

# Symbolic patterns, clusters and hierarchies in spiking systems

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*Campus d'Excel·lència Internacional*

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Chennai, July 2014





- Andres Aragonese



- Taciano Sorrentino



- Carme Torrent

# Where are we?

1. Barcelona
2. Castelldefels
3. Igualada
4. Manresa
5. Mataró
6. Sant Cugat del Vallès
7. Terrassa
8. Vilanova i la Geltrú

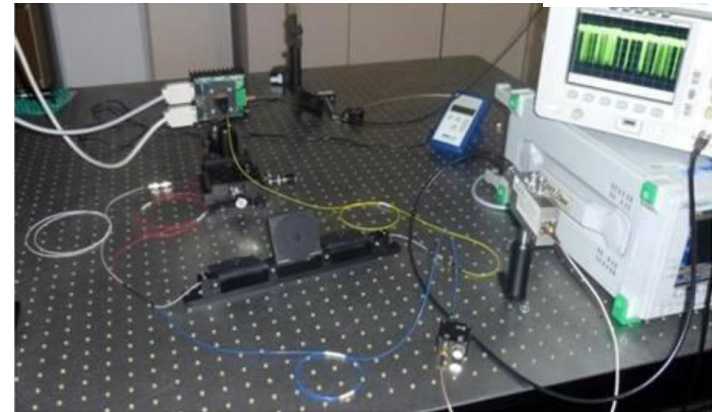
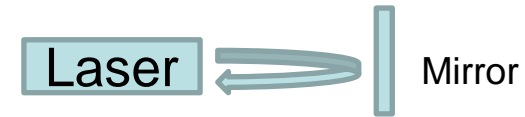
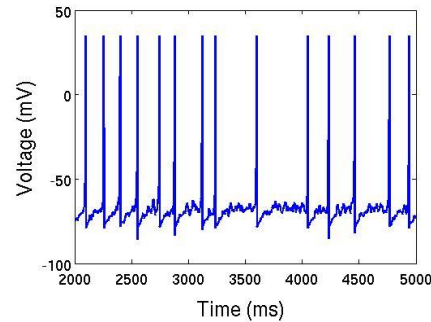
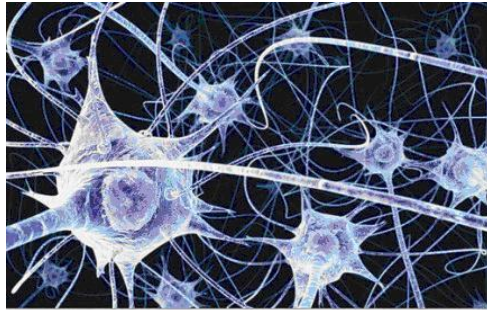


Viernes, 25 de septiembre de 2009. *Diari de Terrassa*

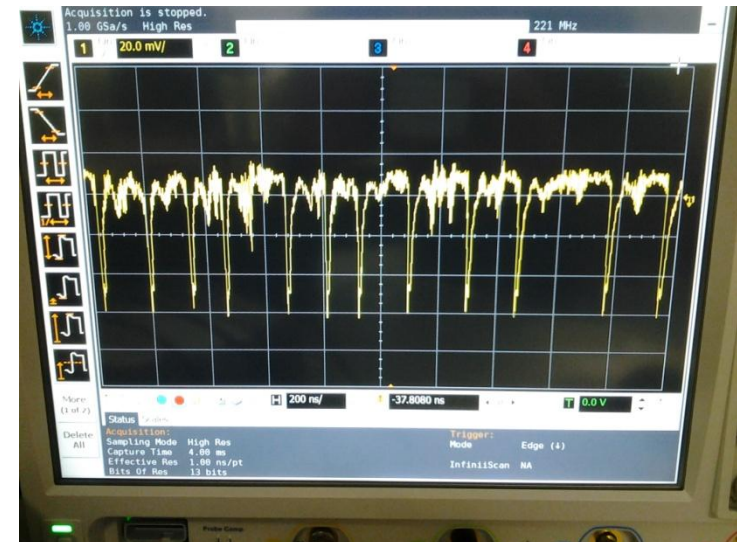


El edifici Gala centraliza grups científics consolidats i emergents.

# Research lab: nonlinear dynamics of semiconductor lasers



- Research goals: are optical spikes similar to neuronal spikes?
- Potential for ultra-fast optical, brain-inspired information processing?



## ■ Introduction

Semiconductor lasers with optical feedback: stochastic and high-dimensional dynamical systems

## ■ Method of symbolic time-series analysis and experimental setup

## ■ Results

- Inferring signatures of determinism
- Clusters in the symbolic dynamics
- Response to external periodic forcing

## ■ Conclusions and take home message



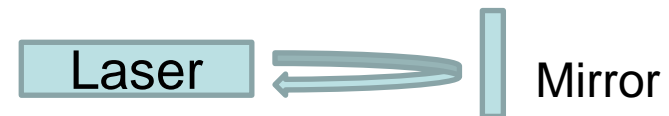
## ■ Used in:

- Fiber-optics communications
- Optical data storage (CDs, DVDs ...)
- Barcode scanners, laser printers, computer mice
- Biomedical applications (imaging, sensing ...)
- etc

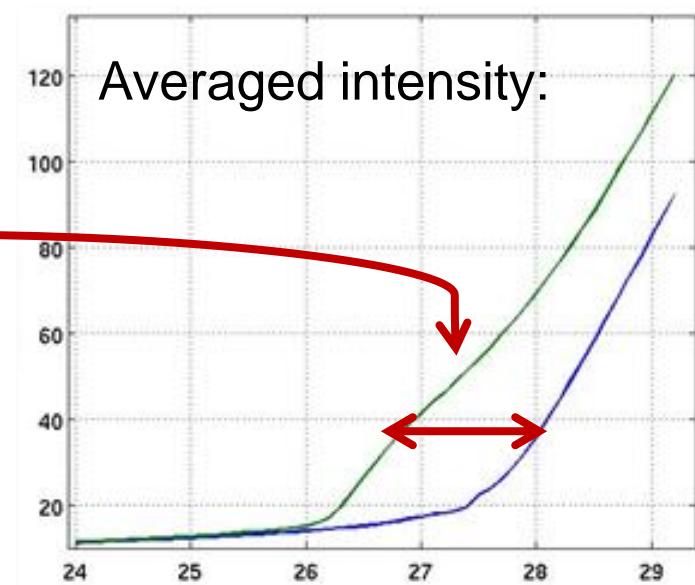
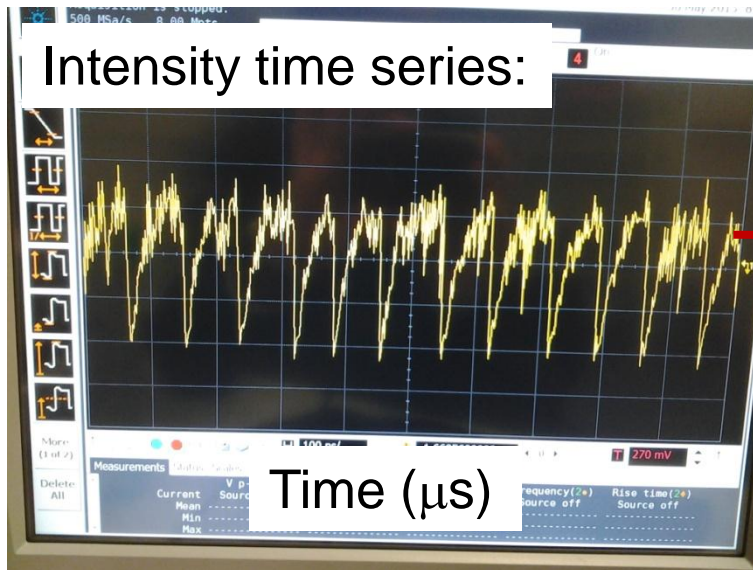


## ■ Feedback induces nonlinear dynamics:

- Multi-stability
- Regular pulses
- Extreme pulses
- Intermittency
- Chaos



- The **intensity dropouts** resemble neuronal **spikes**



with  
feedback

without  
feedback

Pump current (mA)  
**(CONTROL PARAMETER)**

- This spiking dynamics is referred as low-frequency fluctuations

# Why intensity dropouts?

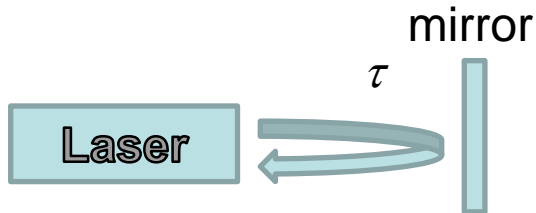
- Complex interplay of:
    - Feedback delay time
    - Various noise sources
    - Nonlinear light-matter interactions $\Rightarrow$  Stochastic and high-dimensional system
  - Over years a lot of work has been devoted to understand the laser spiking behavior and to identify simple models.
- A. Prasad et al, *Low-frequency fluctuations in external cavity semiconductor lasers: understanding based on a simple dynamical model*, J. Opt. B: Quantum Semiclass. Opt. 3 242 (2001).



- To develop a method of time-series analysis for determining signatures of determinism;
- To extract new information;
- To compare model predictions with observations;
- To find a minimal model;
- To explore potential for building optical neurons.

