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Topics on non-equilibrium statistical mechanics and nonlinear physics

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PREFACE

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The exciting field of complexity, chaos and nonlinear science has experienced an impressive growth in the last decade. Not only have classic problems in statistical mechanics, fluids, pattern formation, chaos and dynamical systems received a lot of attention, but also new interdisciplinary fields have been developed, including complexity, networks, social dynamics, biological applications, etc. This issue of *Phil. Trans. R. Soc. A* features an interesting selection of papers by leading researchers working in the field of non-equilibrium statistical mechanics and nonlinear physics and covers a wide range of relevant topics in nonlinear science.

In the first contribution, Burada *et al.* (2009) present an interesting study of biased diffusive transport of Brownian particles through narrow and irregularly shaped channels. The complexity of the diffusive dynamics is reduced to a one-dimensional description by means of the so-called Fick–Jacobs approximation. The elimination of the transverse degrees of freedom, originating from geometrical confinements and/or bottlenecks, results in entropic potential barriers that the Brownian particles have to overcome when moving forward. The validity of the reduced model is tested by comparing the one-dimensional kinetics with simulations of Brownian particles in the full configuration space. The authors also demonstrate a violation of the Sutherland–Einstein fluctuation–dissipation relation occurring for moderate-to-strong bias.

In the next contribution, Romero-Arias *et al.* (2009) study the role of the cooling speed and thermal conductivity during glass transition. By using a one-dimensional Fermi–Pasta–Ulam chain with nonlinear second-neighbour springs placed at random, the authors show the great impact of low-frequency vibrational modes on the thermal properties of a glass, specifically on the relaxation time and on the thermal conductivity. Their results are of interest in glass transition phenomena in real systems, since most of the theories concerning glass transition do not consider such effects.

Hernandez-García *et al.* (2009) address the interesting interdisciplinary problem of coexistence of competing species. The system considered is an ecology composed of species competing for a natural resource, but other systems are equally amenable to the description provided by this work, which makes it of general interest. The authors discuss conditions that lead to the

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One contribution of 14 to a Theme Issue ‘Topics on non-equilibrium statistical mechanics and nonlinear physics’.

stability of clusters of surviving species using the Lotka–Volterra equations as a framework. They extend previous work of an ecology with uniform carrying capacity to the more realistic situation of non-uniform carrying capacity. They discuss how realistic ecological interactions can result in different types of competition coefficients. This contribution yields insight into the mechanisms underlying the clustering phenomenon, i.e. the spontaneous partition, under certain circumstances, of the species systems into separated clusters. Since clustering is a phenomenon that appears in a variety of fields, the results presented are of interest not only for researches working in ecology but also for researchers working in other fields.

The next three contributions address relevant issues in pattern-forming dynamical systems.

The contribution by Yang & Radons (2009) reviews recent work on hydrodynamic Lyapunov modes (HLMs), which provide a natural and very important connection between nonlinear dynamics and statistical physics. HLMs have been observed in many extended systems with translational symmetry, such as hard sphere systems, dynamic XY-models, Lennard-Jones fluids, etc. In the first part of this contribution, the authors discuss the appearance of good and 'vague' HLMs in a model of coupled map lattices. The structural properties of the HLMs are related to the geometry of the phase space and to fluctuations of local Lyapunov exponents. The authors show the relevance of hyperbolicity for observing good HLMs. In the second part, the authors discuss the fascinating idea of extending the concept of phonons and finding their counterpart in fluids. Such an idea can be traced back to Maxwell, who suggested that the dynamics of liquids at short times has some similarities to that of solids. Recent work along this line of research proposed the so-called instantaneous normal modes (INMs), and the authors discuss their relation to the HLMs. It has been pointed out that the appearance of HLMs relies on the same mechanisms as phonons and INMs: a spontaneous breaking of certain symmetries in the systems' Hamiltonians. Thus, the authors discuss the issue of whether HLMs can serve as the counterpart of phonons in systems with strong anharmonic dynamics. The final part of the contribution is devoted to partial differential equations (PDEs) and the analysis of their effective degrees of freedom. The authors show that the Lyapunov instability analysis provides a practical method for obtaining information on the number of the effective degrees of freedom and present numerical results based on simulations of the Kuramoto–Sivashinsky equation. Since similar results have also been found for the complex Ginzburg–Landau equation, the method is expected to give meaningful results for a wide class of dissipative PDEs.

