

# What are complex systems and what techniques can we use to analyze them?

**Cristina Masoller,  
Departament de Física, UPC**

Col·loqui FMC, Universitat de Barcelona  
March 27, 2023



**UNIVERSITAT POLITÈCNICA  
DE CATALUNYA  
BARCELONATECH**

*Campus d'Excel·lència Internacional*



# Presentation

- Bachelor & Master in Physics (1986-1991)
- PhD in Physics (1999, Bryn Mawr College, PA, USA)
- At Physics Department UPC since 2004 (Ramon I Cajal, Prof. Agregat, Prof. Catedratica).



UNIVERSIDAD DE LA REPÚBLICA  
URUGUAY

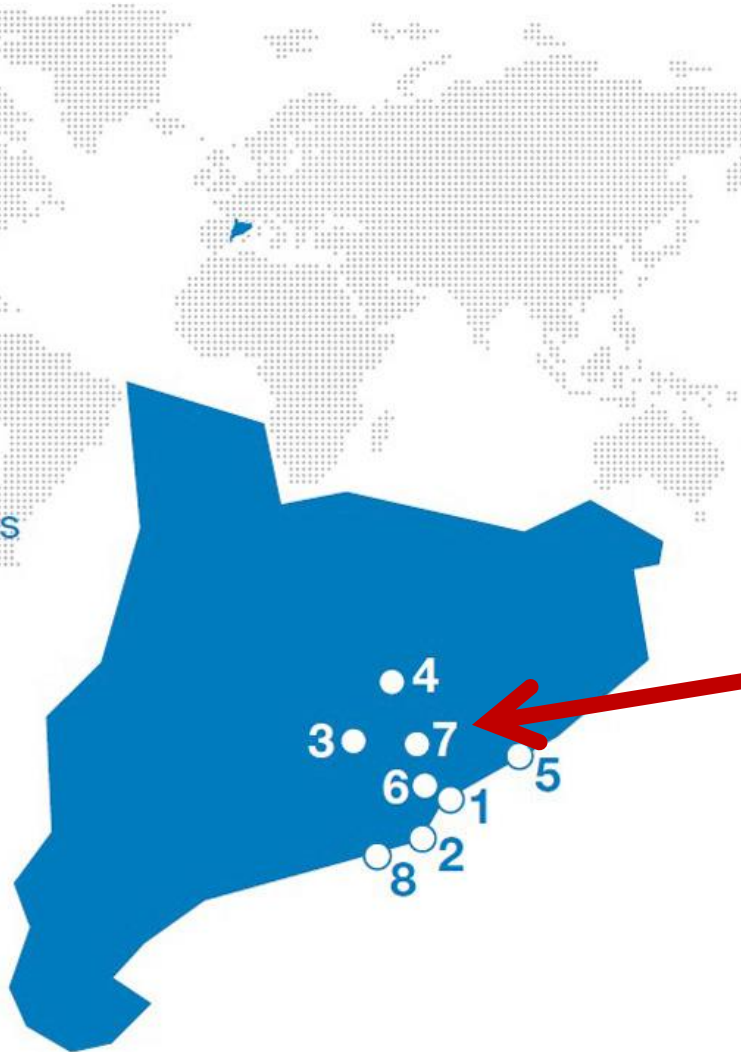
**BRYN MAWR**  
COLLEGE



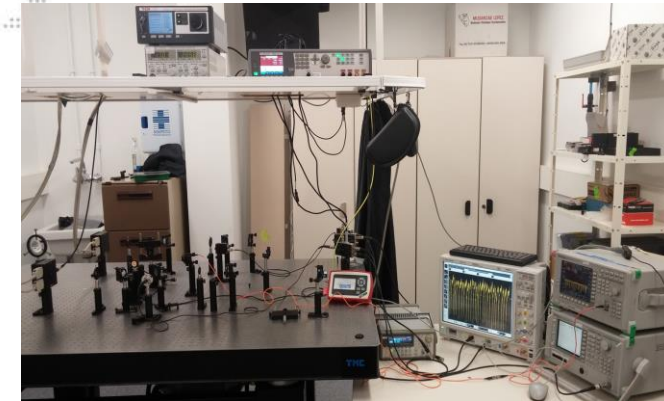
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# Where are we? UPC Campus Terrassa

1. Barcelona
2. Castelldefels
3. Igualada
4. Manresa
5. Mataró
6. Sant Cugat del Vallès
7. Terrassa
8. Vilanova i la Geltrú

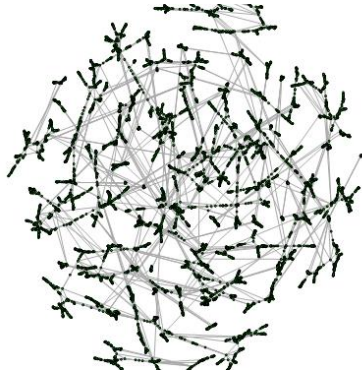
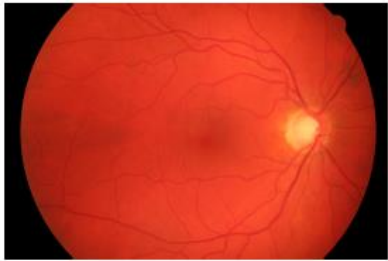
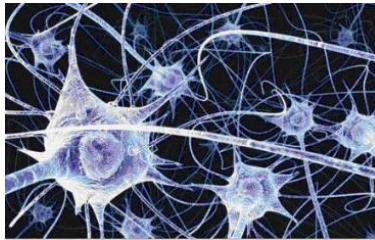


El edificio Gaia centraliza grupos científicos consolidados y emergentes.



Laser lab in Gaia Building,  
UPC Campus Terrassa

# Research lines

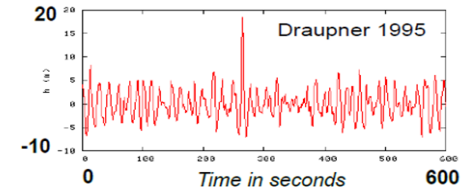
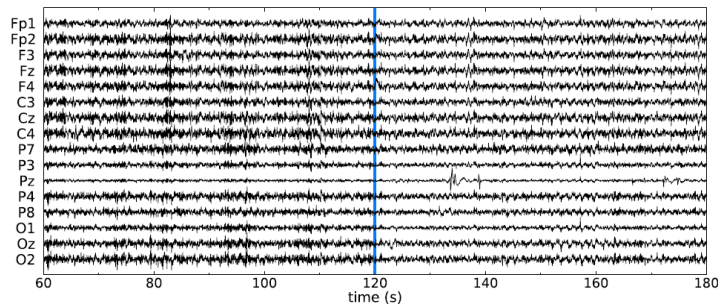
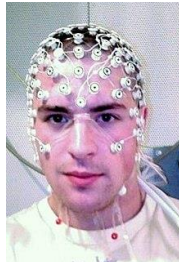
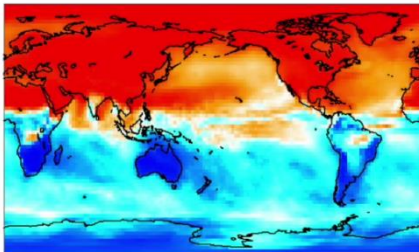


**Nonlinear  
dynamics  
and complex  
systems**

**Data  
analysis  
techniques**

**Applications**

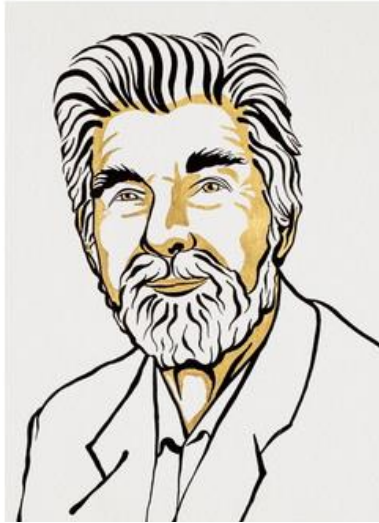
1 July



# Outline

- Complex systems and data analysis
- Ordinal analysis: Lasers and neurons
- Hilbert analysis: Climate data
- Network analysis: Retina fundus images

# The Nobel Prize in Physics 2021

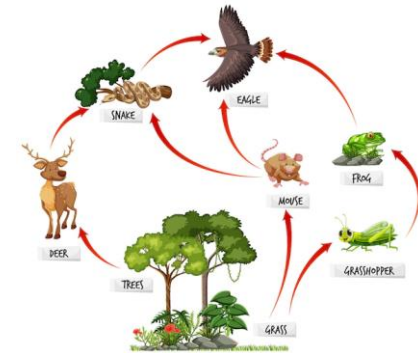
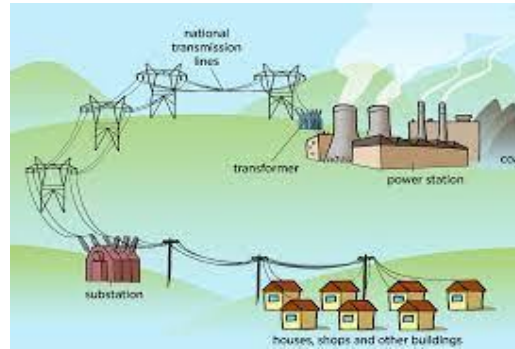
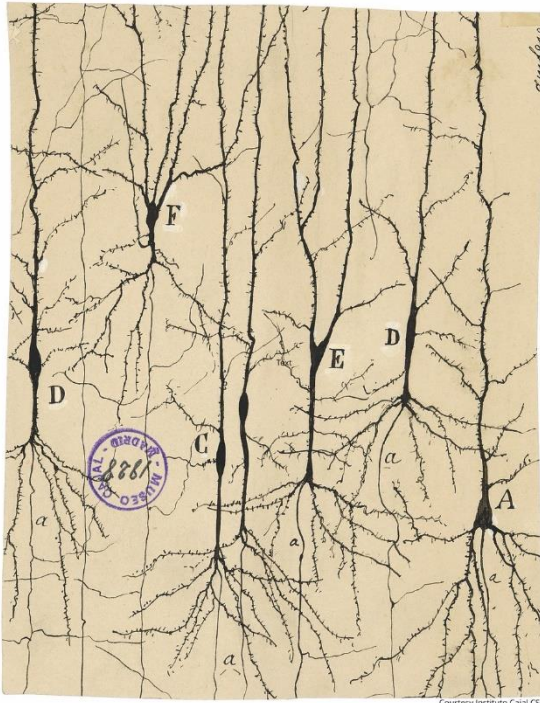


for groundbreaking contributions to our understanding of **complex systems**

½ Syukuro Manabe and Klaus Hasselmann      ½ Giorgio Parisi

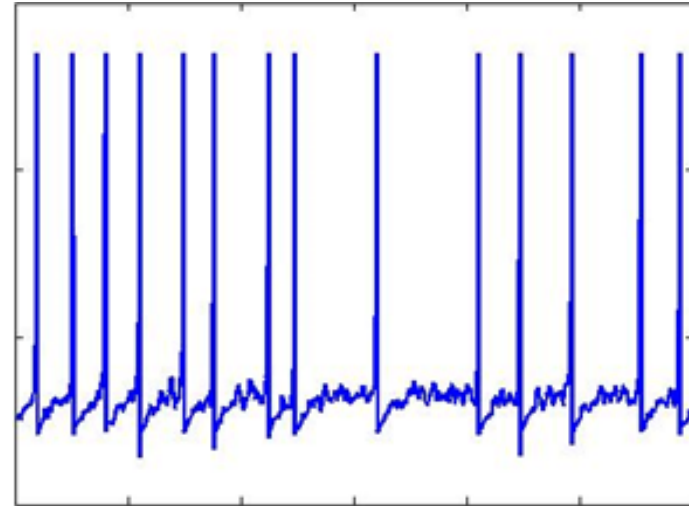
# What is a complex system?

