# On the encoding of weak periodic signals by coupled FitzHugh-Nagumo neurons

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*Campus d'Excel·lència Internacional*

**2nd International Workshop on Neurodynamics (NDy'18)**
**Castro Urdiales, September 2018**

ICREA

GOBIERNO DE ESPAÑA
MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD

*Research group on Dynamics, Nonlinear Optics and Lasers (11 permanent faculty members, 9 PhD students, 2 posdocs)*

# Where are we?

1. Barcelona
2. Castelldefels
3. Igualada
4. Manresa
5. Mataró
6. Sant Cugat del Vallès
7. Terrassa
8. Vilanova i la Geltrú

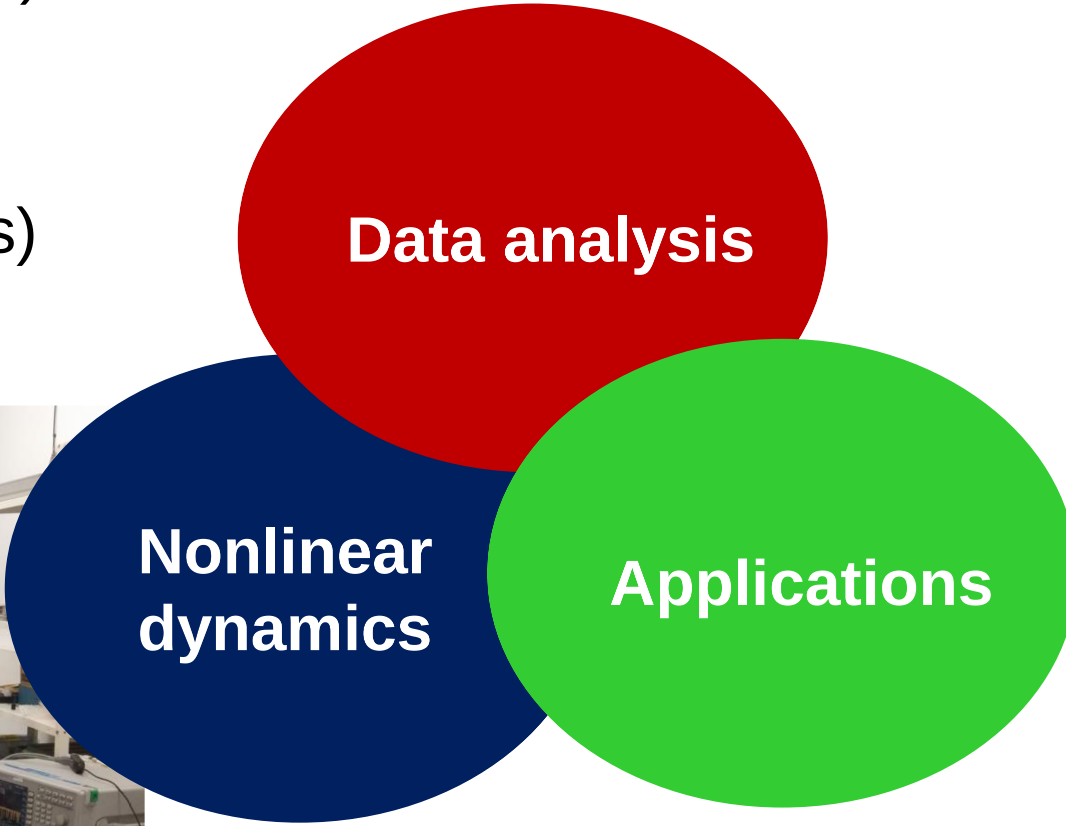


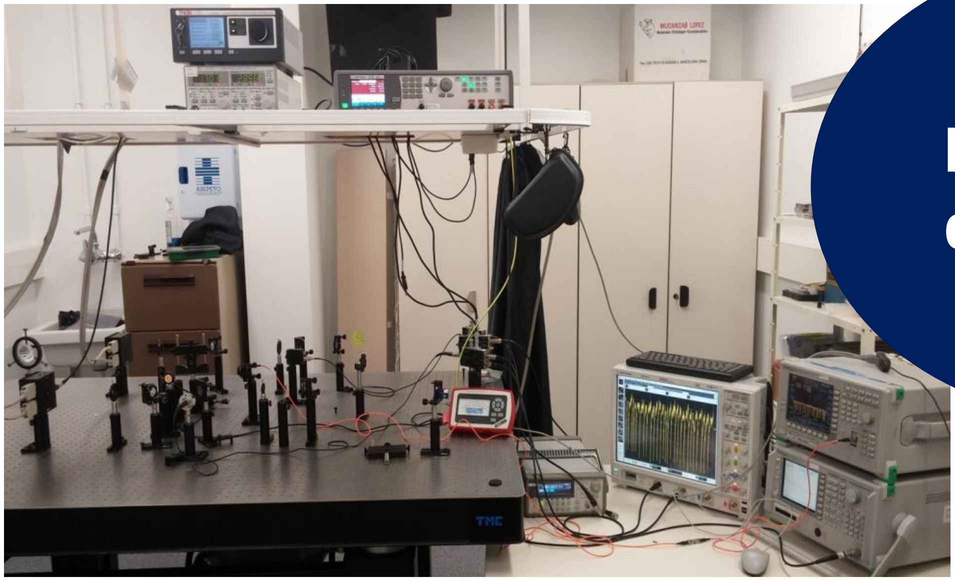
Viernes, 25 de septiembre de 2009 Diari de Terrassa

El edificio Gaia centraliza grupos científicos consolidados y emergentes.

## What do we study?

- laser dynamics (models & experiments)
- neuronal dynamics (models)
- complex networks
- data analysis (climate time series & biomedical images)



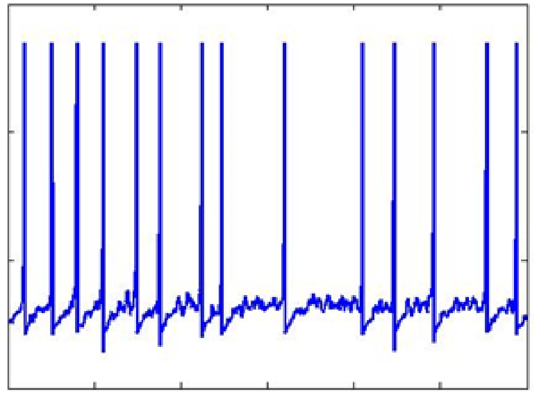



# Lasers and neurons?

**Optical spikes**

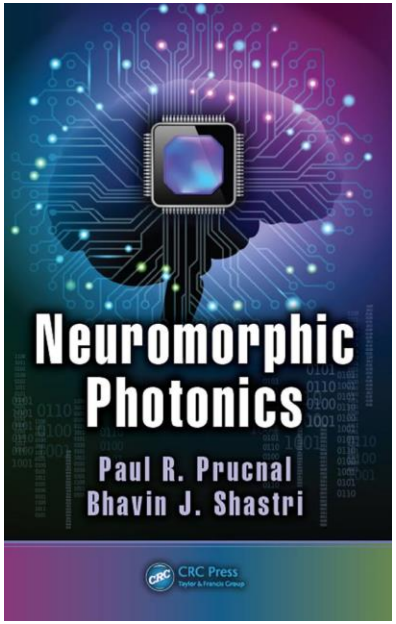
Time (µs)

**Neuronal spikes**

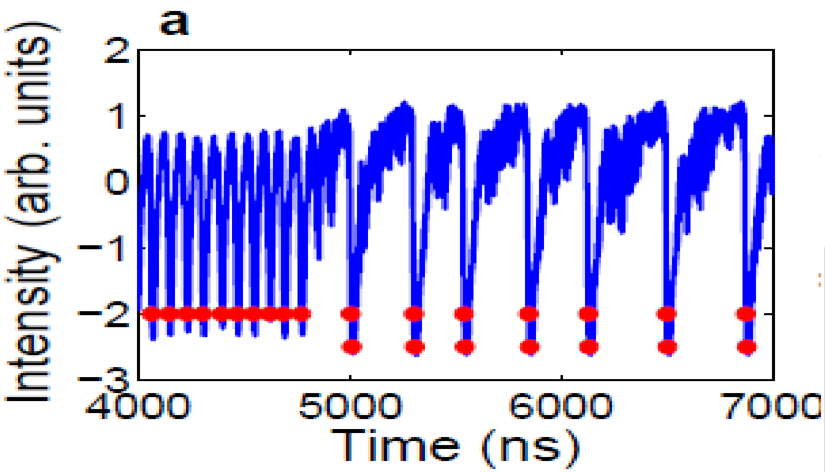


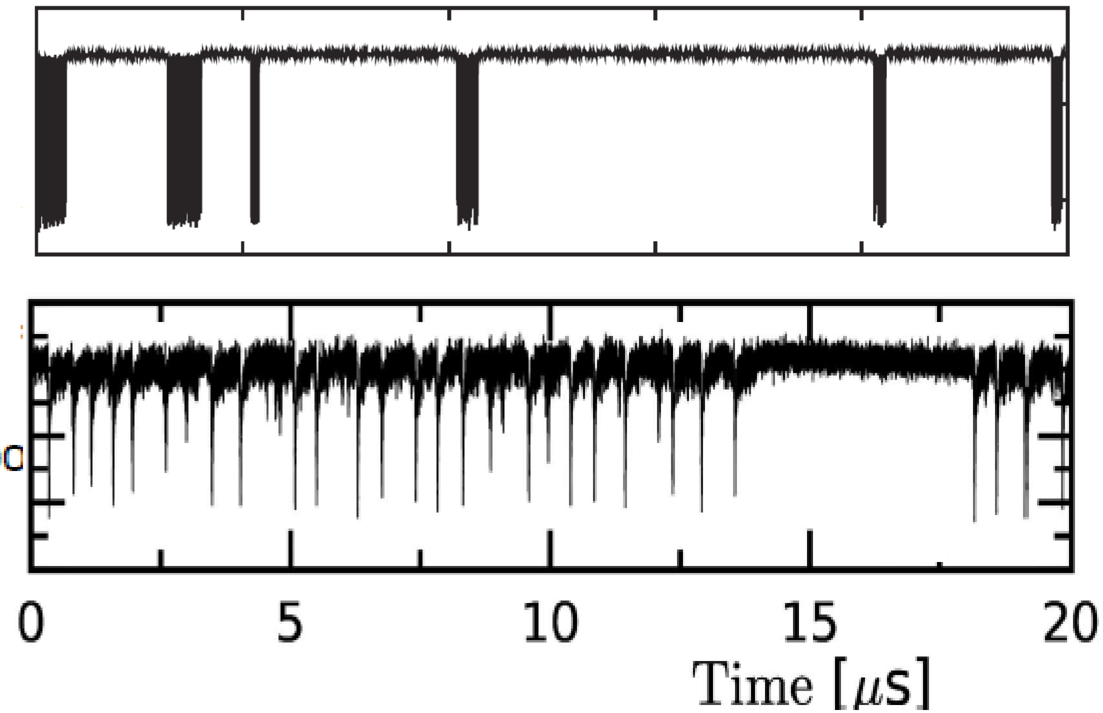
Time (ms)

