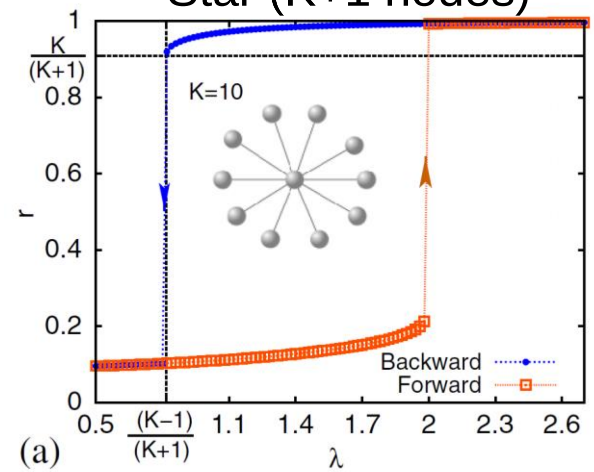
Random



Scale free



Star (K+1 nodes)



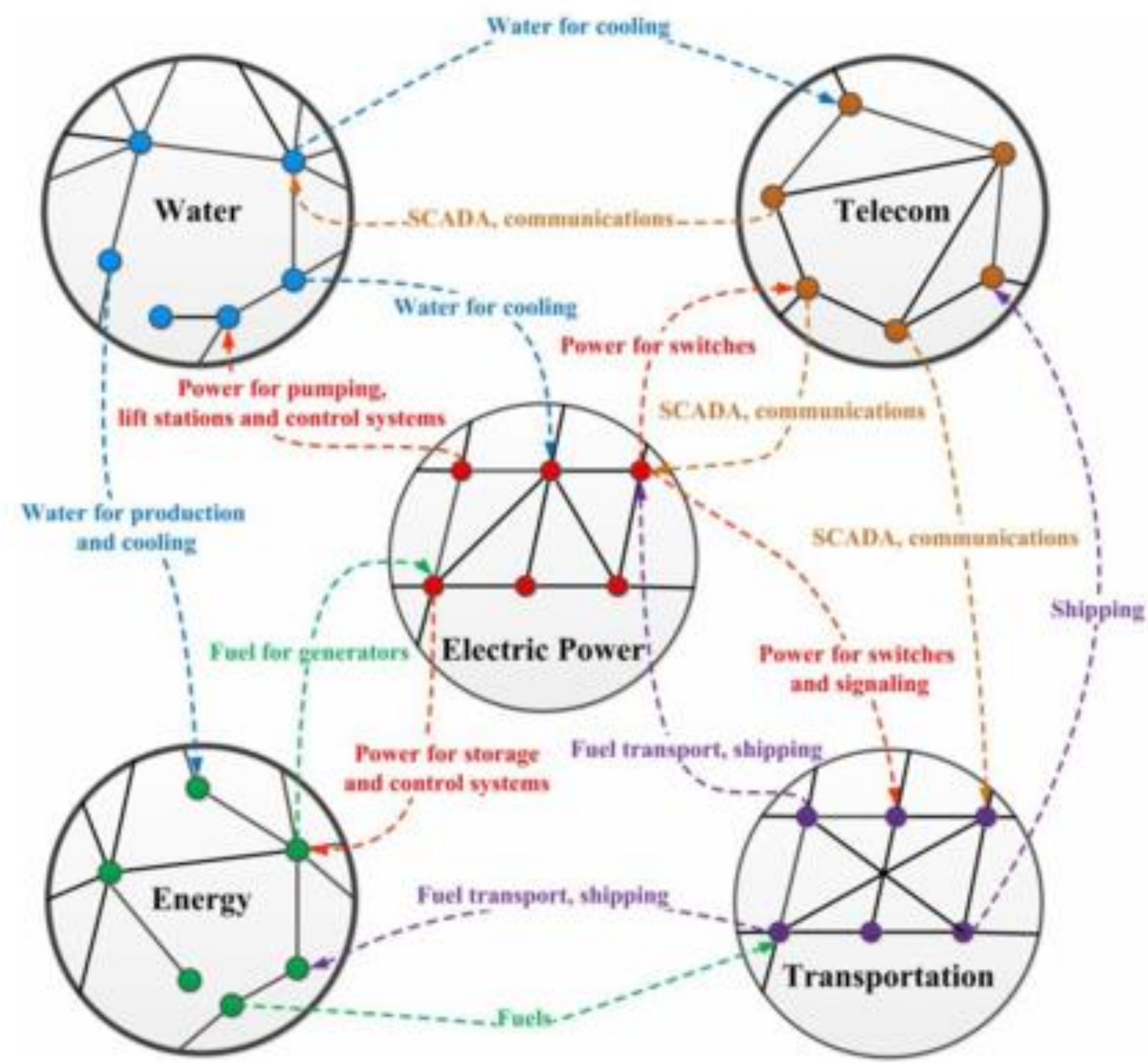
Explosive (phase) synchronization has been observed in coupled lasers and in electronic circuits:

J. Zamora et al., Phys. Rev. Lett. 105, 264101 (2010).
J. Gomez-Gardeñes et al., Phys. Rev. Lett. 106, 128701 (2011).
I. Leyva et al, Phys. Rev. Lett. 108, 168702 (2012).

cristina.masoller@upc.edu @cristinamasoll1

Networks of networks: interdependent networks

Can we predict the effect of a critical (or extreme) event in one network? Cascade of failures?



