

School on Applications of Nonlinear Systems to Socio-Economic Complexity, Oct. 17 – Oct. 22 2022

Nonlinear time series analysis

Cristina Masoller

Departamento de Física
Universitat Politècnica de Catalunya

Class 1: From dynamical systems to complex systems

Class 2: Univariate time series analysis

Class 3: Univariate time series analysis

Class 4: Bivariate and Multivariate analysis



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cristina.masoller@upc.edu



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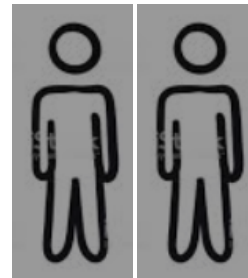
Methods of time series analysis are classified as:

- Univariate analysis

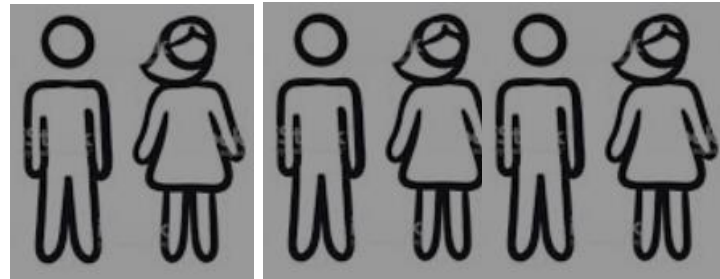


- Bivariate analysis

- Cross Correlation
- Mutual Information
- Event synchronization
- Causality



- Multivariate analysis



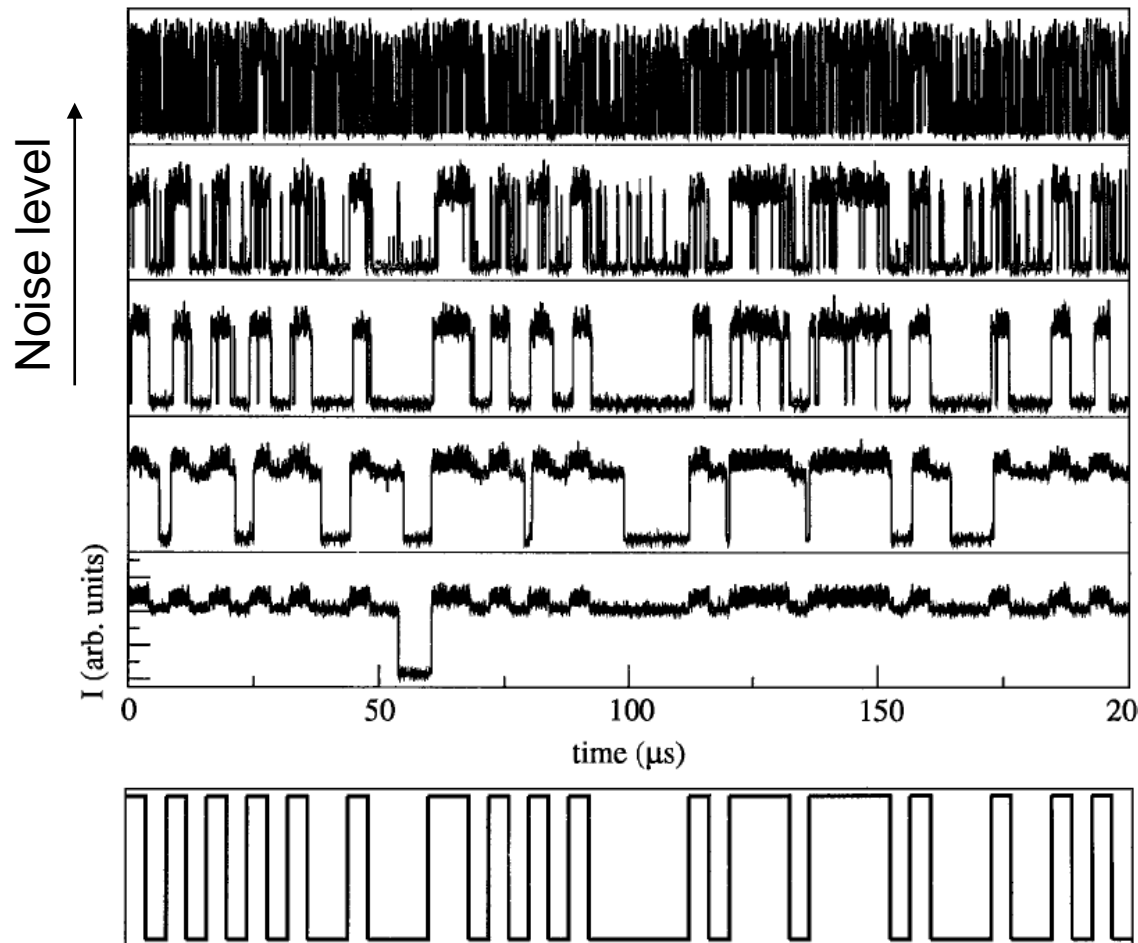
Cross-correlation

$$C_{xy}(\tau) = \frac{1}{N - \tau} \sum_{k=1}^{N-\tau} x(k + \tau)y(k)$$

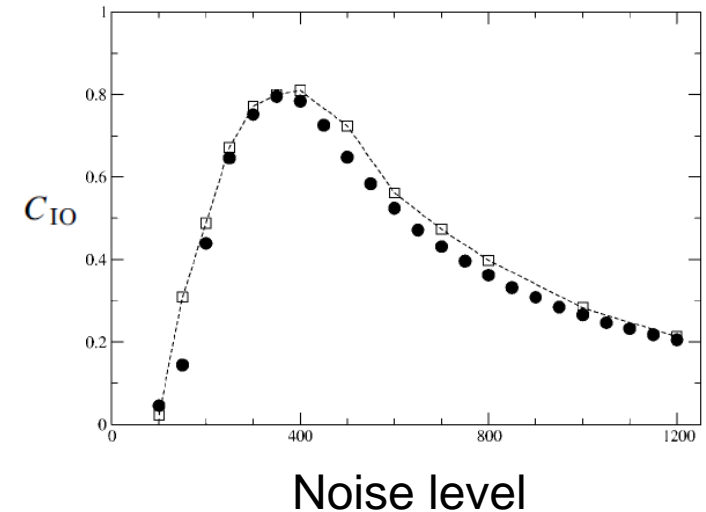
the two time series $x(t)$ and $y(t)$ are normalized to zero-mean $\mu=0$ and unit variance, $\sigma=1$

- $-1 \leq C_{X,Y} \leq 1$
- $C_{X,Y} = C_{Y,X}$
- The maximum of $C_{X,Y}(\tau)$ indicates the **lag** that renders the time series X and Y best aligned.
- Pearson coefficient: $\rho = |C_{X,Y}(0)|$

Example: response of a bistable system to an aperiodic signal (stochastic resonance)



Cross-correlation between input and output signal.



Barbay et al, PRL 85, 4652 (2000)

Lags need to be taken into account when analyzing statistical similarities

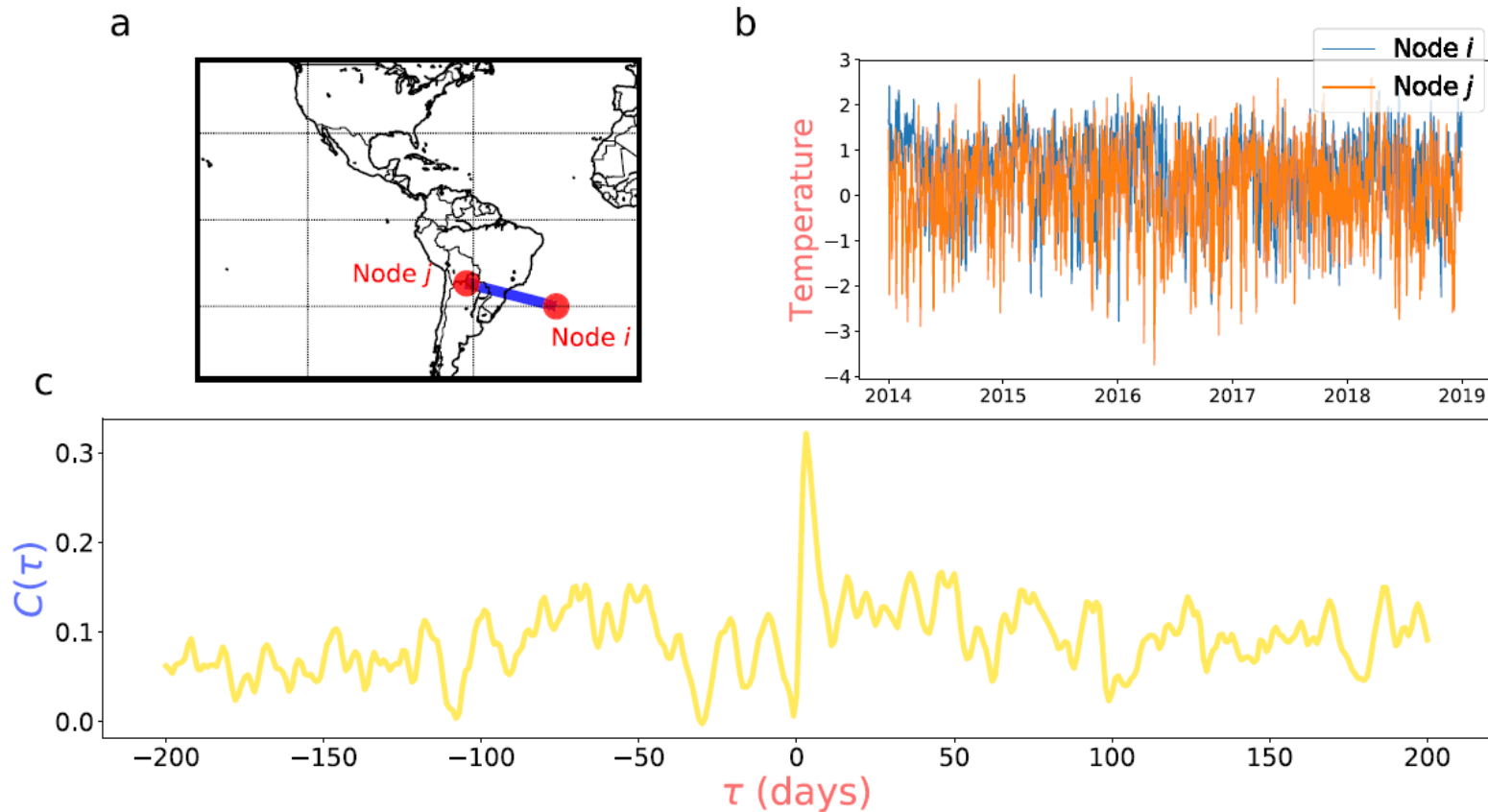
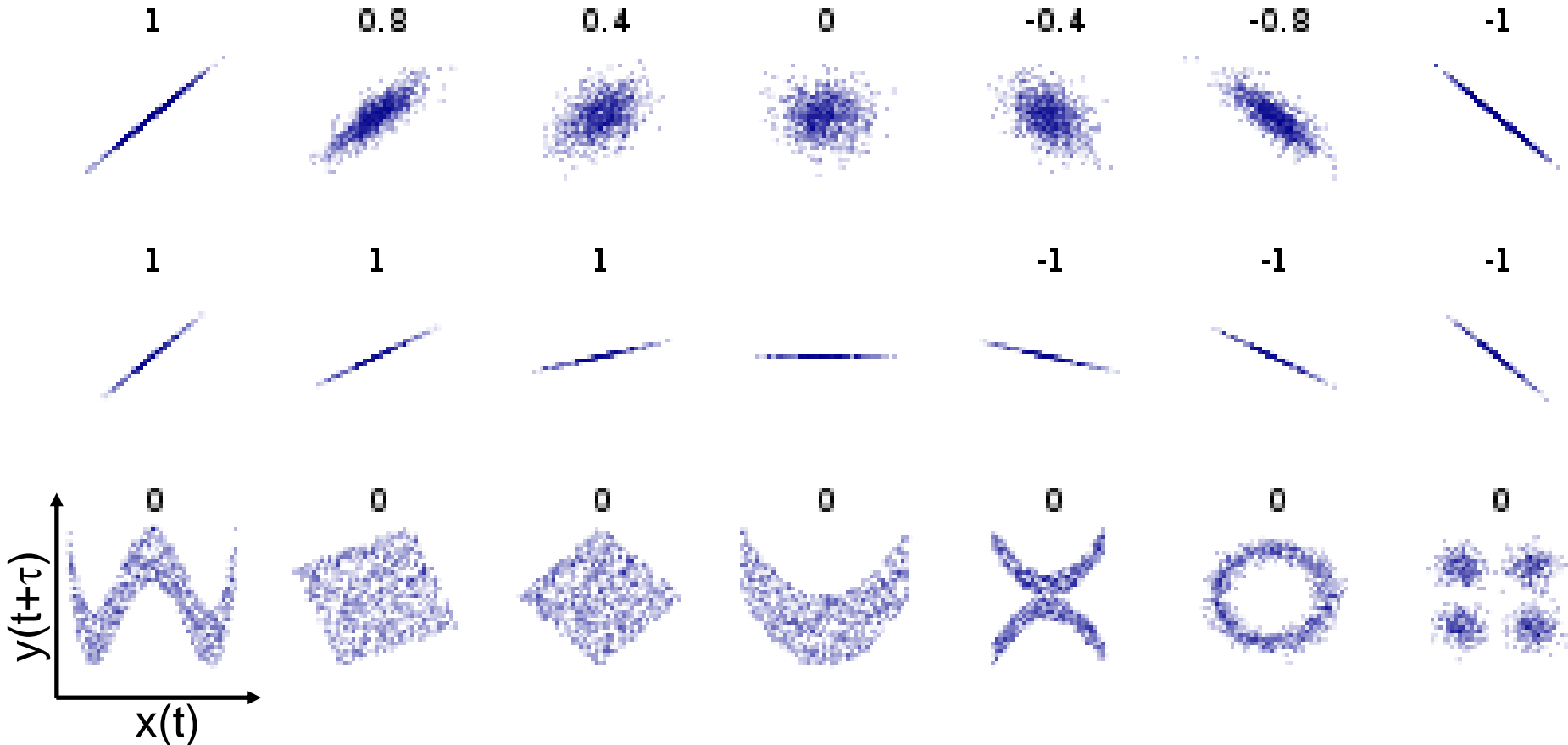


