New indicators for early detection of critical transitions

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Extreme Events: Identification, Analysis and Prediction
808. WE-Heraeus-Seminar, April 25, 2024

Campus d'Excel·lència Internacional

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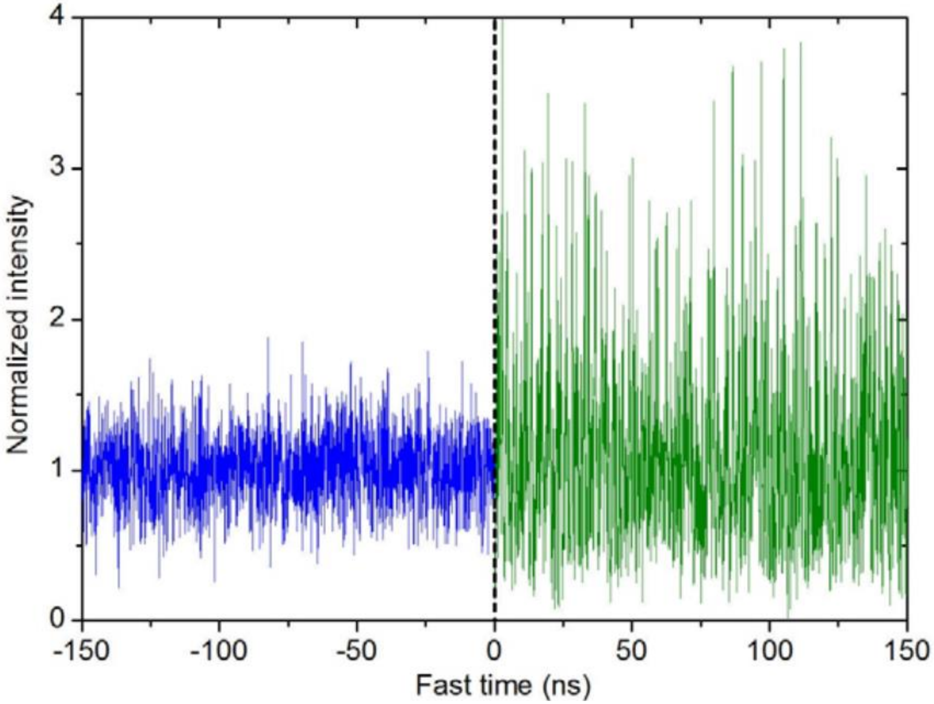
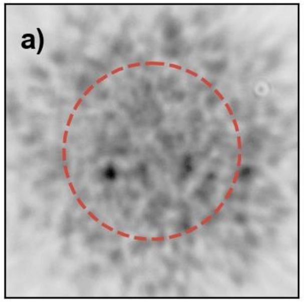
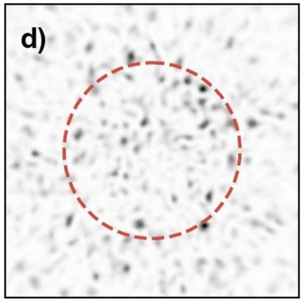
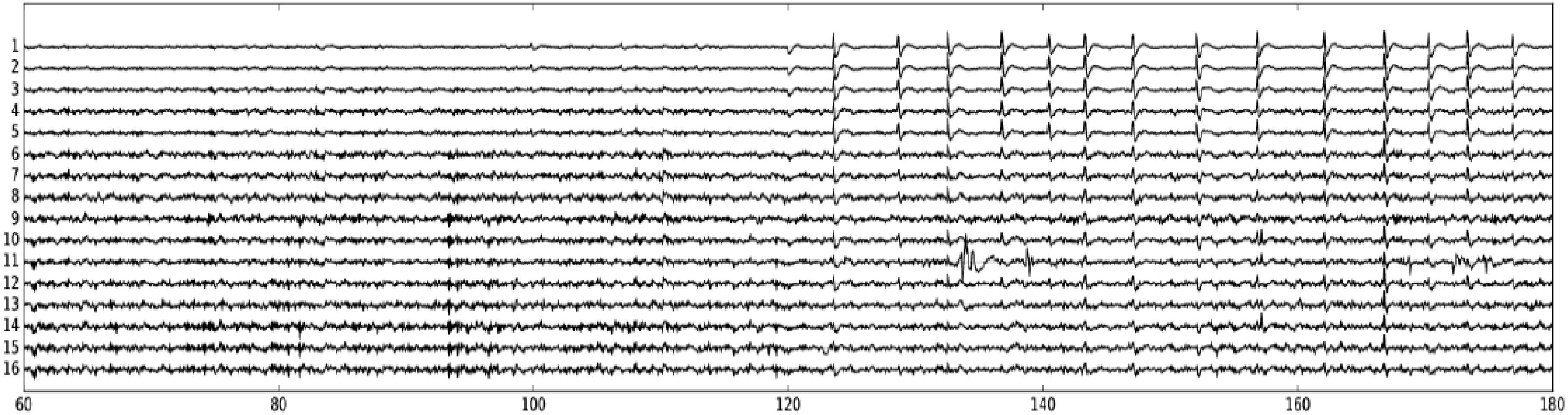
Regime transitions in complex systems









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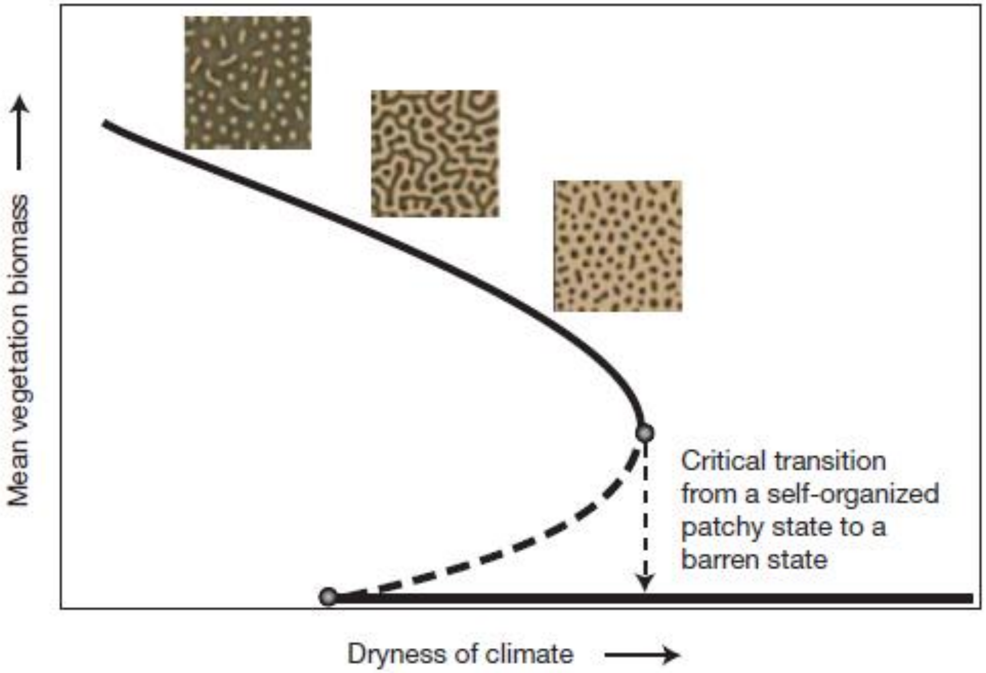
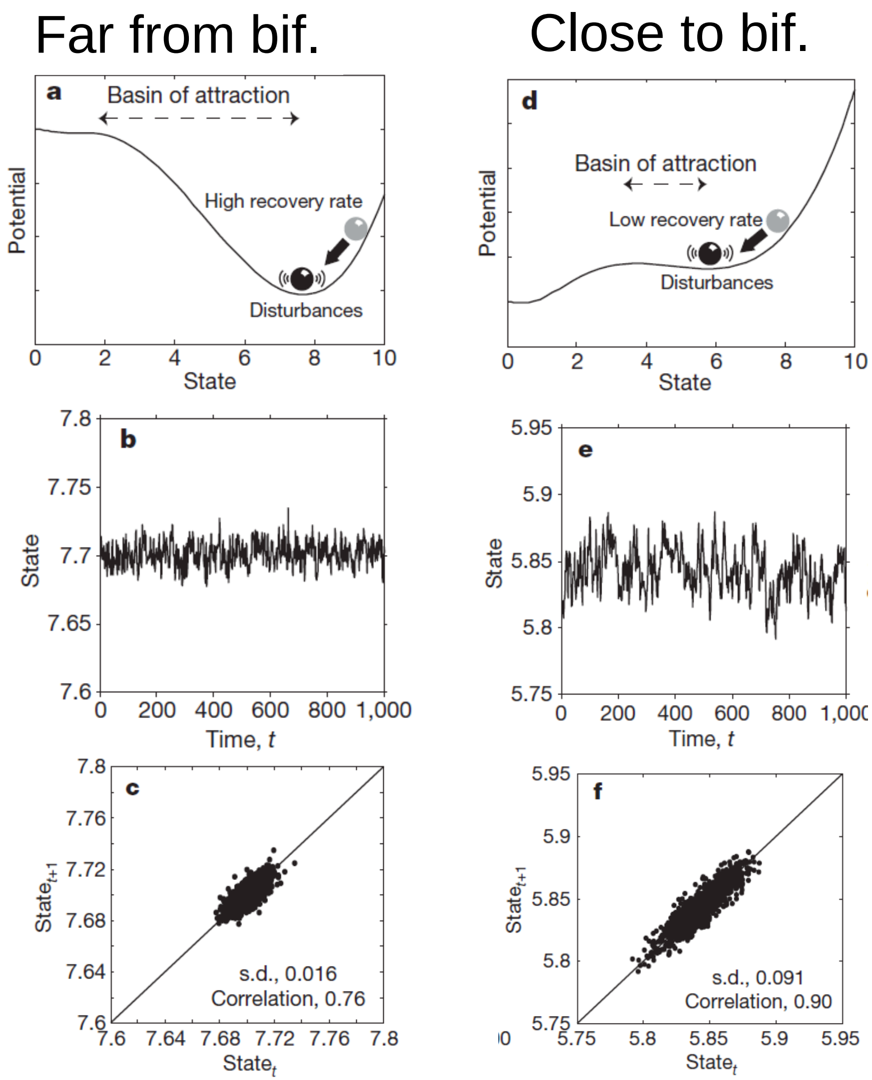
Classical indicators of critical transitions



M. Scheffer et al., Nature 461, 53 (2009)



$\Rightarrow$ increase of variance and autocorrelation (*critical slowing down*)

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Critical slowing down can occur after the bifurcation

PHYSICAL REVIEW LETTERS **125,** 134102 (2020)

Testing Critical Slowing Down as a Bifurcation Indicator in a Low-Dissipation Dynamical System

M. Marconi,[1] C. Métayer,[2] A. Acquaviva,[1] J. M. Boyer,[2] A. Gomel,[3] T. Quiniou,[2] C. Masoller,[4,*] M. Giudici,[1] and J. R. Tredicce[2,3]

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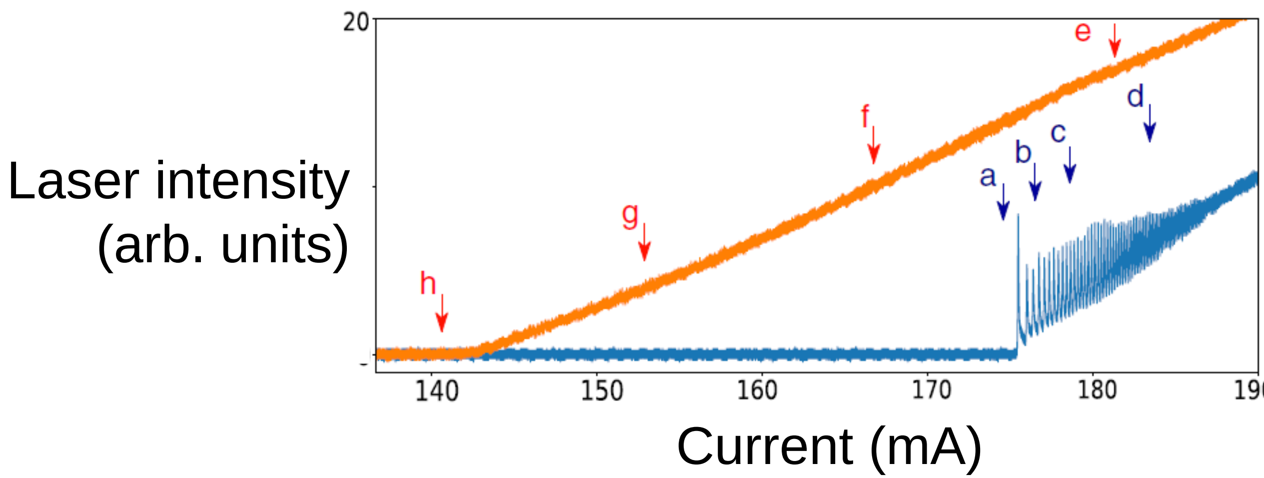
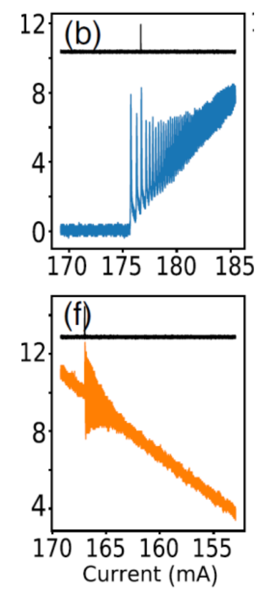
Increasing and Decreasing the laser current





New Preprint March 2024 arXiv:2403.17973

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Outline

- Analysis method: ordinal analysis

- Characterization of the transitions
  - Noise -> extreme events -> chaos in a diode laser (time series)
  - Laminar -> optical turbulence in a fiber laser (time series)
  - High biomass -> low biomass (vegetation 2D data –observational and simulated)
  - Low -> high coherence in a diode laser (speckle images)
  - Eyes closed -> eyes open (multichannel EEG data)

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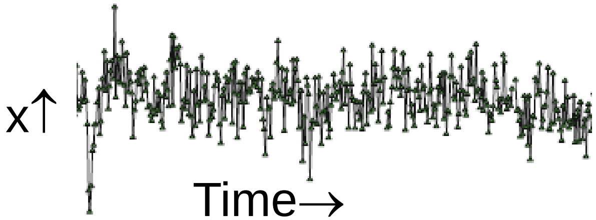
Ordinal time series analysis



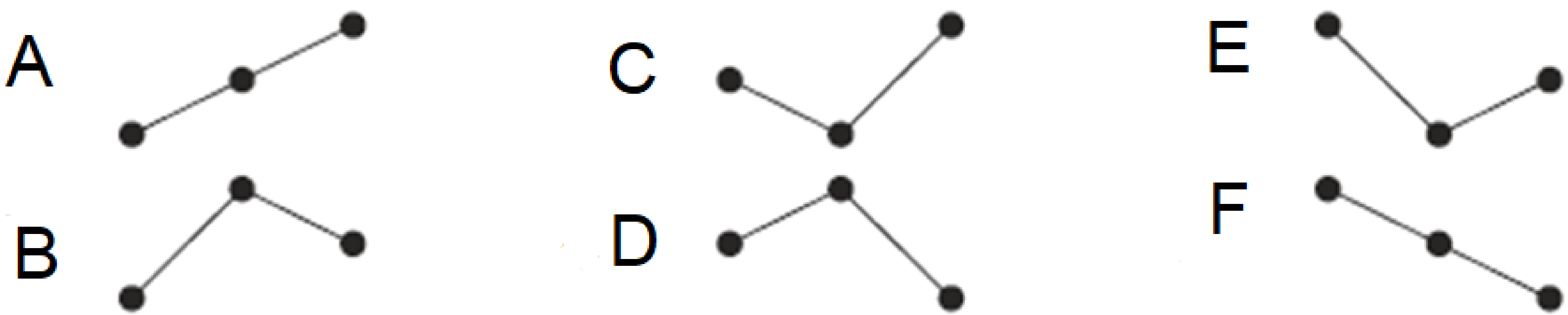
$\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$

Which are the possible order relations?