- High-dimensional, large number of interacting elements, heterogeneous structure, multiscale, memory, adaptation.
- The elements and/or the interactions are **nonlinear**.
- Often display abrupt transitions and extreme events.



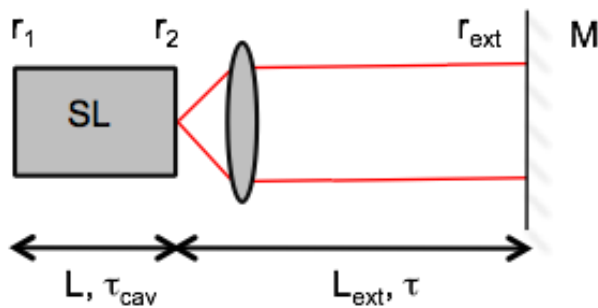
G. Bianconi et al, *Complex systems in the spotlight: next steps after the 2021 Nobel Prize in Physics*, J. of Phys: Complexity 4, 010201 (2023).

# Data analysis methods allow to discover statistical similarities in very different systems



Time  $10^{-9}$  s

Time  $10^{-3}$  s



(High-dimensional, Memory)

$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$

$$\frac{dy}{dt} = x + a + D\xi(t).$$

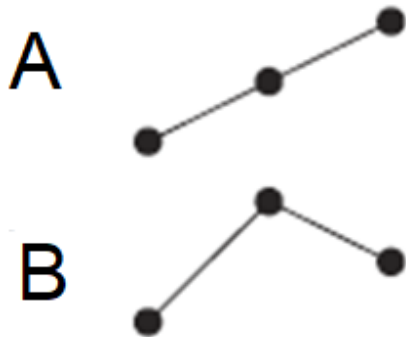


# First data analysis method: ordinal analysis

$$\{\dots X_i, X_{i+1}, X_{i+2}, \dots\}$$

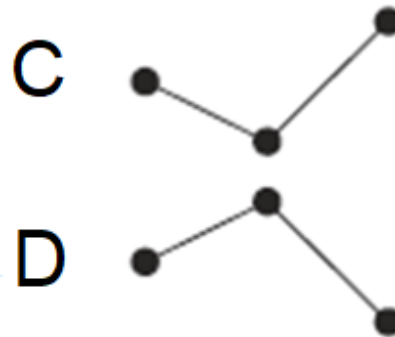
Possible order relations among three numbers (e.g., 2, 5, 7)

$$\{\dots 2, 5, 7 \dots\}$$



$$\{\dots 2, 7, 5 \dots\}$$

$$\{\dots 5, 2, 7 \dots\}$$



$$\{\dots 5, 7, 2 \dots\}$$

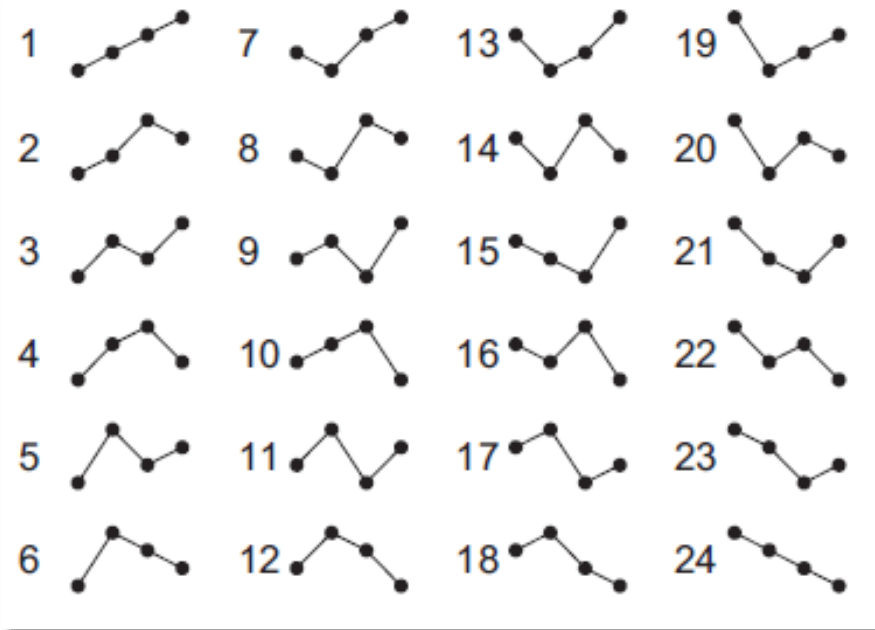
$$\{\dots 7, 2, 5 \dots\}$$



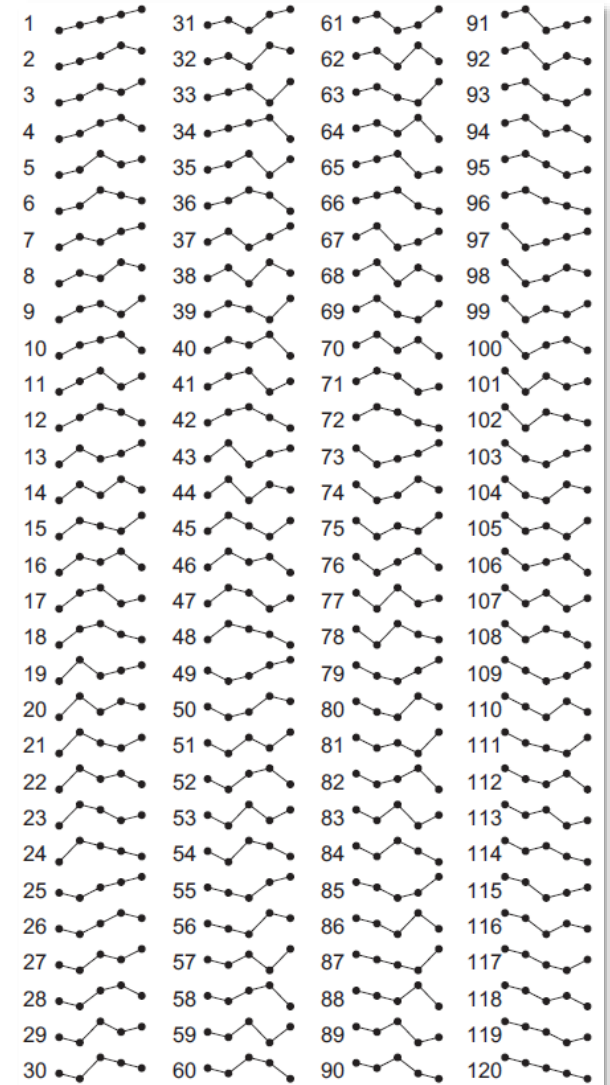
$$\{\dots 7, 5, 2 \dots\}$$

Bandt and Pompe: Phys. Rev. Lett. 2002

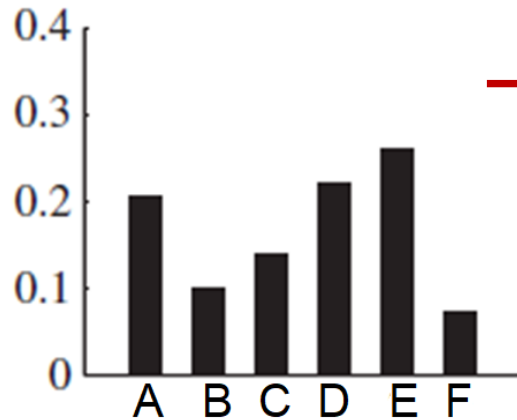
# The number of ordinal patterns increases as D!



A problem for short datasets.



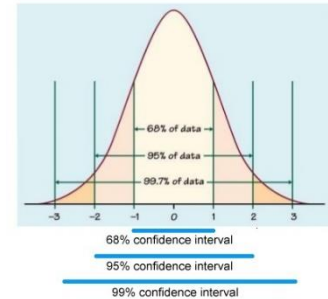
From a time series, by counting the different patterns, we can calculate the set of “ordinal probabilities”



?

- A. Analyze the probabilities (are differences statistically significant?)
- B. Compute information theory measures (entropy, complexity)

Probability from surrogates



$$H = -\sum_{i=1}^N p_i \ln p_i$$

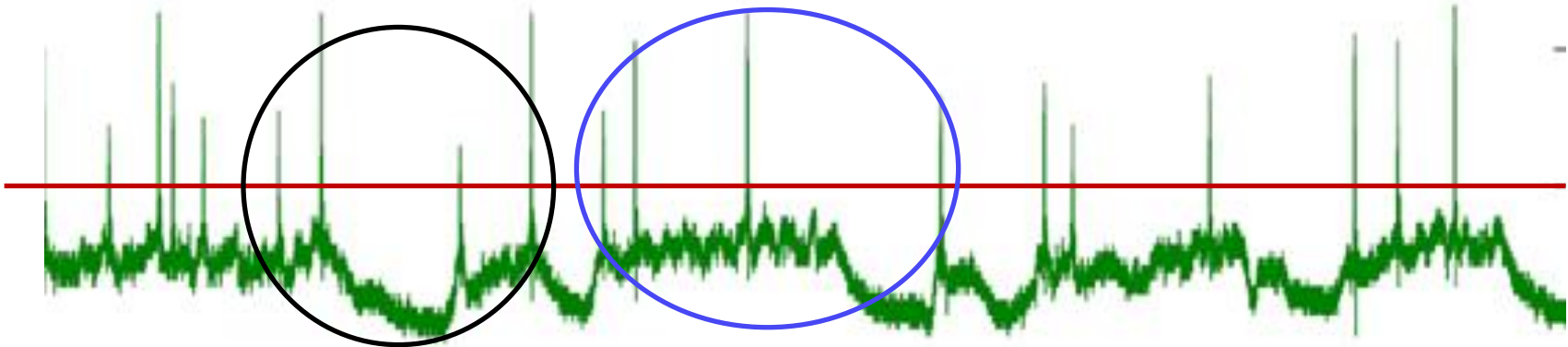
Ordinal analysis has been extensively used:

- to test if a model is good for the data,
- to fit the model's parameters,
- to classify different types of data based on similarities of probabilities of ordinal patterns.

I. Leyva, J. M. Martinez, C. Masoller, O. A. Rosso, M. Zanin, “20 Years of Ordinal Patterns: Perspectives and Challenges”, EPL 138, 31001 (2022).