Fig. 6. Typical weighted and directed link in a Pearson Correlation Climate Network. (a) Node i is located on the Southwest Atlantic and node j is in the South American continent. (b) The near surface daily air temperature anomalies for the period [2014,2018]. (c) The cross-correlation function between the time series shown in (b). The direction of this link is from j to i with weight $W_{ij}^+ = 5.71$.

J. Fan et al. Phys. Rep. 896, 1 (2021).

Cross-correlation detects linear relationships only



Source: wikipedia

Correlation is NOT causality

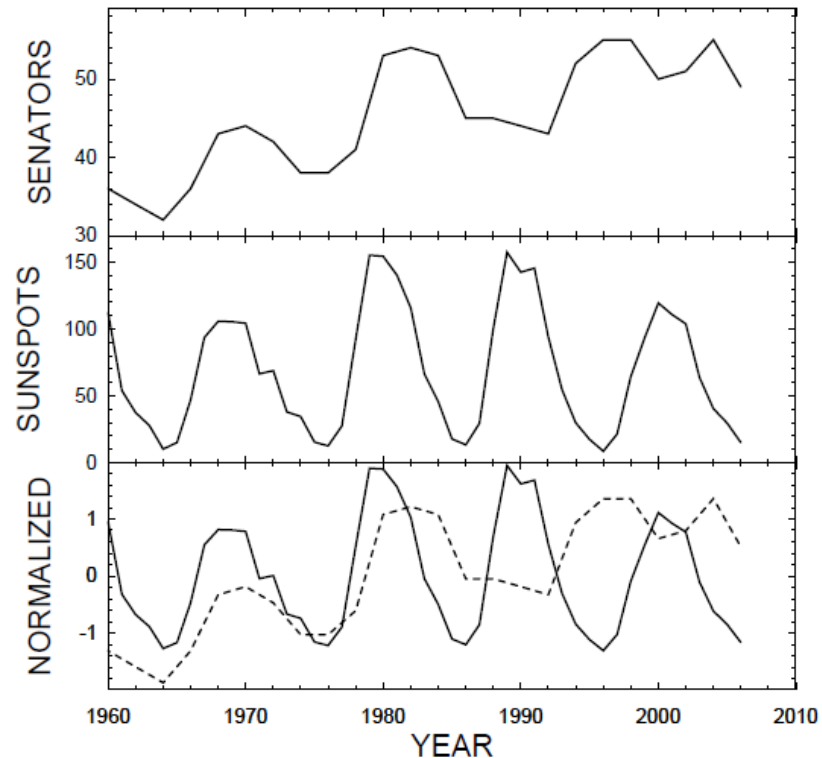
An illustrative example: the number of sunspots and the number of the Republicans in the U.S. Senate in the years 1960 to 1986 (biannual sampling, 14 points).

C=0.52

Appropriate significance test needed!

M. Palus, Contemporary Physics 48, 307 (2007).

<http://tylervigen.com/spurious-correlations>

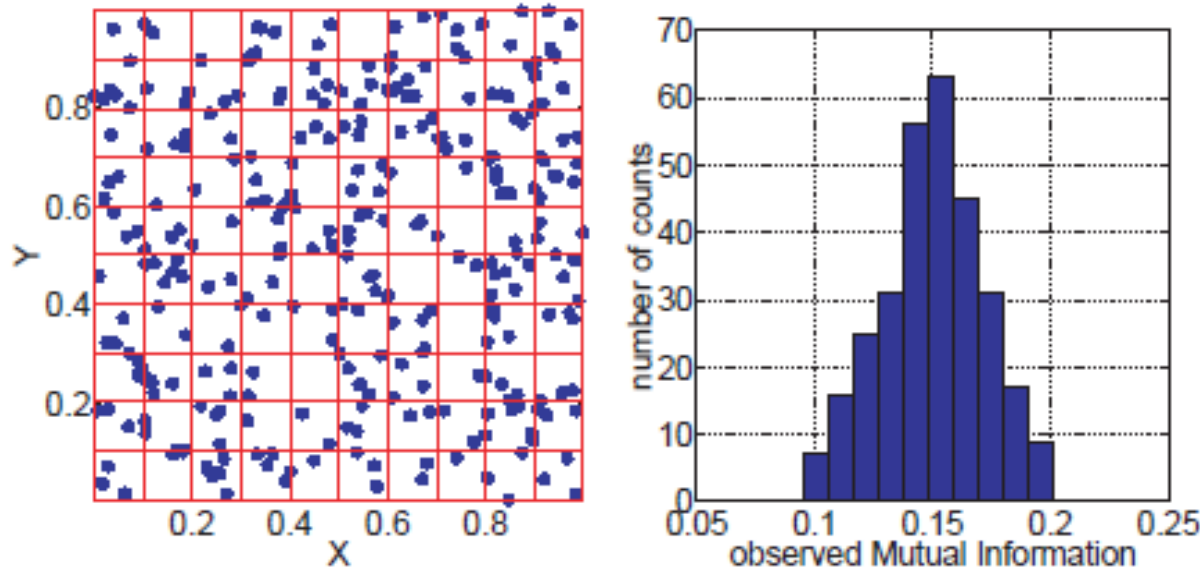


The Mutual Information: a nonlinear correlation measure

$$MI = \sum_{i \in x} \sum_{j \in y} p(x, y) \log \left(\frac{p(x, y)}{p(x)p(y)} \right)$$

- $MI(x, y) = MI(y, x)$
- $p(x, y) = p(x) p(y) \Rightarrow MI = 0$, else **$MI > 0$**
- MI can also be computed with a lag-time.
- MI can also be computed from symbolic probabilities (e.g., probabilities of ordinal patterns).

MI values are systematically overestimated



$$\langle I(X, Y)^{estimated} \rangle \approx 0.15 \pm 0.02$$

Main
problem: a
reliable
estimation of
MI requires a
large amount
of data

Fig. 1. Naive estimation of the mutual information for finite data. Left: The dataset consists of $N = 300$ artificially generated independent and equidistributed random numbers. The probabilities are estimated using a histogram which divides each axis into $M_x = M_y = 10$ bins. Right: The histogram of the estimated mutual information $I(X, Y)$ obtained from 300 independent realizations.

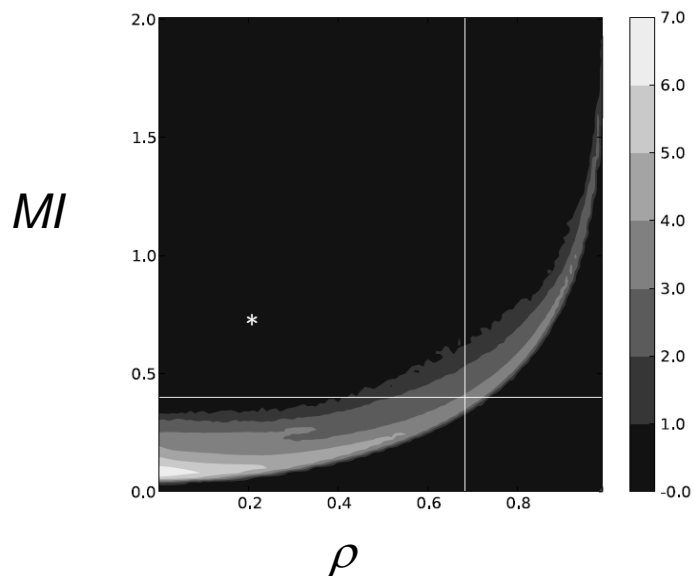
R. Steuer et al, Bioinformatics 18, suppl 2, S231 (2002).

