

# Characterisation of emergent properties during the transition to optical turbulence in a fibre laser

**Laura Carpi and Cristina Masoller**

Universitat Politècnica de Catalunya, Barcelona, Spain

Cristina.masoller@upc.edu

[www.fisica.edu.uy/~cris](http://www.fisica.edu.uy/~cris)



UNIVERSITAT POLITÈCNICA  
DE CATALUNYA  
BARCELONATECH

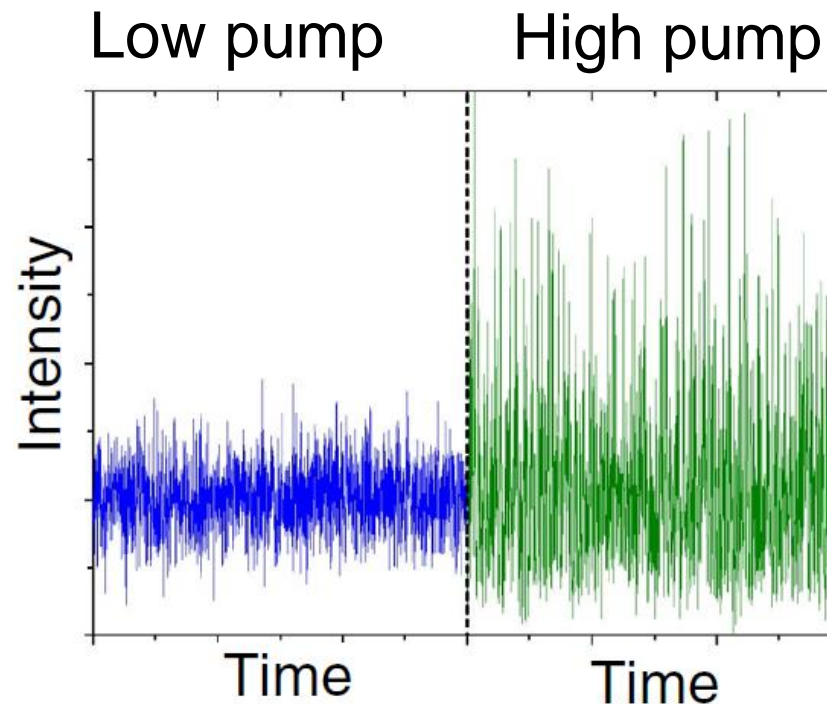
*Campus d'Excel·lència Internacional*

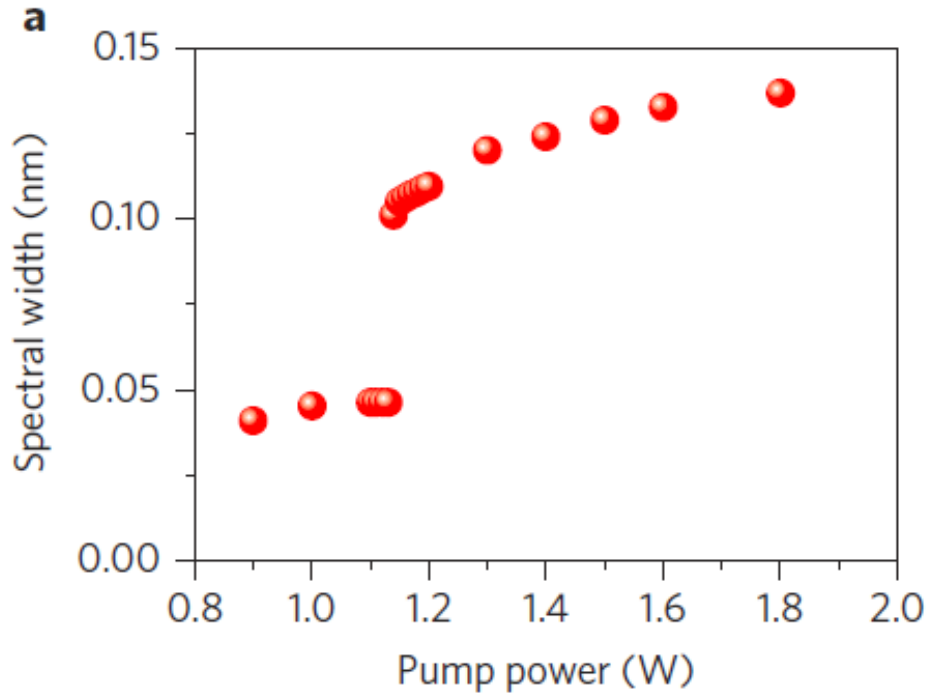


CLEO EUROPE  
Munich, June 2017

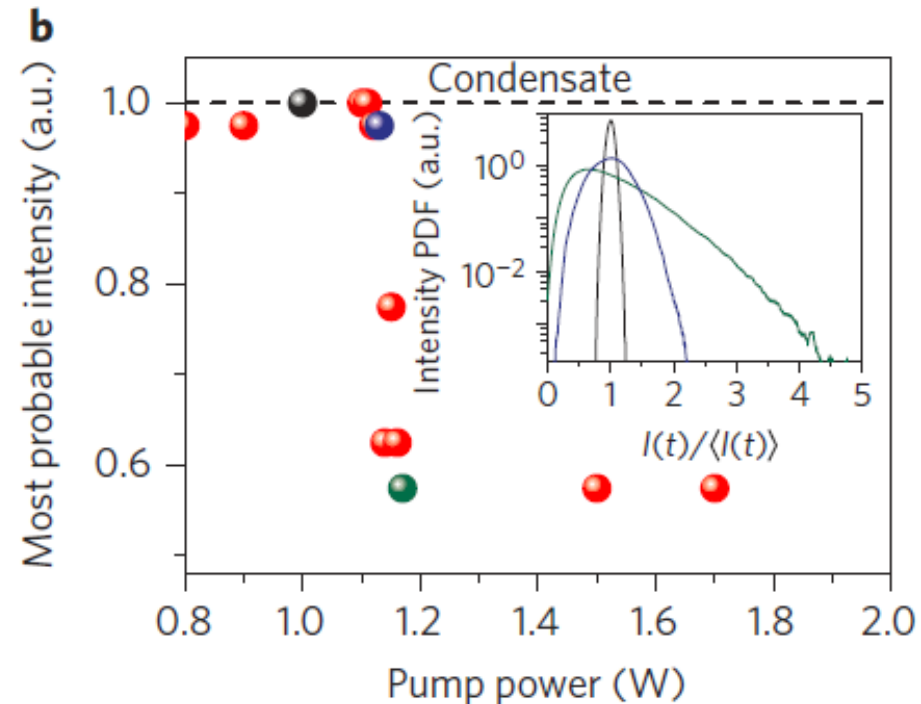


- Transition to optical turbulence in a quasi-cw Raman fiber laser
- 1 km of normal dispersion fiber placed between two fiber Bragg gratings acting as cavity mirrors





Sharp increase of the number of excited modes.



