

Frequency dynamics of semiconductor lasers with atomic absorbers: theory and experiments

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Abstract. In this paper we discuss the spectral behavior of monomode semiconductor laser systems whose output amplitude is constant, but whose frequency may be bistable, multistable, locked or present instabilities. The explored configurations are: (i) a laser diode (Fabry-Pérot) under orthogonal filtered optical feedback; and (ii) an extended-cavity diode laser with an intracavity strongly-saturated resonant vapor. Starting from rate equations for the carrier density and for the radiation field oscillating in the cavity of these systems we describe spectral features which are in very good quantitative agreement with experimental observations.

1 Introduction

In the last decades many interesting non linear dynamical behaviors in optical cavities have been proposed and/or demonstrated in different configurations [1]. Bistability, multi-stability, synchronization, and chaotic behaviors have been studied in systems such as lasers under optical feedback [2]; optical cavities pumped by external sources [3–5]; laser cavities with saturable absorbers [6,7], etc. Electromagnetic fields allow coding information in their amplitude and phase. However, in the optical domain the observed variable has been mainly the radiation amplitude. More recently the study of the spatial component of the phase has been the subject of work of many research groups, seeking the control of spatial optical patterns [8]. However, analysis of the strictly temporal frequency dynamics are scarce in the literature, probably due to practical difficulties to produce spectral dynamics without perturbing the light amplitude. For instance, in 2004 Lenstra and collaborators [9] reported the first observation of the frequency oscillation of a laser, induced by spectrally filtered optical feedback. Such a frequency dynamical behavior was obtained in a regime of stable output power, paving then the way for FM applications in the optical domain.

Further experimental results yielding spectral dynamic behavior were obtained thanks to the use of orthogonal optical feedback [10,11] (i.e., reinjecting light whose polarization is orthogonal to that of the lasing field in the cavity). The strategy in using this configuration is to induce

changes in the carrier density without interfering with the cavity oscillating field. A theoretical model [12], taking into account the temporal evolution of the field amplitude and of the carrier density, together with an equation for the temperature evolution and nonlinear gain terms, allows one to describe the linear response of the laser frequency. Moreover, it accounts for the system dynamical behavior when a nonlinear filter is introduced into the feedback loop. This model also brings new insight on the behavior of semiconductor lasers under filtered orthogonal feedback, later observed in the laboratory [12].

In the search for producing exclusively frequency controlled laser emission, an *intracavity* resonant vapor acting as a nonlinear absorber has also been employed [13]. This type of system has already been used in studies of the amplitude dynamics [14]. However, in [13] the idea has been to use a strongly saturated medium in order to have negligible absorption (therefore no amplitude modulation), while still being able to modify the field phase, i.e., the laser frequency [13]. A theoretical model [15] for the dynamics of a semiconductor laser with an intracavity resonant vapor was developed with the usual rate equations for the carrier and the radiation field and taking into account the propagation and attenuation of the field in the atomic vapor volume. The results account for the very low attenuation observed in the laser intensity and describe the hysteretic cycles measured in the laser frequency.

In the present work, we discuss in detail the observations of the frequency dynamics of laser diodes under filtered orthogonal feedback and of lasers having both a

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1 semiconductor junction as a gain medium and a selective
 2 absorber inside their cavity. We also present numerical
 3 results obtained from simulations of the model proposed
 4 in [12], that are in good agreement with the frequency
 5 multistability observed in [11], in a setup where the ortho-
 6 gonal feedback is filtered by an atomic vapor with a
 7 dispersive lineshape.

8 2 Control of laser frequency 9 with orthogonal light

10 The dynamical behavior of diode lasers has been studied
 11 using the nonlinearity of the semiconductor itself. In those
 12 experiments, part of the laser output (in either coherent
 13 or incoherent regimes of the phase delay) is returned into
 14 the laser cavity. The response of the laser diodes depends
 15 on the delay introduced by the feedback loop compared
 16 to the radiation coherence time [16,17]. Acting only on
 17 the carrier density is another strategy to explore laser dy-
 18 namics and may be achieved by way of a high frequency
 19 *electronic* feedback [18] or through the use of an *optical*
 20 orthogonal polarization feedback. This last technique is
 21 discussed in this section.