Relation between cross-correlation and mutual information

- Depends on the data.
- If the two processes are Gaussian, the MI and the Pearson correlation coefficient are related as

$$MI = -1/2 \log(1-\rho^2).$$

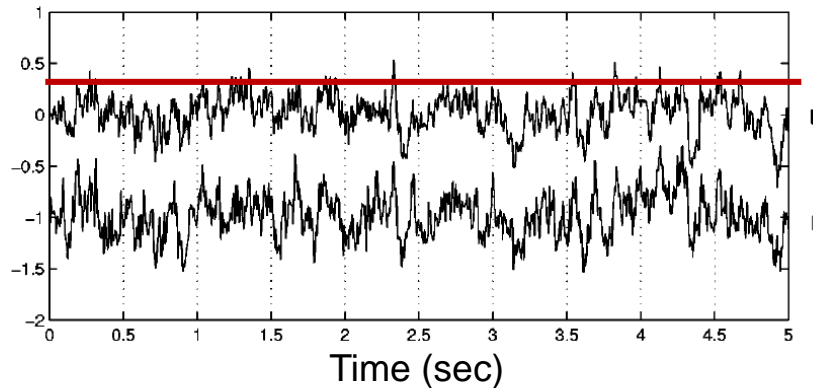
- Example: values computed from 6816 x 6816 surface air temperature (SAT) anomaly series:



2D histogram; the color represents the number of elements in each bin in log scale.

Donges et al, Eur. Phys. J. Special Topics 174, 157 (2009).

How to find “synchronized events” in two time series?



Rat EEG signals from right and left cortical intracranial electrodes. For a better visualization, the left signal is plotted with an offset.

- Define “events” in each time series.
- Count $c^\tau(x|y)$ = number of times an event appears in x shortly after an event appears in y . Analogous for $c^\tau(y|x)$.

- Calculate:

$$Q_\tau = \frac{c^\tau(y|x) + c^\tau(x|y)}{\sqrt{m_x m_y}} \quad q_\tau = \frac{c^\tau(y|x) - c^\tau(x|y)}{\sqrt{m_x m_y}}$$

m_x, m_y are the number of events in each time series.

- $Q_\tau = 1$: the events of the signals are fully synchronized.
- $q_\tau = 1$: the events in x always precede those in y .

Quian Quiroga et al, *PRE* 66, 041904 (2002).

Example

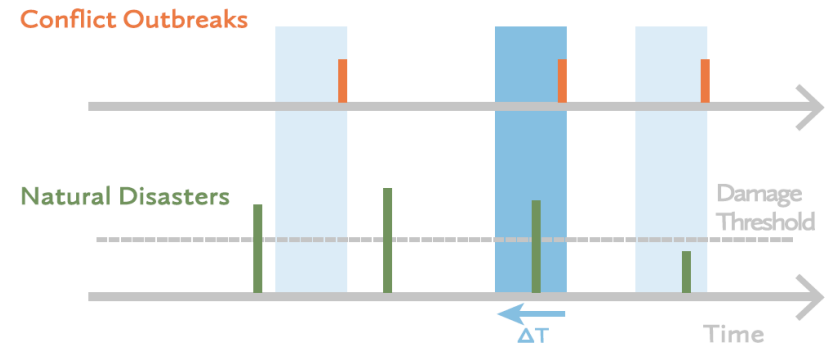
Armed-conflict risks enhanced by climate-related disasters in ethnically fractionalized countries

Carl-Friedrich Schleussner^{a,b,c,1}, Jonathan F. Donges^{a,d}, Reik V. Donner^a, and Hans Joachim Schellnhuber^{a,e,1}

^aPotsdam Institute for Climate Impact Research, 14473 Potsdam, Germany; ^bClimate Analytics, 10969 Berlin, Germany; ^cIntegrative Research Institute on Transformations of Human–Environment Systems, Humboldt University, 10099 Berlin, Germany; ^dStockholm Resilience Centre, Stockholm University, 114 19 Stockholm, Sweden; and ^eSanta Fe Institute, Santa Fe, NM 87501

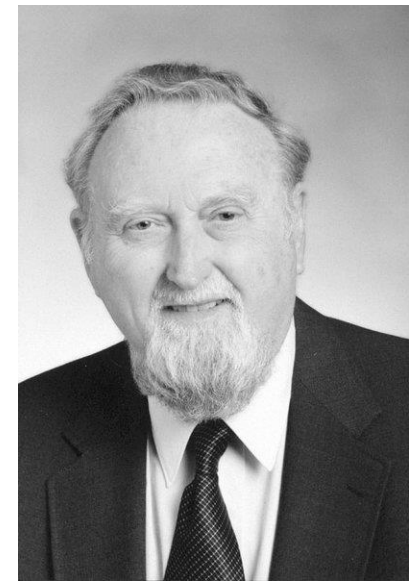
9216–9221 | PNAS | August 16, 2016 | vol. 113 | no. 33

Social and political tensions keep on fueling armed conflicts around the world. Although each conflict is the result of an individual context-specific mixture of interconnected factors, ethnicity appears to play a prominent and almost ubiquitous role in many of them. This overall state of affairs is likely to be exacerbated by anthropogenic climate change and in particular climate-related natural disasters. Ethnic divides might serve as predetermined conflict lines in case of rapidly emerging societal tensions arising from disruptive events like natural disasters. Here, we hypothesize that climate-related disaster occurrence enhances armed-conflict outbreak risk in ethnically fractionalized countries. **Using event coincidence analysis, we test this hypothesis based on data on armed-conflict outbreaks and climate-related natural disasters for the period 1980–2010.** Globally, we find a coincidence rate of 9% regarding armed-conflict outbreak and disaster occurrence such as heat waves or droughts. Our anal-



Granger Causality

Hypothesis: X_1 and X_2 can be described by stationary autoregressive linear models.



$$X_1(t) = \sum_{j=1}^p \text{past of } X_1 A_{11,j} X_1(t-j) + \text{Residual error } E_1(t)$$

$$X_1(t) = \sum_{j=1}^p \text{past of } X_1 A_{11,j} X_1(t-j) + \sum_{j=1}^p \text{past of } X_2 A_{12,j} X_2(t-j) + \text{Residual error } E'_1(t)$$

$$\text{If } \langle E'_1(t) \rangle < \langle E_1(t) \rangle \quad \longrightarrow \quad X_2 \rightarrow X_1$$

C. W. J. Granger *Investigating causal relations by econometric models and cross-spectral methods*. *Econometrica* 37, 424–438 (1969) (> 10000 citations)

Transfer Entropy (TE) and Directionality Index (DI)

- TE: is the *Conditional* Mutual Information, given the “past” of one of the variables.

$$TE(x,y) = MI(x, y|x_{\tau})$$

$$TE(y,x) = MI(y, x|y_{\tau})$$

- $MI(x,y) = MI(y,x)$ but $TE(x,y) \neq TE(y,x)$
- Directionality Index: $TE(x,y) - TE(y,x)$
- TE and GC are equivalent for Gaussian processes.
- TE can be computed from the probabilities of symbols (symbolic TE).

T. Schreiber, Measuring information transfer, Phys. Rev. Lett. 85, 461 (2000).

K. Hlaváčková-Schindler et al. / Physics Reports 441 (2007) 1–46

Problems:



In addition: Transfer Entropy is computationally demanding.

A “simple” solution: to use the expression that is valid for Gaussian processes [$MI = -1/2 \log(1-\rho^2)$]

Does this work? Check it out:

R. Silini, C. Masoller “*Fast and effective pseudo transfer entropy for bivariate data-driven causal inference*”, Sci. Rep. 11, 8423 (2021).

<https://doi.org/10.1038/s41598-021-87818-3>

Data Generating Processes and significance analysis

DGPs: We know whether X and Y are independent or not.

		Model		
Y	X	M0	$x_t = (0.01 + 0.5 x_{t-1}^2)^{0.5} + E_{1t}$	$y_t = 0.5 y_{t-1} + E_{2t}$
		M1		
		M2		
Y	→ X	M3	$x_t = 0.6 x_{t-1} + 0.5 y_{t-1} + E_{1t}$	$y_t = 0.5 y_{t-1} + E_{2t}$
		M4		
		M5		
		M6		
		M7		
		M8		
		M9		
		M10		
		M11		
		M12		
Y	↔ X	M13	$x_t = 0.15 x_{t-1} + 0.7 y_{t-1} + E_{1t}$	$y_t = 0.1 y_{t-1} + 0.8 x_{t-1} + E_{2t}$
		M14		

Significance analysis: time-shifted surrogates (cheap for causality testing)

Quiroga et al., Phys. Rev. E 65, 041903 (2002).

Results

Power: there is causality and we find causality (True Positives)

Size: there is no causality but we find causality (False Positives)

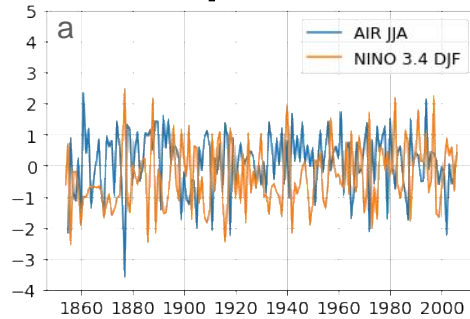
		Model	pTE					
			$Y \rightarrow X$	$X \rightarrow Y$				
$Y \quad X$	{	M0	3.8	3.9	✓			
		M1	2.3	2.6				
		M2	4.2	4.7				
$Y \rightarrow X$	{	M3	100	4.5	✓			
		M4	80.7	3.8				
		M5	100	2.2				
		M6	100	1.8				
		M7	100	2.8				
		M8	100	4.5				
		M9	100	0.1				
		M10	62.6	3.1		✗		
		M11	46.1	43.1				
		M12	99.9	1.0				
		$Y \Leftrightarrow X$	{	M13		100	100	✓
				M14		100	100	

