Transverse-Mode Dynamics in Directly Modulated Vertical-Cavity Surface-Emitting Lasers With Optical Feedback

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***Abstract*—Vertical-cavity surface-emitting lasers (VCSELs) with optical feedback are known to exhibit different transverse-mode regimes depending on the injection current. Close to threshold a VCSEL operates on the fundamental transverse mode, while for larger injection the dynamics is often multimode, with the optical feedback inducing either in-phase or anti-phase transverse mode oscillations. In this paper, we study numerically the influence of current modulation on these different feedback-induced transverse-mode regimes. The modulation amplitude and period are taken as control parameters. We find that the in-phase and anti-phase regimes are robust under weak modulation. As the modulation amplitude increases, there is a transition to a dynamics governed by the current modulation, where the total output power follows the injection current and there is either single-mode or in-phase multimode behavior. However, the effect of the current modulation depends on the modulation period. Under fast modulation, the laser cannot follow the modulation and the optical-feedback-induced effects are dominant. On the contrary, under slow modulation there is a superposition of modulation and feedback effects, with the total output following the modulated current and an underlying transverse-mode behavior mainly determined by the optical feedback. A resonant behavior was observed for modulation periods close to the internal oscillation period. In this case, current modulation induces pulsing output intensity with single-mode or in-phase multimode behavior.**

***Index Terms*—Anti-phase dynamics, dynamical laser instabilities, laser diodes, modulation response, optical chaos, semiconductor lasers, vertical-cavity surface-emitting lasers (VCSELs).**

I. Introduction

THE increasing demand for high data-rate fiber-optic communication systems has motivated the development of a new generation of low-cost, high-performance semiconductor lasers. Vertical-cavity surface-emitting lasers (VCSELs) are attractive sources for high-bit-rate optical data transmission. The advantages of VCSELs over conventional, edge-emitting semiconductor lasers are single-longitudinal-mode operation, low threshold, high modulation efficiency, dense packing capability, and narrow circular beam profile [1].

Near-threshold VCSELs typically emit linearly polarized light in the fundamental transverse mode. However, in many devices, it is observed that the polarization state selected at threshold becomes unstable as the injection current is increased, and a switch to the orthogonal polarization state occurs. The polarization bistability, switching, and competition has been studied by many authors (see, e.g., [2] and references therein). For high-power operation, high-order transverse modes are excited and the VCSEL usually emits multiple transverse modes. An important consideration for the application of multimode VCSELs in high-speed multimode fiber links using multiplexed modulation schemes is the understanding of the effects of external optical feedback on the dynamics of current-modulated multimode VCSELs.

The nonlinear dynamics of edge-emitting semiconductor lasers subjected to both optical feedback and current modulation has been studied by several authors [3]–[11]. While recent attention has been paid to the dynamics of external-cavity VCSELs [12]–[15] and the dynamics of current-modulated VCSELs [16], [17], to the best of our knowledge the combined effects of optical feedback and current modulation have not been studied in VCSELs. The aim of this paper is to study these effects with particular emphasis on multi-transverse-mode operation. As a first step to understand the behavior of current-modulated external-cavity VCSELs, we do not take into account the polarization dynamics, which we plan to include as a second step in future work. Our model is therefore valid for studying transverse effects such as instabilities induced by spatial inhomogeneities, but not for polarization-induced instabilities.

It was recently shown numerically that weak optical feedback from a distant reflector induces distinct transverse-mode dynamical regimes [18]. Close to threshold the VCSEL dynamics is single-mode and optical feedback induces periodic oscillations of the laser output. For larger injection current, the VCSELs dynamics is multimode, and optical feedback induces either in-phase (for low current) or anti-phase (for larger current) transverse mode oscillations. In this paper, we study numerically the effect of current modulation in these dynamical regimes. Our simulations are based on the model of [18], which is an extension of the model originally proposed by Valle *et al.* [19], [20]. The model includes spatial profiles for both the

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transverse modes and the carrier density. The model applies to index-guided VCSELs, where the modal spatial profiles and frequencies are determined by the built-in refraction index distribution. The model includes external optical feedback as in the Lang–Kobayashi approach [21], taking into account a single reflection in the external cavity.

The modulation amplitude and the modulation period were taken as control parameters. Our results show that weak modulation preserves the feedback-induced regimes (periodic single-mode oscillations, as well as in-phase and anti-phase multimode oscillations), but as the modulation amplitude increases there is a transition to a dynamics that is governed by the current modulation. For sufficiently strong modulation, the laser output follows the injection current and the dynamics is be either single-mode or multi-mode. In the latter case, transverse-mode oscillations are in phase. However, the effect of the modulation depends on the modulation period: when the modulation period is close to the period of the feedback-induced oscillations, a resonant pulsing behavior is observed. This resonance was recently studied numerically in the framework of a simplified rate equation model for a single-mode laser with optical feedback and optical injection [22].

This paper is organized as follows. The model used in this study is described in Section II. Section III presents the results of the numerical simulations on the effects of current modulation on the single-mode, in-phase, and anti-phase optical feedback-induced regimes. Section IV contains a summary and the conclusion.

II. Model

We consider a cylindrically symmetric structure, whose active region [consisting of several quantum wells (QWs)] is modeled as a single effective QW of radius a and thickness $d_{\rm qw}$. Barrier regions of thickness d_b limit the QW region. Two highly reflecting mirrors separated by a distance L along the longitudinal z axis define the laser cavity. The injected current is

$$\begin{aligned} j(r,t) &= j_o\left[1+\delta\sin\left(\frac{2\pi t}{T_{\rm mod}}\right)\right], \qquad \text{for } r<a \\ j(r,t) &= 0, \qquad \text{otherwise} \end{aligned} \tag{1}$$

where δ is the amplitude of the modulation $(0<\delta<1)$ and $T_{\rm mod}$ is the period of the modulation.

