Network dissimilarity measure and application to brain network differentiation

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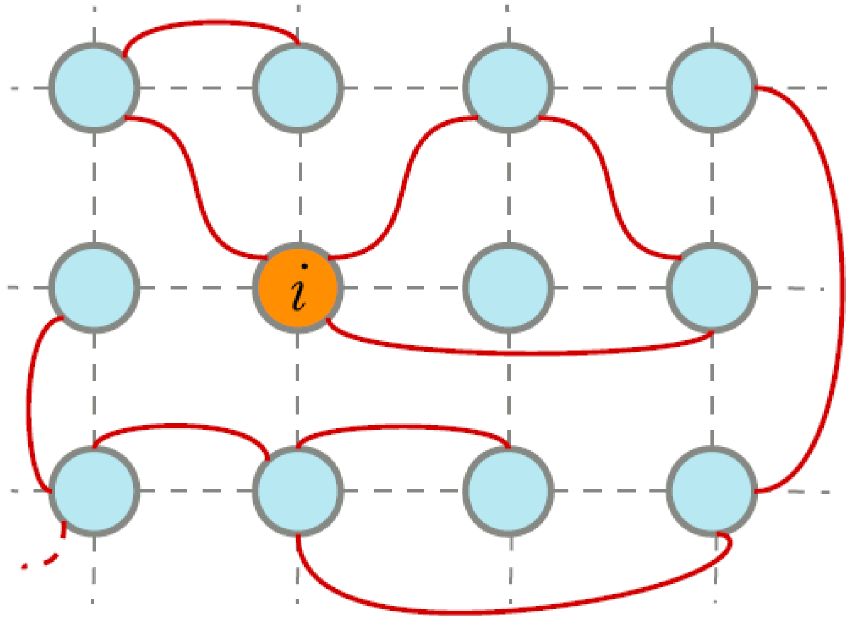
- Motivation
- Dissimilarity measure
- Applications

Motivation: how to compare time-evolving correlation networks

$$A_{ij} = \mathrm{H}(|\mathcal{C}(B_i, B_j)| - \theta)$$

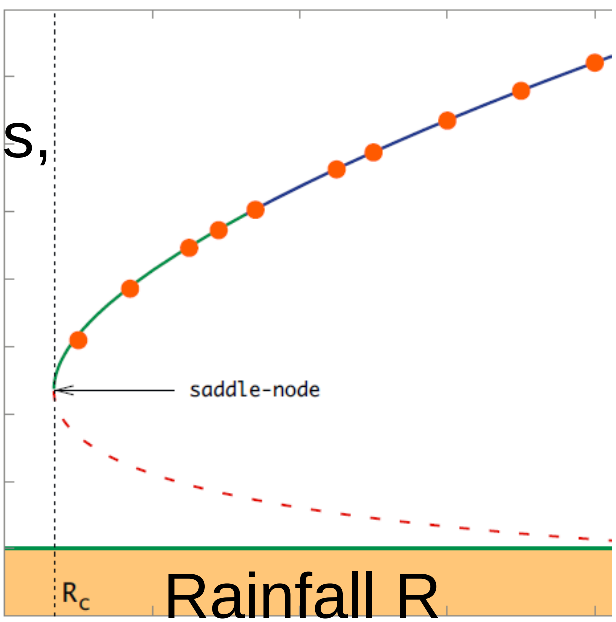


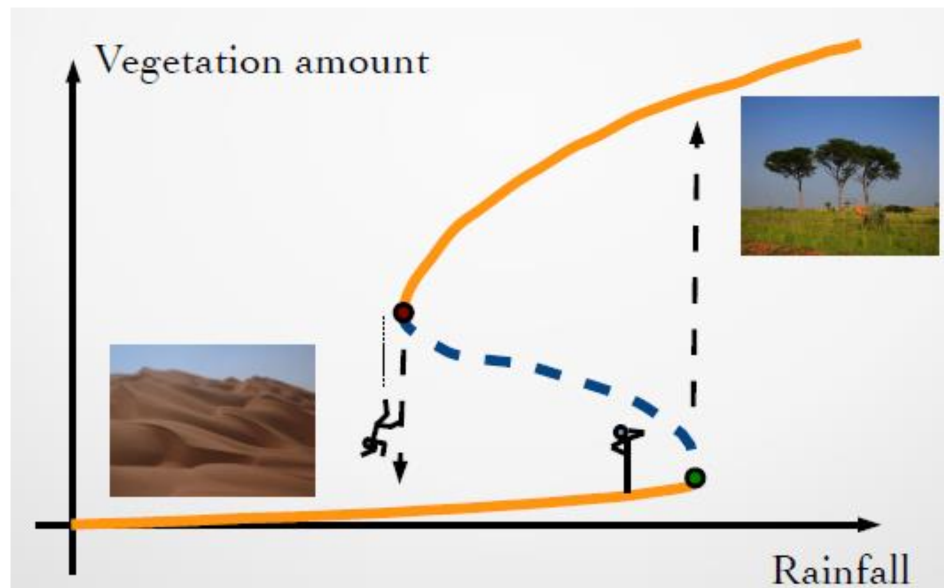
Example: desertification transition under the lens of correlation network

$$\frac{\partial w}{\partial t} = R - \frac{w}{\tau_w} - \Lambda w B + D\nabla^2 w + \sigma_w w_0 \xi^w(t),$$

$$\frac{\partial B}{\partial t} = \rho B\left(\frac{w}{w_0} - \frac{B}{B_c}\right) - \mu \frac{B}{B + B_0} + D\nabla^2 B + \sigma_B B_0 \xi^B(t)$$

Biomass,
B





G. Tirabassi et al., Ecological Complexity (2014)

Network analysis

- **Degree** (number of links of a node)

$$k_i \equiv \sum_{j=1}^{N} A_{ij}$$

- **Assortativity** (average degree of the neighbors of a node)

$$a_i \equiv \frac{1}{k_i} \sum_{j=1}^{N} A_{ij} k_j$$

- **Clustering coefficient** (fraction of neighbors of a node that are also neighbors among them)

$$c_i \equiv \frac{1}{k_i(k_i - 1)} \sum_{j=1}^{N} \sum_{l=1}^{N} A_{ij} A_{jl} A_{li}$$

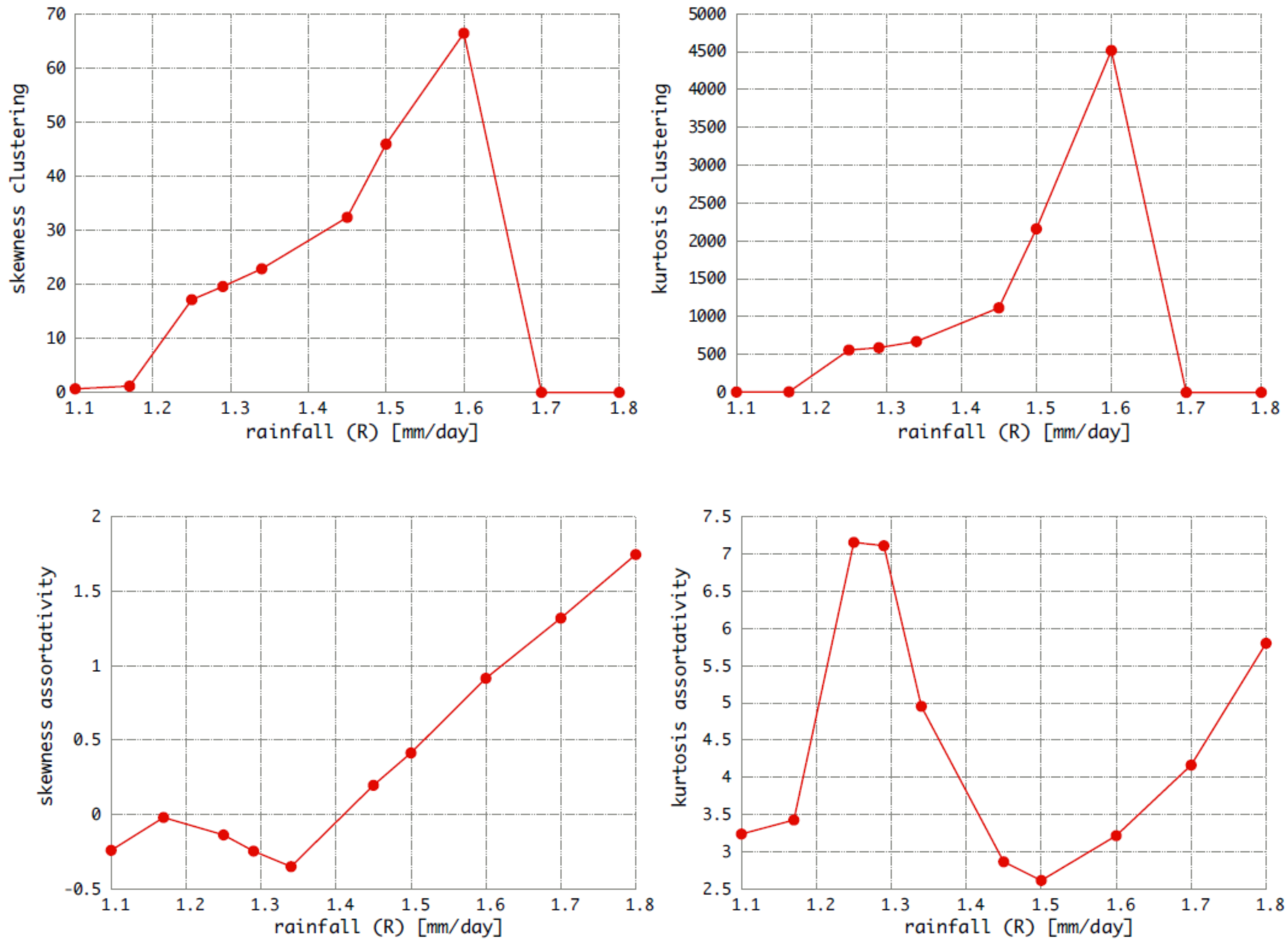
“Randomization” of the network when the tipping point is approached