Comparison with Granger Causality and Transfer Entropy

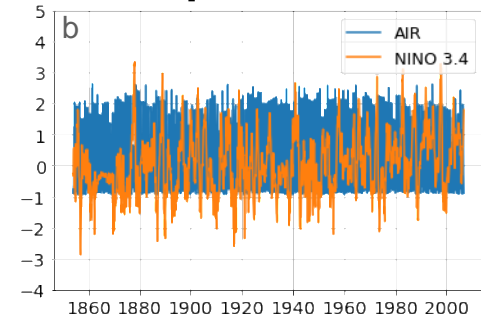
		Model	pTE		GC		TE		DI		
			$Y \rightarrow X$	$X \rightarrow Y$	$Y \rightarrow X$	$X \rightarrow Y$	$Y \rightarrow X$	$X \rightarrow Y$	pTE	GC	TE
$Y \rightleftharpoons X$	{	M0	3.8	3.9	5.1	5.0	4.4	4.4	-0.01	0.01	0.00
		M1	2.3	2.6	3.3	3.1	100	100	-0.06	0.03	0.00
		M2	4.2	4.7	5.5	5.9	4.7	4.9	-0.06	-0.04	-0.02
$Y \rightarrow X$	{	M3	100	4.5	100	4.8	70.2	5.6	0.91	0.91	0.85
		M4	80.7	3.8	84.2	4.9	96.0	4.7	0.91	0.89	0.91
		M5	100	2.2	100	3.1	100	3.8	0.96	0.94	0.93
		M6	100	1.8	100	2.8	100	4.3	0.96	0.95	0.92
		M7	100	2.8	100	3.4	100	4.0	0.95	0.93	0.92
		M8	100	4.5	100	5.6	100	100	0.91	0.89	0.00
		M9	100	0.1	100	0.1	100	100	1.00	1.00	0.00
		M10	62.6	3.1	67.3	4.3	12.2	4.5	0.91	0.88	0.46
		M11	46.1	43.1	53.1	49.8	37.8	45.0	0.03	0.03	-0.09
		M12	99.9	1.0	100	0.9	100	0	1.0	1.0	1.0
$Y \Leftrightarrow X$	{	M13	100	100	100	100	100	100	0.00	0.00	0.00
		M14	100	100	100	100	100	100	0.00	0.00	0.00

Application to real data NINO3.4 \leftrightarrow All India Rainfall

Yearly
sampled (152)



Monthly
sampled (1836)



pTE

NINO3.4 \rightarrow AIR

0.04 s

GC

NINO3.4 \rightarrow AIR

0.4 s

TE

NINO3.4 \leftrightarrow AIR

1 s

NINO3.4 \leftarrow AIR

0.5 s

NINO3.4 \leftarrow AIR

0.9 s

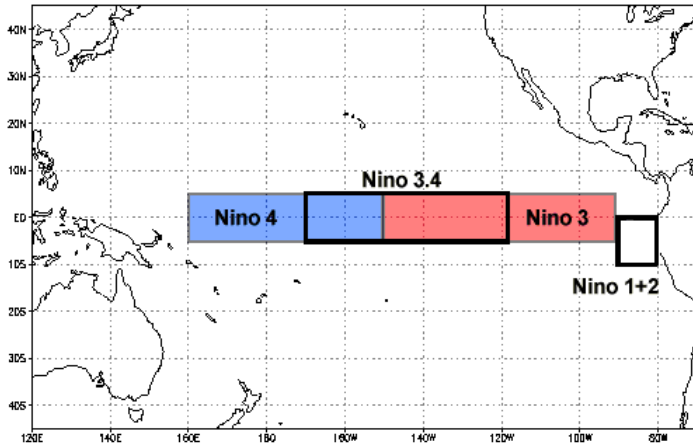
NINO3.4 \leftrightarrow AIR

40₉
3
68 s

How much time we save by using the “pseudo Transfer Entropy”?

For two time-series of 500 data points (1 data point per month, 40 years):

TE: **112 ms** but pTE: **4 ms**



8000 grid points (high resolution)
⇒ 64×10^6 pairs

⇒ **829 days** (TE) vs. **29 days** (pTE).

(without “surrogate” analysis)

But, there is a price to pay, no “free lunch”.

<https://github.com/riccardosilini/pTE>

Besides Granger causality and transfer entropy, many methods have proposed (and extensively used in social, economic and financial data).

- Symbolic Transfer Entropy
- Partial Correlation
- Partial Directed Coherence
- Cross Mapping
- Partial Cross Mapping
- Etc.

Read more: A. Krakovska et al., *Comparison of six methods for the detection of causality in a bivariate time series*, Phys. Rev. E 97, 042207 (2018)

Example

RESEARCH ARTICLE

INFORMATION SCIENCE

The dynamics of information-driven coordination phenomena: A transfer entropy analysis

Javier Borge-Holthoefer,^{1*†} Nicola Perra,^{2*} Bruno Gonçalves,^{3‡} Sandra González-Bailón,⁴ Alex Arenas,^{5*} Yamir Moreno,^{6,7,8*} Alessandro Vespignani^{2,8,9*}

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10.1126/sciadv.1501158

Data from social media provide unprecedented opportunities to investigate the processes that govern the dynamics of collective social phenomena. We consider an information theoretical approach to define and measure the temporal and structural signatures typical of collective social events as they arise and gain prominence. We use the symbolic transfer entropy analysis of microblogging time series to extract directed networks of influence among geolocalized subunits in social systems. This methodology captures the emergence of system-level dynamics close to the onset of socially relevant collective phenomena. The framework is validated against a detailed empirical analysis of five case studies. In particular, we identify a change in the characteristic time scale of the information transfer that flags the onset of information-driven collective phenomena. Furthermore, our approach identifies an order-disorder transition in the directed network of in-

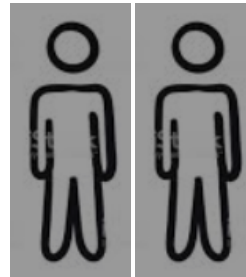
Borge-Holthoefer et al. Sci. Adv. 2, e1501158 (2016).

Methods of time series analysis are classified as:

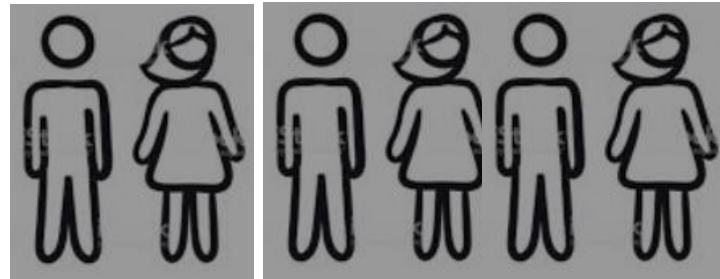
- Univariate analysis



- Bivariate analysis

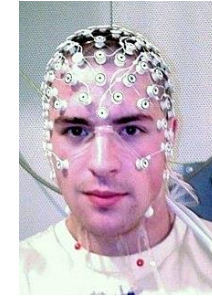
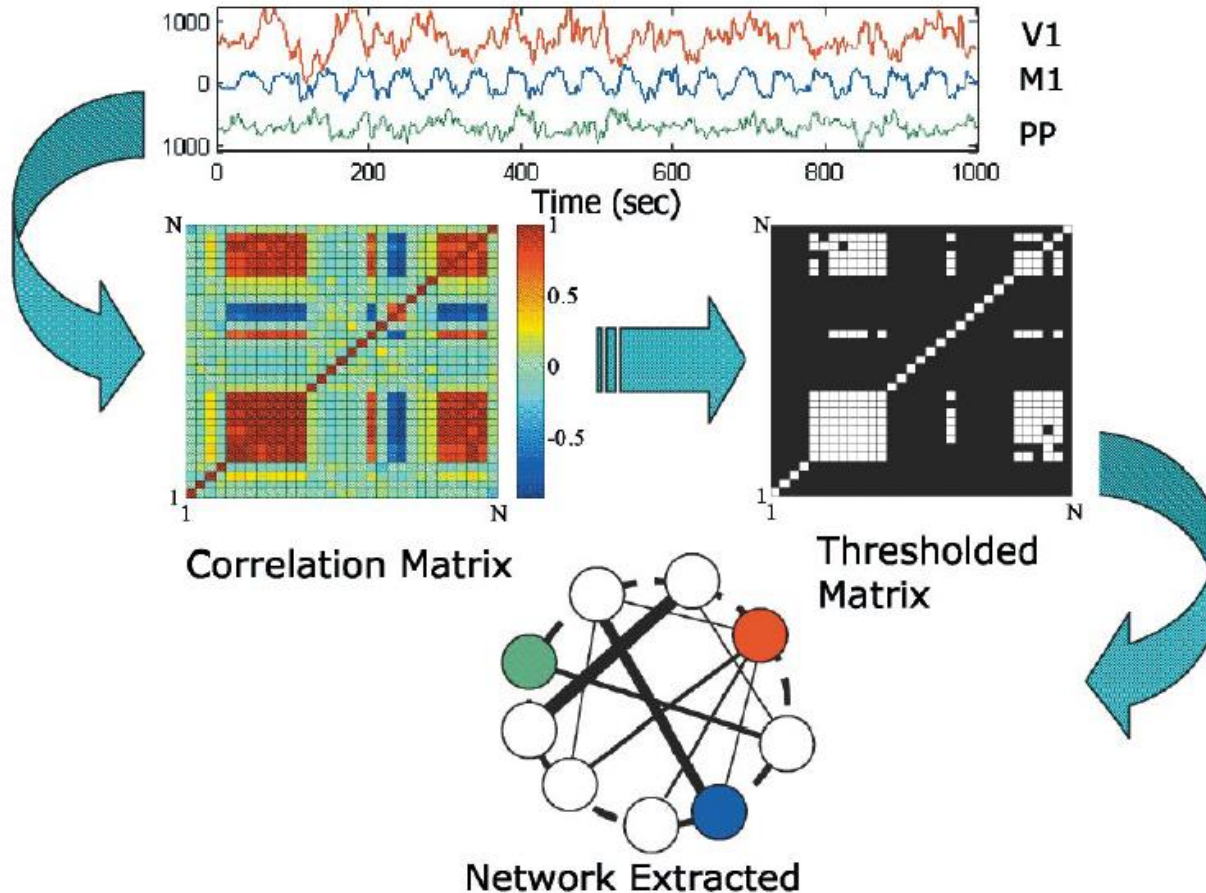


- Multivariate analysis
 - Functional networks
 - Network inference



“Functional networks” are obtained by using bivariate correlation or causality measures

