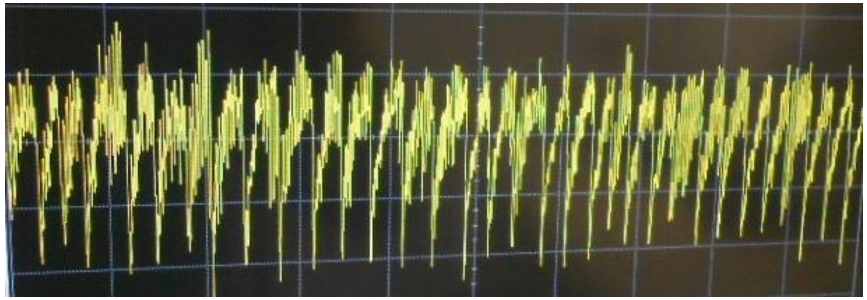

Time crystal like oscillations in a weakly modulated stochastic time delayed system

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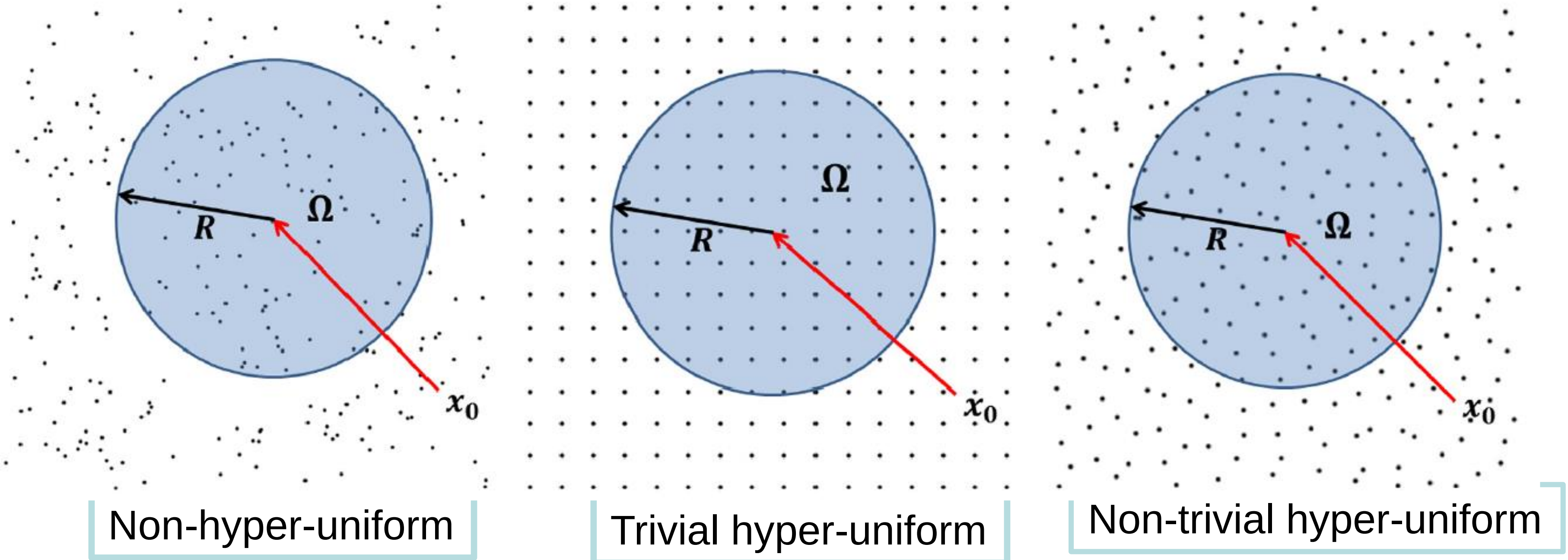
Nonlinear Dynamics of Oscillatory Systems (NWP-1)
Topical Problems in Nonlinear Wave Physics (NWP-2021)
Nizhny Novgorod, Russia, 20/9/2021

Outline

- Motivation: hyper-regular and time-crystal states
- Analogy between time-delayed systems and spatially-extended systems
- Dynamics of diode lasers with time-delayed feedback
  - Without modulation: irregular optical spikes
  - With weak periodic modulation of the laser current
- Quantification of the temporal regularity of the timing of the spikes using the Fano Factor
- Conclusions and open questions

Not all "disordered systems" are equally disordered



How does the number of particles inside the circle varies with the radius of the circle?

$$\sigma_N^2(R) \sim R^d$$

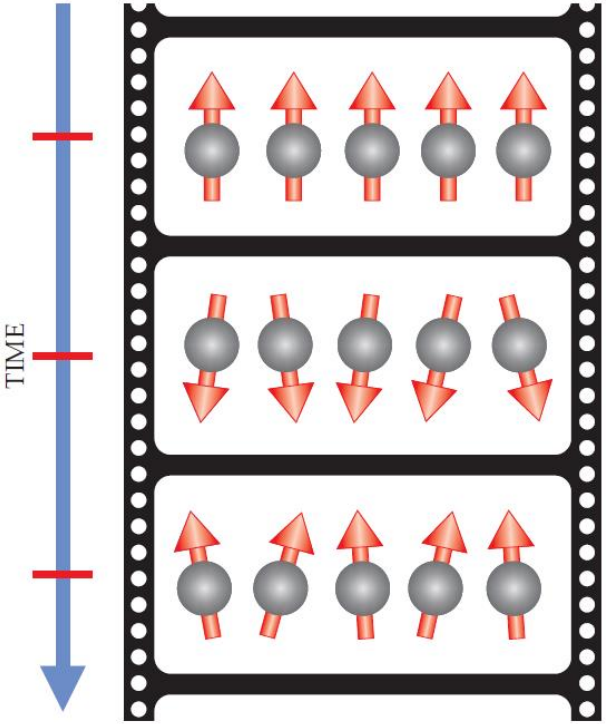
But some disordered systems show a slower grow of density fluctuations (asymptotic scaling between surface and volume growth).

S. Torquato / Physics Reports 745 (2018) 1–95

Time-crystal and hyper-uniform states: peculiar states of some disordered systems

- **Hyper-uniform states** exhibit an anomalously long-ranged suppression of density fluctuations. Many observations.
- **Time-crystal states** in ***periodically driven*** systems exhibit:
  - **highly regular oscillations** (in space and in time) that persist over long time intervals,
  - these oscillations are robust under small variations of the initial conditions or parameters ("**rigidity**"),
  - **break time-translation symmetry** because the period of the oscillations differs from the period of the driving signal: subharmonic locking but **no harmonic locking**.
  - Observed in many-particle quantum systems.

S. Torquato, Phys. Rep. 745, 1 (2018).
F. Wilczek, Phys. Rev. Lett. 109, 160401 (2012).
N. Y. Yao, and C. Nayak, Physics Today 71, 9, 40 (2018).



- The spin orientation flips during each driving period, so it takes two periods for the spins to return to something resembling their initial state.
- But to someone viewing the system at fixed intervals (that is, stroboscopically), the system appears to be in equilibrium.

Time crystals' defining traits

	Time crystal	Period-doubled nonlinear dynamical system	Mode-locked laser	Parametric down-conversion	NMR spin echo	Belousov–Zhabotinsky reaction	Convection cells	AC Josephson effect
Many-body interactions	✓	X	✓	✓	X	X	✓	✓
Long-range order	✓	X	X	X	X	X	X	X
Crypto-equilibrium	✓	X	X	X	X	X	X	X

N. Y. Yao and C. Nayak, Physics Today Sep. 2018

Can we find time crystal behavior in classical, high-dimensional dynamical systems?