The emission behavior is determined by the built-in index guiding introduced by transverse refractive index step in the surrounding region. The core (cladding) refractive index is taken to be $n_{\rm core}(n_{\rm clad})$, i.e., the transverse refractive index profile is $n(r)=n_{\rm core}$ for $r>a$ and $n(r)=n_{\rm clad}$ for $r>a$. For this geometry, the appropriate transverse modes are the linearly polarized LP_{mn} modes [23]. To simplify the calculations, we consider that only three modes, having azimuthal symmetry, are excited in the range of parameters considered in this paper as follows:

$$\begin{aligned} \psi_1(r) &\equiv \psi_{01}(r,\theta) \\ \psi_2(r) &\equiv \psi_{02}(r,\theta) \\ \psi_3(r) &\equiv \psi_{03}(r,\theta). \end{aligned} \tag{2}$$

The mode profiles are normalized such that $\int_0^\infty |\psi_i(r)|^2 r dr = 1$. Since the mode profiles are exponentially small outside the active region, this normalization hardly differs from the physical normalization $\int_0^a |\psi_i(r)|^2 r dr = 1$.

The equations for the slowly varying complex amplitude of the ith mode, $e_i(t)$, the density of carriers confined in the QW region, $n_w(r,t)$, and the density of (unconfined) carriers in the barrier region, $n_b(r,t)$ are [18]

$$\begin{aligned} \frac{de_i}{dt} &= \frac{1+j\alpha}{2}\left(g_i-\frac{1}{\tau_{pi}}\right)e_i(t) \\ &\quad + k_i e_i(t-\tau)\exp(-j\omega_i\tau) \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{\partial n_b}{\partial t} &= \frac{j(r,t)}{ed_b}-\frac{n_b}{\tau_{\rm cap}}+\frac{V_{\rm qw}}{V_b}\frac{n_w}{\tau_{\rm esc}}-\frac{n_b}{\tau_n} \\ &\quad + D_b\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n_b}{\partial r}\right) \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{\partial n_w}{\partial t} &= \frac{V_b}{V_{\rm qw}}\frac{n_b}{\tau_{\rm cap}}-\frac{n_w}{\tau_{\rm esc}}-\frac{n_w}{\tau_n}-g_o(n_w-n_t) \\ &\quad \times\sum|e_i|^2|\psi_i|^2+D_w\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n_w}{\partial r}\right). \end{aligned} \tag{5}$$

The first term on the right-hand side of (3) accounts for optical gain, losses, and phase-amplitude coupling. Here, α is the linewidth enhancement factor and g_i is the modal gain, given as follows:

$$g_i(t)=\int_0^\infty g_o\Gamma_i(n_w-n_t)|\psi_i|^2 r dr \tag{6}$$

where g_o is the linear gain coefficient, Γ_i is the confinement factor for the ith mode, and n_t is the transparency carrier density. τ_{pi} is the photon lifetime for the ith mode.

The second term on the right-hand side of (3) takes into account the field reflected from the external cavity. We consider a single reflection and therefore the model is valid for weak to moderate feedback levels, k_i is the feedback coefficient of the ith mode, ω_i is the optical frequency of the ith mode in the absence of feedback, and τ is the external-cavity round-trip time.

The terms on the right-hand side of (4) correspond, from left to right, to: 1) the rate at which carriers are injected into the barrier region; 2) the rate at which carriers are captured into the QWs; 3) the rate at which carriers escape out of the QWs; 4) the carrier loss owing to various nonradiative recombination processes; and 5) the last term accounts for carrier diffusion across the barrier region. The transport effects are included by a capture time $\tau_{\rm cap}$, an escape time $\tau_{\rm esc}$, and a diffusion coefficient D_b. The carrier loss is included by a carrier lifetime τ_n. Since the variables n_b and n_w refer to carrier densities, the different sizes of the barrier and QW regions must be taken into account. This is done by the ratio $V_b/V_{\rm qw}$, where $V_b=d_b\pi a^2$ is the volume of the barrier region and $V_{\rm qw}=d_{\rm qw}\pi a^2$ is the volume of the QW region.

The terms on the right-hand side of (5) correspond, from left to right, to: 1) the carriers captured into the QWs; 2) the carriers that escape out of the QWs; 3) the nonradiative carrier loss; 4) the carrier loss owing to stimulated recombination; and 5) the final term to carrier diffusion across the QWs.

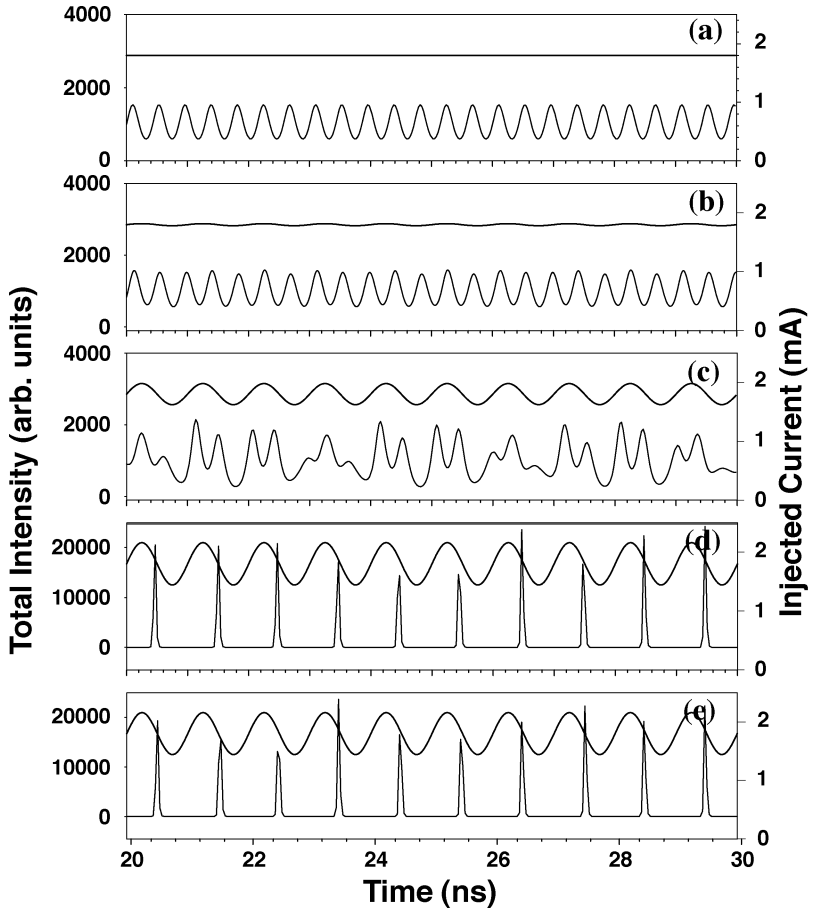

Fig. 1. Laser output power as a function of time for different values of the modulation amplitude. The dc injection current is such that the laser operates on the fundamental transverse mode (I_0 =1.8 mA). The modulation period is $T_{\rm mod}$ = 1 ns. (a) $\delta = 0.0$. (b) $\delta = 0.01$. (c) $\delta = 0.1$. (d) $\delta = 0.2$. (e) $\delta = 0.2$ and no optical feedback ($k = 0$). The thin (thick) line shows the value of the intensity (injection current).