Example: brain functional network

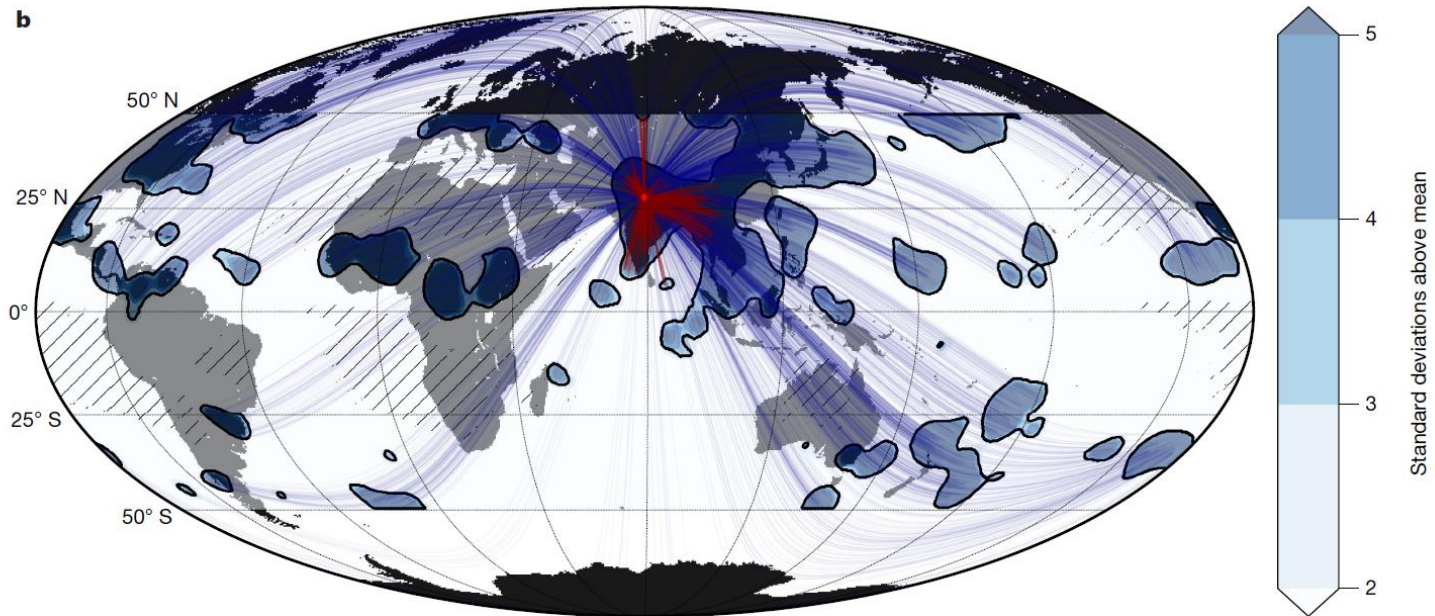


The adjacency matrix is obtained by “thresholding”

$$S_{ij} > Th \\ \Rightarrow A_{ij} = 1, \\ \text{else } A_{ij}=0$$

V. M. Eguiluz et al, *Phys. Rev. Lett.* 94, 018102 (2005).

Example: functional network constructed from extreme rainfall events, using the “event synchronization” measure



Teleconnection pattern for south-central Asia for events above the 95th percentile (with a maximum delay of ten days). Links shorter (longer) than 2,500 km are shown in red (blue).

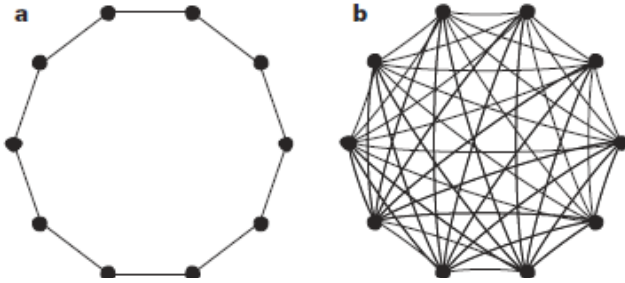
N. Boers et al, “Complex networks reveal global pattern of extreme-rainfall teleconnections”, Nature 566, 373 (2019).

How to characterize the graph?

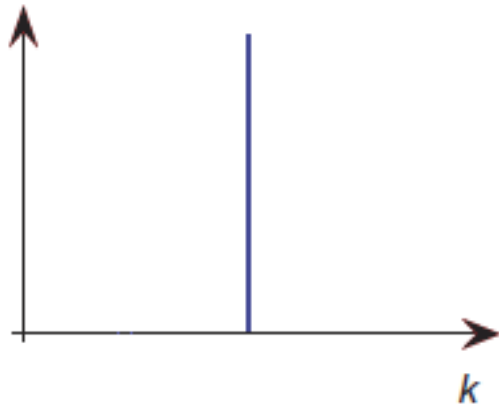
Begin with the degree distribution

Degree of node i : $k_i = \sum_j A_{ij}$

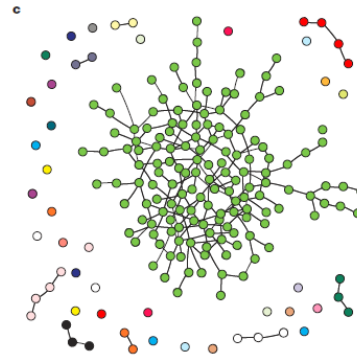
Regular



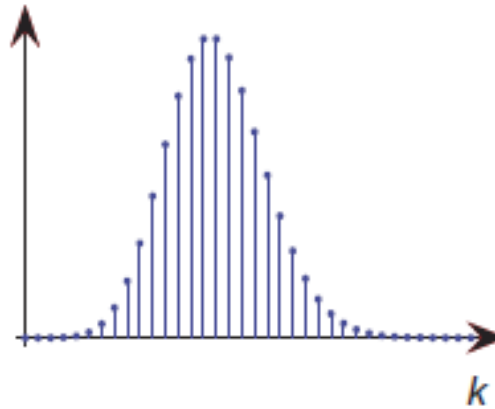
$P(k)$



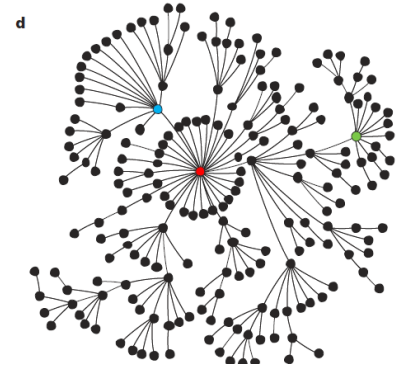
Random



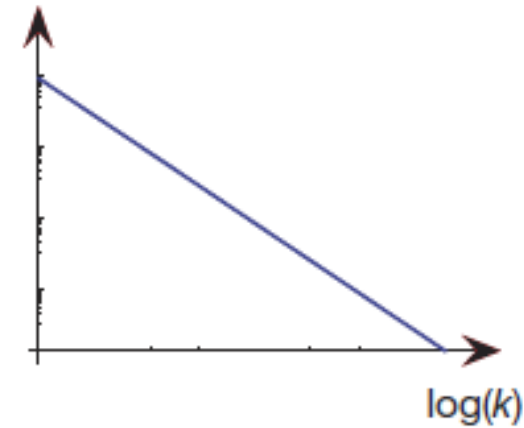
$P(k)$



Scale-free



$\log[P(k)]$



S. H. Strogatz, *Nature* 410, 268 (2001).

How to characterize the degree distribution?

- **Mean** degree: μ
- **Variance**: $\sigma^2 = \text{Var}(X) = E[(X-\mu)^2]$
- **Skewness**: $Z = \frac{X - \mu}{\sigma}$ $S = E[Z^3]$
- **Kurtosis**: measures the "tailedness" of the distribution.
Are there "hubs", i.e., nodes with lots of links?

$$K = E[Z^4]$$

How to compare two distributions?

Distance between two distributions P and P_e

Euclidean $D_E[P, P_e] = \|P - P_e\|_E = \sum_i (p_i - p_{i,e})^2$

Kullback $D_K[P, P_e] = K[P|P_e] = I[P_e] - I[P]$

Jensen divergence $D_J[P, P_e] = \frac{K[P|P_e] + K[P_e|P]}{2}$

S-H Cha: Comprehensive Survey on Distance/Similarity Measures between Probability Density Functions, Int. J of. Math. Models and Meth. 1, 300 (2007).

Other distributions that characterize a network (unweighted and undirected)

- **Clustering coefficient:** measures the fraction of a node's neighbors that are neighbors also among themselves

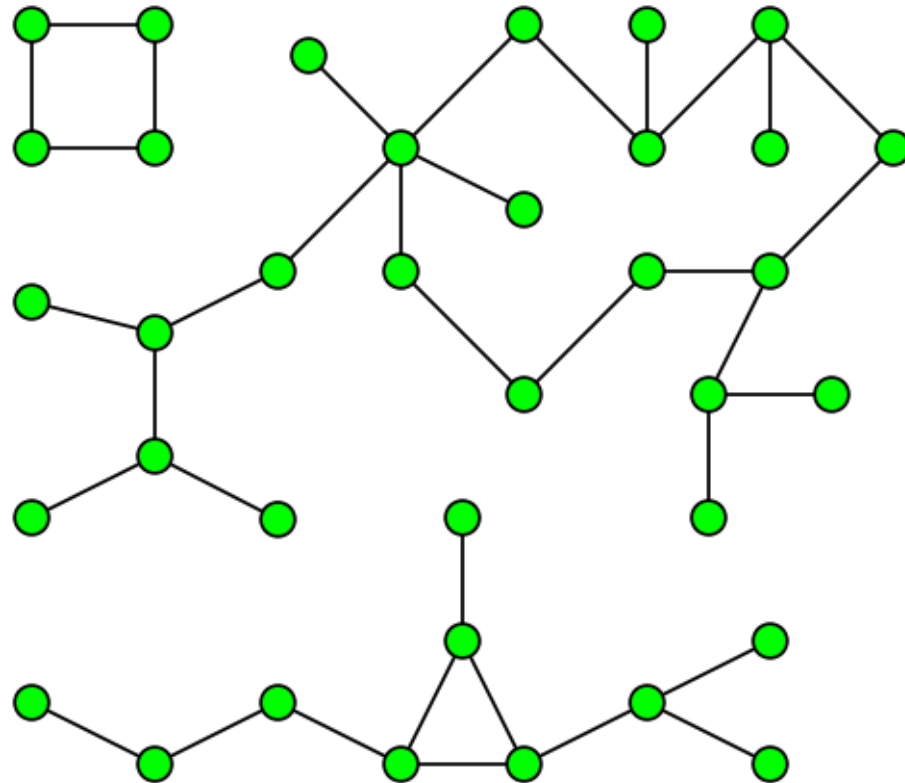
$$C_i = \frac{2R_i}{k_i(k_i - 1)} = \frac{1}{k_i(k_i - 1)} \sum_{j=1}^N \sum_{l=1}^N \mathcal{A}_{ij} \mathcal{A}_{jl} \mathcal{A}_{li}$$

R_i is the number of connected pairs in the set of neighbors of node i

- **Assortativity:** measures the tendency of a node with high/low degree to be connected to other nodes with high/low degree