Stochastic **time delayed systems** (TDSs) represented by

$$du(t)/dt = f(u(t), t) + K\, u(t-\tau) + \xi(t)$$

are infinite dimensional because the initial condition is the function u(t) **defined in [-τ,0].**

The dynamics of some TDSs has similarities to the dynamics of some one-dimensional **spatially extended systems** (1D SESs)

$$\partial u(x,t)/\partial t = f(u,x,t) + D\, \partial^2 u/\partial x^2 + \xi(x,t) \quad \text{with } x(t) \text{ in } [0, L]$$

C. Quintero-Quiroz, M. C. Torrent, and C. Masoller, Chaos 28, 075504 (2018)

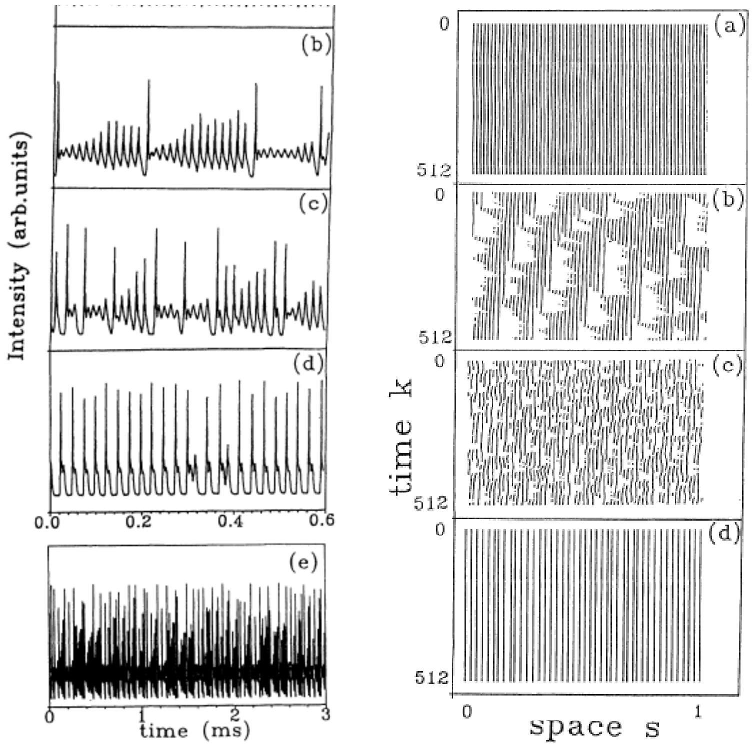
Similarities between time-delayed and spatially-extended systems (pattern formation, wave propagation) can be visualized using a 2D representation.

RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 45, NUMBER 7 1 APRIL 1992

Two-dimensional representation of a delayed dynamical system

F. T. Arecchi,* G. Giacomelli, A. Lapucci, and R. Meucci
Istituto Nazionale di Ottica, Largo E. Fermi 6, 50125 Firenze, Italy
(Received 31 July 1991; revised manuscript received 10 December 1991)

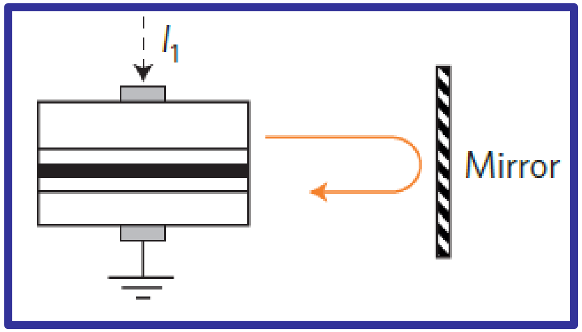


Main question

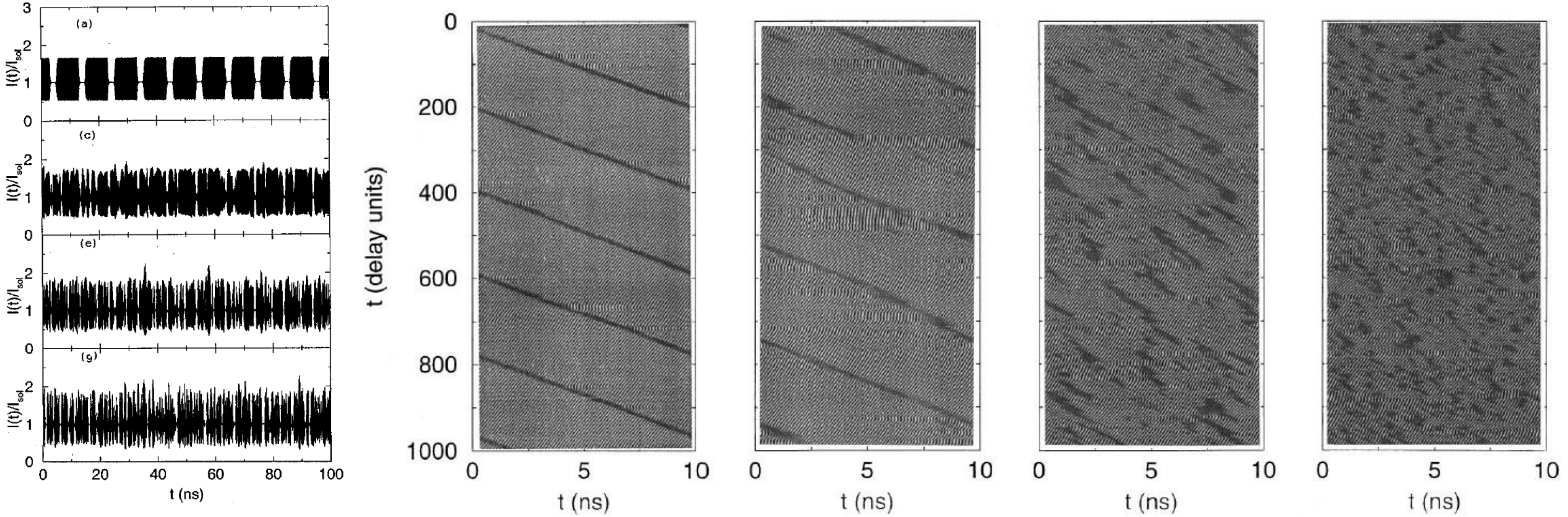
Can we find a stochastic time delayed system that has peculiar states which are analogous to hyper-uniform or time-crystal states?

Semiconductor laser with feedback light

- Time-delay due to propagation time (ns)
- Laser current can be modulated with a small-amplitude signal.
- Near threshold: stochastic dynamics (quantum spontaneous emission).

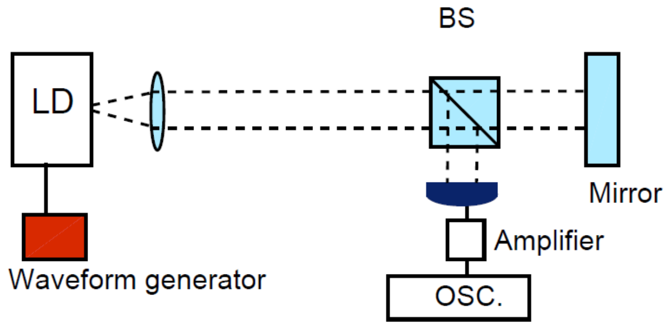


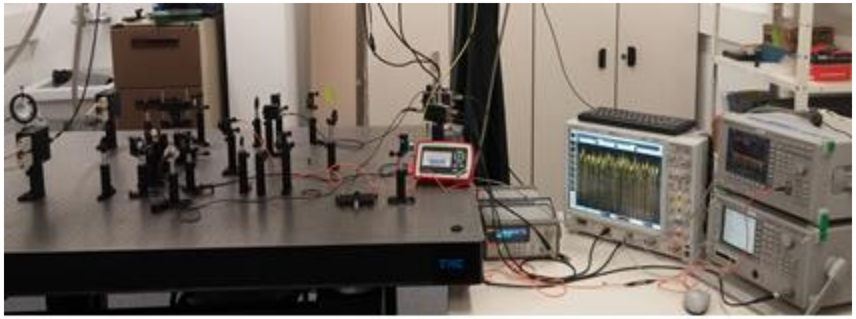
Model simulations (years ago): increasing the feedback strength → complex dynamics