- Photonic neurons: potential for building blocks for ultra-fast, energy-efficient neuron-inspired information processing systems.
- Miliseconds vs micro-nano seconds.
- Inexpensive laser diodes




# Dynamics of the laser output intensity

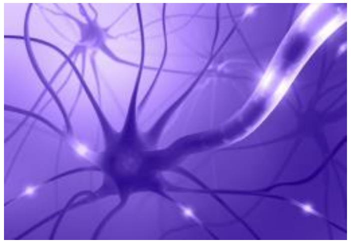


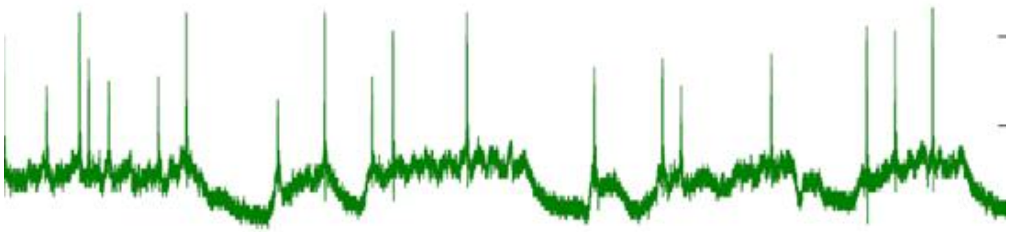




A. Aragoneses, S. Perrone, T. Sorrentino, M. C. Torrent and C. Masoller, "*Unveiling the complex organization of recurrent patterns in spiking dynamical systems*", Sci. Rep. **4**, 4696 (2014).

C. Quintero-Quiroz, J. Tiana-Alsina, J. Roma, M. C. Torrent, and C. Masoller, "*Characterizing how complex optical signals emerge from noisy intensity fluctuations*", Sci. Rep. **6** 37510 (2016).

# How neurons encode information?



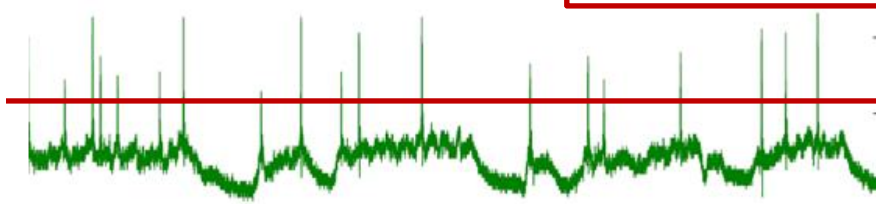


- In the spike **rate**?
- In the relative **timing** of the spikes?
- **Single** neuron encoding or **ensemble** encoding?
- If there are temporal correlations, how can they be detected and quantified?
- Our goal: try to understand how neurons encode a periodic weak (subthreshold) signal in the presence of noise.

# Outline

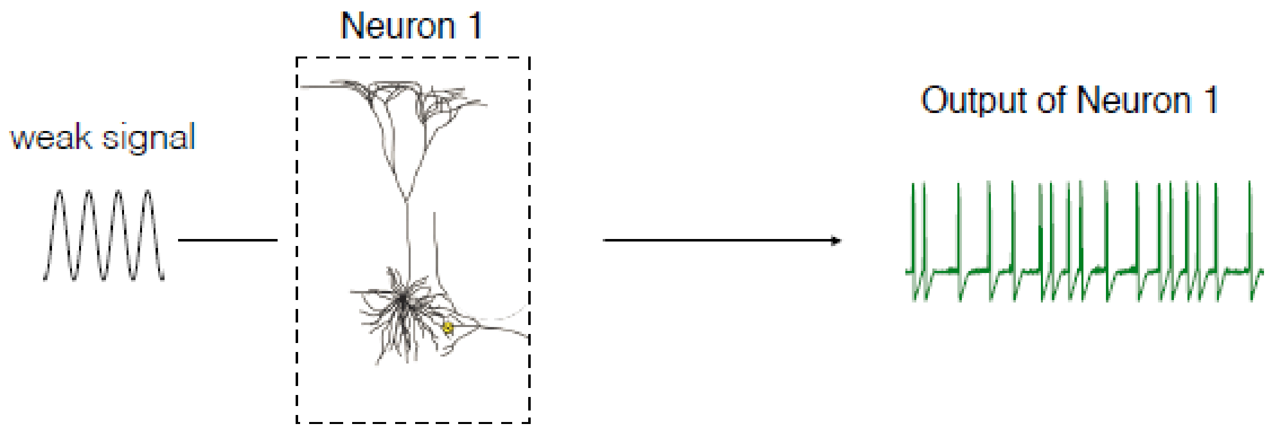
$$\{\ldots I_{i-1}, I_i, I_{i+1} \ldots\}$$

*inter-spike-intervals* $I_i = t_{i+1} - t_i$

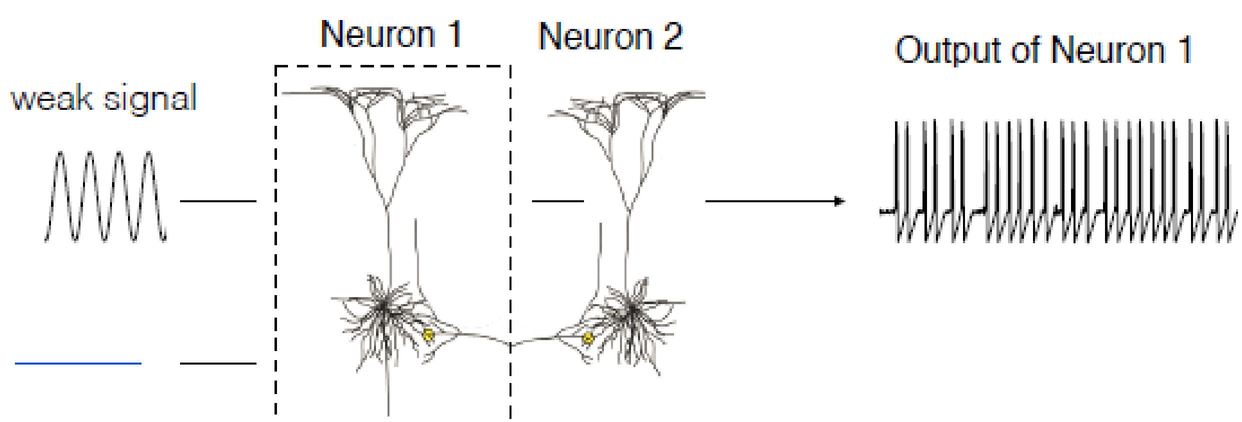


- Symbolic method of analysis of ISI sequences

- Single neuron




- Two neurons

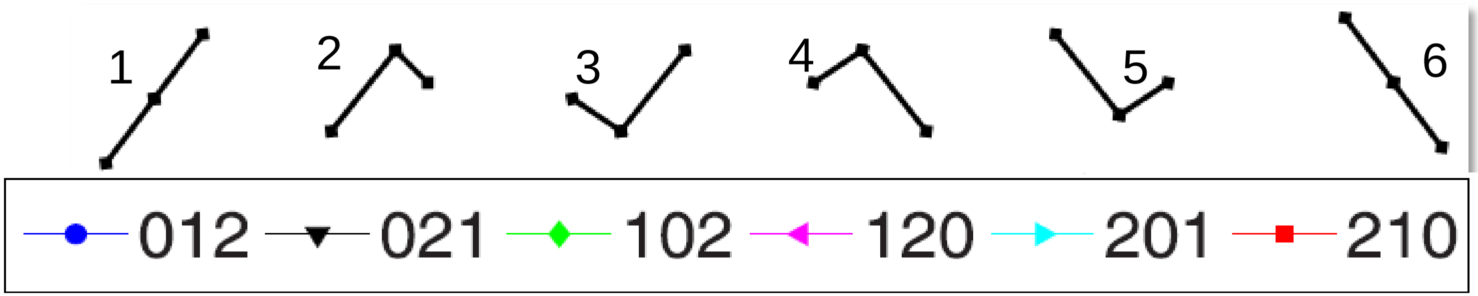


# Symbolic method of time-series analysis

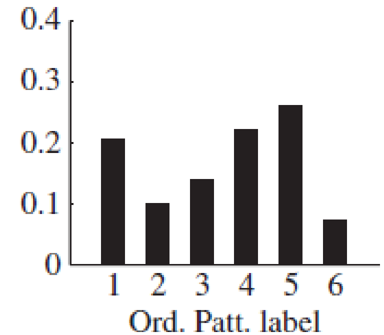
# Relative order of three consecutive intervals

