

Neural coding of subthreshold periodic signals in symbolic spike patterns

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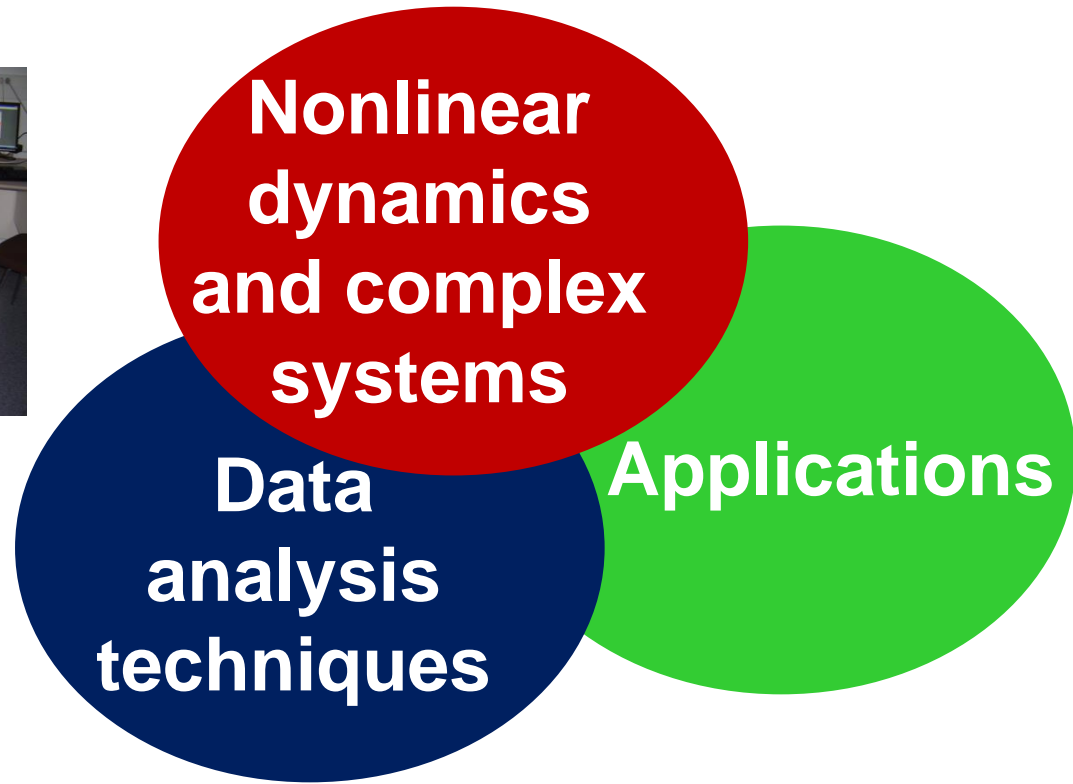
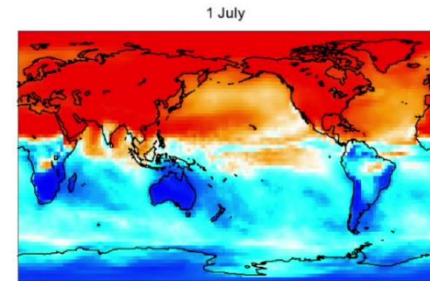
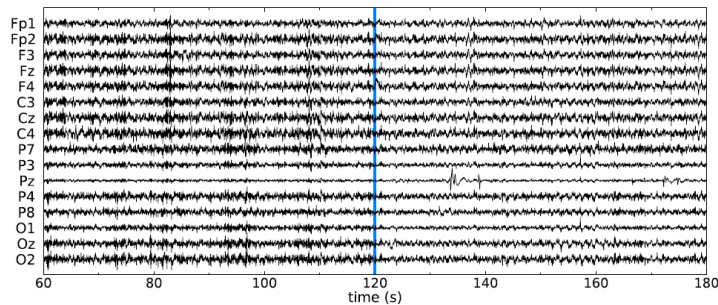
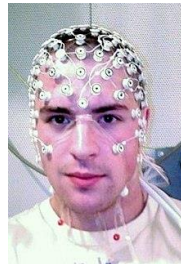
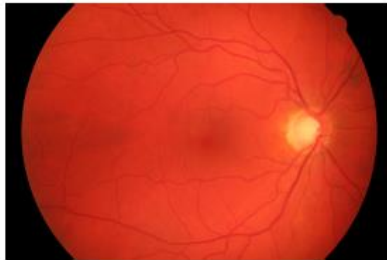
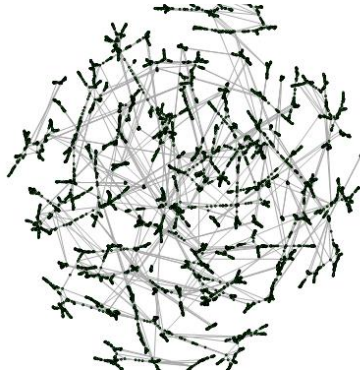
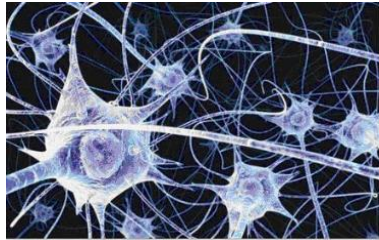
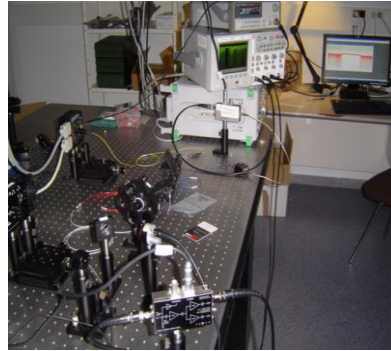
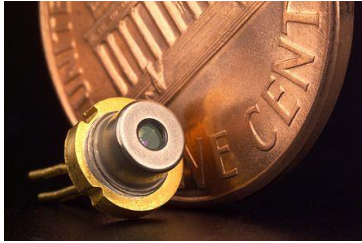
ICREA



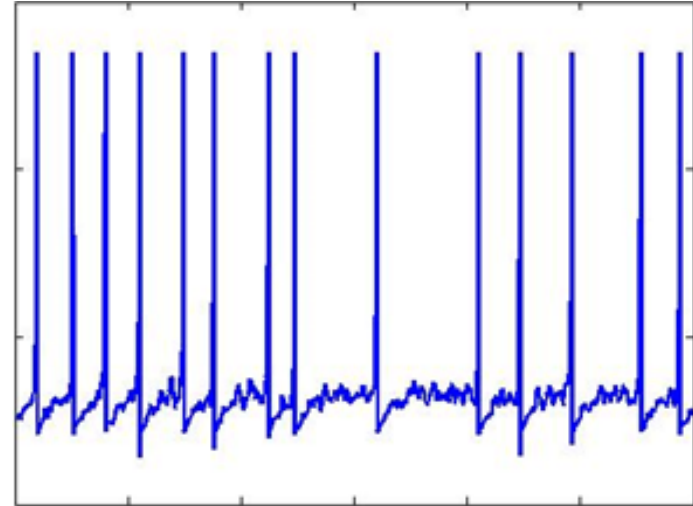
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DE ESPAÑA