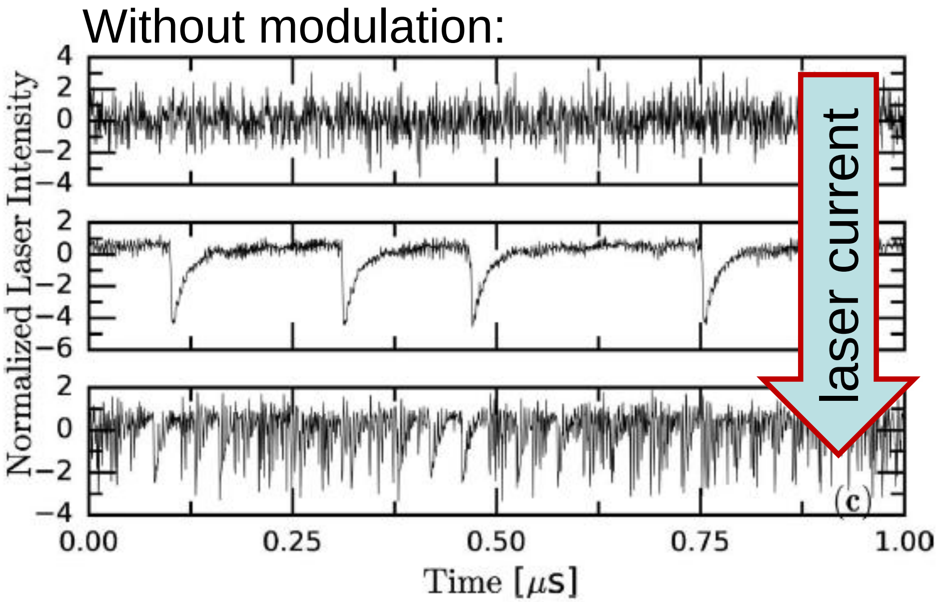


C. Masoller, Chaos 7, 455 (1997)

Key advantage: experiments can be done with precise control of different parameters (here: modulation amplitude, modulation frequency and dc value of the laser current)



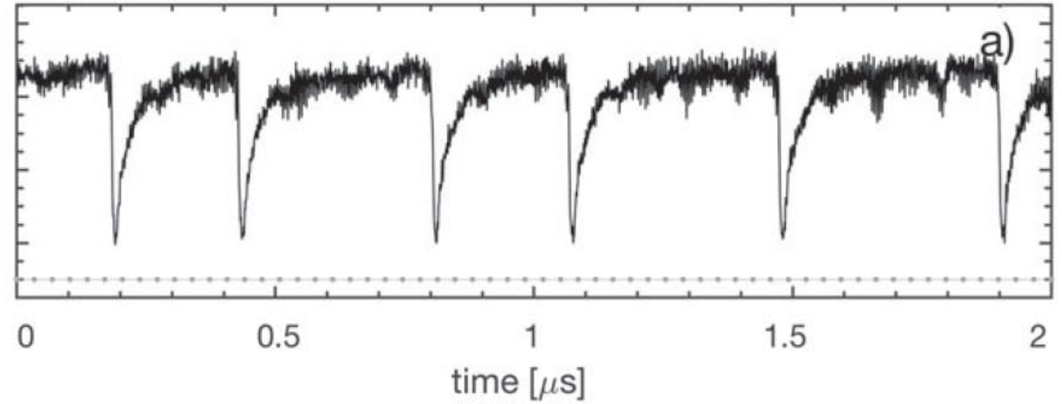




We focus on the parameter region where the laser emits "**spikes**". Questions:

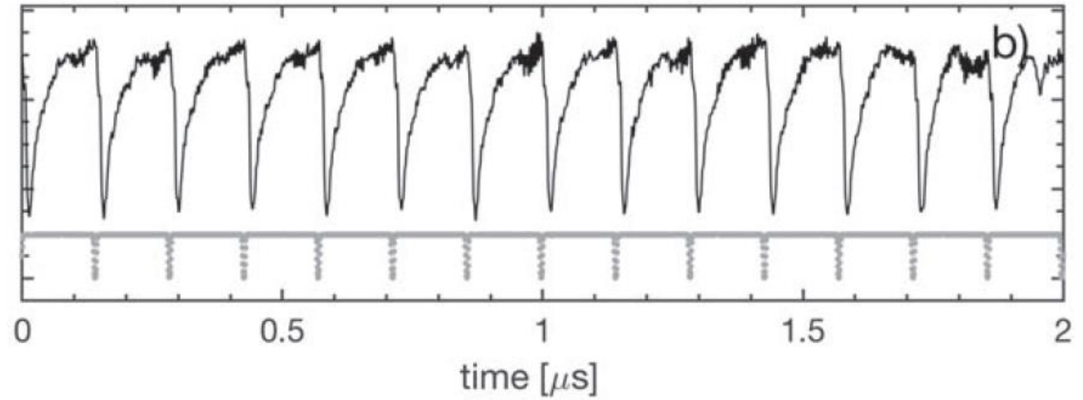
- Can we **lock the spikes** to a **weak** periodic signal that drives the laser current?
- Which waveform is best for observing highly regular spikes?
- How regular can the spikes be?
- How can we quantify the regularity of the spike timing?

Without modulation: irregular spike timing



With pulsed modulation (2.4% of I_{dc}):

- **regular** 1:1 locked spikes;
- **irregular** oscillations in between the spikes



J. Tiana-Alsina et al, New J. Phys. 21 103039 (2019)

Early experiments with sinusoidal modulation

I_{dc} = 39 mA
f_{mod} = 17 MHz

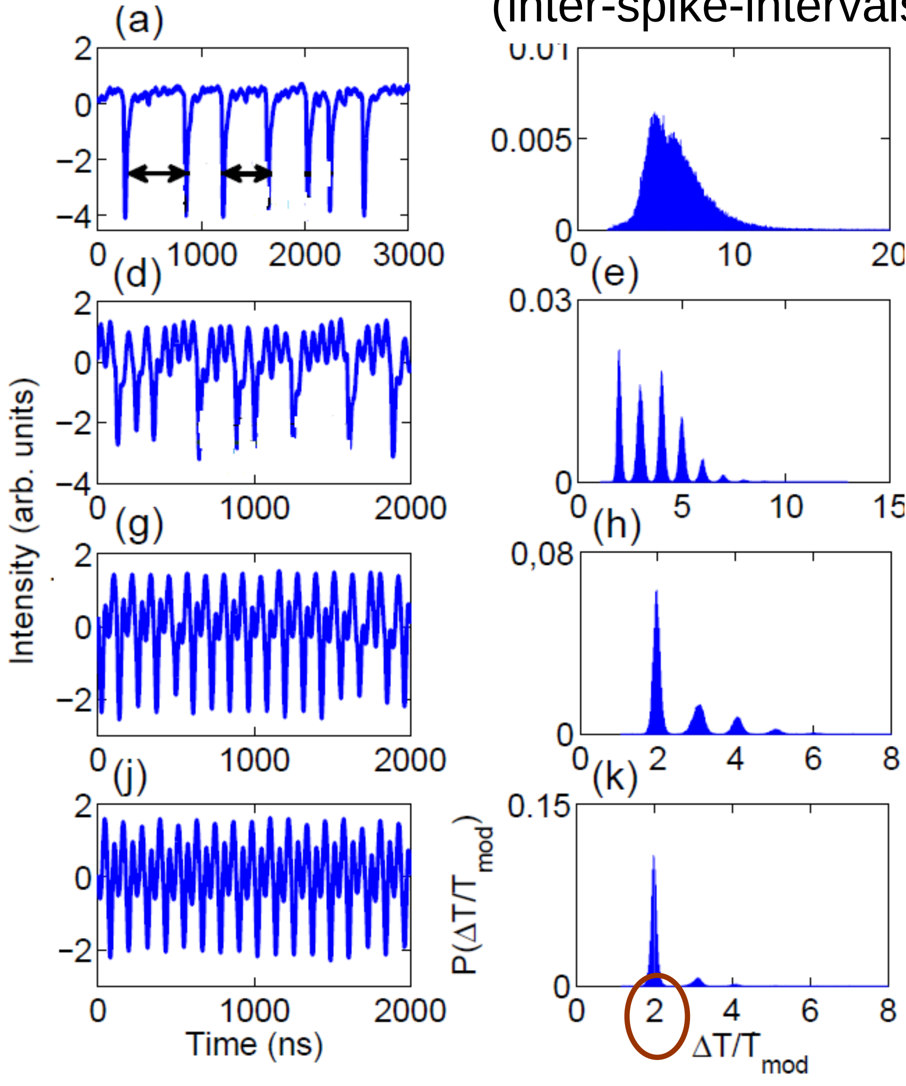
Distribution of intervals between spikes (inter-spike-intervals)

No modulation

1.2%

1.6%

2%



A. Aragoneses et al., Optics Express **22**, 4705 (2014).

Sinusoidal modulation: varying the modulation frequency while keeping constant the modulation amplitude (1.2 % I_{DC})