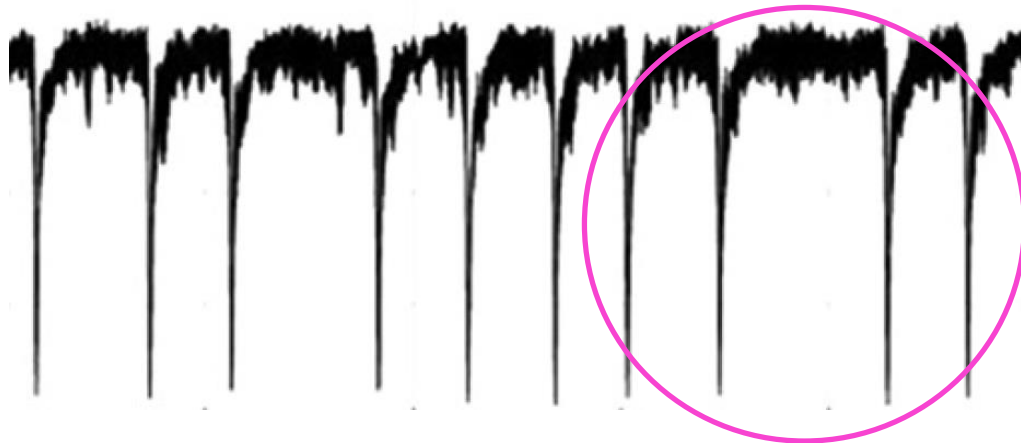
# Sequence of inter-spike-intervals (ISIs) $\Rightarrow$ sequence of ordinal patterns

**D=3**

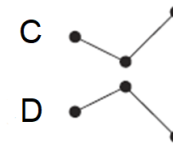
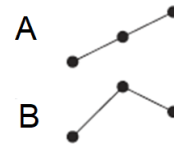


**021=B**

**012=A**



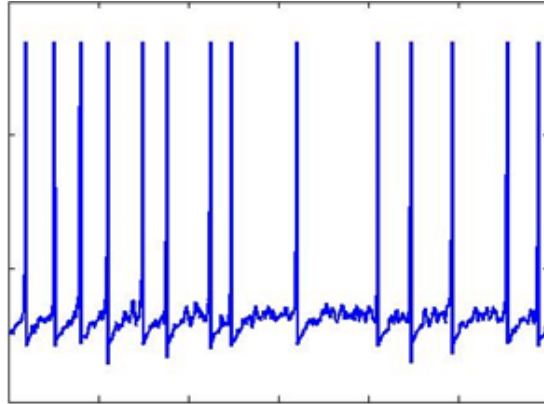
**120=D**



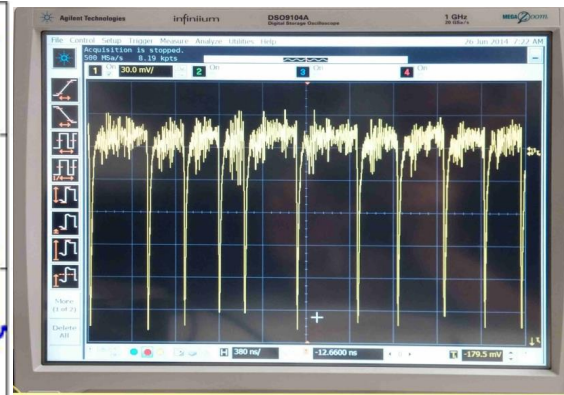
# The analysis of the ordinal probabilities uncovers similarities in ISI sequences



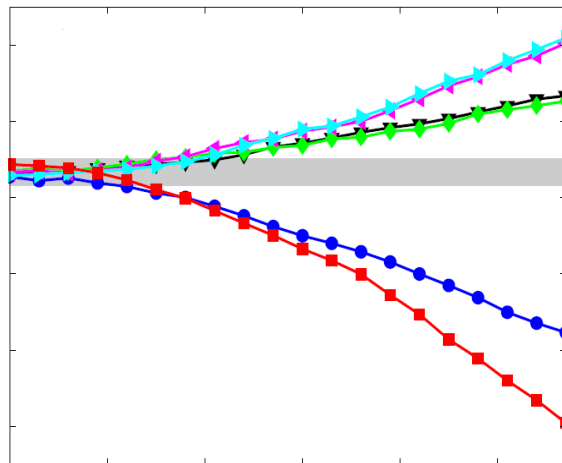
## Neuron model



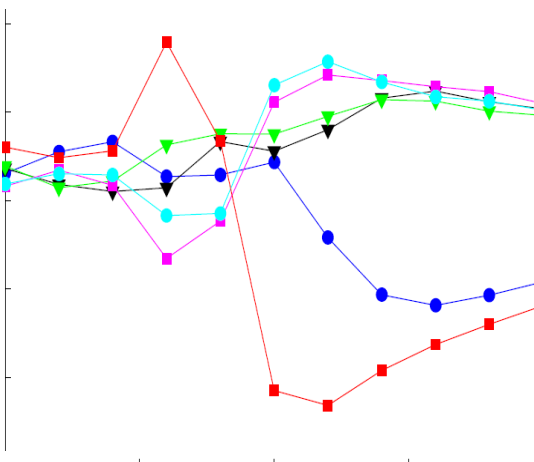
## Diode laser



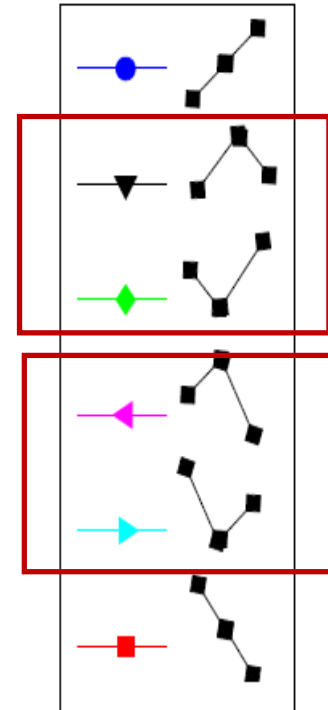
Ordinal probabilities



Forcing amplitude



Forcing amplitude



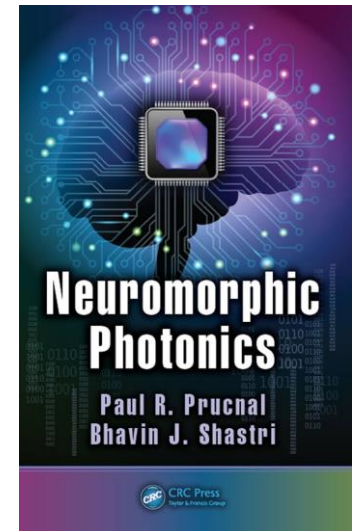
J. M. Aparicio-Reinoso et al PRE 94, 032218 (2016) A. Aragonese et al, Sci. Rep. 4, 4696 (2014)

# Uncovering similarities between neurons and lasers... Interesting but relevant?

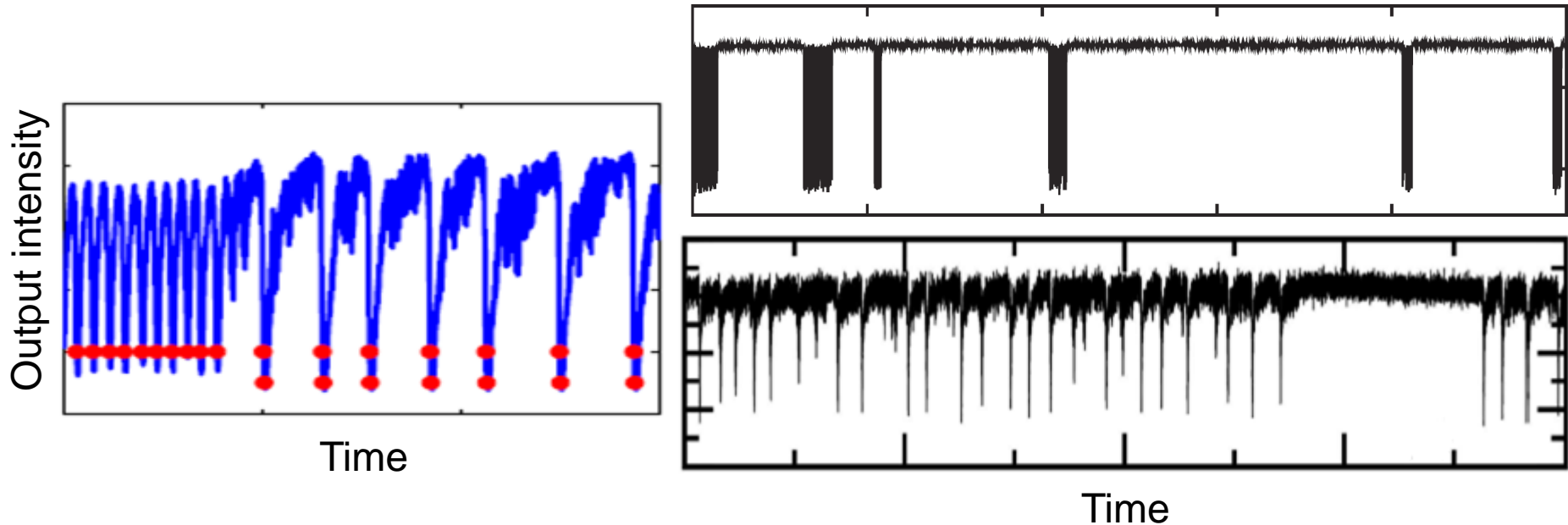
- Data centers, AI systems, HPC consume huge amounts of energy.
- Big concern in the context of climate change.
- The human brain processes huge amounts of information using only 19 Watts.
- Uncovering genuine similarities between neurons and lasers will allow to develop **photonic neurons**, able to process information as real neurons do, but
  - much faster,
  - with much less energy consumption.



*European Centre for Medium-Range Weather Forecasts, Reading, UK*



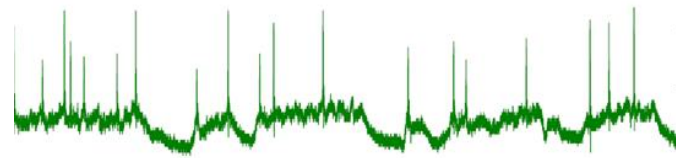
# Time series recorded in our lab show excitability, tonic spikes, and bursting. Similar to real neurons?



A. Aragonese, S. Perrone, T. Sorrentino, M. C. Torrent and C. Masoller, "*Unveiling the complex organization of recurrent patterns in spiking dynamical systems*", Sci. Rep. **4**, 4696 (2014).

C. Quintero-Quiroz, J. Tiana-Alsina, J. Roma, M. C. Torrent, and C. Masoller, "*Characterizing how complex optical signals emerge from noisy intensity fluctuations*", Sci. Rep. **6** 37510 (2016).