22 A stable output intensity is observed in lasers under
 23 orthogonal feedback when the operation current is higher
 24 than about two times the laser threshold current [19,20].
 25 In this stable output regime, the frequency ν of the laser
 26 emission varies linearly with the light power P_f , sent back
 27 into the laser cavity [12],

$$\nu = \nu_{sol} + \beta P_f, \quad (1)$$

28 where ν_{sol} is the frequency of the solitary laser and β is
 29 the coefficient of the orthogonal power coupling into the
 30 gain volume. Such a linear frequency shift as a function
 31 of the feedback power is observed in a relatively large fre-
 32 quency range, up to 10 GHz. Further efforts to improve
 33 the coupling of the feedback beam into the laser gain vol-
 34 ume of sub-micrometric transversal section should result
 35 in larger values of the β coefficient.

36 2.1 Optical locking of a laser on an atomic line 37 through orthogonal feedback

38 The laser response to orthogonal feedback may be used
 39 to stabilize the laser frequency, sending part of the output
 40 beam through a frequency discriminator (i.e., a frequency-
 41 to-amplitude converter) and then sending it back into the
 42 laser with an orthogonal polarization. Atomic resonances
 43 are convenient spectral filters because they are absolute
 44 references. An experimental set-up is schematically shown
 45 in Figure 1, where orthogonal feedback is used to stabilize
 46 a laser diode frequency on an atomic line. The laser diode
 47 is a monomode AsAlGa Fabry-Pérot type with a linewidth
 48 of 40 MHz. Its junction is stabilized in current and tem-
 49 perature, which reduces the drift of the laser line center.
 50 The laser operation current is about 100 mA, resulting in
 51 an output power of 45 mW. The emission frequency may

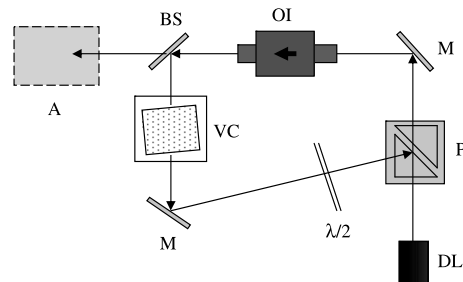


Fig. 1. (Color online) Set-up. Control of a laser diode frequency by an orthogonal filtered feedback. Part of the laser beam, modulated by the Cs vapor resonance, has its polarization rotated by a half-wave plate and is reflected toward the laser by a high-rejection polarizer. DL: laser diode; P: polarizer; M: mirror; OI: optical isolator; BS: beam splitter; VC: vapor cell; A: analysis device (Fabry-Pérot or atomic vapor cell transmission).

be scanned around 852 nm, hence probing the Cs D_2 resonance transition ($6S_{1/2}-6P_{3/2}$), the line of the spectral filter used in this experiment.

Scanning through the atomic resonance, one can follow the response of the laser frequency to the feedback modulated by the filter transmission (an absorption line will feed more or less power back into the laser depending on the frequency) and observe the system avoiding the unstable flank and tending to stay on the stable side of the absorption line (see Fig. 2). In this simple configuration, the laser may be coupled to a point on the stable side of the atomic line and self-correct the laser frequency drifts [21], because variations to higher (lower) frequencies will be followed by the increase (decrease) of the feedback level. Notice that this is an intrinsically fast and simple stabilization technique, where the phase of the field fed back to the laser does not need to be controlled, as it should be in coherent locking methods [22,23]. The optimized response of this locking system depends on the coupling of the feedback beam into the laser gain medium and on the filter lineshape, both determining the gain of the feedback loop. The resulting frequency shift is given by

$$\nu - \nu_{sol} = \beta[(1 - \epsilon f(\nu)]P_o, \quad (2)$$

where ϵ is the amplitude coefficient of the filter normalized lineshape, $f(\nu)$, and P_o is the out-of-resonance feedback power. With this setup, a reduction in the laser frequency drift by a factor of four was reported in [21].