# Governing equations

R. Lang and K. Kobayashi, IEEE J. Quantum Electron. 16, 347 (1980)



$|E|^2 \sim$  photon number (output intensity)

$N \sim$  number of carriers (electron-holes)

$$\frac{dE}{dt} = \frac{1}{2\tau_p} (1 + i\alpha)(G - 1)E + \underbrace{\eta E(t - \tau)e^{-i\omega_0\tau}}_{\text{feedback}} + \underbrace{\sqrt{\beta_{sp}}\xi}_{\text{noise}}$$

$$\frac{dN}{dt} = \frac{1}{\tau_N} (\underbrace{\mu}_{\text{pump current}} - N - G|E|^2)$$

feedback

noise

$\eta$  = feedback strength

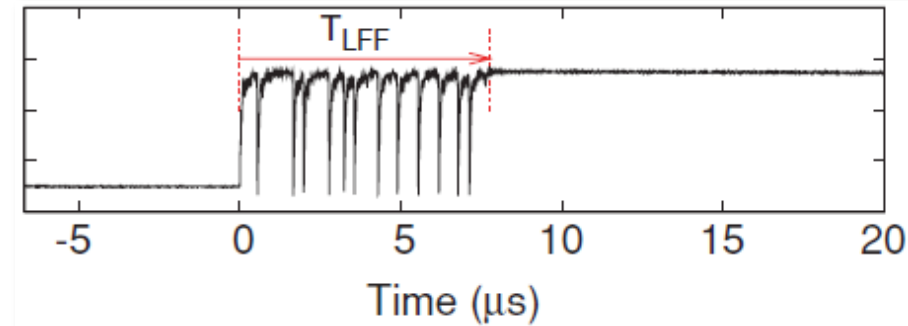
$\tau$  = feedback delay time

$\mu$  = pump current

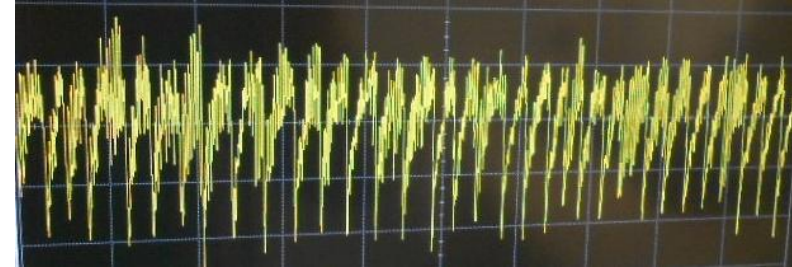
**(control parameter)**

Gain:  $G = N / (1 + \varepsilon|E|^2)$

Laser  
intensity



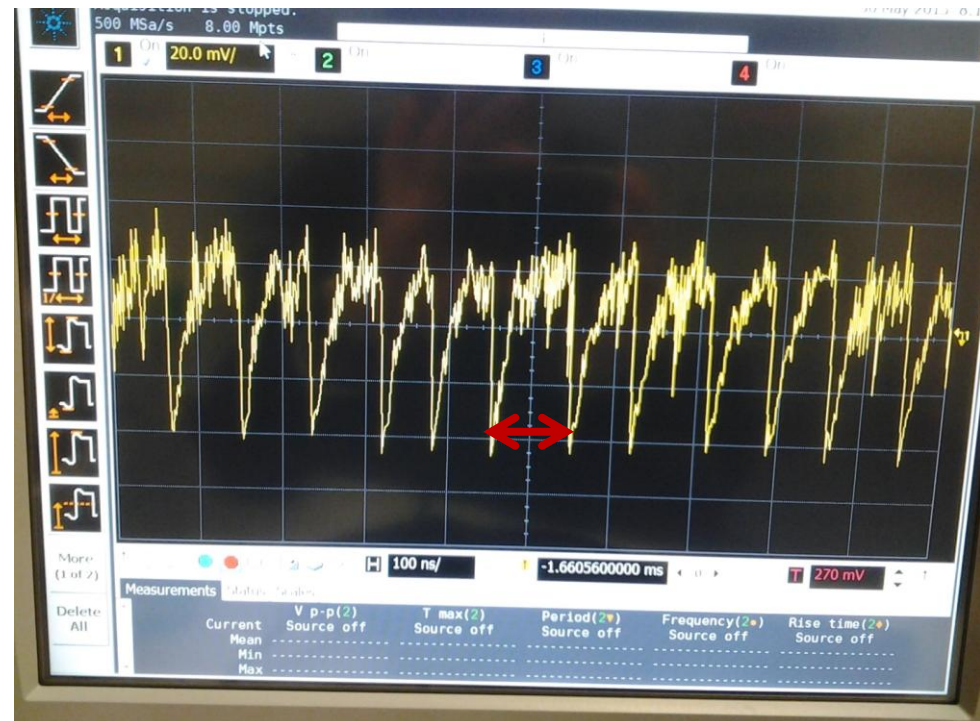
- In **deterministic** simulations: the spikes (dropouts) are **transient**.
- But in **stochastic** model simulations: **bursts** of dropouts.
- In the experiments: which dropouts are triggered by **noise** and which ones are **deterministic**?
- **Information** in the spike sequence?



# Problems and strategy

- Main problem: we can measure only one variable (the laser intensity)
- Also a problem: the detection system (photodiode, oscilloscope) has a finite *bandwidth* that gives limited temporal resolution.
- Our strategy: we analyze the sequence of **inter-dropout-intervals (IDIs)**:

$$\Delta T_i = t_{i+1} - t_i$$



# Event level description of complex systems

- Examples of sequences of events:
  - Intervals between threshold crossings, barrier crossings,
  - Neurons: inter-spike intervals (ISIs),
  - Human communication: inter-event user times (SMS, emails, Twitters).
  - Earth and climate: intervals between earthquakes, extreme events (tornados, rainfalls) etc.
  
- The identification of patterns in the sequence of events allows for:
  - Model verification
  - Parameter estimation
  - Classification of different types of dynamical behaviors
  - Predictability - forecasting



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# Symbolic Ordinal Analysis

- Has been widely used to analyze data generated from complex systems
  - **Financial, economical**
  - **Biological, life sciences**
  - **Geosciences, climate**
  - **Physics, chemistry, etc**
  
- Able to:
  - **Distinguish stochasticity and determinism**
  - **Classify different types of dynamical behaviors (pathological, healthy)**
  - **Quantify complexity**
  - **Identify coupling and directionality.**

Brandt & Pompe, Phys. Rev. Lett. 88, 174102, (2002).

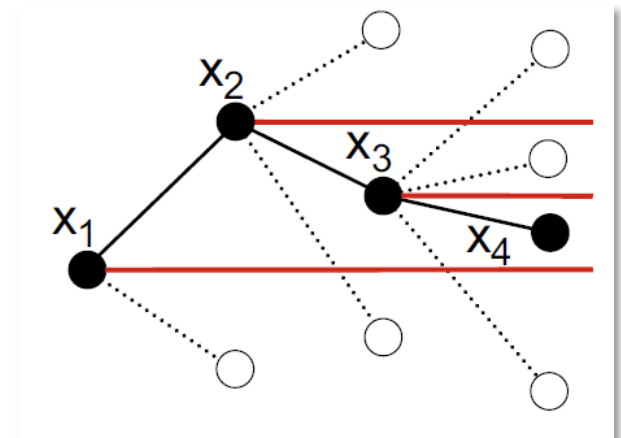
# Ordinal Patterns (“words”)

- “Words” of **D letters** can be formed by considering the **order relation** between sets of D values  $\{\dots x_i, x_{i+1}, x_{i+2}, \dots\}$ .
- For **D=3** there are 6 possible words



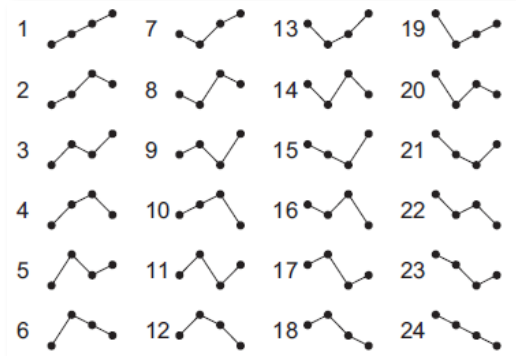
Example: the set (5, 1, 7) gives “102” because  $1 < 5 < 7$

- Advantage: the transformation keeps information about correlations in the time-series.
- Drawback: the set (5,1,100) also gives word “102”.

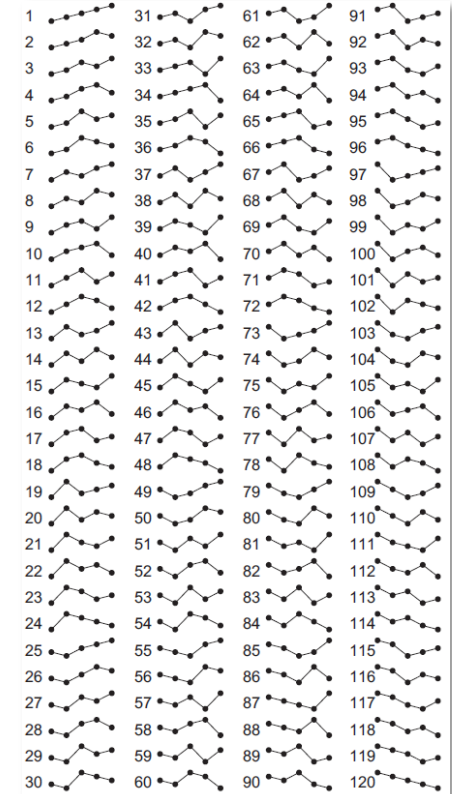


# Number of possible ordinal patterns: D!

**D=4**

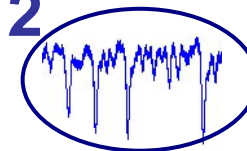


**D=5**

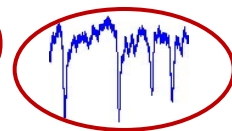


- The probabilities of the words unveil correlations between events.
- How to select D? Optimal D depends on:
  - The length of the time series.
  - The time scale of correlations.
- For optical spikes: D=2 (D=3) reveal correlations among 3 (4) spikes