# Main challenge

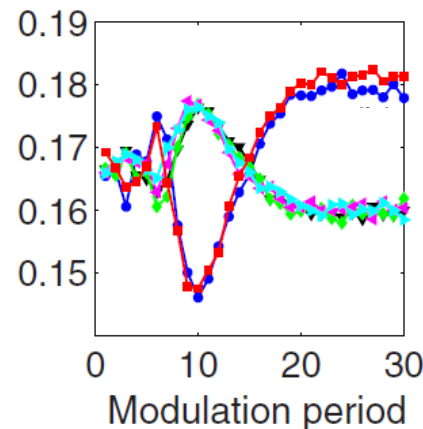
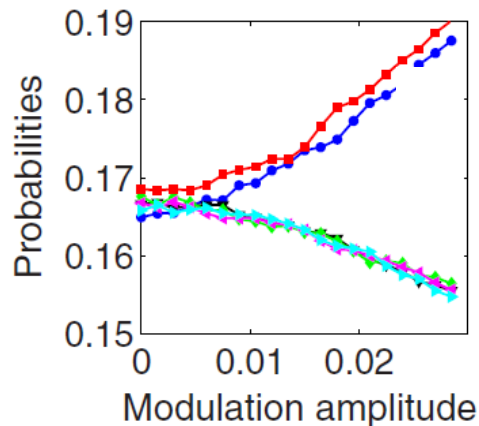


Understand how to mimic with lasers the way neurons encode and process information.

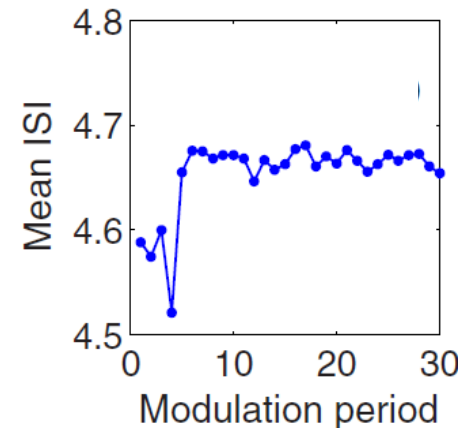
$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$

Weak, subthreshold signal

$$\frac{dy}{dt} = x + a + a_o \cos(2\pi t/T) + D\xi(t),$$



Rate coding?



J. A. Reinoso, M. C. Torrent, and C. Masoller, “*Emergence of spike correlations in periodically forced excitable systems*”, Phys. Rev. E. 94, 032218 (2016).





# Single-neuron vs ensemble encoding

- Single-neuron encoding: **slow** because long spike sequences are needed to estimate the ordinal probabilities.
- Ensemble encoding: can be **fast** because, from the ISI sequences of all the neurons, few spikes per neuron can be enough to accurately estimate the probabilities.

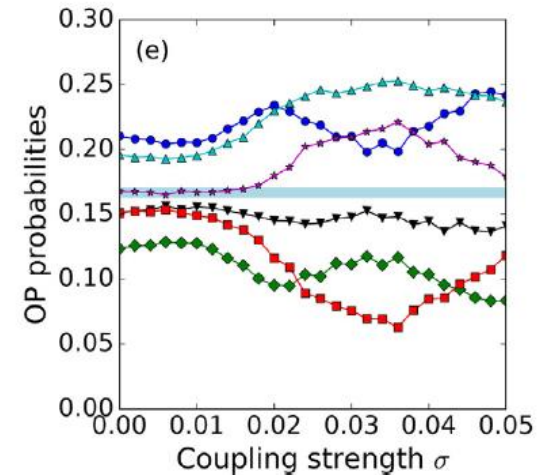
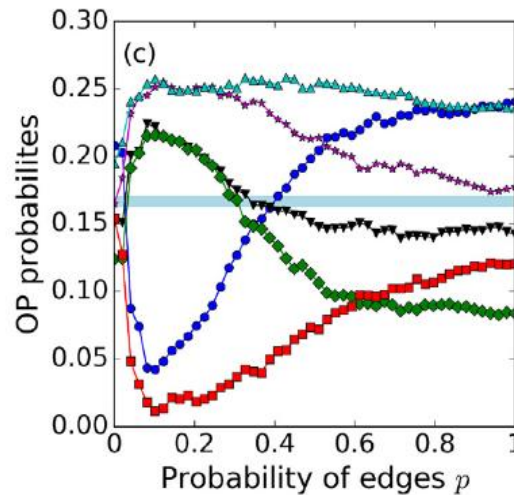
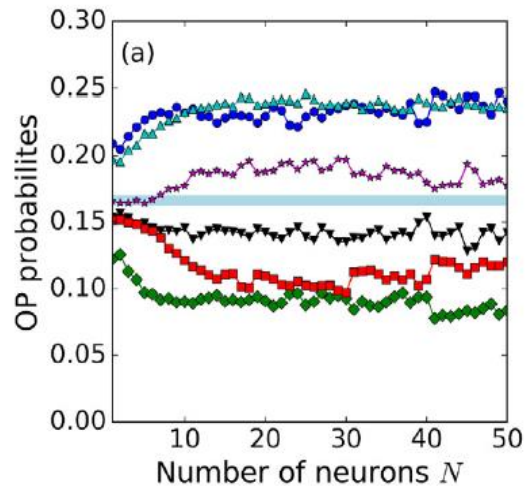
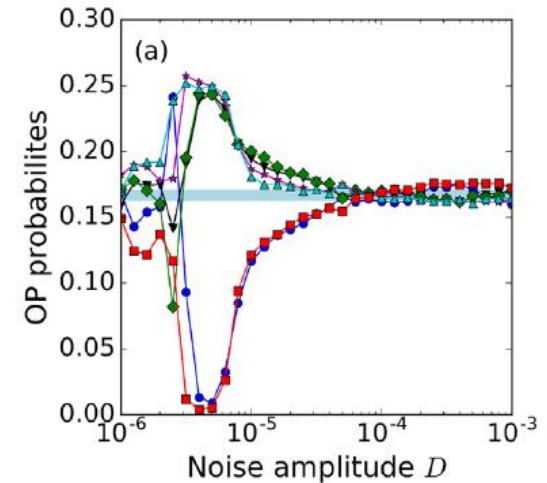
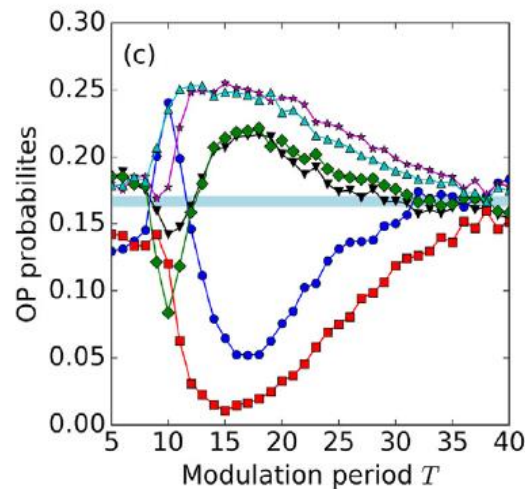
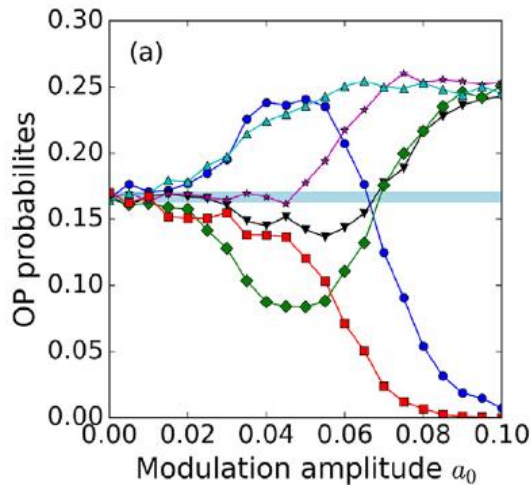
$$\epsilon \dot{u}_i = u_i - \frac{u_i^3}{3} - v_i + a_0 \cos(2\pi t/T) + \frac{\sigma}{k_i} \sum_j^N a_{ij} (u_j - u_i) + \sqrt{2D} \xi_i(t), \quad i \neq j$$
$$\dot{v}_i = u_i + a.$$

Weak, subthreshold signal

$$k_i = \sum_j a_{ij}$$
$$a_{ij} = a_{ji} = 1$$
$$a_{ij} = a_{ji} = 0$$

M. Masoliver and C. Masoller, “Neuronal coupling benefits the encoding of weak periodic signals in symbolic spike patterns”, Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).

# Ensemble encoding of a weak sinusoidal signal in the frequencies of occurrence of ordinal patterns



M. Masoliver and C. Masoller, Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).

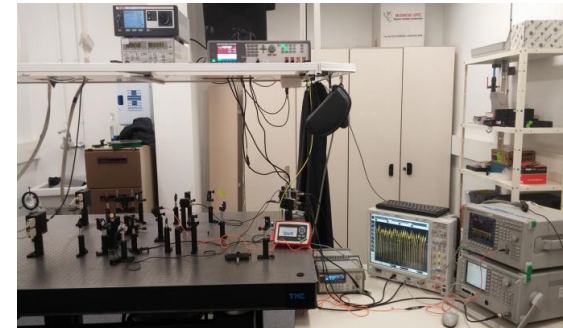
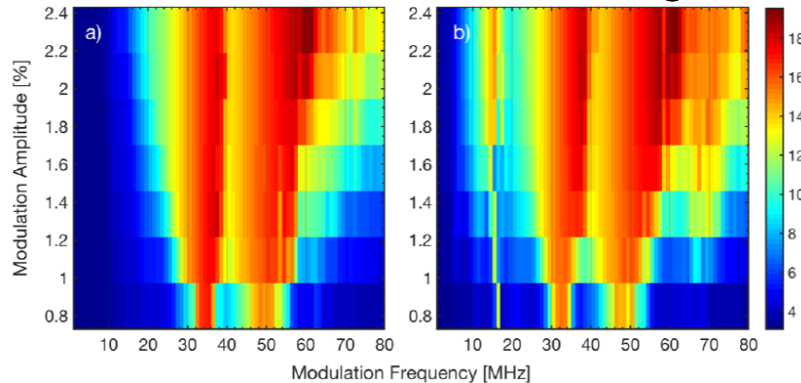
# Laser-neuron comparison: encoding a weak periodic signal using spike rate code



Spike rate in color code

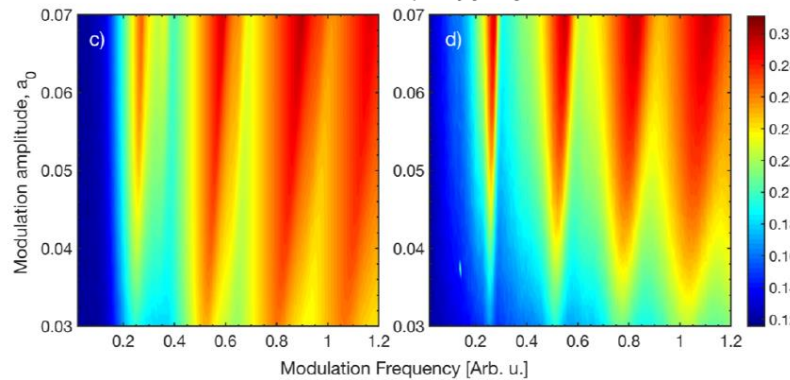
Sinusoidal

Pulsed signal



Experiments modulating the laser current

Neuron model with the same input signal



$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$

$$\frac{dy}{dt} = x + a + D\xi(t).$$

J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, “Comparing the dynamics of periodically forced lasers and neurons”, *New J. of Phys.* 21, 103039 (2019) (2019).