Clerc *et al.* (2009) present an up-to-date theoretical and experimental study of localized excitations in a vertically driven small rectangular water container. They aim at characterizing the interaction of the dissipative solitons close to the Faraday instability, where the parametrically driven damped nonlinear Schrödinger equation (PDDNSE) provides a unifying description of the system. This equation, which has been used to study patterns in fluids, nonlinear optics, ferromagnetic media, nonlinear lattices, chains of pendulums, etc., exhibits coherent states that are usually referred to as dissipative solitons. Based on this model, the authors study the pair interaction law between localized states, which decreases exponentially with the distance between the dissipative solitons and is attractive or repulsive depending on whether they are in phase or out of phase, respectively. The merging of two dissipative solitons is characterized by the radiation of two small perturbations and the appearance of only one dissipative soliton. An excellent agreement is demonstrated between the experimental data and the pair interaction law deduced within the framework of the PDDNSE. Since the PDDNSE is the prototype model of conservative or time-reversible systems perturbed with small injection and dissipation of energy (the so-called quasi-reversal systems), the results presented in this contribution are expected to apply to other parametrically driven quasi-reversal systems, close to the parametric resonance.

The contribution by Gutiérrez *et al.* (2009) is devoted to the important problem of bifurcation from pulses to fronts in the one-dimensional cubic-quintic Ginzburg–Landau equation. For pulse-type localized structures, the authors review an approximation scheme that allows one to compute some properties of the structures, like their existence range. From that scheme, the authors obtain mathematical conditions for the existence of pulses in the upper limit of a control parameter. In that limit, they found that the width of pulses diverges, leading to a transition between pulses and fronts. Using this simplified approach, the authors obtained approximate bifurcation diagrams for such transitions that agree with numerical simulations.

The next two contributions are devoted to interesting biologically motivated problems.

The contribution by Trevisan & Mindlin (2009) presents a fascinating study of the physics of birdsong and specifically aims to answer the question of the basic physical ingredients to construct a model for the generation of bird sounds. In songbirds, articulation between the neural code and the actual song is exerted by an organ, the syrinx, which is a versatile organ that can display extremely rich dynamics when driven at different regimes. The authors show that the birds' vocal organ is not a mere instrument of the nervous system but rather a central source of complex acoustical behaviour. Driving their model with real physiological instructions recorded during spontaneous singing, the authors test the model that produces synthetic songs very similar to the real ones. The results and methodology presented are very interesting and motivate further interdisciplinary research among physics, mathematics and biology.

The next contribution by Masoller *et al.* (2009) addresses the effects of communication time delays in the behaviour of a population of globally coupled neurons that display intrinsic subthreshold oscillations. The coupling is of the type of time-delayed collective mean field, while the individual neuron is modelled using a Hodgkin–Huxley-type conductance model with parameters chosen such that the uncoupled neuron displays autonomous subthreshold oscillations of the membrane potential. The authors show that the ensemble can display collective synchrony or clustering behaviour depending on the coupling delay time. The synchrony is also controlled by the time delay that induces either in-phase or out-of-phase oscillations. The authors interpret the behaviour of the neuronal ensemble in terms of a phase flip bifurcation observed in mutually delay-coupled oscillators.

Synchronization phenomena represent nowadays a huge research field, and the next contribution by Rubido *et al.* (2009) also focuses on synchrony, in this case arising in a set of light-controlled oscillators (LCOs). An LCO is a realistic pulse oscillator whose behaviour resembles the integrate-and-fire oscillator, but differs by the fact that its discharge is not instantaneous. The authors study experimentally and numerically a system composed of coupled LCOs in different configurations, including local coupling. Synchronization times are quantified as a function of the initial conditions and the coupling strength. For each configuration, the number of stable states is determined by varying the different parameters that characterize each oscillator.

