Time-series similarity analysis of coupled nonlinear oscillators and application to climate data

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Dynamic Days Europe 2016
MS on Advanced Time-Series Analysis
Corfu, Greece, June 2016

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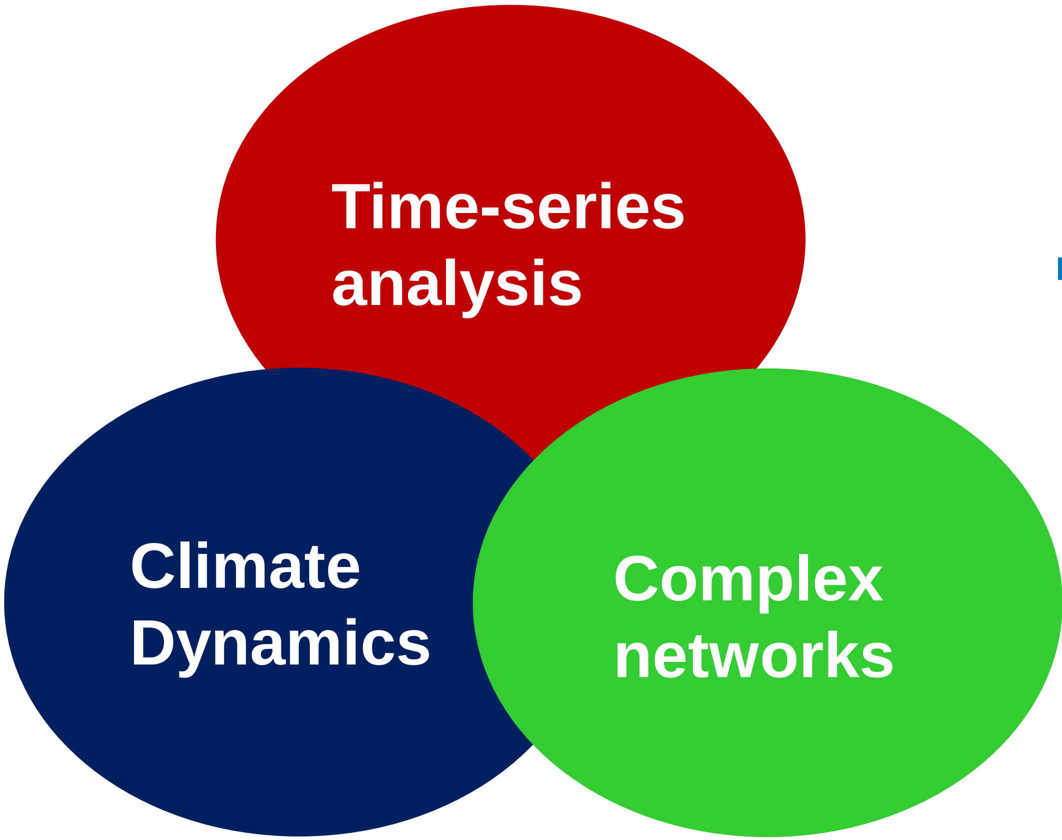
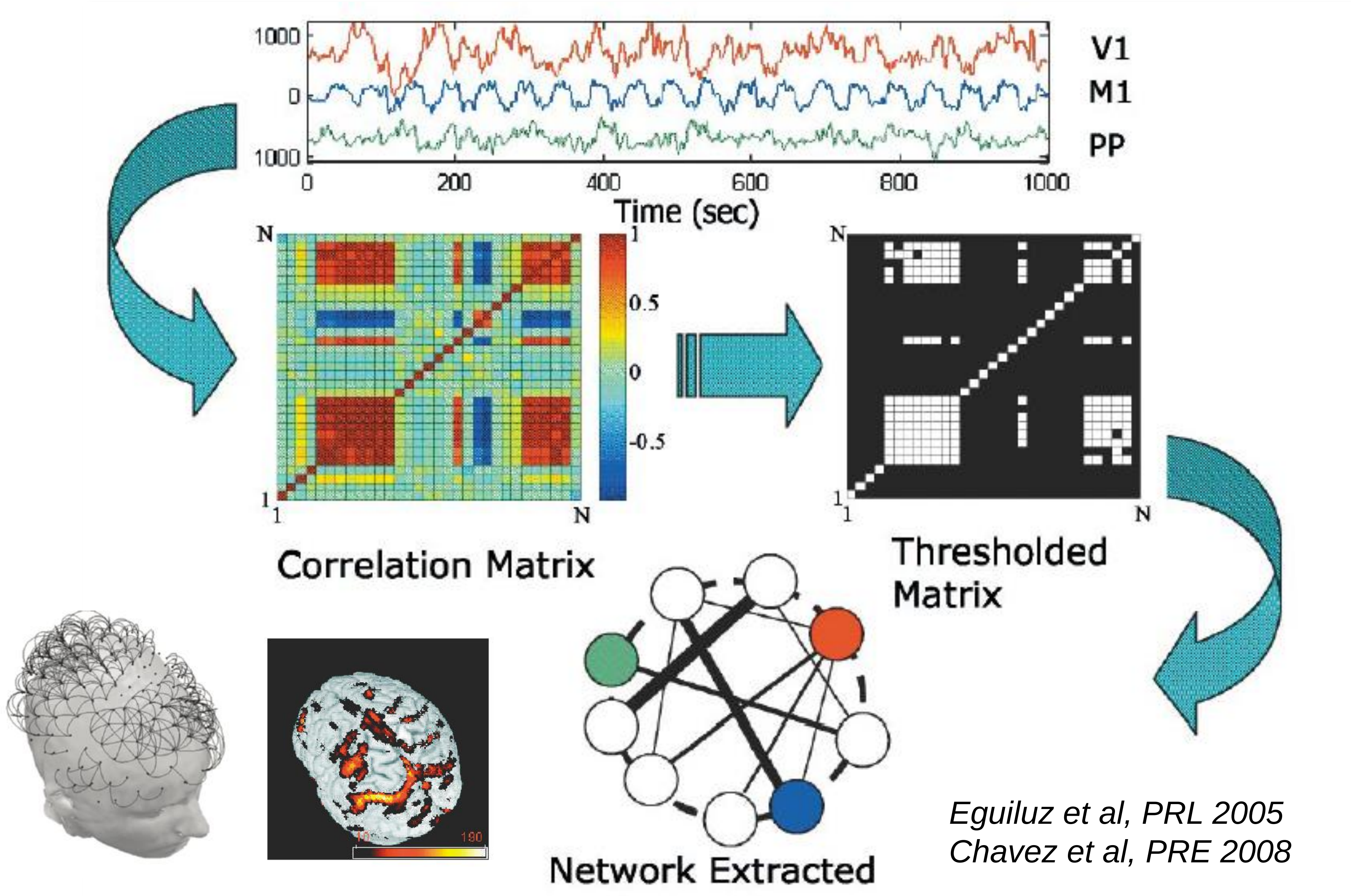
MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD

Outline



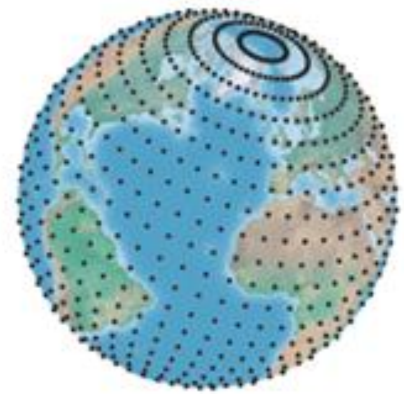
- Introduction
  - Climate networks
  - Symbolic method of time-series analysis
- Results:
  - Inferring the connectivity of Kuramoto oscillators
  - Application to experimental data: electronic circuits
  - Application to climate networks
- Conclusions

Brain functional network

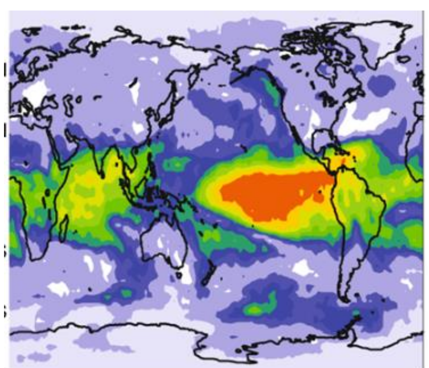


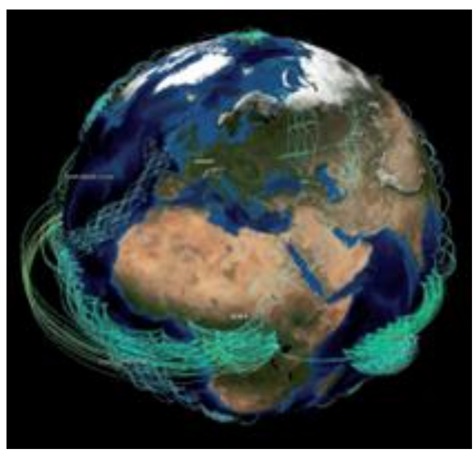
Eguiluz et al, PRL 2005
Chavez et al, PRE 2008

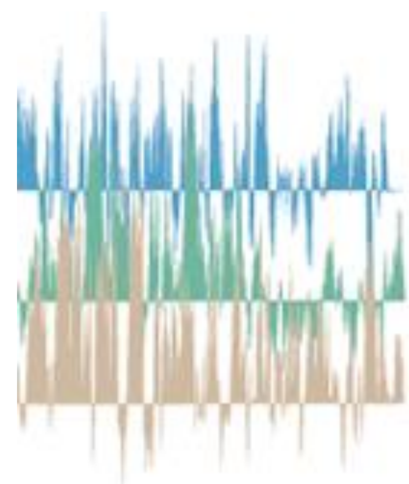
Climate networks



Area-weighted connectivity (weighted degree)