MINISTERIO
DE CIENCIA
E INNOVACIÓN

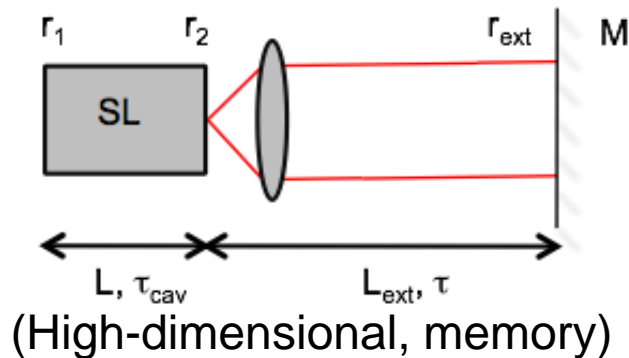
Research lines



The intensity of a semiconductor laser vs. simulated spikes



Time 10^{-9} s



Time 10^{-3} s

$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$

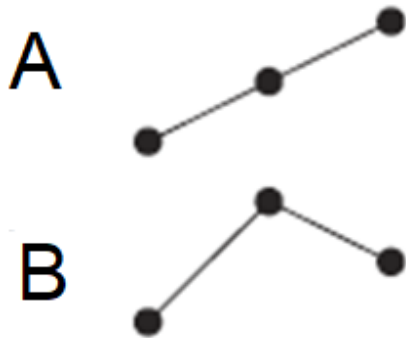
$$\frac{dy}{dt} = x + a + D\xi(t).$$

Symbolic analysis method: ordinal analysis

$$\{\dots X_i, X_{i+1}, X_{i+2}, \dots\}$$

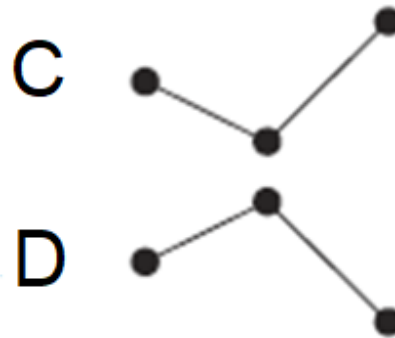
Possible order relations among three numbers (e.g., 2, 5, 7)

$$\{\dots 2, 5, 7 \dots\}$$



$$\{\dots 2, 7, 5 \dots\}$$

$$\{\dots 5, 2, 7 \dots\}$$



$$\{\dots 5, 7, 2 \dots\}$$

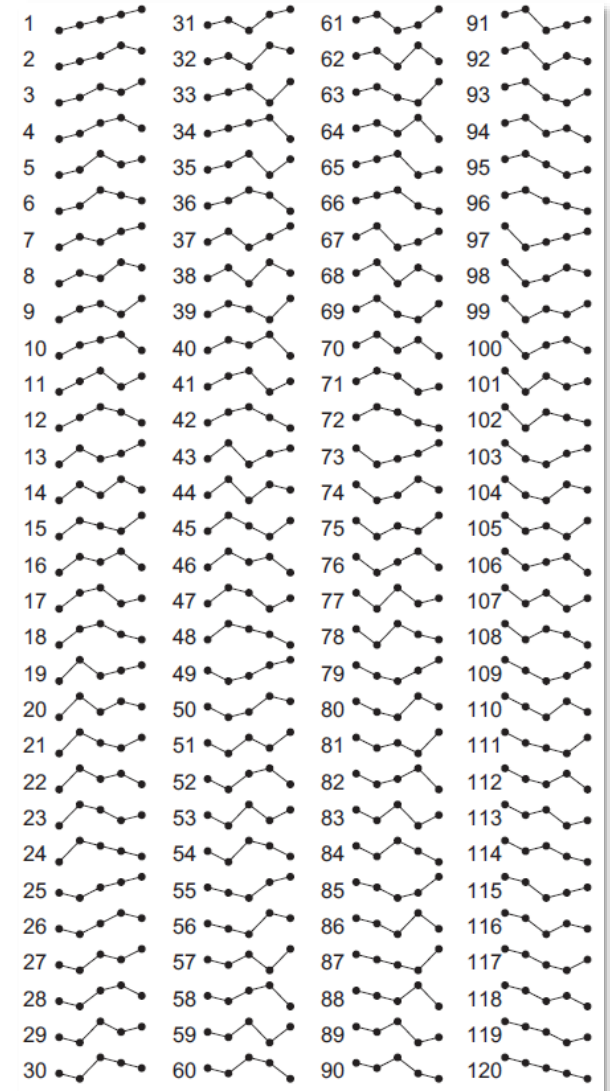
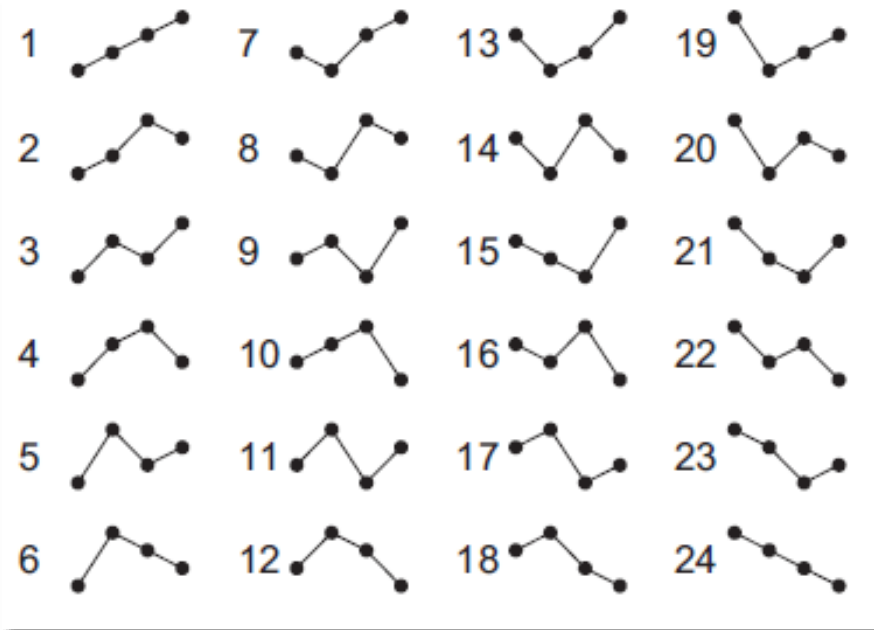
$$\{\dots 7, 2, 5 \dots\}$$



$$\{\dots 7, 5, 2 \dots\}$$

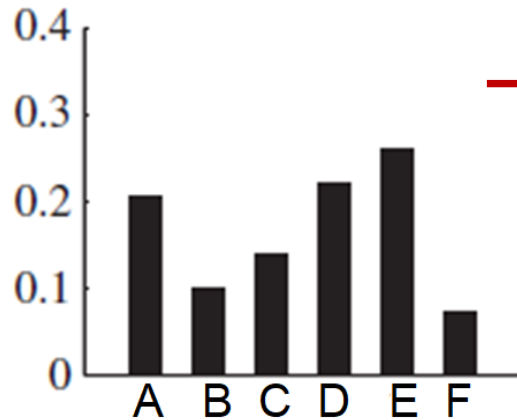
Bandt and Pompe: Phys. Rev. Lett. 2002

The number of ordinal patterns increases as D!



A problem for short datasets.

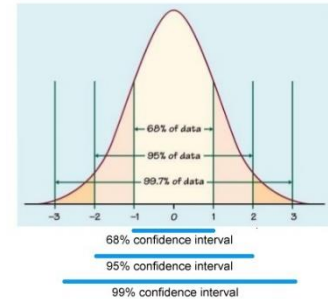
From a time series, by counting the different patterns, we can calculate the set of “ordinal probabilities”



?