$I_i = t_{i+1} - t_i$

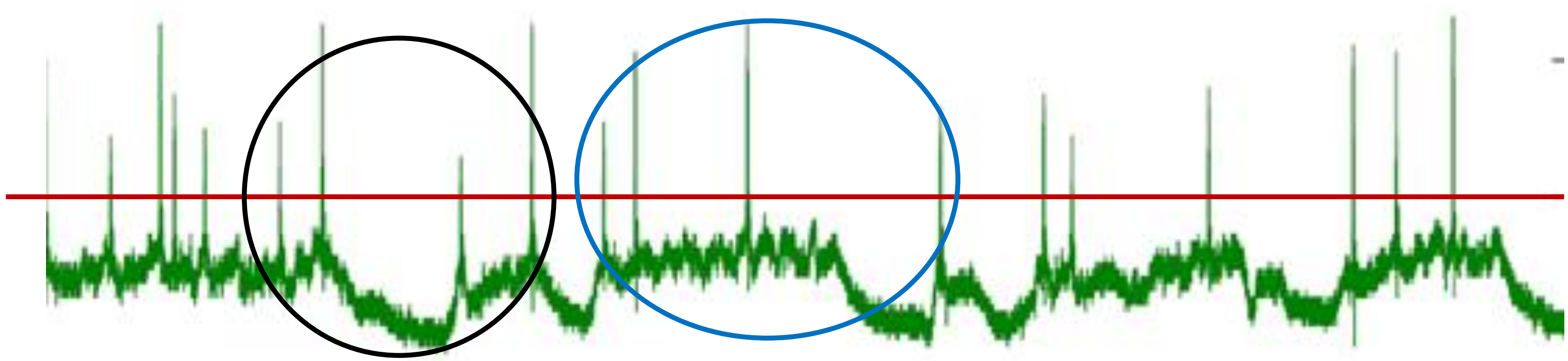
$\{\ldots I_i, I_{i+1}, I_{i+2}, \ldots\}$




Ordinal probabilities




Example: (5, 1, 7) gives “102” because 1 < 5 < 7



**021** **012**

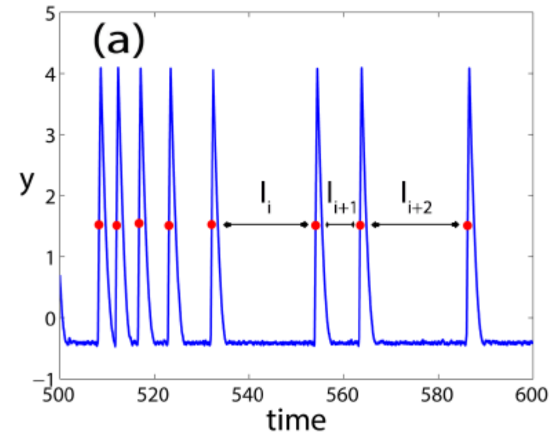
*Brandt & Pompe, PRL 88, 174102 (2002)*

# Analysis of single-neuron ISI sequences simulated with FitzHugh-Nagumo model

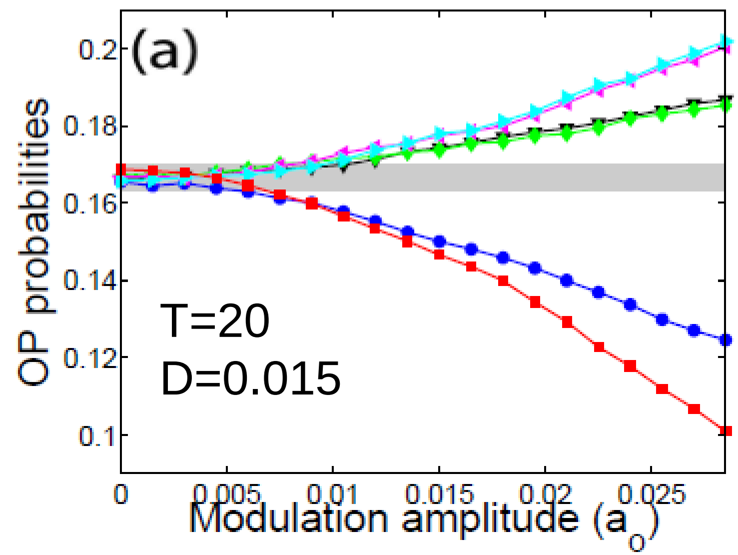
**- more/less frequent patterns encode information about subthreshold signal?**

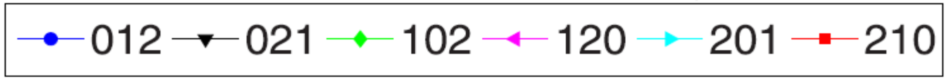
# FitzHugh-Nagumo model

$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o \cos(2\pi t/T)} + D\xi(t),$$



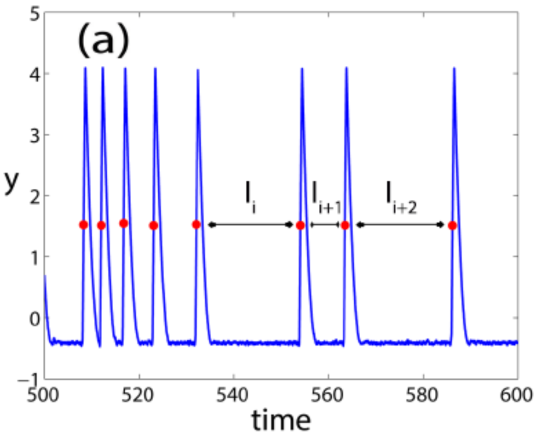

- Gaussian white noise and <u>subthreshold</u> signal: **$a_0$** and **T** such that spikes are **noise-induced**.
- Time series with 100,000 ISIs simulated (a=1.05, ε=0.01).
- Gray region: significance analysis with surrogates, 3σ confidence level.

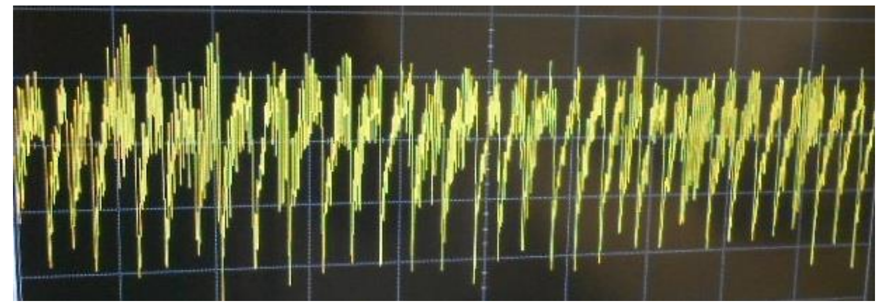


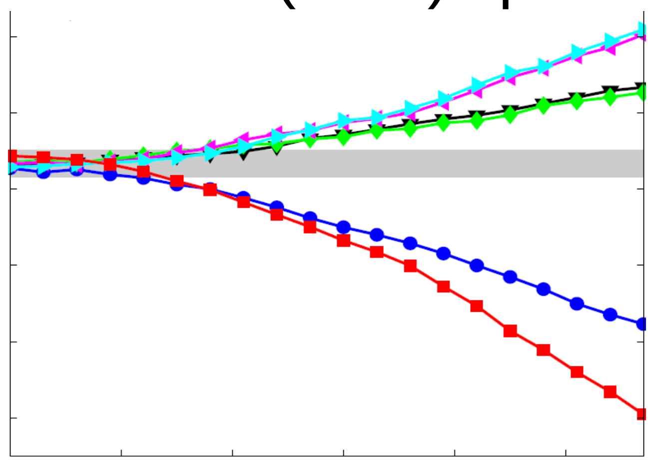

J. M. Aparicio-Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016).

# Side note: comparison with the laser spikes (exp. data)



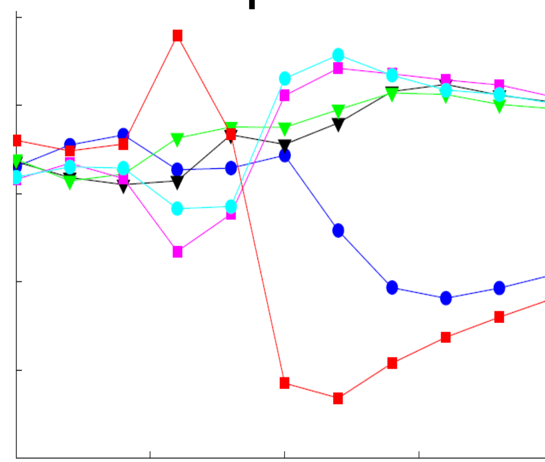
Neuronal (FHN) spikes



Modulation amplitude

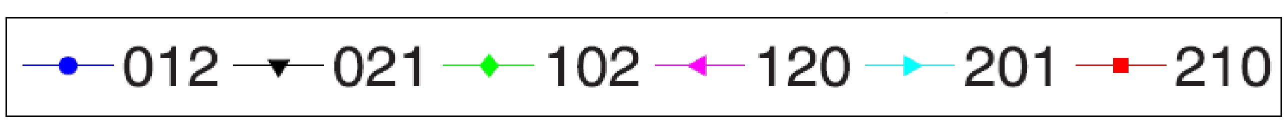
J. M. Aparicio-Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016).