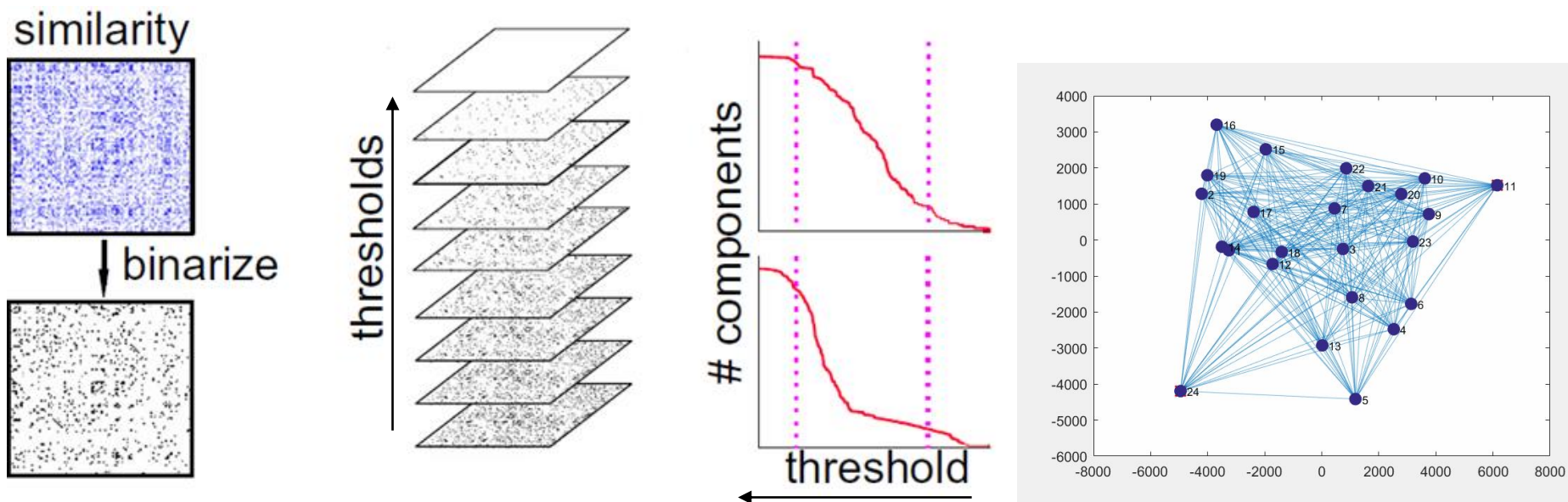
$$a_i \equiv \frac{1}{k_i} \sum_{j=1}^N \mathcal{A}_{ij} k_j$$

Number of connected components (“communities”)



A graph with three connected components.
Source: Wikipedia

Problems with networks obtained by thresholding



- The variation of the number of connected components with the threshold reveals different structures.
- Thresholding near the dotted lines would suggest inaccurately that these two networks have similar structures.
- Persistent “features” are “true” features.

Giusti et al., J. Comput. Neurosci. 4, 1 (2016).

Network inference: How to reconstruct the network from observations?

$$S_{ij} > Th \Rightarrow A_{ij} = 1 \text{ else } A_{ij}=0$$

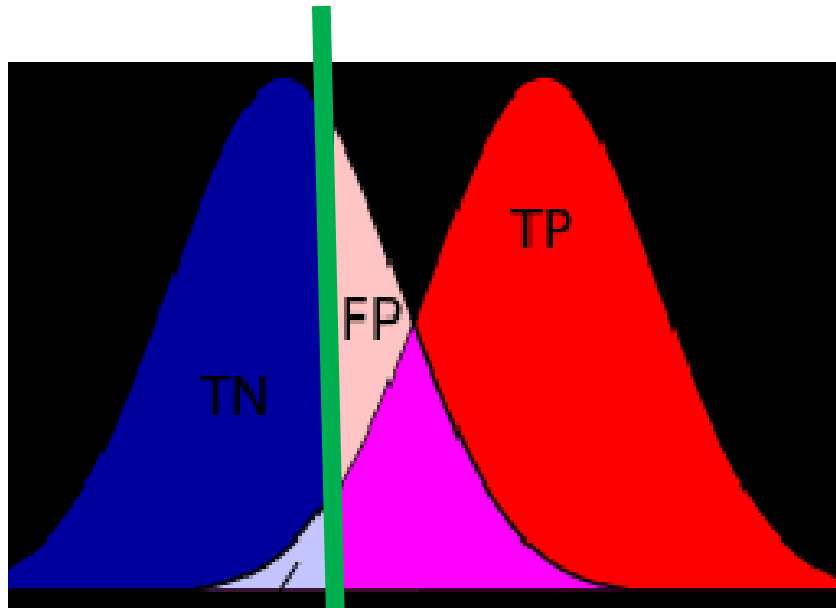
- How to select the “optimal” threshold?
- How to keep **weak-but-significant** links?
- A classification problem:
 - the interaction exists (is significant)
 - the interaction does not exist (or is not significant)

Confusion matrix

	Predicted: NO	Predicted: YES
Actual: NO	TN	FP
Actual: YES	FN	TP

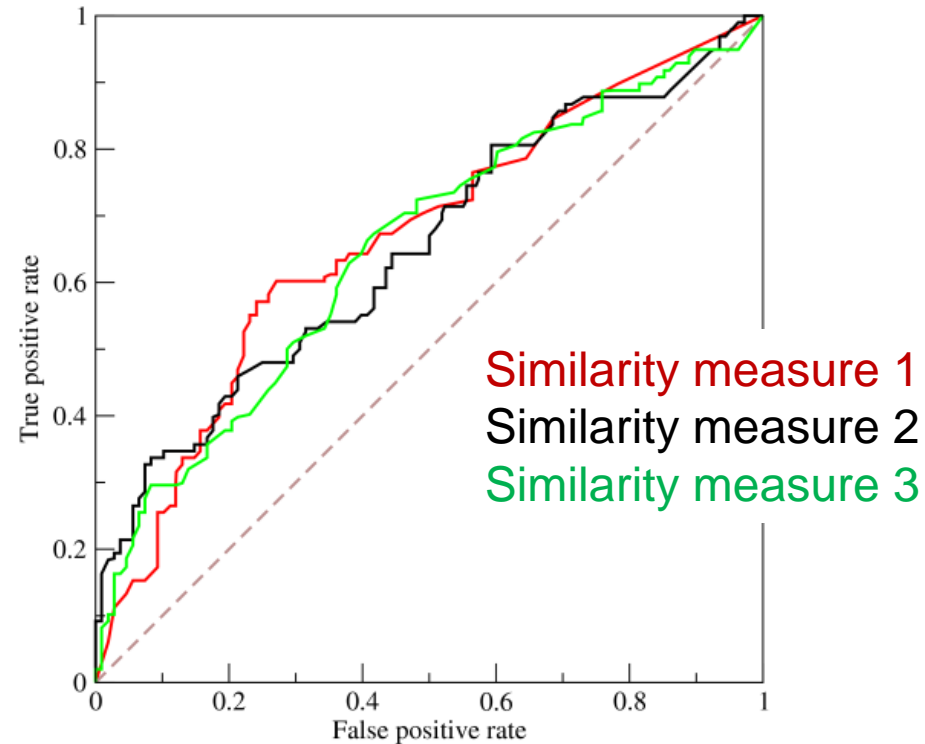
- **Accuracy**: How often is the classifier correct? **$(TP+TN)/total$**
- **Misclassification** (Error Rate): How often is it wrong? **$(FP+FN)/total$**
- **True Positive Rate** (TPR, Sensitivity or Recall): When it's yes, how often does it predict yes? **$TP/actual\ yes$**
- **False Positive Rate** (FPR) : When it's no, how often does it predict yes? **$FP/actual\ no$**
- **Specificity** ($1 - FPR$) : When it's no, how often it predicts no? **$TN/actual\ no$**
- **Precision**: When it predicts yes, how often is it correct? **$TP/predicted\ yes$**
- **Negative Predictive Value**: When it predicts no, how often is it correct? **$TN/predicted\ no$**
- **Prevalence**: How often does the yes condition actually occur in the sample? **$actual\ yes/total$**

Receiver operating characteristic (ROC curve) and Precision-Recall (PR curve)



Source: Wikipedia

TP	FP
FN	TN



For reconstructing sparse networks (with a small % of links) the “Precision-Recall” curve is more informative because it does not depend on the # of true negatives.

$$\text{Precision} = \text{TP} / \text{predicted yes (TP+FP)}$$

$$\text{Recall} = \text{TP} / \text{actual yes (TP+FN)}$$

How to compare the performance of different statistical similarity measures for inferring interactions from data?

- Use a “toy model” where **we know** the “ground truth”, i.e., we know the underlying equations and interactions and so we can check the performance of the different measures in inferring the interactions.
- Problem: results will depend on the “toy model” used as the performance of the statistical similarity measure depends on the characteristics of the data.

Kuramoto oscillators in a random network

$$d\theta_i = \omega_i dt + \frac{K}{N} \sum_{j=1}^N A_{ij} \sin(\theta_j - \theta_i) dt + D dW_t^i$$

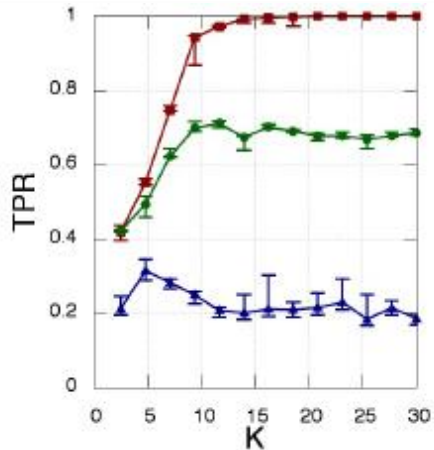
A_{ij} is a symmetric random matrix;
 $N=12$ time-series, each with 10^4 data points.

Phases (θ)