It should be noticed that equation (2) holds when the laser dynamics is stable, i.e., when both the laser frequency and the output power are constant in time; in a dynamic situation, the orthogonal feedback power depends on both the delayed value of the lasing intensity, $P(t - \tau)$, and the delayed value of the frequency, $\nu(t - \tau)$, where τ is the delay time in the external cavity ($\tau = 2L/c$, with L being the length of the external cavity and c the velocity of light) [12].

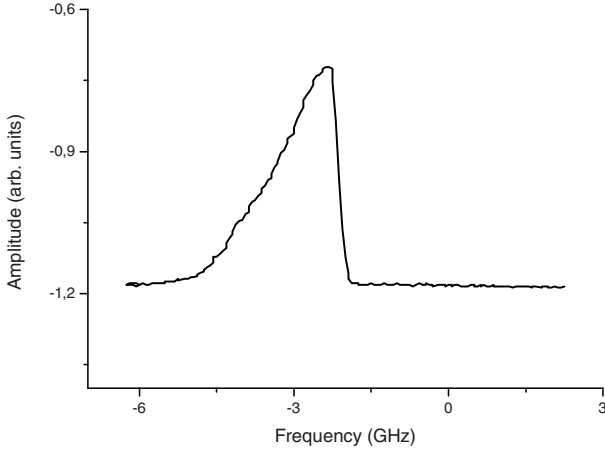


Fig. 2. Modified absorption of the Cs D_2 line (linear absorption in the analysis cell, Fig. 1) as a signature of the laser frequency dynamical behavior, due to the filtered orthogonal feedback. The frequency scale is centered on the atomic line resonance when the laser is feedback-free.

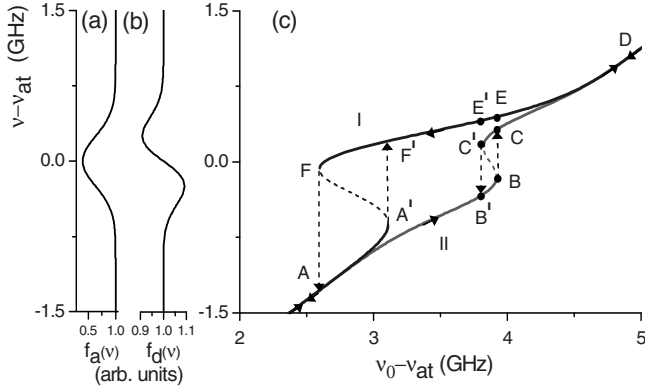


Fig. 3. (Color online) (a) Absorptive and (b) dispersive filter lineshape. (c) Frequency (ν) of the laser under feedback versus the free laser frequency (ν_0), both centered in the atomic frequency, ν_{at} . Multistability cycle of the laser frequency due to the filtered orthogonal feedback from an absorptive filter (blue line, cycle AA' F'D and DFA). A dispersive (ABCD and DC'B'A) or a mixed role of the two filters may produce multiple states as B'B, C'C and E'E. The dotted parts of the blue and red curves are the regions avoided by the system, yielding bistability zones.

2.2 Hysteresis and bistability

One should expect spectral filters to induce nonlinear responses in the laser frequency. Indeed, frequency-dependent orthogonal feedback (FOF) results in frequency hysteresis and therefore in the possibility of multistability, depending on the filter lineshape, as shown in Figure 3. Scanning back and forth across the absorptive atomic line, one can observe bistable states for the laser frequency [10]. Moreover, the use of both the absorptive (Fig. 3a) and dispersive (Fig. 3b) responses of the atomic medium, which may be manipulated by a second laser field (not in the scheme of Fig. 3), leads to an enormous number of possible configurations and effects in the laser spectral response.