III. Numerical Results

We numerically solve the model equations with typical VCSEL parameters [18]: $a = 6$ μm, $d_{\rm qw} = 0.024$ μm (three quantum wells of thickness 0.08 μm each), $d_b = 1.2$ μm, index step = 0.1, $\alpha = 3$, $g_o = v_g \partial g/\partial n$ with $v_g = 0.0715$ μm/ns and $\partial g/\partial n = 5.95 \times 10^{-8}$ μm^2, $n_t = 1.33 \times 10^6$ μm^{-3}, $\tau_{\rm cap} = 5$ ps, $\tau_{\rm esc} = 25.5$ ps, $\tau_n = 1.52$ ns, and $D_w = D_b = 0.5$ μm^2/ns. All modes have the same confinement factor $\Gamma_i = 0.038$ and losses $\tau_{pi} = 2.2$ ps. For these parameters, the threshold current is $I_{\rm th} = j_{\rm th}\pi a^2$ ~1.5 mA. If not otherwise stated, the external cavity parameters are the same as in [18], which correspond to weak optical feedback: $k_i = k$ =1 ns^{-1}, τ =1 ns, $\omega_i \tau = 0$ rad.

The equations were integrated with a time integration step $\Delta t = 10^{-4}$ ps and a space integration step $\Delta r = 0.02$ μm. Due to carrier diffusion, we needed to integrate (4) and (5) over a space interval $0 < r < na$ with n large enough to assure that $n_w(r = na) \sim 0$ and $n_b(r = na) \sim 0$. The initial conditions are all taken through the paper with the modal fields off (i.e., $e_i(t) = \xi(t)$ for $-\tau < t < 0$, where ξ is a small complex noise source), and the carrier densities are at the transparency value.

We analyzed the effect of current modulation on three different feedback-induced transverse-mode regimes, which occur for increasing values of the dc injection current, $I_0 = j_0 \pi a^2$: (a) single-mode operation, with periodic oscillations of the fundamental LP$_{01}$ mode; (b) two-mode operation, with in-phase oscillations of the LP$_{01}$ and LP$_{02}$ modes; and (c) three-mode operation, with anti-phase oscillations of the LP$_{01}$ and LP$_{02}$ modes. The modulation amplitude and period are taken as control parameters.

First we consider a dc injection current such that the laser operates in the fundamental LP$_{01}$ mode (I_o =1.8 mA). Fig. 1(a) displays the laser output as a function of time in the absence of current modulation ($\delta = 0$) and shows that weak optical feedback induces periodic oscillations of the laser output power. The period of the oscillations is close to the period of the relaxation oscillations of the solitary laser ($T_{\rm RO}$ ~0.43 ns for I_o =1.8 mA). Let us now include current modulation: first we fix the modulation period ($T_{\rm mod}$ =1 ns) and vary the modulation amplitude. As δ increases, Fig.1 (b)–(d) displays a transition from a dynamics that is induced by the optical feedback to a dynamics that is induced by the current modulation. It can be observed that weak modulation [$\delta = 0.01$, Fig. 1(b)] modifies only slightly the periodic oscillations. For larger modulation, there is competition between the effects of optical feedback and current modulation [$\delta = 0.1$, Fig. 1(c)]. Under strong modulation [$\delta = 0.2$, Fig. 1(d)], the laser exhibits a periodic pulsing behavior, with period $\sim T_{\rm mod}$. The dynamics is governed by the modulation and the laser output is the same as that in the absence of optical

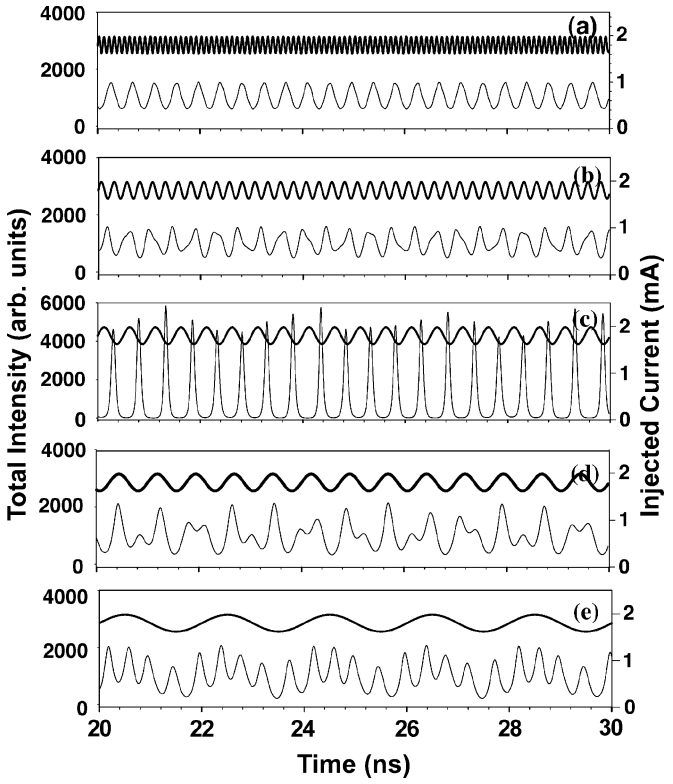

Fig. 2. Laser output power as a function of time for different values of the modulation period. $I_0 = 1.8$ mA and $\delta = 0.1$. (a) $T_{\rm mod}$ =0.1 ns. (b) $T_{\rm mod}$ =0.25 ns. (c) $T_{\rm mod}$ =0.5 ns. (d) $T_{\rm mod}$ =0.75 ns. (e) $T_{\rm mod}$ =2 ns. The thin (thick) line shows the value of the intensity (injection current).