skewness

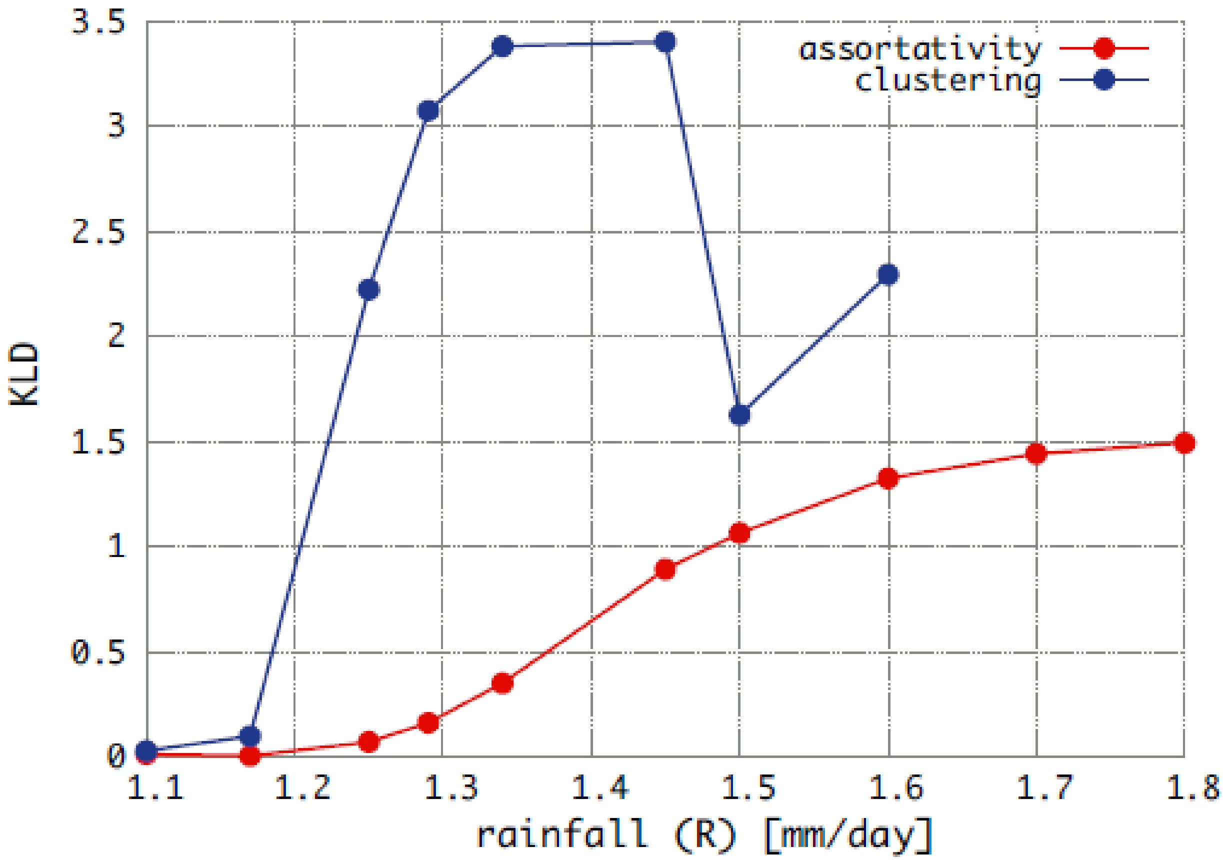
kurtosis

clustering

assortativity



The "randomization" can be quantified by the Kullback–Leibler Distance



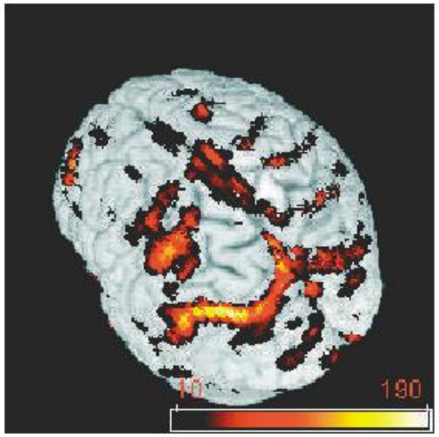
$$\mathrm{KLD} \equiv \int_{-\infty}^{\infty} \ln\left(\frac{P(x)}{Z(x)}\right) P(x)\, \mathrm{d}x.$$

- Open issue: the "Gaussianisation" might be a model-specific feature.

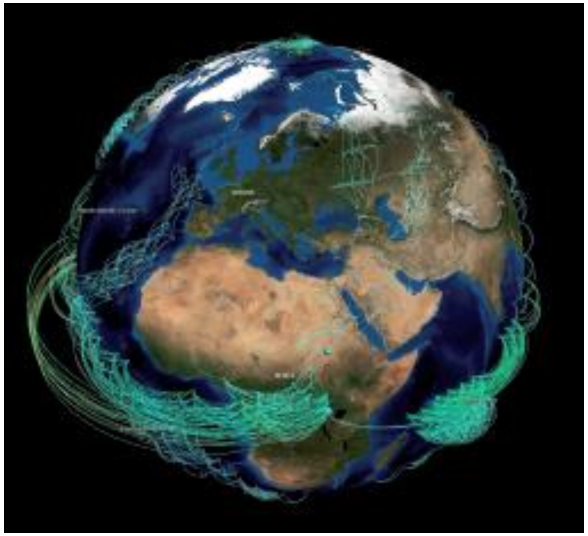
G. Tirabassi et al., Ecological Complexity 19, 148 (2014)

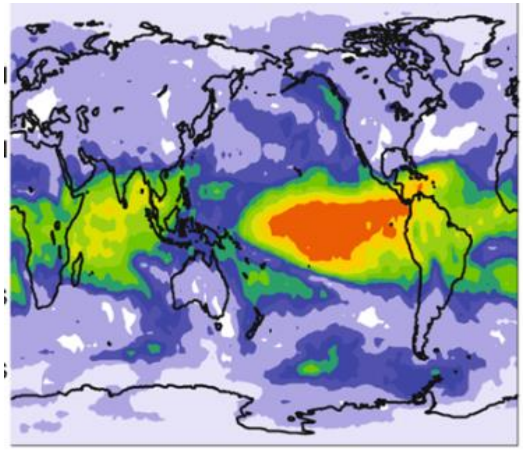
Other examples of time-evolving correlation networks

Brain



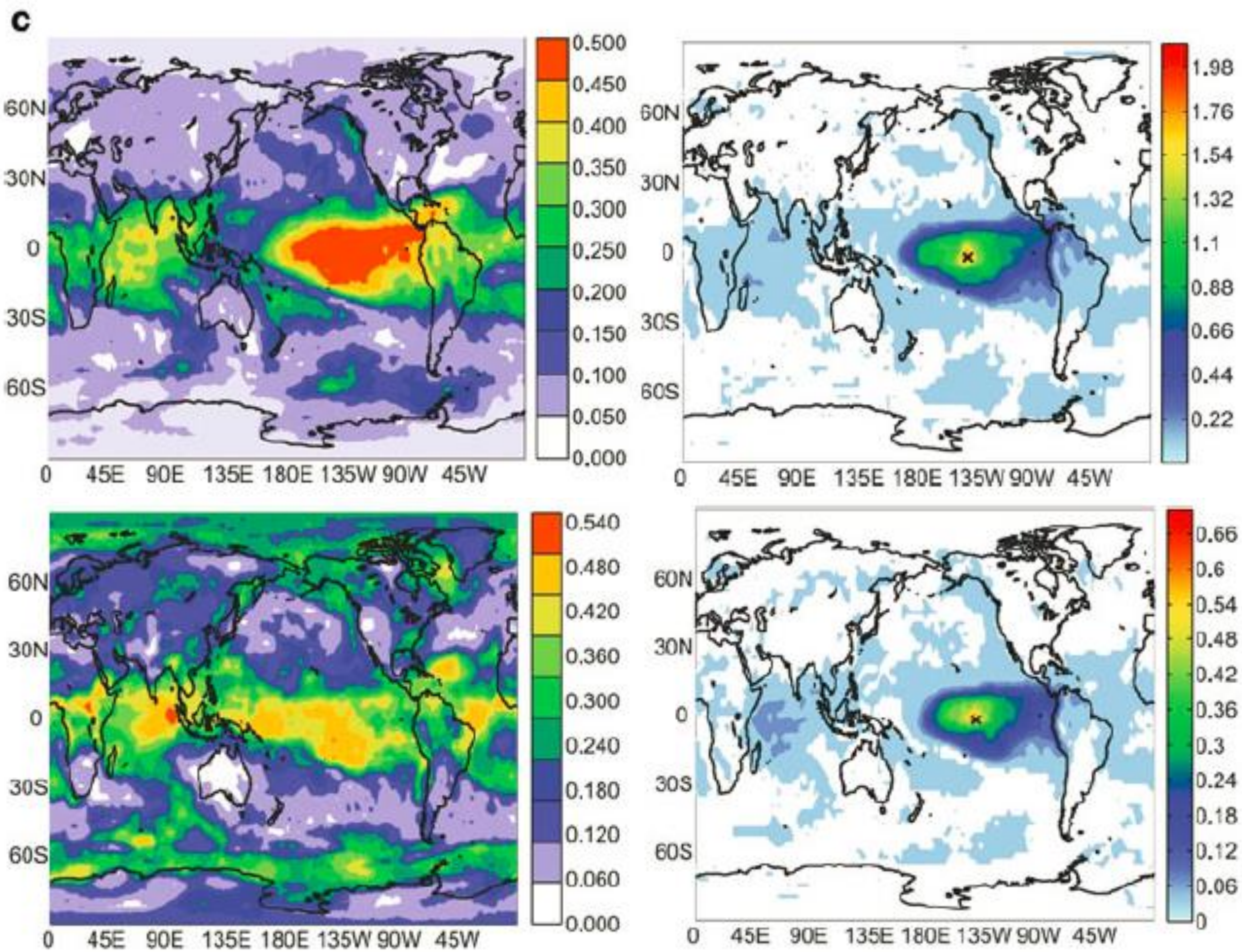
Climate





Weighted degree

Climate networks: how to detect relevant changes in the connectivity paths of the network?