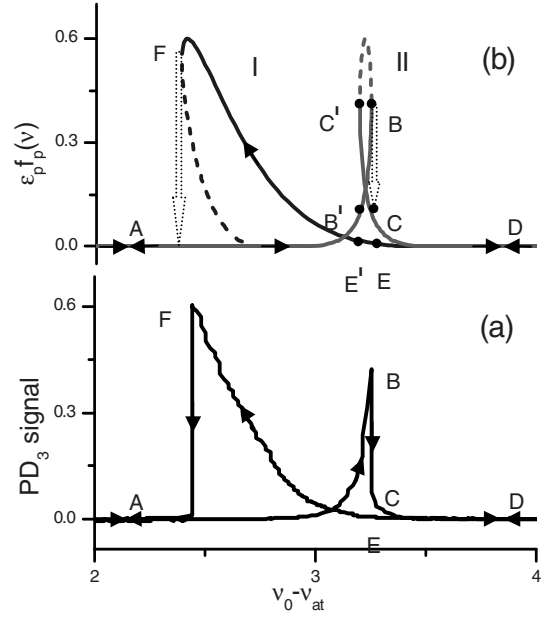


Fig. 4. (Color online) (a) Experimental and (b) calculated transmission of the analysis cell, where the probe laser is under filtered orthogonal feedback. The filter does both dispersive and absorptive-like modulations on the beam returning into the laser. Capital letters mark points correspondent to those of Figure 3.

Figure 3c clearly shows the origin of multistabilities by indicating the trajectories of the system as it avoids instabilities (negative derivatives), resulting from the modulation of the feedback beam by a resonant medium with complex refractive index.

The analysis of the spectral behavior of lasers under FOF is made through a frequency discriminator, such as a Fabry-Pérot interferometer. More conveniently, a cell of analysis containing a vapor of the same atomic element than the one of the feedback filter allows the measurement of the spectral dynamics (see Fig. 2), without the need of adjusting the frequency of this analyzer. Figure 4a shows spectra of frequency multistability, measured through an absorptive line, the Doppler-broadened Cs D_2 line at room temperature (analysis device displayed in Fig. 1). Figure 4b shows the calculated laser frequency analyzed through a Gaussian frequency-amplitude converter.

The results of Figures 3 and 4 are obtained for a Doppler-broadened Cs D_2 line as a filter, which has a linewidth of about twenty times the 40 MHz laser linewidth. However, narrower lasers are commercially available, with linewidth (few MHz) comparable to the linewidth of resonant atomic transitions such as the alkali atoms D_2 line of about 5 MHz. Therefore, as an experimental extension of the results presented above, coherent resonance [24,25] may be explored with highly contrasted electromagnetically induced transparency or absorption (EIT or EIA). An example is shown in Figure 5 for an EIT lineshape (Fig. 5a), where it has been considered that the filter is interacting with a second field coherent with the feedback one. The frequency of the coupled laser will

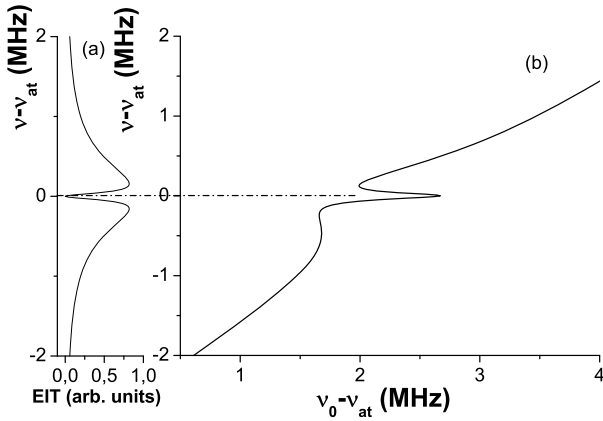


Fig. 5. (a) Electromagnetically induced transparency (EIT) signal. (b) Calculated response of the laser frequency under orthogonal feedback filtered by the spectrum in (a), as a function of the solitary laser frequency ν_0 .

1 follow a non linear trajectory, as depicted in Figure 5b,
 2 as a function of the free laser frequency. The sharp varia-
 3 tion of this curve represents, for instance, a high gain for
 4 a stabilization loop. The trajectory stretches with nega-
 5 tive derivative are avoided by the system and may result
 6 in bistable cycles. Let us stress here that magnetic sub-
 7 levels of the atomic transition may also be explored [26]
 8 in a more complex and rich configuration.