Source: Wikipedia

cristina.masoller@upc.edu @cristinamasoll1

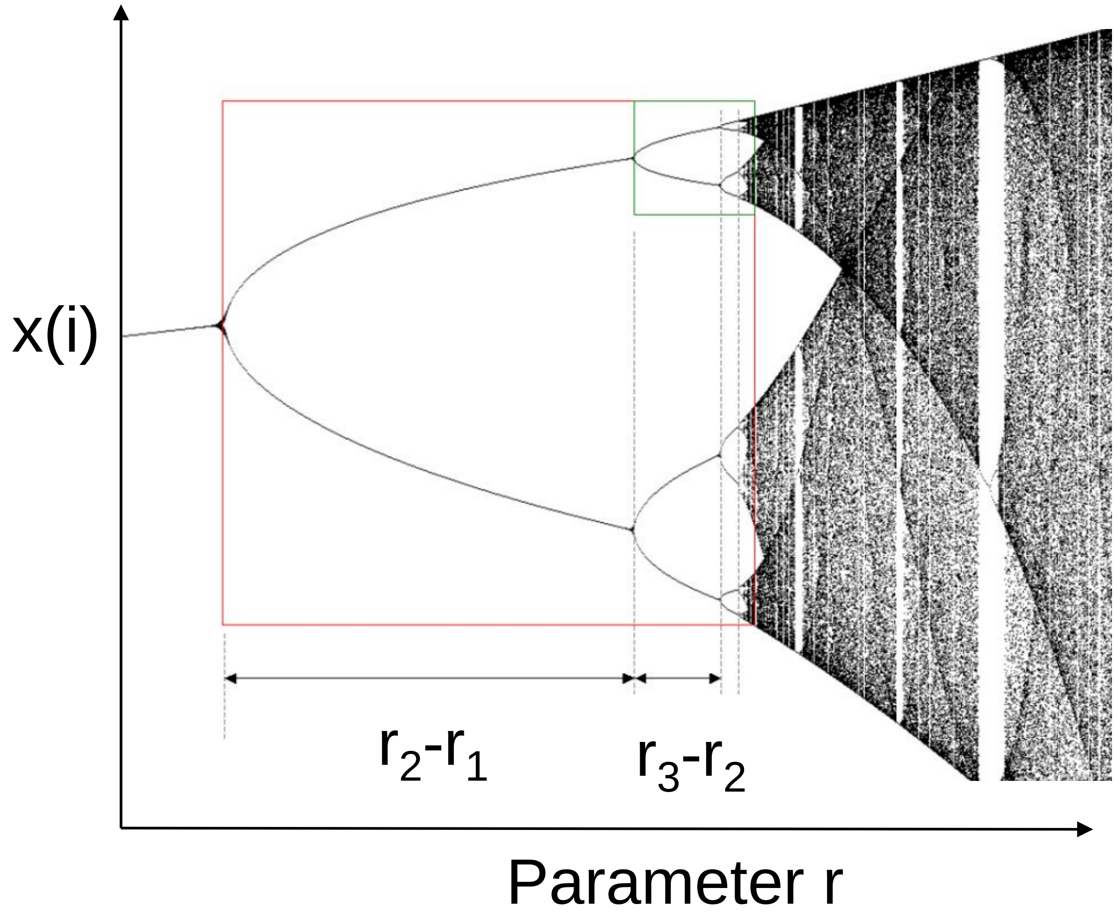
From dynamical systems to complex systems & data science

- Dynamical systems theory (bifurcations, low-dimensional attractors) allows to
  - uncover patterns and “order within chaos”,
  - uncover universal characteristics
- Synchronization emerges in interacting systems
- Complexity science: study “emergent” phenomena in large sets of nonlinear interacting units (tipping points, critical transitions).
- Time series analysis allows to characterize signals and to “obtain features” that encapsulate properties of the signals.
- Data science: feature selection, classification, forecasting.

cristina.masoller@upc.edu @cristinamasoll1

Hands-on exercise 1: work with the logistic map

$$x(i+1) = r\,x(i)[1 - x(i)]$$



- Plot the bifurcation diagram
- Estimate $\delta=(r_2-r_1)/(r_3-r_2)$
- Role of transient time?
- Continuous variation of r?

cristina.masoller@upc.edu @cristinamasoll1