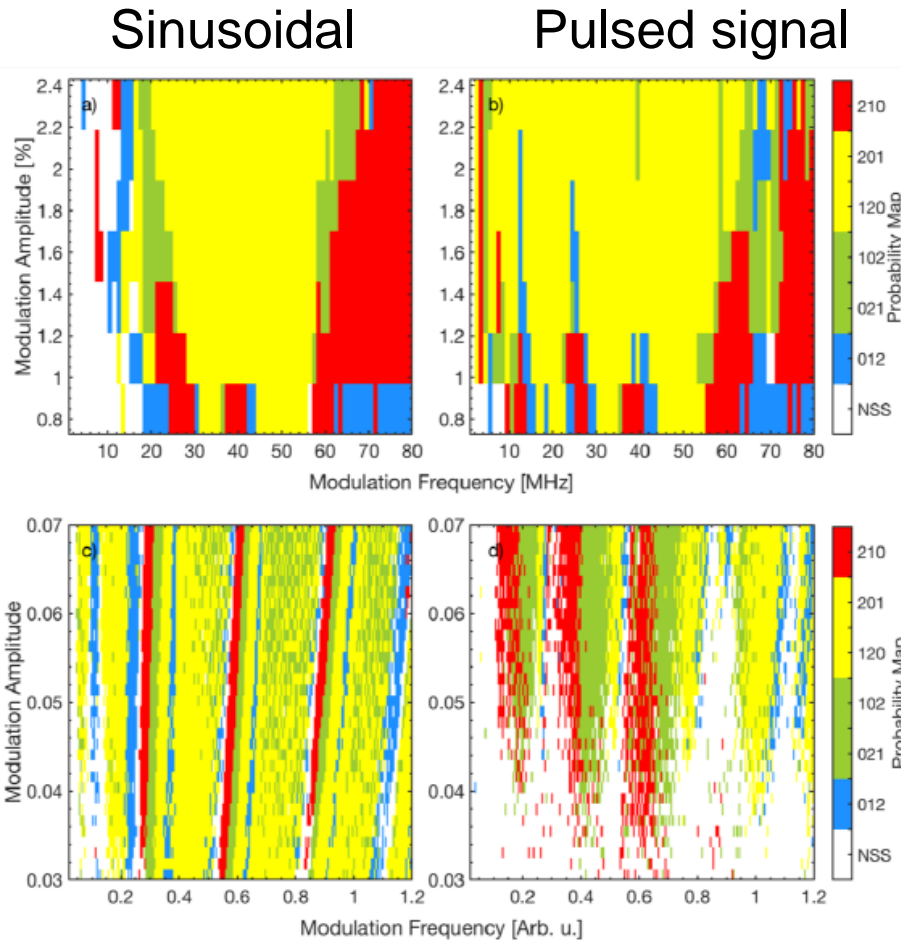
J. Tiana-Alsina, C. Masoller, “Time crystal dynamics in a weakly modulated stochastic time delayed system”, *Sci. Rep.* 12, 4914 (2022).

# How about the temporal code?

Ordinal analysis unveils differences in spike timing.

Diode  
laser with  
optical  
feedback

FitzHugh-  
Nagumo  
model



**Most  
probable  
pattern in  
color  
code**

J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, New J. of Phys. 21, 103039 (2019).

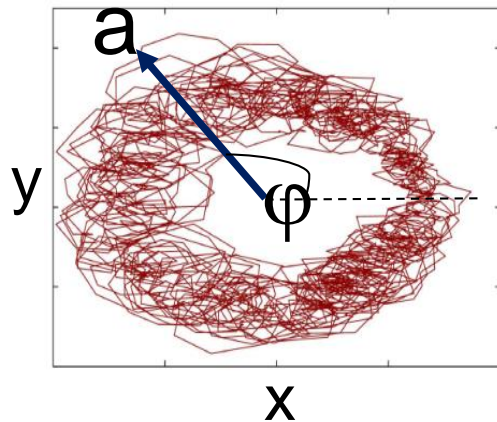
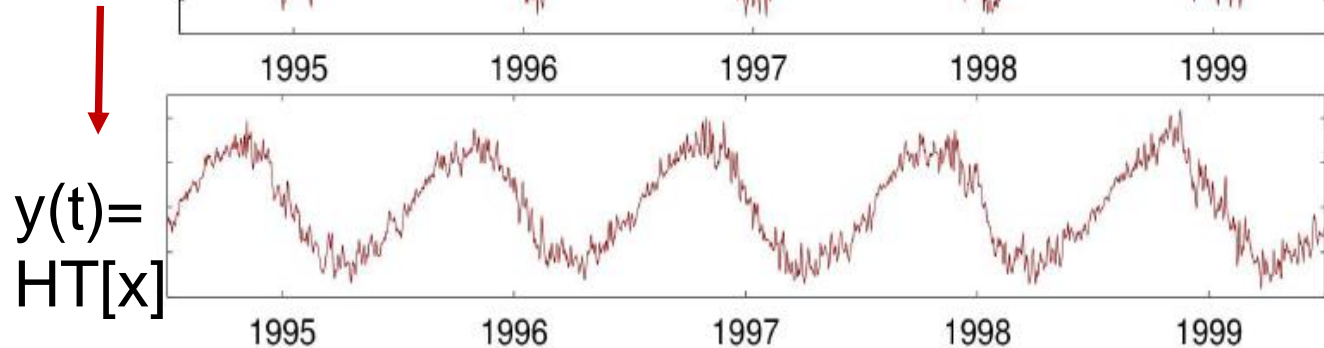
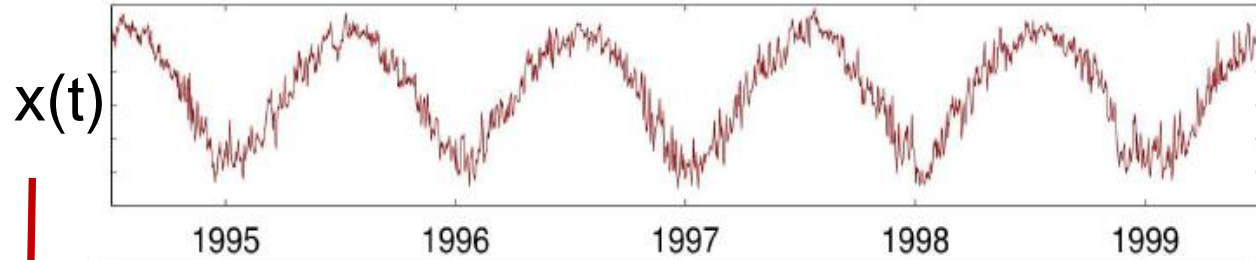
# Outline

- Complex systems and time series analysis
- Ordinal analysis: Lasers and neurons
- Hilbert analysis: Climate data
- Network analysis: Retina fundus images

# Hilbert Transform applied to Surface Air Temperature (SAT)

SAT in a geographical region

$$\text{HT}[\sin(x)] = \cos(x)$$



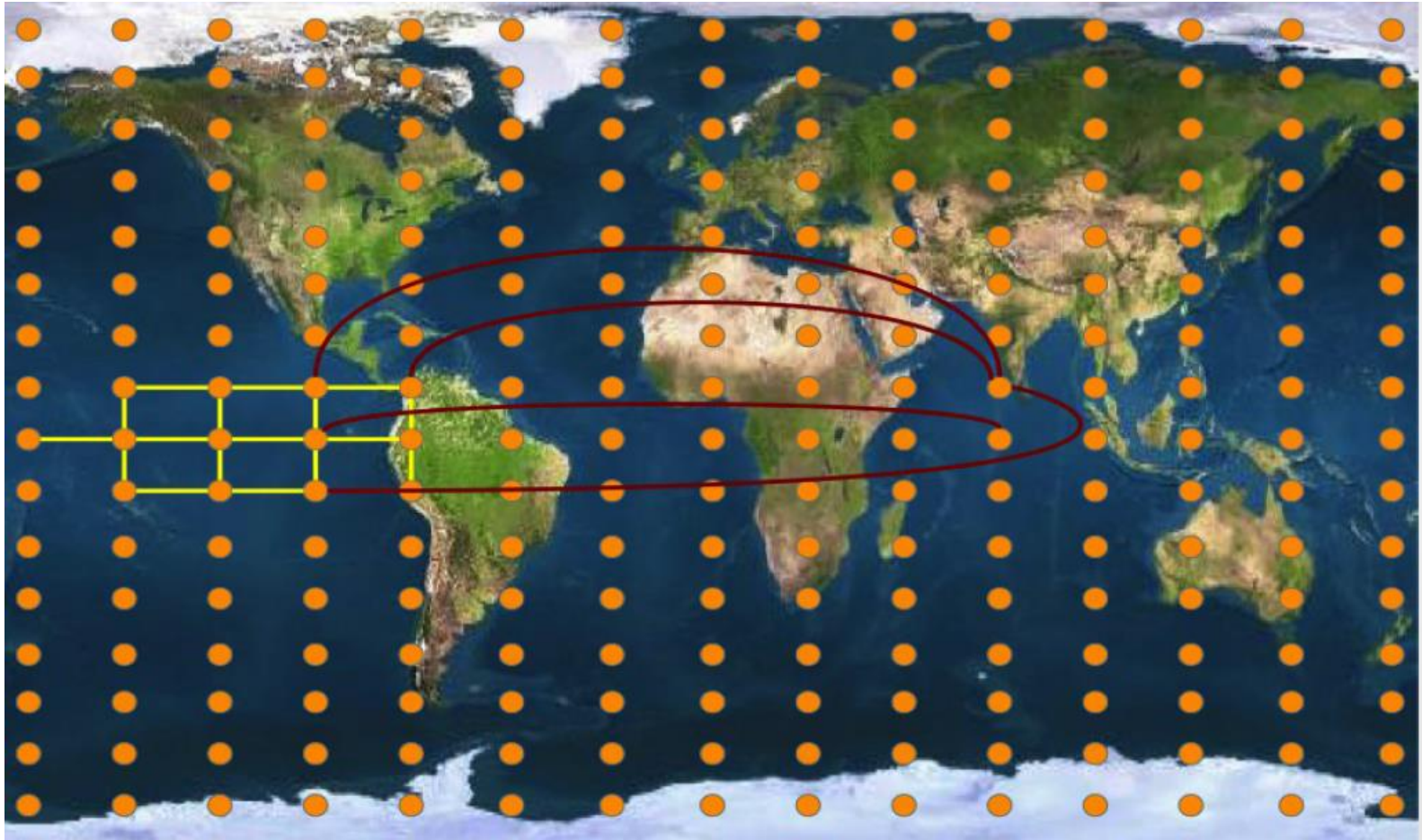
*Instantaneous amplitude and phase*

$$a(t) = \sqrt{[x(t)]^2 + [y(t)]^2}$$

$$\varphi(t) = \arctan[y(t)/x(t)]$$

Clear physical meaning only if  $x(t)$  is a narrow-band signal. Then,  $a(t)$  coincides with the **envelope** of  $x(t)$  and  $\omega(t) = d\phi/dt$ , coincides with the **main frequency** in the spectrum.