9 Another extension of this study of the spectral behav-
 10 ior of lasers under filtered orthogonal feedback is to use
 11 the rate-equation model developed in [12] to describe the
 12 multistable behavior provoked by a dispersive filter (in [12]
 13 an absorptive filter was considered). The rate equations
 14 for the electromagnetic field, carrier density and junction
 15 temperature are given by:

$$\frac{d\mathcal{E}}{dt} = ik\theta\mathcal{E} + k(1 + i\alpha)(\Gamma G - 1)\mathcal{E}, \quad (3)$$

$$\frac{d\mathcal{N}}{dt} = -\gamma_N [\mathcal{N} - J + G(\mathcal{P} + \mathcal{P}_f)], \quad (4)$$

$$\frac{dT}{dt} = -\gamma_T(T - 1 - Z\mathcal{N} - PJ^2), \quad (5)$$

16 together with a saturated gain

$$G(\mathcal{N}, \mathcal{P}, \mathcal{P}_f) = (\mathcal{N} - 1)/(1 + \epsilon_1\mathcal{P} + \epsilon_2\mathcal{P}_f). \quad (6)$$

17 In equations (4) and (6) \mathcal{P} represents the density of pho-
 18 tons with TE polarization, $\mathcal{P}(t) = |\mathcal{E}(t)|^2$, \mathcal{P}_f represents
 19 the density of photons with TM polarization,

$$\mathcal{P}_f(t) = \kappa|\mathcal{E}(t - \tau)|^2, \quad (7)$$

20 where κ is the feedback strength and τ is the delay time,
 21 $\tau = 2L/c$, with L being the length of the external cavity
 22 and c the velocity of light. ϵ_1 and ϵ_2 are self- and cross-
 23 saturation coefficients respectively.

24 Also, the model assumes a linear variation of the laser
 25 detuning with the temperature of the semiconductor junc-
 26 tion,

$$\theta = \theta_0 - \epsilon(T - 1), \quad (8)$$

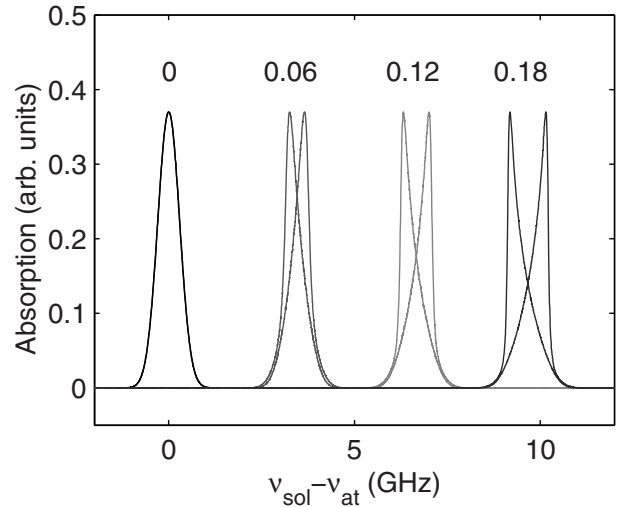


Fig. 6. (Color online) Calculated laser response to an orthog-
 onal feedback modulated by a dispersive filter, as in Figure 3b,
 for different feedback strengths, as indicated in the figure.

where ϵ is the coupling strength between the optical field
 and the temperature. For a detailed discussion of the
 model equations and parameter values, see [12].

From the rate equations above we calculate the laser
 response (Fig. 6) to a modulation with a *dispersive* line-
 shape filter (see Fig. 3b). The calculated laser frequency is
 analyzed through a Gaussian frequency discriminator and
 can thus be compared with experimental spectra (Fig. 7),
 which are done through measurements of an analysis cell
 transmission (see Fig. 1 and Ref. [11] for experimental de-
 tails). Here we point out differences between calculated
 and experimental curves (Figs. 6 and 7, respectively): the
 experimental high frequency peak is narrower and smaller
 than the calculated one. These features are due, on the
 one hand, to the experimental contribution of the filter
 absorptive modulation, that becomes more important at
 higher feedback levels; and, on the other hand, to the laser
 linewidth that “averages” narrow structures, resulting in
 an effective smaller peak.