Laser spikes

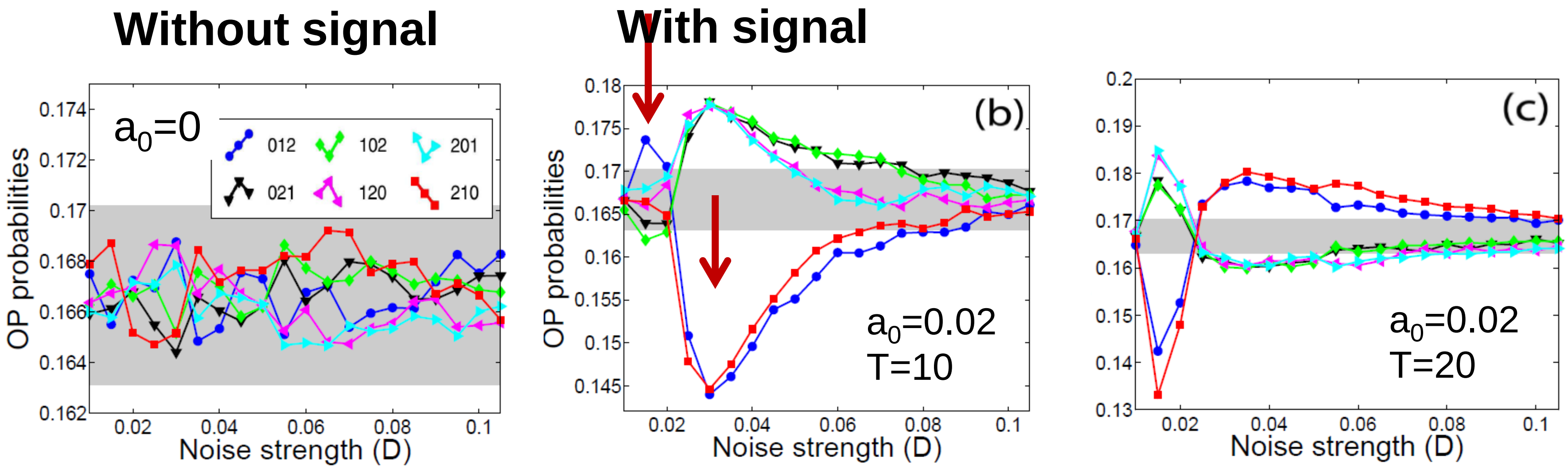


Modulation amplitude

A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014).



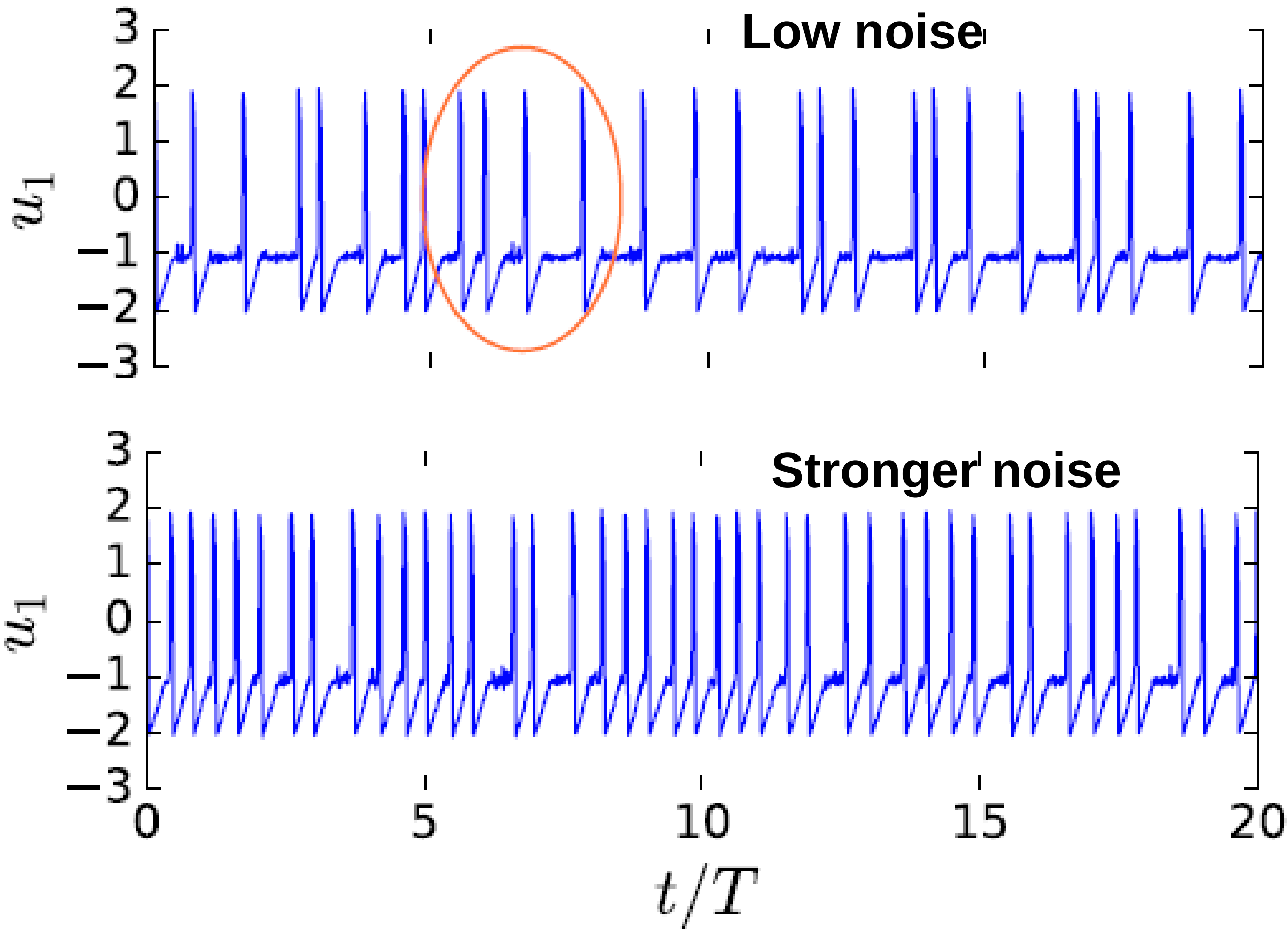
# Role of the noise strength




- No signal ⇒ no temporal ordering.
- Subthreshold periodic input induces preferred and infrequent patterns.
- They depend on the period and on the noise strength.
- Resonant-like behavior.

J. M. Aparicio-Reinoso M. C. Torrent and C. Masoller, PRE 94, 032218 (2016).

# Time series with different P(012)

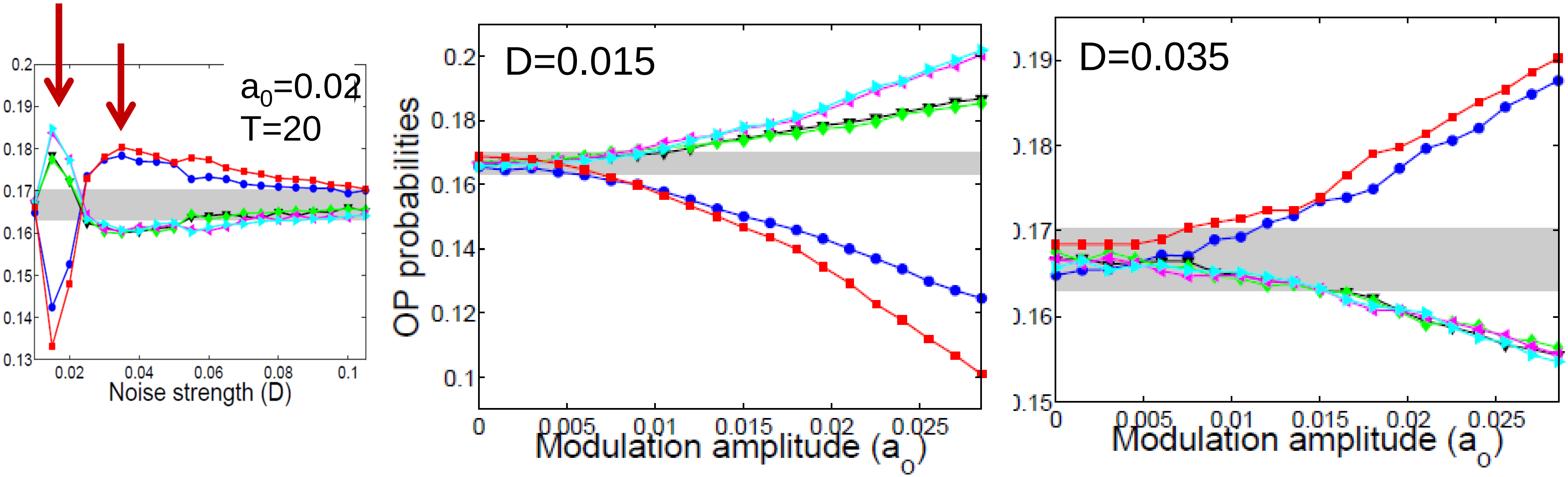



**P(012)>1/6**

**P(012)<1/6**

~ 40 spikes in 20 T

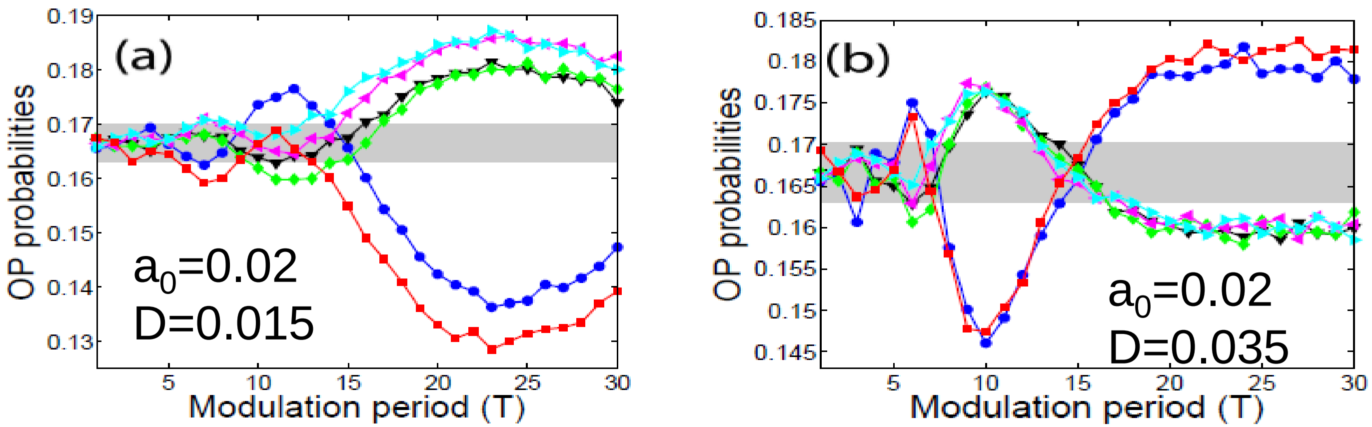
# Role of the signal amplitude




- The amplitude of the (subthreshold) signal does not modify the preferred or the infrequent patterns.
- The values of the probabilities encode information about the amplitude of the signal.