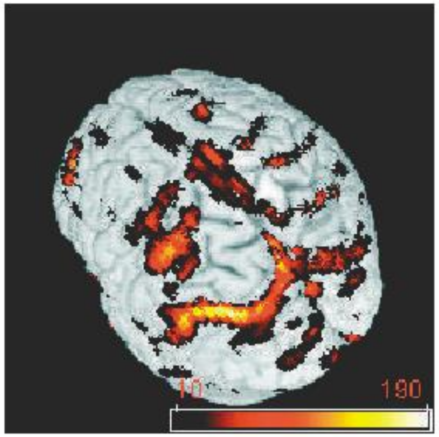




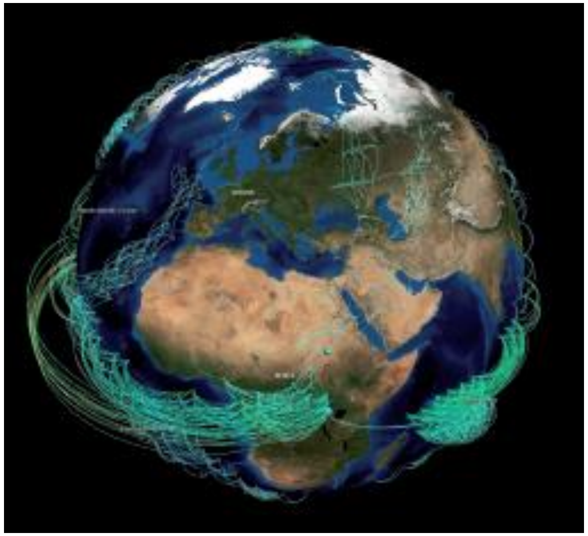
Deza et al, Chaos 2013

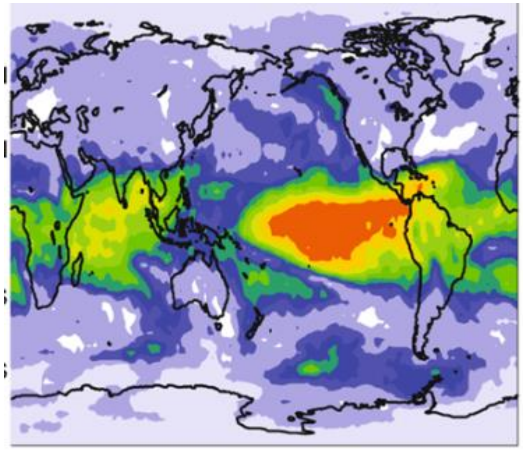
Donges et al, Chaos 2015

Brain network



Climate network

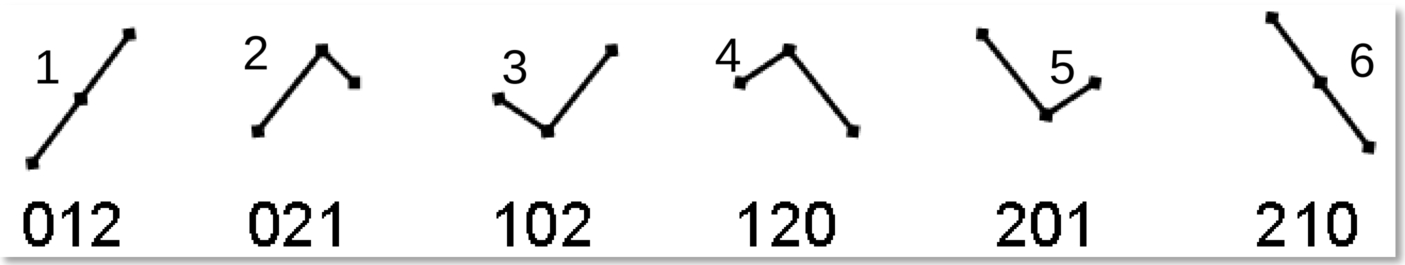


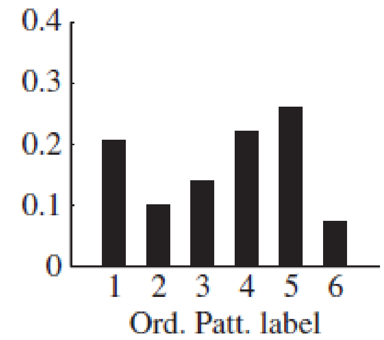


Method of symbolic time-series analysis: ordinal patterns

- $X = \{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$

Brandt & Pompe, PRL 88, 174102 (2002)