- A. Analyze the probabilities (are differences statistically significant?)
- B. Compute information theory measures (entropy, complexity)

Probability from surrogates



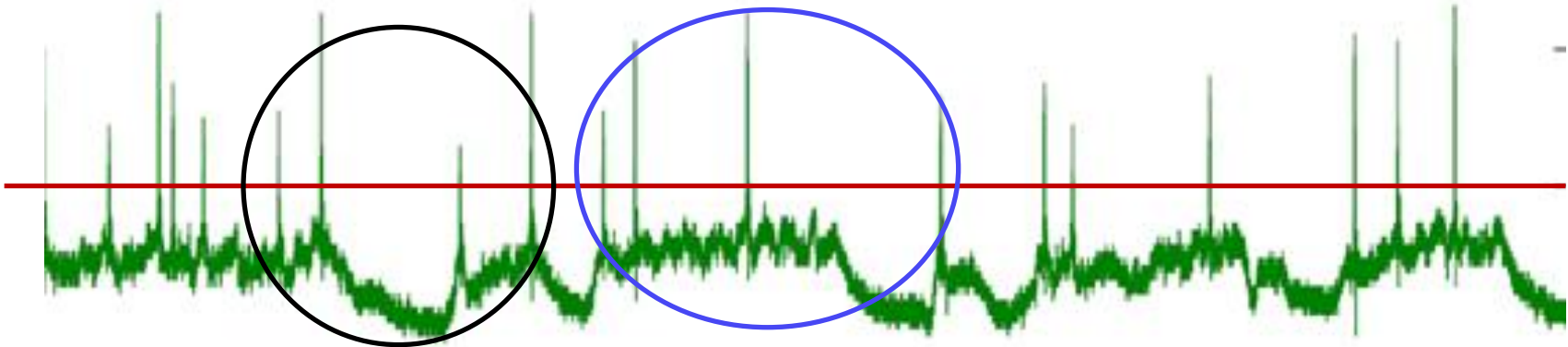
Ordinal analysis has been extensively used:

- to test if a model is good for the data,
- to fit the model's parameters,
- to classify different types of data based on similarities of probabilities of ordinal patterns.

I. Leyva, J. M. Martinez, C. Masoller, O. A. Rosso, M. Zanin, “20 Years of Ordinal Patterns: Perspectives and Challenges”, EPL 138, 31001 (2022).

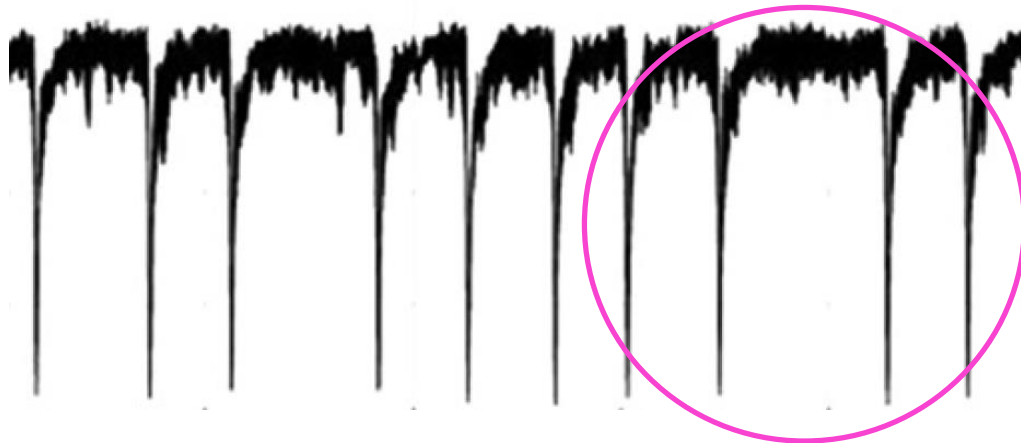
Sequence of inter-spike-intervals (ISIs) \Rightarrow sequence of ordinal patterns

D=3

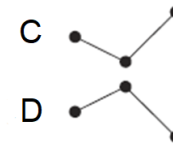
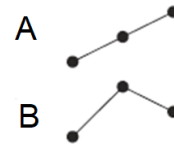