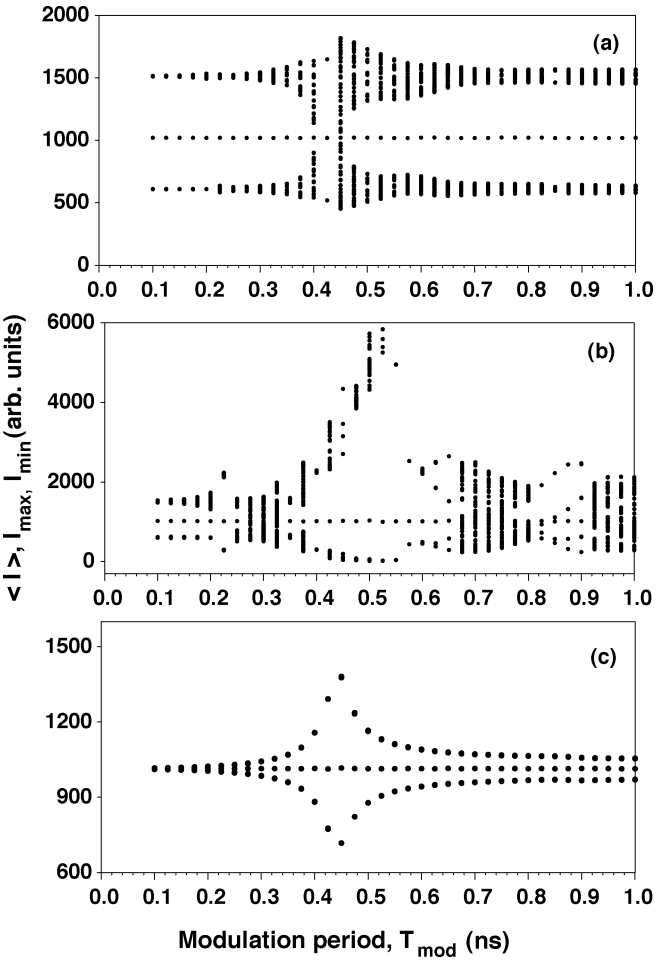

Fig. 3. Maximum, minimum, and average values of the intensity oscillations as a function of the modulation period. I_0 =1.8 mA. (a) $\delta = 0.01, k = 1$ ns^{-1}. (b) $\delta = 0.1, k = 1$ ns^{-1}. (c) $\delta = 0.01, k = 0.2$ ns^{-1}.

feedback [shown for comparison in Fig. 1(e), where $\delta = 0.2$ and $k = 0$].

Next we fix the modulation amplitude and vary the modulation period. Fig. 2 displays the laser output as a function of time for $\delta = 0.1$ and different $T_{\rm mod}$. For very fast modulation [Fig. 2(a)], the laser cannot follow the modulation and its output is the same as that in the absence of modulation. As the modulation becomes slower, the effect of the modulation becomes increasingly relevant [Fig. 2(b)–(e)]. For sufficiently slow modulation, the dynamics is quasi-periodic: the current modulation induces a slow envelope in the feedback-induced fast oscillations [Fig. 2(e)]. It can be observed that the envelope is in-phase with the injection current. In this case, there is a superposition of optical feedback and current modulation effects, with one frequency predominantly determined by the modulation and the other frequency predominantly determined by the feedback.

It can be observed in Fig. 2(c) that the laser output exhibits large pulses of period $T_{\rm mod}$, which suggests a resonant behavior. We note also that this occurs for $T_{\rm mod}$ =0.5 ns, which is close to the period of the relaxation oscillations (T_{RO} ~0.43 ns). To explore in more detail this behavior, we display in Fig. 3 the average ($\langle I \rangle$), the maximum ($I_{\rm max}$), and the minimum ($I_{\rm min}$) values of the intensity oscillations as a function of the modulation period $T_{\rm mod}$ for various values of the modulation amplitude δ and the feedback level k. To clearly distinguish the regions where the oscillations are periodic from regions where the oscillations are more complex, we plot all maxima and minima. When the dynamics is not periodic, there are several (local) maxima and minima. On the contrary, when the dynamics is periodic, there is a single (global) maximum and minimum.

We found that $\langle I \rangle$ is independent of $T_{\rm mod}$ while $I_{\rm max}$ and $I_{\rm min}$ show a complicated dependence on $T_{\rm mod}$. Fig. 3(a) displays results for weak modulation ($\delta = 0.01$) and the same feedback level as in Figs. 1 and 2 ($k = 1$ ns^{-1}). A resonance peak can be observed at $T_{\rm mod} \sim T_{RO} \sim 0.43$ ns, such that the maximum value of $I_{\rm max}$ coincides with the minimum value of $I_{\rm min}$. The resonance peak becomes asymmetric for larger modulation amplitude [see Fig. 3(b) which is for $\delta = 0.1$ and $k = 1$ ns^{-1}]. The complicated structure of the resonance is due to the optical feedback. Fig. 3(c), which is obtained for the same modulation amplitude as Fig. 3(a) but with a lower feedback level ($\delta = 0.01$, $k = 0.2$ ns^{-1}), reveals a simple and symmetric peak, which is what one expects, based on the behavior of a nonlinear oscillator under weak periodic forcing. The resonant behavior for particular values of the modulation period was recently studied in detail in [22], based on a simplified model for a single-mode laser.