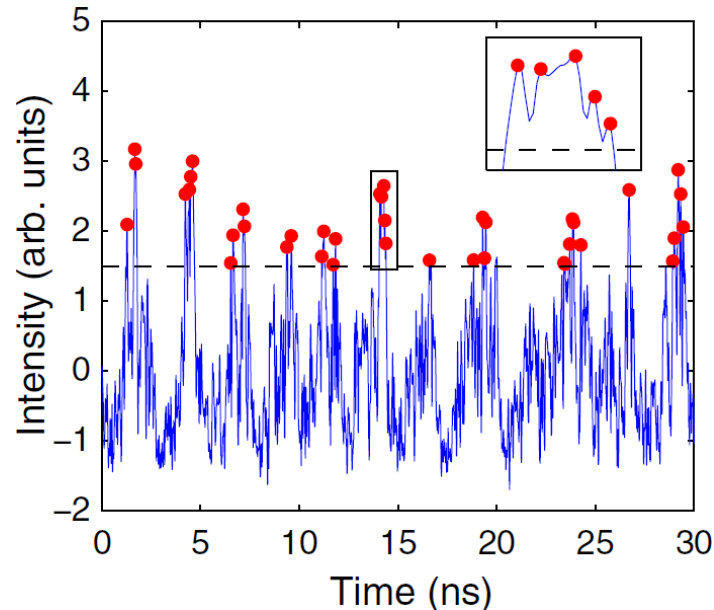
Sharpe decrease of the most probable intensity

## TEMPORAL CORRELATIONS?

- “Big data”: novel diagnostic tools for the analysis of output signals of complex systems.
- These tools can provide new insight into optical instabilities and complex dynamics.
- Outline
  - Methods
  - Results
  - Summary

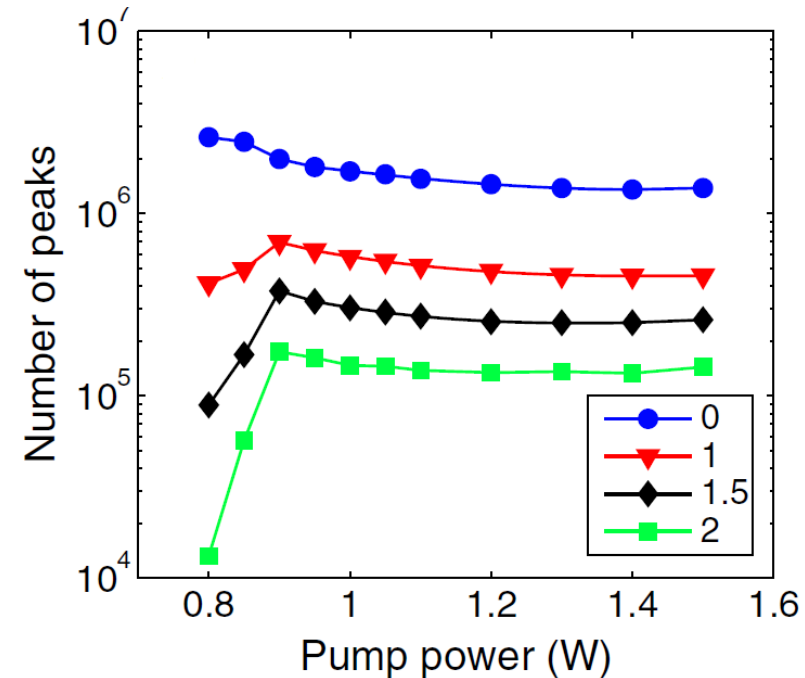
# Analysis of the intensity dynamics

Preprocessing: each time series is normalized to  $\langle I \rangle = 0$  and  $\sigma = 1$



$$\{I_{\max,i}\}$$

time series with  $5 \times 10^7$  data points, 12.5 ps resolution



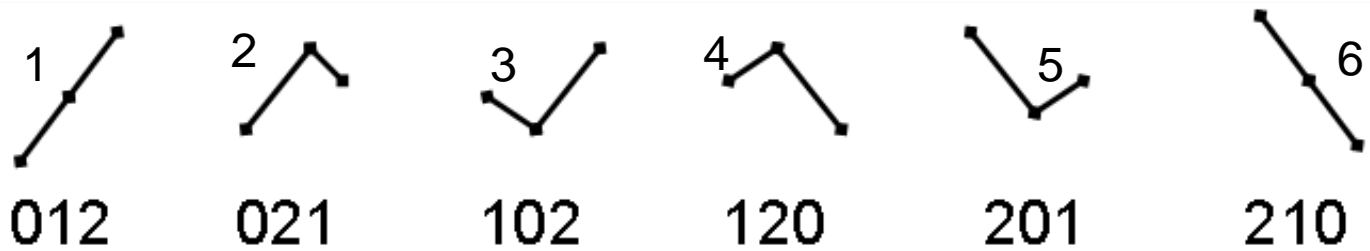
number of pulses above  **$2\sigma$** : larger than  $10^4$  for all pump powers

*Experimental data from Prof. Turitsyn' group, Aston Univ., UK.*

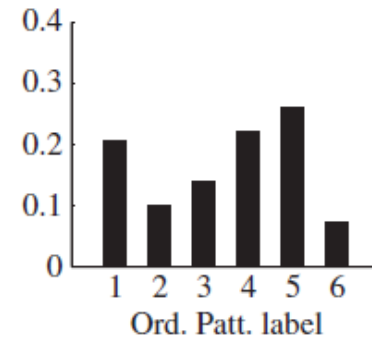
# Symbolic method of time-series analysis: ordinal Patterns