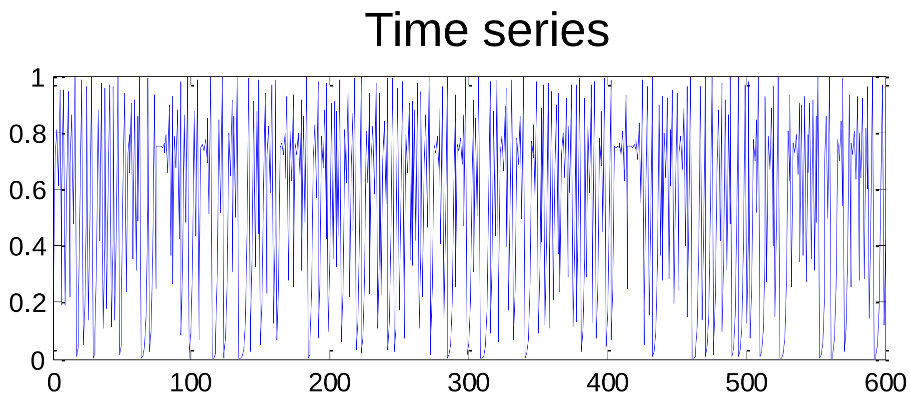


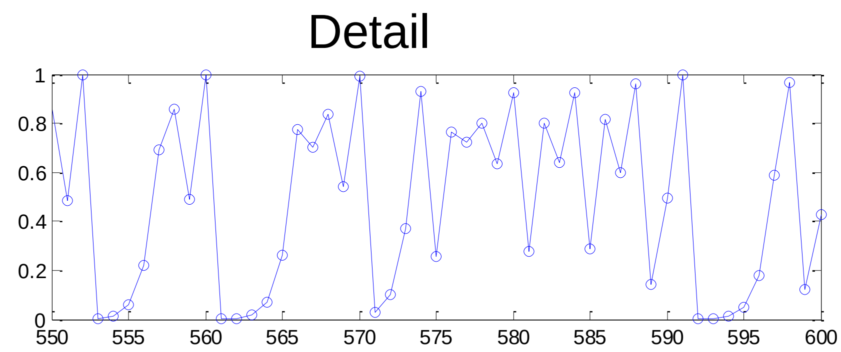
The OP probabilities allow to identify frequent patterns in the *ordering* of the data points

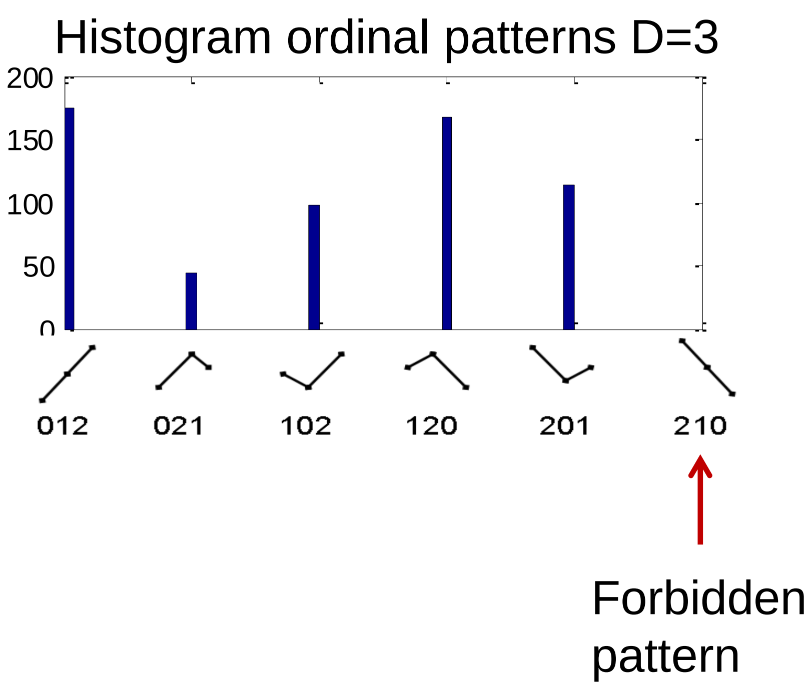
Random data $\Rightarrow$ OPs are equally probable

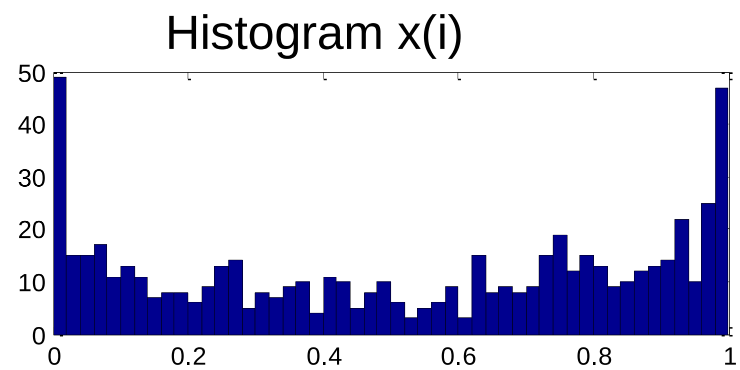
- Advantage: the probabilities uncover temporal correlations.
- Drawback: we lose information about the actual values.