It is interesting to study the nature of the transition from "feedback-dominant" to "modulation-dominant" dynamics in the resonant situation $T_{\rm mod} = T_{RO}$. Fig. 4 displays the laser output for fixed $T_{\rm mod}$ =0.43 ns and increasing δ. It can be observed that the optical feedback and the current modulation do

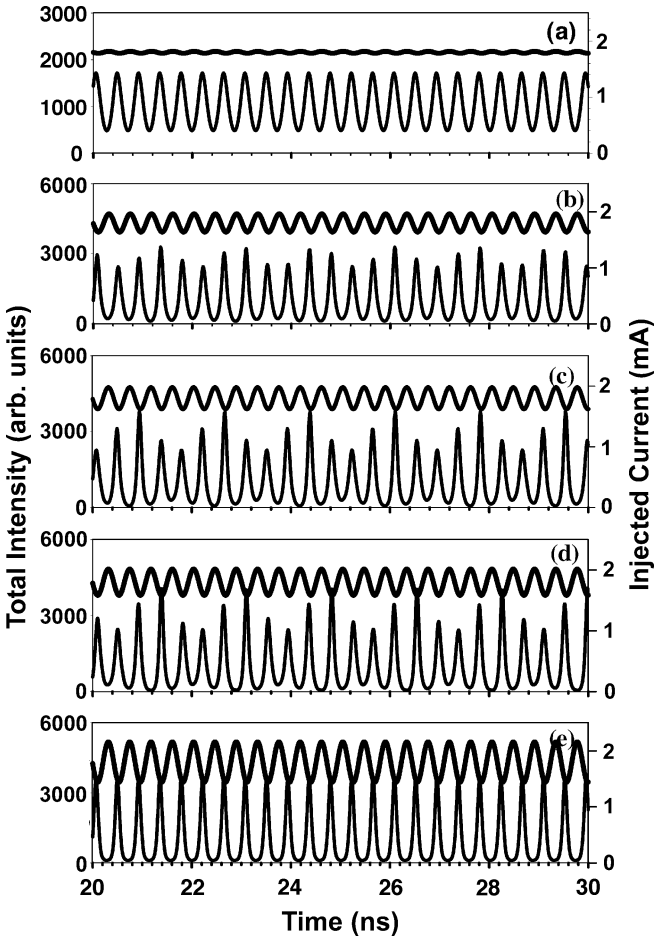

Fig. 4. Laser output power as a function of time for different values of the modulation amplitude. I_0 =1.8 mA and $T_{\rm mod} = T_{\rm RO}$ =0.43 ns. (a) $\delta = 0.01$. (b) $\delta = 0.05$. (c) $\delta = 0.1$. (d) $\delta = 0.15$. (e) $\delta = 0.2$. The thick (thin) line shows the value of the intensity (injection current).

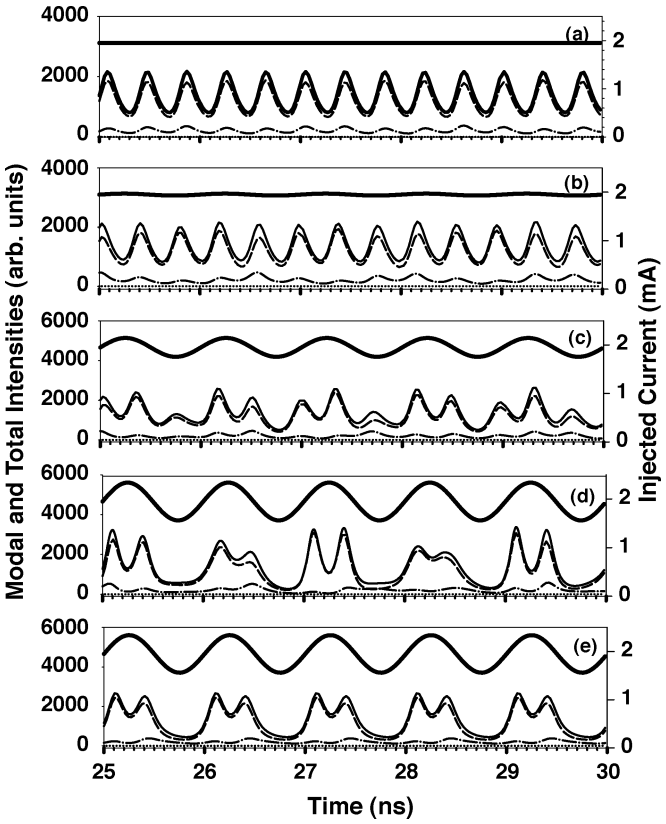

Fig. 5. Total intensity (thin solid line) and modal intensities (LP_{01}: dashed line; LP_{02}: dot–dashed line) as a function of time for different modulation amplitudes. I_0 =1.95 mA and $T_{\rm mod}$ =1 ns. (a) $\delta = 0.0$. (b) $\delta = 0.01$. (c) $\delta = 0.1$. (d) $\delta = 0.2$. (e) $\delta = 0.2$ and no optical feedback ($k = 0$). The thick line shows the injection current.

not compete but cooperate to induce periodic pulses (of period $T_{\rm mod} = T_{RO}$). It can also be observed that the injection current and the laser output power are in exact anti-phase.

Next we consider a larger dc injection current, such that the laser supports two transverse modes (the LP_{01} and LP_{02} modes). Fig. 5(a) shows the dynamics in the absence of current modulation. It can be observed that weak optical feedback induces in-phase oscillations of the modal intensities (the total output power exhibits periodic oscillations of period $\sim$0.4 ns). Let us first increase δ while keeping $T_{\rm mod}$ fixed. Fig. 5(b)–(d) illustrates a transition from feedback-induced dynamics to modulation-induced dynamics. Fig. 5(e) displays the dynamics of the solitary laser ($k = 0$) under strong modulation. By comparing Fig. 5(d) and (e), it can be observed that the effect of weak optical feedback is still relevant under strong modulation: without feedback the laser output is quasi-periodic [Fig. 5(e)], while in the presence of optical feedback the laser output exhibits more irregular oscillations. Let us now fix δ and vary $T_{\rm mod}$. Fig. 6 shows that varying $T_{\rm mod}$ has the same effect as in the single-mode regime: 1) fast modulation does not affect the laser output, which is nearly the same as that of the laser without modulation [Fig. 6(a)]; 2) resonant pulses of period $T_{\rm mod}$ are observed for $T_{\rm mod} = T_{RO}$ $\sim$0.4 ns [Fig. 6(c)]; and 3) a quasi-periodic output occurs for large enough $T_{\rm mod}$ [Fig. 6(e)], with one frequency determined by the optical feedback and the other frequency determined by the current modulation.