7 MHz

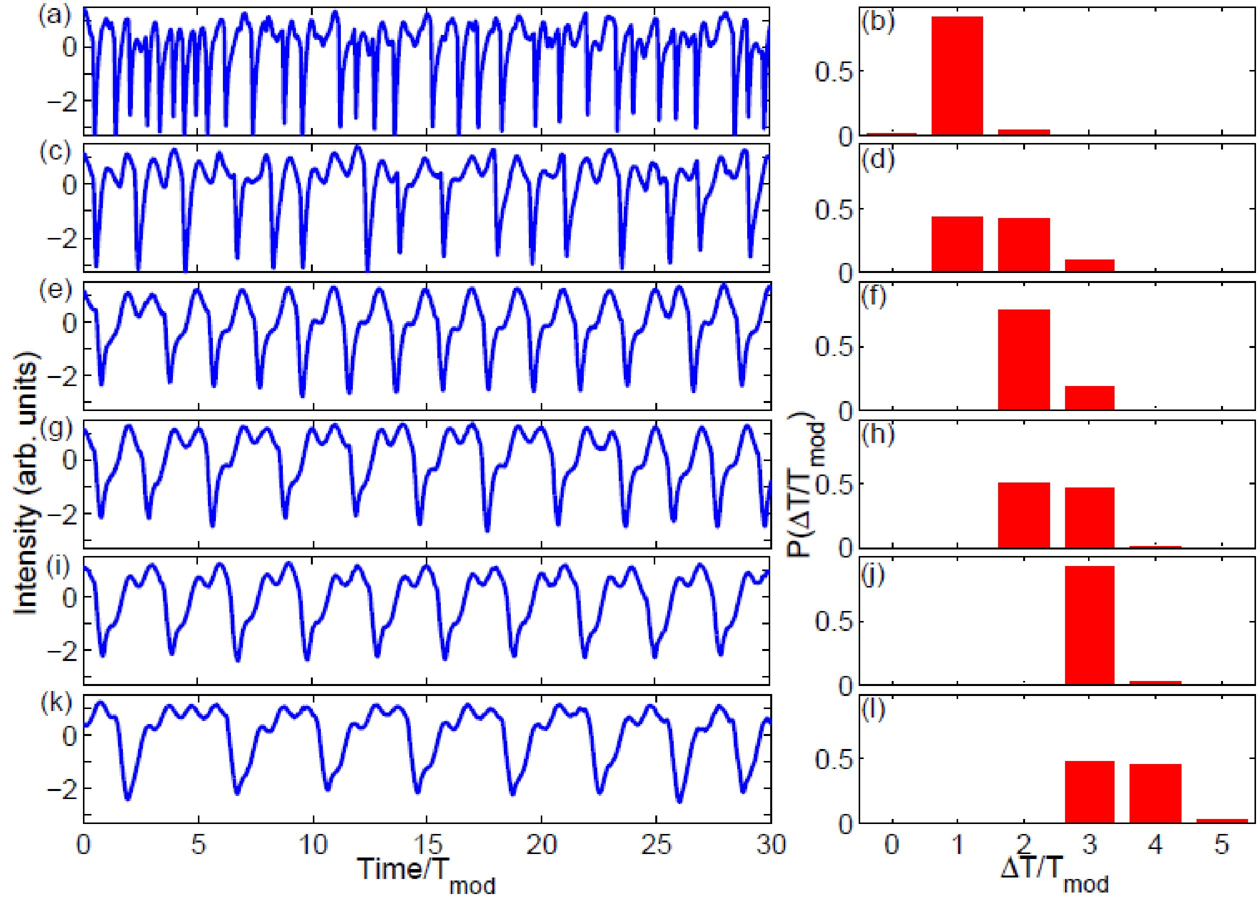
14 MHz

26 MHz

31 MHz

39 MHz

49 MHz

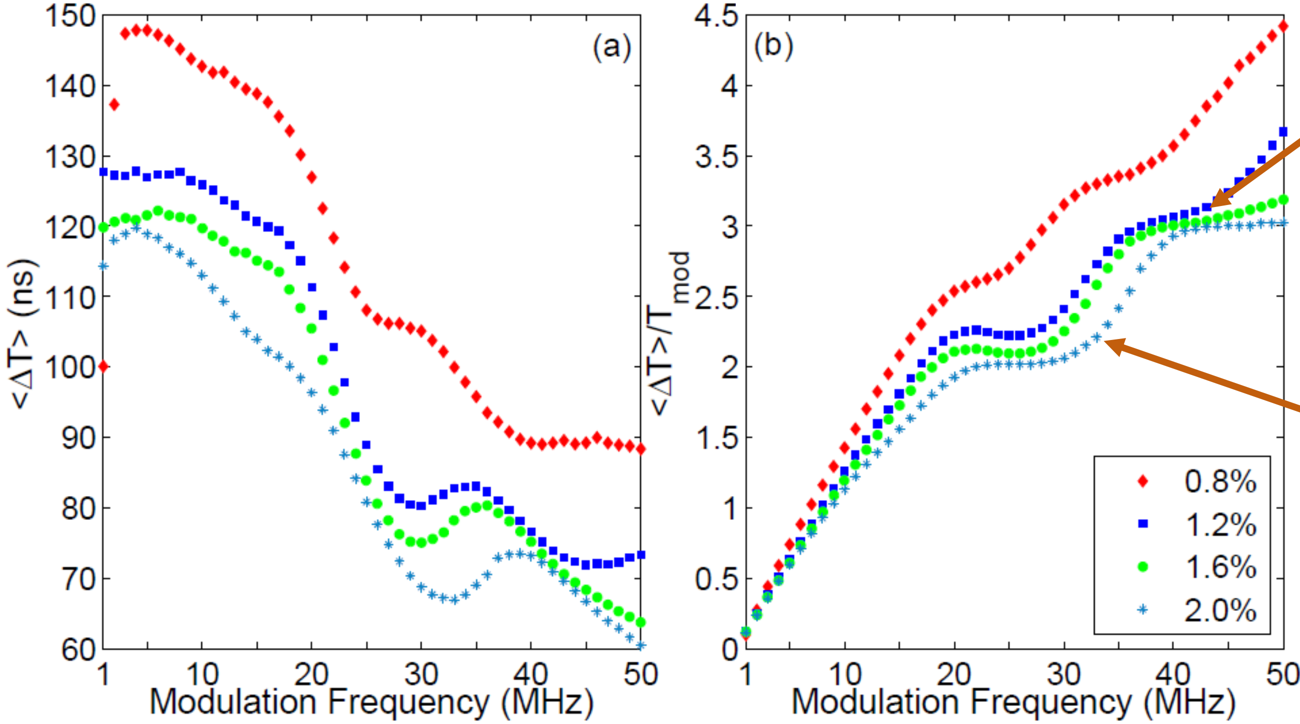


T. Sorrentino et al., Optics Express **23**, 5571 (2015).

Locking "plateaus"

Average time interval between consecutive spikes.

Average time interval between consecutive spikes, *normalized to the modulation period*.