3 Resonant filter inside the laser cavity

The dynamic behavior of lasers with intracavity absorbers
 has been studied mainly for molecular CO₂ lasers [27,28].
 Semiconductor lasers have also been built with two sec-
 tions having independent current supplies [29]. Amplitude
 dynamics were observed, particularly oscillations (self pul-
 sation) due to cycles of absorption, saturation and gain
 recovery. Laser diodes under filtered feedback have been
 theoretically and experimentally studied by Arimondo and
 collaborators [30], where they used an atomic vapor to fil-
 ter the power sent back into the laser. Multimode opera-
 tion was observed as a result of the laser coupling to this
 frequency selective feedback. However, a laser diode with
 a true intracavity absorber was difficult to set up because
 the semiconductor cavity was very hard to eliminate using
 anti-reflection coatings. Nowadays, laser diodes with

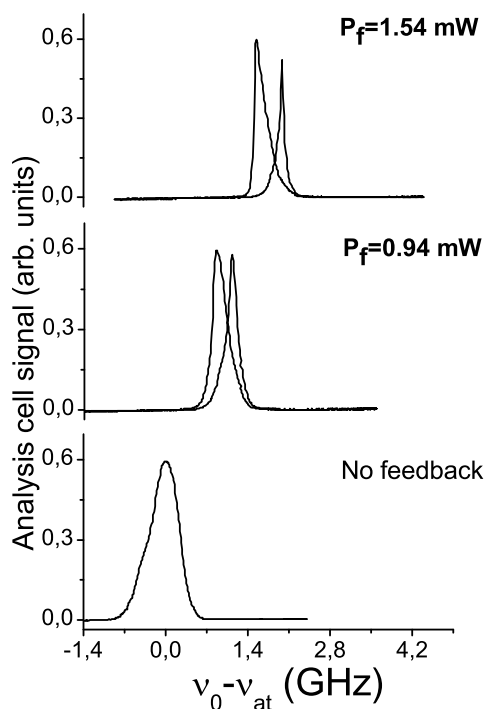


Fig. 7. Experimental measurement of the frequency laser by the analysis cell in Figure 1 set up. Through filtered orthogonal feedback the laser is submitted to dispersive effects in the atomic vapor resonance, the Doppler-broadened Cs D_2 line. The spectra are shown for three values of feedback power. Notice that the shape deviates from the calculated one (Fig. 6) for higher feedback strength because of the absorptive effects.

1 anti-reflection coating are commercially available (Fabry-
 2 Pérot type, with reflectivity less than 5×10^{-4} , from
 3 Eagleyard Photonics, for instance) and they need an external
 4 reflector to close the optical cavity and produce
 5 oscillation (Fig. 8). Using an external diffraction grating
 6 (1200 lines/mm) it was possible for us to get laser oscillation
 7 even with a resonant vapor cell placed into the
 8 extended cavity. In our set up, a low density vapor is
 9 employed in order to keep the absorption very low, so that
 10 the laser *amplitude* is hardly modified around the filter
 11 resonance. However, the analysis of the output laser *fre-*
 12 *quency* reveals a very rich spectral behavior [13]. Particu-
 13 larly, clear signatures of frequency locking and bistability
 14 are observed.

15 The goal of using an atomic-vapor filter is to take
 16 advantage of the Doppler line substructures because the
 17 atoms of the vapor interact selectively with the cavity
 18 fields. The stationary field in the cavity may be seen as
 19 two counterpropagating beams, therefore at the frequen-
 20 cies for which both beams are resonant with the same
 21 atoms the vapor transmission is slightly modified, mean-
 22 ing that the laser selectivity is increased at this particular
 23 frequencies. As a result we show in Figure 9 the linear ab-
 24 sorption spectrum of a probe cell which presents a much
 25 more pronounced absorption at the frequency of one spec-
 26 ific dip.

