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Antiphase dynamics in a multimode Fabry–Perot semiconductor laser with external feedback

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Abstract

In the context of the multimode Fabry–Perot generalization of the Lang–Kobayashi model describing a semiconductor with external feedback, we illustrate antiphase dynamics by showing examples of dynamical compensation in the periodic regime, multiple coexisting attractors in the LFF regime and in the chaotic regime.

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Keywords: Semiconductor laser; Antiphase dynamics; Multimode oscillation

1. Introduction

Since the pioneering work of Huygens [1] on the phase locking of coupled pendulums, the topic of interacting oscillators has remained an active subject of study. In the case of lasers, two problems have been focal points of interest: arrays of single mode lasers and uncoupled multimode lasers. These represent two limiting cases of interactions: arrays are chains of nonlinear oscillators coupled to nearest neighbours via an evanescent field, while multimode lasers are examples of globally coupled nonlinear oscillators since all the cavity modes interact with the same nonlinear medium. The properties of these two types of couplings are quite different and their use is motivated

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by different considerations. In this paper, we focus on the dynamics of a multimode semiconductor laser (SCL).

In many applications, the light emitted by a SCL is partially backreflected into the laser. The small size of these lasers and the trend to overall integration make it difficult to avoid this backreflection. Early experiments clearly indicated that an external feedback may significantly change the laser dynamics. The first signature of this change was the observation of a new peak in the low-frequency domain of the RF spectrum [2]. It was shown that this peak corresponds to irregular drop-offs in the laser output [3]. In an attempt to model this peculiar dynamics which they also studied experimentally, Lang and Kobayashi [4] proposed a simple model that includes in the laser rate equations the effect of external feedback. It amounts to add a *delayed* coherent source to the complex electric field equations. This source is simply the field which is backreflected by an external mirror. The resulting Lang–Kobayachi equations in dimensionless formulation are

$$\frac{\mathrm{d}E}{\mathrm{d}t} = (1 + \mathrm{i}\alpha)EN + \eta \mathrm{e}^{-\mathrm{i}v\tau}E(t - \tau) , \tag{1}$$

$$T\frac{\mathrm{d}N}{\mathrm{d}t} = P - N - (1 + 2N)|E|^2 , \tag{2}$$

where E is the electric field, N the excess free carrier density above threshold, α the linewidth enhancement factor, η the attenuation factor of the backreflected field, v is the lasing frequency for $\eta = 0$, τ is the delay time in the external cavity, and P is the excess pump rate above threshold. For N and P, the threshold refers to the lasing threshold with $\eta = 0$. Time is measured in units of the photon cavity life time, $1/\gamma_{\mathrm{p}}$, and T is the free carrier relaxation time. These equations have been used successfully to explain in a unified framework a wide variety of properties displayed by SCL with external cavities [5–8].

Recently, it has been realized that the generic mode of operation of a SCL with external cavity (SCLEC) is multimode [9–11], rather than single mode as assumed by Lang and Kobayashi. Considering only edge emitting SCL and longitudinal modes, there have been at least three directions along which the single mode rate equations have been generalized to the multimode regime: the modal ring cavity model [12], the PDE rate equation model [13], and the modal Fabry–Perot cavity model [14,15]. Thus, in the multimode regime, some confusion still reigns and each of these three generalizations can claim some partial success. In this paper, we focus on the multimode Fabry–Perot generalization and report on some properties of that model which is described by the dynamical equations

$$\frac{\mathrm{d}E_k}{\mathrm{d}t} = (1 + \mathrm{i}\alpha)E_k N_k + \eta_k \mathrm{e}^{-\mathrm{i}\Omega_k\tau}E_k(t - \tau) , \tag{3}$$

$$T\frac{\mathrm{d}N_k}{\mathrm{d}t} = P - N_k - (1 + 2N_k)\sum_j \beta_{kj}|E_j|^2 , \tag{4}$$

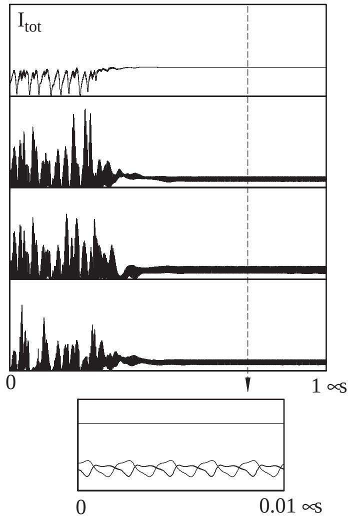

Fig. 1. Dynamical compensation in the periodic regime for Eqs. (3) and (4) with three modes. The total intensity is averaged, assuming a 1 GHz bandwidth detector. Parameters are $P = 10^{-3}$, $T = 10^3$, $\gamma_p = 1$ THz, $\tau = 1.9$, $\alpha = 5$, $\beta = \frac{2}{3}$, $\eta = 7.5 \times 10^{-3}$, and $\Omega\tau = 0$ (mod 2π).

where the new parameters are $\beta_{kk} \equiv 1$ and the cross-saturation parameters $0 < \beta_{nm} \leqslant 1$ which account for the free carrier grating.