$$X = \{\dots \mathbf{x}_i, \mathbf{x}_{i+1}, \mathbf{x}_{i+2}, \dots\}$$

**D=3**

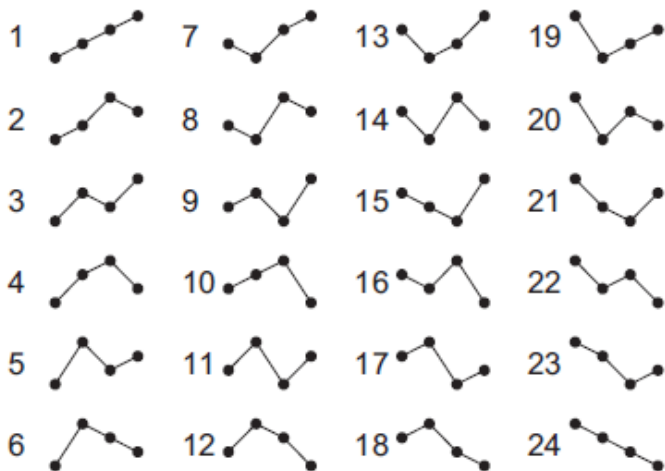


Ordinal  
probabilities



Example: (5, 1, 7) gives “102” because  $1 < 5 < 7$

**D=4:**



**D=5,  
120**

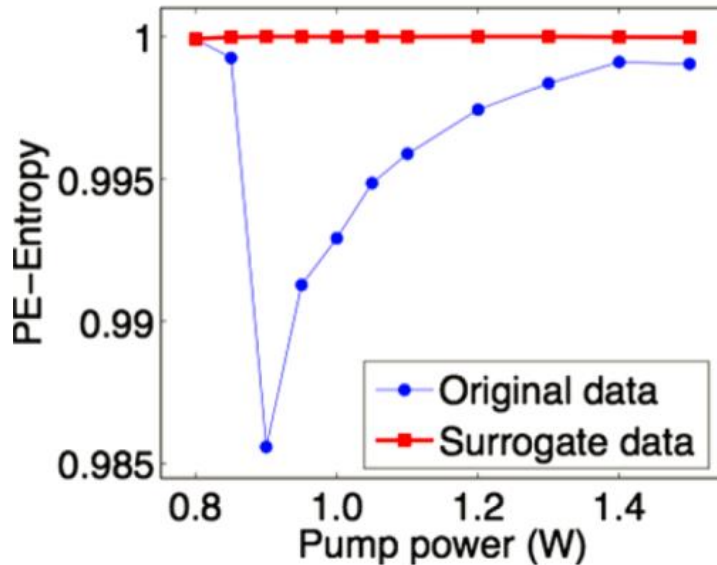
Permutation entropy

$$s_p = -\sum p_i \log p_i$$

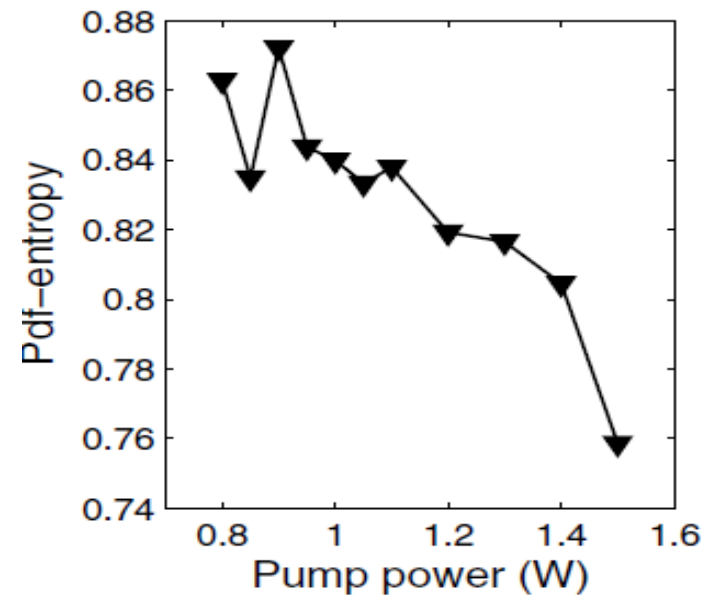
We will also use a lag

$$\{\dots \mathbf{x}_i, \mathbf{x}_{i+1}, \mathbf{x}_{i+2}, \mathbf{x}_{i+3}, \mathbf{x}_{i+4}, \mathbf{x}_{i+5} \dots\}$$

## Entropy from the pattern probabilities, intensity pulses $> 2\sigma$



## Entropy from pdf of intensity values



- Sharp transition is not captured when the entropy is calculated from the intensity pdf.
- Different entropy behavior.

A. Aragoneses, L. Carpi, N. Tarasov, D. V. Churkin, M. C. Torrent, C. Masoller, and S. K. Turitsyn, Phys. Rev. Lett. 116, 033902 (2016).

# Second diagnostic tool: time-series is represented as a graph

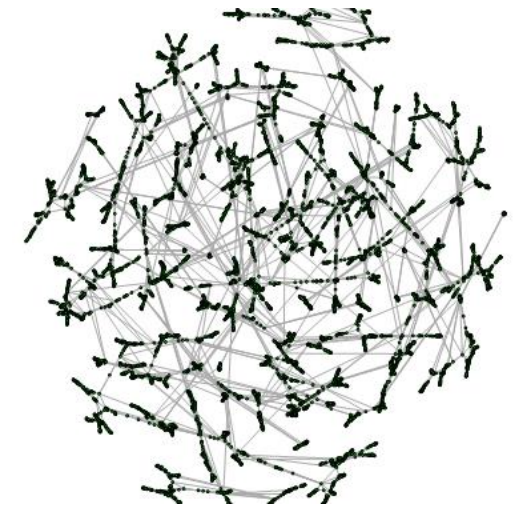
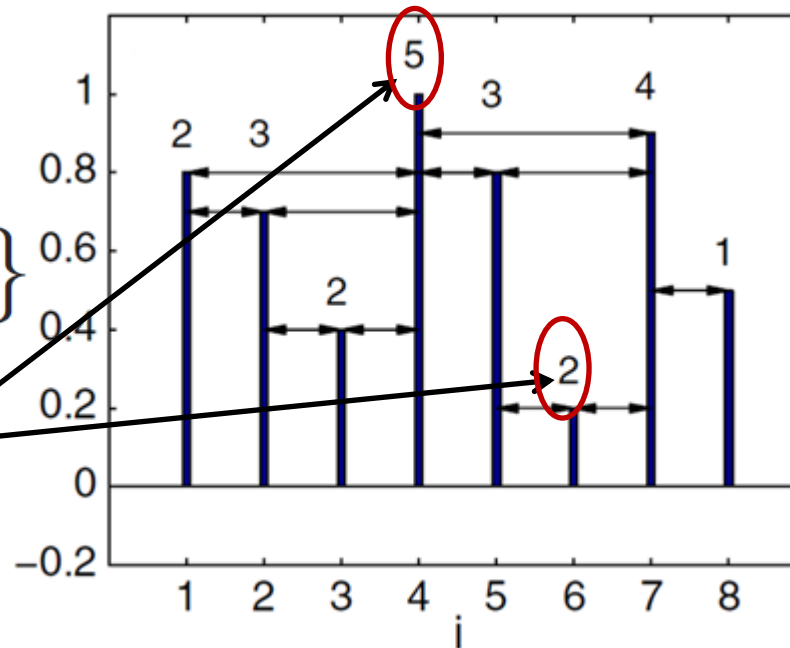
B. Luque et al, PRE 80, 046103 (2009)

## Horizontal Visibility Graph (HVG)

intensity  
pulses  $> 2\sigma$

$\{I_{\max,i}\}$

Number of links



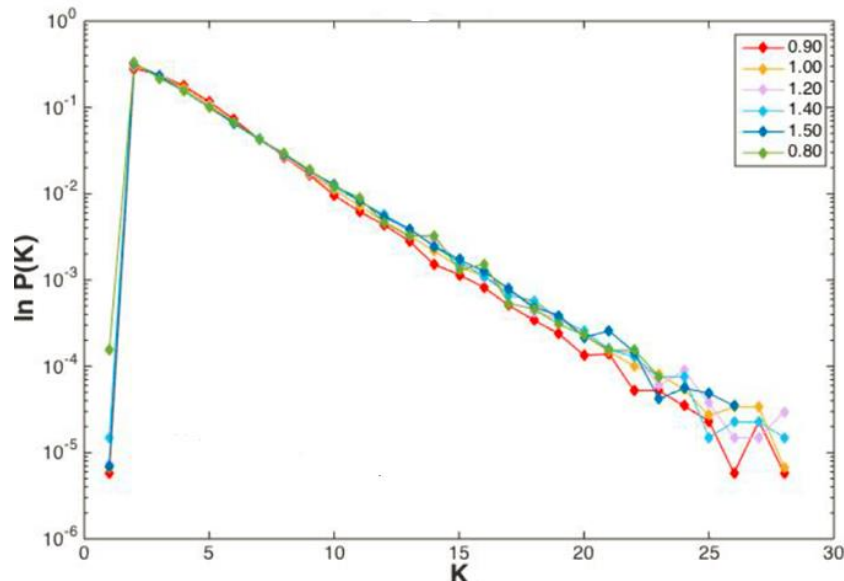
- Rule: pulses  $i$  and  $j$  are connected if there is “**horizontal visibility**” between them:  $I_{\max,i}$  and  $I_{\max,j} > I_{\max,n}$  for all  $n, i < n < j$
- The sequence  $\{ \dots 2, 3, 2, 5, 3, 2, 4, 1 \dots \}$  contains information about the pulses **relative heights**  $\{ \dots I_{\max,i} \dots \}$