021=B

012=A

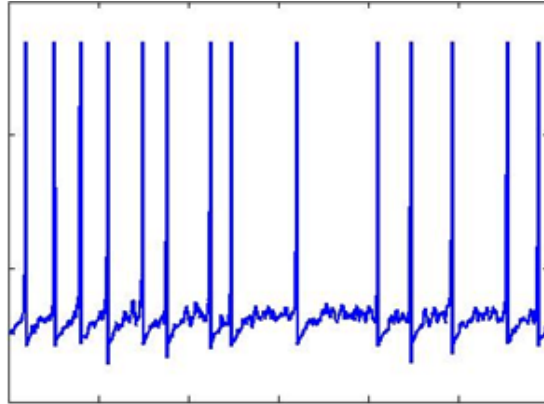


120=D

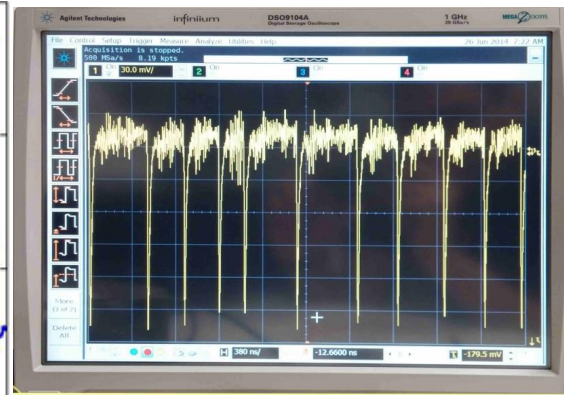


The analysis of the ordinal probabilities uncovers similarities in ISI sequences

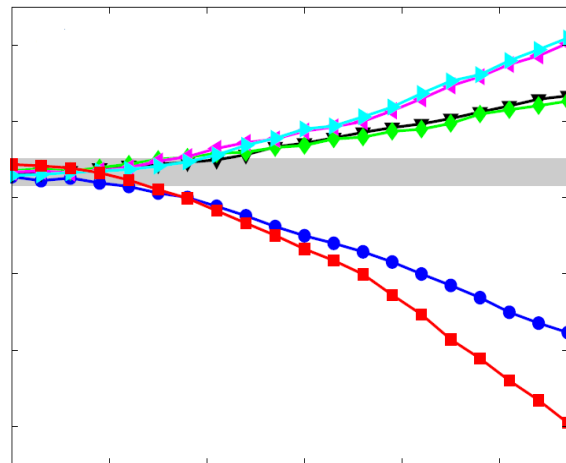
Neuron FHN model



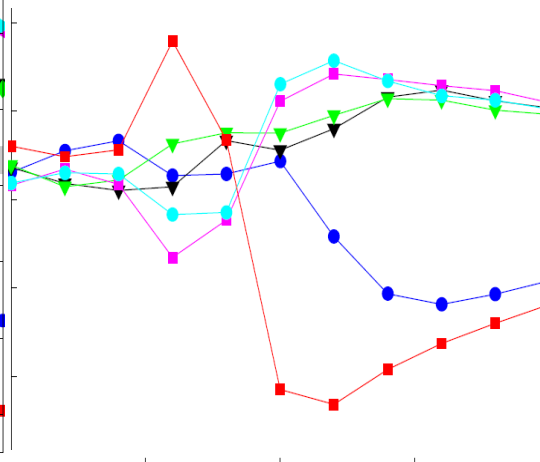
Semiconductor laser



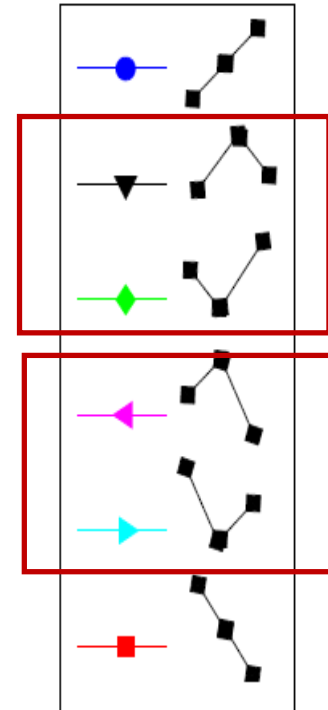
Ordinal probabilities



Forcing amplitude



Forcing amplitude



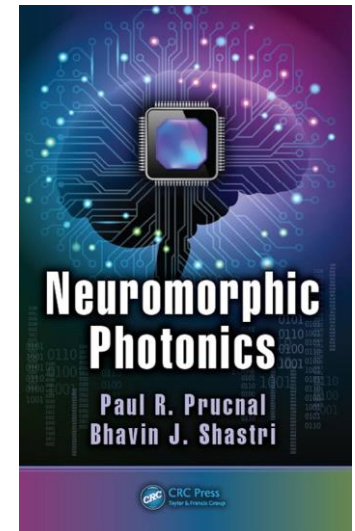
J. M. Aparicio-Reinoso et al PRE 94, 032218 (2016) A. Aragoneses et al, Sci. Rep. 4, 4696 (2014)

Uncovering similarities between neurons and lasers... Interesting but relevant?

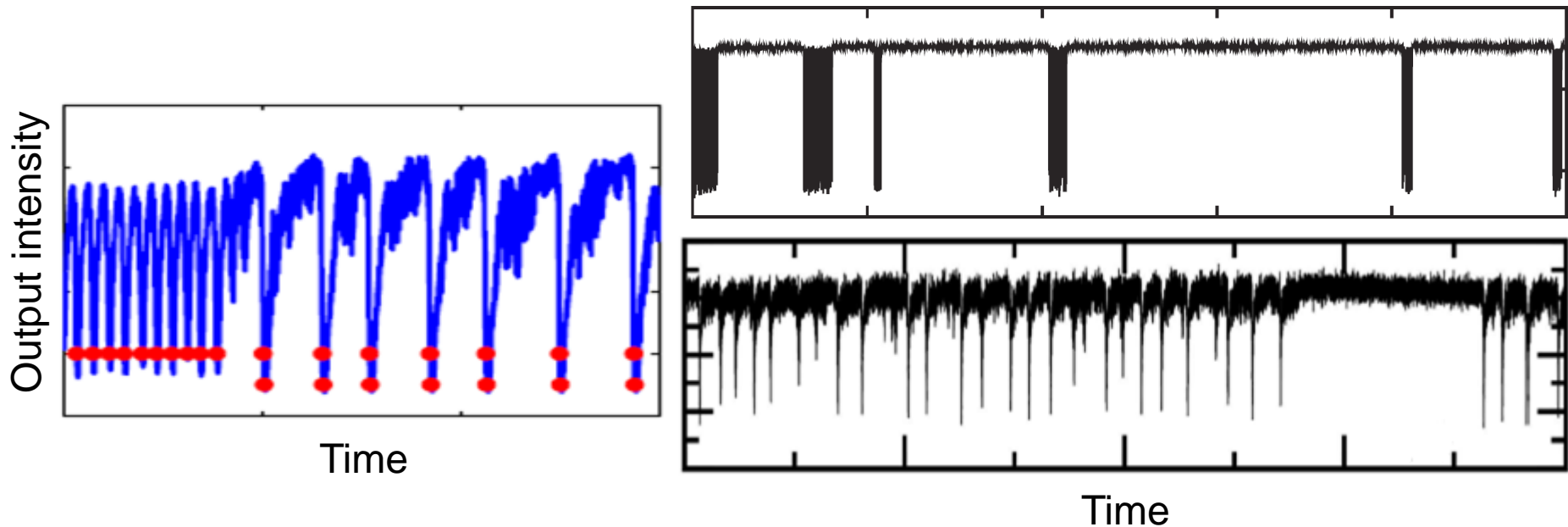
- Data centers, AI systems, HPC consume huge amounts of energy.
- Big concern in the context of climate change.
- The human brain processes huge amounts of information using only 19 Watts.
- Uncovering genuine similarities between neurons and lasers will allow to develop **photonic neurons**, able to process information as real neurons do, but
 - much faster,
 - with much less energy consumption.



European Centre for Medium-Range Weather Forecasts, Reading, UK



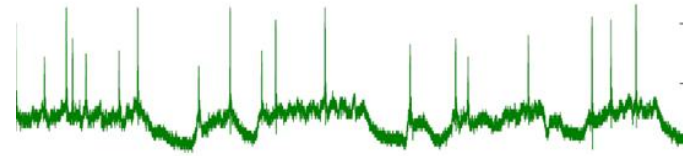
Time series recorded in our lab show excitability, tonic spikes, and bursting. Similar to real neurons?



A. Aragonese, S. Perrone, T. Sorrentino, M. C. Torrent and C. Masoller, "*Unveiling the complex organization of recurrent patterns in spiking dynamical systems*", Sci. Rep. **4**, 4696 (2014).

C. Quintero-Quiroz, J. Tiana-Alsina, J. Roma, M. C. Torrent, and C. Masoller, "*Characterizing how complex optical signals emerge from noisy intensity fluctuations*", Sci. Rep. **6** 37510 (2016).

Main challenge

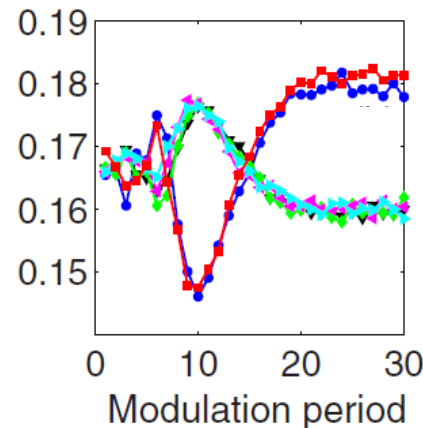
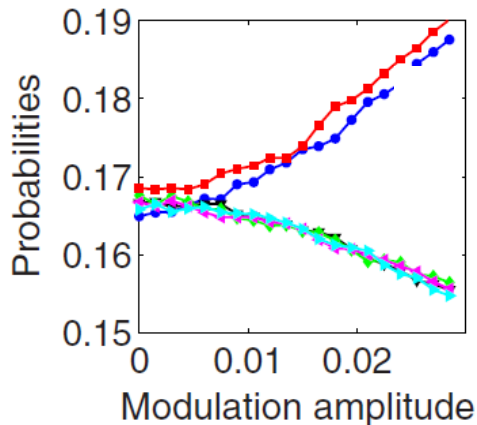


Understand how to mimic with lasers the way neurons encode and process information.

