Symbolic patterns, clusters and hierarchies in spiking systems

Cristina Masoller
UPC, Barcelona, Spain

www.fisica.edu.uy/~cris

Dynamics Days Asia Pacific 08
Chennai, July 2014
Collaborators

- Andres Aragoneses
- Taciano Sorrentino
- Carme Torrent
Where are we?

1. Barcelona
2. Castelldefels
3. Igualada
4. Manresa
5. Mataró
6. Sant Cugat del Vallès
7. Terrassa
8. Vilanova i la Geltrú
- **Research goals:** Are optical spikes similar to neuronal spikes?

- **Potential for ultra-fast optical, brain-inspired information processing?**
Introduction
Semiconductor lasers with optical feedback: stochastic and high-dimensional dynamical systems

Method of symbolic time-series analysis and experimental setup

Results
• Inferring signatures of determinism
• Clusters in the symbolic dynamics
• Response to external periodic forcing

Conclusions and take home message
Semiconductor lasers

- **Used in:**
  - Fiber-optics communications
  - Optical data storage (CDs, DVDs …)
  - Barcode scanners, laser printers, computer mice
  - Biomedical applications (imaging, sensing …)
  - etc

- **Feedback induces nonlinear dynamics:**
  - Multi-stability
  - Regular pulses
  - Extreme pulses
  - Intermittency
  - Chaos
The **intensity dropouts** resemble neuronal **spikes**

This spiking dynamics is referred as low-frequency fluctuations
Why intensity dropouts?

- Complex interplay of:
  - Feedback delay time
  - Various noise sources
  - Nonlinear light-matter interactions

  ⇒ Stochastic and high-dimensional system

- Over years a lot of work has been devoted to understand the laser spiking behavior and to identify simple models.

To develop a method of time-series analysis for determining signatures of determinism;

To extract new information;

To compare model predictions with observations;

To find a minimal model;

To explore potential for building optical neurons.
Governing equations


\[ |E|^2 \sim \text{photon number (output intensity)} \]

\[ N \sim \text{number of carriers (electron-holes)} \]

\[
\frac{dE}{dt} = \frac{1}{2\tau_p} (1 + i\alpha)(G - 1)E + \eta E(t - \tau)e^{-i\omega_0\tau} + \sqrt{\beta_{sp}} \xi
\]

\[
\frac{dN}{dt} = \frac{1}{\tau_N} \left( \mu - N - G|E|^2 \right)
\]

Gain: \[ G = N / (1 + \varepsilon|E|^2) \]

feedback \hspace{1cm} noise

\[ \eta = \text{feedback strength} \]
\[ \tau = \text{feedback delay time} \]
\[ \mu = \text{pump current} \]

(control parameter)
Model predictions

- In **deterministic** simulations: the spikes (dropouts) are **transient**.

- But in **stochastic** model simulations: **bursts** of dropouts.

- In the experiments: which dropouts are triggered by **noise** and which ones are **deterministic**?

- **Information** in the spike sequence?

Problems and strategy

- Main problem: we can measure only one variable (the laser intensity)
- Also a problem: the detection system (photodiode, oscilloscope) has a finite *bandwidth* that gives limited temporal resolution.
- Our strategy: we analyze the sequence of *inter-dropout-intervals* (IDIs):

\[ \Delta T_i = t_{i+1} - t_i \]
Examples of sequences of events:

- Intervals between threshold crossings, barrier crossings,
- Neurons: inter-spike intervals (ISIs),
- Human communication: inter-event user times (SMS, emails, Twitters).
- Earth and climate: intervals between earthquakes, extreme events (tornados, rainfalls) etc.

The identification of patterns in the sequence of events allows for:

- Model verification
- Parameter estimation
- Classification of different types of dynamical behaviors
- Predictability - forecasting
Outline

- Introduction
- Method of symbolic time-series analysis and experimental setup
- Results
- Conclusions and take home message
Symbolic Ordinal Analysis

- Has been widely used to analyze data generated from complex systems
  - Financial, economical
  - Biological, life sciences
  - Geosciences, climate
  - Physics, chemistry, etc

- Able to:
  - Distinguish stochasticity and determinism
  - Classify different types of dynamical behaviors (pathological, healthy)
  - Quantify complexity
  - Identify coupling and directionality.

“Words” of D letters can be formed by considering the order relation between sets of D values \{...x_i, x_{i+1}, x_{i+2}, ...\}.

For D=3 there are 6 possible words

Example: the set (5, 1, 7) gives “102” because 1 < 5 < 7

Advantage: the transformation keeps information about correlations in the time-series.

Drawback: the set (5,1,100) also gives word “102”.

12/09/2014

C. Masoller
The probabilities of the words unveil correlations between events.

How to select D? Optimal D depends on:
- The length of the time series.
- The time scale of correlations.