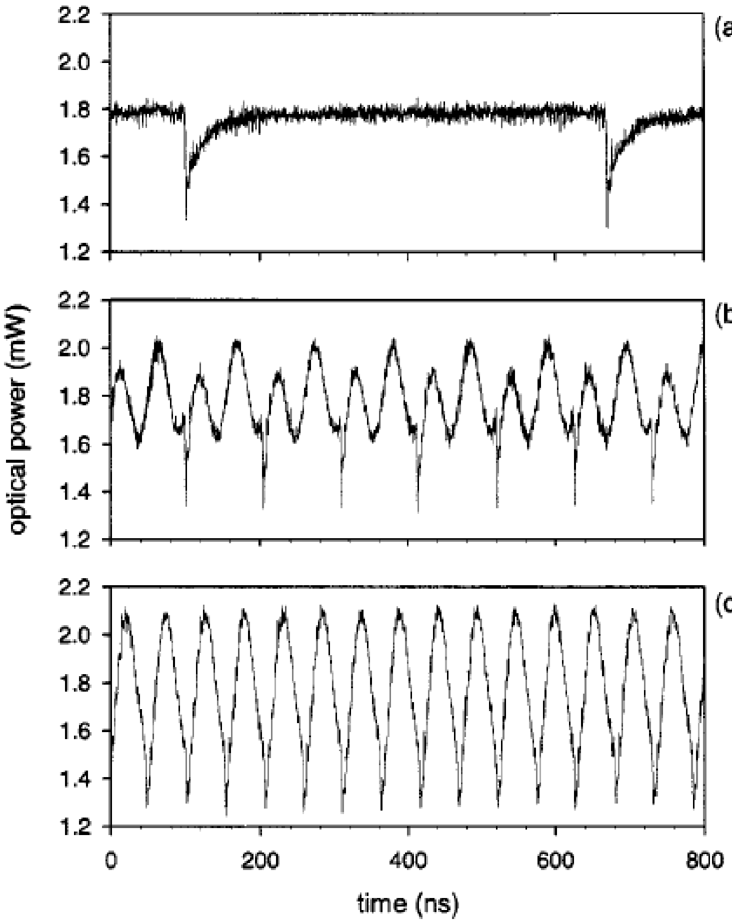
1 spike every 3 modulation cycles

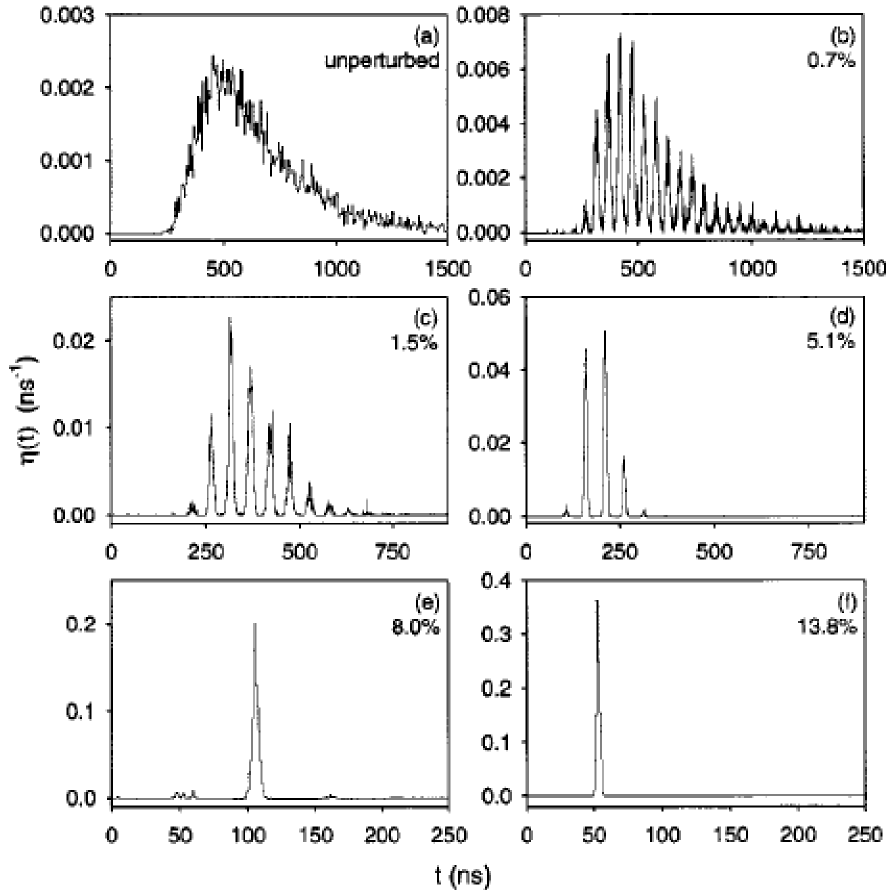
1 spike every 2 modulation cycles

Why no 1:1 locking plateau?

T. Sorrentino et al., Optics Express **23**, 5571 (2015).

Earlier work





1:1 locking found with large modulation amplitude (~14%)

D. W. Sukow and D. J. Gauthier, IEEE J. Quantum Electron 36 (2000).

How regular can the *timing* of the spikes be?

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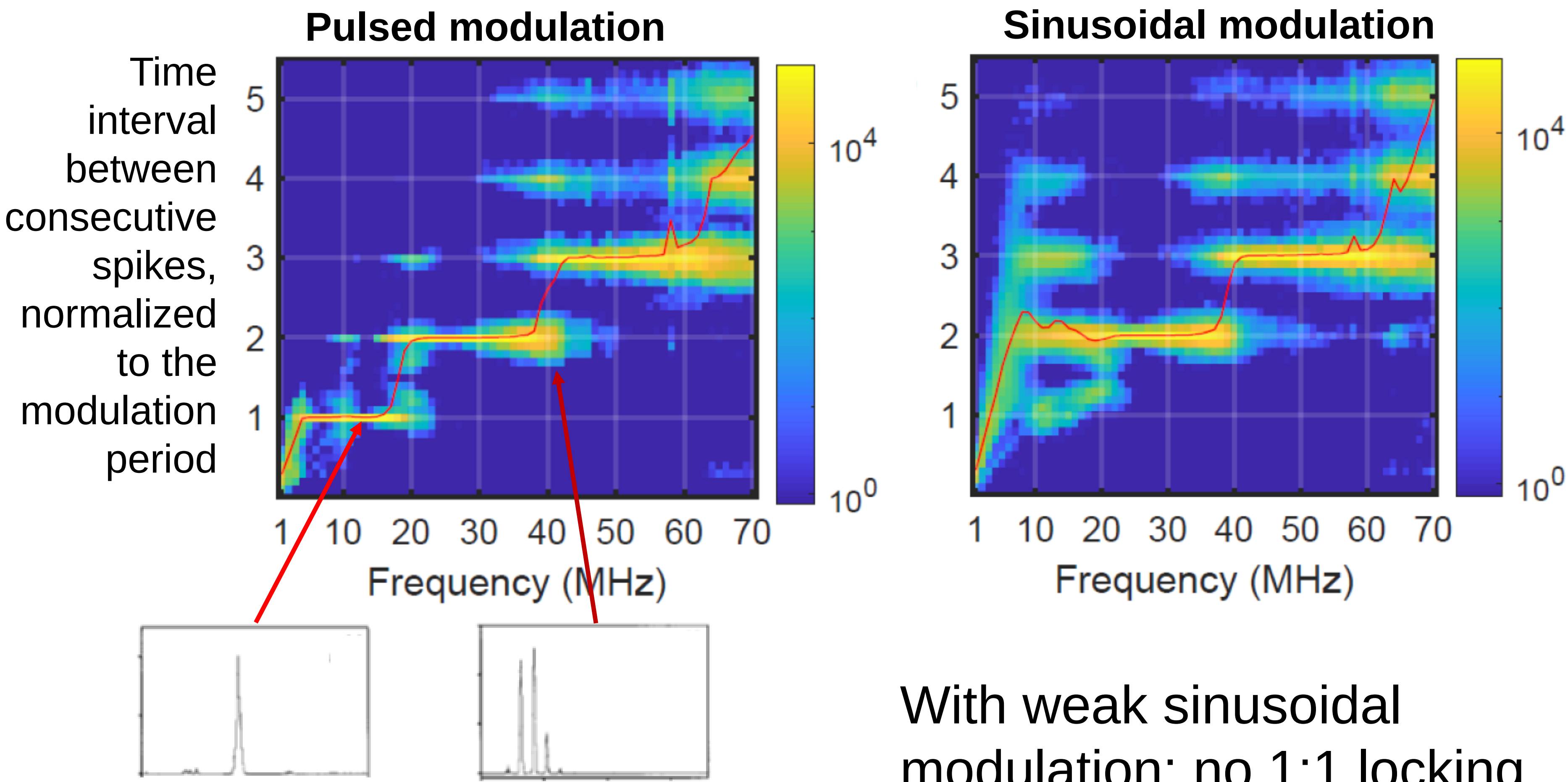
50 ANYS sense límits

Distribution of inter-spike-intervals (log color code) for different modulation frequencies (I_{dc}=26 mA, mod. amplitude=0.631 mA ≈ **2.4%**)



With weak sinusoidal modulation: no 1:1 locking plateau (consistent with earlier experiments).