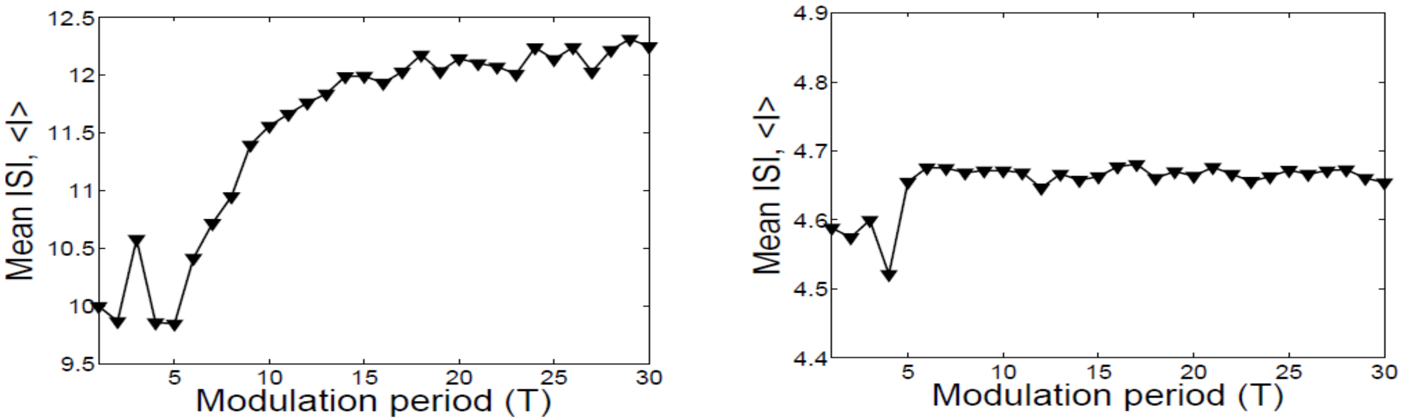
J. M. Aparicio-Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016).

# Role of the signal period



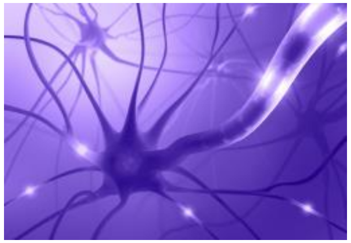

- More probable patterns depend on period and noise strength.

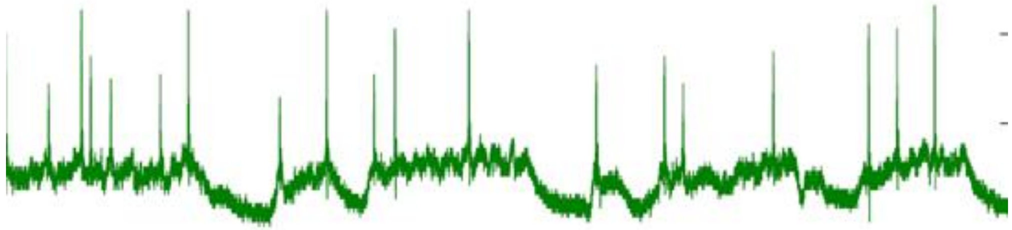
Which is the underlying mechanism? A change of the spike rate?



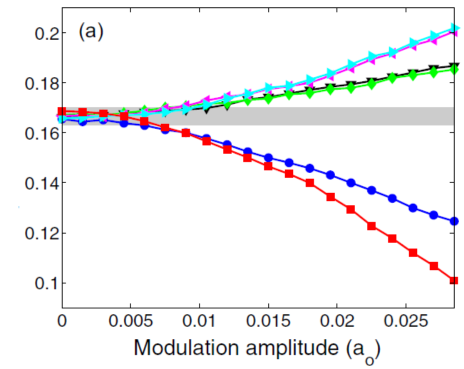

$\Rightarrow$ No direct relation.

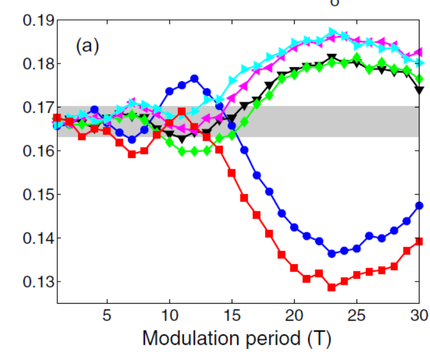
# So… how neurons might encode characteristic features of a weak external stimulus?

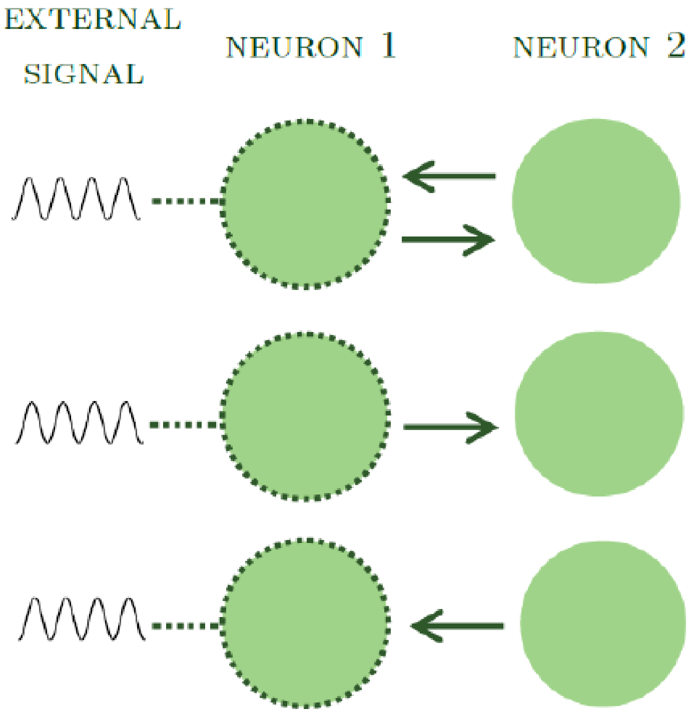




- Periodic signal: amplitude and period might be encoded in more and less expressed patterns.
- <u>Single-neuron encoding</u>: very **slow** because long spike sequences are needed to estimate the probabilities.
- <u>Ensemble encoding</u>: can be **fast** because few spikes are enough to compute the probabilities.






# Coupling to a second neuron

**- how does it affect signal encoding?**

## Model

$$\epsilon \dot{u}_1 = u_1 - \frac{u_1^3}{3} - v_1 + a_0 \cos(2\pi t/T) + \sigma_1 u_2 + \sqrt{2D}\xi_1(t)$$