{…2, 5, 7…}

{…5, 2, 7…}

{…7, 2, 5 …}



{…2, 7, 5…}

{…5, 7, 2…}

{…7, 5, 2…}

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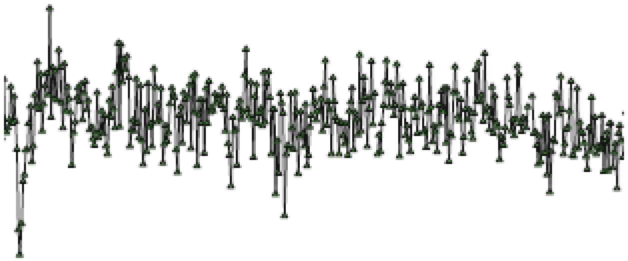
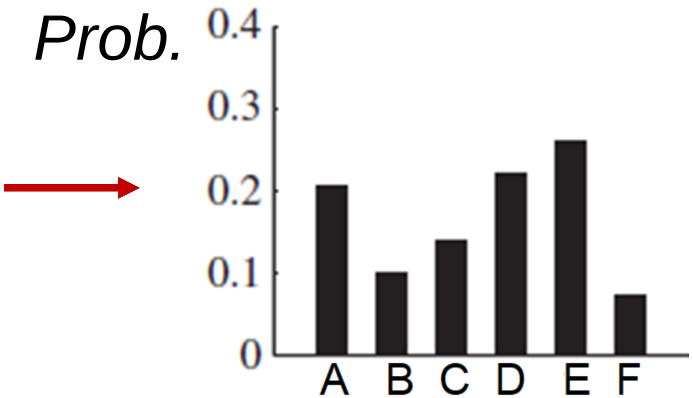
From a time series, by counting the number of different patterns, we calculate the "ordinal probabilities"





?

1. Analyze the probabilities
2. Permutation Entropy: $$H = -\sum_{i=1}^{N} p_i \ln p_i$$

$p_i = p_j$ for all $i, j \Rightarrow H$ max

$p_i = 1, p_j = 0$ for all $j \neq i \Rightarrow H=0$

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Ordinal analysis was first proposed to characterize complex time series.

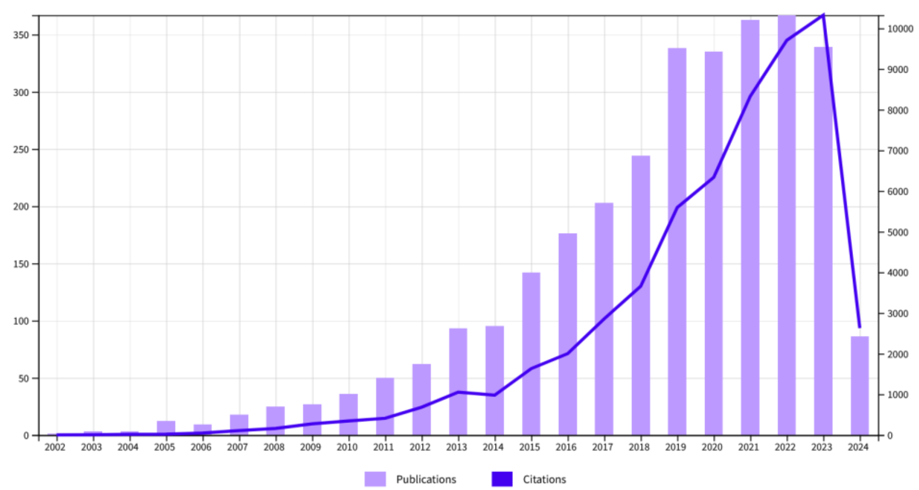
VOLUME 88, NUMBER 17 PHYSICAL REVIEW LETTERS 29 APRIL 2002

Permutation Entropy: A Natural Complexity Measure for Time Series

Christoph Bandt and Bernd Pompe

Institute of Mathematics and Institute of Physics, University of Greifswald, Greifswald, Germany

(Received 19 June 2001; revised manuscript received 20 December 2001; published 11 April 2002)



I. Leyva, J. M. Martinez, C. Masoller, O. A. Rosso, M. Zanin, "20 Years of Ordinal Patterns: Perspectives and Challenges", EPL 138, 31001 (2022).

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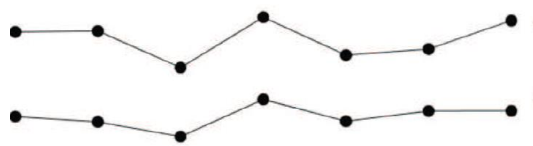
With two time series

x_i

x_j



$\longrightarrow$ $p_i(m)$, $p_j(n)$, $p_{ij}(m,n)$ $\longrightarrow$ M_{ij}

Mutual Information: $$M_{ij} = \sum_{m,n} p_{ij}(m,n) \log \frac{p_{ij}(m,n)}{p_i(m)p_j(n)}$$

x_i, x_j statistically independent: $p_{ij} = p_i p_j \Rightarrow MI=0$

To use patterns of length 3 (6 possible patterns, 36 combinations for p_{ij}) we need at least 400 data points in each time series.

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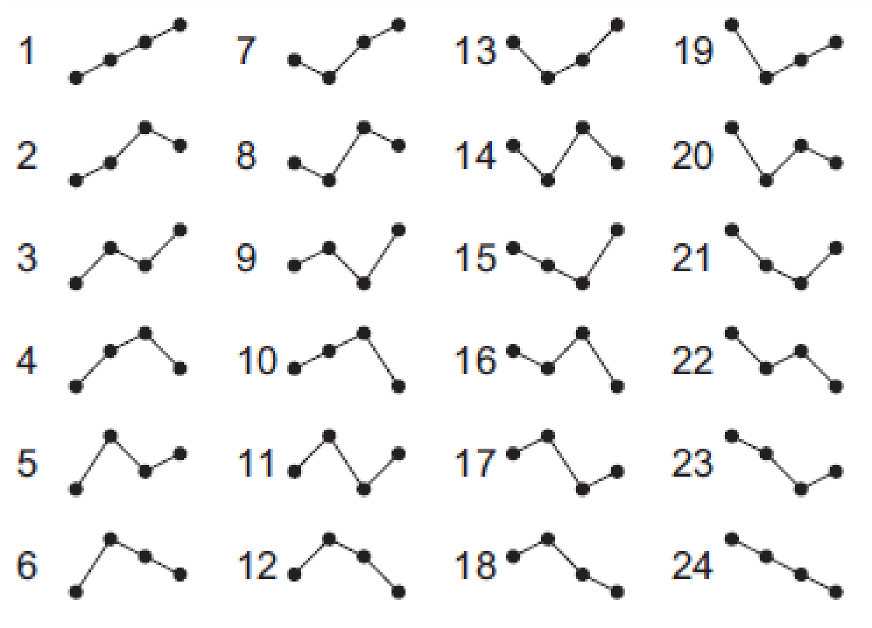
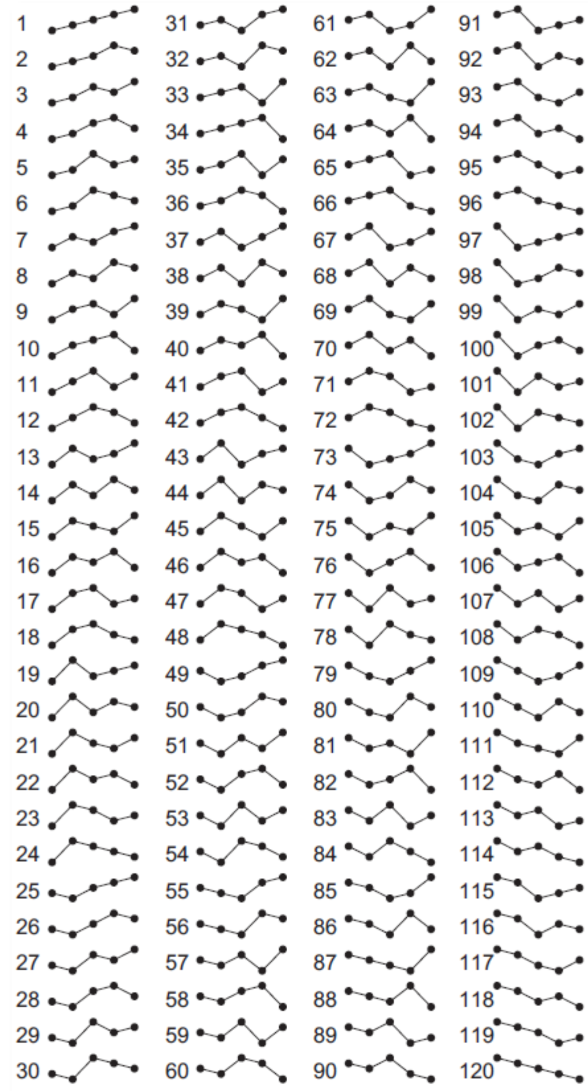
The number of possible patterns increases with the length of the pattern as D!





U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

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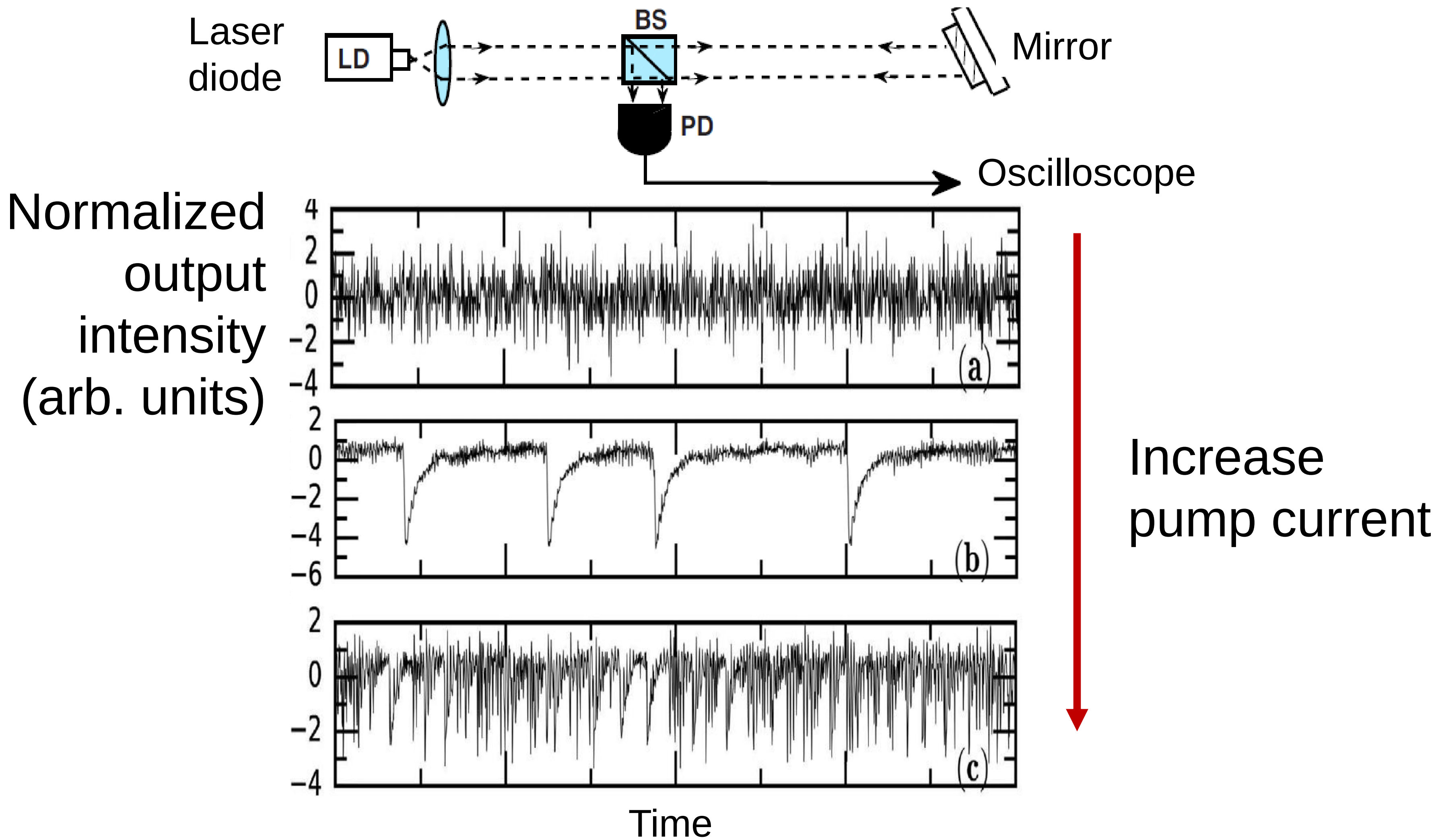
Analysis of regime transitions in a laser with feedback: optical noise → extreme events → chaos