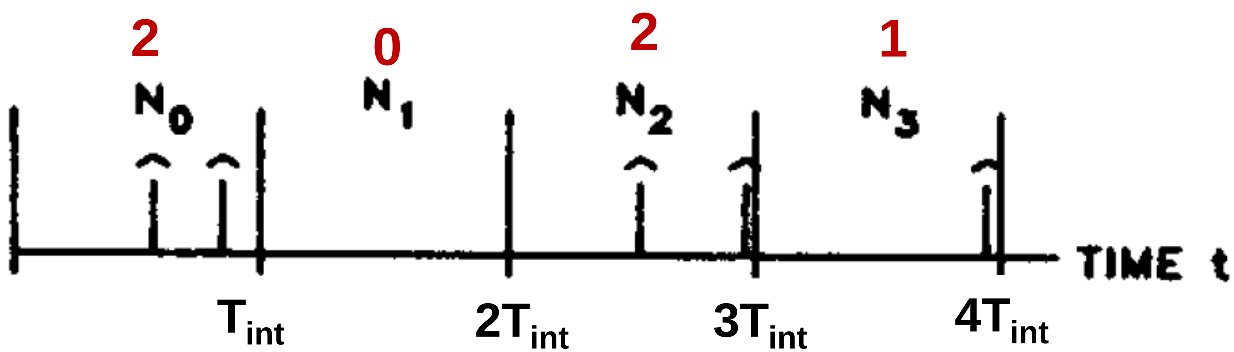
The Fano Factor: a precise measure of spike timing regularity

How to calculate the Fano Factor?

- Divide the intensity time trace in N_{int} non-overlapping segments of duration T_{int}.
- Count the number of spikes in each segment, $\{N_1, N_2, \ldots N_{Nint}\}$.
- Calculate the mean and the variance, $\langle N_i \rangle$, σ^2
- Calculate the Fano factor as $F = \sigma^2(N_i)/\langle N_i \rangle$
- *F* depends on the duration of the counting interval, T_{int}.
- If T_{int} is very small, *F=1* because the sequence of counts is a sequence of 0s and 1s.
- If the process that triggers the spikes is fully random, $F=1 \; \forall T_{int}$.
- To test the presence of correlations in the timing of the spikes:
  - Shuffle the inter-spike intervals
  - Recalculate the spike times
  - Recalculate *F*
  - Compare the *F* values of the original and shuffled spike times.

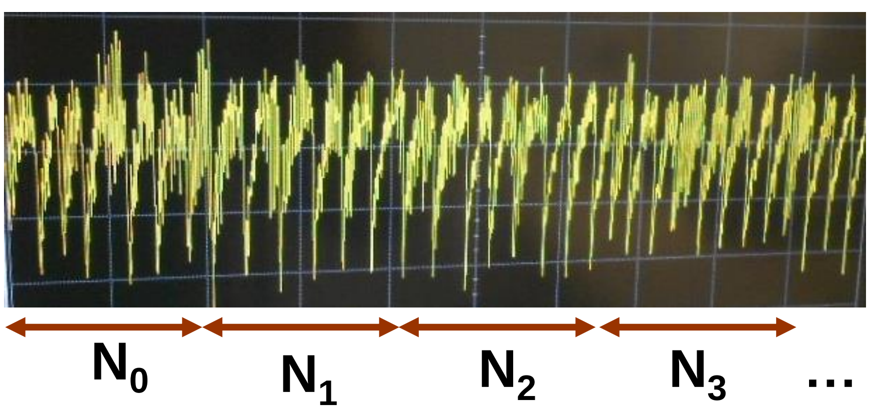
The Fano Factor has been widely used to analyze the timing of neural spike trains

$N_{int} = 4$



Sequence of counts: $\{N_i\} = \{2, 0, 2, 1\}$

$$F = \sigma^2(N_i) / \langle N_i \rangle$$



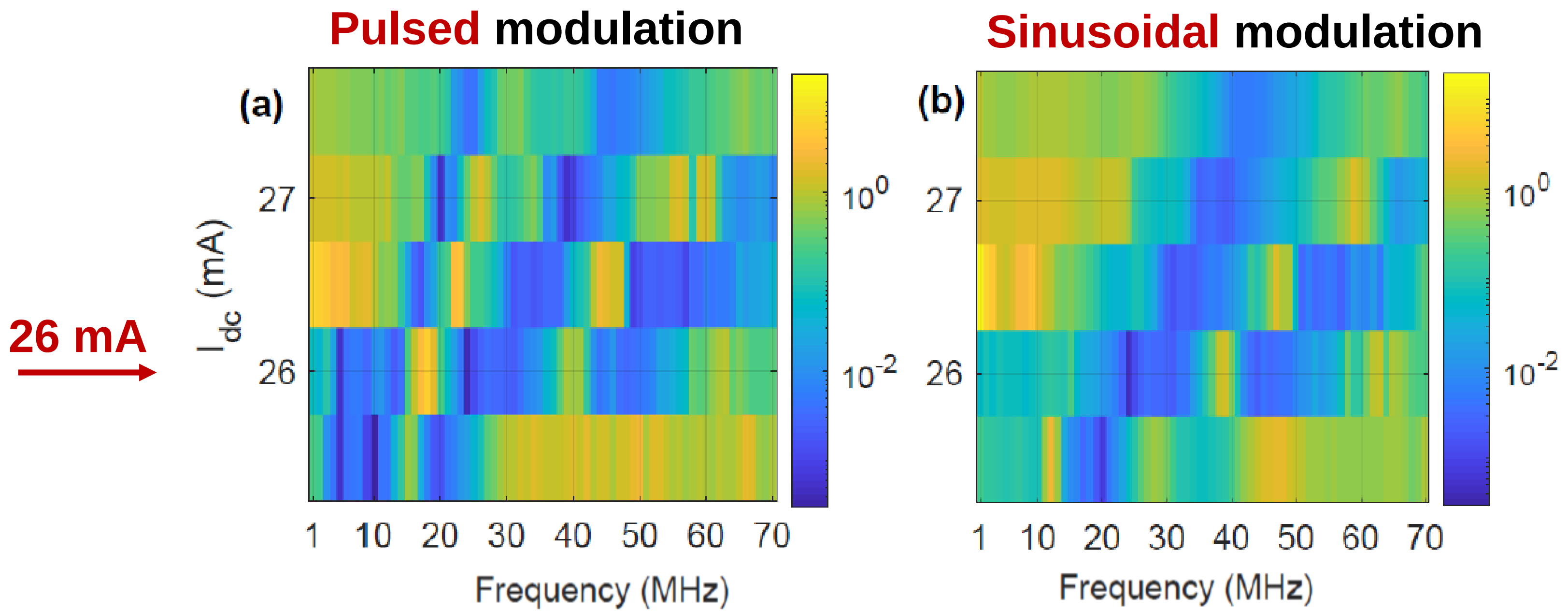
In our experiments the total recorded time, **5 ms**, contains 9000-120000 spikes, depending on the parameters.

Number of intervals: N_{int} = **1000**,
Duration of each interval: T_{int} = **5 μs**