2. Dynamical compensation in the periodic regime

One property of Eqs. (3) and (4) is that they display antiphase dynamics. It has been shown analytically in solid-state lasers that close to the Hopf bifurcation leading to the antiphase period regime, the sum of all modal intensities is a constant [16,17]. This result also holds for Eqs. (3) and (4). However, to be useful, this result must remain valid at a finite distance from the bifurcation, where the perturbation expansion in powers of the distance to the bifurcation may no longer hold. We have verified numerically that this is indeed the case for a three mode SCL with external feedback, as shown in Fig. 1.

For the parameters of that figure, the Hopf bifurcation occurs at $\eta_H = 6 \times 10^{-4}$. The lowest panel of the figure shows a hundredfold enlargement of the time traces. It indicates that a high degree of dynamical compensation is taking place. Although all modes have the same gain, two modes are exactly in phase and have indistinguishable traces after the initial transients, while the third mode antiphases with the other two modes, resulting in a practically flat total output.

The time scales of SCL being very short, the electronic detection means are too slow to record directly the time traces of the lasers, except for streak cameras. Therefore, the detection device unavoidably averages the signal over its bandwidth. This is taken

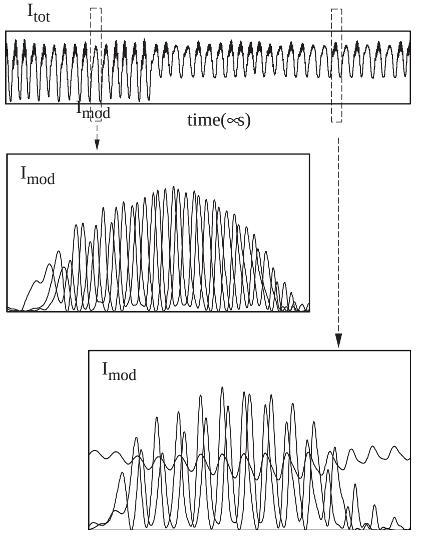

Fig. 2. Coexisting attractors in the weakly chaotic regime for Eqs. (3) and (4) with three modes. The total intensity is averaged, assuming a 1 GHz bandwidth detector. Parameters are $P = 10^{-3}$, $T = 10^3$, $\gamma_p = 1$ THz, $\tau = 0.73$, $\alpha = 5$, $\beta = 0.1$, $\eta = 7.5 \times 10^{-3}$, and $\Omega\tau = 0 \pmod{2\pi}$.

into account in the numerical simulations and explains the difference between the total intensity traces, which are averaged, and the modal intensity traces, which are drawn before averaging because this representation is better adapted to the comparison.

3. Coexisting attractors

Another property which is to be expected in antiphase dynamics is the multiplicity of attractors [18–21]. An example of this property is displayed in Fig. 2. What is observed in the total intensity is a jump between two regimes. To understand what happened, it suffices to look at the modal intensities. This shows that the large amplitude oscillations correspond to a regime where, within each pulse, the three modes antiphase in such a way that all three modes are in the same periodic regime but shifted by one third of the period with respect to the other two modes. On the contrary, the smaller amplitude oscillations in the total intensity correspond to a regime displaying the so-called localized solutions, where one mode has a much smaller oscillation amplitude than the other two modes [22].

4. Multiple attractors in the LFF regime

In the low-frequency fluctuation (LFF) regime, the averaged total intensity displays sudden drop-offs followed by gradual recoveries, as shown in the upper panel of Fig. 3. The lower panels show an enlarged view of the first two drop-offs for the averaged total intensity and for the non averaged modal intensities. It clearly appears

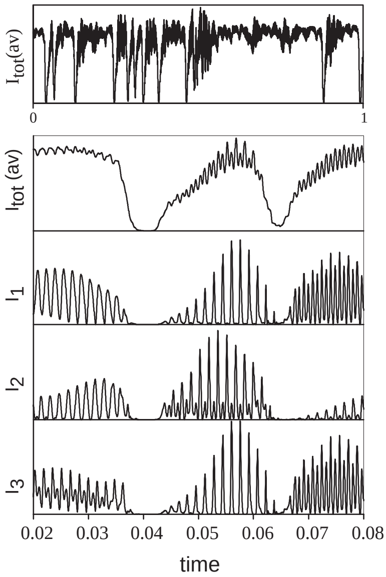

Fig. 3. LFF regime for Eqs. (3) and (4) with three modes. The total intensity is averaged, assuming a 1 GHz bandwidth detector. Parameters are $P = 10^{-3}$, $T = 10^3$, $\gamma_p = 1$ THz, $\tau = 1.5$, $\alpha = 5$, $\beta = \frac{2}{3}$, $\eta = 7.5 \times 10^{-3}$, and $\Omega\tau = 0$ (mod 2π).

that for the modal intensities there can be a change of attractor after each drop-off. For instance, for I_1 we observe the sequence $P1 \to P1 \to P2$ where Pn refers to the basic pattern that is repeated (but not exactly since this is not a periodic regime but a chaotic regime). For I_2, the sequence is $P1 \to P2 \to P2$ while for I_3 the pattern is $P2 \to P1 \to P2$. The patterns Pn belong to different attractors and the drop-offs are opportunities for the modes to switch (chaotically) from one attractor to another attractor. In the presence of noise, this switching will be a truly random process.

To summarize, we have shown that a multimode semiconductor laser with external optical feedback, modelled by the rate equations (3) and (4), can exhibit a rich variety of complex nonlinear behaviours.

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