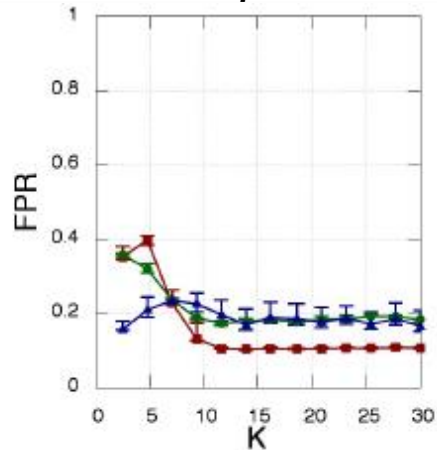
CC MI MIOP

“Observable” $Y=\sin(\theta)$

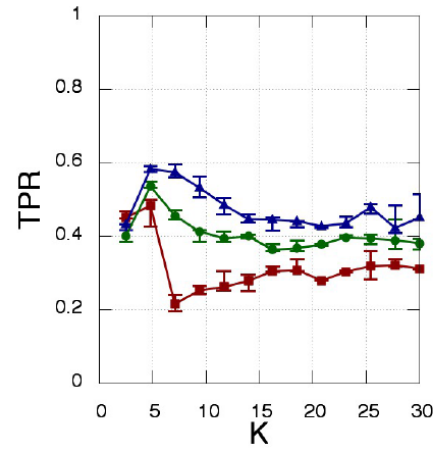
True positives



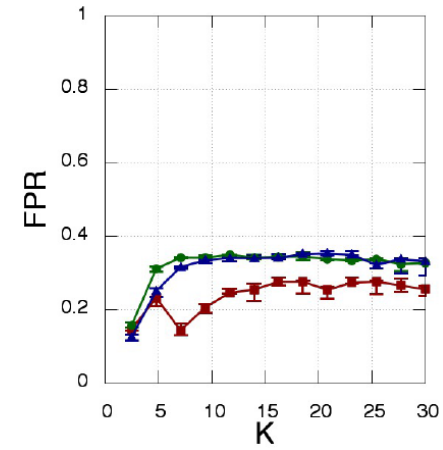
False positives



True positives



False positives

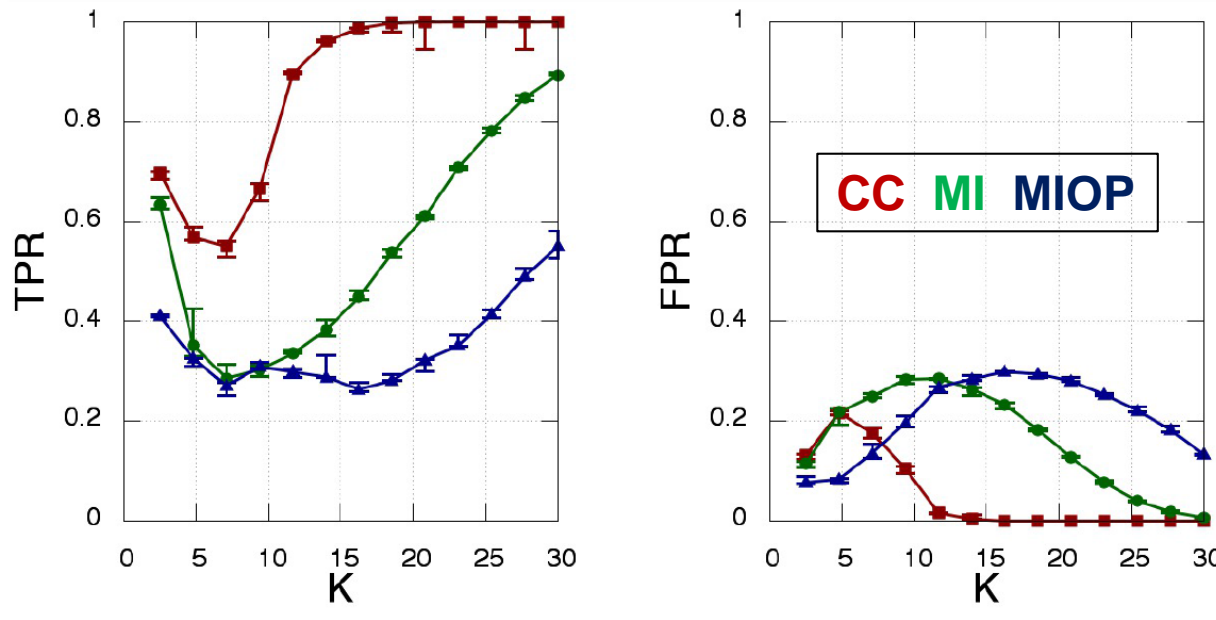


Results of a 100 simulations with different oscillators' frequencies, random matrices, noise realizations and initial conditions.

For each K , the threshold was varied to obtain optimal reconstruction.

G. Tirabassi et al, Sci. Rep. 5 10829 (2015).

Instantaneous frequencies ($d\theta/dt$)



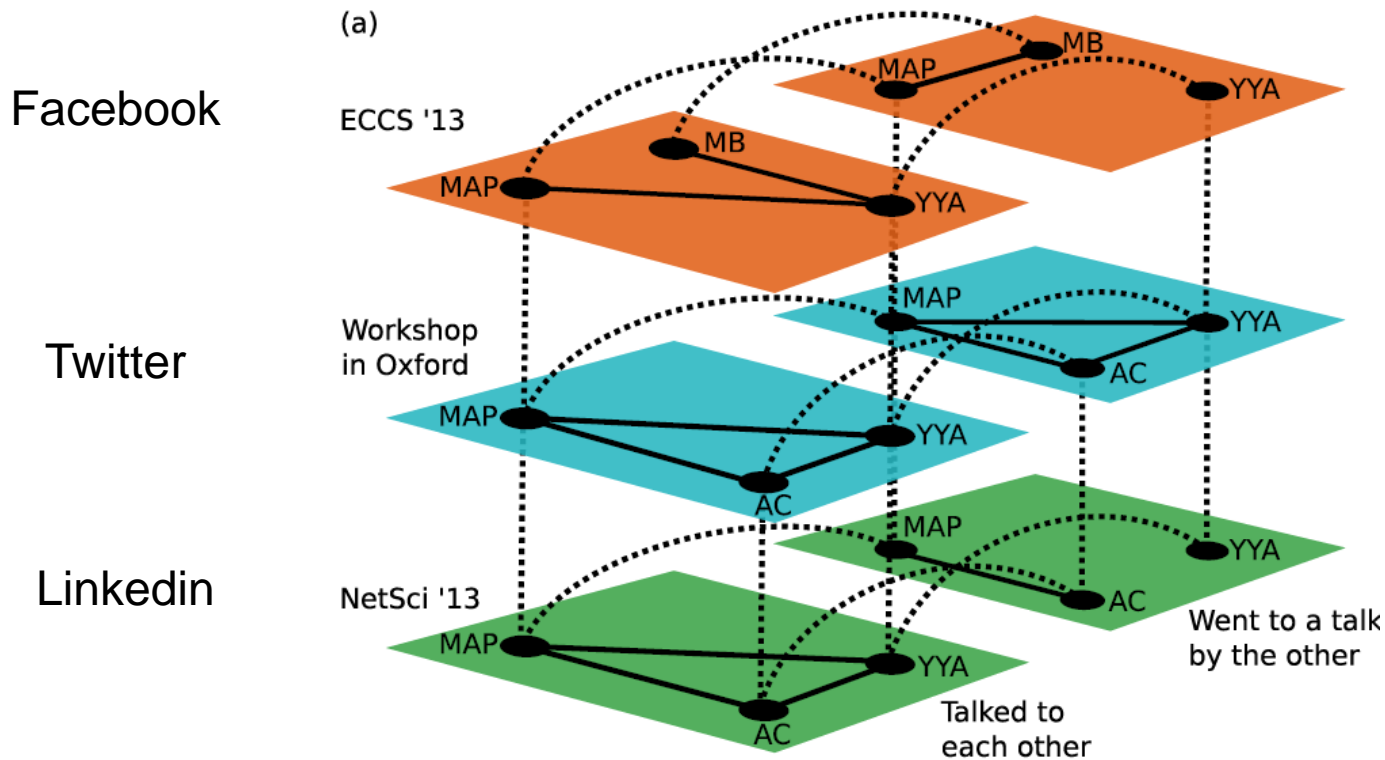
Perfect network inference is possible!

BUT

- the number of oscillators is small (12),
- the coupling is symmetric (\Rightarrow only 66 possible links) and
- the data sets are long (10^4 points)

G. Tirabassi et al, Sci. Rep. 5 10829 (2015).

Complicated structures (such as multilayer networks), require of advanced methods for characterization / reconstruction.



Kivela et al, J. Complex Netw. 2, 203 (2014).

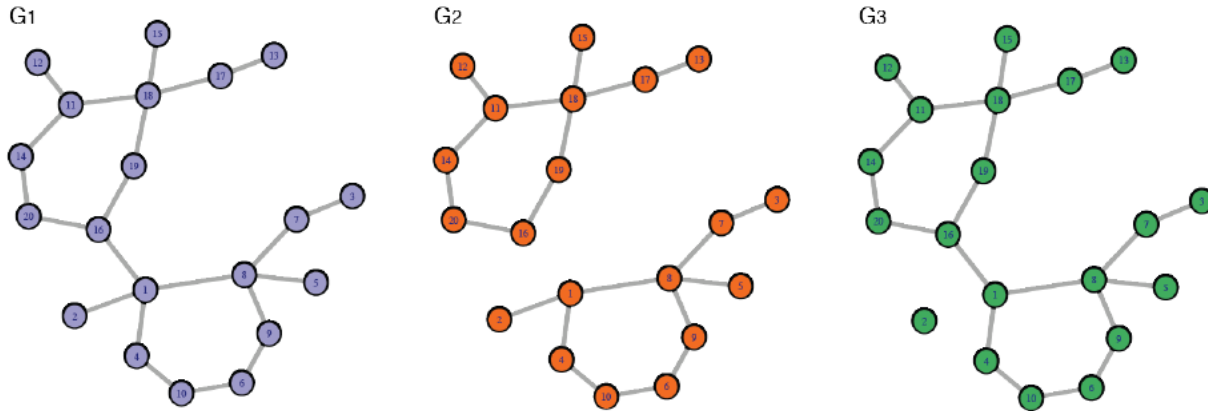
Multilayer networks in social systems and economy

Resume of topics and references		
Field	Topic	References
Social	Online communities	Pardus: [63, 421–424] Netflix: [425, 426] Flickr: [66, 88, 427] Facebook: [68, 428–430] Youtube: [431] Other online communities: [54, 89, 432] Merging multiple communities: [122, 123, 433, 434]
	Internet	[109, 110, 435]
	Citation networks	DBLP: [31, 33, 436–441] Scottish Community Alliance: [442] Politics: [68, 443]
	Other social networks	Terrorism: [23] Bible: [444] Mobile communication: [445]
Economy	Trade networks	International Trade Network: [70, 71, 450] Maritime flows: [451]
	Interbank market	[452]
	Organizational networks	[453–455]

Boccaletti et al., Phys. Reps. 544, 1 (2014).

How similar are two graphs?