Main Goal: to develop a measure that allows a precise comparison of complex networks (including different sizes)

A.A. Tsonis (ed.), *Advances in Nonlinear Geosciences*,
DOI 10.1007/978-3-319-58895-7_4

In order to detect structural changes we need a precise measure to compare networks

- Degree, centrality, assortativity distributions etc. provide *partial* information.
- How to define a measure that contains detailed information about the *global topology* of a network, in a *compact* way?

$\Rightarrow$ Node Distance Distributions (NDDs)

- $p_i(j)$ of node “i“ is the fraction of nodes that are connected to node i at distance j
- If a network has N nodes:

$$\text{NDDs} = \text{vector of N pdfs } \{p_1, p_2, \ldots, p_N\}$$

- If two networks have the same set of NDDs $\Rightarrow$ they have the same diameter, average path length, etc.

How to condense the information contained in the node distance distributions?

- The *Network Node Dispersion (NND)* measures the heterogeneity of the N pdfs $\{p_1, p_2, \ldots, p_N\}$
- Quantifies the heterogeneity of connectivity distances.

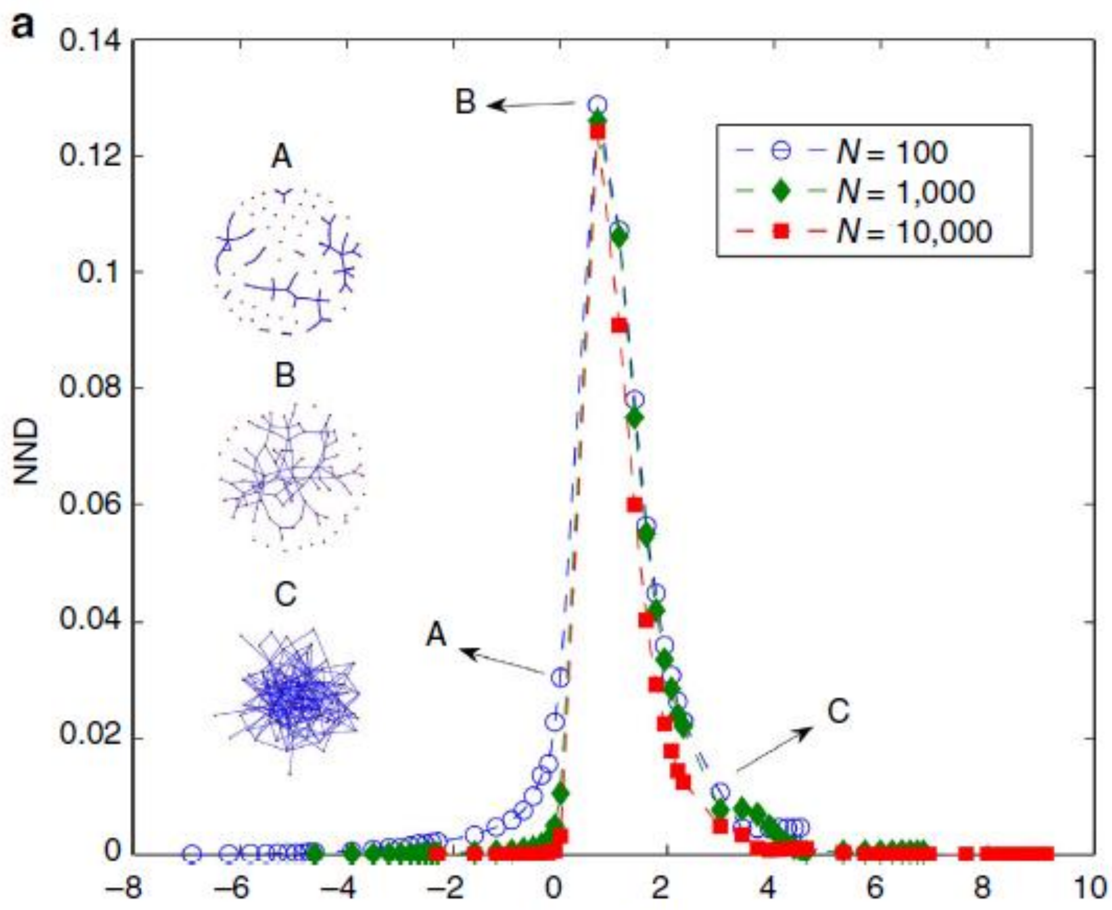
$$\mathrm{NND}(G) = \frac{\mathcal{J}(\mathbf{P}_1, \ldots, \mathbf{P}_\mathrm{N})}{\log(d+1)}$$

d = diameter

$$\mathcal{J}(\mathbf{P}_1, \ldots, \mathbf{P}_\mathrm{N}) = \frac{1}{N} \sum_{\mathrm{i,j}} p_\mathrm{i}(j) \log\left(\frac{p_\mathrm{i}(j)}{\mu_\mathrm{j}}\right)$$

$$\mu_\mathrm{j} = \left(\sum_{\mathrm{i}=1}^{N} p_\mathrm{i}(j)\right) / N$$

Example of application: in a random network the node-distance-distribution detects the percolation transition



Log(PN) P=connection probability

[T. A. Schieber et al, Nat. Comm. (2017)]

Dissimilarity between two networks

$$D(G,G') = w_1\sqrt{\frac{\mathcal{J}(\mu_G,\mu_{G'})}{\log 2}} + w_2\left|\sqrt{\mathrm{NND}(G)} - \sqrt{\mathrm{NND}(G')}\right|$$

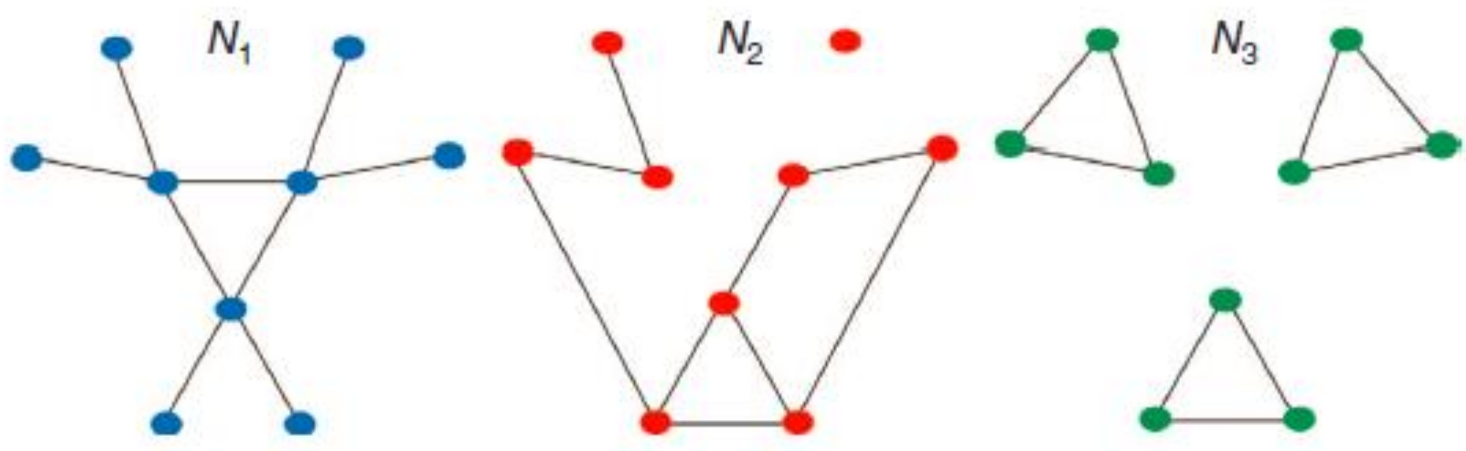
$w_1 = w_2 = 0.5$

compares the averaged connectivity

compares the heterogeneity of the connectivity distances

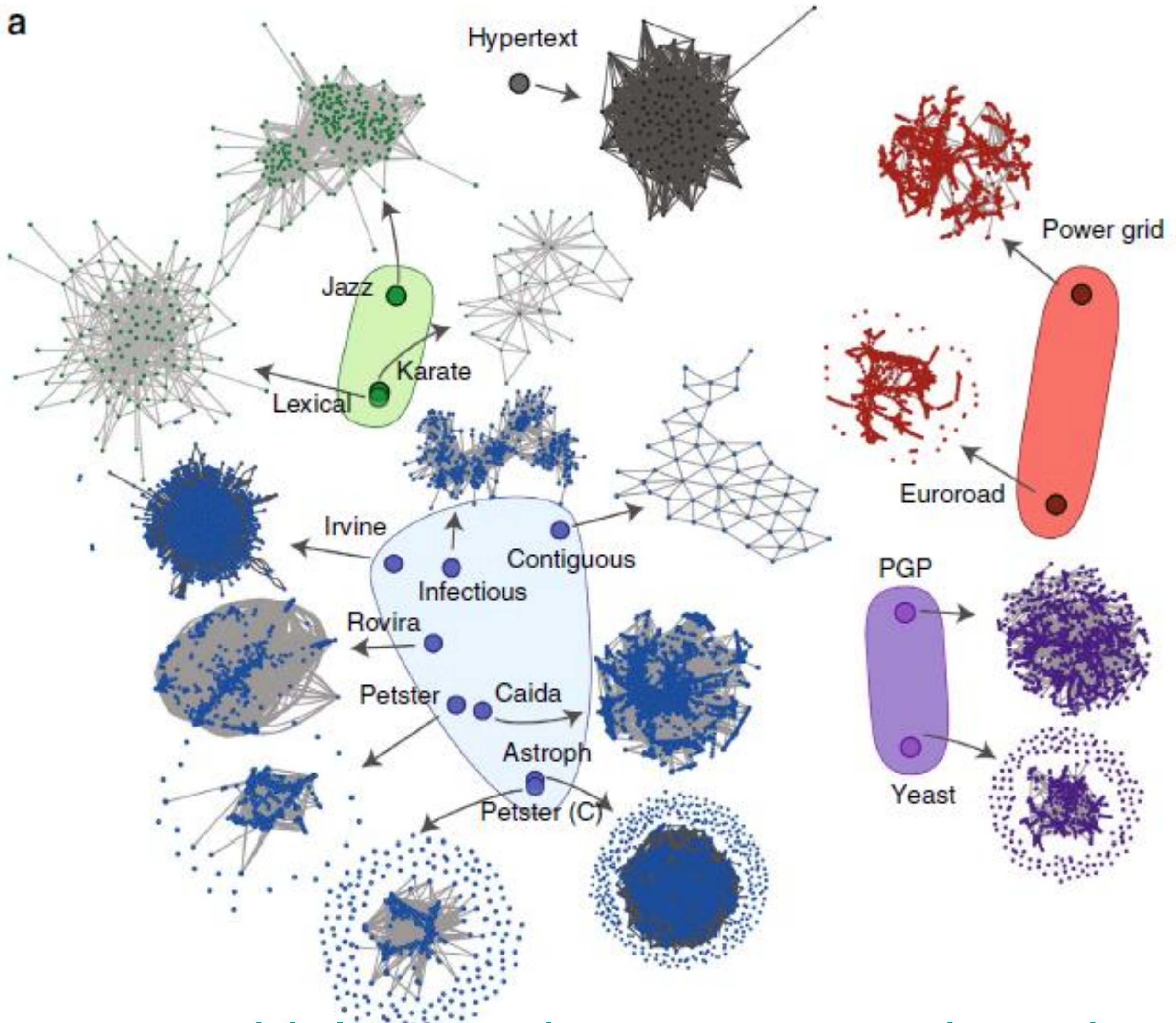
- Extensive numerical experiments demonstrate that isomorphic graphs return ***D=0.***
- Computationally efficient.