**012**

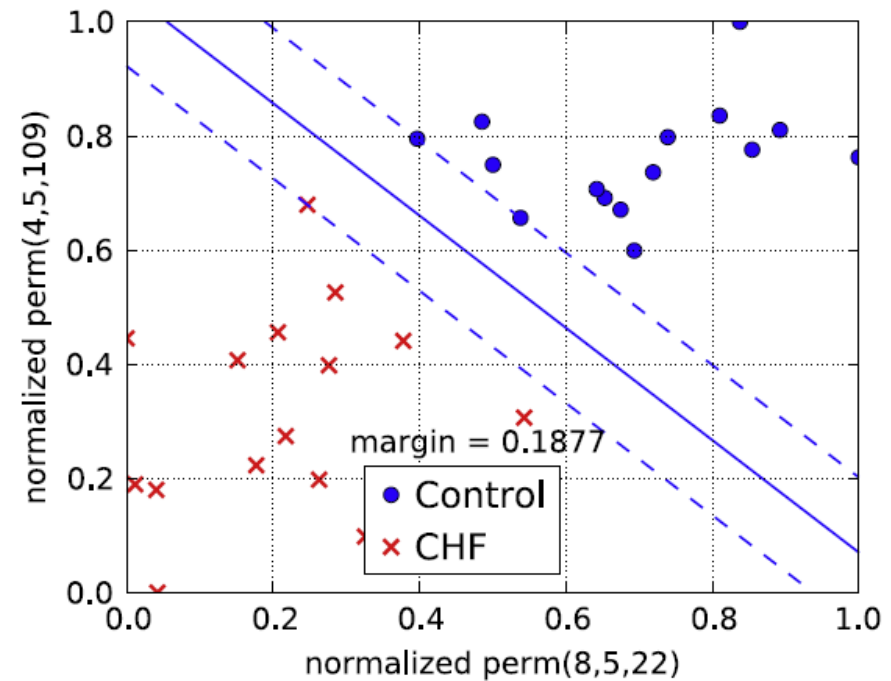
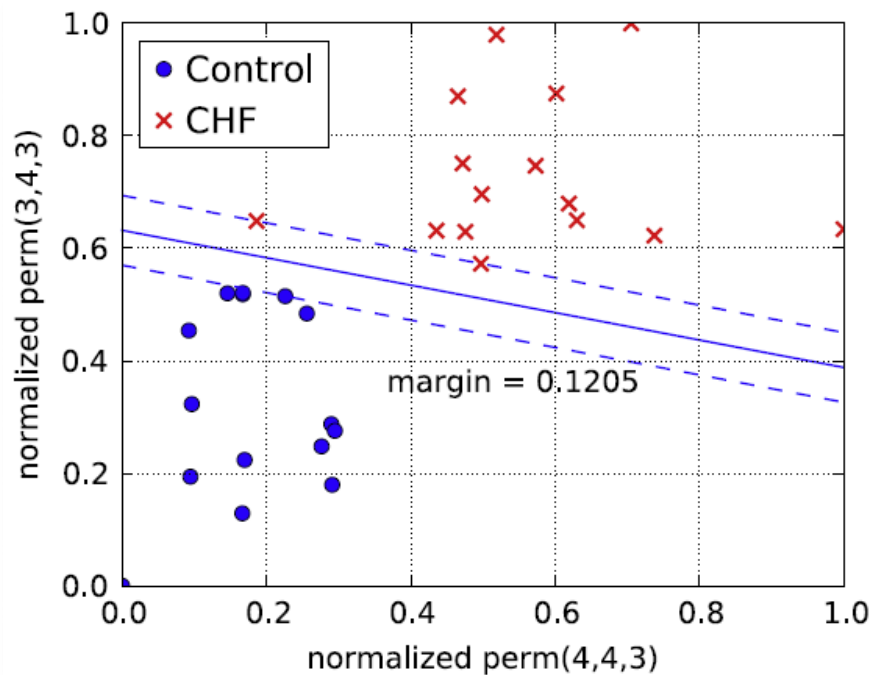


**210**



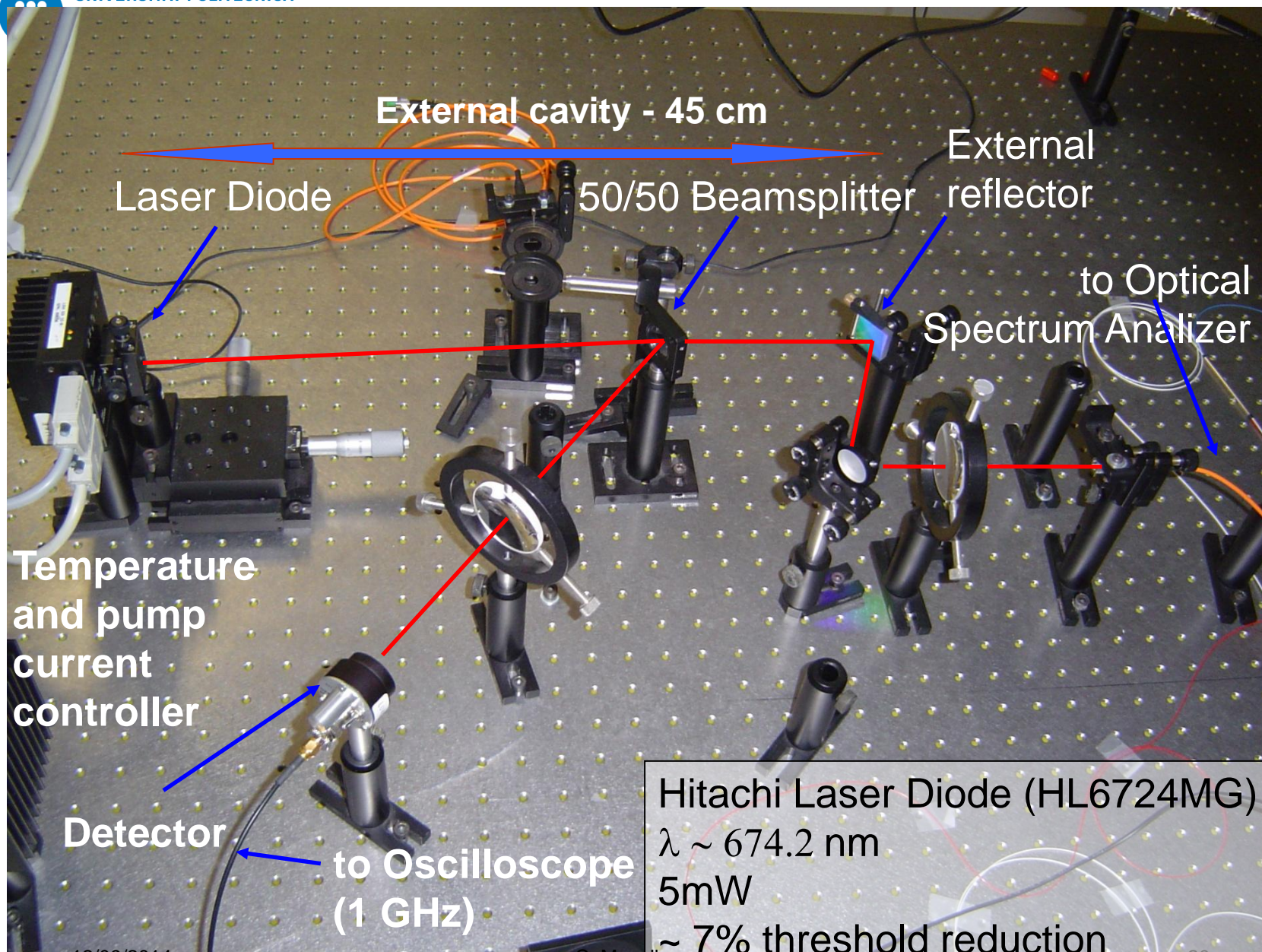
# An illustrative example

Classifying cardiac biosignals using ordinal pattern statistics  
congestive heart failure (CHF) vs healthy subjects.



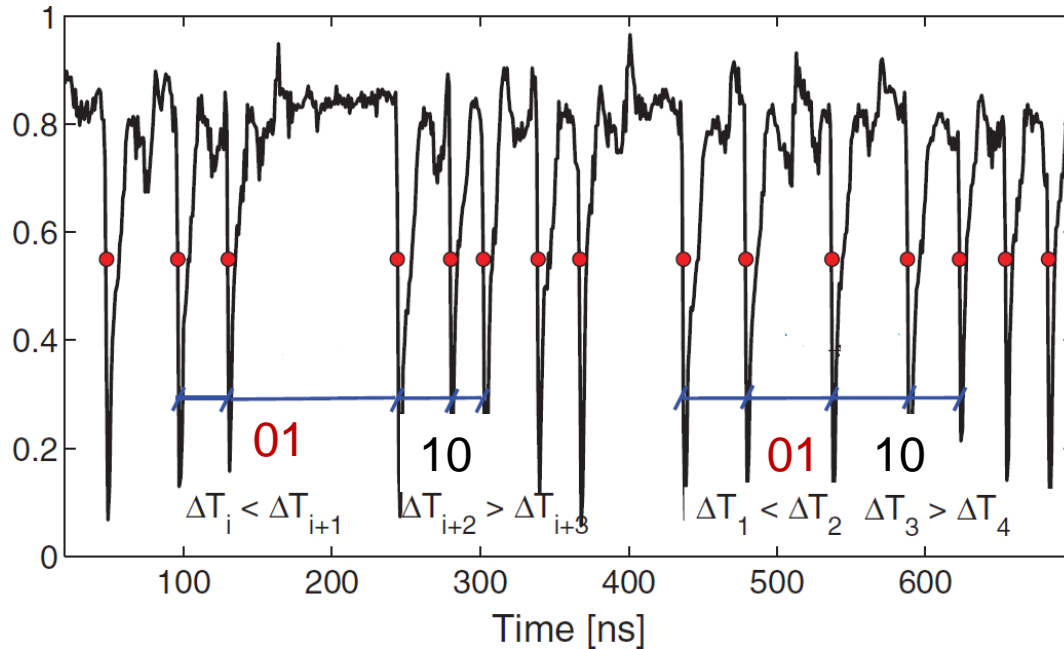


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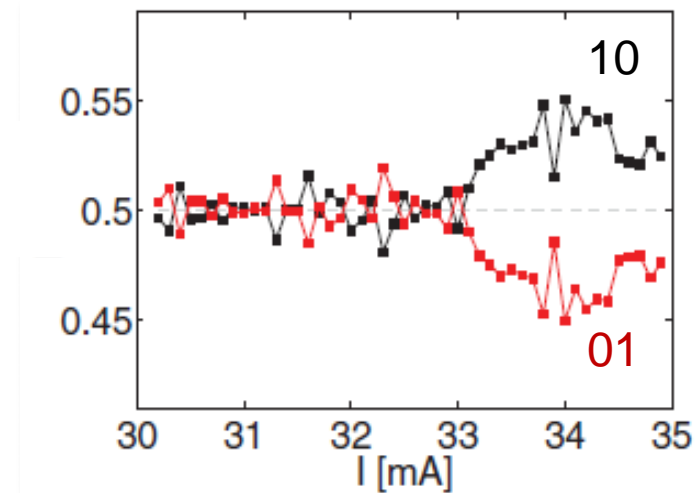
# Correlations between **3** consecutive spikes