M. C. Teich et al, J. Opt. Soc. Am. A 14, 529 (1997)

Fano Factor (color code) of sequences of optical spikes

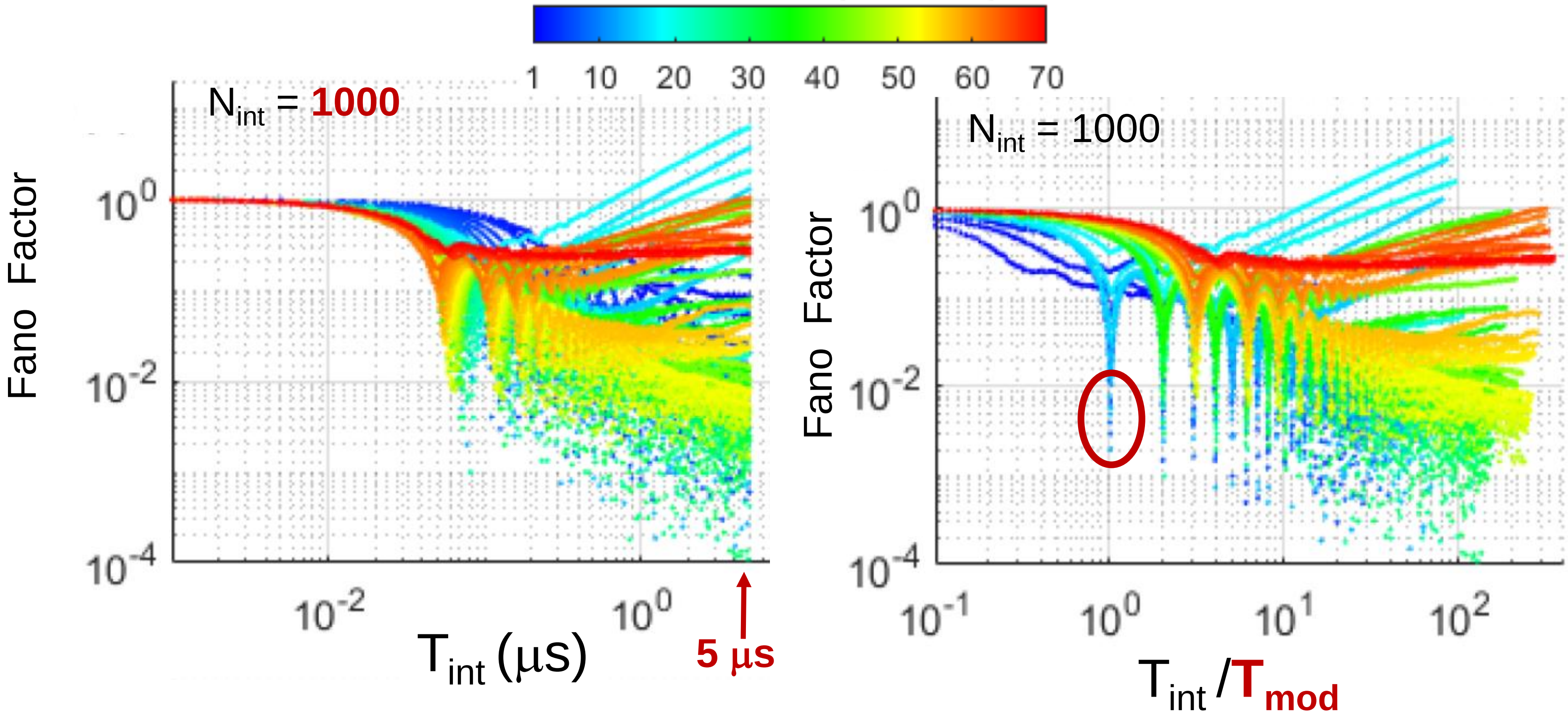
recorded for different I_{dc} and f_{mod}, keeping fixed A_{mod} (≈1-2.5 % of I_{dc})



- **Blue** regions: small $F \Rightarrow$ small $\sigma \Rightarrow$ regular sequence of counts $\Rightarrow$ regular spikes.
- Yellow regions: large $F \Rightarrow$ large $\sigma \Rightarrow$ high variability in the sequence of counts.
- For the pulsed signal there are three blue regions; for the sinusoidal signal, only two.

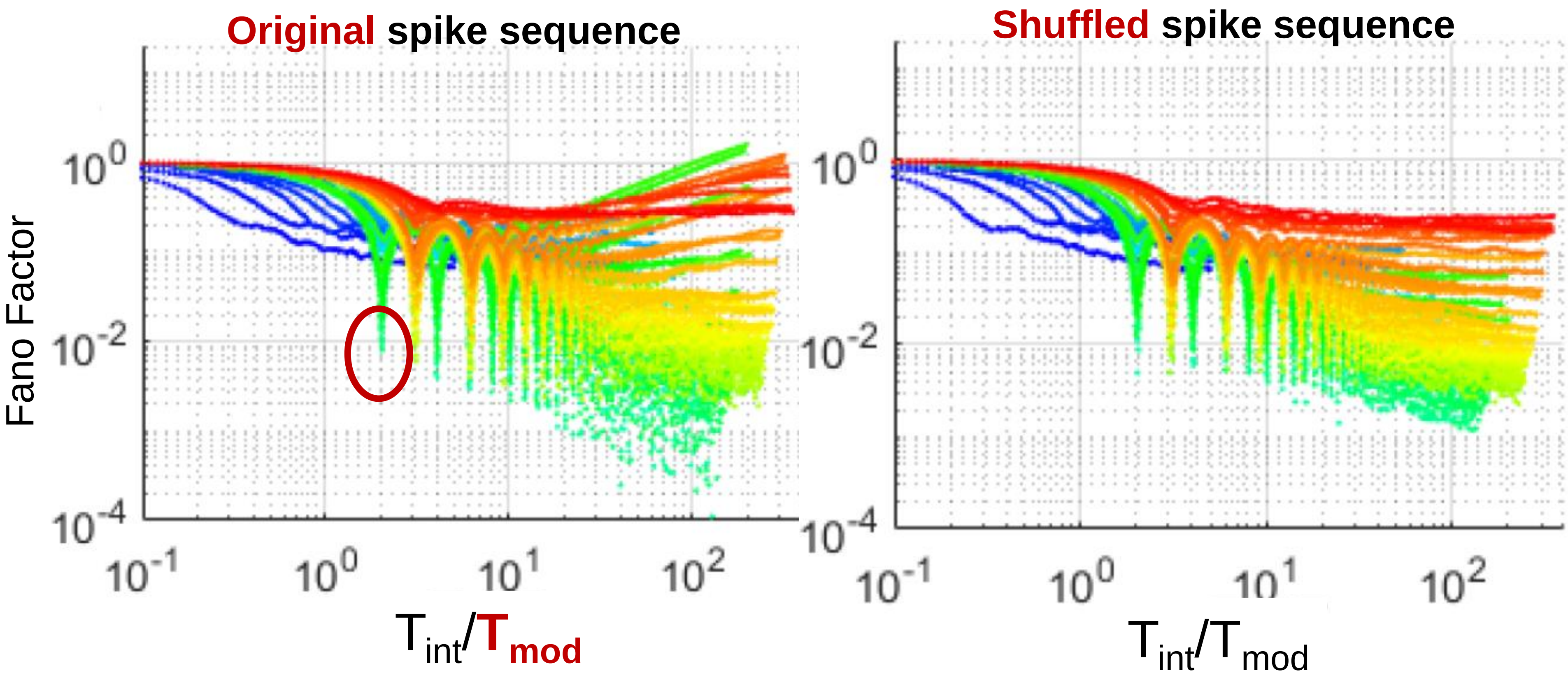
How the Fano Factor depends on the duration of the counting interval?

Here: **pulsed** modulation, I_{dc}=26 mA, the color represents the mod. frequency (MHz).



- Sharp minima reveal that the sequence of counts is very regular when the counting interval contains an integer number of modulation periods.
- $T_{int}=T_{mod}$: the sequence of counts is (1,1,…,1), i.e., 1000 intervals with one spike in each interval.

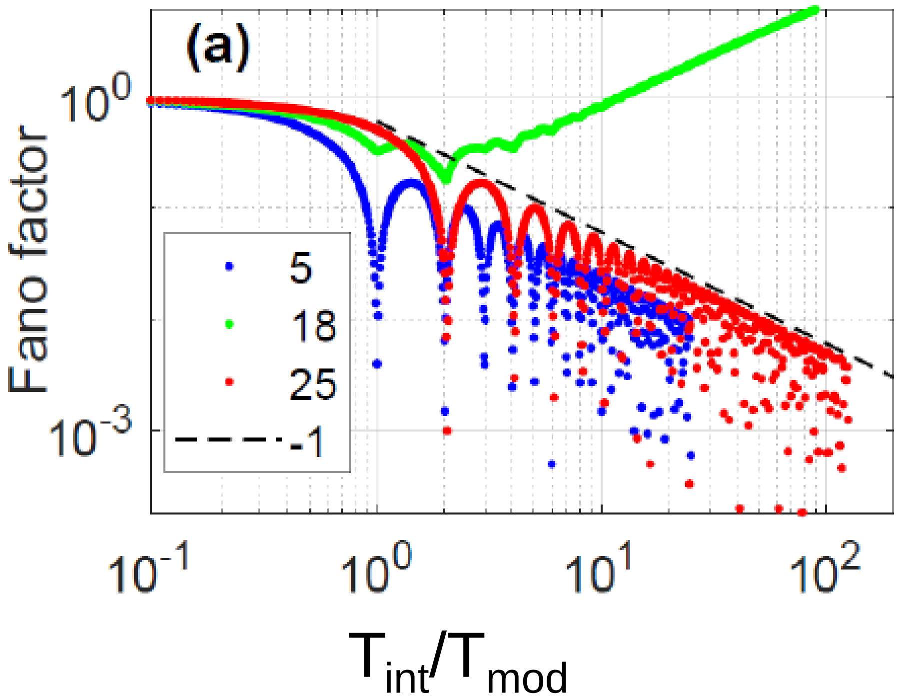
With sinusoidal modulation:



- First minima at T_{int}=2 T_{mod}
- Minima are more pronounced in the original spike sequence than in the shuffled one (temporal correlations are removed when shuffling the spikes).