Meaningful comparison of networks with the same number of nodes and links



	D	Hamming	Graph Edit Distance
N_1,N_2	0.25	12	6
N_1,N_3	0.56	12	6
N_2,N_3	0.47	12	6

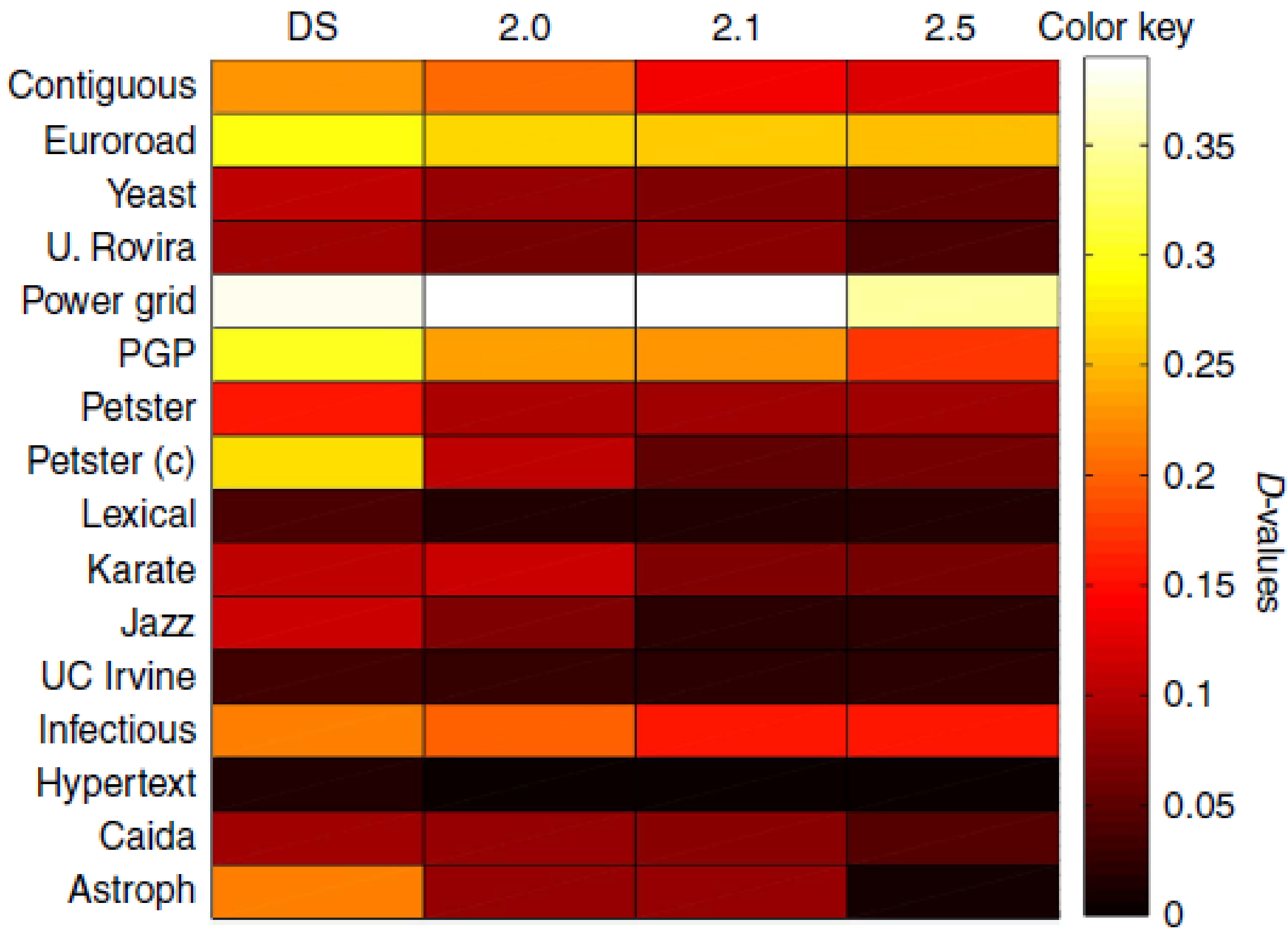
Distances between real networks (Koblenz Network Collection)



[T. A. Schieber et al, Nat. Comm. (2017)]

Comparing real networks to null models

DS preserves the degree sequence; **2.0** also preserves the degree correlation; **2.1** also the clustering coefficient; **2.5** also the clustering spectrum



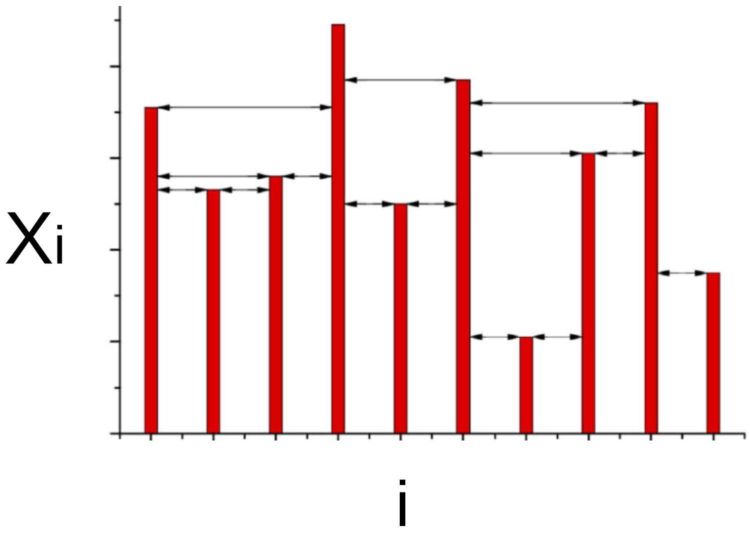
Comparing brain networks

- EEG data
  - https://archive.ics.uci.edu/ml/datasets/eeg+database
  - 64 electrodes placed on the subject’s scalp sampled at 256 Hz during 1s
  - 107 subjects: 39 control and 68 alcoholic

- Use Horizontal Visibility Graph to transform **each** EEG Time Series into a network.

- The HVG method is applied to the **raw** data (no pre-filtering to extract a particular frequency band).

The horizontal visibility graph (HVG) method: transforms a time series into a unweighted and undirected graph

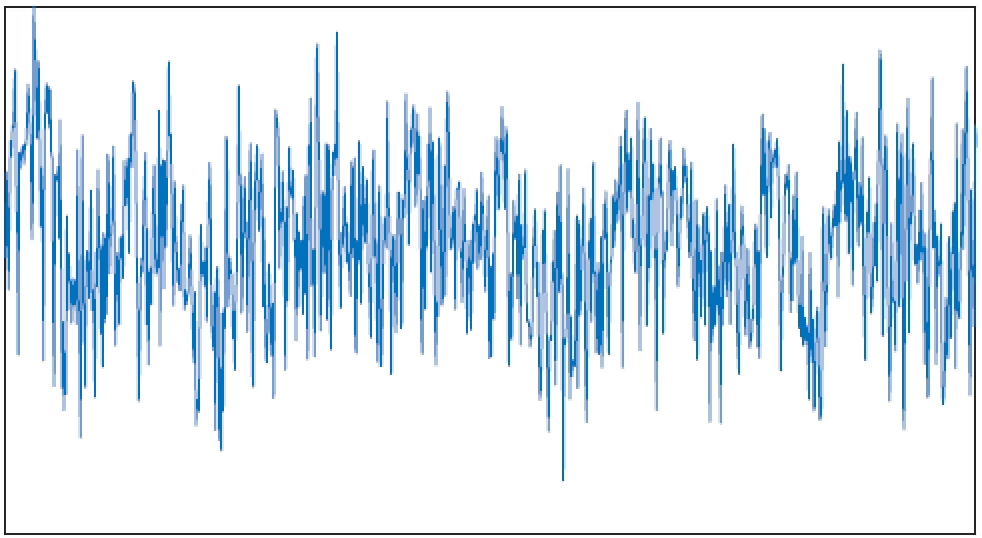


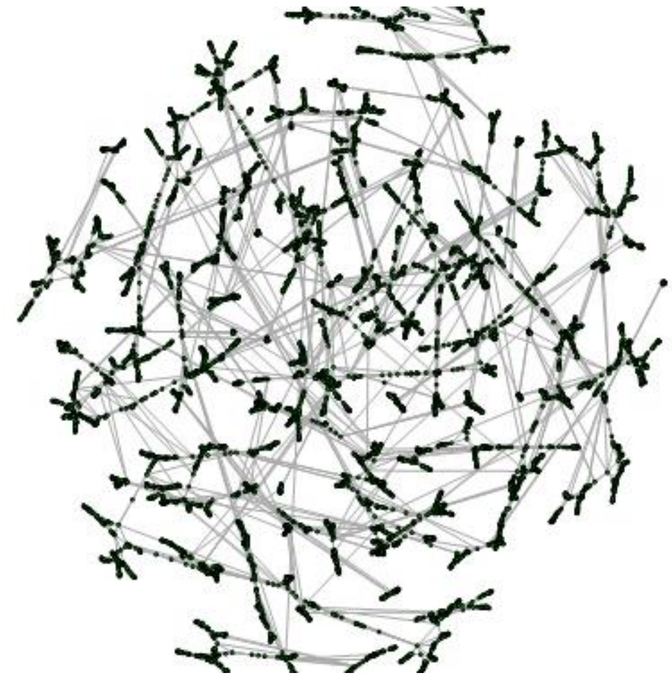
- Each data point is a node.
- <u>Rule</u>: data points *i* and *j* are connected if there is "visibility" between them (Xi and Xj > Xn for all n i<n<j)