Example: the logistic map
$x(i+1)=4x(i)[1-x(i)]$









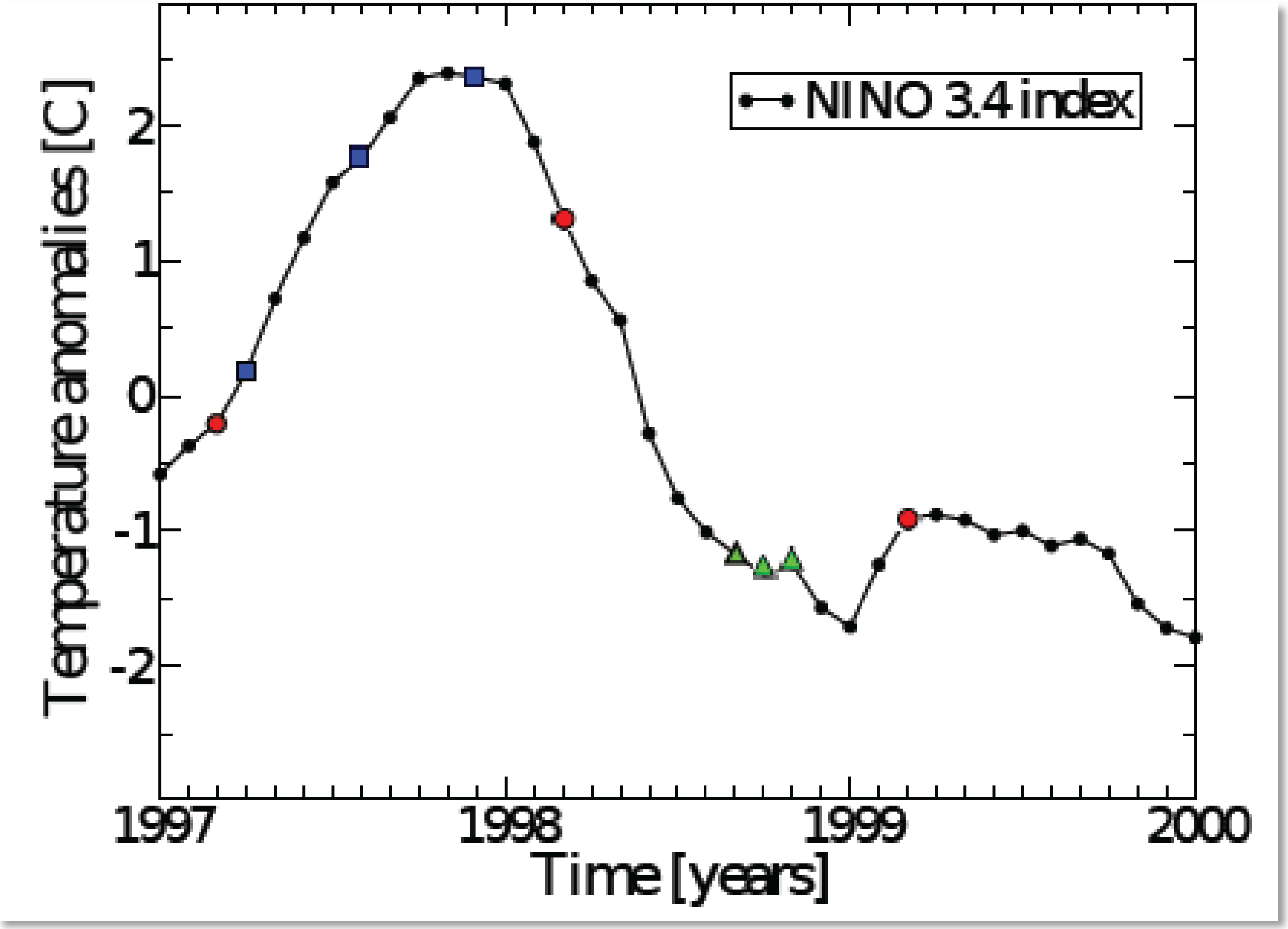
Ordinal analysis provides complementary information.

Ordinal analysis allows selecting the time scale of the analysis

Intra-season 102

Intra-annual 012

Inter-annual 120



Bivariate statistical similarity measures (SSM)

$a_i(t)$, $a_j(t)$, t=1, …,T

- Cross correlation

$$CC_{ij}(\tau_{ij}) = \frac{1}{T - \tau_{max}} \left| \sum_{t=0}^{T-\tau_{max}} a_i(t) a_j(t + \tau_{ij}) \right|$$

- Mutual information
  - Histograms
  - Ordinal patterns

$$MI_{ij}(\tau_{ij}) = \sum_{m,n} p_{ij}(m, n) \log_2 \left(\frac{p_{ij}(m, n)}{p_i(m) p_j(n)} \right)$$

p_i is associated to $a_i(t)$;
p_j is associated to $a_j(t+\tau_{ij})$

$$SSM_{ij} = \max_{\tau_{ij}} SSM_{ij}(\tau_{ij}) \qquad \tau_{max} = T/5$$

Kuramoto oscillators in a random network

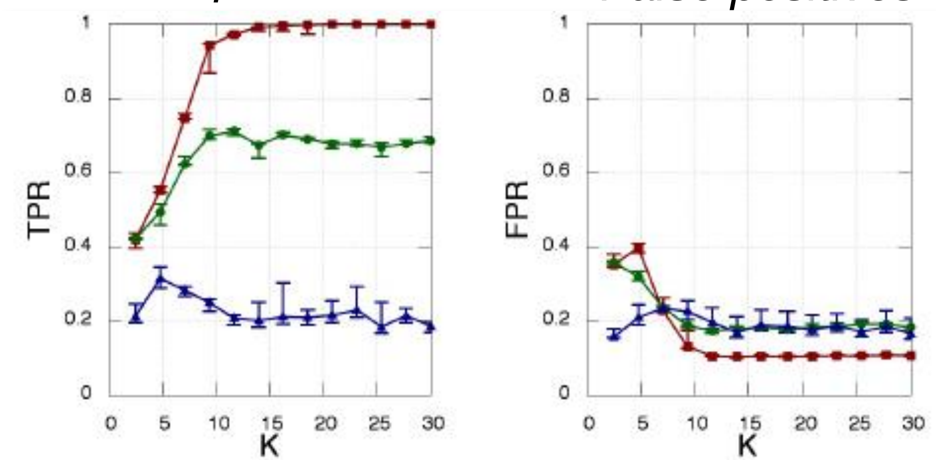
$$d\theta_i = \omega_i dt + \frac{K}{N}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i)dt + D\, dW_t^i$$

A_{ij} is a symmetric random matrix;
N=12 time-series, each with 10^4 data points.