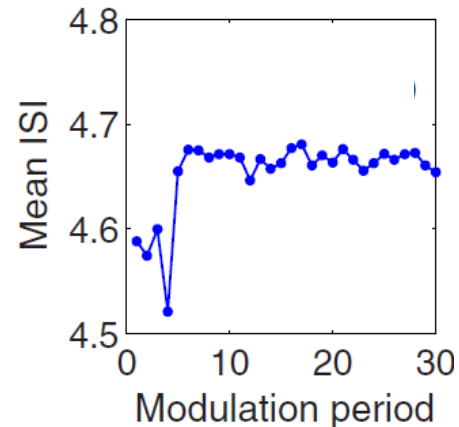
$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$

$$\frac{dy}{dt} = x + a + a_o \cos(2\pi t/T) + D\xi(t),$$

Weak, subthreshold signal



Rate coding?



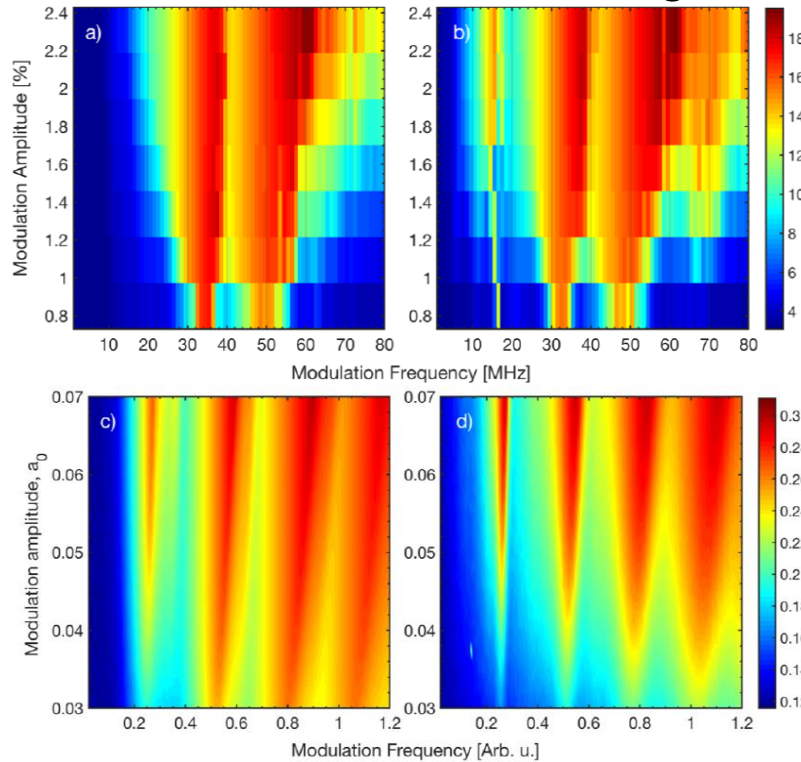
J. A. Reinoso, M. C. Torrent, and C. Masoller, “*Emergence of spike correlations in periodically forced excitable systems*”, Phys. Rev. E. 94, 032218 (2016).

Laser-neuron comparison: encoding a weak periodic signal using spike rate code

Spike rate in color code

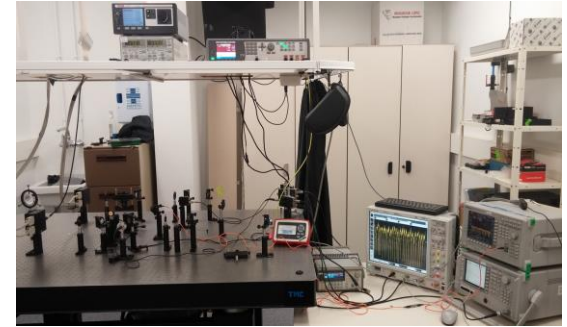
Sinusoidal

Pulsed signal



Experiments modulating the laser current

Neuron model with the same input signal



$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + D\xi(t).$$

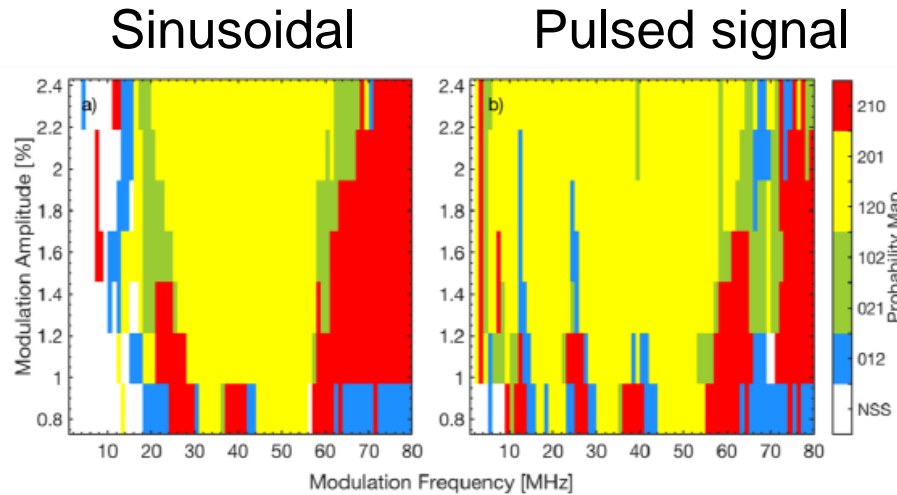
J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, “Comparing the dynamics of periodically forced lasers and neurons”, *New J. of Phys.* 21, 103039 (2019) (2019).

J. Tiana-Alsina, C. Masoller, “Time crystal dynamics in a weakly modulated stochastic time delayed system”, *Sci. Rep.* 12, 4914 (2022).

How about the temporal code?

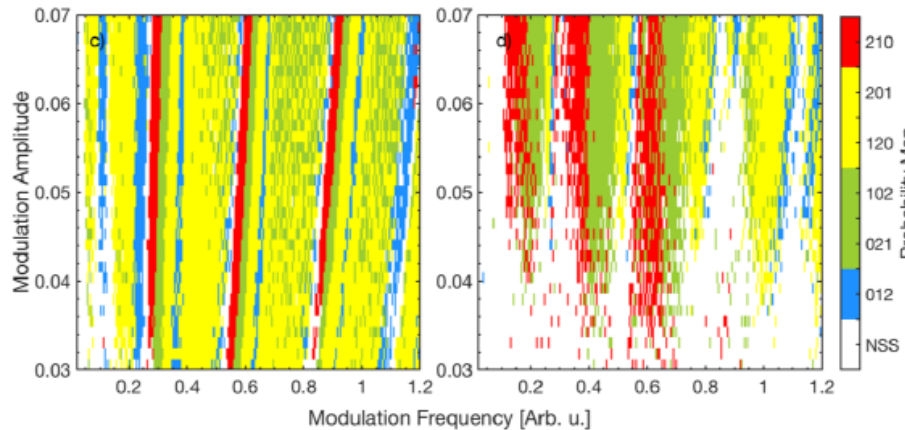
Ordinal analysis unveils differences in spike timing.

Diode
laser with
optical
feedback



**Most
probable
patterns
in color
code**

FitzHugh-
Nagumo
model



J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, New J. of Phys. 21, 103039 (2019).