Luque et al PRE (2009); Gomez Ravetti et al, PLOS one (2014)

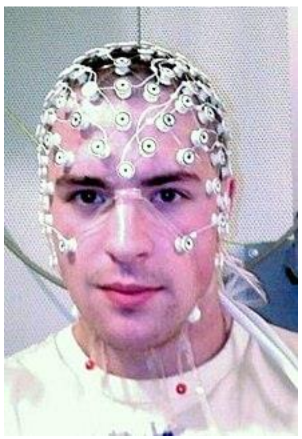
Horizontal Visibility Graph

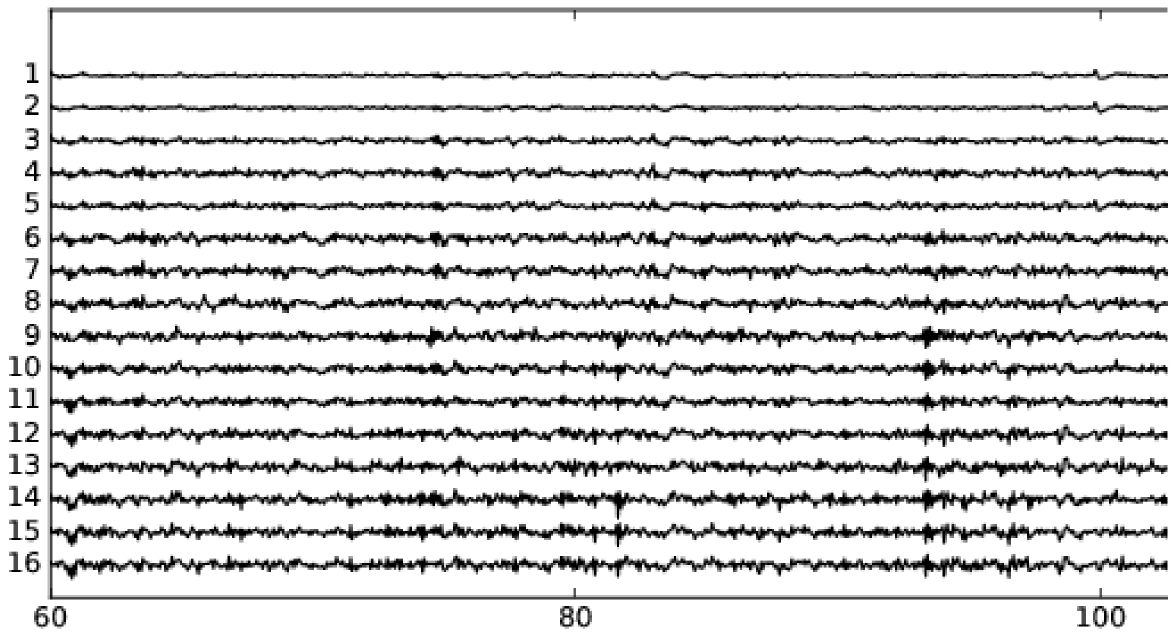
A time series

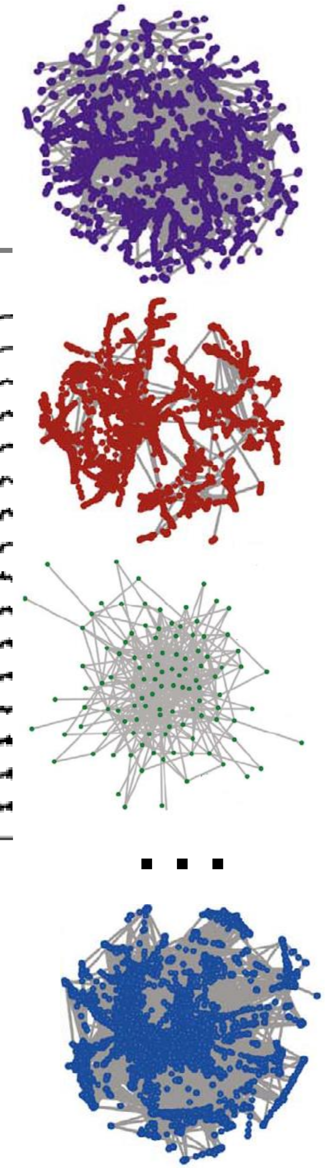




For each subject, the time series recorded at each electrode is transformed into a graph



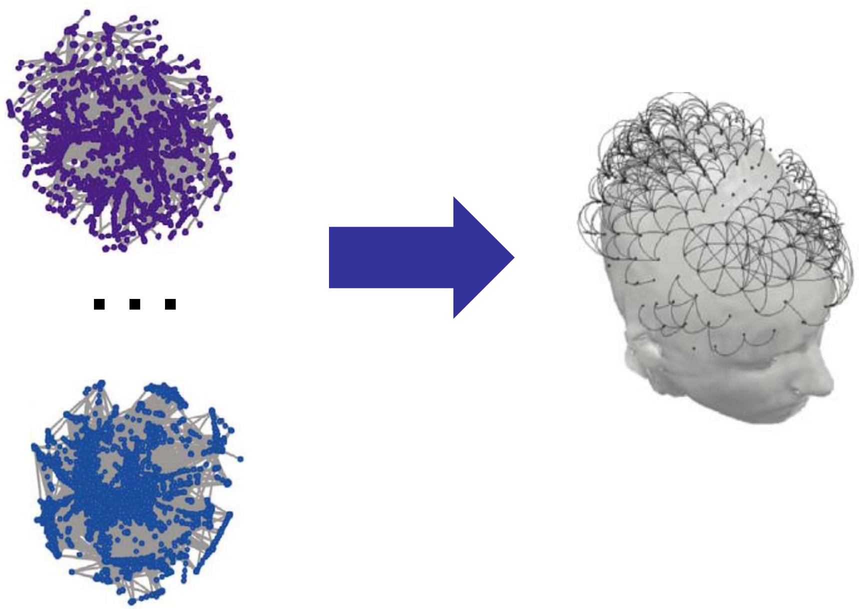




Dataset has 64 channels $\Rightarrow$ 64 networks

The brain network of each subject

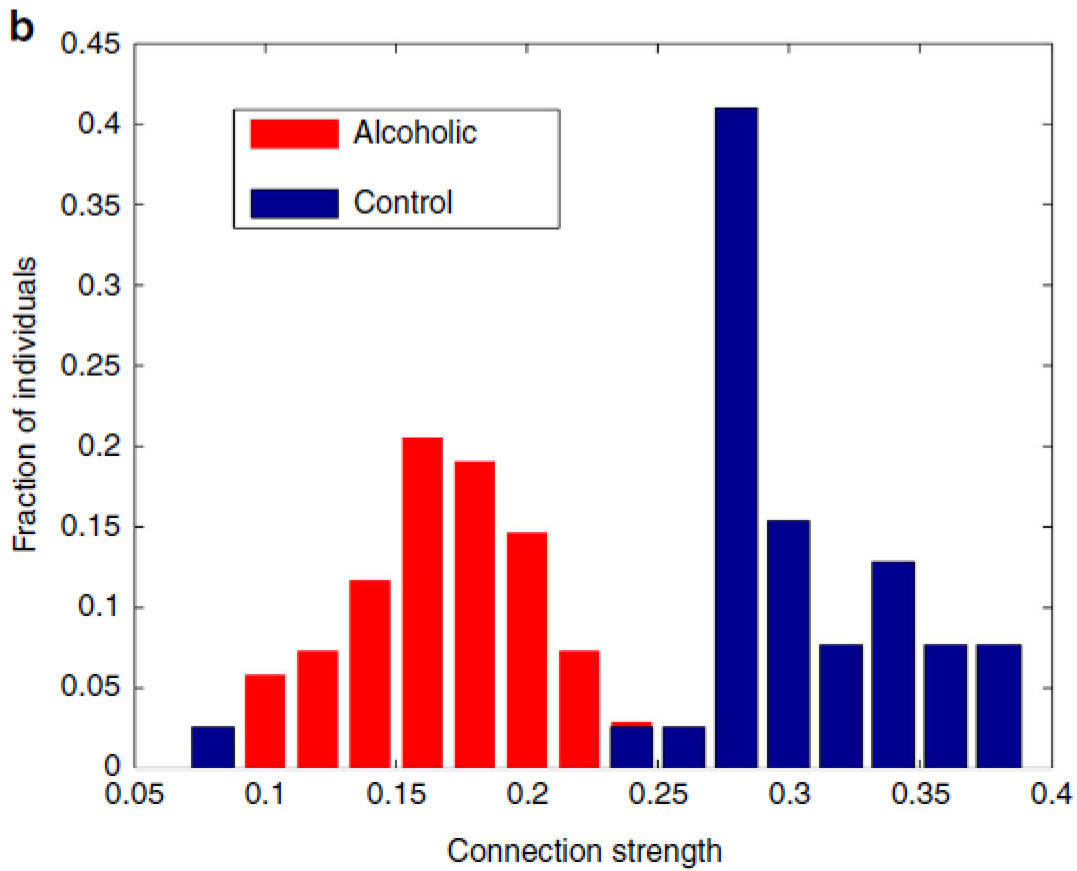
- The weight of the link between two graphs (G, G') representing two brain regions is defined as: **1-*D*(G,G')**



- The resulting network (with 64 nodes=electrodes, all-to-all coupled with weighted links) represents the similarity between the EEG signals in different brain regions of one subject.