CC MI MIOP

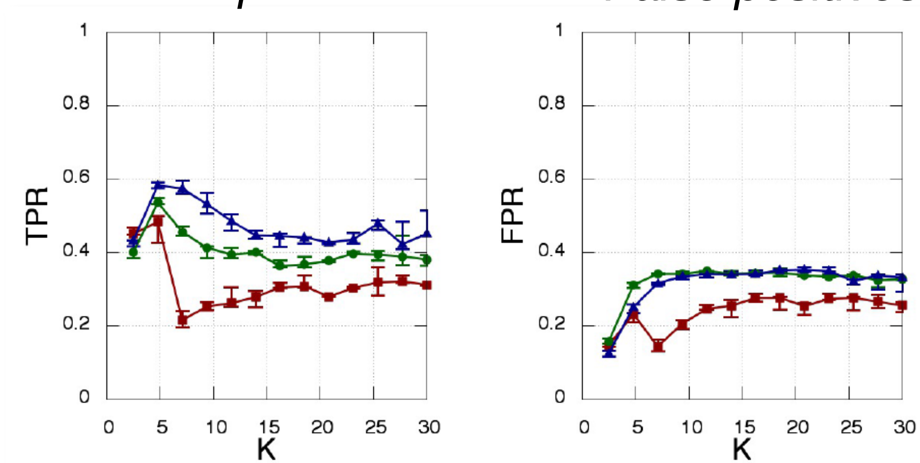
Phases (θ)

True positives *False positives*



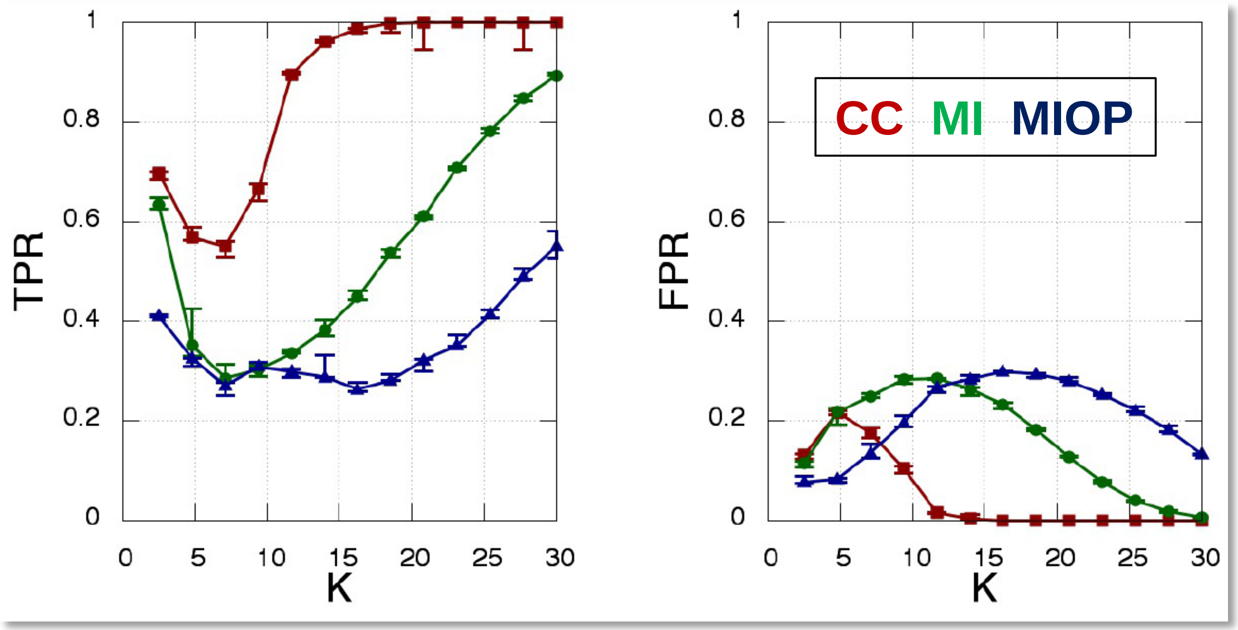
"Observable" Y=sin(θ)

True positives *False positives*



Results of a 100 simulations with different oscillators' frequencies, random matrices, noise realizations and initial conditions.
For each K, the threshold was varied to obtain optimal reconstruction.

Instantaneous frequencies (dθ/dt)



Perfect network inference is possible!