Using the HT we analyzed “re-analysis data” from the *European Centre for Medium-Range Weather Forecasts*, with high spatial and temporal resolution in the period 1979-2016

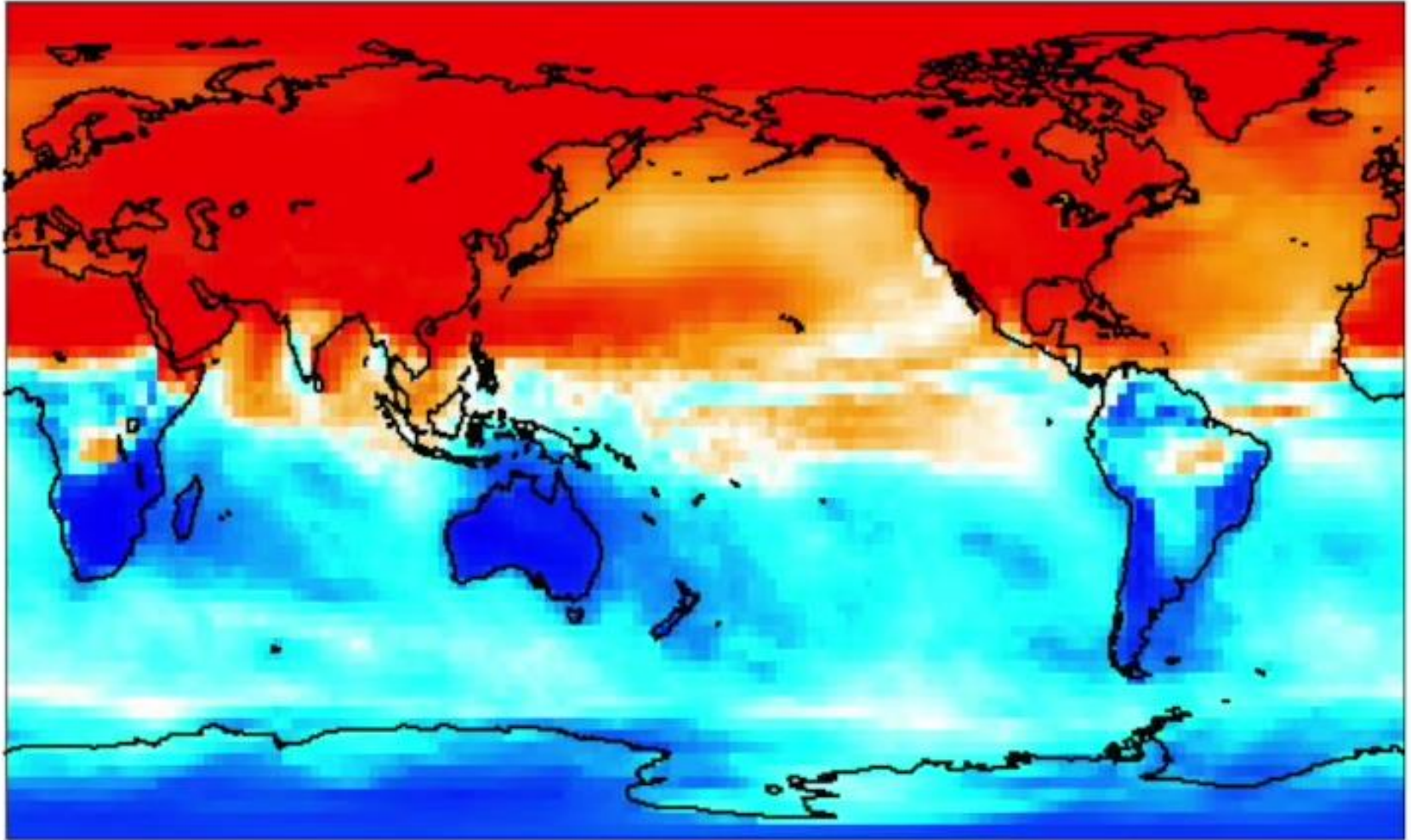


$73 \times 144 = 10\,512$  geographical sites, in each site the SAT time series has 13696 days

# Average of the cosine of the Hilbert phase



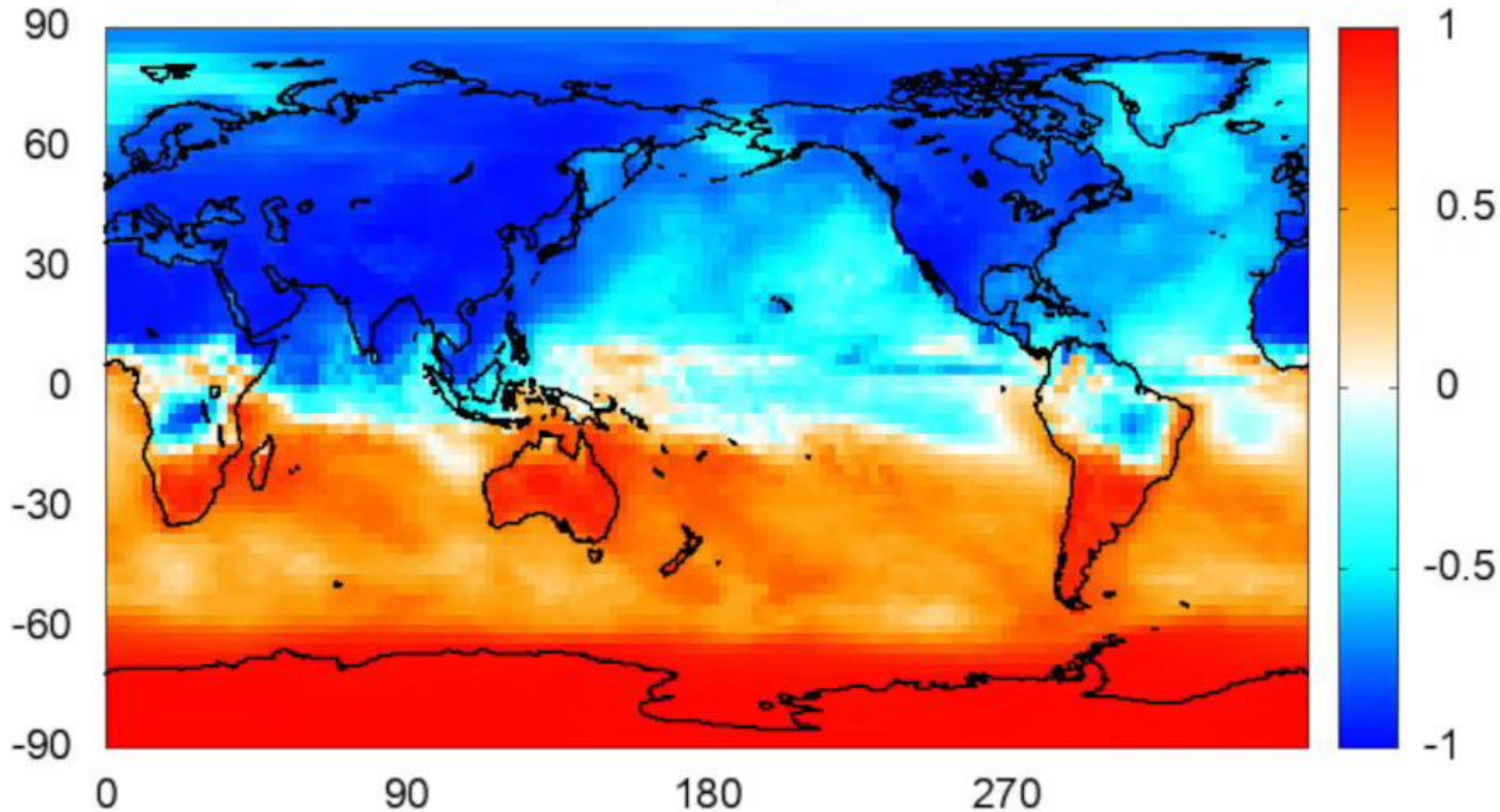
1 July





# How can we visualize the passing of the seasons? Average annual evolution of $\cos(\varphi)$ .

1 January

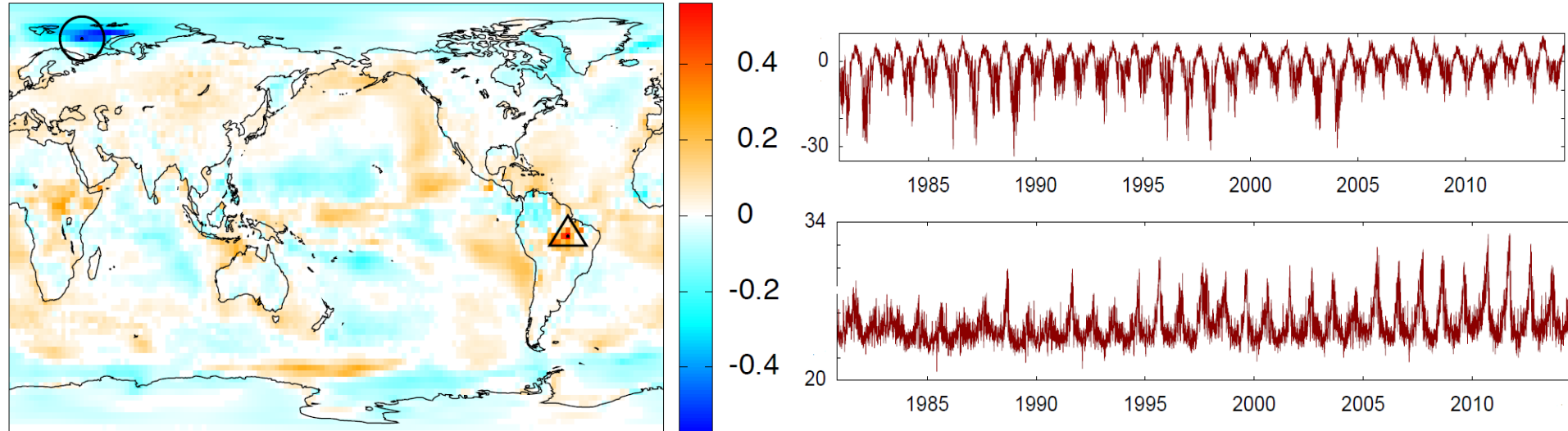


# How to detect significant changes in the last 30 years?

$$\Delta a = \langle a \rangle_{2016-2007} - \langle a \rangle_{1988-1979} \quad \frac{\Delta a}{\langle a \rangle_{2016-1979}}$$

$$\text{Significant if: } \frac{\Delta a}{\langle a \rangle} \geq \langle \cdot \rangle_s + 2\sigma_s \quad \text{or} \quad \frac{\Delta a}{\langle a \rangle} \leq \langle \cdot \rangle_s - 2\sigma_s$$

with  $\sigma_s$  computed from 100 “surrogates”