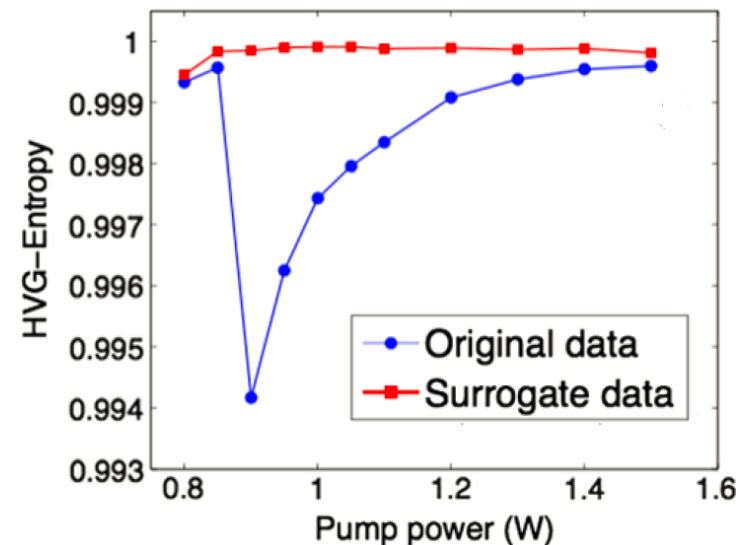
How to characterize the graph obtained?

⇒ Distribution of the number of links (degree distribution)

- $P(k)$  = degree distribution various pump powers



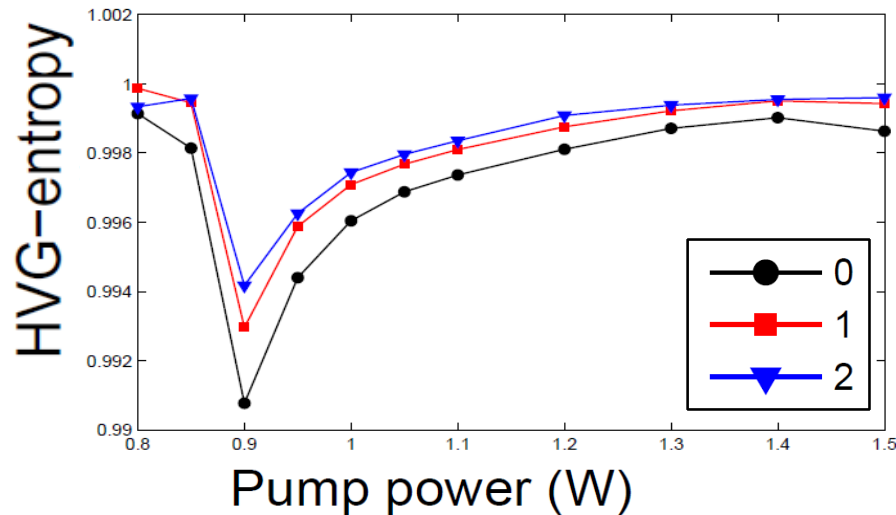
- Entropy of  $P(k)$  (normalized to the entropy of Gaussian white noise)



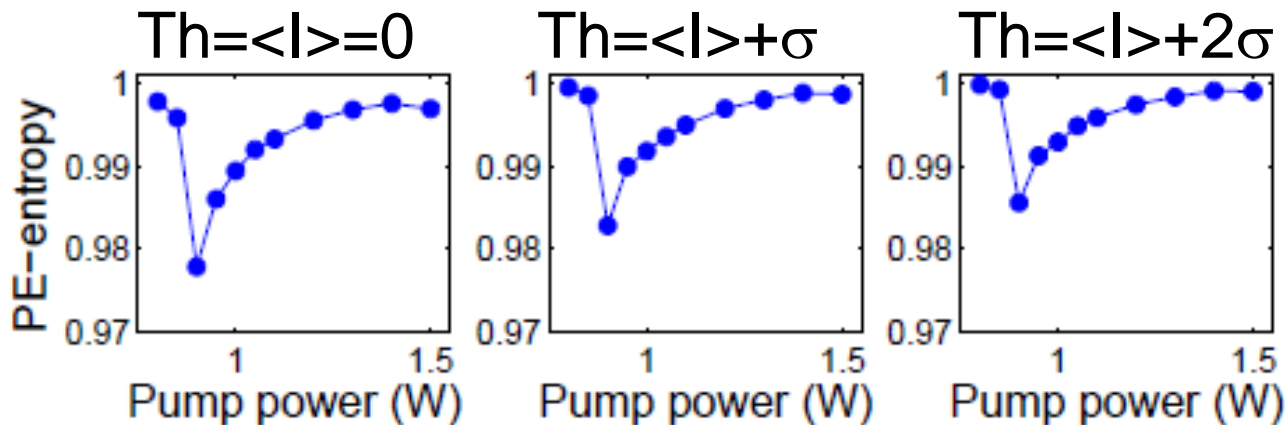
⇒ sharp transition detected, entropy decreases at the transition.

# Influence of the threshold

Raw data  $\{.../i...\} \Rightarrow Th \Rightarrow \{.../_{max,i}...\}$



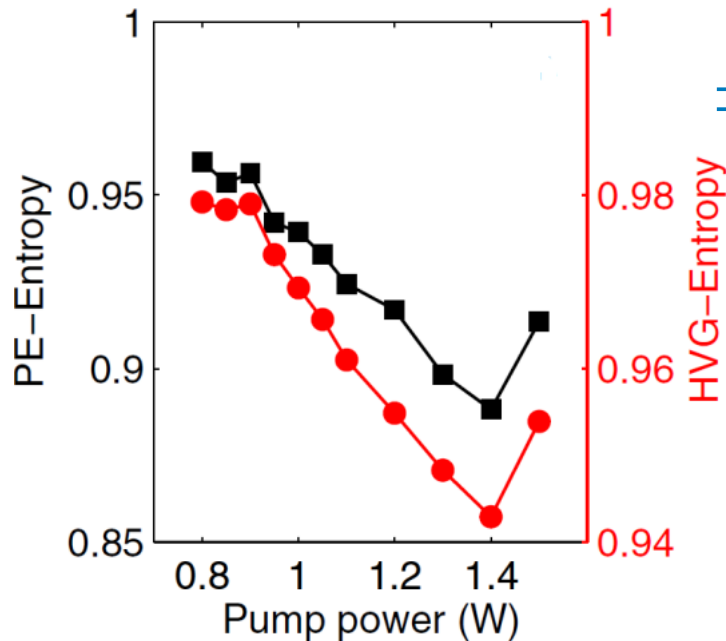
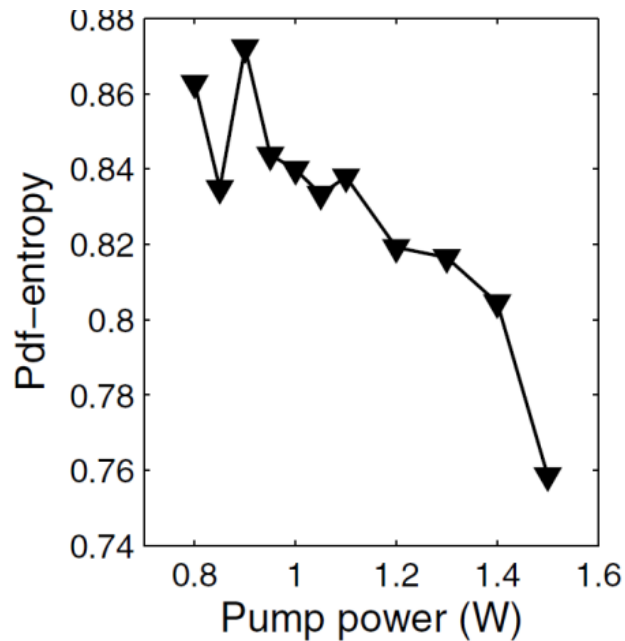
⇒ sharp transition detected with different thresholds.