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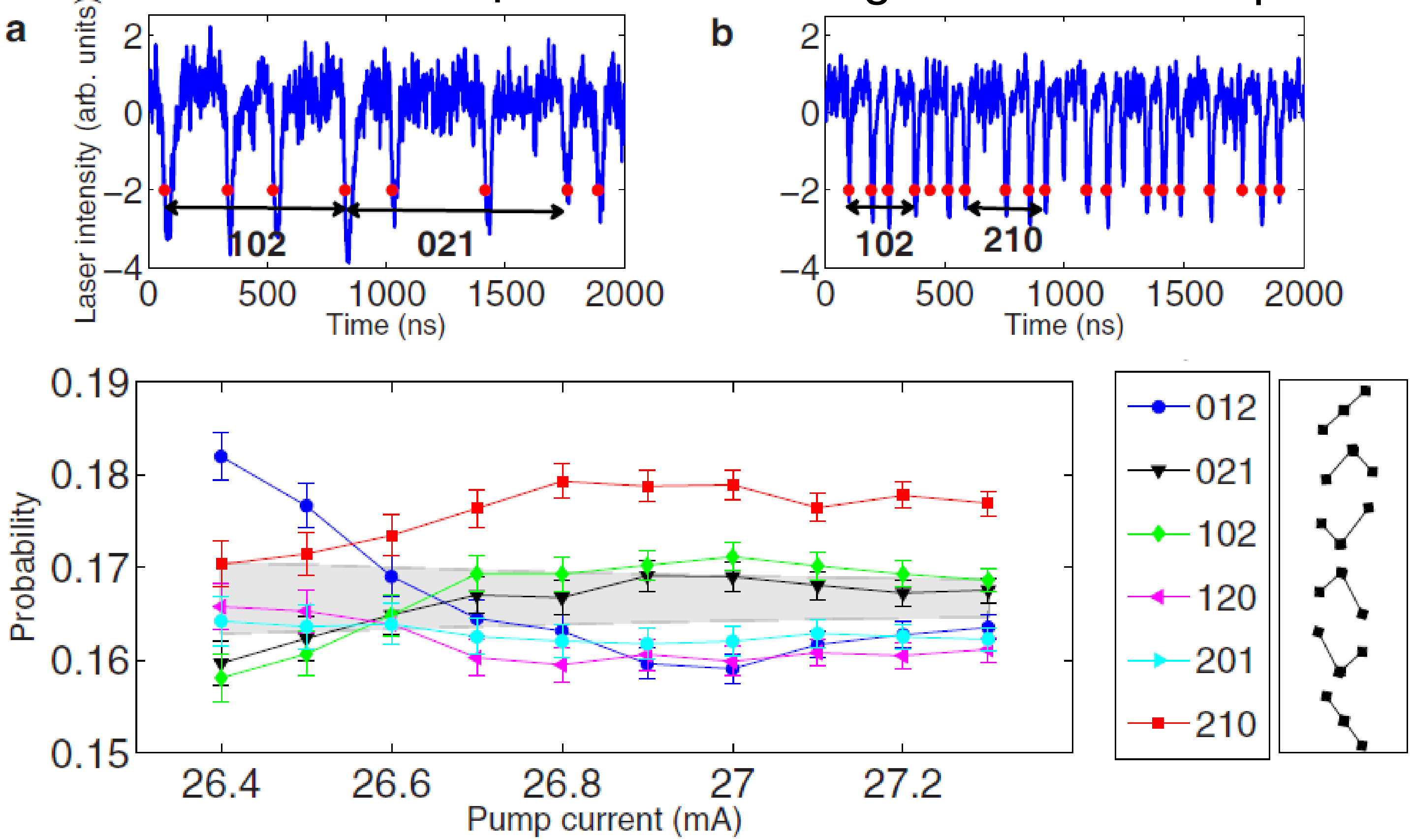
The ordinal probabilities uncover a change in spike timing

Low current: slow spike rate High current: fast spike rate



A. Aragones et el., "Unveiling the complex organization of recurrent patterns in spiking dynamical systems", Sci. Rep. 4, 4696 (2014).

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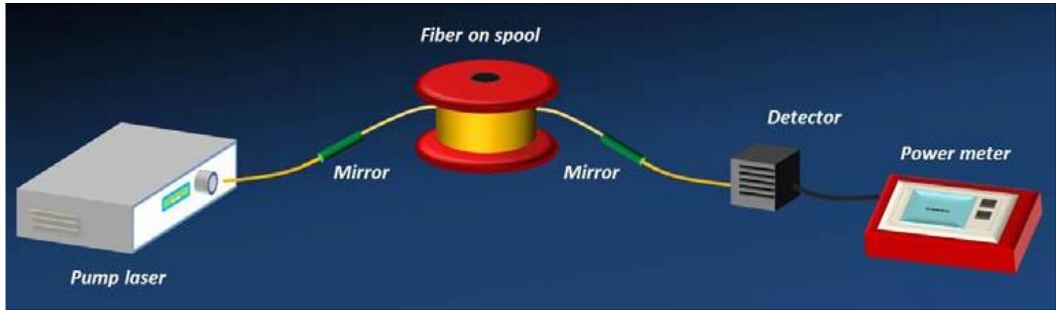
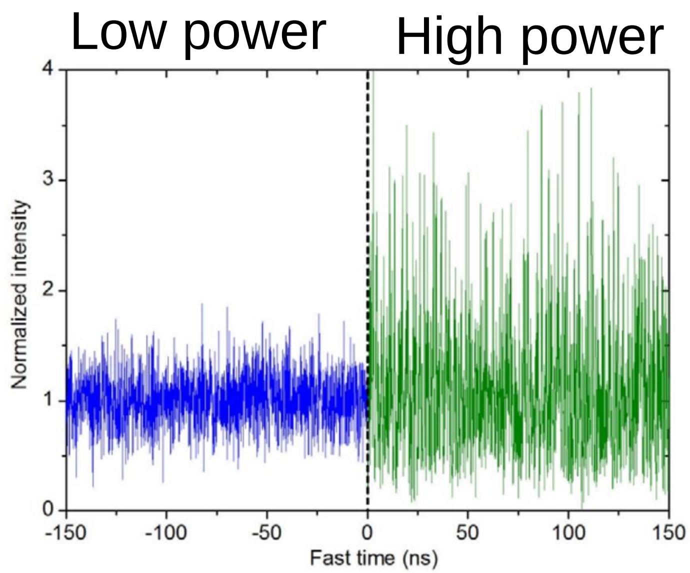
Transition laminar → optical turbulence in a fiber laser
(governing equations similar to hydrodynamics)

Control parameter: power of pump laser





E. G. Turitsyna et. al, Nat. Photonics 7, 783 (2013).

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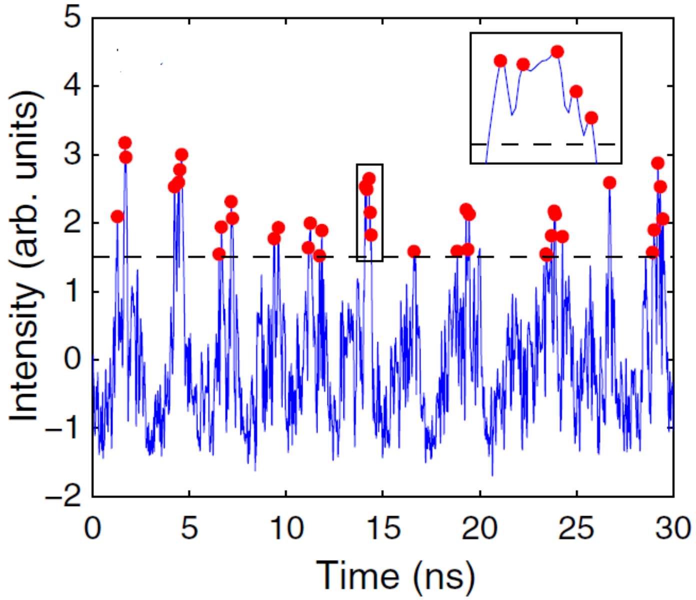
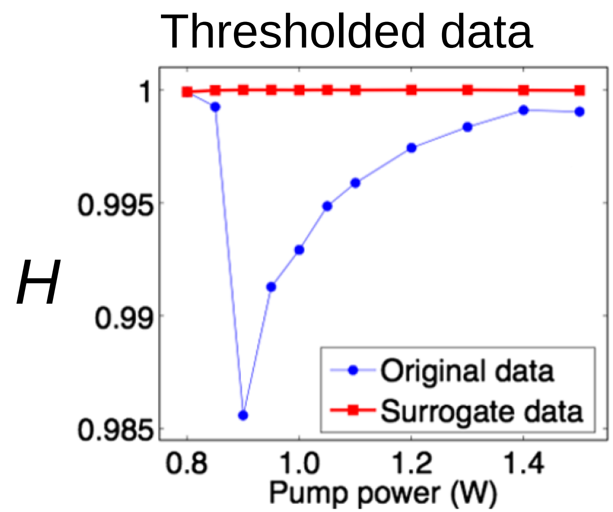
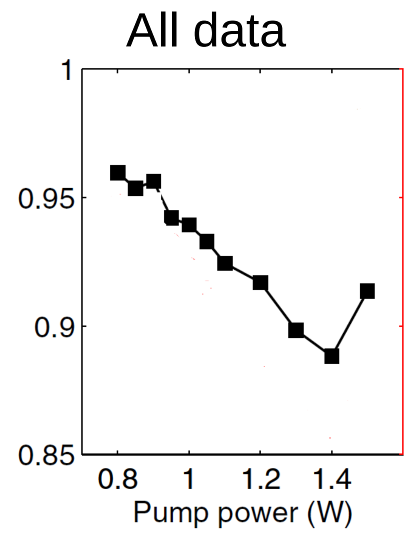
Entropy characterization of the transition

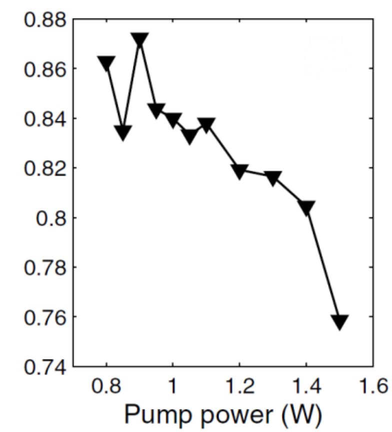
$$H = -\sum_{i=1}^{N} p_i \ln p_i$$







Entropy of the distribution of values. The distribution develops a tail (extreme events) and H decreases.



A. Aragoneses et al., "Unveiling temporal correlations characteristic of a phase transition in the output intensity of a fiber laser", PRL 116, 033902 (2016).

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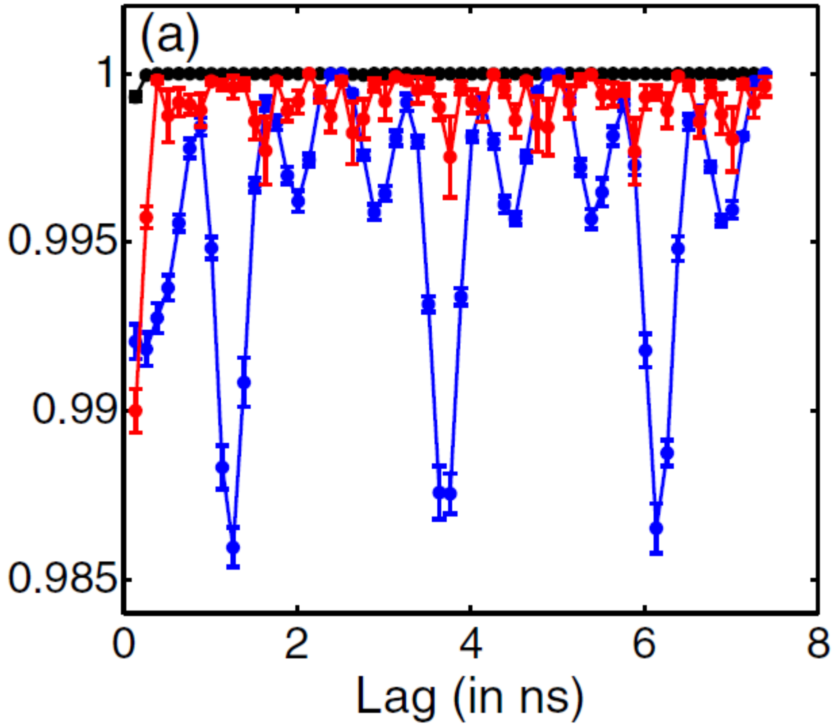
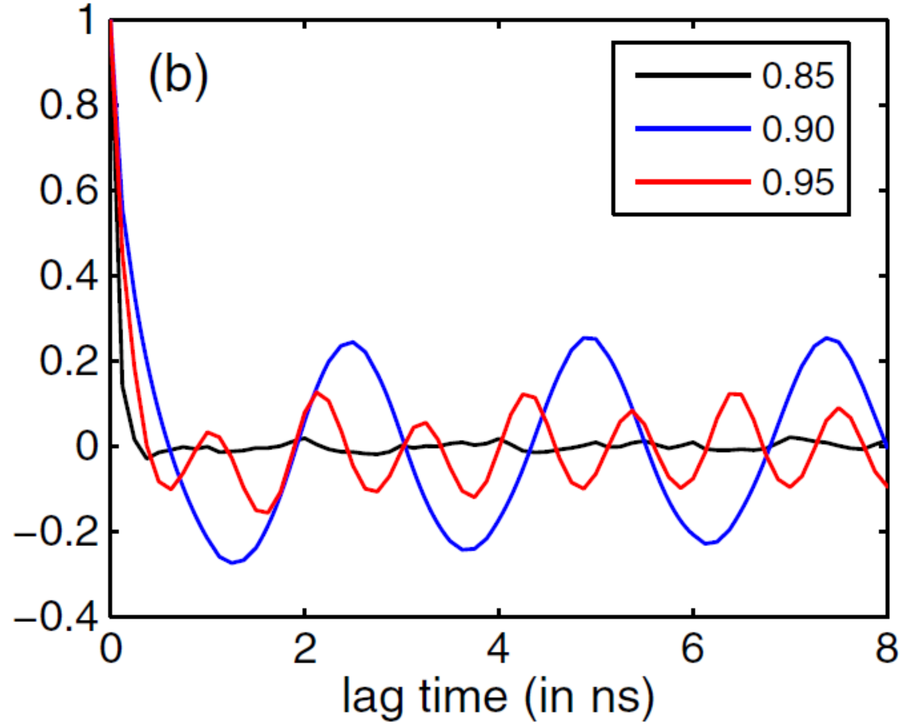
Ordinal patterns can be defined using a time lag (capture order relations between not-consecutive points)

Permutation entropy **below**, **at**, and **above** the transition



Autocorrelation function

$$C(\tau) = \frac{\langle [x(t)-\mu][x(t+\tau)-\mu] \rangle}{\sigma^2}$$



A. Aragoneses et al., "Unveiling temporal correlations characteristic of a phase transition in the output intensity of a fiber laser", PRL 116, 033902 (2016).