**D=2:** 3-spike correlations?  
**Null hypothesis:** fully random  
 sequence of spikes  $\Rightarrow P(01) = P(10)$

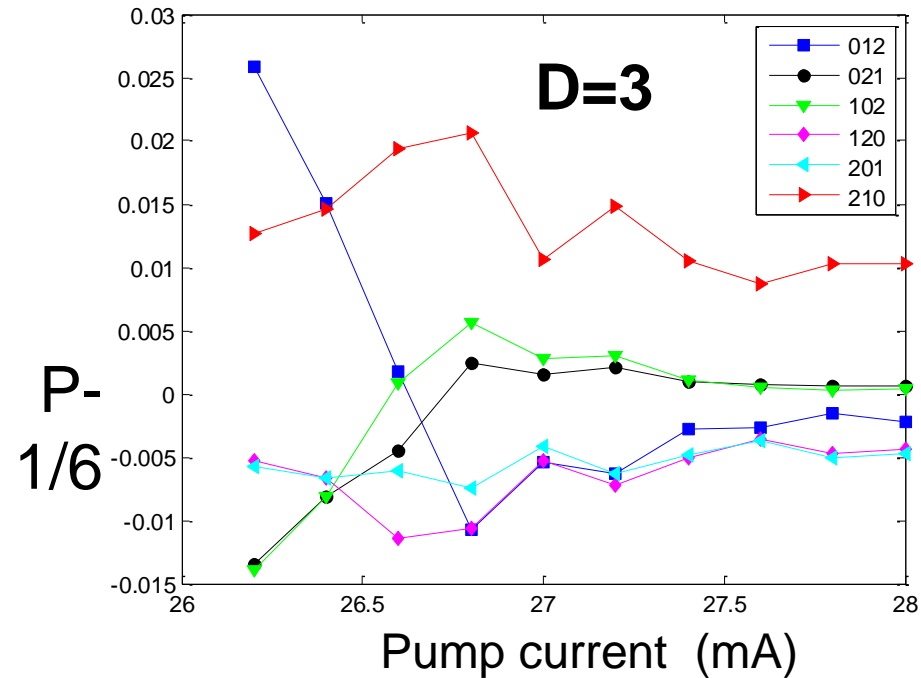
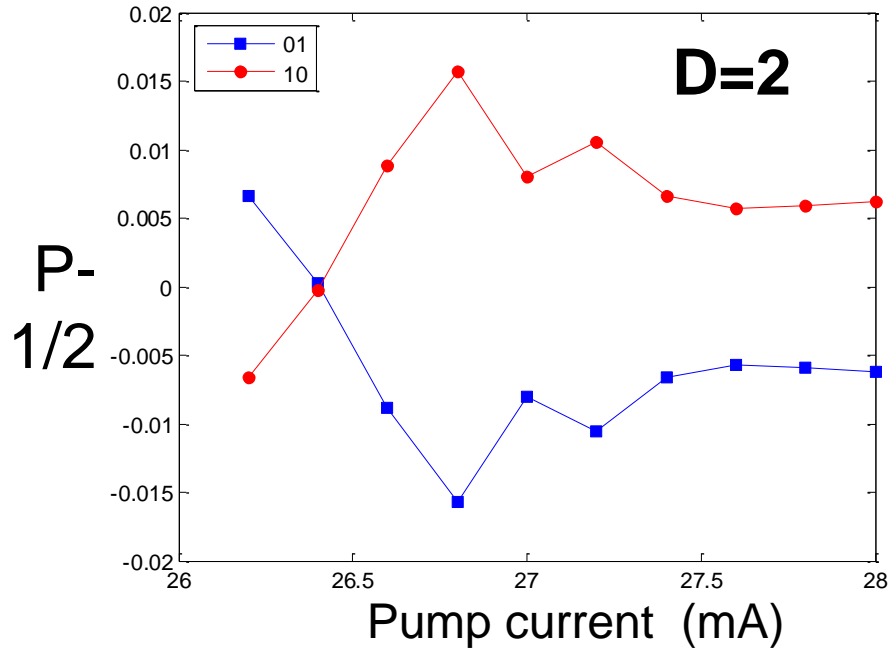
N. Rubido et al, Phys. Rev. E 84, 026202 (2011)

Word probabilities:



Consistent with  
 stochastic at low pump  
 current, but signatures  
 of determinism at high  
 pump current.

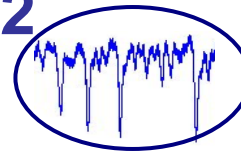
# At low pump current: are the spikes fully random? New experiment



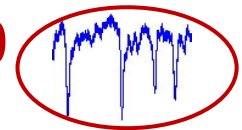
45000 - 220000 spikes

A. Aragonese et al, Scientific Reports 3, 1778 (2013)

012

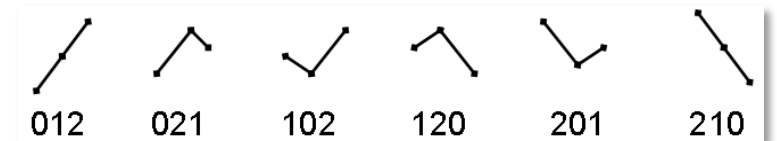
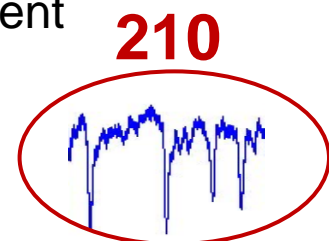
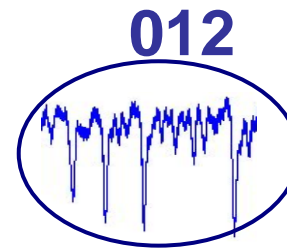
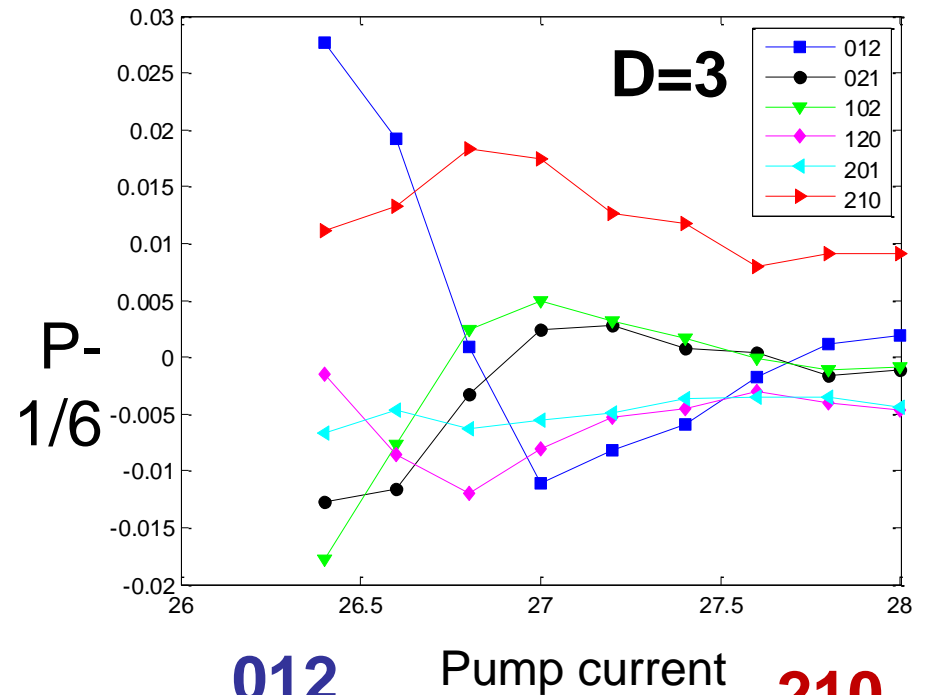
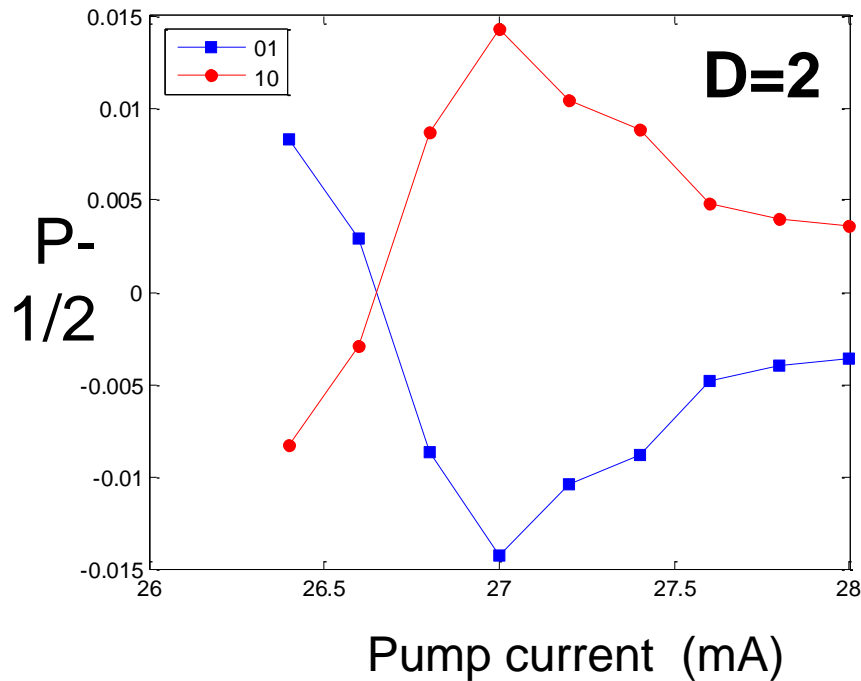