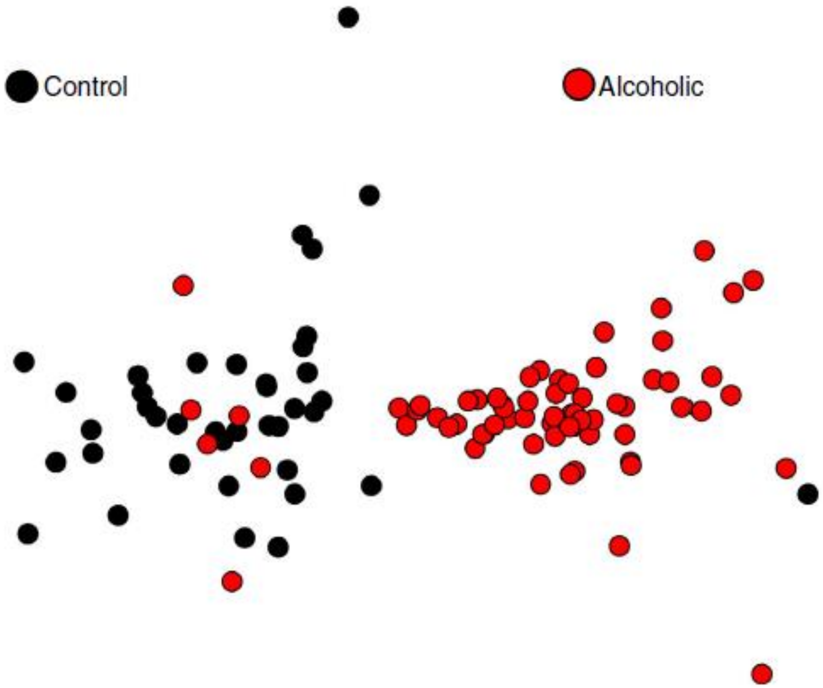
  $\Rightarrow$ We can then compare different subjects.

We identify two brain regions (called ‘nd’ and ‘y’), where the connection strength between these regions is higher in control than in alcoholic subjects.

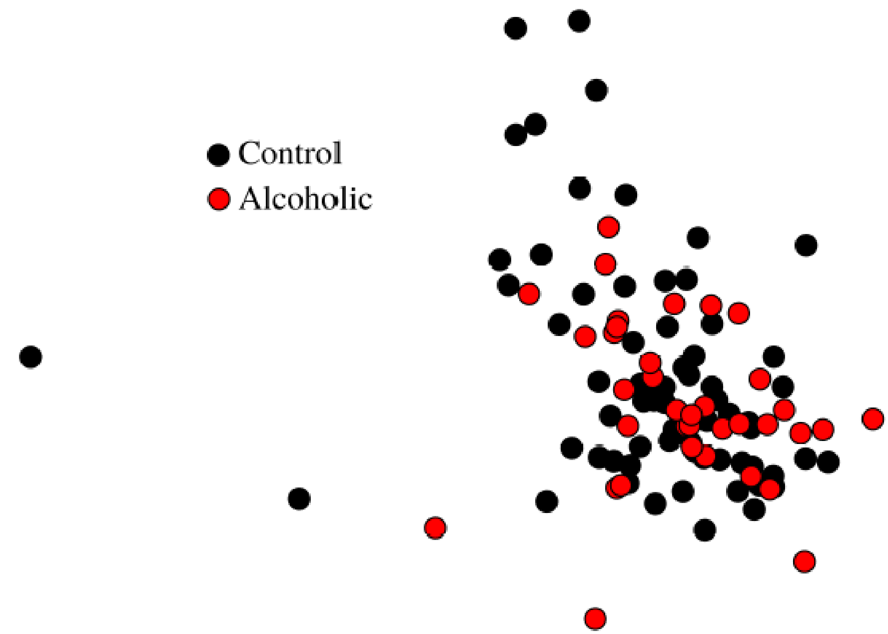


Using the Hamming distance we can not distinguish.

Dissimilarity measure



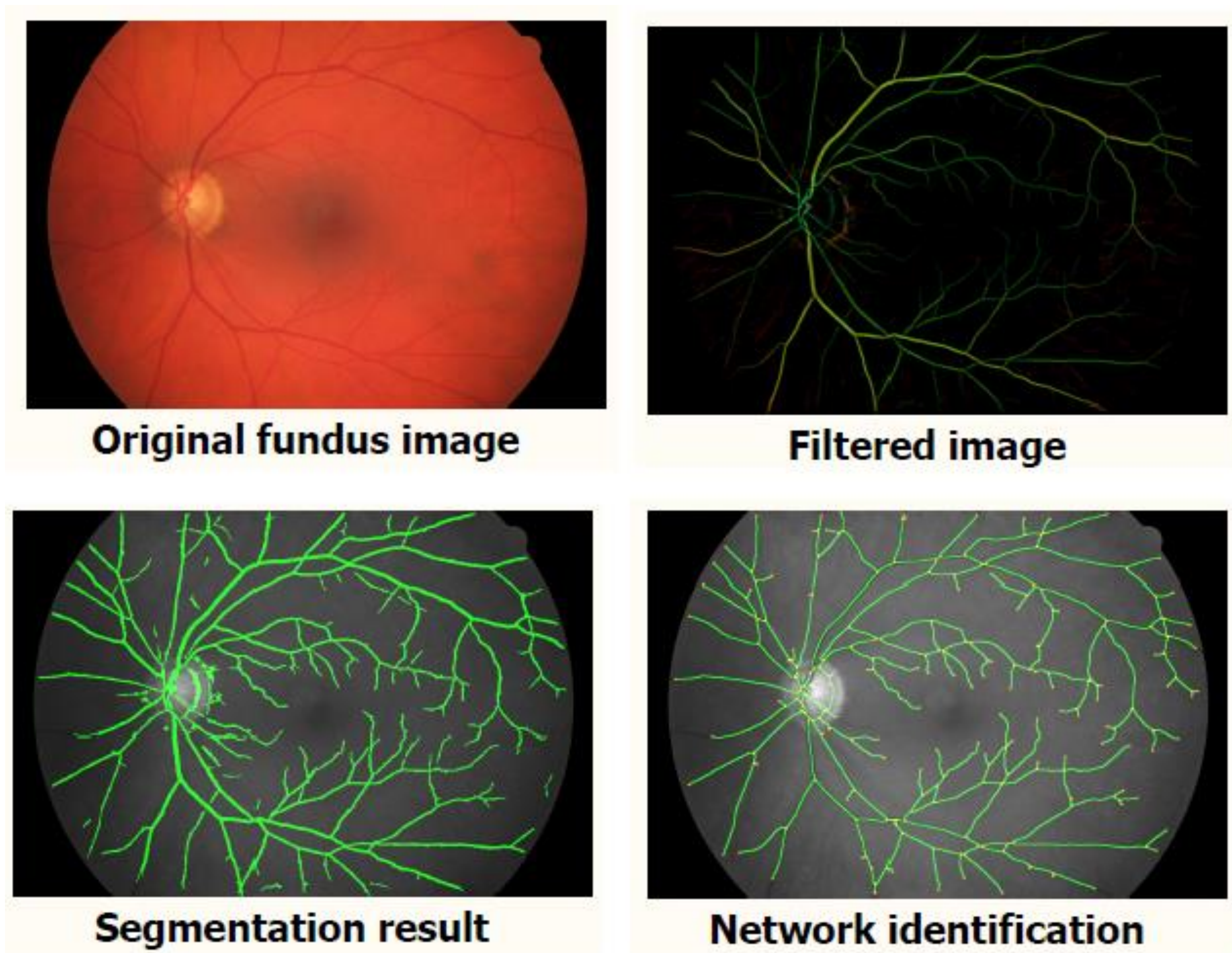
Hamming distance



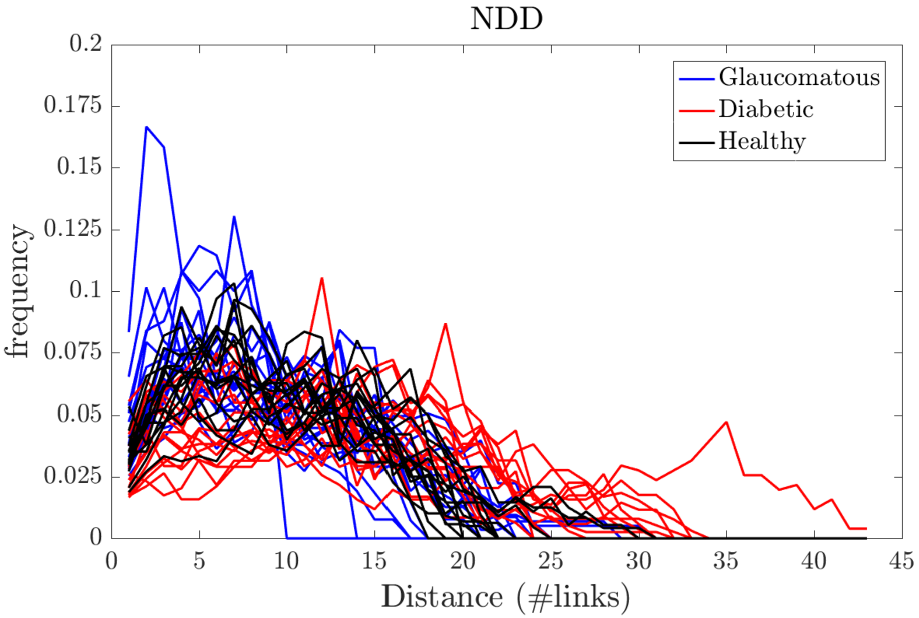
T. A. Schieber et al, Nat. Comm. 8, 13928 (2017)