# Influence of the threshold

Raw data  $\{.../i...\} \Rightarrow \text{Th} \Rightarrow \{.../_{\max,i}...\}$

With the raw data



⇒ sharp  
transition not  
detected.

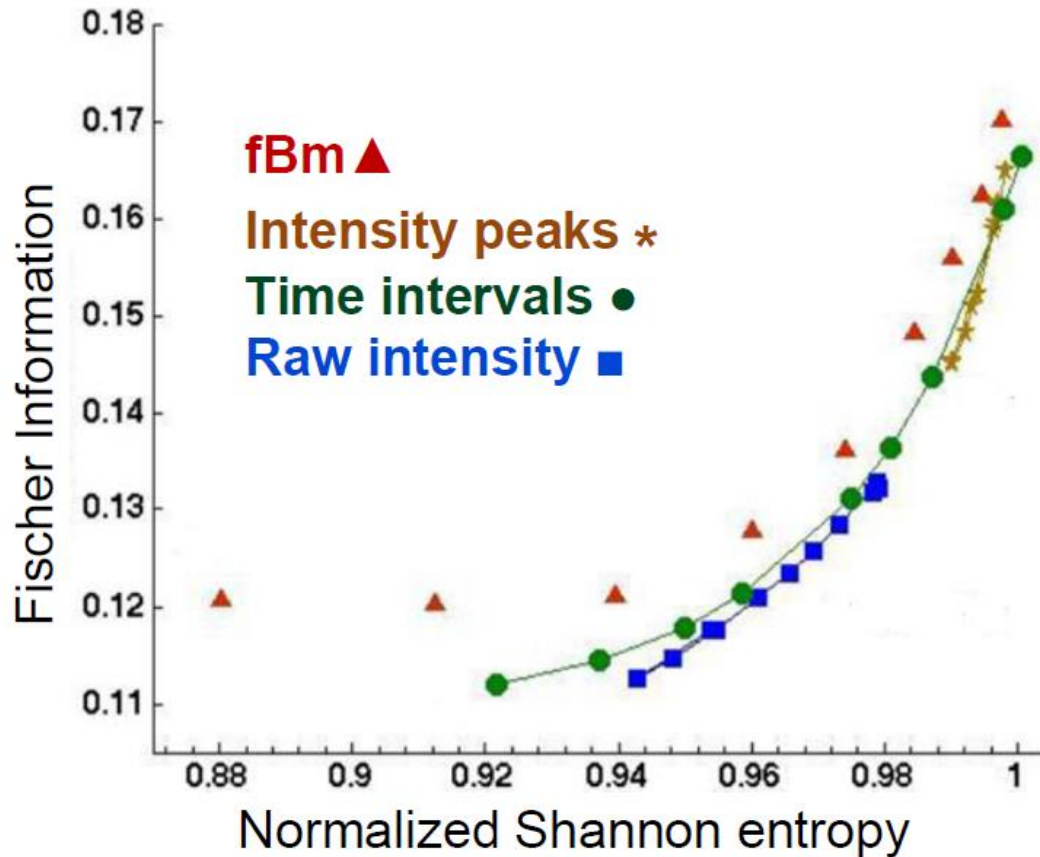
Can we obtain more information from the raw data?

# Comparison with **synthetic** data: fractional Brownian motion (fBm)

Gaussian stochastic process with a control parameter (the Hurst exponent  $H$ ) that allows to vary temporal correlations

- $H > 0.5$ , consecutive increments  $(x_{i+1} - x_i)$ ,  $(x_{i+2} - x_{i+1})$  tend to have the same sign  $\Rightarrow$  the process is **persistent**.
- $H < 0.5$ , consecutive increments tend to have opposite signs  $\Rightarrow$  the process is **anti-persistent**.
- $H = 0.5$ , consecutive increments are **uncorrelated**  
 $\Rightarrow$  Gaussian white noise.

# Information-theory approach: the Shannon-Fischer plane



weaker  
correlations

⇒ At all pump powers  
the intensity  
dynamics is highly  
stochastic, very  
close to the noise  
“frontier”.

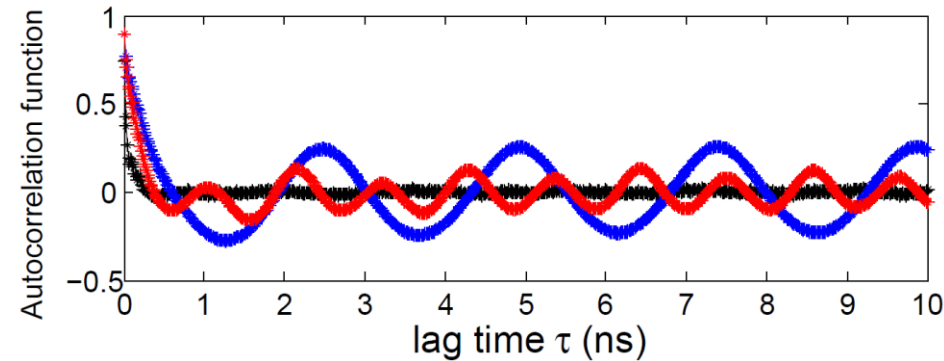
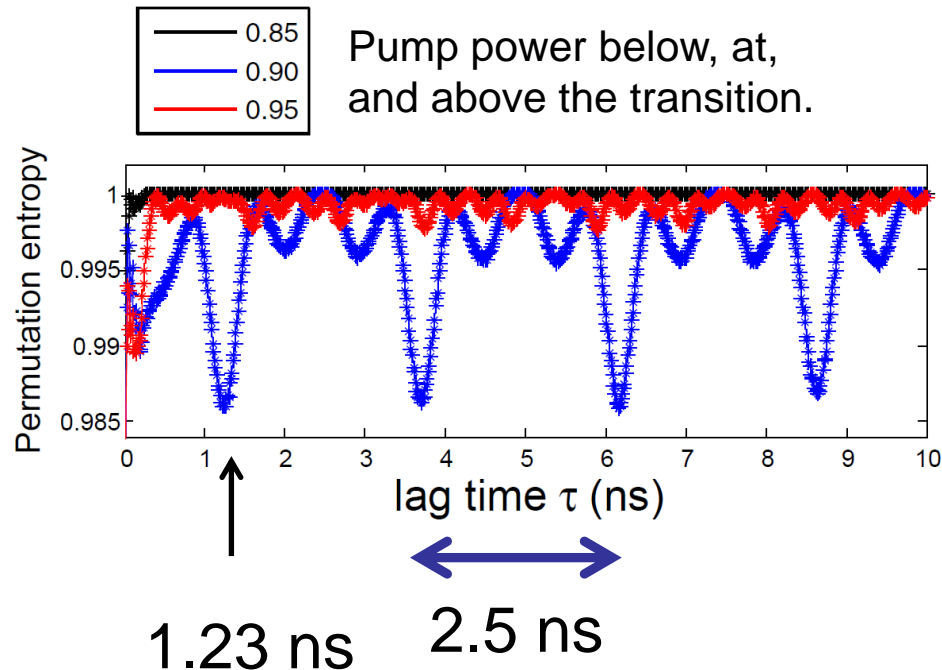
stronger  
correlations

Can we extract more  
information from the  
data?