Single-neuron vs ensemble encoding

- Single-neuron encoding: **slow** because long spike sequences are needed to estimate the ordinal probabilities.
- Ensemble encoding: can be **fast** because, from the ISI sequences of all the neurons, few spikes per neuron can be enough to accurately estimate the probabilities.

subthreshold external input

$$\epsilon \dot{u}_i = u_i - \frac{u_i^3}{3} - v_i + a_0 \cos(2\pi t/T) + \frac{\sigma}{k_i} \sum_j^N a_{ij} (u_j - u_i) + \sqrt{2D} \xi_i(t), \quad i \neq j$$

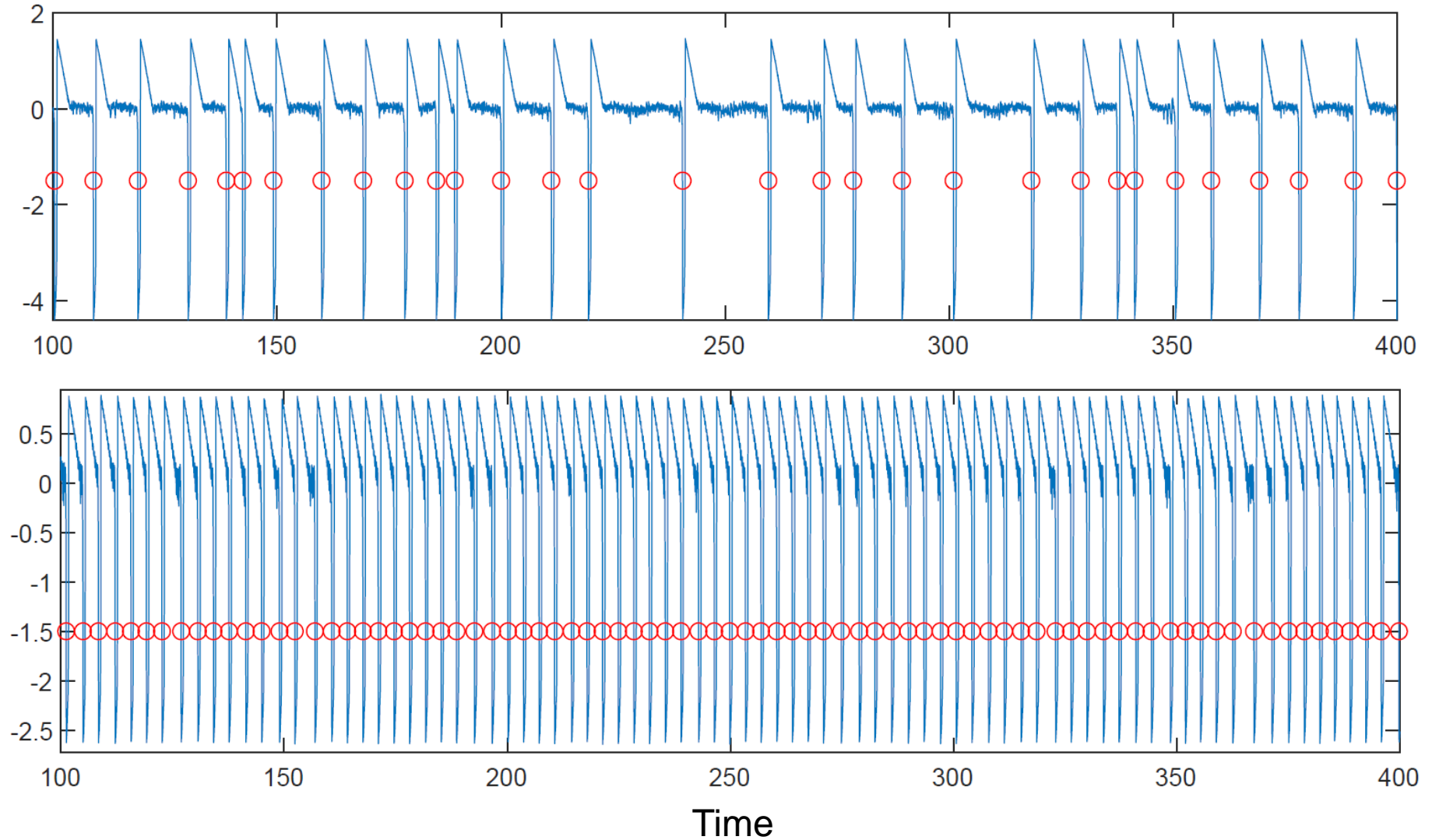
$$\dot{v}_i = u_i + a.$$

$k_i = \sum_j a_{ij}$

$a_{ij} = a_{ji} = 1$
 $a_{ij} = a_{ji} = 0$ Random with prob. **p**

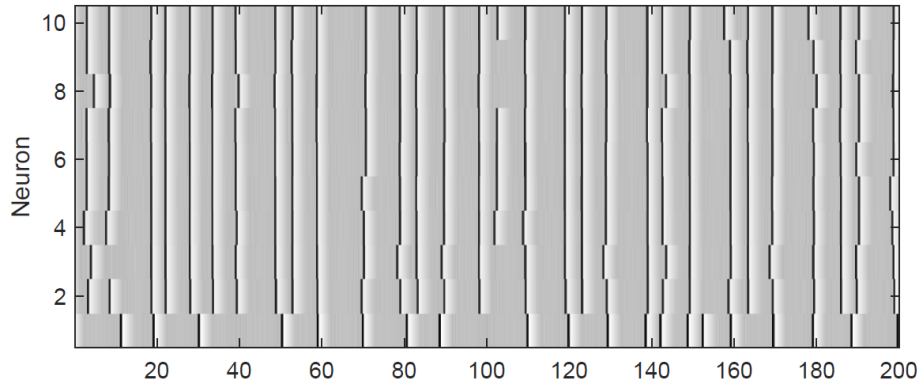
M. Masoliver and C. Masoller, “Neuronal coupling benefits the encoding of weak periodic signals in symbolic spike patterns”, Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).