210



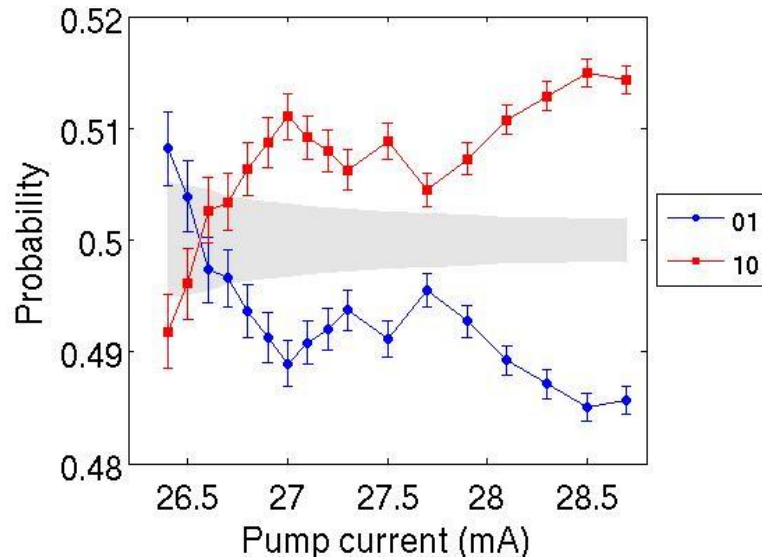


# Also in another data set recorded at a different temperature ( $T=20$ C)

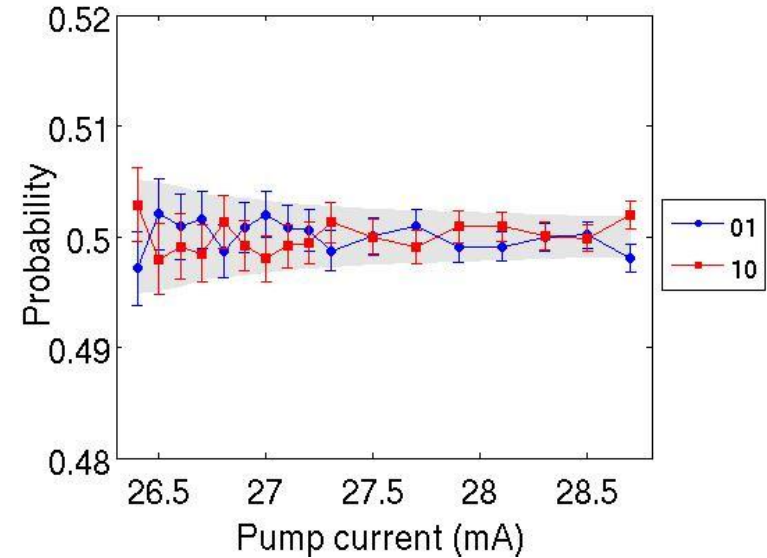


# Are the deviations from the null hypothesis **significant**?

## Recorded data



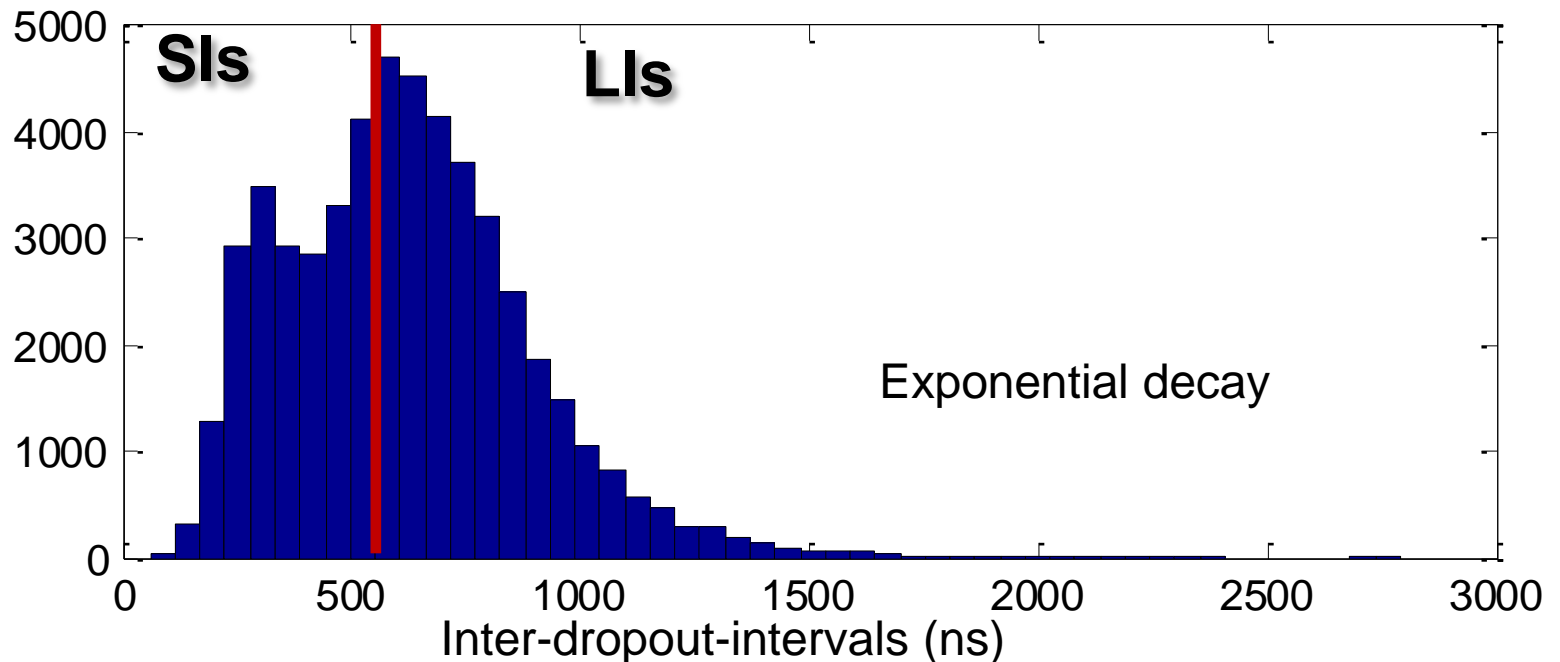
## Surrogated data



Error bars computed with a binomial test, gray region is consistent with N.H.

# Which dropouts are triggered by noise?

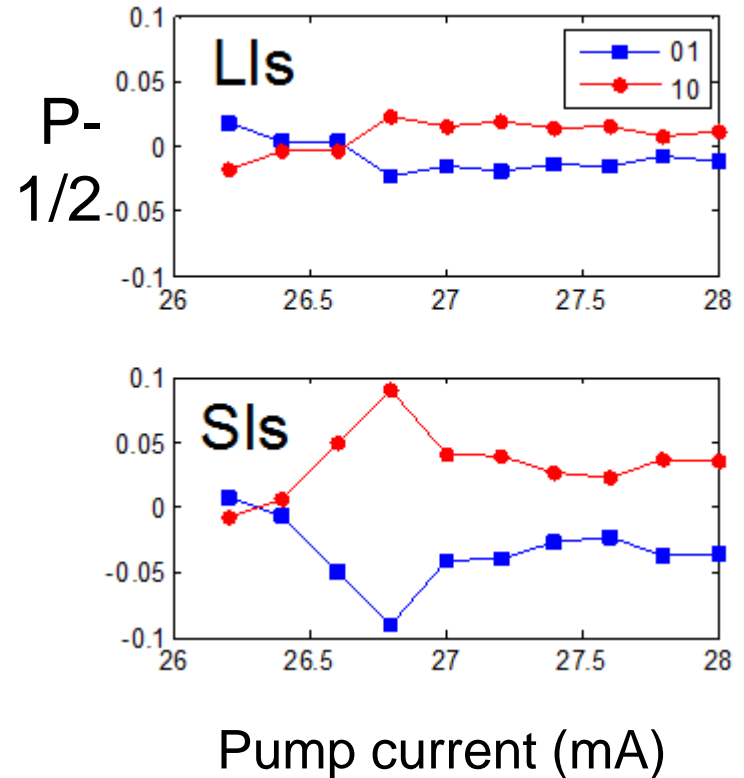
Histogram of Inter-dropout-intervals (IDIs)



We use a **threshold** to classify the inter-dropout-intervals as **short** and **long** intervals

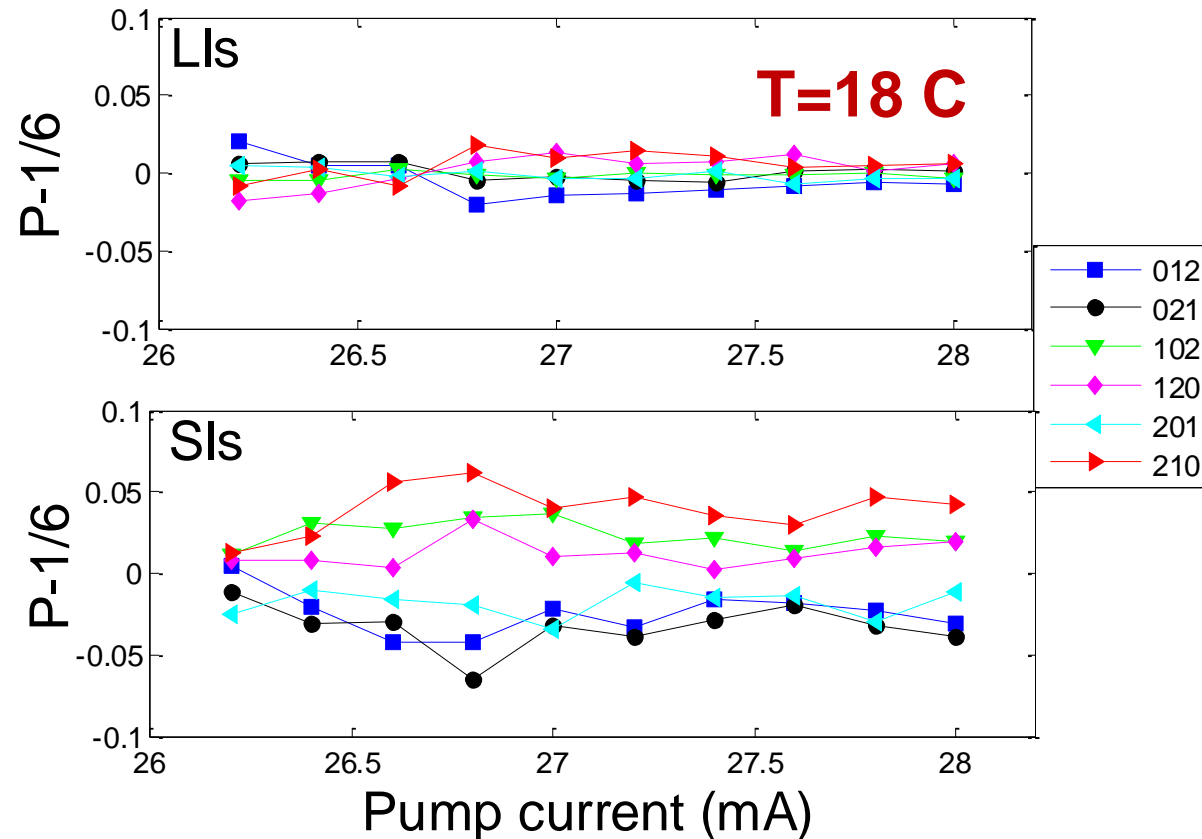
# Constructing the words with **2** consecutive SIs or LIs

- At high currents: significant differences
  - LIs consistent with random events
  - SIs more deterministic.
- But at low currents, the spikes can not be classified in two types with significant differences.