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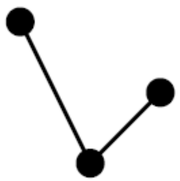
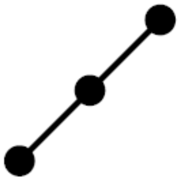
Using lagged points to define the patterns allows to select the time scale of the analysis, useful for seasonal data

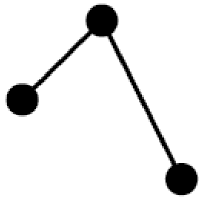
Intra-season

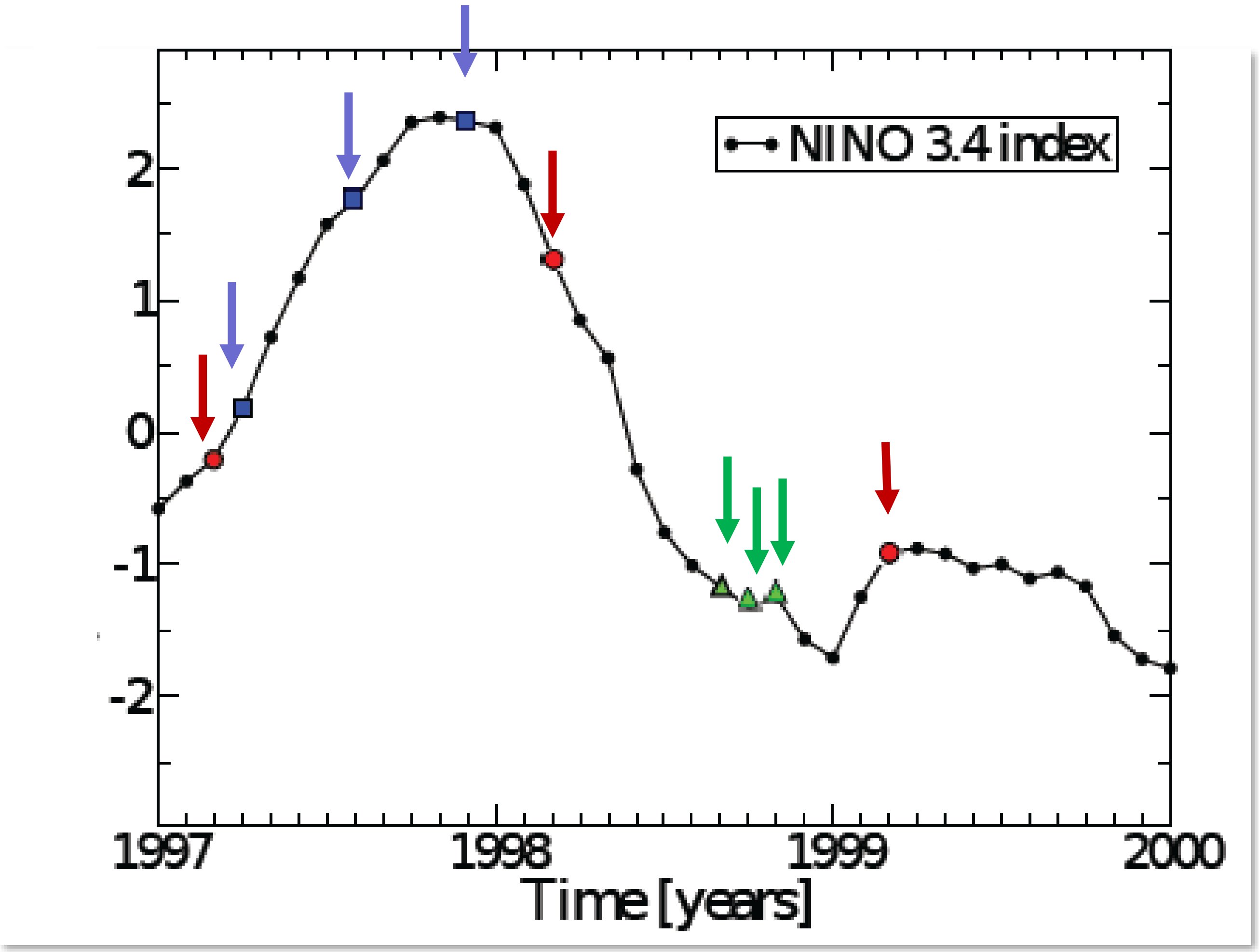


Intra-annual



Inter-annual





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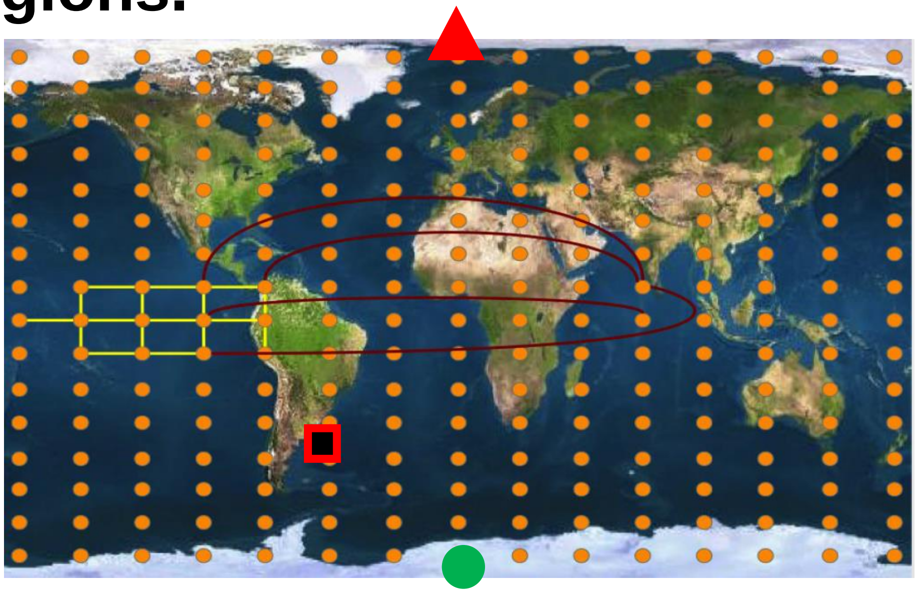
Application: analysis of surface air temperature (SAT) anomaly in two geographical regions.

Anomaly = annual solar cycle removed



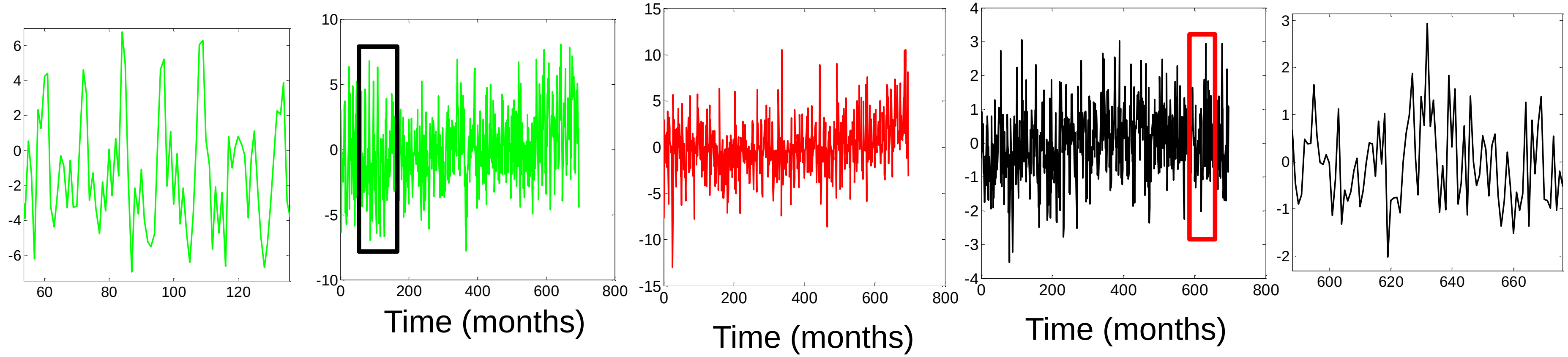
Reanalysis (data assimilation)

2.5º x 2.5º =10226 grid points.

In each point 696 anomaly values

(1949-2006: 58 years x 12 months)

How does the data look like?



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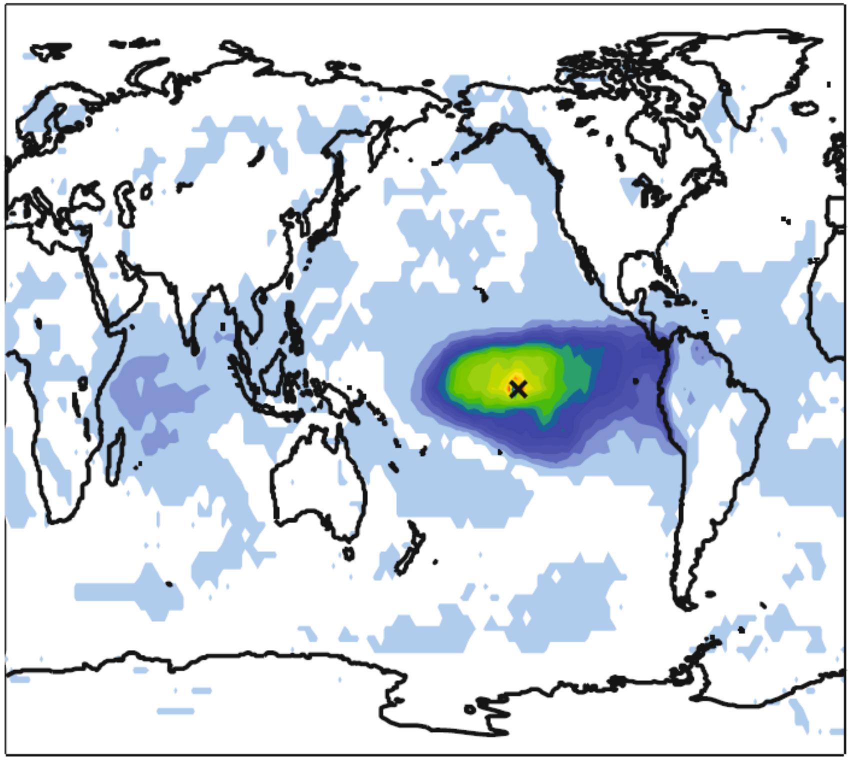
Mutual Information (color code) of SAT anomaly in El Niño region and other regions (white: MI not significant)

$$M_{ij} = \sum_{m,n} p_{ij}(m,n) \log \frac{p_{ij}(m,n)}{p_i(m)p_j(n)}$$

MI from probabilities of SAT values



J. I. Deza, M. Barreiro, C. Masoller, "Inferring interdependencies in climate networks constructed at inter-annual, intra-season and longer time scales", Eur. Phys. J. ST 222, 511 (2013).

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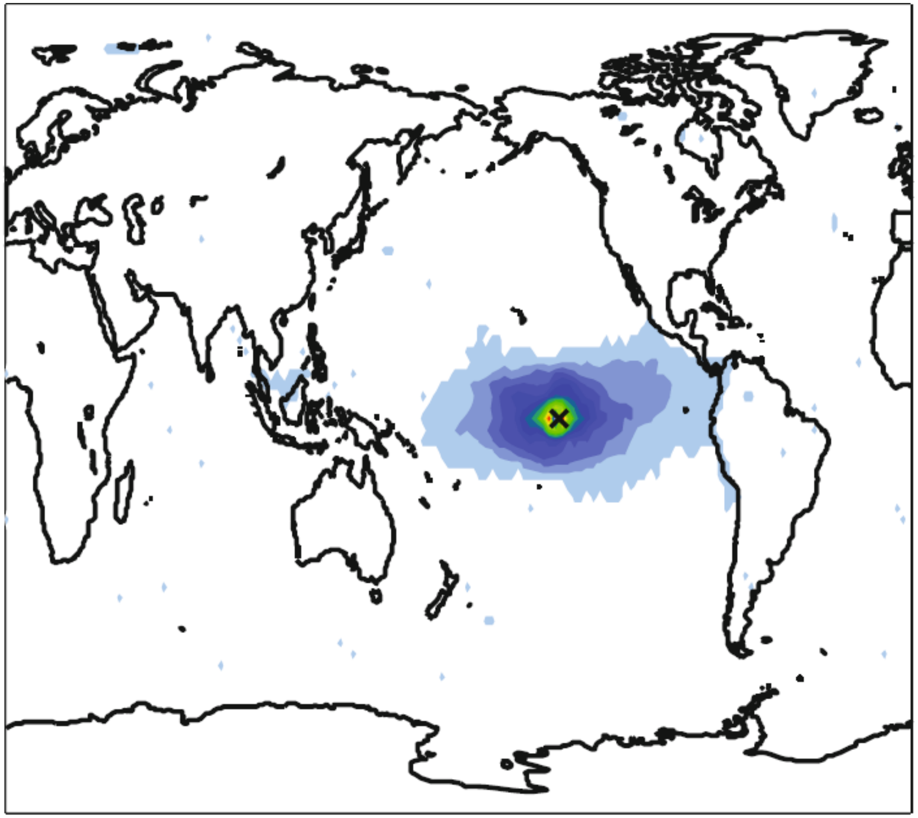
Mutual Information (color code) of SAT anomaly in El Niño region and other regions (white: MI not significant)

$$M_{ij} = \sum_{m,n} p_{ij}(m,n) \log \frac{p_{ij}(m,n)}{p_i(m)p_j(n)}$$

MI from probabilities of ordinal patterns defined by values in 3 consecutive months.



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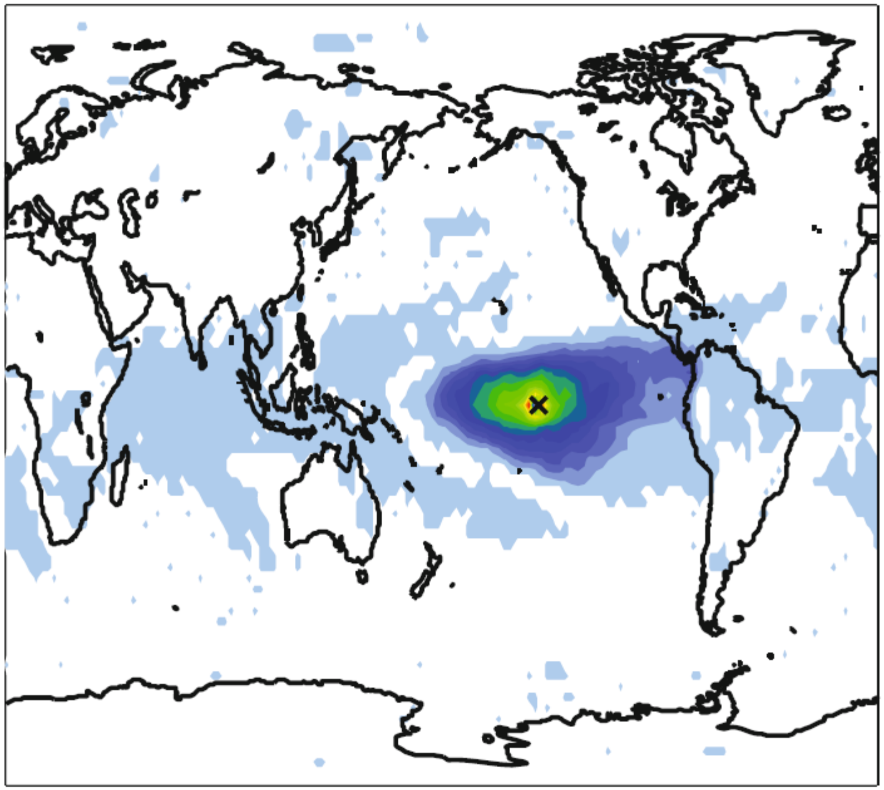
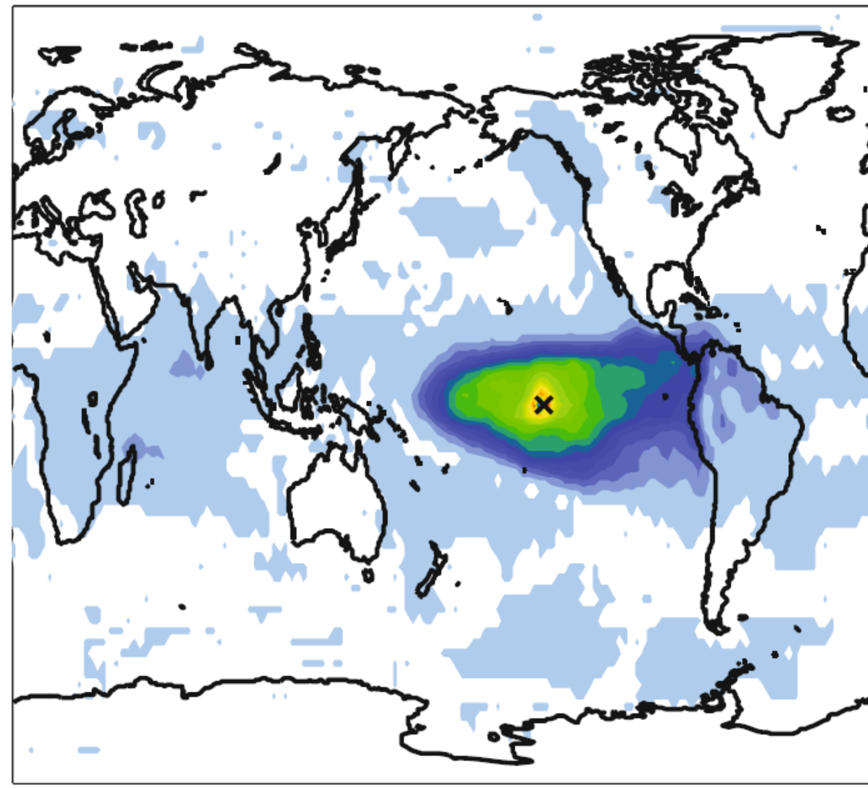
Mutual Information (color code) of SAT anomaly in El Niño region and other regions (white: MI not significant)

Patterns defined by 3 values in a year



Patterns defined by values in 3 consecutive years



J. I. Deza, M. Barreiro, C. Masoller, “Inferring interdependencies in climate networks constructed at inter-annual, intra-season and longer time scales”, Eur. Phys. J. ST 222, 511 (2013).