The next two contributions show that information theory provides novel effective tools to investigate important multi-disciplinary problems such as the characterization of pseudo-random number generators (PRNGs) via complexity and randomness quantifiers and the characterization of high-order correlations in the information transmitted by neuronal systems via Shannon information theory.

De Micco *et al.* (2009) present an excellent review of randomness quantifiers, focusing on chaos in time series used in connection with PRNGs. Their work sheds a didactic light on the importance of the main statistical characteristics of a chaotic map, namely, its invariant measure and the mixing constant. In particular, the authors show that two classes of quantifiers are required for the evaluation of the quality of a PRNG: (i) quantifiers depending on mixing constant only and (ii) quantifiers depending on the invariant measure only. Representation planes with one quantifier of each class as coordinate axis allow for different chaotic PRNGs to be compared with each other so as to determine the best one. Furthermore, these representation planes allow a judicious selection of the best randomizing procedure.

Understanding the operations of neural networks in the brain requires an understanding of whether interactions among neurons can be described by a pairwise interaction model, or whether a higher order interaction model is needed. In the next contribution, Montani *et al.* (2009) apply the maximum entropy principle to study the rate of synchronous discharge of a local population of neurons, a macroscopic index of the activation of the neural network that can be measured experimentally. The authors evaluate whether the probability of synchronous discharge can be described by interactions up to any given order. They consider how a homogeneous-pool model containing interactions of arbitrary order fits real distributions of the rate of coincident firing in real *in vivo* neural networks. When compared with real neural population activity obtained from the rat somatosensory cortex, the model shows that interactions of at least order 3 or 4 are necessary to explain the data. They use Shannon information theory to compute the impact of higher order correlations on the amount of somatosensory information transmitted by the rate of synchronous discharge and found that correlations of higher order progressively decreased the information available through the neural population. These results are compatible with the hypothesis that higher order interactions play a role in shaping the dynamics of neural networks and that they should be taken into account when computing the representational capacity of neural populations.

The last two contributions consider up-to-date problems in the field of complex networks.

The contribution by Zanette (2009) is of interest to scientists working in networks, opinion dynamics and complex systems, in general, since it describes a generalization of the concept of a network towards a ‘multiplet’-based structure. The process of opinion formation is simulated in a population of agents whose interaction pattern is defined on the basis of randomly distributed groups of three agents, or 'triplets', in contrast to networks, which are defined on the basis of agent pairs. The time to reach consensus is compared with a random network with the same number of triangles. Important differences between the two cases are demonstrated: the full-consensus time in the triplet structure is systematically lower than that in the network, which highlights the relevance of different topological structures. Analytical calculations are also presented, which allow one to trace the origin of the differences to different shapes of the probability distribution for the number of triplets and triangles per agent in each interaction pattern.

Wang & Gonzalez (2009) in their contribution analyse human mobility data from the point of view of complex networks. The authors find large clusters that emerge in urban areas that are surrounded by less-dense areas. The authors describe the mechanisms underlying the formation of large clusters and discuss the impact of human mobility in large-scale epidemics. Not only the complex network approach but also the data analysed are very original: the data were collected by a mobile phone carrier and consist of the full mobile communication pattern of 6.2 million users, recording the location of each user with a mobile tower level resolution each time the user makes or receives a phone call or an SMS, resulting in over 1.1 billion location data points.

Concluding, this issue of *Phil. Trans. R. Soc. A* covers a wide range of up-to-date topics in nonlinear science and biological systems, such as Brownian motion, glass transitions, species competition, patterns and localized structures, synchronization, complexity measures, networks, etc.

The authors have made a special effort in writing high-level papers with an introduction general enough so as to reach a wide audience and introduce people outside their fields to their high-quality research. We thank all the authors for their very interesting contributions and we are grateful for the invaluable help of the referees who critically evaluated the papers that now form the present issue of *Phil. Trans. R. Soc. A*.

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