# Constructing the words with 3 consecutive SIs or LIs

- At high currents: significant differences
- But at low currents, the spikes can not be classified.



Similar results were obtained in the other dataset (**T=20 C**)



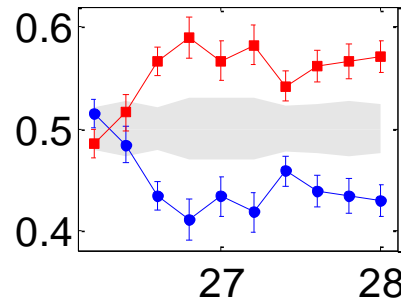
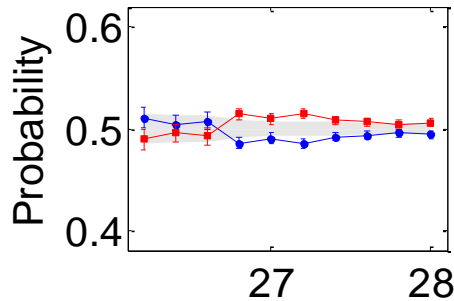
# Influence of the threshold used to classify IDIs as LIs and SIs

**LIs**

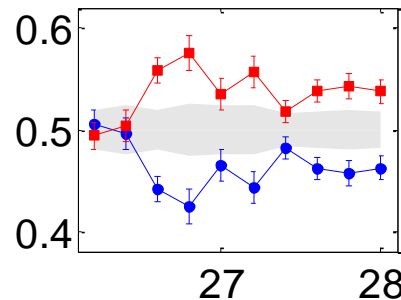
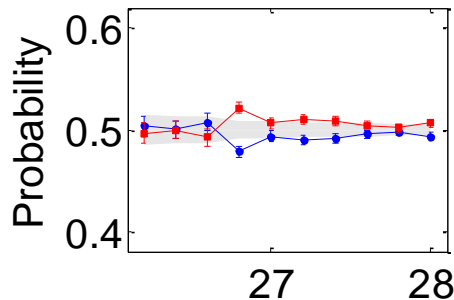
**SIs**

$\Delta T^*$  = most  
probable value

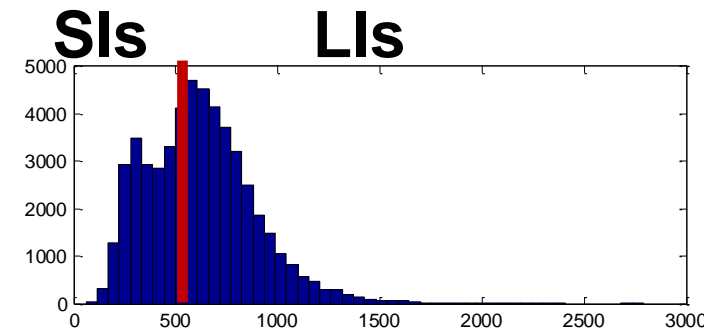
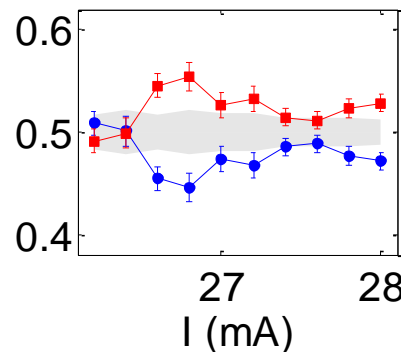
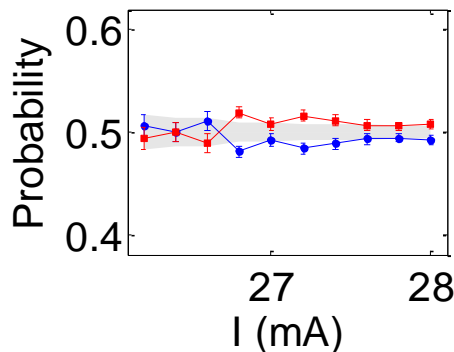
0.85  $\Delta T^*$



0.90  $\Delta T^*$



0.95  $\Delta T^*$



Error bars computed with  
a binomial test, gray  
region consistent with NH

# Tips to chose a good threshold

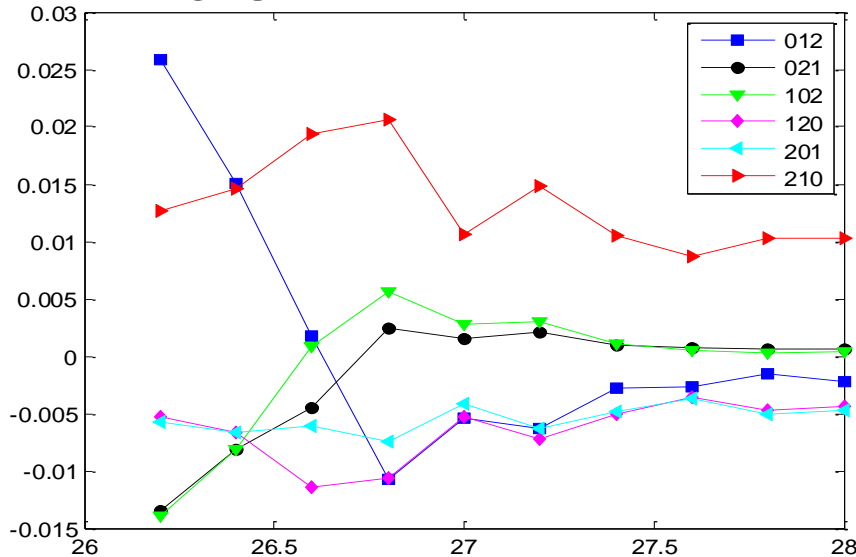
- LIs have statistical features as close as possible to random events:
  - Exponential distribution of values
  - Uniform distribution of word probabilities
- Good statistics: there are enough consecutive LIs and SIs
  - The NH region is sufficiently narrow
  - For the LIs, the error bars are in the NH region
  - For the SIs, the error bars are out of the NH region.

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# Ordinal analysis unveils new information

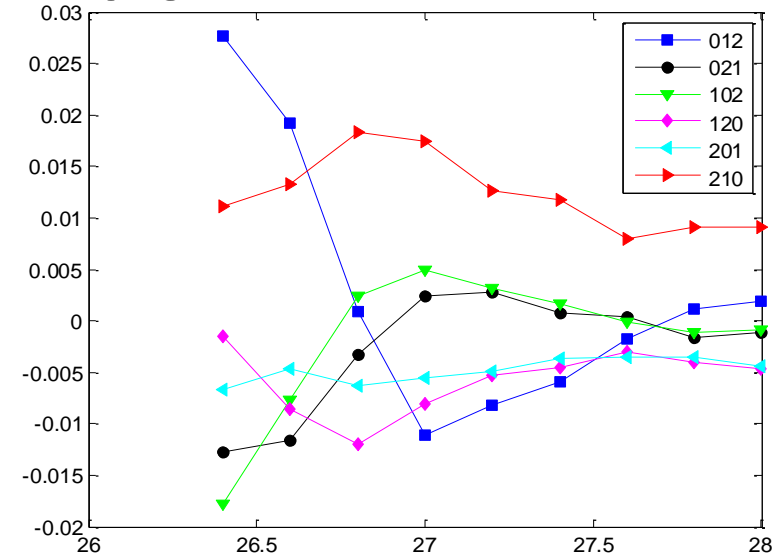
**T=18 C**

**P-  
1/6**



Pump current (mA)

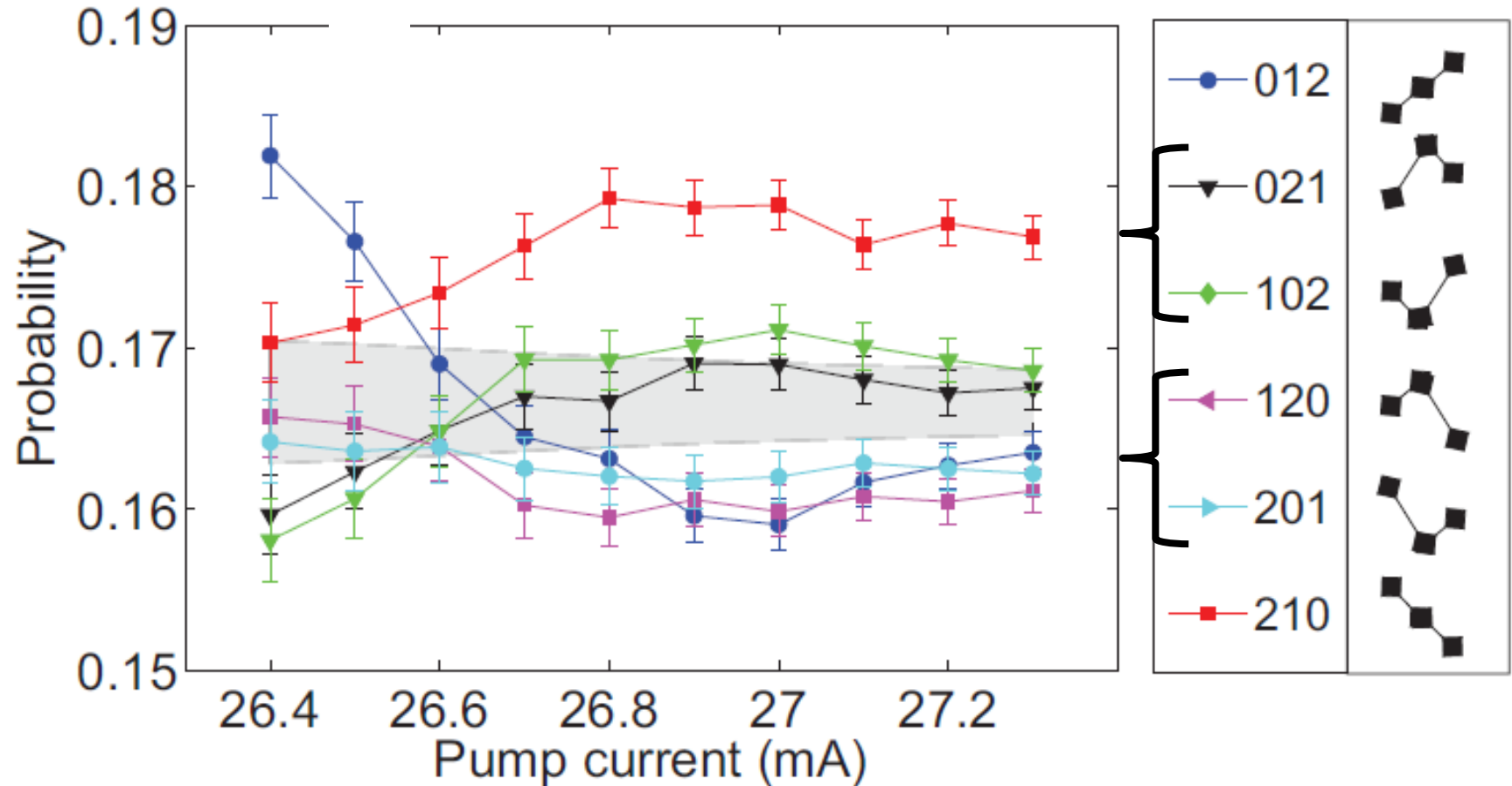
**T=20 C**



Pump current (mA)