For optical spikes: D=2 (D=3) reveal correlations among 3 (4) spikes.
Classifying cardiac biosignals using ordinal pattern statistics

congestive heart failure (CHF) vs healthy subjects.

Outline

- Introduction
- Method of time-series analysis and experimental setup
- Results
- Conclusions and take home message
Hitachi Laser Diode (HL6724MG)

- \( \lambda \sim 674.2 \text{ nm} \)
- 5mW
- \( \sim 7\% \) threshold reduction
Outline

- Introduction
- Method of time-series analysis and experimental setup
- Results
  - Inferring signatures of determinism
  - Clusters in the symbolic dynamics
  - Response to external periodic forcing
- Conclusions and take home message
Correlations between 3 consecutive spikes

Consistent with stochastic at low pump current, but signatures of determinism at high pump current.

**D=2**: 3-spike correlations?

**Null hypothesis**: fully random sequence of spikes $\Rightarrow P(01) = P(10)$

At low pump current: are the spikes fully random? New experiment

A. Aragoneses et al, Scientific Reports 3, 1778 (2013)
Also in another data set recorded at a different temperature (T=20 C)

Pump current (mA)

D=2

D=3

Pump current

012

210

12/09/2014  C. Masoller
Are the deviations from the null hypothesis significant?

Recorded data

![Chart showing recorded data with error bars and probability distribution against pump current.]

Surrogated data

![Chart showing surrogated data with error bars and probability distribution against pump current.]

Error bars computed with a binomial test, gray region is consistent with N.H.

12/09/2014

C. Masoller
Which dropouts are triggered by noise?

Histogram of Inter-dropout-intervals (IDIs)

We use a **threshold** to classify the inter-dropout-intervals as **short** and **long** intervals.
Constructing the words with 2 consecutive SIs or LIs

- At high currents: significant differences
  - LIs consistent with random events
  - SIs more deterministic.

- But at low currents, the spikes can not be classified in two types with significant differences.
— At high currents: significant differences

— But at low currents, the spikes can not be classified.

Similar results were obtained in the other dataset (T=20 C)
Influence of the threshold used to classify IDIs as LIs and SIs

$\Delta T^* = \text{most probable value}$

Error bars computed with a binomial test, gray region consistent with NH
### Tips to chose a good threshold

- **LIs** have statistical features as close as possible to random events:
  - Exponential distribution of values
  - Uniform distribution of word probabilities

- **Good statistics**: there are enough consecutive LIs and SIs
  - The NH region is sufficiently narrow
  - For the LIs, the error bars are in the NH region
  - For the SIs, the error bars are out of the NH region.
Introduction

Method of time-series analysis and experimental setup

Results

• Inferring signatures of determinism
• Clusters in the symbolic dynamics
• Response to external periodic forcing

Conclusions and take home message
Ordinal analysis unveils new information

There is a hierarchical and clustered organization of the probabilities of the words
In another experiment: also the same hierarchy and the same 2 clusters

75,000 – 880,000 spikes
(different laser, new oscilloscope)
Can we find a minimal model that displays these features?
A modified circle map: minimal phenomenological model

\[ \phi_{i+1} = \phi_i + \rho + \frac{K}{2\pi} \left[ \sin(2\pi \phi_i) + \alpha \sin(4\pi \phi_i) \right] \]

\[ X_i = \phi_{i+1} - \phi_i \]

Outline

- Introduction
- Method of time-series analysis and experimental setup
- Results
  - Inferring signatures of determinism
  - Clusters in the symbolic dynamics
  - Response to external periodic forcing
- Conclusions and take home message
Response to external periodic modulation

Laser intensity:

Increasing modulation amplitude

Tendency to lock

Two sets of experiments:

$\lambda=660\ \text{nm}$

$\lambda=1550\ \text{nm}$

Relevant for understanding neuronal encoding of external stimuli
Similar observations @ 1550 nm
Interpretation: locking to external forcing
- Introduction
- Method of time-series analysis and experimental setup
- Results
- Conclusions and take home message
Novel method for identifying signatures of determinism in complex time series.

Spikes were classified in two categories: one stochastic; the other displaying signatures of determinism.

We found new symbolic states with an hierarchical and clustered organization.

We identified a minimal model. Robust under external forcing.

Potential breakthrough: optical neurons for neuro-inspired information processing.

Present work: towards understanding why the modified circle map is a good minimal model.
Ordinal analysis is a powerful method of symbolic time-series analysis

— useful for understanding data, uncovering patterns,
— for improving system modeling, model comparison, parameter estimation,
— for classifying data,
— for improving predictability and forecasting.
You for your attention!

Andres Aragoneses

Taciano Sorrentino

Carme Torrent

Papers @ www.fisica.edu.uy/~cris