Next, we consider an even larger dc injection current such that the laser supports three transverse modes (the LP_{01}, LP_{02}, and LP_{03} modes). Fig. 7(a) shows that weak optical feedback induces anti-phased oscillations of the LP_{01} and LP_{02} transverse modes. Fig. 7(b)–(d) show the effect of increasing δ for fixed $T_{\rm mod}$. The anti-phase dynamics is robust under the modulation [Fig. 7(b)–(d)]. Notice that in Fig. 7(c) and (d) the total output power and the injection current are in phase, while there is an underlying anti-phase dynamics of the LP_{01} and LP_{02} modes. It can be observed that the effect of the feedback is still important under strong modulation: without optical feedback, there is an almost in-phase dynamics of the three transverse modes [see Fig. 7(e)].

Fig. 8 shows the effect of varying the modulation period for fixed modulation amplitude. As expected, given the complexity of the multitransverse-mode dynamics, several interesting features can be observed. Fast modulation [Fig. 8(a)] preserves the anti-phase dynamics (but induces small amplitude oscillations at the driving frequency in the modal intensities). For $T_{\rm mod}$ =0.25 ns, the total intensity exhibits large pulses of period $T_{\rm mod}$ while there are in-phase pulses of the three transverse modes [Fig. 8(b)]. As the modulation becomes slower, the total intensity follows the injection current while there are two different underlying modal behaviors. In Fig. 8(d) the three modes are nearly in-phase; in Fig. 8(e) the LP_{01} and LP_{02} modes are

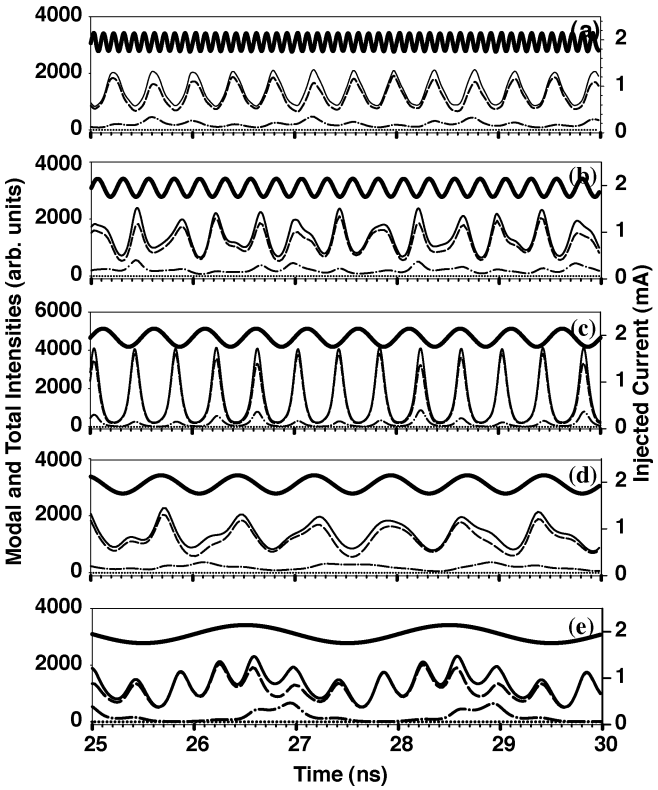

Fig. 6. Total intensity (thin solid line) and modal intensities (LP_{01}: dashed line; LP_{02}: dot–dashed line) as a function of time for different modulation periods. I_0 =1.95 mA and δ =0.1 ns. (a) $T_{\rm mod}$ =0.1 ns. (b) $T_{\rm mod}$ =0.25 ns. (c) $T_{\rm mod}$ =0.4 ns. (d) $T_{\rm mod}$ =0.75 ns. (e) $T_{\rm mod}$ =2 ns. The thick line shows the injection current.

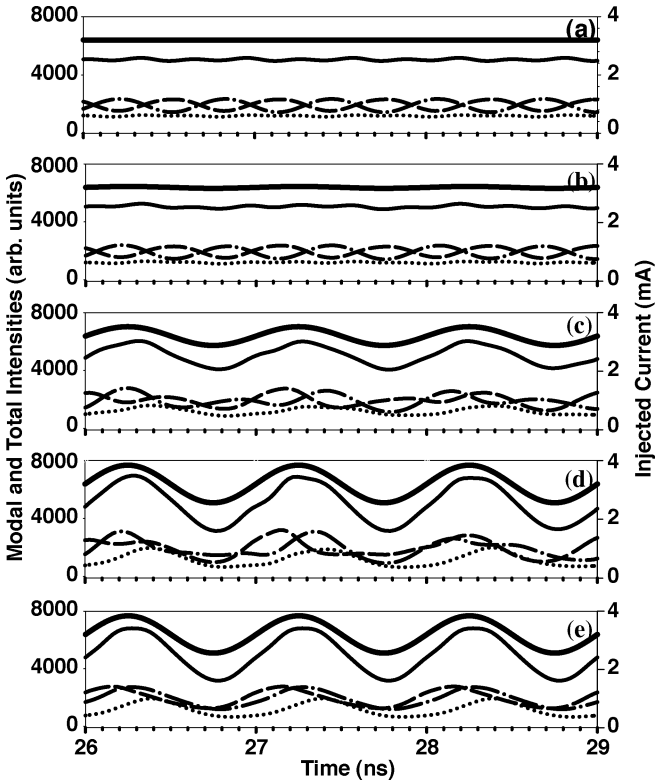

Fig. 7. Total output intensity (thin solid line) and modal intensities (LP_{01}: dashed line; LP_{02}: dot–dashed line; LP_{03}: dotted line) as a function of time for different modulation amplitudes. I_0 =3.2 mA and $T_{\rm mod}$ =1 ns. (a) $\delta = 0$. (b) $\delta = 0.01$. (c) $\delta = 0.1$. (d) $\delta = 0.2$. (e) $\delta = 0.2$ and no feedback ($k = 0$). The thick line shows the injection current.

in anti-phase while the LP_{03} oscillates in-phase with the sum of the LP_{01} and LP_{02} modal intensities.

In multimode operation, there is a resonant behavior which is similar to that found in the single-mode case. This resonance is revealed by plotting the average, maximum, and minimum value of the total intensity as a function of the modulation period [shown in Fig. 9 for the two injection currents considered, I_0 =1.95 mA and I_0 =3.2 mA]. For low amplitude modulation [Fig. 9(a) and (b)], there is a symmetric peak: the maximum value of $I_{\rm max}$ and the minimum value of $I_{\rm min}$ both occur for $T \sim T_{\rm RO}$; for larger amplitude modulation [Fig. 9(c) and (d)], there is a distortion of the resonance that suggests that for even larger δ bistability might occur.