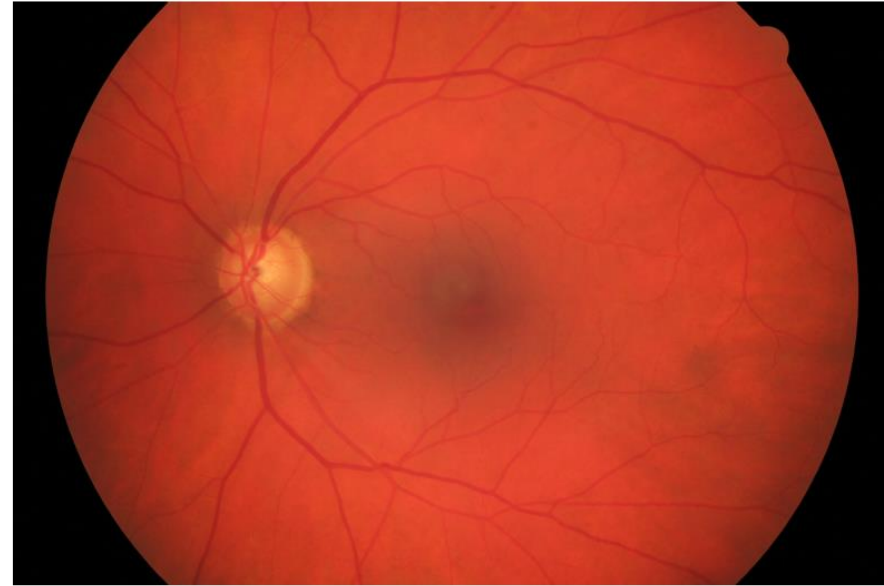
*D. A. Zappala, M. Barreiro, C. Masoller, “Quantifying changes in spatial patterns of surface air temperature dynamics over several decades”, Earth Syst. Dynam. 9, 383–391 (2018).*

# Outline

- Complex systems and time series analysis
- Ordinal analysis: Lasers and neurons
- Hilbert analysis: Climate data
- Network analysis: Retina fundus images

# Analysis of retina fundus images

- For the diagnosis of eye diseases & follow up of treatments.
- Biometric identity identification.
- Opportunity to detect other diseases (alterations in retina network may reflect alterations in other arterial systems).



**BE-OPTICAL**

*Advanced Biomedical Optical  
Imaging and Data Analysis*



H2020-675512



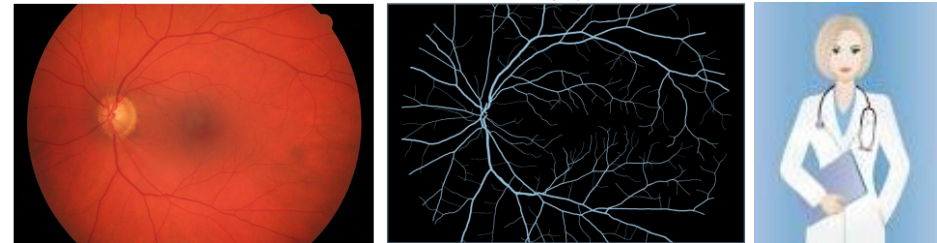
[cristina.masoller@upc.edu](mailto:cristina.masoller@upc.edu)



[@cristinamasoll1](https://twitter.com/cristinamasoll1)

# Data and image analysis steps

- 45 high resolution images (3504 × 2336 pixels)
  - 15 healthy subjects
  - 15 glaucoma
  - 15 diabetic retinopathy
- For every subject we had:
  - fundus photography
  - manual segmentation done by an expert ophthalmologist.

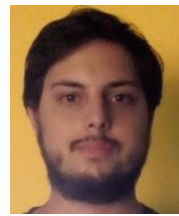
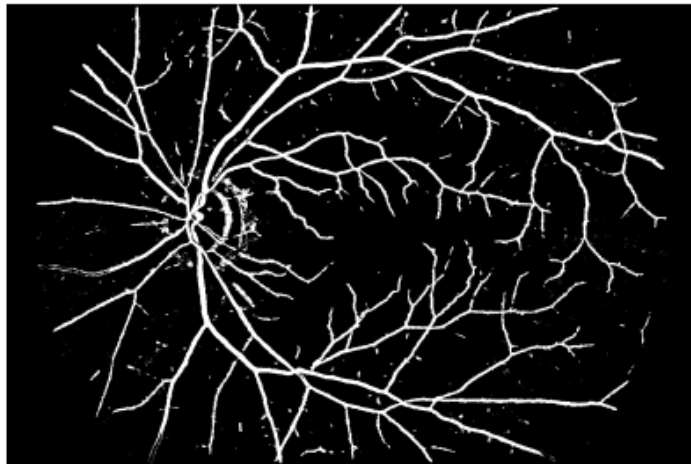
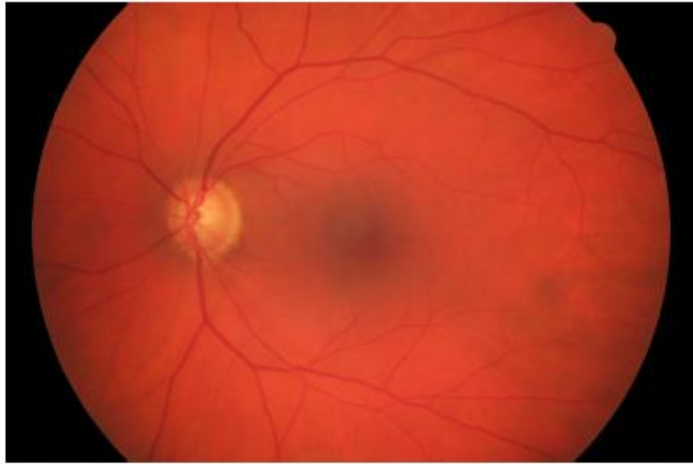


## Steps:

1. Pre-process and un-supervisedly, segment the images.
2. Extract network.
3. Compare networks obtained from different images.
4. Classify the images.

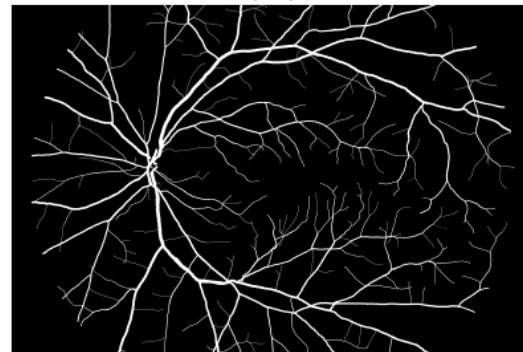
<https://www5.cs.fau.de/research/data/fundus-images/>

# Step 1: Pre-process and segmentation



We adapted an *unsupervised* algorithm, originally developed for segmenting images of cultured neuronal networks.

Manual segmentation

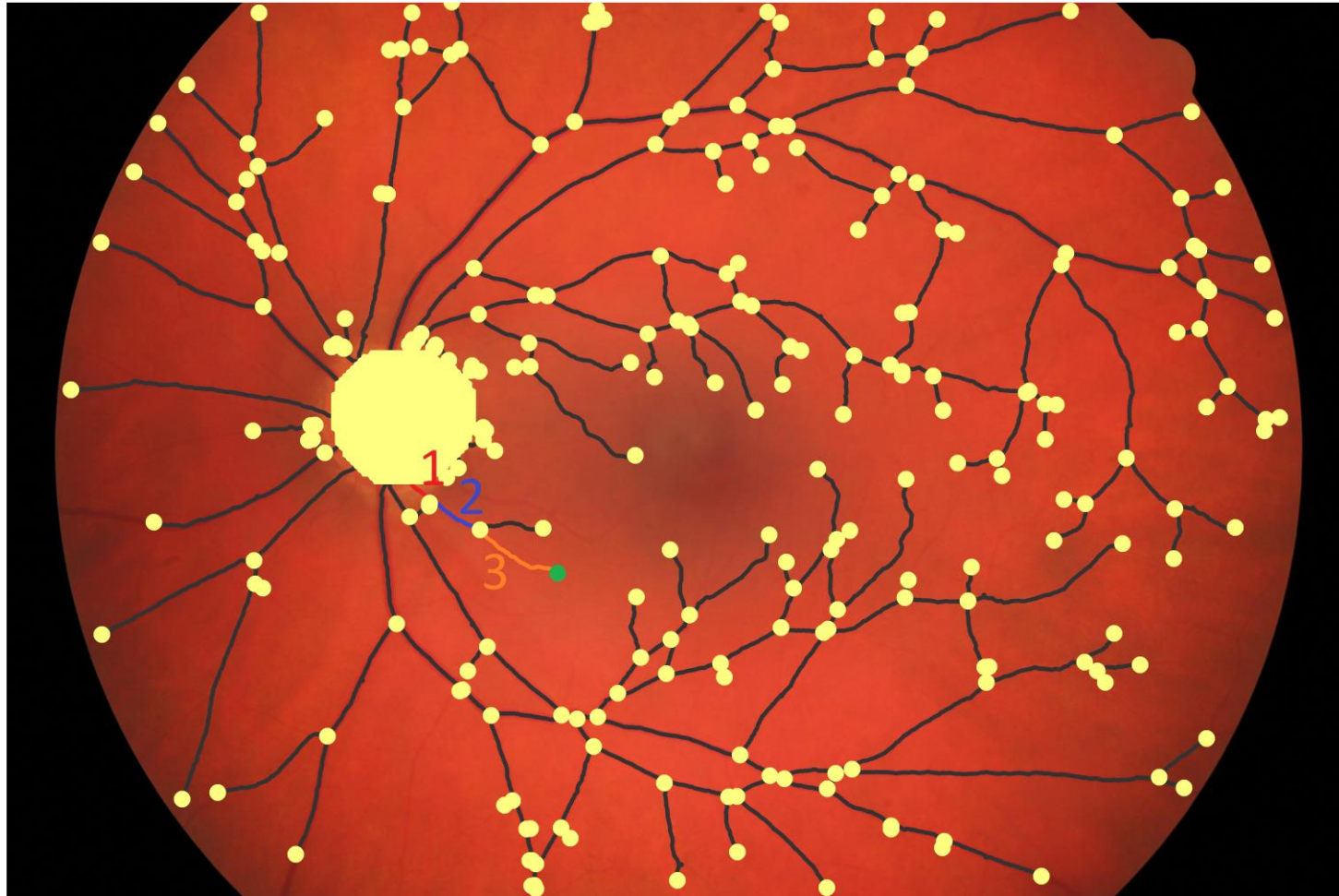


D. Santos-Sierra, I. Sendiña-Nadal, I. Leyva et al. Cytometry Part A. 87, 513 (2015).

P. Amil, F. Reyes-Manzano, L. Guzmán-Vargas, I. Sendiña-Nadal, C. Masoller, “*Network-based features for retinal fundus vessel structure analysis*”, PLoS ONE 14, e0220132 (2019).