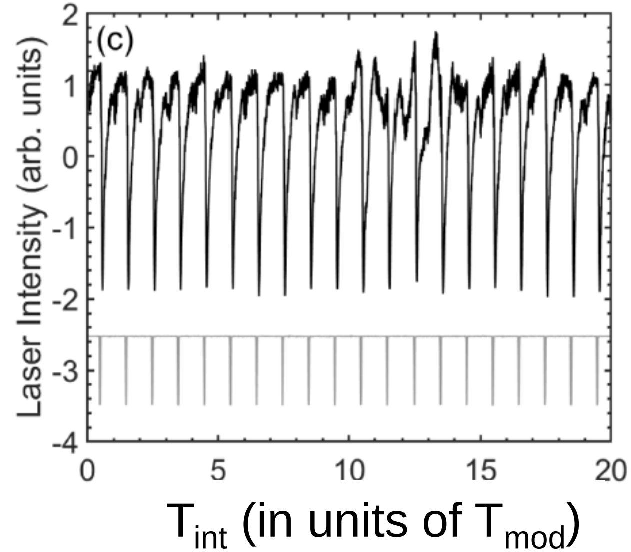
For some modulation frequencies: power law variation of the Fano factor with the size of the counting window

Pulsed modulation

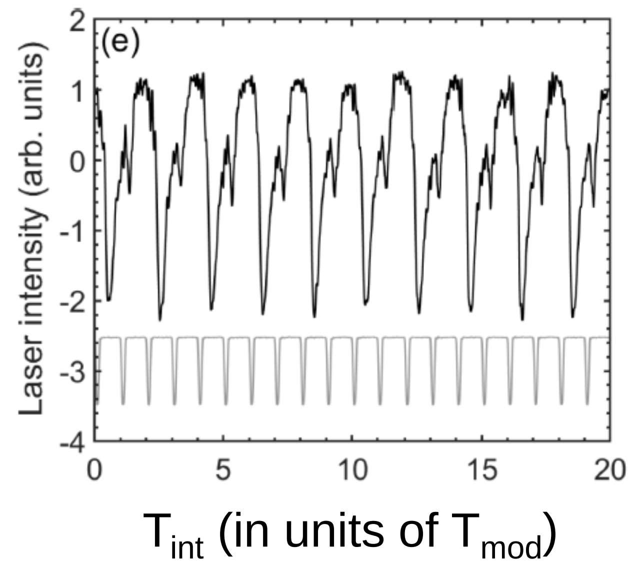


f_{mod}

5 MHz



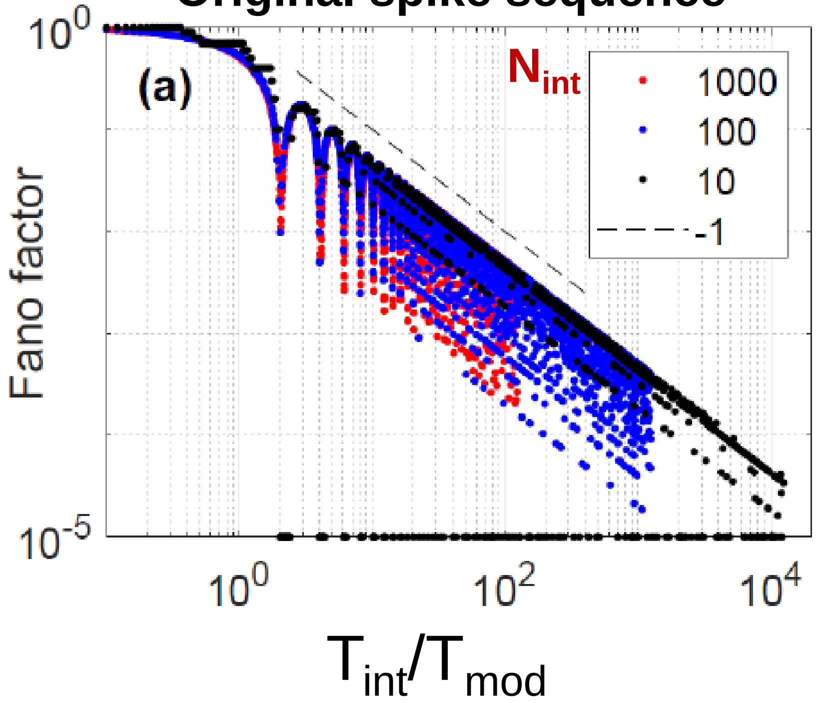
25 MHz



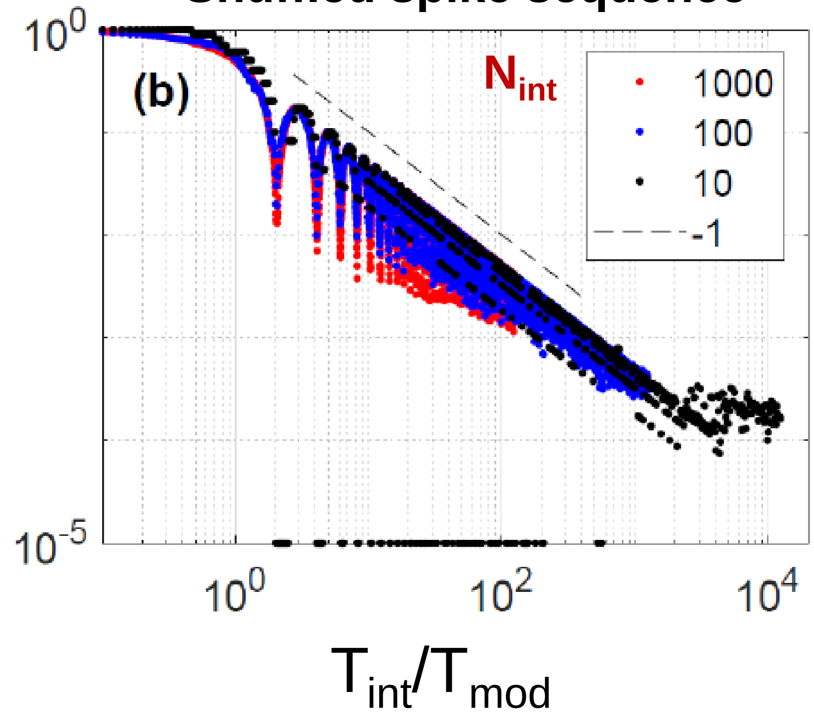
Only the timing of the spikes is regular; the fluctuations between spikes are irregular.

Power law persists for longer counting windows?

Original spike sequence



Shuffled spike sequence



Spike timing is regular over time intervals that contain 10^4 cycles of the modulation.

Power law variation saturates when the sequence of inter-spike intervals is suffled.

Conclusions and open questions

- Pulsed modulation generates locked spikes (1:1 and 2:1) with long-range regularity (analogous to “hyper-uniform” states).
- Sinusoidal modulation generates sub-harmonically locked spikes with long-range regularity (no 1:1 locking, analogous to “time-crystal” states).
- This is in contrast with classical nonlinear oscillators that show both, 1:1 and higher order lockings.
- Which mechanisms induce long-range order?
- Why the sinusoidal signal does not produce 1:1 locking?
- Model simulations are in good agreement with the observations

  [J. Tiana-Alsina and C. Masoller, Appl. Sci. 11, 7871 (2021)]
- Generic phenomena that may be observed in other periodically modulated stochastic time delayed systems?
- Influence of the feedback strength and the delay time?

Thank you for your attention!
Cristina.masoller@upc.edu

J. Tiana-Alsina et al., Phys. Rev. E 99, 022207 (2019)
J. Tiana-Alsina and C. Masoller, Appl. Sci. 11, 7871 (2021)