There is a hierarchical and clustered organization of the probabilities of the words

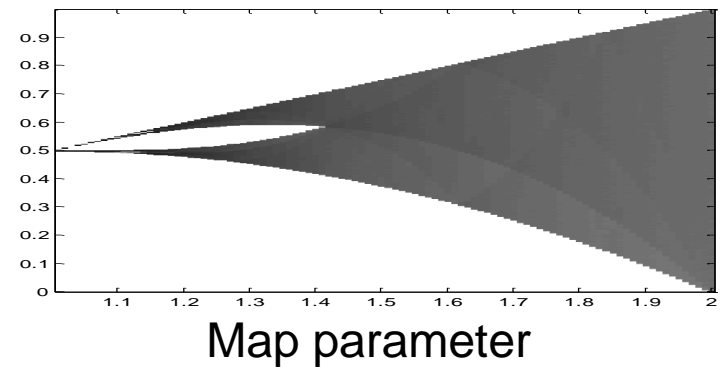
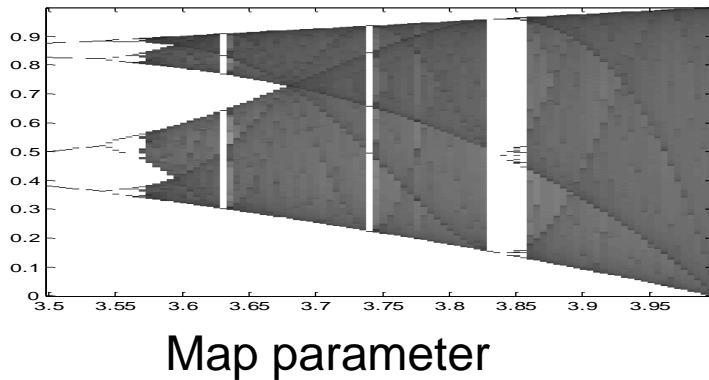
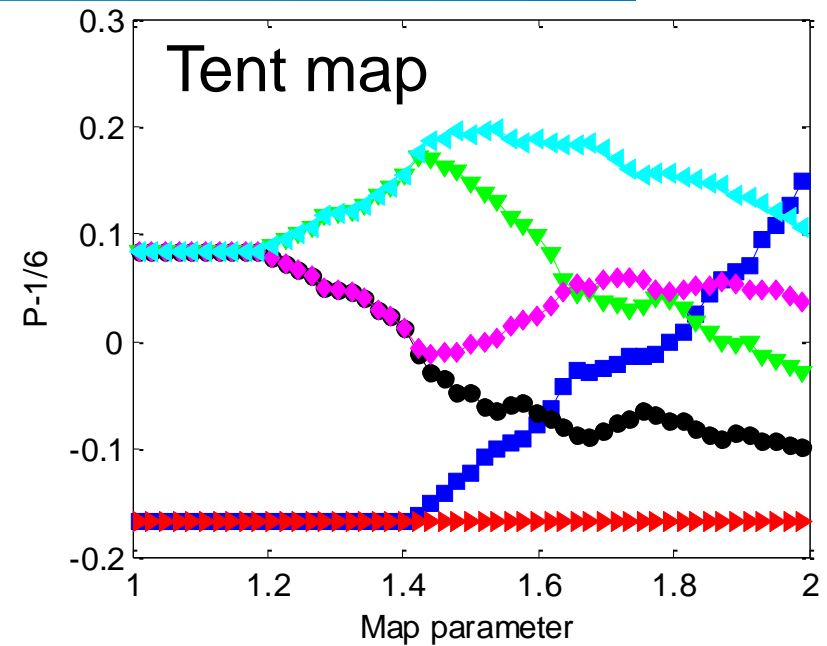
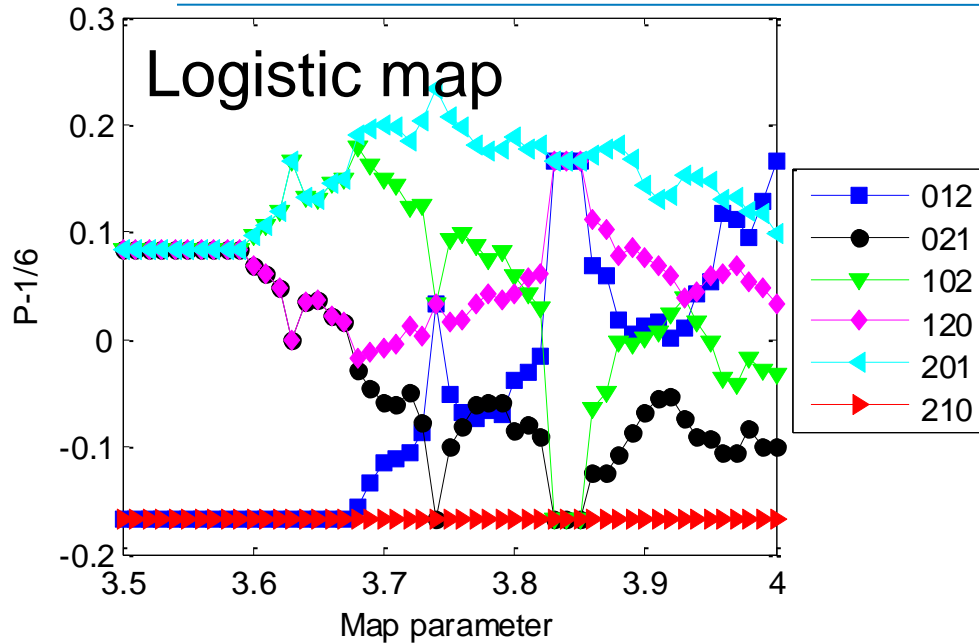
**In another experiment: also the same hierarchy and the same 2 clusters**



75,000 – 880,000 spikes  
(different laser, new oscilloscope)

(Prof. Struzik't talk)

# Can we find a minimal model that displays these features?

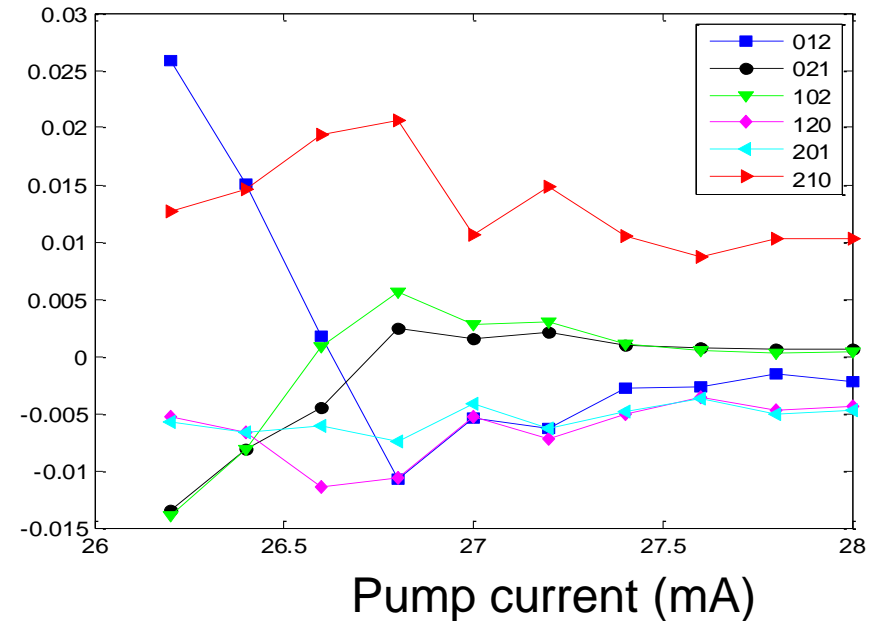
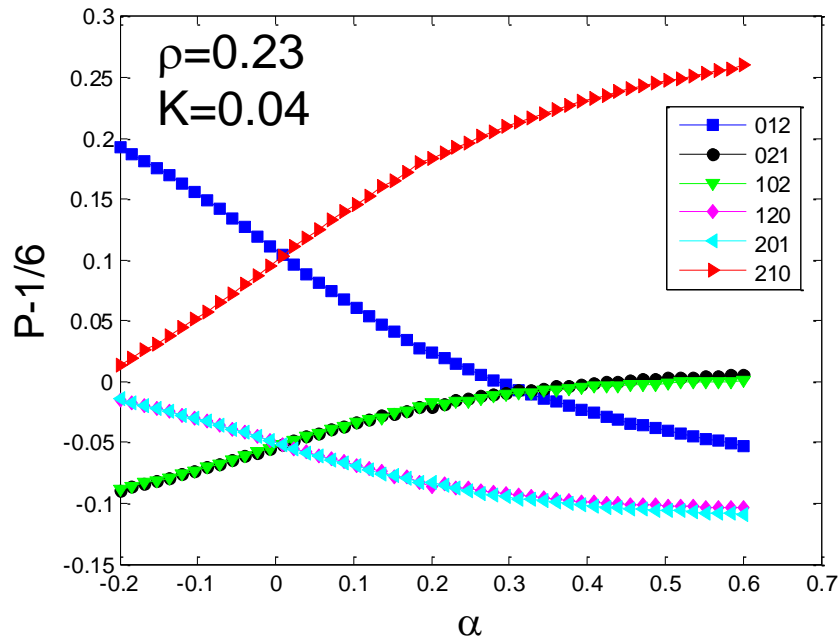