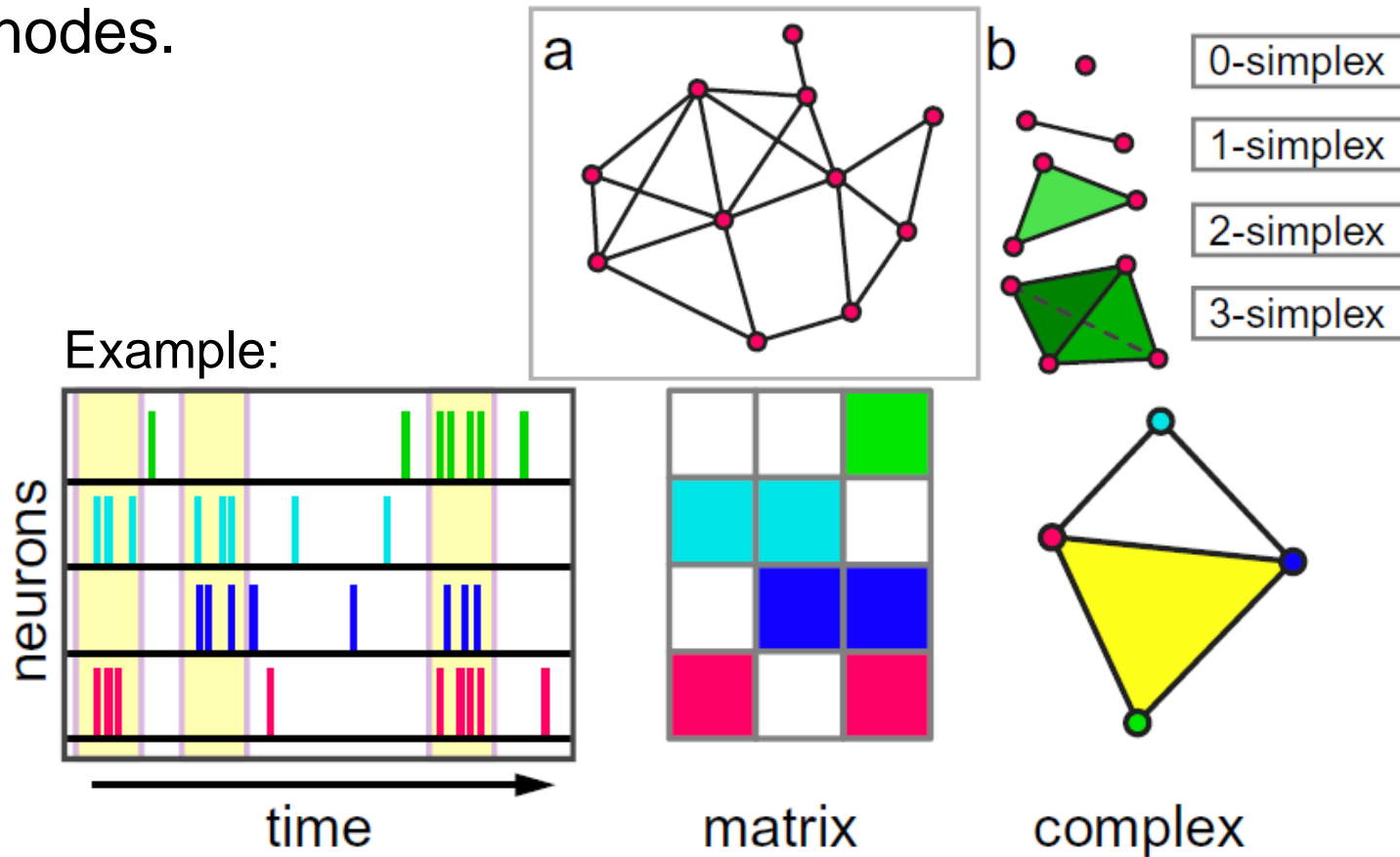
- Hamming distance $d_{\text{Hamming}}(\mathbf{y}_1, \mathbf{y}_2) = \sum_{i \neq j}^N [A_{ij}^{(1)} \neq A_{ij}^{(2)}]$
- Can be used to compare two graphs of the same size.
- Main problem: not all the links have the same importance.



- A “dissimilarity” measure that can be used to compare graphs with different sizes, based in the comparison of distributions extracted from the graphs: Schieber et al, Nat. Comm. 8, 13928 (2017).

A basic limitation of network analysis

- Links represent interactions between pairs of nodes.
- **Simplicial complexes** represent interactions among several nodes.



Giusti et al., *J Comput Neurosci* 41, 1 (2016).

Battiston et al., *Phys. Rep.* 874, 1–92 (2020).

Summary

- Multivariate analysis uncovers inter-relationships in datasets
- Different similarity measures are available for inferring the connectivity of a complex system from observations.
- Different measures can uncover different properties.
- Thresholding, unobserved variables, hidden “nodes” can difficult or make impossible the inference of the network structure.
- Network science: fast evolving field, with many applications and challenges!

Collaborators, references and **announcement**



Andrés Aragoneses



Carlos Quintero



Riccardo Silini



Giulio Tirabassi

G. Tirabassi et al, Sci. Rep. 5 10829 (2015).

A. Aragoneses et al., Phys. Rev. Lett. 116, 033902 (2016).

M. Panozzo, C. Quintero-Quiroz et al, Chaos 27, 114315 (2017).

C. Quintero-Quiroz et al., Chaos 28, 106307 (2018).

R. Silini and C. Masoller, Sci. Rep. 11, 8423 (2021).

<http://www.fisica.edu.uy/~cris/>



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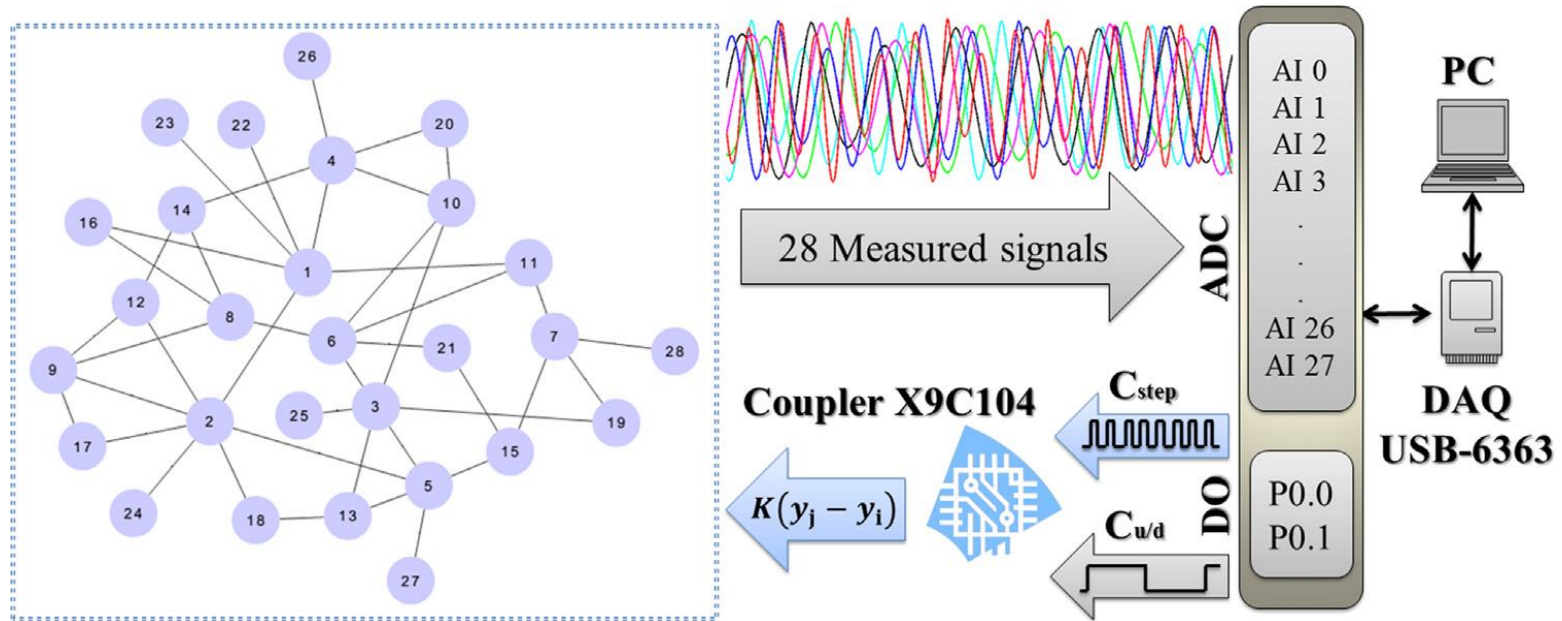


cristina.masoller@upc.edu



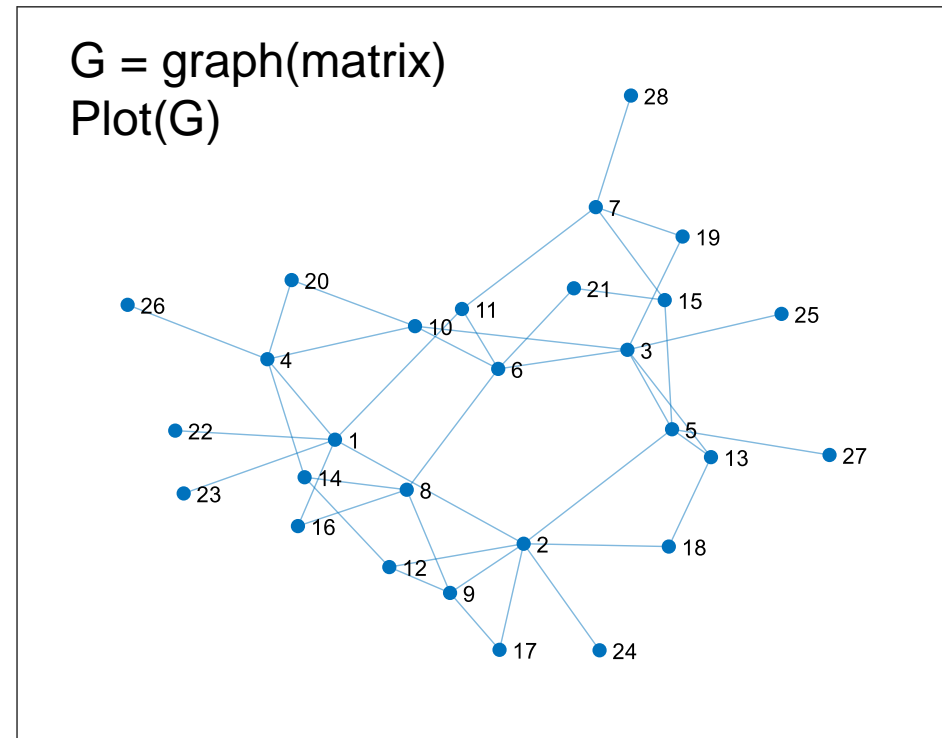
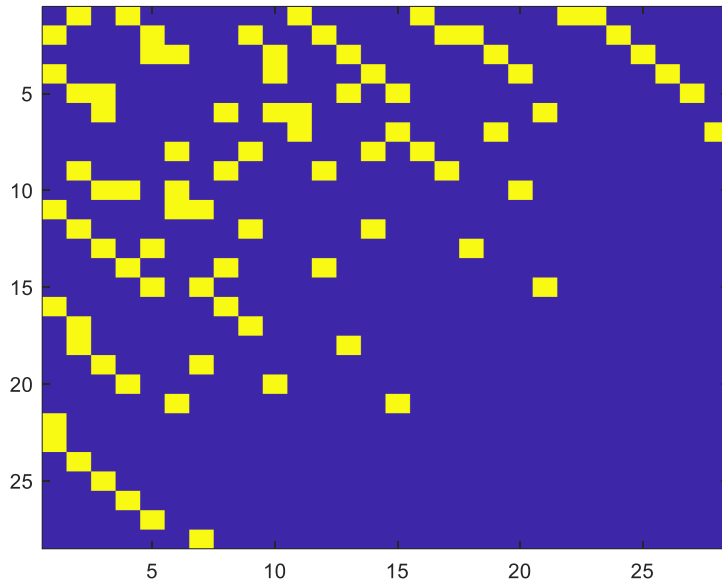
[@cristinamasoll1](https://twitter.com/cristinamasoll1)

Hands-on exercise 4: analyze a random network of chaotic Rossler-like electronic oscillators



Reference: [Data in Brief 7 \(2016\)1185–1189](#)

1. Represent graphically the structural network.
2. Calculate the degree distribution of the structural network.
3. Reconstruct the network using the Pearson coefficient ($\rho = |CC(0)|$) as statistical similarity measure, keeping only the 10% strongest links.
4. Reconstruct the network using the threshold condition $\rho > 0.5$.
5. Calculate the mean degree as a function of the threshold $\rho_{th} = 0 \dots 1$.



Mean degree = number of links / number of nodes

Ts_25, only first 5000 datapoints