$$\dot{v}_1 = u_1 + a,$$

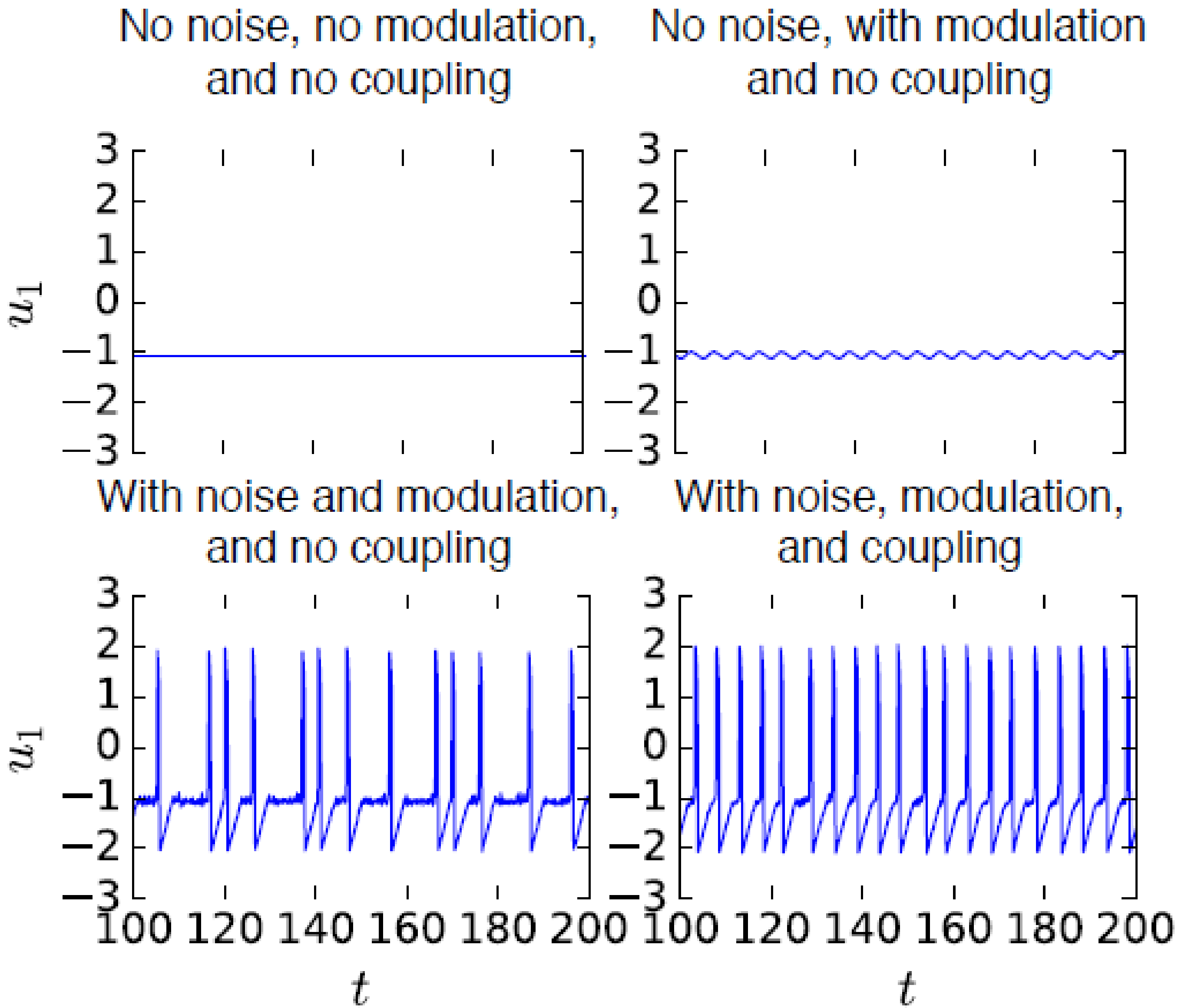
$$\epsilon \dot{u}_2 = u_2 - \frac{u_2^3}{3} - v_2 + \sigma_2 u_1 + \sqrt{2D}\xi_2(t)$$

$$\dot{v}_2 = u_2 + a$$

- Identical neurons.
- Linear & instantaneous & asymmetric coupling
- Signal, coupling and noise in the fast variable.
- a=1.05 and $\varepsilon$=0.01; parameters: $a_0$, T, D, $\sigma_1$, $\sigma_2$