# A modified circle map: minimal phenomenological model

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi} [\sin(2\pi\varphi_i) + \alpha \sin(4\pi\varphi_i)]$$

$$X_i = \varphi_{i+1} - \varphi_i$$

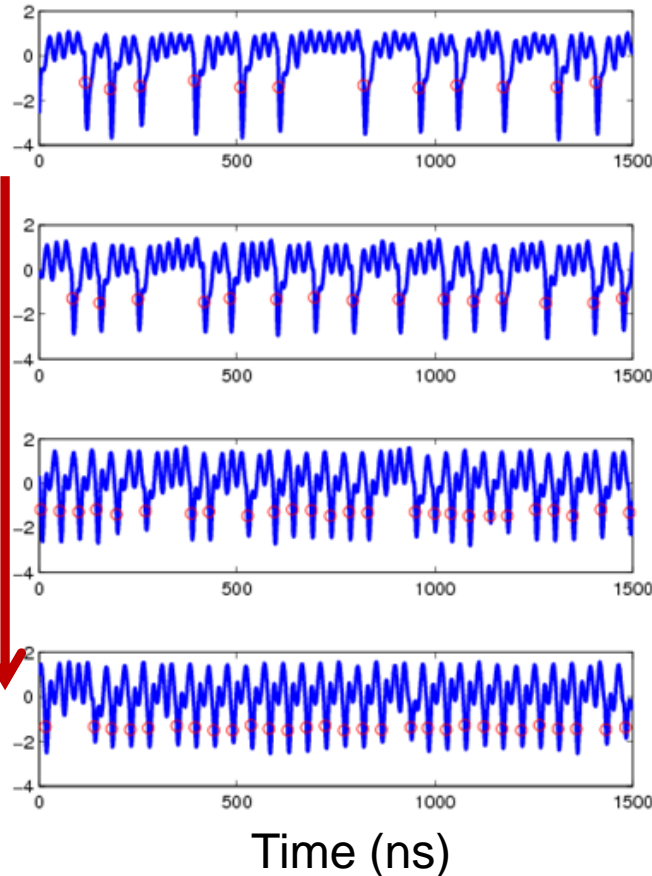


A. B. Neiman and D. F. Russell, *Minimal model for electroreceptors of paddlefish*, PRE 71, 061915 (2005)

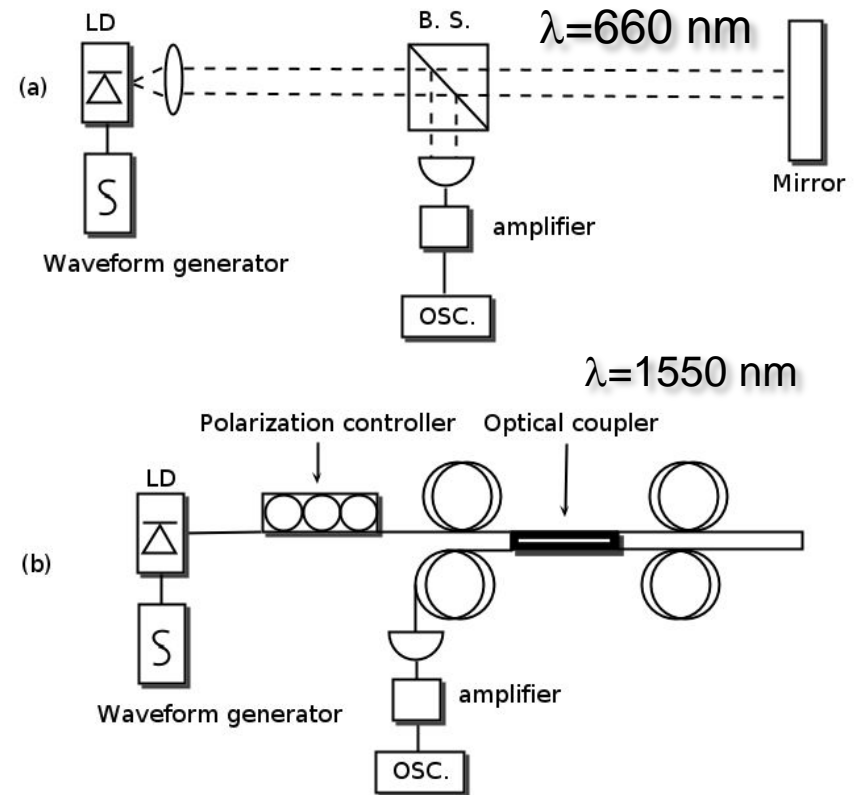
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# Response to external periodic modulation

Laser intensity:



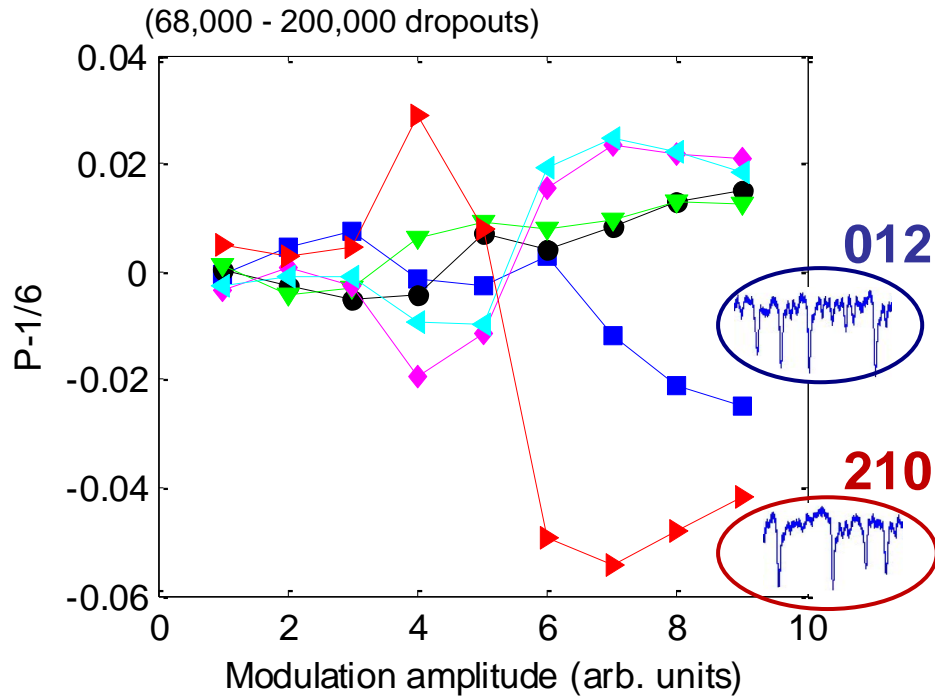
Two sets of experiments:



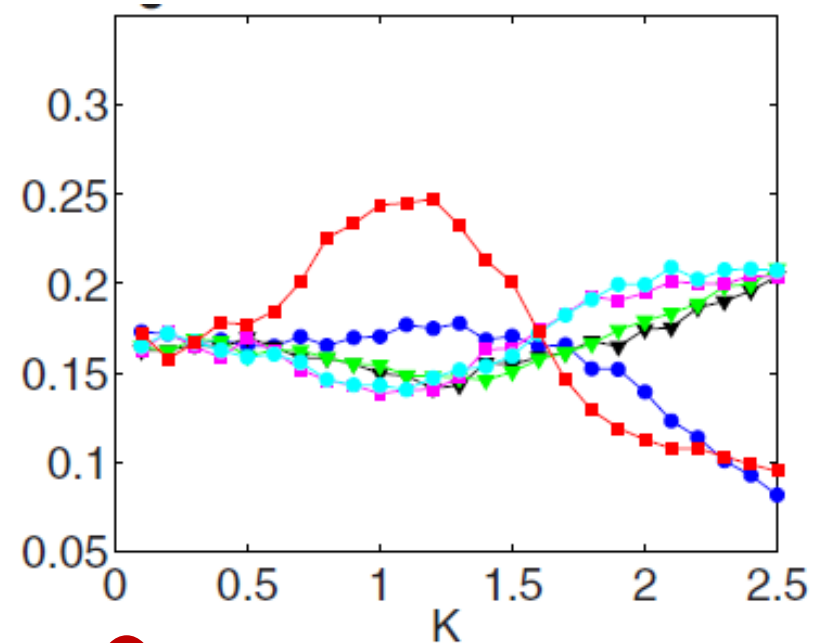
Relevant for understanding neuronal encoding of external stimuli

# Experiment-model comparison

## Experiments @ 660 nm



## Minimal circle-map model



$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi} [\sin(2\pi\varphi_i) + \alpha \sin(4\pi\varphi_i)] + D\zeta$$

Similar observations @ 1550 nm

Interpretation: locking to external forcing

- Introduction
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- Novel method for identifying signatures of determinism in complex time series.
- Spikes were classified in two categories: one stochastic; the other displaying signatures of determinism.
- We found new symbolic states with an hierarchical and clustered organization.
- We identified a minimal model. Robust under external forcing.
- Potential breakthrough: optical neurons for neuro-inspired information processing.
- Present work: towards understanding why the modified circle map is a good minimal model.

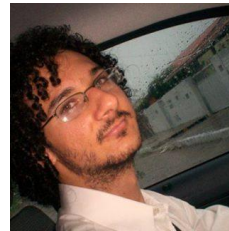
- Ordinal analysis is a powerful method of symbolic time-series analysis
  - useful for understanding data, uncovering patterns,
  - for improving system modeling, model comparison, parameter estimation,
  - for classifying data,
  - for improving predictability and forecasting.



# You for your attention!



Andres Aragoneses



Taciano Sorrentino



Carme Torrent

Papers @ [www.fisica.edu.uy/~cris](http://www.fisica.edu.uy/~cris)

- A. Aragoneses et al, Scientific Reports 3, 1778 (2013).
- A. Aragoneses et al, Scientific Reports 4, 4696 (2014).