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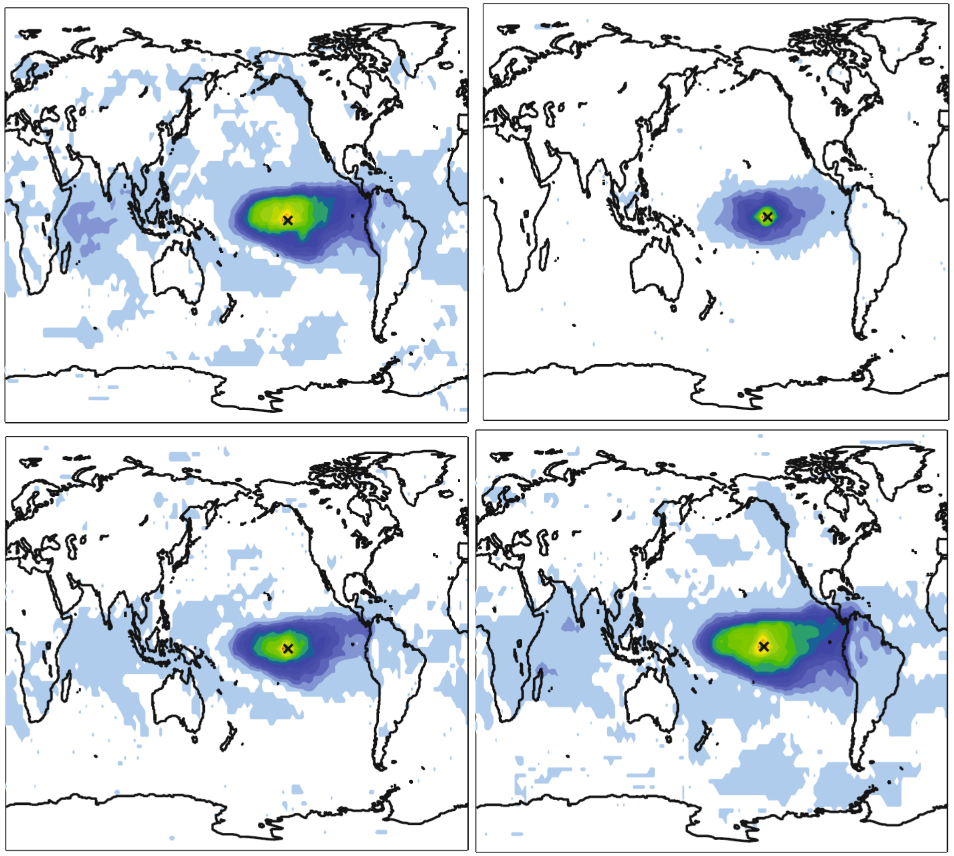
Comparison

$$M_{ij} = \sum_{m,n} p_{ij}(m,n) \log \frac{p_{ij}(m,n)}{p_i(m)p_j(n)}$$

probabilities of SAT values

probabilities of ordinal patterns defined by values in 3 consecutive months.



probabilities of patterns defined by 3 values in a year.

probabilities of patterns defined by values in 3 consecutive years.

J. I. Deza, M. Barreiro, C. Masoller, "Inferring interdependencies in climate networks constructed at inter-annual, intra-season and longer time scales", Eur. Phys. J. ST 222, 511 (2013).

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Permutation entropy of two dimensional patterns (images)

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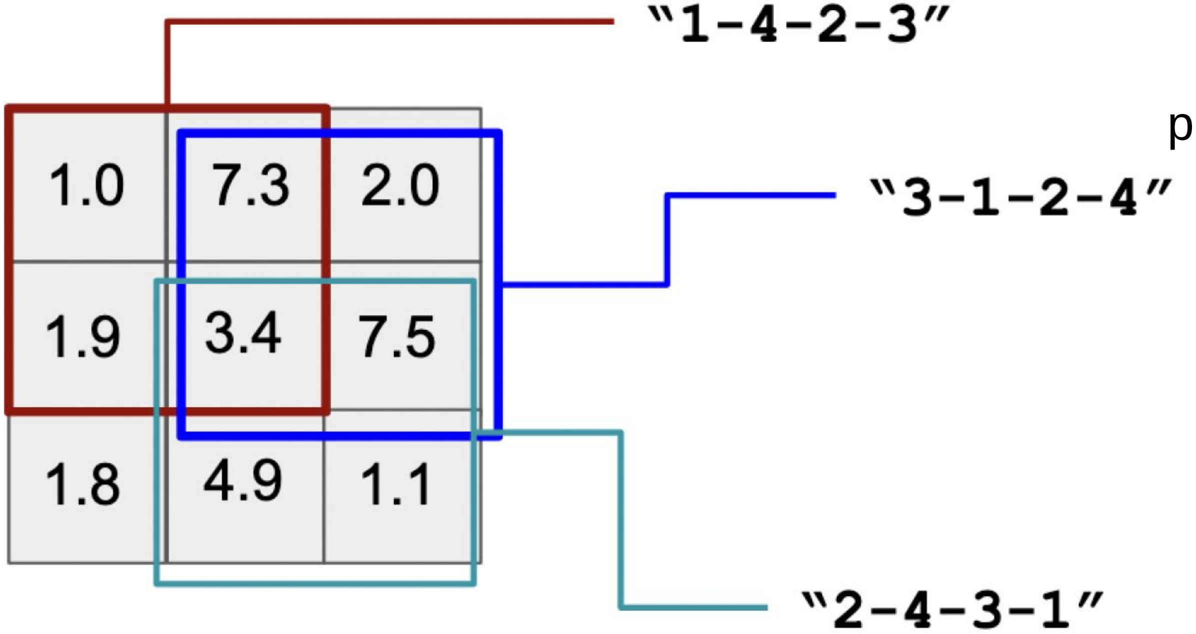
Ordinal patterns defined on two-dimensional data



Compute probabilities

Permutation entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

2x2 pixels: 24 possible patterns

H. V. Ribeiro et. al, PLoS ONE 7, e40689 (2012).

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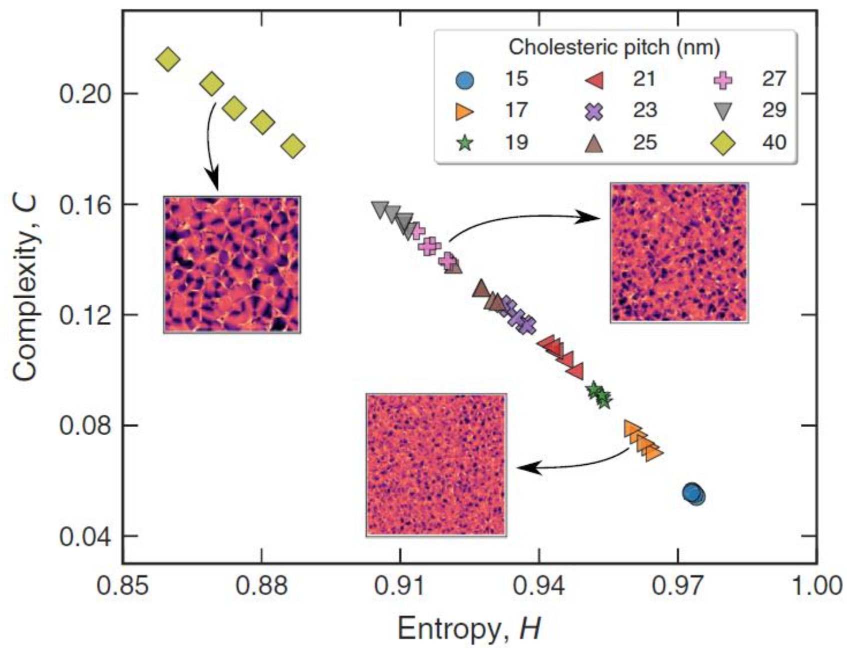
The “spatial” permutation entropy was proposed to characterize two dimensional patterns and images.

PHYSICAL REVIEW E **99**, 013311 (2019)

Estimating physical properties from liquid crystal textures via machine learning and complexity-entropy methods

H. Y. D. Sigaki,[1] R. F. de Souza,[1] R. T. de Souza,[1,2] R. S. Zola,[1,2,*] and H. V. Ribeiro[1,†]
[1]*Departamento de Física, Universidade Estadual de Maringá, Maringá, PR 87020-900, Brazil*
[2]*Departamento de Física, Universidade Tecnológica Federal do Paraná, Apucarana, PR 86812-460, Brazil*



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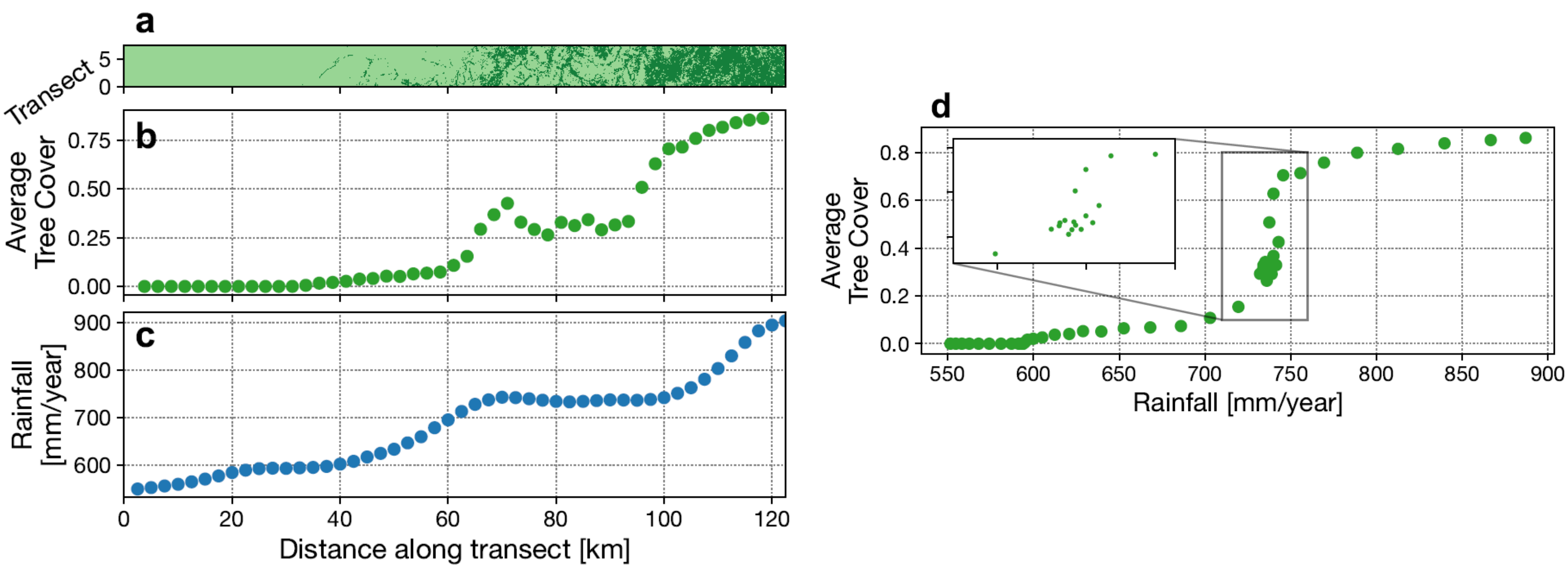
The variation of the spatial permutation entropy can give an early indicator of a vegetation transition.

High-resolution vegetation data from the Serengeti–Mara ecosystem in northern Tanzania and southern Kenya.



G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2023).

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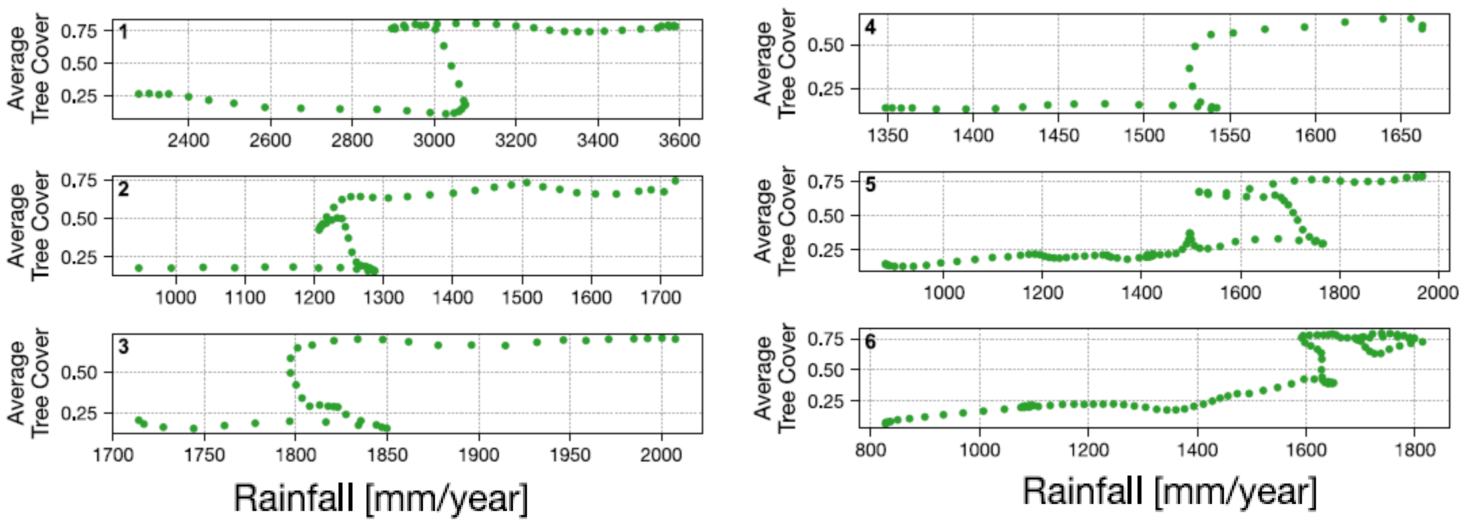
We also analyzed **low-resolution** satellite (MODIS) vegetation data, combined with data from the Tropical Rainfall Measuring Mission (TRMM)



G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2023).