M. Masoliver and C. Masoller, Scientific Reports 8, 8276 (2018).

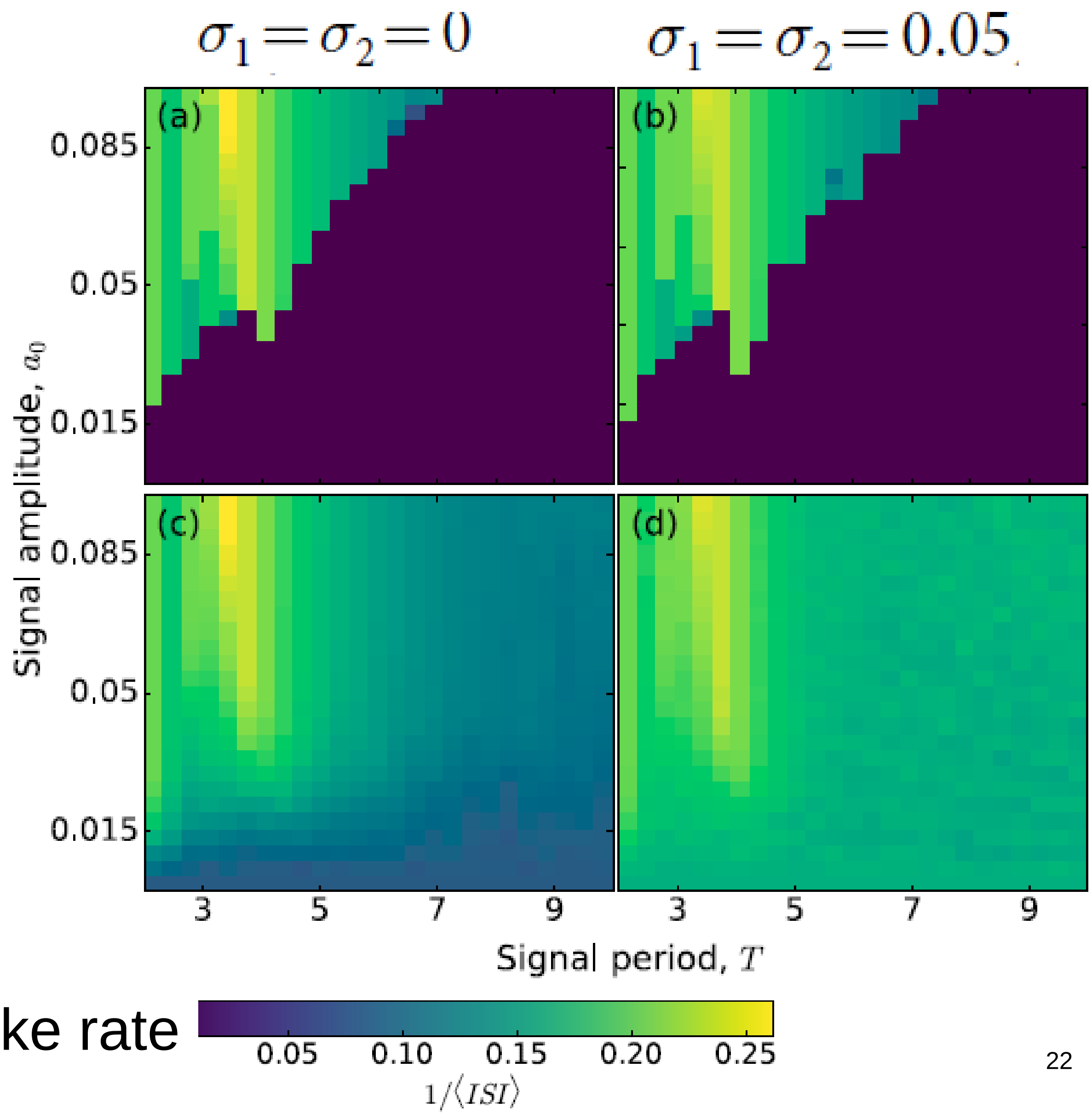
# We analyze the output of neuron 1



# Identification of the subthreshold region

**Without noise**: signal is subthreshold if $a_0$ small and/or T long

**With noise**: coupling increases the spike rate



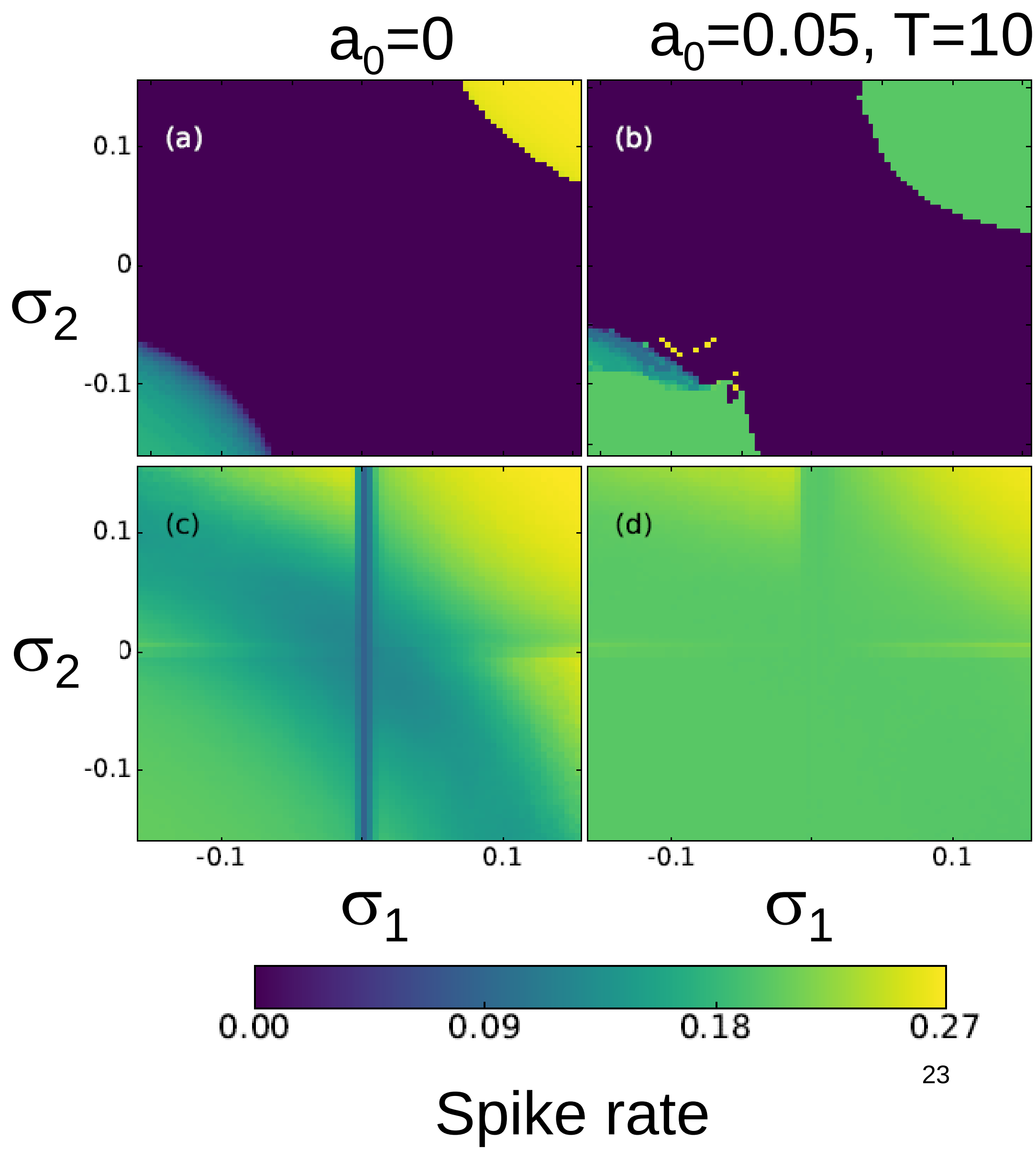

Spike rate

# Role of coupling coefficients

**Without noise**: large enough $|\sigma|$ induces spikes.

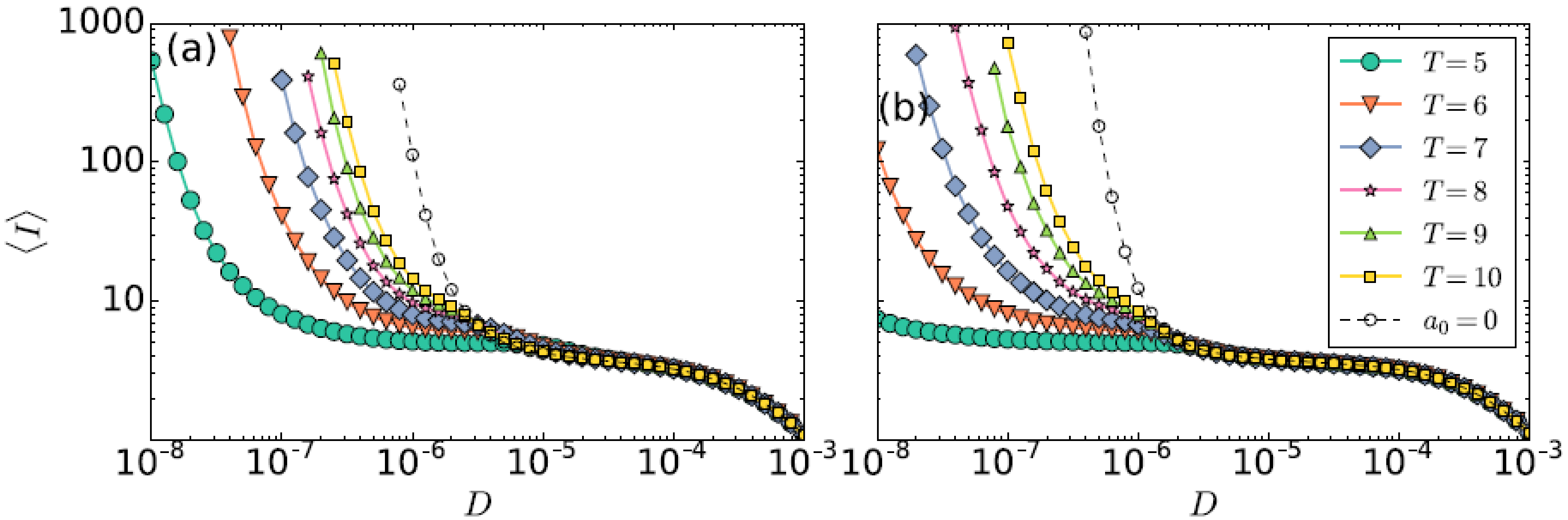
**With noise**: In the region of noise-induced spikes, the signal increases the spike rate.

If $\sigma_1=0$, the spike rate of neuron 1 does not depend of neuron 2



# Influence of noise and the signal period in the spike rate

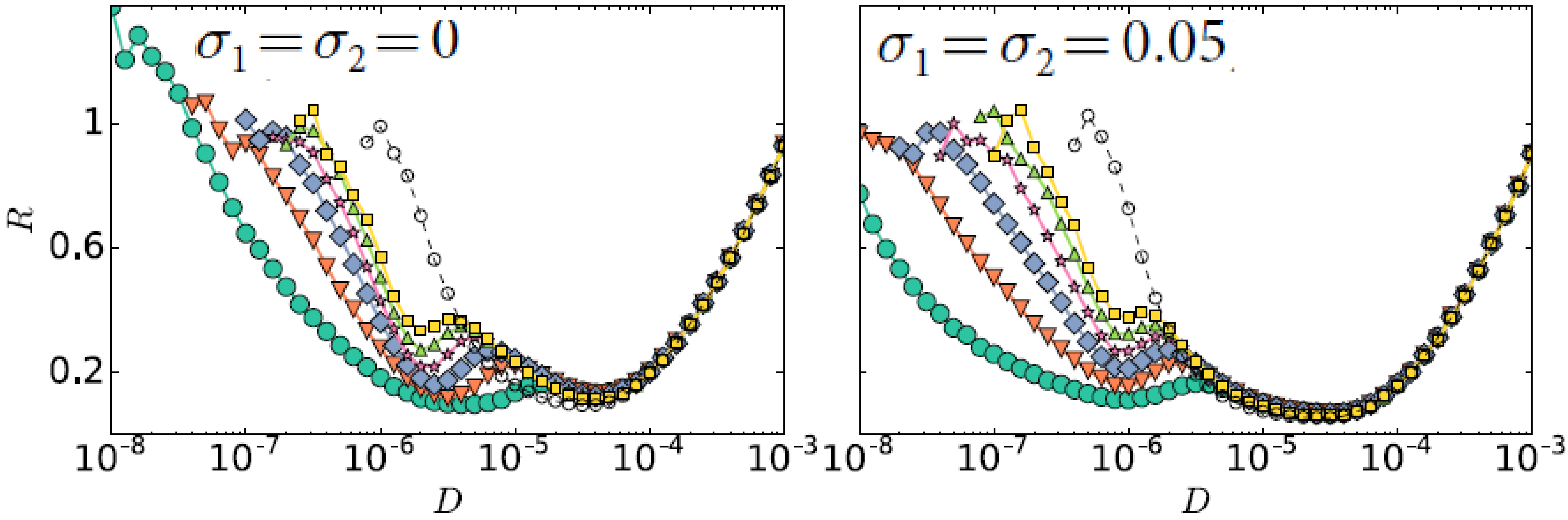
$\sigma_1 = \sigma_2 = 0$ $\sigma_1 = \sigma_2 = 0.05$




With coupling the neuron fires at a lower noise level ⇒ less noise is needed to encode the external signal.

# Coherence and stochastic resonances

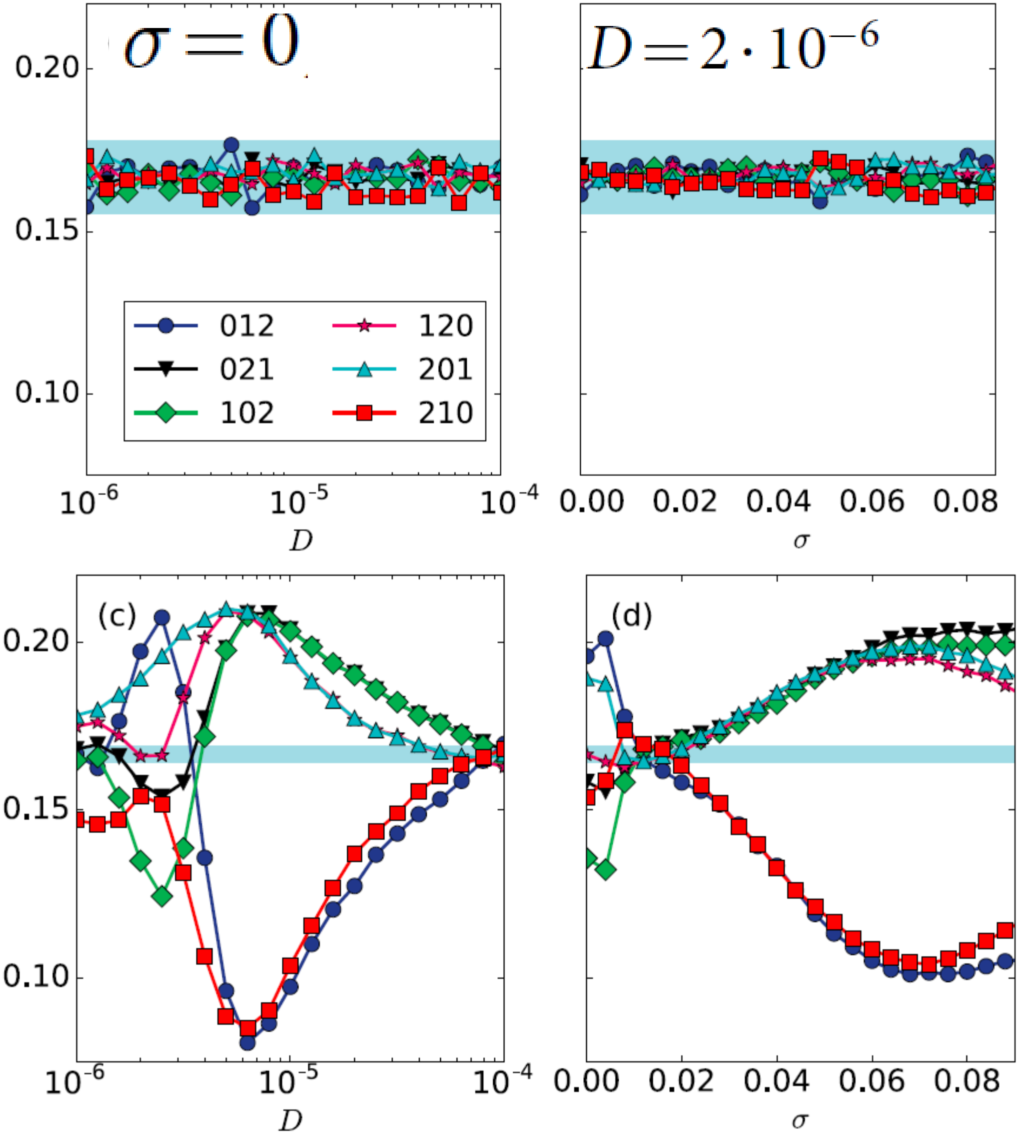
$$R = \frac{\sqrt{\langle I^2 \rangle - \langle I \rangle^2}}{\langle I \rangle}$$