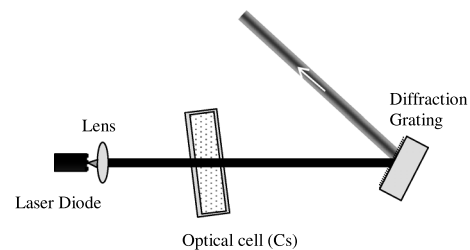


Fig. 8. (Color online) Set-up of a semiconductor laser with a non linear intracavity absorber. Laser diode with a anti-reflection coating and a diffraction grating returning the diffraction first order into the semiconductor gain media. A glass cell containing a low-pressure atomic vapor is placed inside the cavity at a small angle to avoid multiple reflections in the cavity. The diffraction zero order is the laser output.

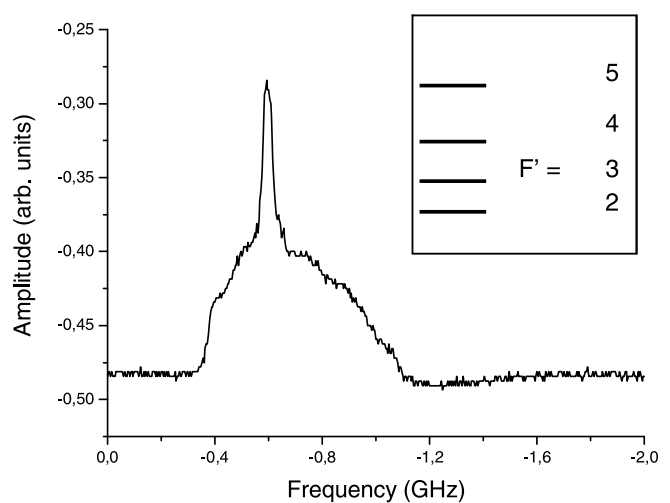


Fig. 9. Spectrum of the laser absorption in an external probe cell, using a linear absorption configuration. The frequency is quite sensitive to the atomic frequencies where the laser cavity counterpropagating beams interact with atoms of the intracavity vapor with the same velocity class (see Fig. 8). As a result, there is a high spectral power for the frequencies around the saturation absorption dips. Inset: Cs D_2 line excited sublevels. The room-temperature Doppler broadening is larger than these hyperfine sublevels intervals.

27 To interpret this result we first recall that the probe
 28 linear absorption is not sensitive to sub-Doppler features.
 29 Second, the power measured directly in a photodetector
 30 keep its level constant during the cavity scan. Therefore
 31 the laser spectral characteristics have changed during the
 32 scan around the atomic vapor resonance. In other words,
 33 the increase of the laser absorption in the probe cell
 34 indicates that the laser becomes narrower around the
 35 dips of the intracavity saturation absorption. Such frequen-
 36 cy singularities are seen in the calculated spectra of our
 37 modeling for this system [15]. The spectrum of Figure 9
 38 shows this frequency exclusive effect, due to the nonlinear
 39 intracavity filter. A very sensitive dependence with the
 40 diode current changes the spectrum shape, eventually en-
 41 hancing another resonant peak.
 42

4 Conclusions

In this paper we discussed a few observations of dynamics occurring in the frequency domain in optical systems, which are equivalent to those where amplitude dynamics of the optical field has been explored for decades, particularly, lasers under feedback and lasers with an intracavity absorber. For these first two cases we have experimentally observed a new behavior occurring exclusively in the laser frequency.

We have developed the theoretical models describing the frequency dynamics for these laser systems, either under filtered orthogonal feedback or with intracavity cavity saturated absorber. The frequency dynamics obtained through numerical calculations are consistent with the observed stability of the output intensity. I.e., the dynamical behavior occurs only in the frequency space. The numerical curves reproduce very well the bistable behavior of the laser output when we consider a nonlinear dependence for the laser gain and when we take into account a temporal evolution for the temperature of the semiconductor junction. Moreover, we present here new results for a dispersive filter, that reproduces previous observations. More specifically, in the case of a laser with an intracavity nonlinear filter, we describe behaviors of frequency locking and bistability, introducing into the rate equation for the field the terms of damping and propagation due to the intracavity atomic vapor volume. For this calculation we carefully considered the Doppler-broadened lineshape containing the sub-Doppler features of the saturation absorption spectra. However, in order to develop a better multimodal model to describe non periodic oscillations observed in preliminary experiments in the intracavity set up, more systematic measurements are needed and we are currently working on these developments.

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