# Ordinal analysis of **lagged** raw data

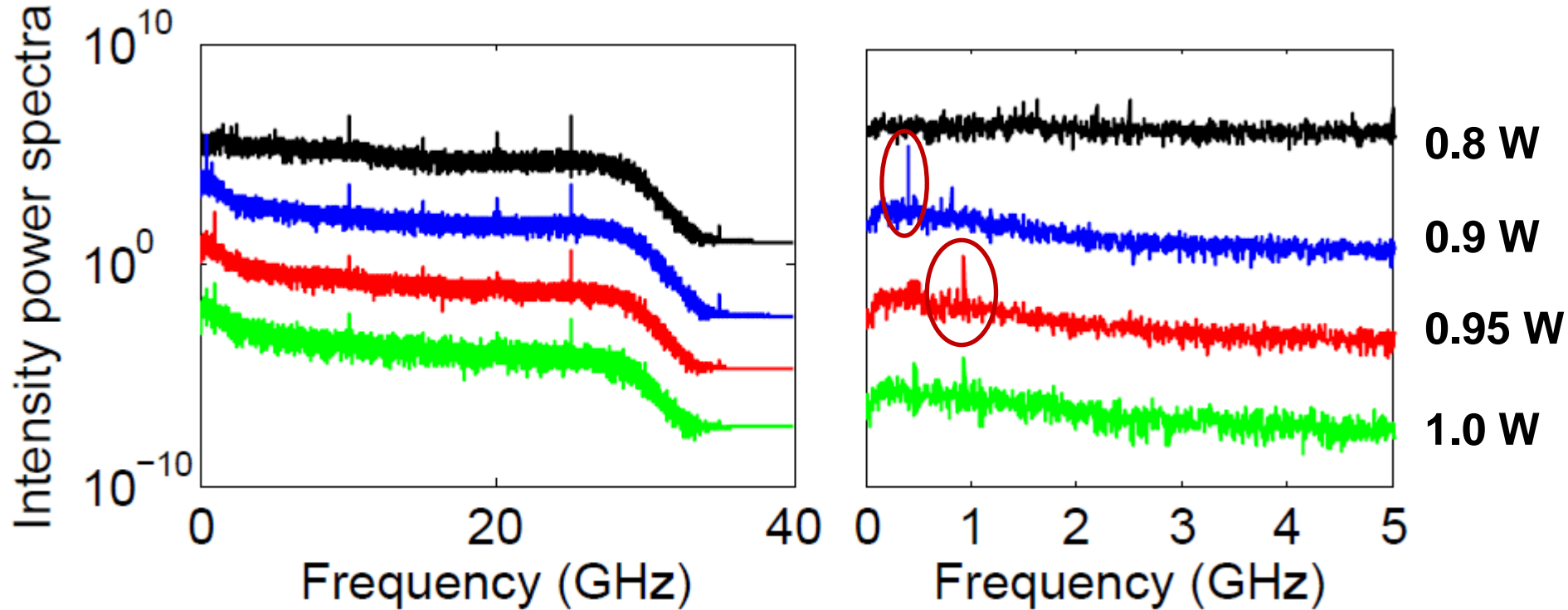
$$\{I_i, I_{i+\tau}, I_{i+2\tau}, \dots\},$$

all data points, no threshold used



$\Rightarrow$  Sharp variation not captured by correlation analysis.

# Intensity power spectra (displaced vertically for clarity)



At 0.9 W:  $\nu_0 = 0.4$  GHz ( $T_0 = 2.5$  ns)  
0.81 GHz  $\rightarrow$  1.23 ns

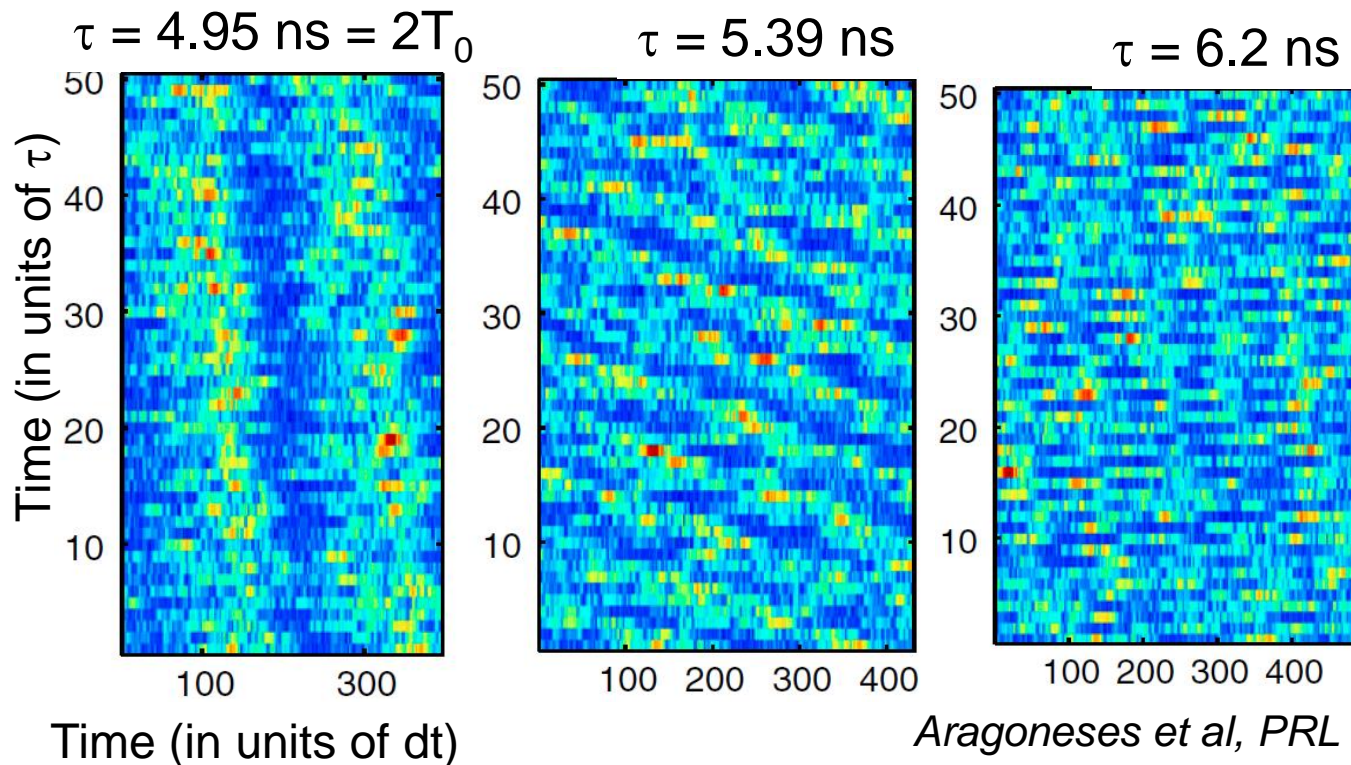
At 0.95W: 0.93 GHz



# Space time representation

$$\left\{ \begin{array}{c} \xleftrightarrow{\tau} \\ I_1, I_2, I_3, \dots, I_\tau, I_{\tau+1}, \dots, I_{2\tau}, I_{2\tau+1}, \dots \end{array} \right\} \Rightarrow \begin{bmatrix} \dots & \dots & \dots & \dots \\ I_{2\tau+1} & I_{2\tau+2} & \dots & I_{3\tau} \\ I_{\tau+1} & I_{\tau+2} & \dots & I_{2\tau} \\ I_1 & I_2 & \dots & I_\tau \end{bmatrix}$$