Finally, let us assess the generality of the above results by considering different delay times and modal frequencies. Fig. 10 displays results for fixed $T_{\rm mod}(T_{\rm mod} \sim T_{\rm RO})$ and different τ. We show results corresponding to a large injection current (such that the laser supports three transverse modes), but qualitatively the same effect of the delay time was observed when the laser operates in single mode or in two transverse modes. Under the resonant modulation condition $T_{\rm mod} \sim T_{\rm RO}$, the laser output exhibits pulses of period $T_{\rm mod}$ for all τ, which are in anti-phase with the injection current. It is observed that for small τ the pulses are regular and the transverse modes are in-phase [Fig. 10(a)], while for large τ the pulses are irregular and a more complex transverse-mode dynamics occurs, with fast in-phase pulses of the three modes combined with a longer-time scale anti-phase behavior of the LP_{01} and LP_{02}

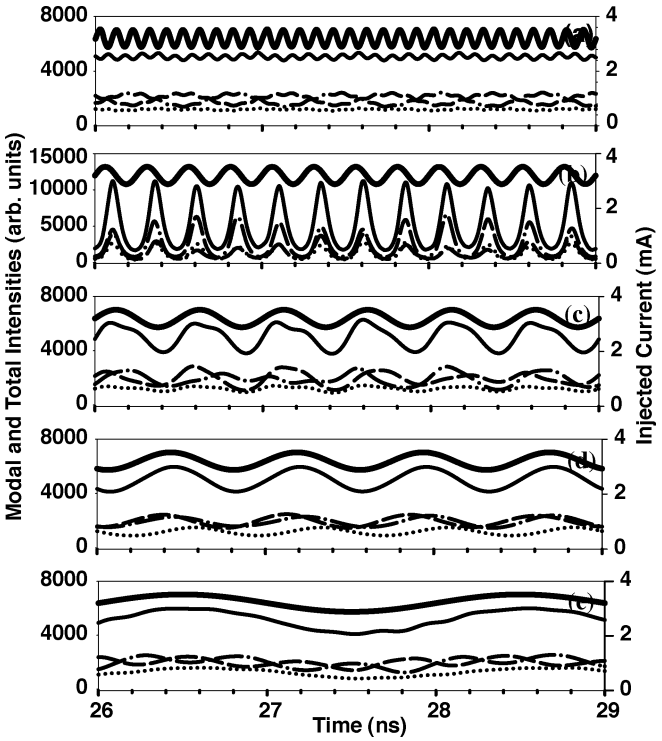

Fig. 8. Total output intensity (thin solid line) and modal intensities (LP_{01}: dashed line; LP_{02}: dot–dashed line; LP_{03}: dotted line) as a function of time for different modulation modulation periods. I_0 =3.2 mA and $\delta = 0.1$. (a) $T_{\rm mod}$ =01 ns. (b) $T_{\rm mod}$ =0 25 ns. (c) $T_{\rm mod}$ =0.5 ns. (d) $T_{\rm mod}$ =0.75 ns. (e) $T_{\rm mod}$ =2 ns. The thick line shows the injection current.

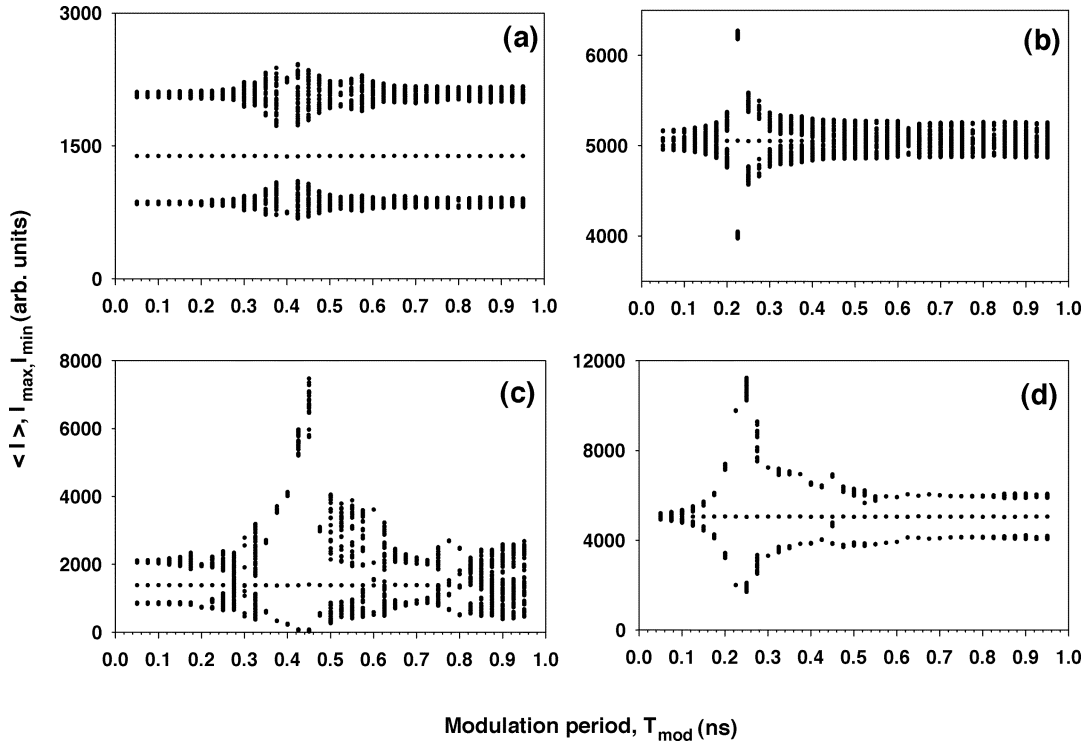

Fig. 9. Maximum, minimum, and average values of the total intensity oscillations as a function of the modulation period. (a) I_0 =1.95 mA, $\delta = 0.01$. (b) I_0 =3.2 mA, $\delta = 0.01$. (c) I_0 =1.95 mA, $\delta = 0.1$. (d) I_0 =3.2 mA, $\delta = 0.1$.