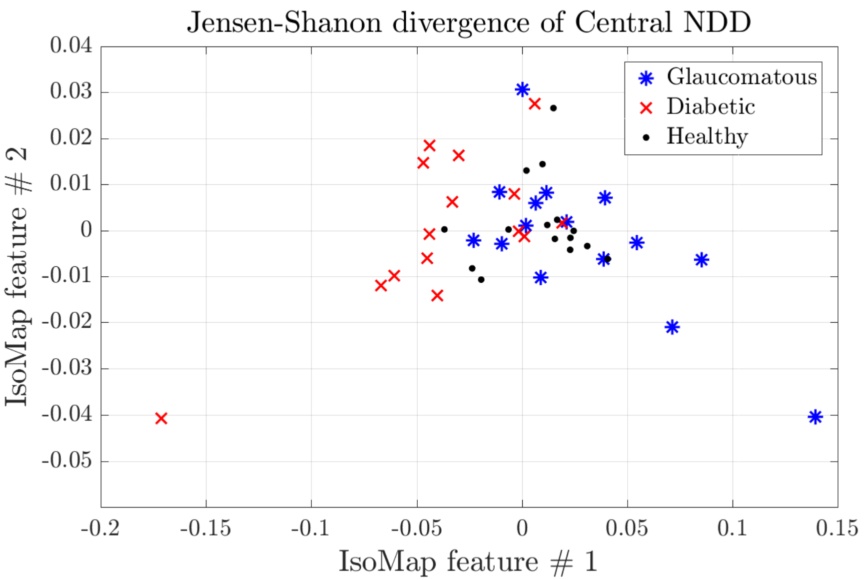
Retina image classification using node distance distribution

Pablo Amil, ITN BE-OPTICAL, ongoing work in collaboration with Irene Sendiña, IRJC

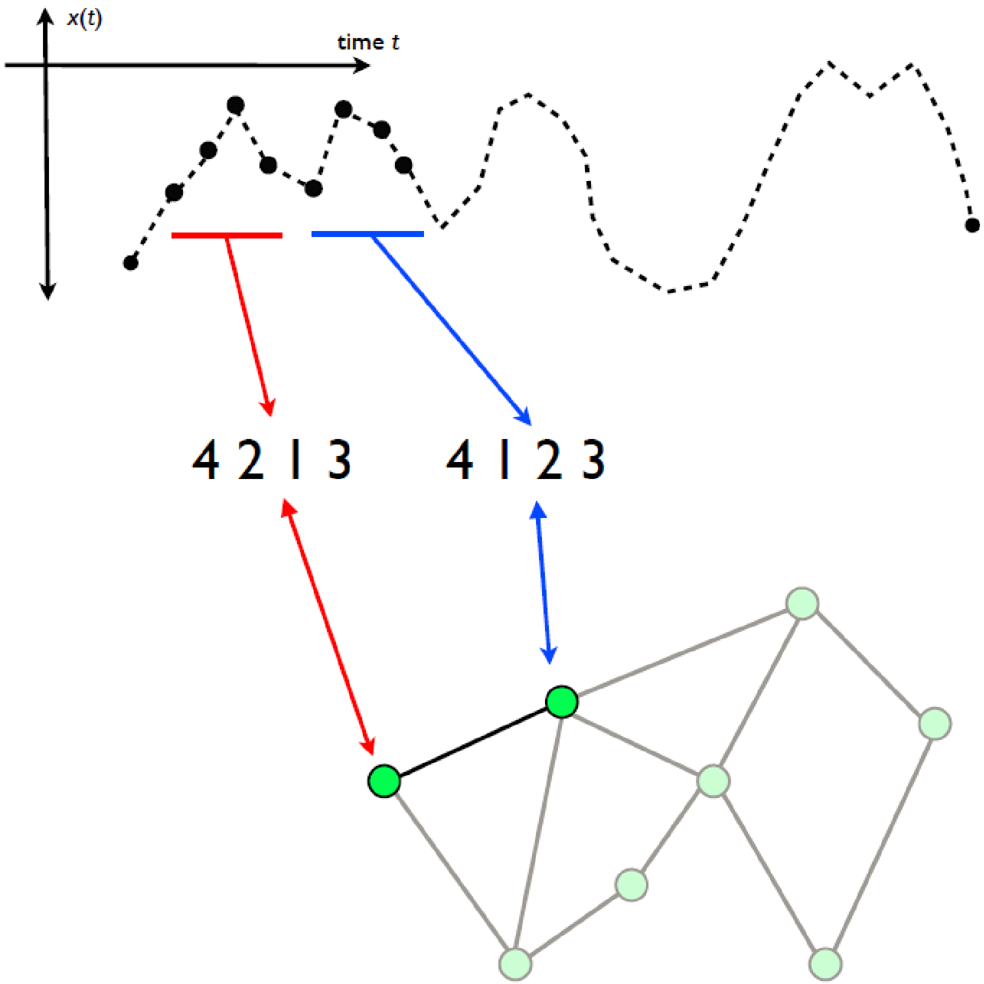


The distance distribution of the central node: a promising classification tool

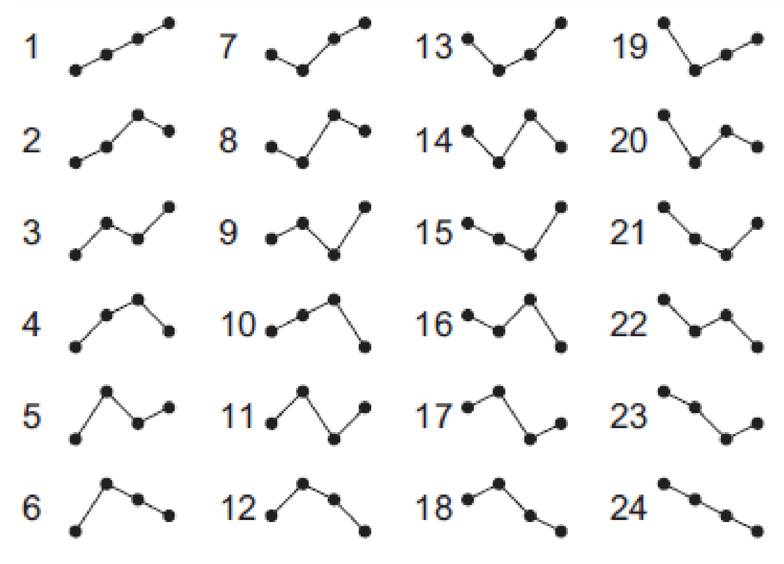




Another method to transform a time series into a graph: symbolic analysis



Ordinal patterns of 4 “letters”

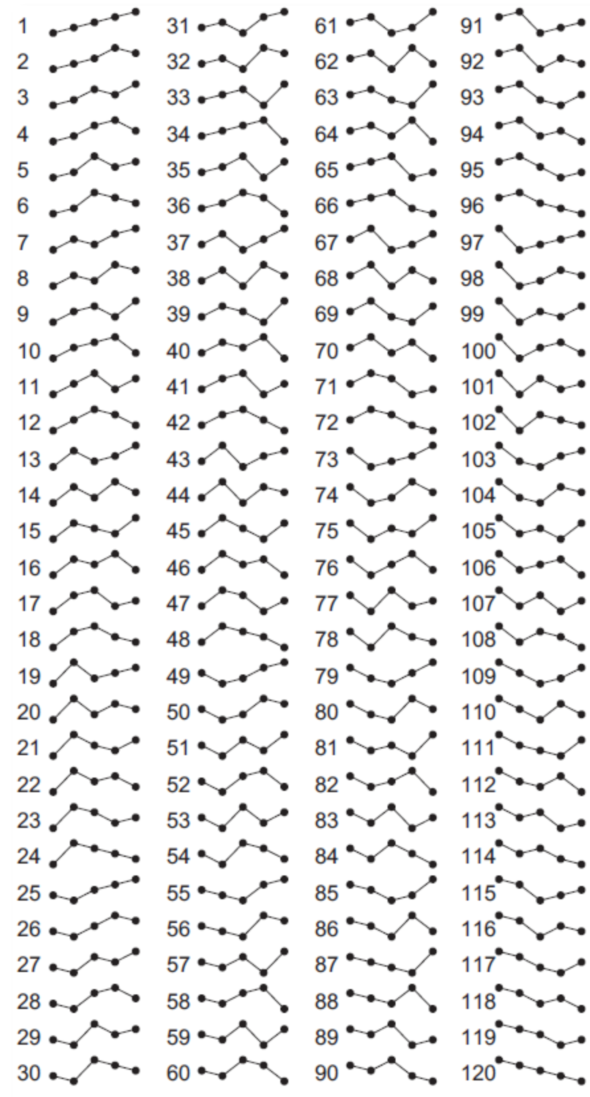


U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Adapted from M. Small
(The University of Western Australia)

This method gives a weighted and directed graph

- D! nodes
- Weigh of node i: the probability of pattern i ($\sum_i p_i=1$)
- Weight of the link i→j: probability of the transition i→j (for each *i*: $\sum_j w_{ij}=1$)



U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Measures used to characterize the graph

- Entropy computed from node weights (**permutation entropy**)

$$s_p = -\sum p_i \log p_i$$

- Average node entropy (entropy of the link weights)

$$s_n = \frac{1}{M}\sum_{i=1}^{M} s_i \qquad s_i = -\sum_{j=1}^{M} w_{ij} \log w_{ij}$$

- Asymmetry coefficient: normalized difference of transition probabilities, P('01'→ '10') - P('10'→ '01'), etc.

$$a_c = \frac{\sum_i \sum_{j \neq i} \left| w_{ij} - w_{ji} \right|}{\sum_i \sum_{j \neq i} \left(w_{ij} + w_{ji} \right)}$$