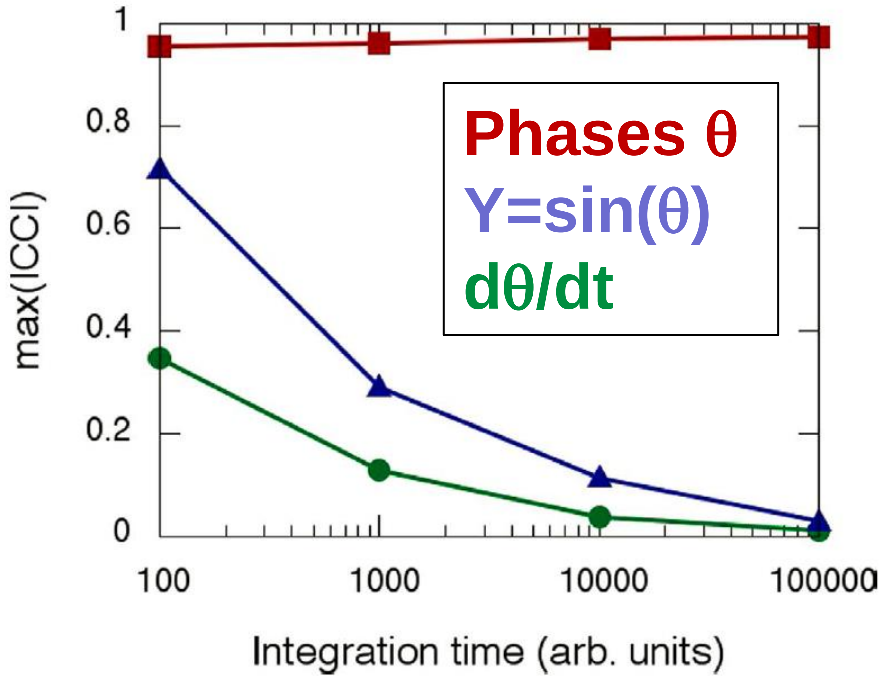
BUT

- the number of oscillators is small (12),
- the coupling is symmetric (⇒ only 66 possible links) and
- the data sets are long (10^4 points)

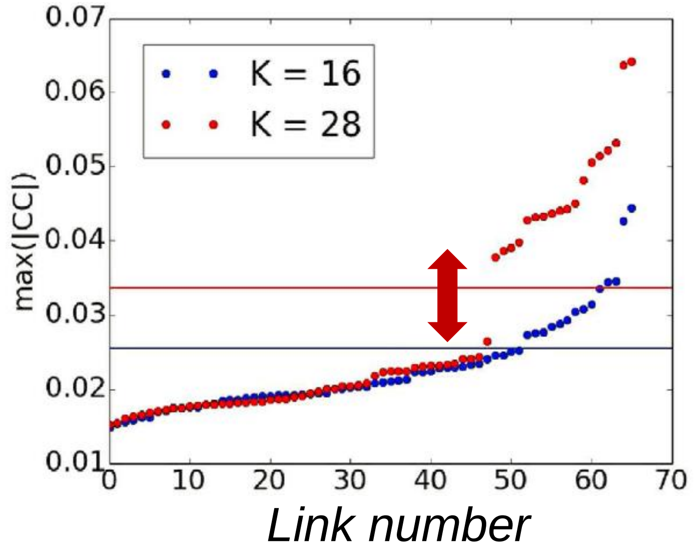
G. Tirabassi et al, Sci. Rep. ***5*** *10829 (2015)*

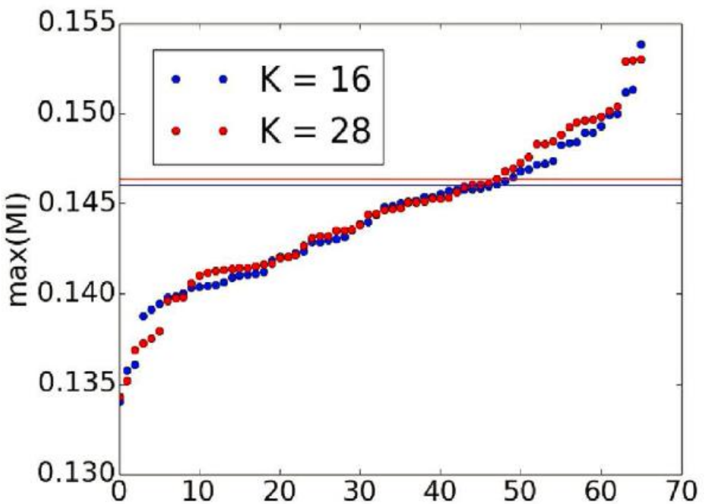
Why instantaneous frequencies are better than phases and "observables"?

Correlation analysis of two UNCOUPLED oscillators (K=0)

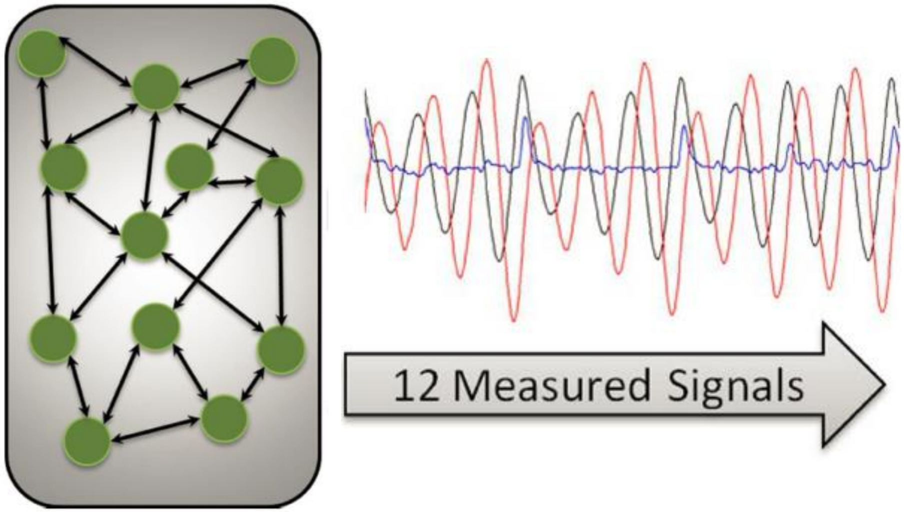


Why does CC outperforms MI?