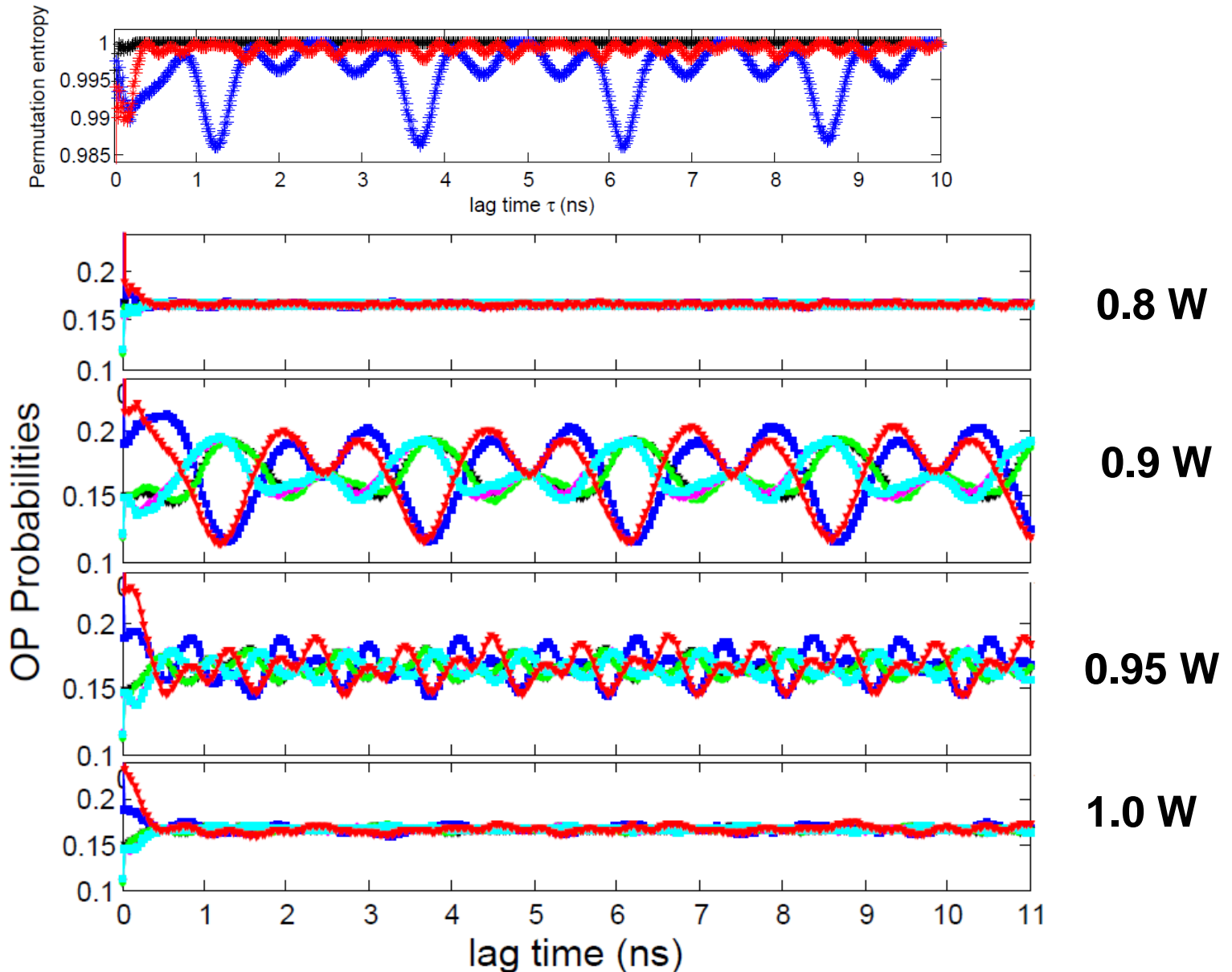
Color:  $I_i$



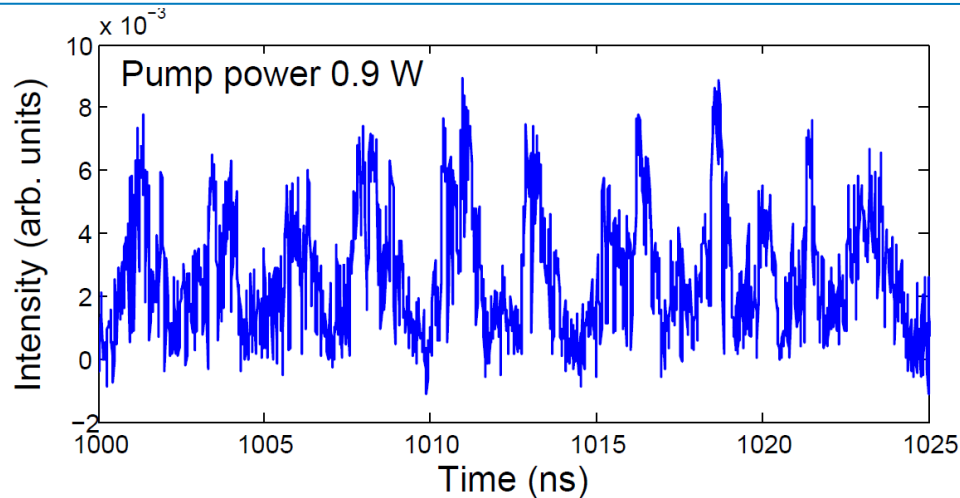
$\Rightarrow$  Different structures uncovered with different lags (sampling times).