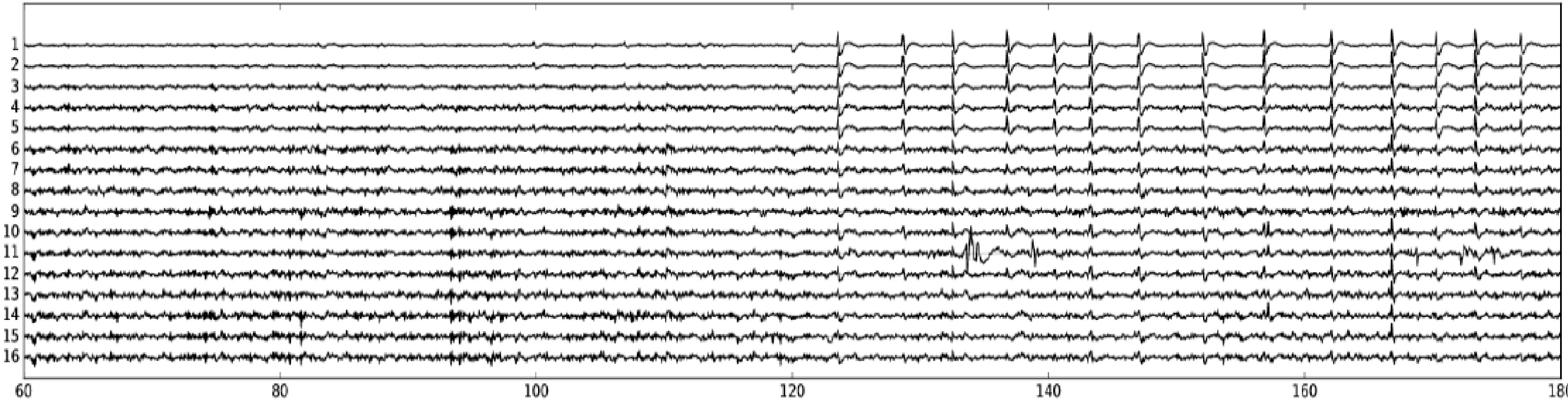
(0 in a fully symmetric network;
1 in a fully directed network)

Application: distinguishing eyes closed (EC) and eyes open (EO) brain states

Carlos Quintero ITN NETT, ongoing work in collaboration with A. Pons, M. C. Torrent, J. Garcia-Ojalvo and BitBrain.

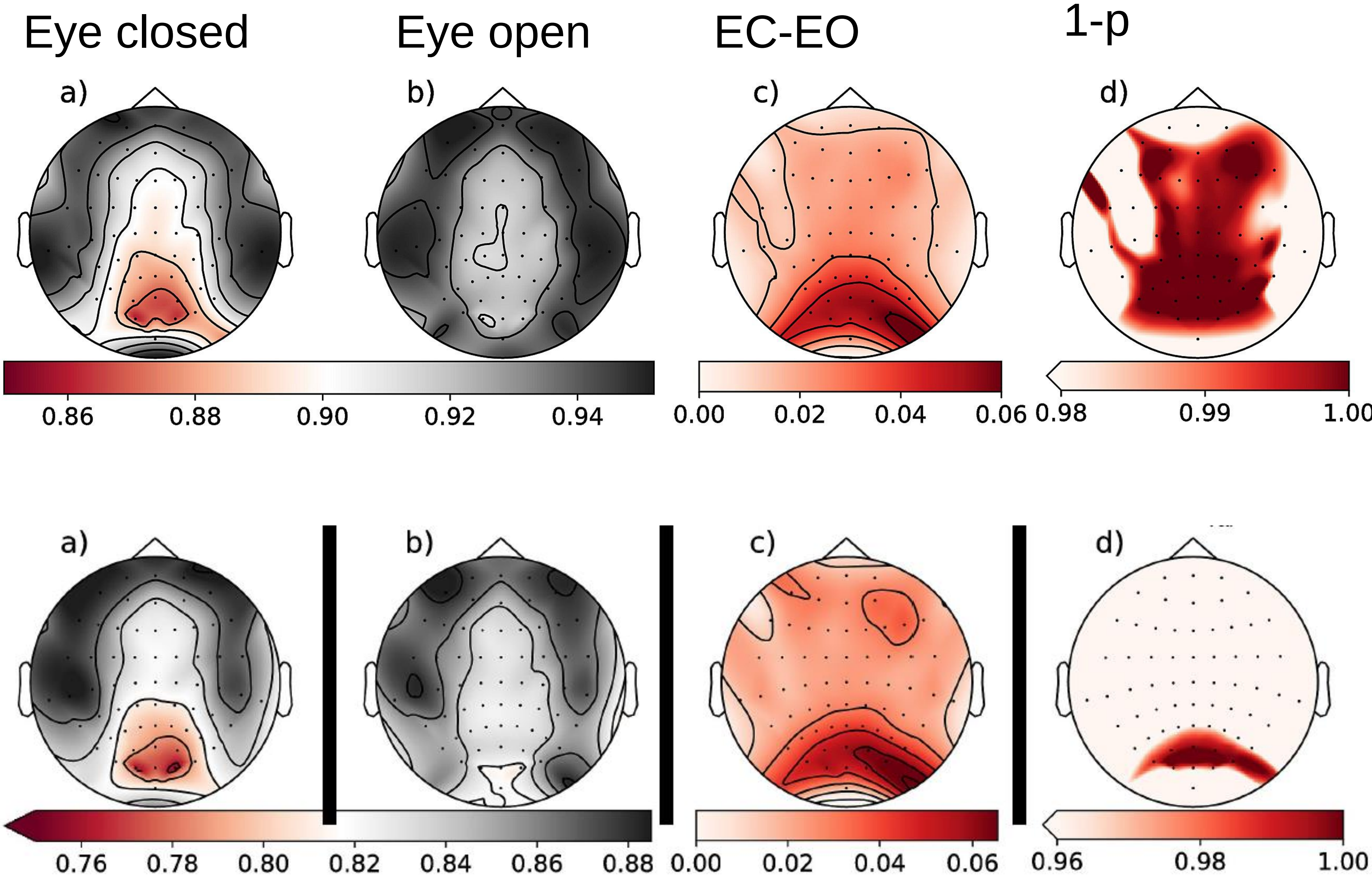
	BitBrain	PhysioNet
	DTS1	DTS2
Sampling rate(Hz)	256	160
Time task(seg)	120	60
Total points	30720	9600
Number of electrodes	16	64
Number of subjects	70	109

Eye closed   Eye open

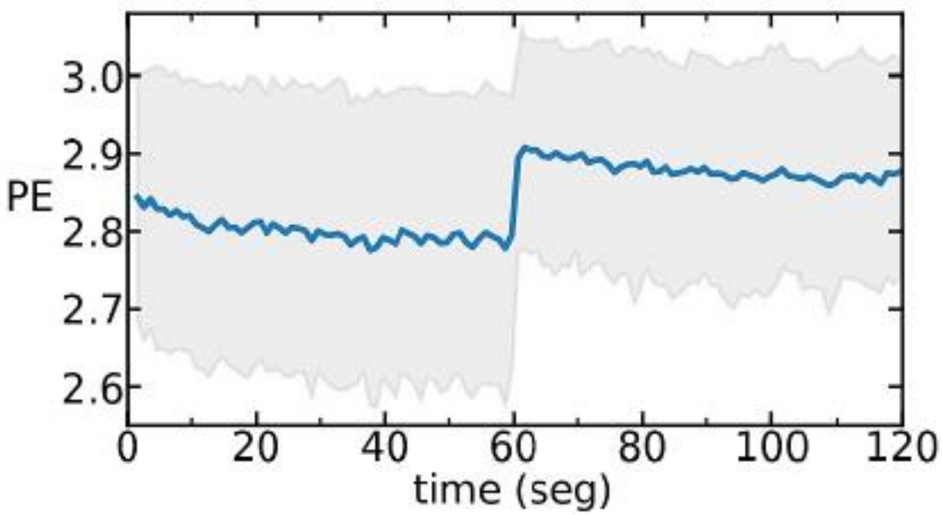


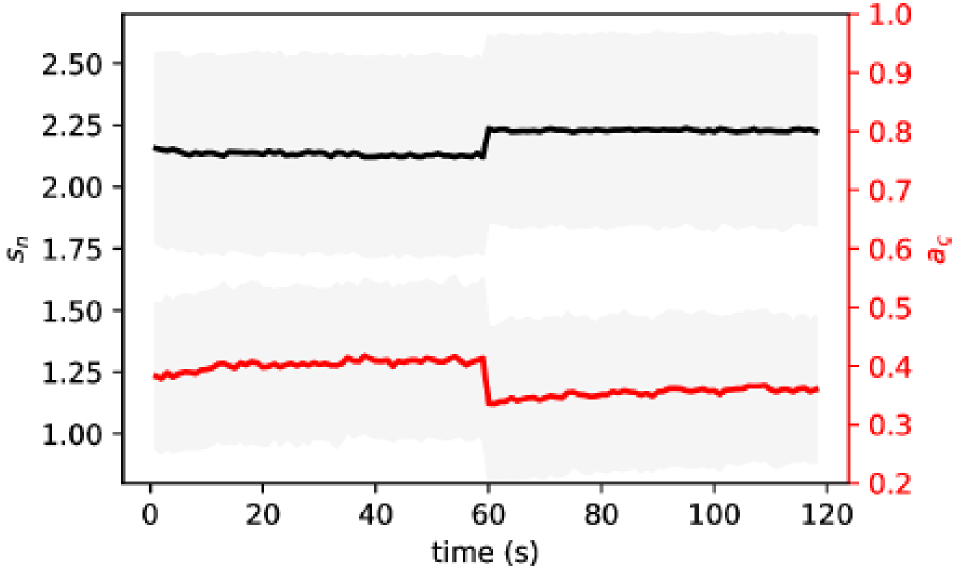
- Symbolic analysis is applied to the raw data; similar results were found with filtered data using independent component analysis.

Permutation entropy and node entropy (PhysioNet)



“Randomization”: the entropies increase and the asymmetry coefficient decreases





Time window = 1 s
(160 data points)

Concluding

Summary

- New measure to quantify the heterogeneity of the connectivity paths of a single network.
  - detects the percolation transition in a random network.
- New measure to calculate distance between graphs
  - can be applied to graphs of different sizes.
  - returns $D=0$ only if they are isomorphic.
- Used to differentiate brain networks (alcoholic/non alcoholic) constructed using the horizontal visibility graph (raw EEG).
- Many possible applications for characterizing time-evolving networks, classification of biomedical data, etc.
- Symbolic analysis also applied to raw EEG data seems promising for differentiating brain states.

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http://www.fisica.edu.uy/~cris/

G. Tirabassi et al., Ecological Complexity 19, 148 (2014)

T. A. Schieber et al, Nat. Comm. 8, 13928 (2017)