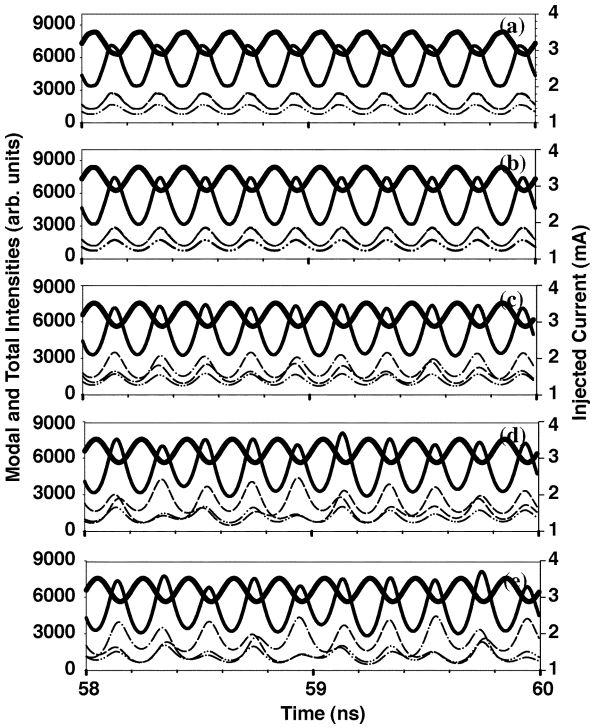

Fig. 10. Total output intensity (thin solid line) and modal intensities (LP_{01}: dashed line; LP_{02}: dot–dashed line; LP_{03}: dotted line) as a function of time for different delay times. I_0 =3.2 mA, $\delta = 0.1$, $T_{\rm mod}$ =0.2 ns. (a) τ =0.1 ns. (b) τ =0.7 ns. (c) τ = 1 ns; (d) τ =3 ns. (e) τ = 5 ns. The thick line shows the injection current.

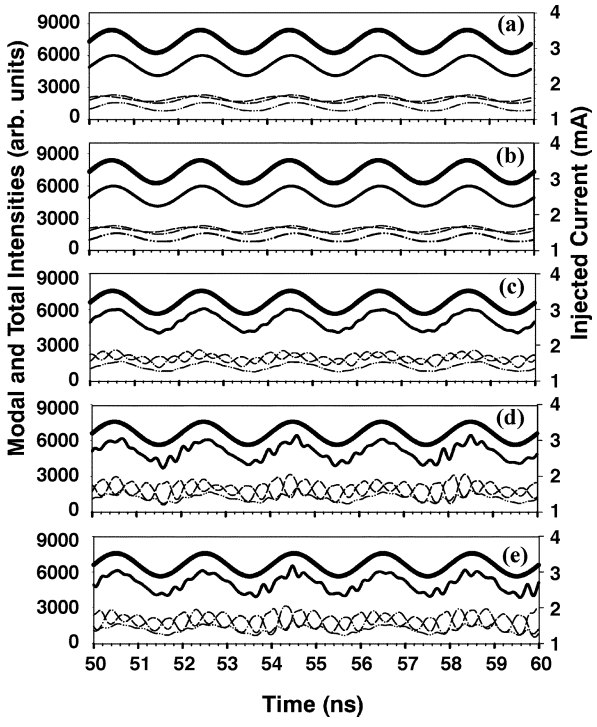

Fig. 11. The same is in Fig. 10 but $T_{\rm mod}$ =2 ns.

modes [Fig. 10(e)]. Fig. 11 displays results for the same values of τ and $T_{\rm mod} \gg T_{\rm RO}$. It can be observed that the total output is in-phase with the injection current for all τ, the value of τ affecting the underlying transverse-mode dynamics, which is nearly in-phase for short τ and more complex (with anti-phase oscillations of the LP_{01} and LP_{02} modes) for longer τ. For short delay times, the plot of $I_{\rm max}$, $I_{\rm min}$, and $\langle I \rangle$ versus $T_{\rm mod}$ reveals periodic behavior with a simple resonant peak [similar to that shown in Fig. 3(a)], while for long delay times the dynamics is quasi-periodic and the resonant peak is more complex [as that shown in Fig. 9(b)].

In the simulations presented so far, we have assumed that the feedback field is in-phase with the cavity field, i.e., that the feedback phases are $\omega_i\tau = 2n\pi$. In single-mode operation, the selection of a particular value of the feedback phase does not influence the laser dynamics (it leads to a global shift of the frequencies of the external cavity modes, as was shown, e.g., in [24]–[26]). In multimode operation, the feedback phases could, in principle, have a more important influence. Our simulations indicate that in general the values of $\omega_i\tau$ do not, in fact, play a significant role in the laser dynamics. Simulations performed with realistic values for the modal frequencies ω_i yield similar results to those presented in Figs. 10 and 11. However, for slow modulation and short delay times, we have observed a pulsing behavior which occurs for very particular irrational values of $\omega_i\tau$.

IV. Summary and Conclusion

The effect of direct modulation on the transverse-mode dynamics of VCSELs with weak optical feedback has been studied numerically. We have analyzed three different feedback-induced regimes: single-mode periodic oscillations of the fundamental LP_{01} mode, in-phase oscillations of the LP_{01} and LP_{02} modes, and anti-phase oscillations of the LP_{01} and LP_{02} modes. The modulation amplitude and period were taken as control parameters. We found that these feedback-induced transverse-mode regimes are generally robust under weak current modulation. When the modulation is slow, the feedback and modulation effects superpose such that some features of the dynamics are induced by the modulation and others are induced by the feedback. For example, in Fig. 7(c) and (d), there are anti-phase oscillations of the LP_{01} and LP_{02} modes which are due to the optical feedback, and there is a modulation of the total output which is due to the current modulation. When the period of the modulation is close to the internal relaxation oscillation period, a resonant pulsing behavior of the laser output was observed even for weak modulation. In the pulsing regime, the laser operates either in a single-transverse mode or in several transverse modes. In the latter case, the transverse modes exhibit in-phase pulses. These results might be of interest for high-bit-rate optical data transmission systems based on multimode VCSELs.

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