Spike sequences for a low frequency or for a high frequency input

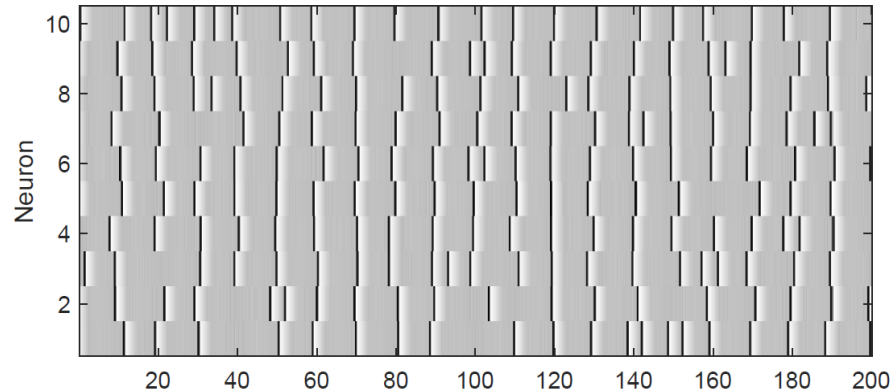


Spiking dynamics with/without coupling, with/without external input

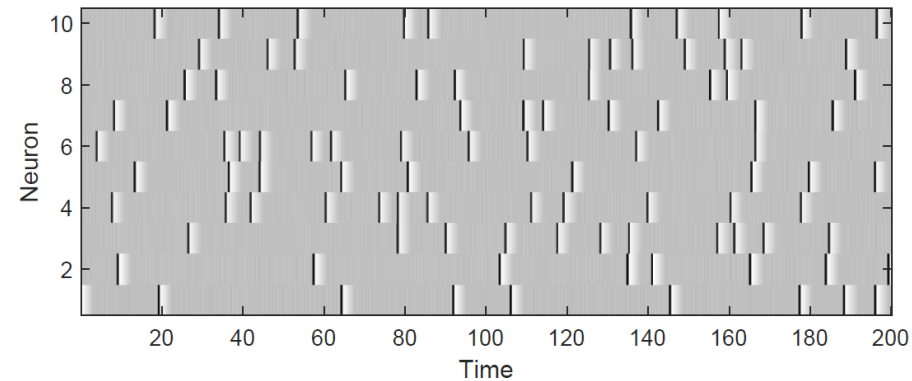
$\sigma \neq 0$
 $a \neq 0$



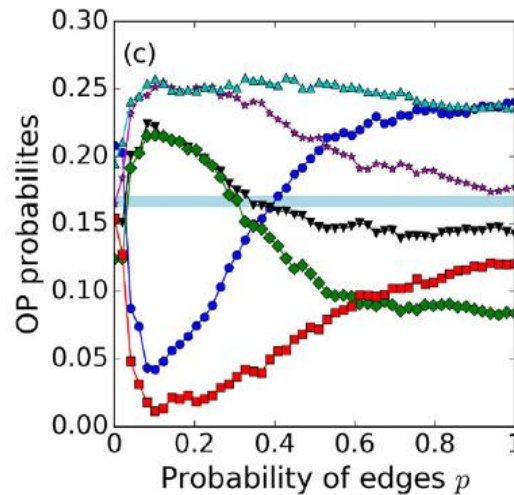
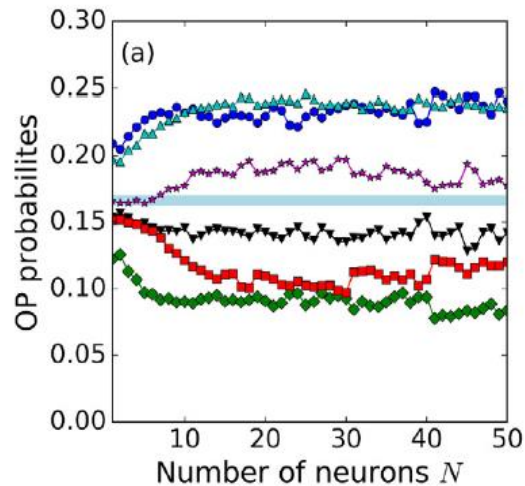
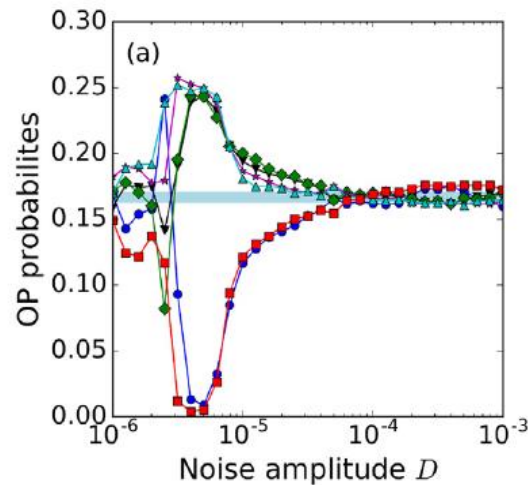
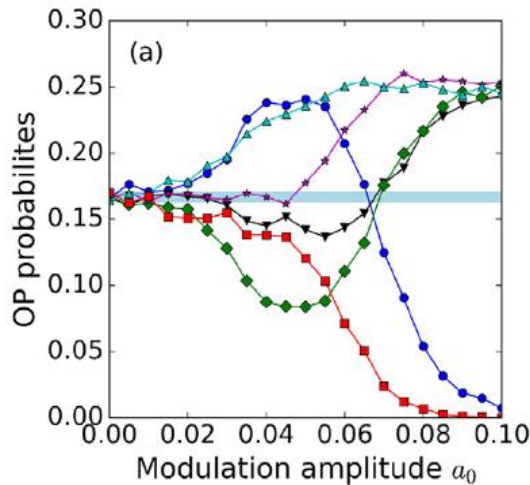
$\sigma = 0$
 $a \neq 0$



$\sigma = 0$
 $a = 0$



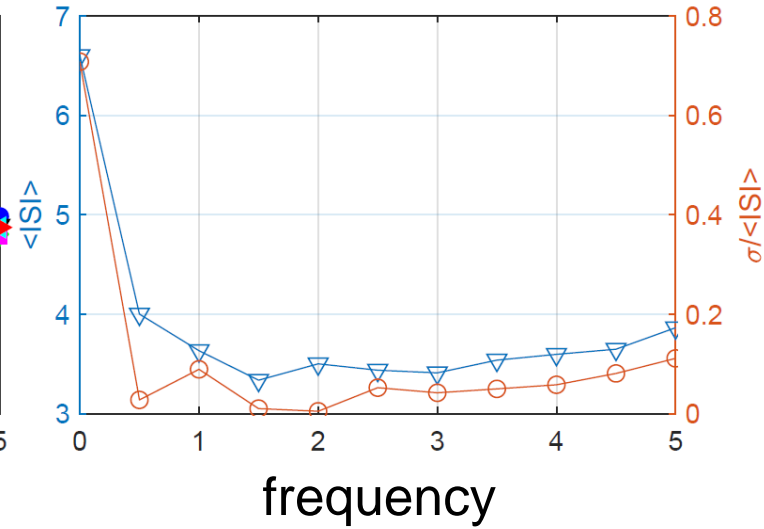
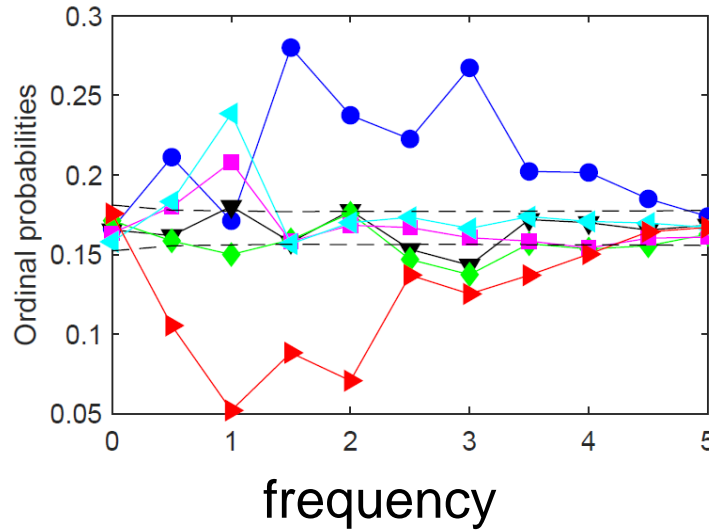
Ensemble encoding of a weak sinusoidal signal in the frequencies of occurrence of ordinal patterns