# Ordinal probabilities vs. lag



# Minimal model that represents ordinal probabilities at the transition



Stochastic limit cycle:  
phase rate equation

$$\frac{d\varphi}{dt} = \omega_0 + f(\varphi, t) + \xi$$

Stroboscopic sampling at time-interval  $\tau$

$$\varphi(t_0 + \tau) = \varphi(t_0) + \omega_0 \tau + F(\varphi(t_0)) + \xi$$

$$\varphi_{i+1} = \varphi_i + \underbrace{\rho}_{\text{red circle}} + \frac{K}{2\pi} \sin(2\pi\varphi_i) + D\xi$$

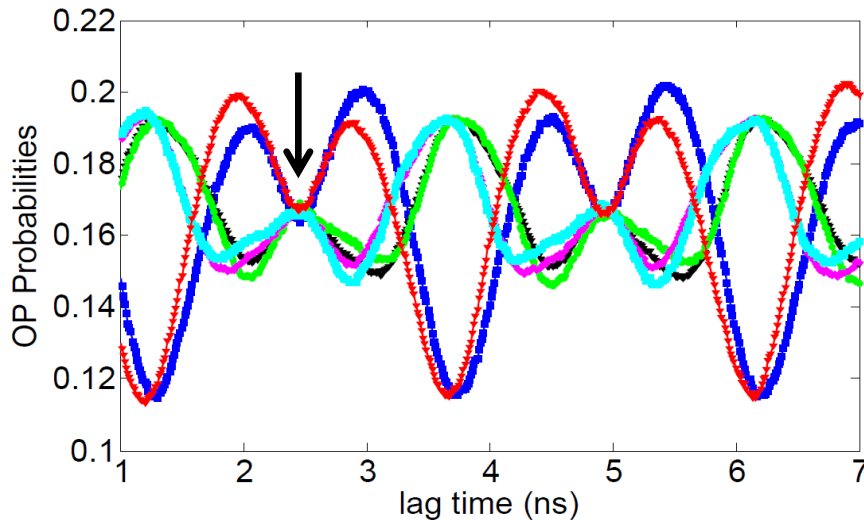
# Circle map: minimal model of ordinal probabilities at the transition

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi} \sin(2\pi\varphi_i) + D\xi$$

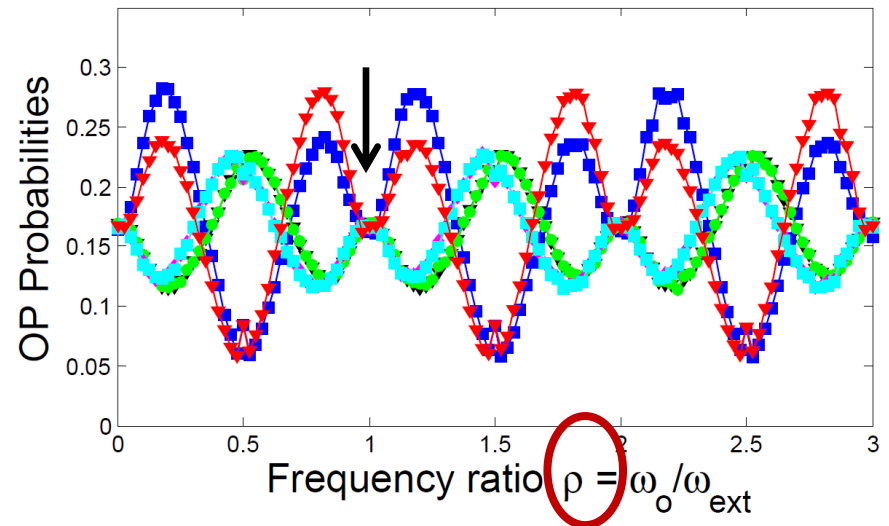
$$\{X_i, X_{i+1}, X_{i+2}, \dots\}$$

$$X_i = \varphi_{i+1} - \varphi_i$$

**Fiber laser data**  $\{I_i, I_{i+\tau}, I_{i+2\tau}, \dots\}$   
(at the transition, pump power 0.9 W)



**Synthetic data (phase increments)**  
(K=0.23 & D=0.02)



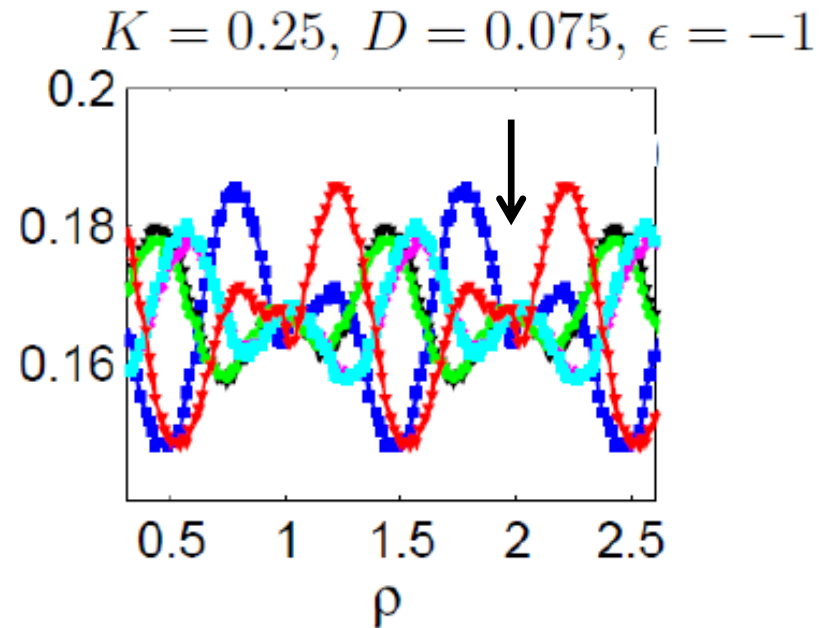
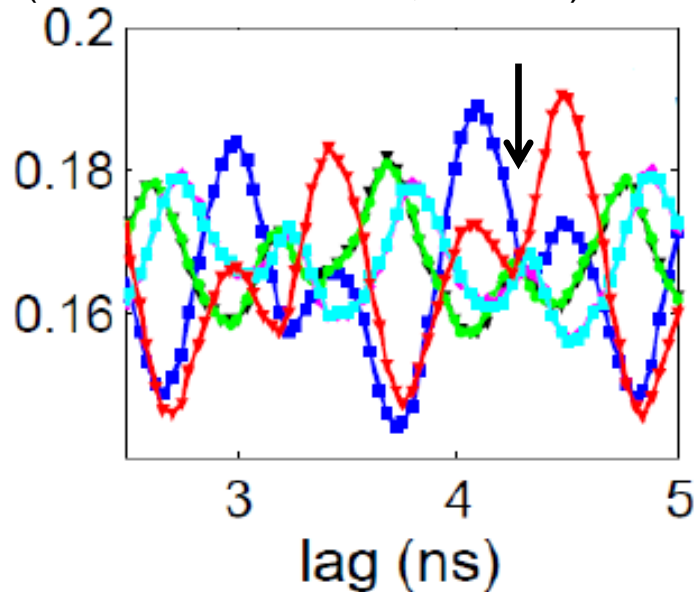
$$\tau = 2.5 \text{ ns} \Leftrightarrow \rho = 1 = \nu_0 \tau \Rightarrow \nu_0 = 1/\tau = 0.4 \text{ GHz}$$

# Good agreement also at higher pump power

$$\phi_{i+1} = \phi_i + \epsilon\rho + (K/2\pi) \sin(2\pi\phi_i) + D\xi_i$$

## Fiber laser data

(above the transition, 0.95 W)



$$\tau = 4.3 \text{ ns} \Leftrightarrow \rho = 2 = \nu_0 \tau \Rightarrow \nu_0 = 1/(2\tau) = 0.46 \text{ GHz}$$

The intensity spectrum has a peak at about 0.93 GHz,  
consistent with  $2\nu_0$ .

- Nonlinear data analysis tools were applied to study the intensity dynamics during the transition to turbulence in a quasi-cw Raman fiber laser (normal dispersion fiber).
- Sharp transition seen in thresholded data but not in raw data.
- Specific time-scales detected at the transition, not captured by linear analysis.
- At the transition, minimal model identified: stochastic limit cycle dynamics.

**THANK YOU FOR YOUR ATTENTION !**

Experimental data from Prof. Turitsyn (Aston University, UK)