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Results

Permutation entropy

(ordinal patterns defined by the values of 2x2 pixels)

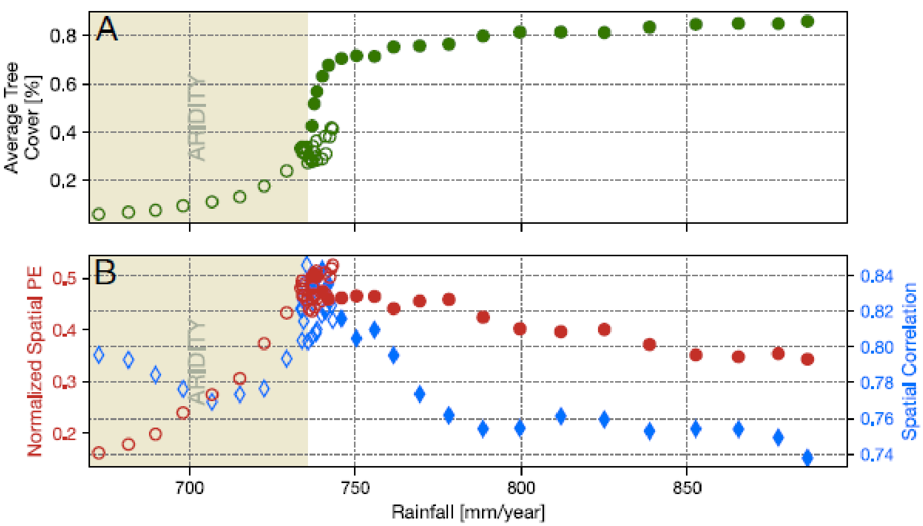
$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

Spatial correlation

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

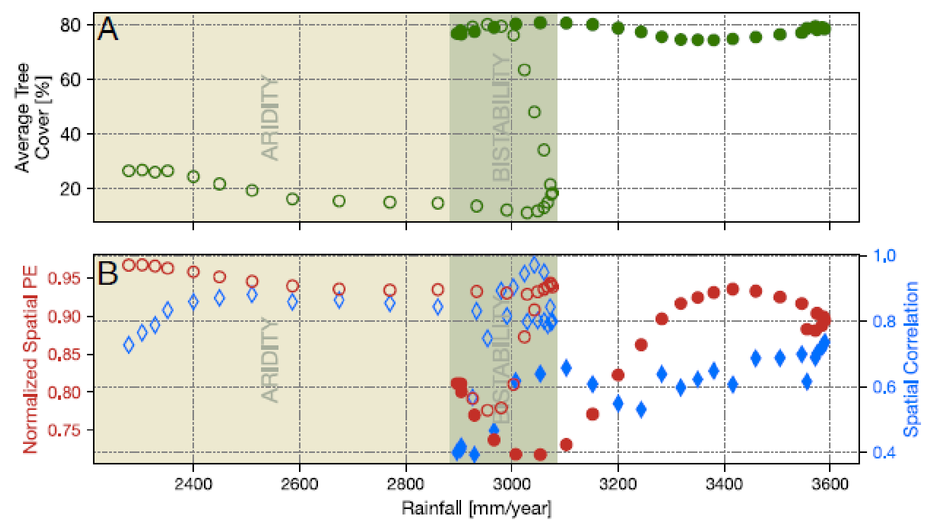
w_{ij}=1 if i, j first neighbors, else 0

High-resolution data



Low-resolution data

(transect 1; large variability across transects)



G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2023).

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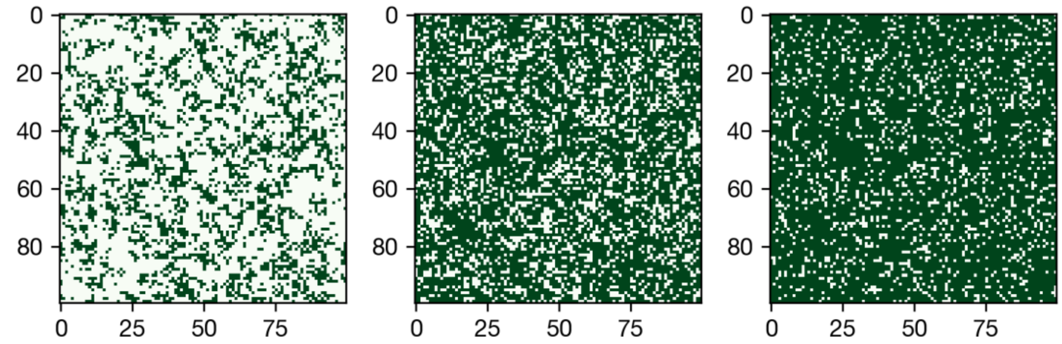
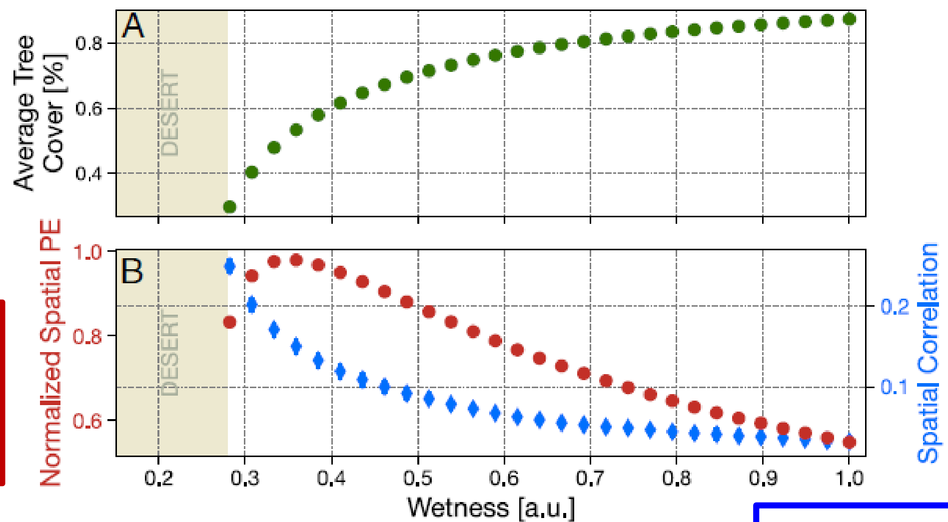
To gain insight: simulations of vegetation models

A) Cellular automata model





$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

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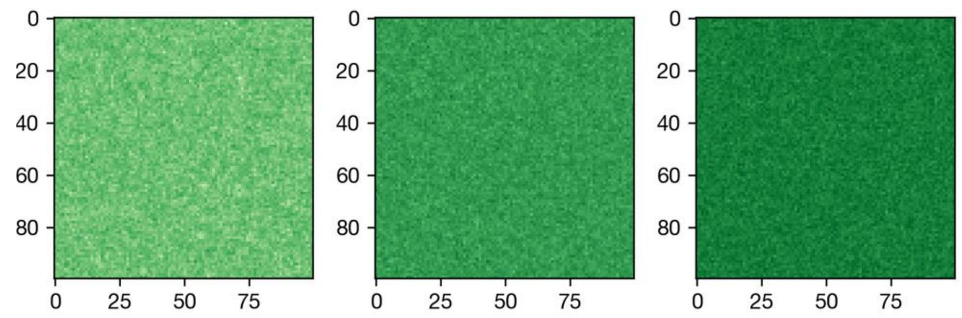
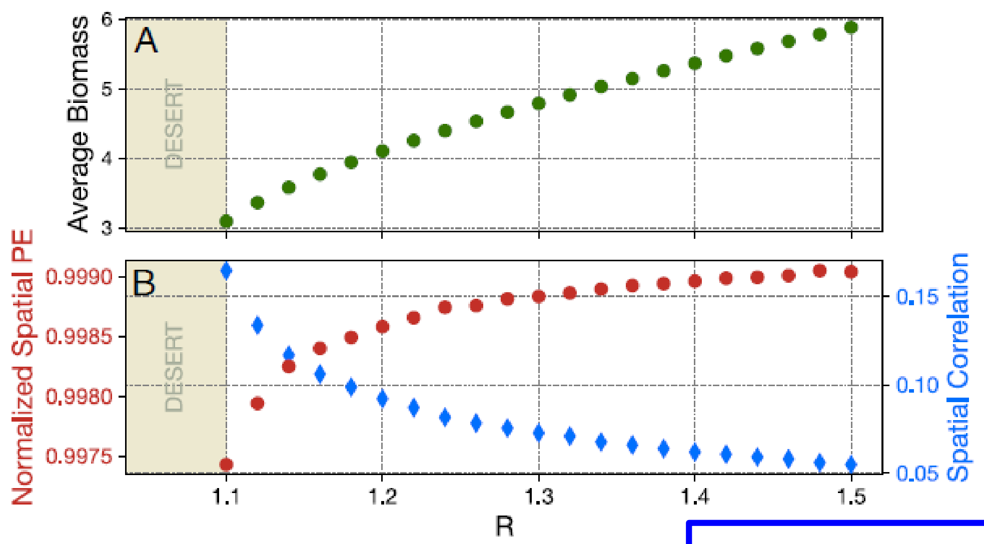
To gain insight: simulations of vegetation models

B) Local Positive Feedback model
(two partial differential equations)





$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

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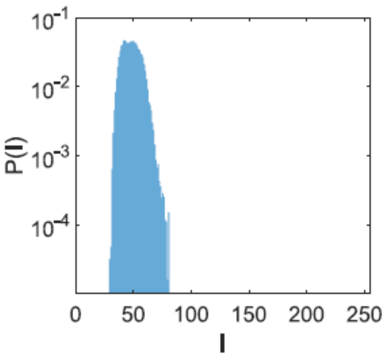
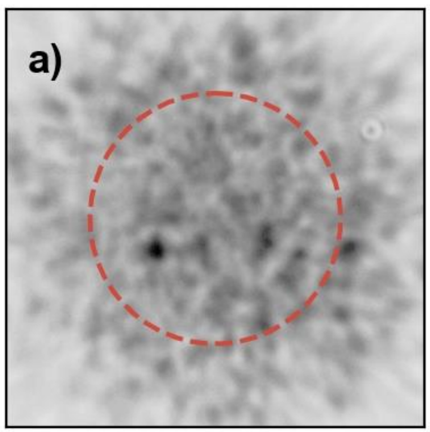
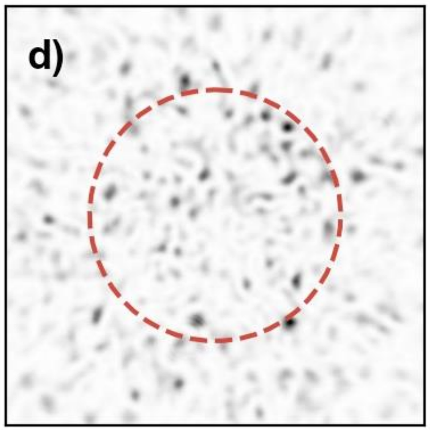
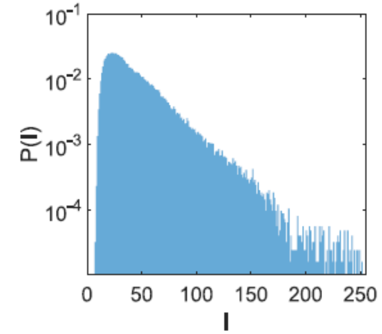
Examples of speckle images

Below threshold **Above threshold**









Quantification of speckle contrast: $\boxed{SC = \sigma/\langle I \rangle}$

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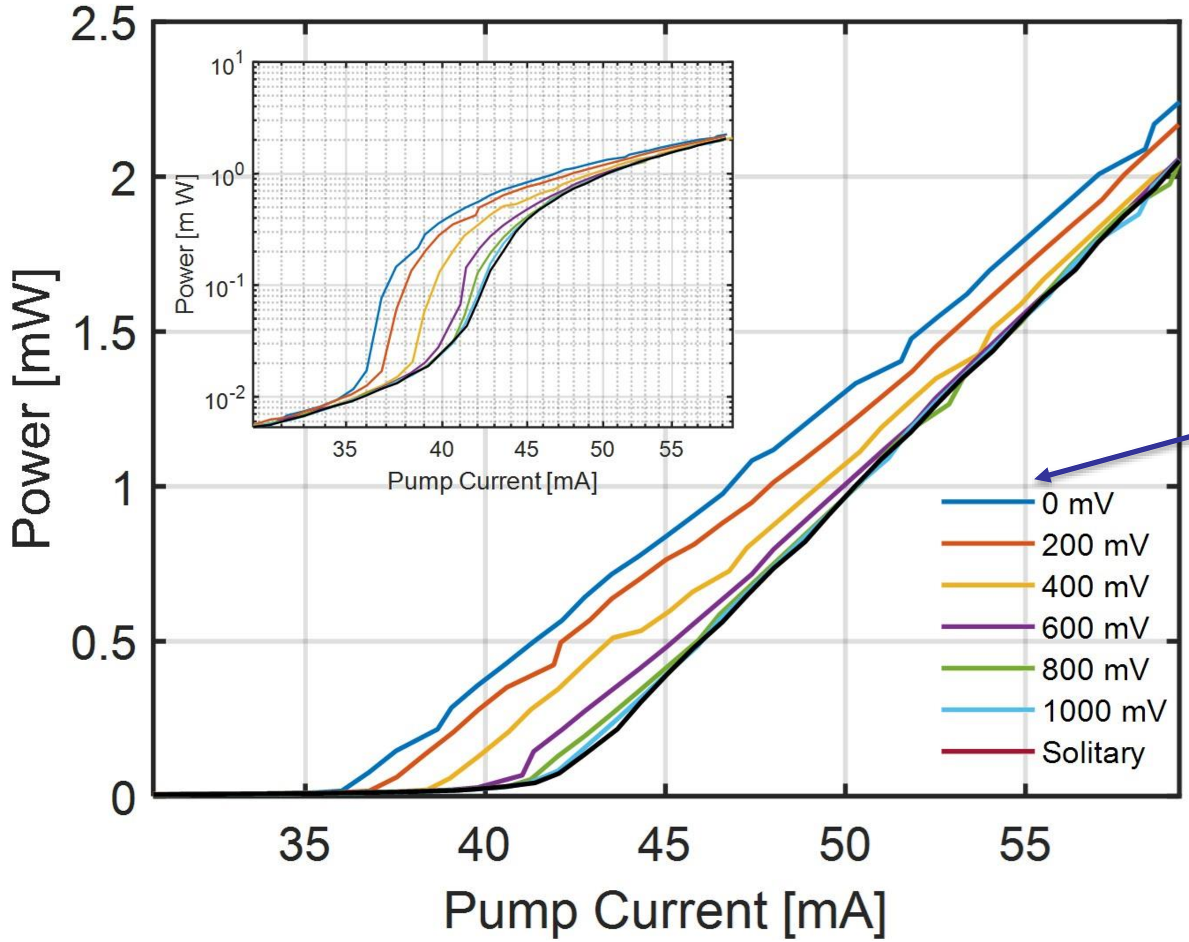
Optical feedback: reduces the lasing threshold