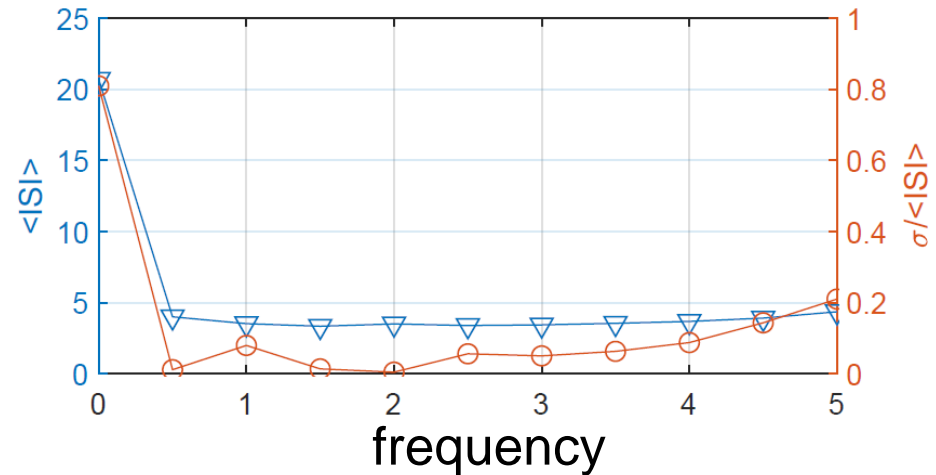
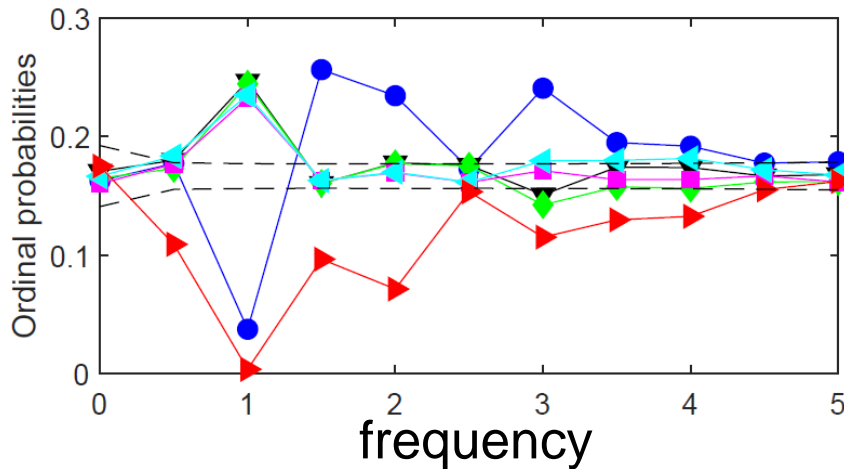
M. Masoliver and C. Masoller, Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).

Effects of the frequency of the external input and the coupling strength?

$N=10$
 $a=0.05$
 $D=6 \times 10^{-6}$
 $\rho=0.2, \sigma=0.01$



$\sigma=0$



Which is the role of the coupling topology?

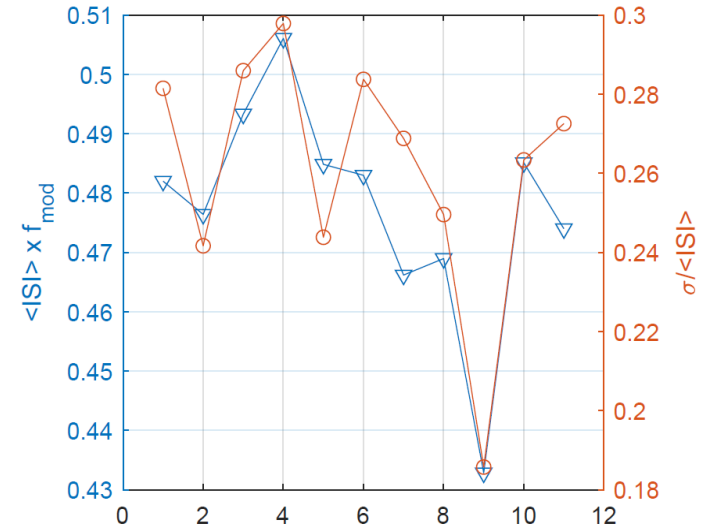
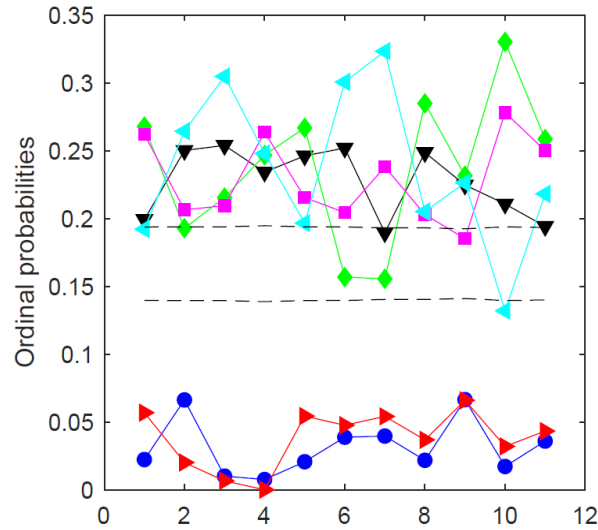
$N=100$, $a=0.05$

$f = 0.1$

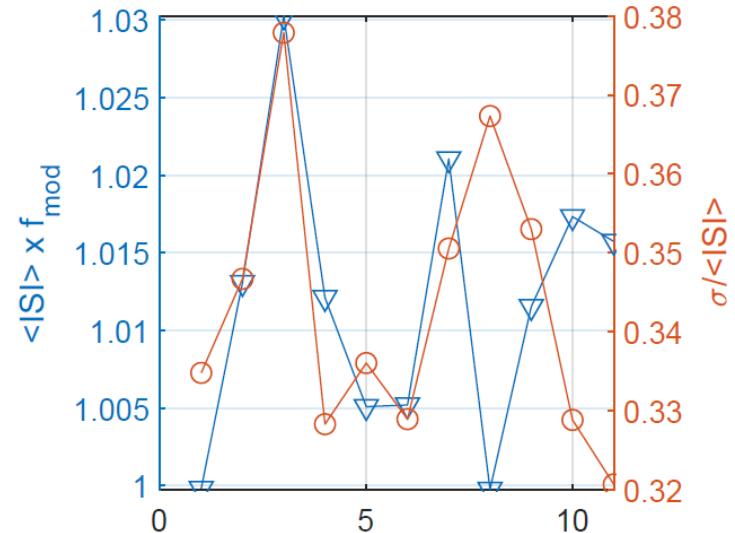
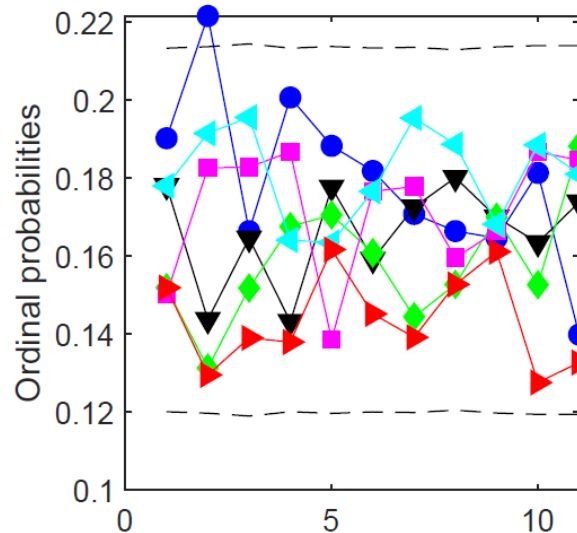
$D=6 \times 10^{-6}$

$\rho=0.2$, $\sigma=0.01$

11 random
matrices A_{ij}



$\sigma=0$



Take home messages and outlook

- A subthreshold sinusoidal signal can be encoded in the probabilities of occurrence of symbols in the spike trains.
- The signal should not be too slow or too fast.
- Neural noise should not be too weak or too strong.
- Key question: can we “decode” information from the analysis of the spike sequences? From the ordinal probabilities?

M. Masoliver and C. Masoller,
Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).

Thank you for your attention!