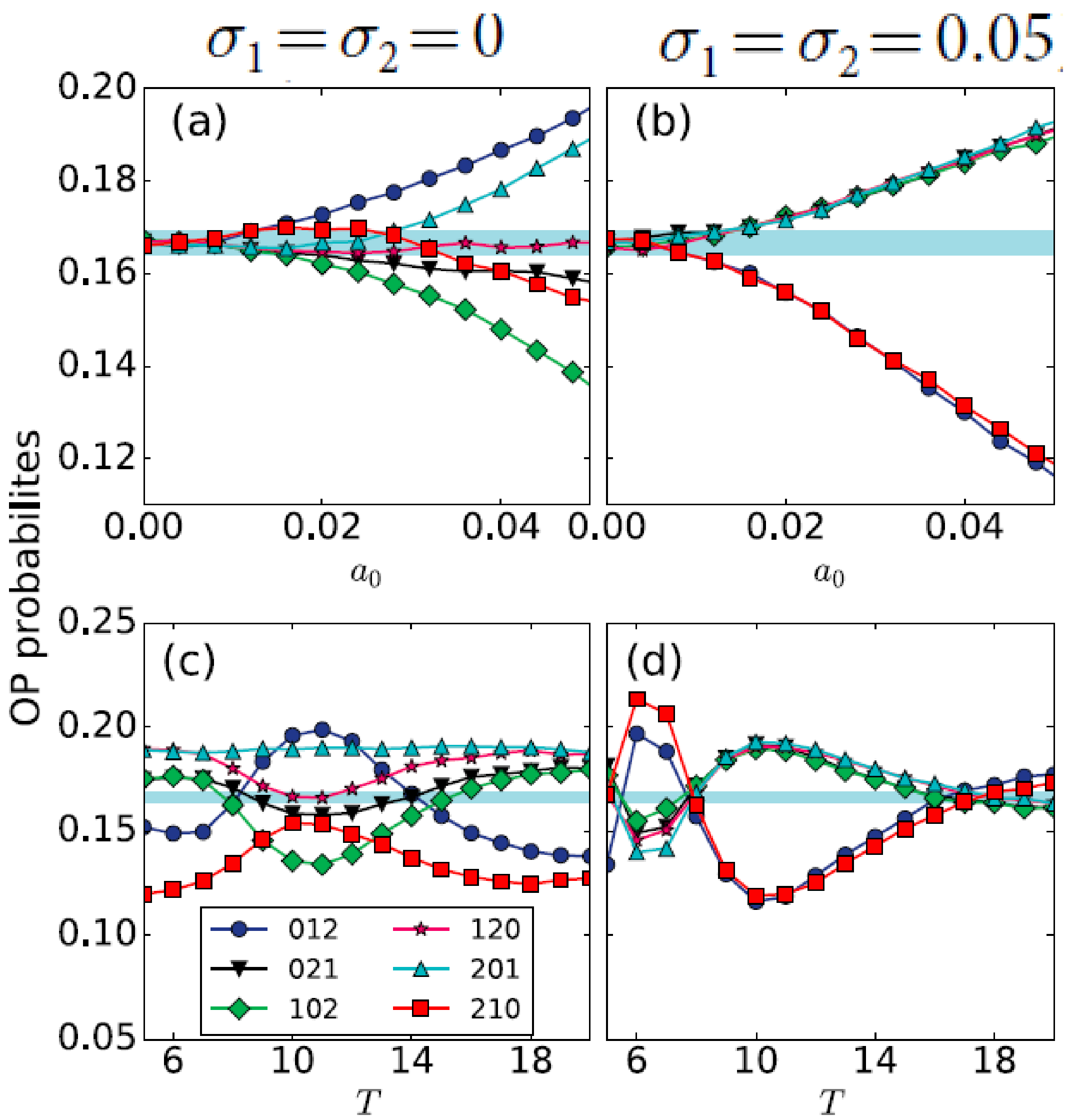
# The signal induces spike correlations

$a_0$=0

$a_0$=0.05
T=10



# Spike correlations depend on the signal period and amplitude



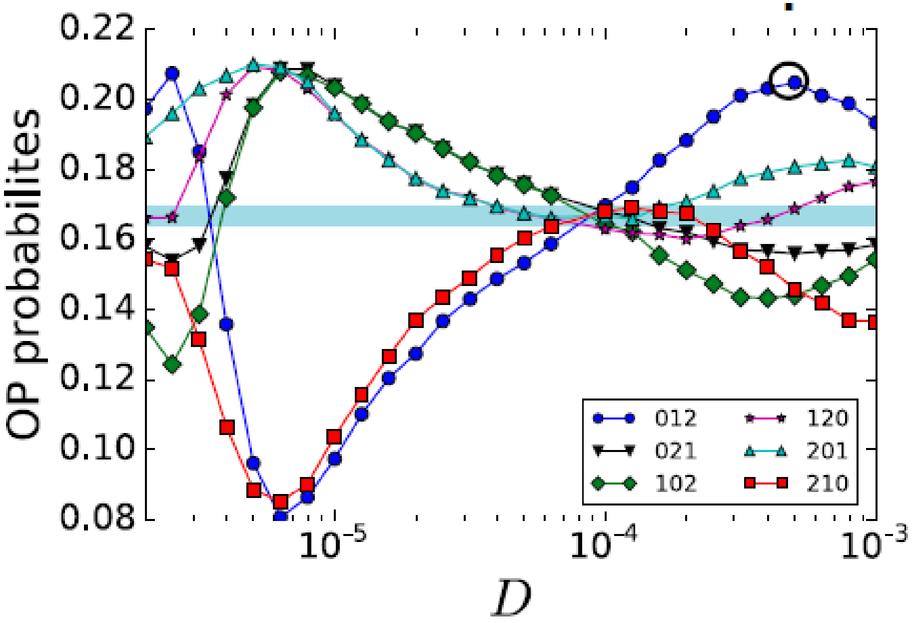

⇒ With coupling the signal is still encoded in the ordinal probabilities.

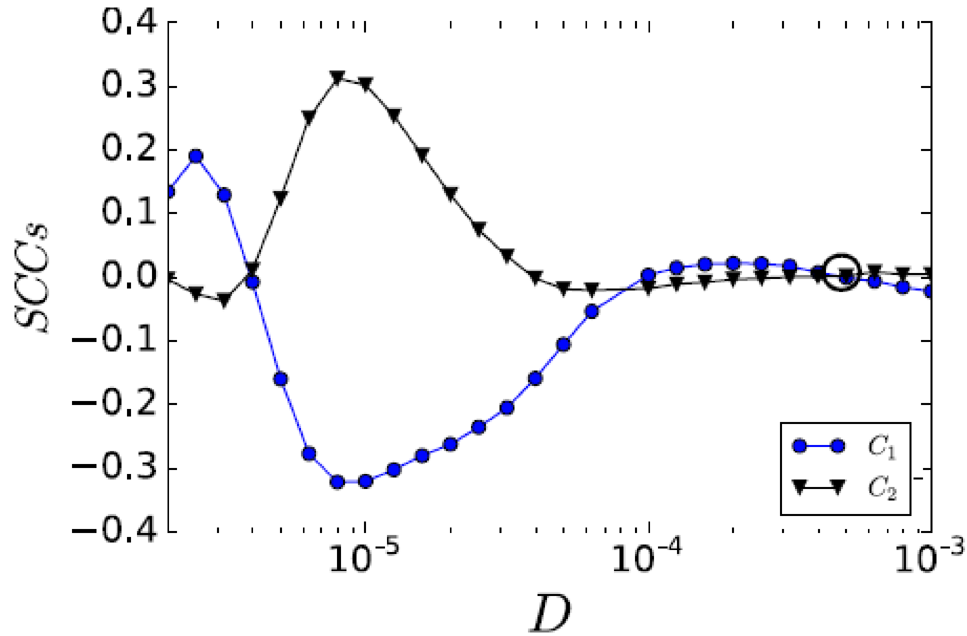
⇒ Coupling changes the preferred & infrequent patterns.

# Are spike correlations captured by linear correlation analysis?

$a_0 = \sigma = 0.05$, T=8

$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$




⇒ For strong noise, correlation coefficients at lag 1 and 2 vanish but ordinal analysis detects more / less expressed patterns.

# Conclusions

## What did we learn?

- Take home message:
  - Ordinal time-series analysis uncovers patterns in data.
  - It detects correlations that might not be captured by linear analysis.
- Main conclusions:
  - Neuron fires at lower noise level when coupled.
  - The ordinal probabilities carry information about the signal features (amplitude, period), with or without coupling.
  - Coupling changes the preferred/infrequent patterns.
- Ongoing work:
  - Similar results with other models and other types of coupling (collaboration with C. Estarellas and C.R. Mirasso)
- Future work:
  - Signal encoding by neuronal ensembles.

## THANK YOU FOR YOUR ATTENTION !

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http://www.fisica.edu.uy/~cris/

***Emergence of spike correlations in periodically forced excitable systems***

J. A. Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016).

***Subthreshold signal encoding in coupled FitzHugh-Nagumo neurons***

M. Masoliver and C. Masoller, Scientific Reports 8, 8276 (2018).