The emitted power (proportional to the number of photons emitted per unit time) characterizes the turn-on transition (off-on)



No attenuation (maximum feedback)

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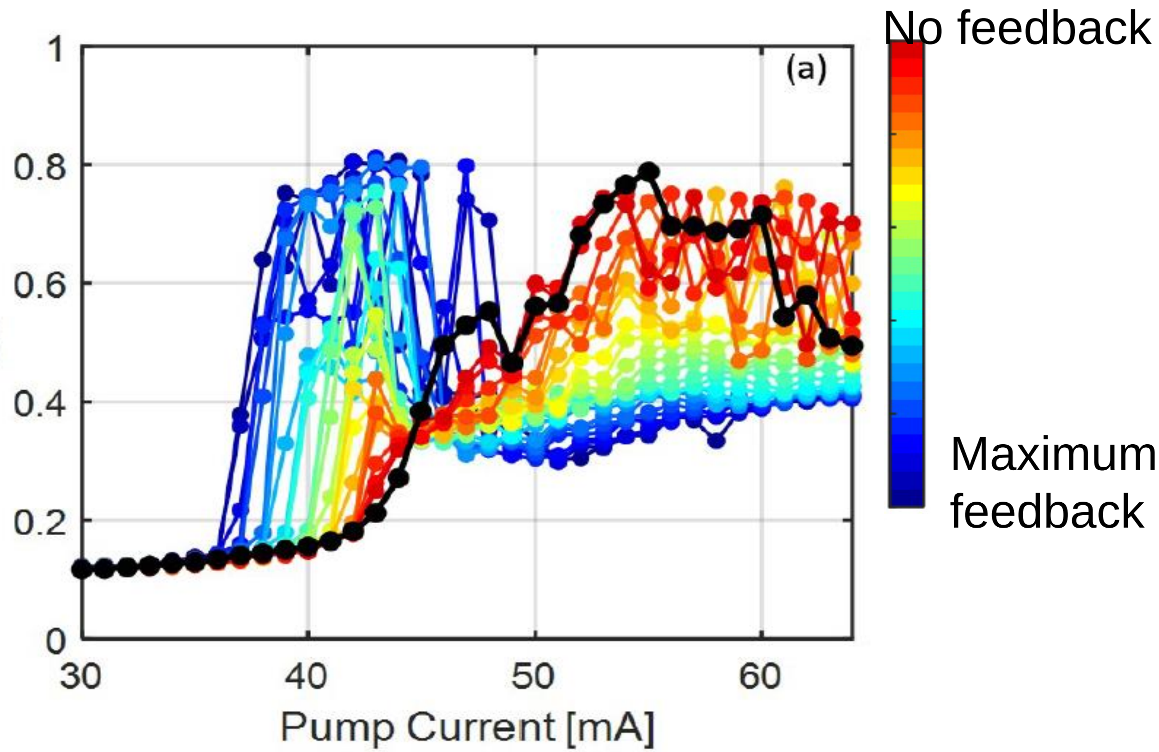
Is the emitted light beam coherent?

The analysis of speckle images allows to quantify the transition spontaneous emission (LED light) → stimulated emission (laser light)

$$\boxed{SC = \sigma/\langle I \rangle}$$



M. Duque-Gijon, C. Masoller, J. Tiana-Alsina, "Abrupt transition from low-coherence to high-coherence radiation in a semiconductor laser with optical feedback," Opt. Exp. 31, 3857 (2023).

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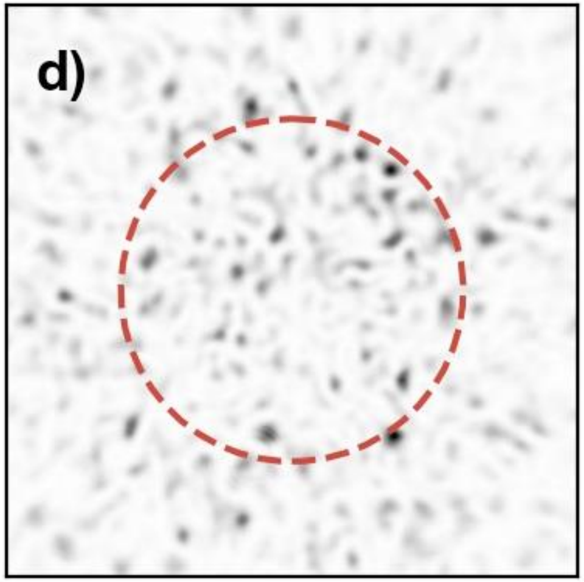
Analysis of speckle images using permutation entropy



70 692 pixels inside the circle

Pattern: x x x x	70093 patterns 4!=24 possible patterns

Pattern: x x x x x	69846 patterns 5! =120 possible patterns

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Results

Pattern:
x x
x x

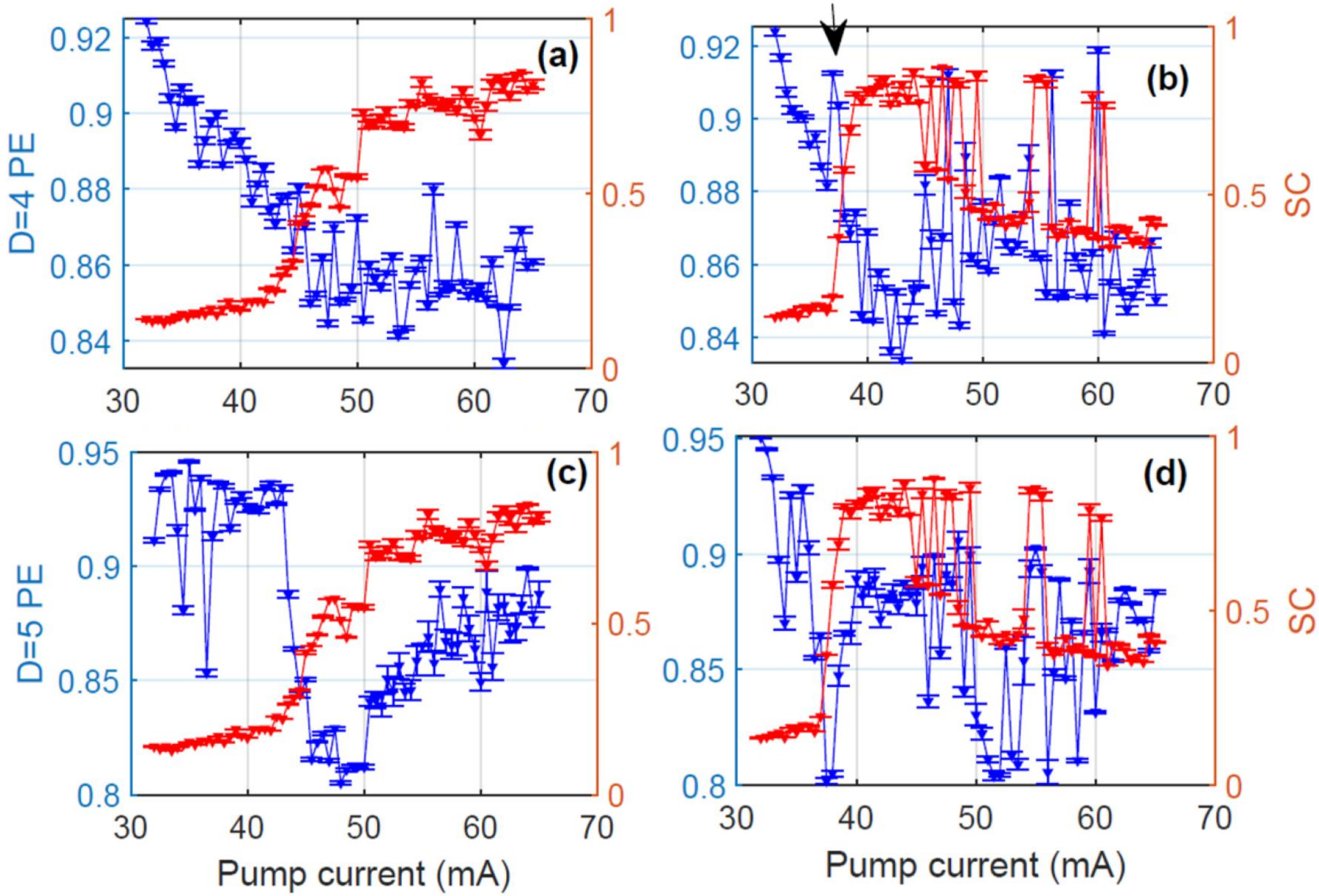
$SC = \sigma/\langle I \rangle$

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

Pattern: x
x x x
x

Laser without feedback

Laser with optical feedback



G. Tirabassi et al., "Permutation entropy-based characterization of speckle patterns generated by semiconductor laser light", APL Photonics 8, 126112 (2023).

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Permutation entropy of multivariate time series

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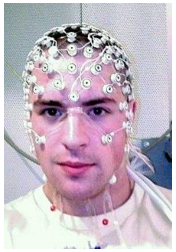
Analysis of eyes-closed eyes-open transition in EEG recordings of healthy subjects.



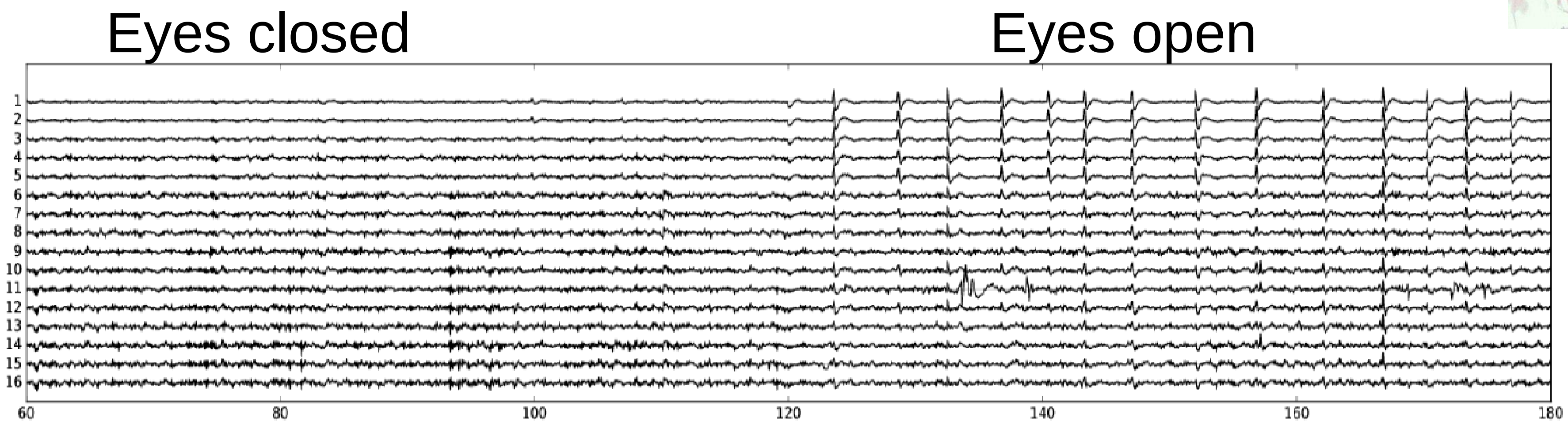


TABLE I. Description of the datasets used.

	DTS1	DTS2
Sampling rate (Hz)	256	160
Time task (seg)	120	60
Total points	30 720	9600
Number of electrodes	16	64
Number of subjects	71	109

DTS1: Bitbrain (Zaragoza)
DTS2: Physionet

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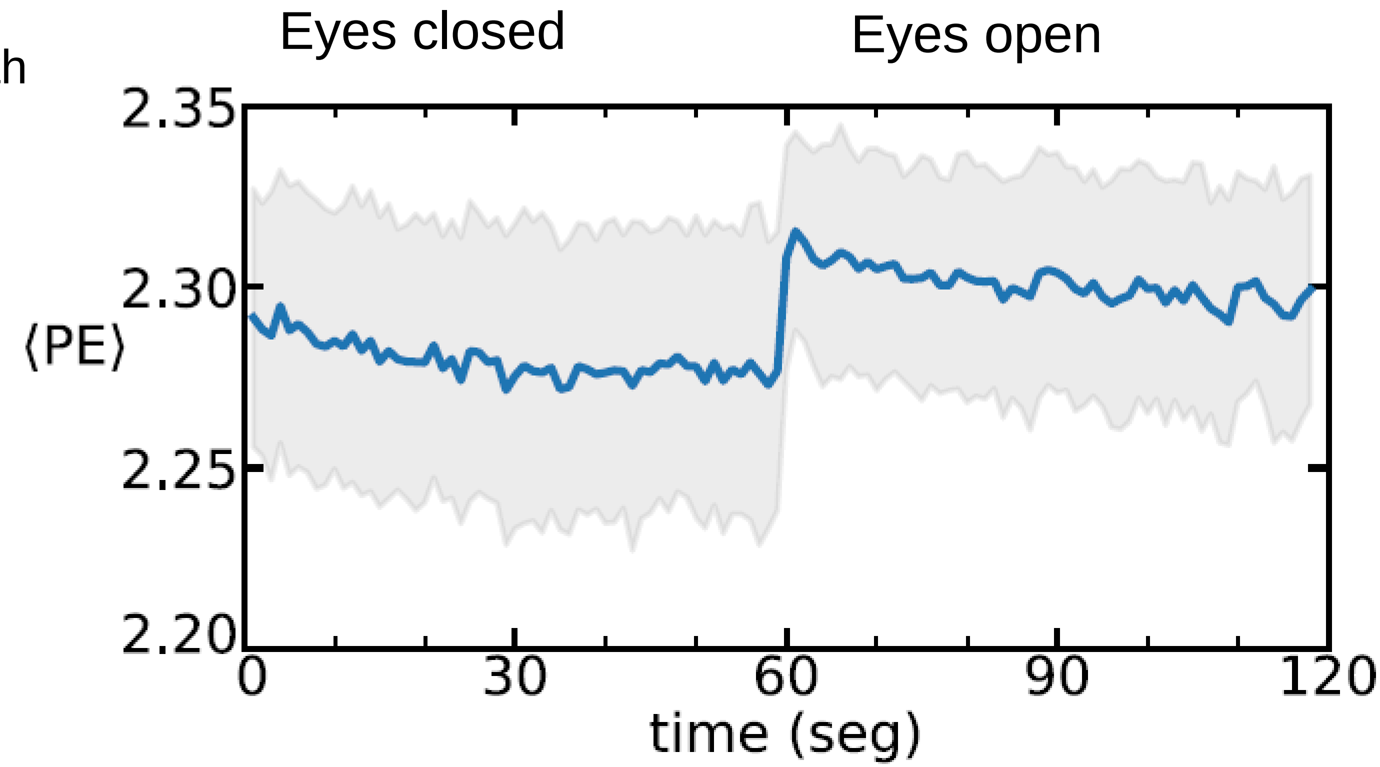
The Permutation Entropy increases in the eyes open state

$$\langle \mathrm{PE} \rangle = \frac{1}{N[\text{electrodes}]} \sum_i \mathrm{PE}^i$$

PE was calculated with patterns of length 4 (# of possible patterns 24) in time windows containing >4000 patterns

Gray region: σ of ⟨PE⟩ values across subjects



C. Quintero-Quiroz, L. Montesano, A. J. Pons, M. C. Torrent, J. García-Ojalvo, C. Masoller, "Differentiating resting brain states using ordinal symbolic analysis", Chaos 28, 106307 (2018).