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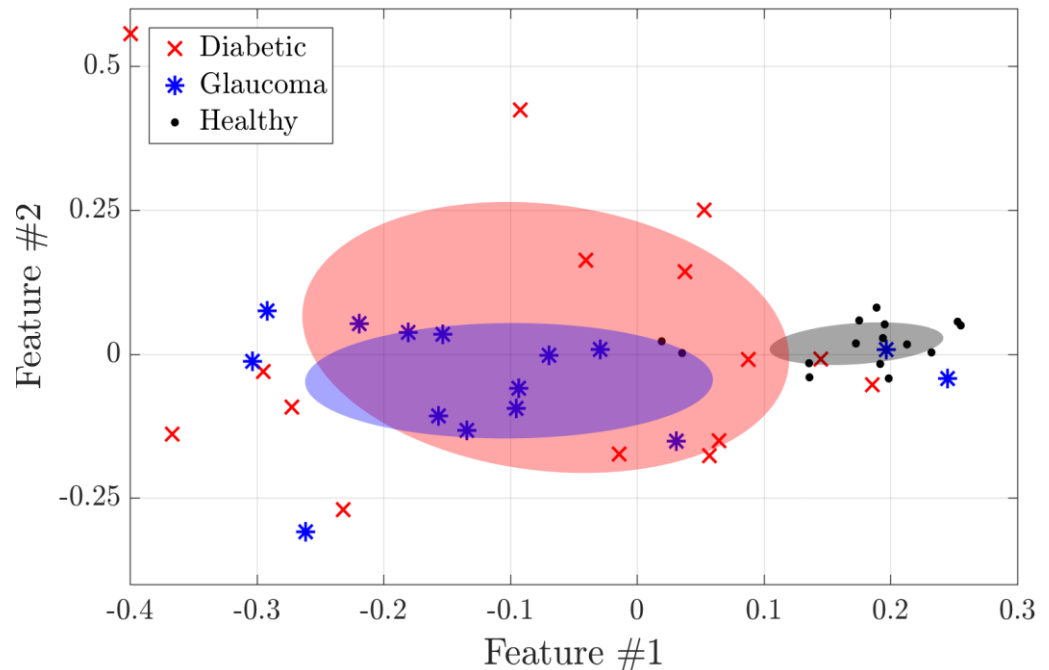
**Step 2: extract the network (identification of the optical nerve, nodes and links and assign weights to the links).**



## Steps 3 and 4: Compare the networks extracted from different images and classify the images.

- $\{p_{i,j}\}$ : distances between probability distributions that characterize the networks obtained from images  $i$  and  $j$ .
- We used nonlinear dimensionality reduction (*Isomap*) to reduce the set of  $45 \times 45$   $\{p_{i,j}\}$  values to only two features.

Distance distribution to the central node in the *manual* segmentation

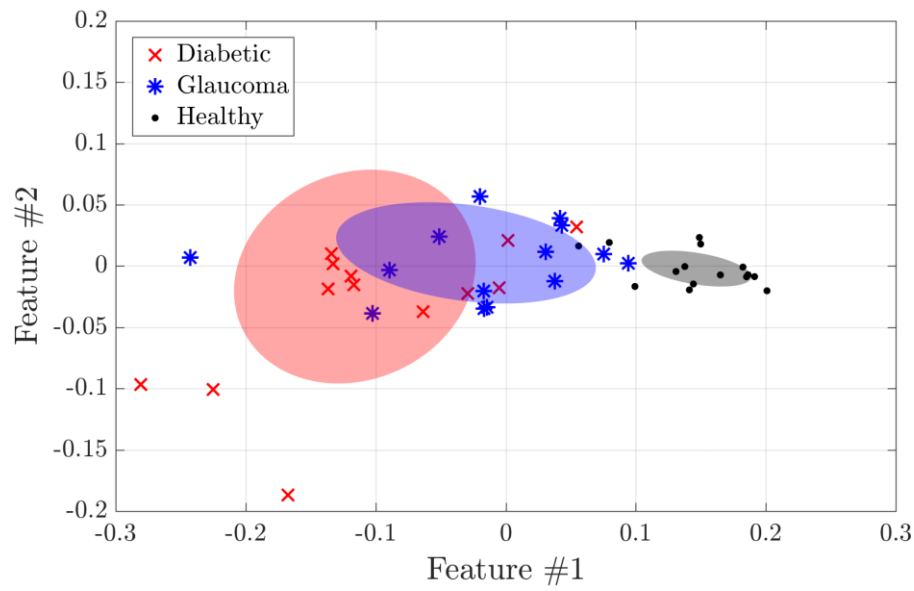


*P. Amil et al, Network-based features for retinal fundus vessel structure analysis, PLoS ONE 14 e0220132 (2019).*

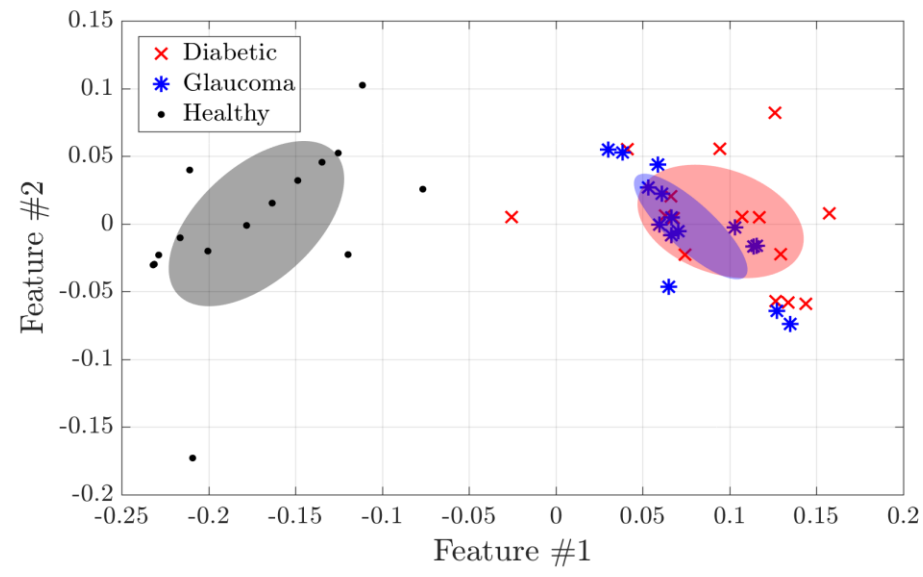


# Performance of network features in the *manual* segmentation

## Distribution of weights along the shortest path to central node



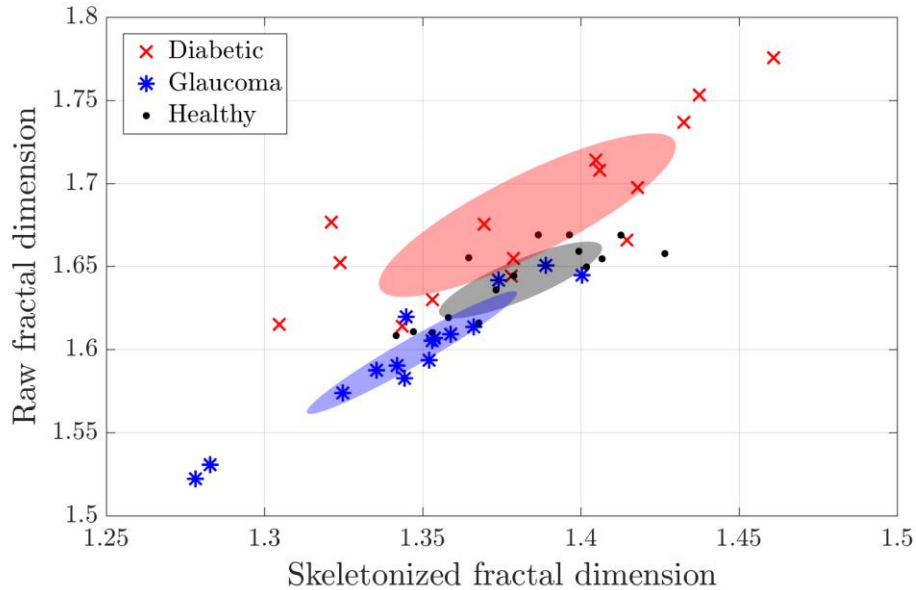
## Distribution of weighted degrees



P. Amil et al, Network-based features for retinal fundus vessel structure analysis, PLoS ONE 14 e0220132 (2019).

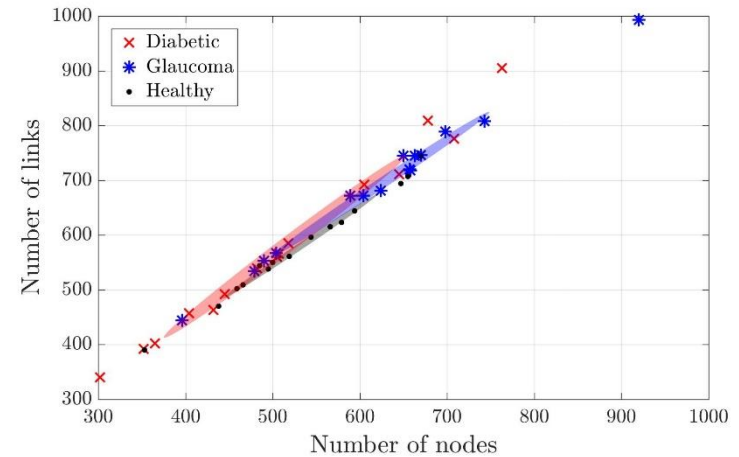
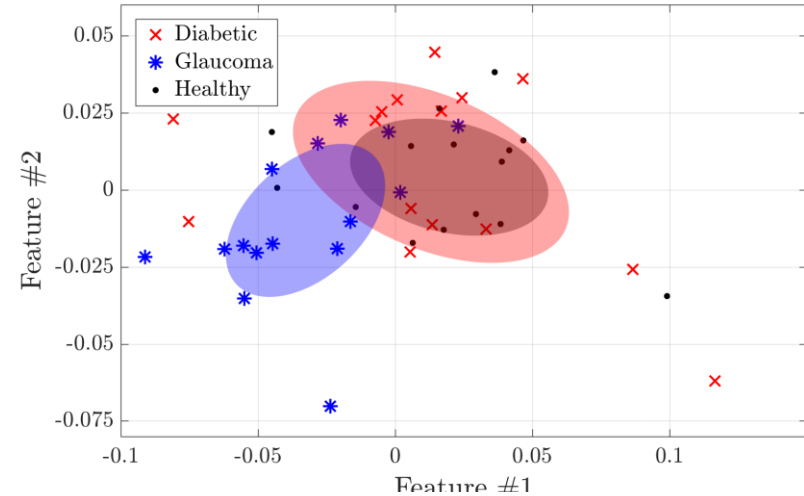
# In the automated segmentation

Fractal dimension  $D = \lim_{\varepsilon \rightarrow 0} \frac{\log(N(\varepsilon))}{\log(1/\varepsilon)}$



Simple network features do not differentiate

Mean weight distribution along the shortest path to central node



*P. Amil et al, Network-based features for retinal fundus vessel structure analysis, PLoS ONE 14 e0220132 (2019).*

# Take home messages

- Data analysis techniques allow us to uncover patterns and relationships in data, which characterize (and sometimes predict) the behavior of complex systems.
- Even when the data does not meet the mathematical or algorithmic requirements, the results can give useful info.
- Different methods provide *complementary* information.
- “Surrogate” tests are needed to determine if the numerical values are statistically significant.
- Data analysis is a fast growing field with many applications.

Holger Kantz: “*Every data set bears its own difficulties: data analysis is never routine*”

# Thanks!



ICREA



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## Thank you for your attention!

 [cristina.masoller@upc.edu](mailto:cristina.masoller@upc.edu)  [@cristinamasoll1](https://twitter.com/cristinamasoll1)