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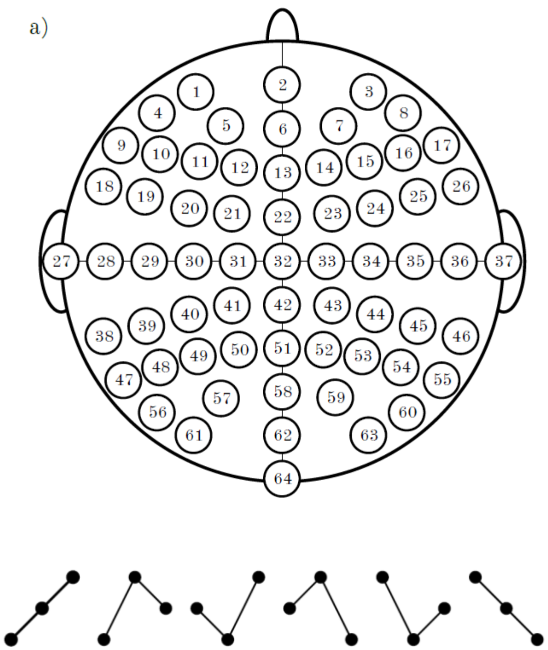
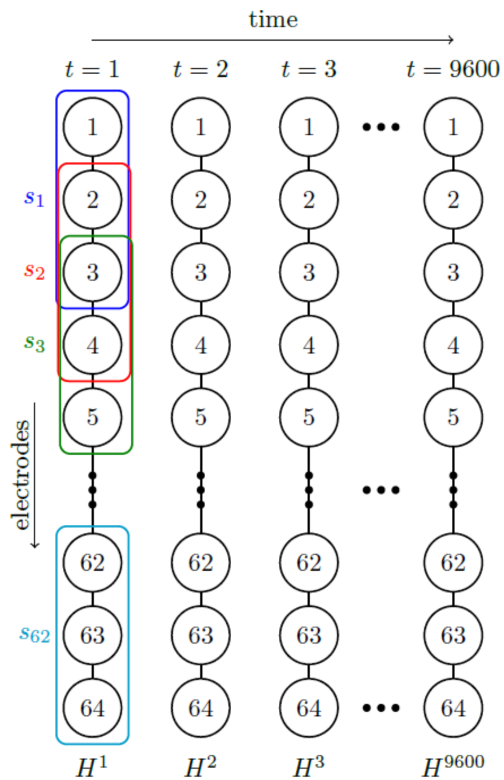
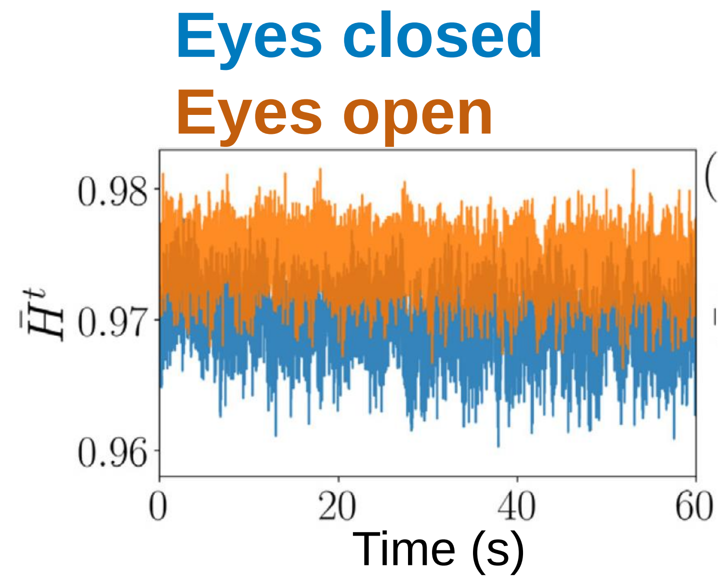
Spatial approach to compute the Permutation Entropy







(H_t is averaged over subjects)

At each time: data values of 64 channels $\Rightarrow$ 62 ordinal patterns to calculate 6 probabilities.

B. R. R. Boaretto, R. C. Budzinski, K. L. Rossi, C. Masoller, E. E. N. Macau, "Spatial permutation entropy distinguishes resting brain states", Chaos, Solitons & Fractals 171, 113453 (2023).

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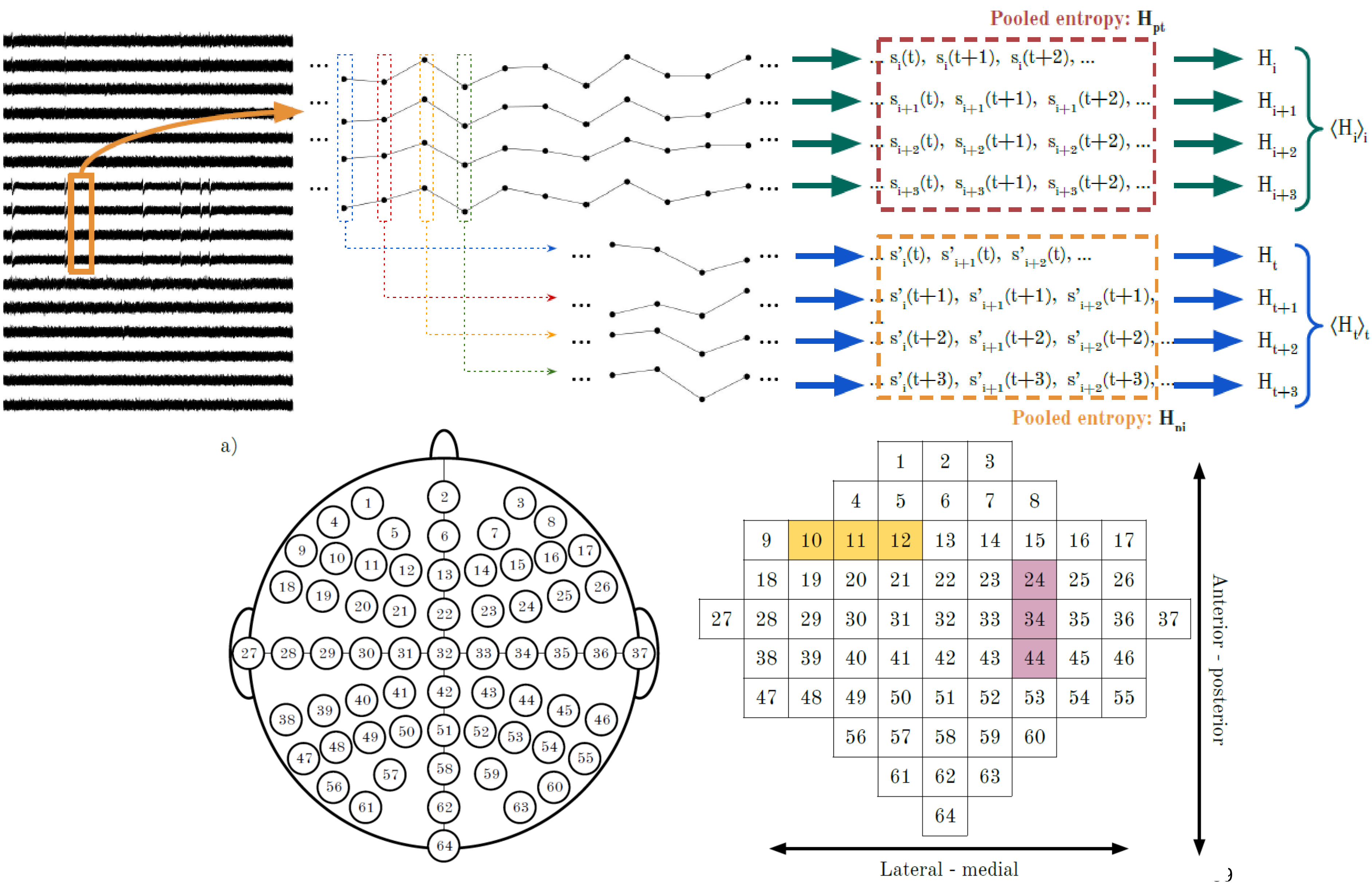
Four approaches to calculate the permutation entropy



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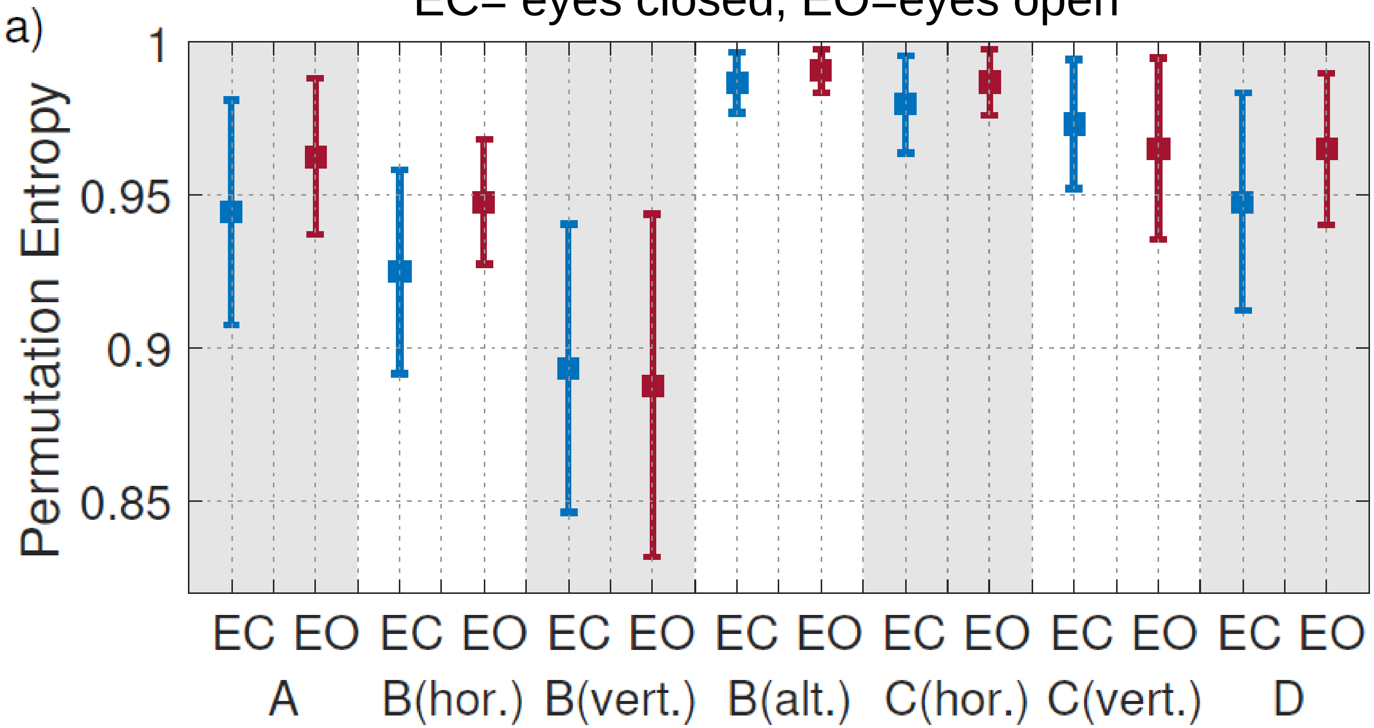
Results

EC= eyes closed, EO=eyes open



error bars:
σ over 107
subjects

A: temporal coding
B: spatial coding (horizontal, vertical, and alt. symbols)
C: spatial pooling (horizontal and vertical symbols)
D: temporal pooling

J. Gancio, C. Masoller, G. Tirabassi, “Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches”, Chaos 34, 043130 (2024).

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Random forest classification of eyes open-eyes closed states

		Accuracy	F1 score	Precision	Recall	Specificity
Horizontal	$\langle H^s_t \rangle_t$	61 ± 7	59 ± 10	63 ± 10	57 ± 16	65 ± 16
	$\sigma\,(H^s_t)$	66 ± 7	65 ± 9	66 ± 9	$\mathbf{67 \pm 15}$	65 ± 15
	H^s_{pi}	58 ± 8	54 ± 12	61 ± 11	50 ± 16	66 ± 15
Vertical	$\langle H^s_t \rangle_t$	54 ± 9	55 ± 12	54 ± 10	59 ± 17	50 ± 15
	$\sigma\,(H^s_t)$	56 ± 9	59 ± 10	56 ± 9	64 ± 15	48 ± 16
	H^s_{pi}	55 ± 9	56 ± 11	55 ± 10	59 ± 17	51 ± 16
Temporal	$\langle H^s_i \rangle_i$	63 ± 8	56 ± 13	70 ± 15	49 ± 16	77 ± 15
	$\sigma\,(H^s_i)$	$\mathbf{69 \pm 8}$	$\mathbf{66 \pm 10}$	$\mathbf{73 \pm 12}$	62 ± 14	76 ± 13
	H^s_{pt}	64 ± 8	58 ± 13	72 ± 14	51 ± 16	$\mathbf{78 \pm 14}$

Using filtered data tends to improve the performance.

Performance is as good as that of other statistical measures.

J. Gancio, C. Masoller, G. Tirabassi, “Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches”, Chaos 34, 043130 (2024).

Take home messages and outlook

1. We have found (in empirical, experimental and synthetic data) that the permutation entropy may give an indication of an approaching transition.

2. It can be used to identify differences in high-dimensional multivariate datasets.

3. Ongoing work: climate data, synchronization transition.

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