We also analyzed experimental data recorded from 12 chaotic Rössler electronic oscillators (symmetric and random coupling)



The Hilbert Transform was used to obtain phases from experimental data

Experiments by J. Buldu & R. Sevilla-Escoboza.

- Kuramoto Oscillators' Network

$$\theta_i$$

$$f_i = \dot{\theta}_i$$

$$Y_i = \sin(\theta_i)$$

- Rössler Oscillators' Network

$$\varphi_i = HT(x_i)$$

$$f_i = \dot{\varphi}_i$$

$$x_i$$

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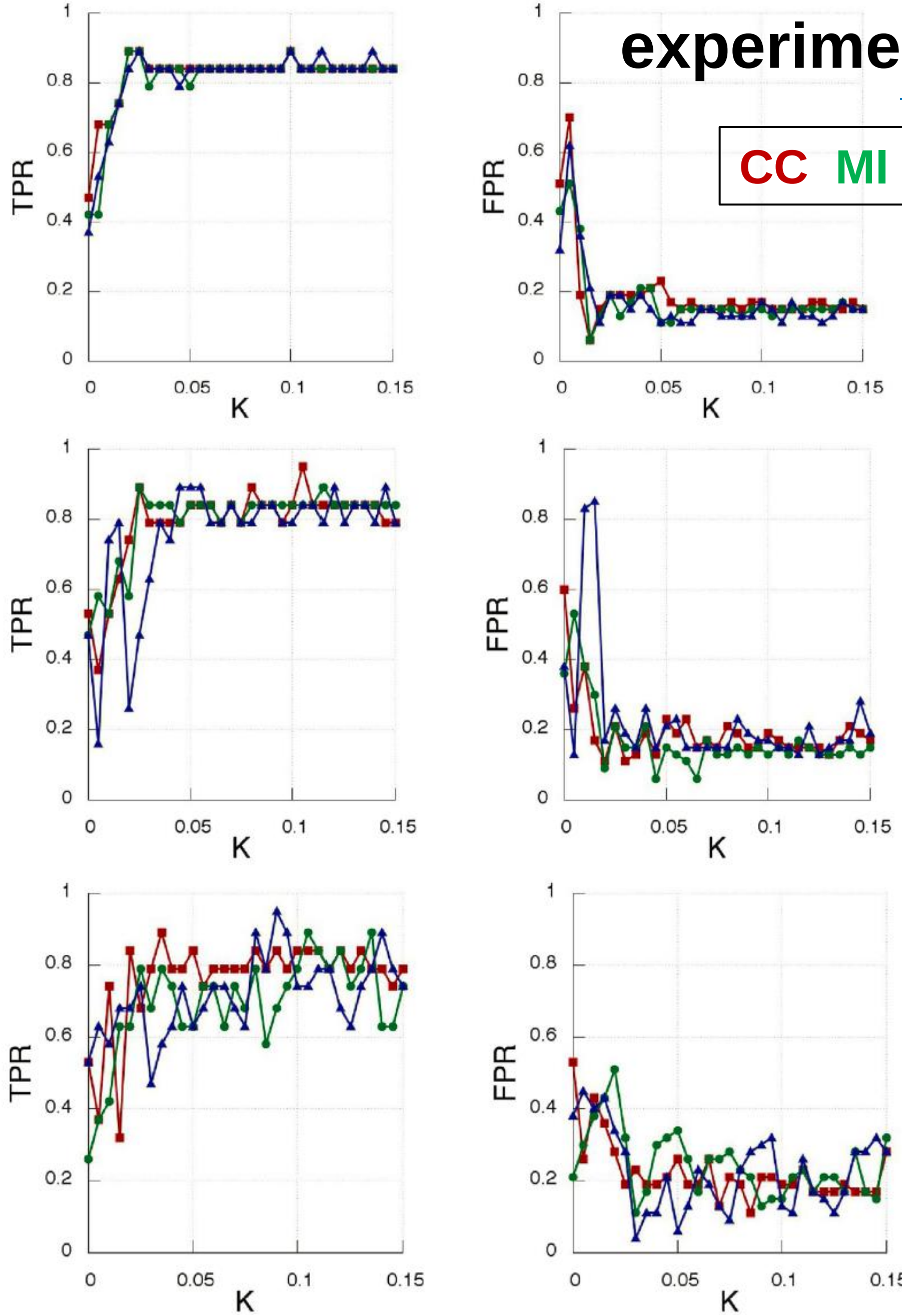
Results obtained with experimental data

CC MI MIOP

Observed variable (x)

Hilbert phase

Hilbert frequency



- No perfect reconstruction
- No important difference among the 3 methods & 3 variables

Application to climate networks

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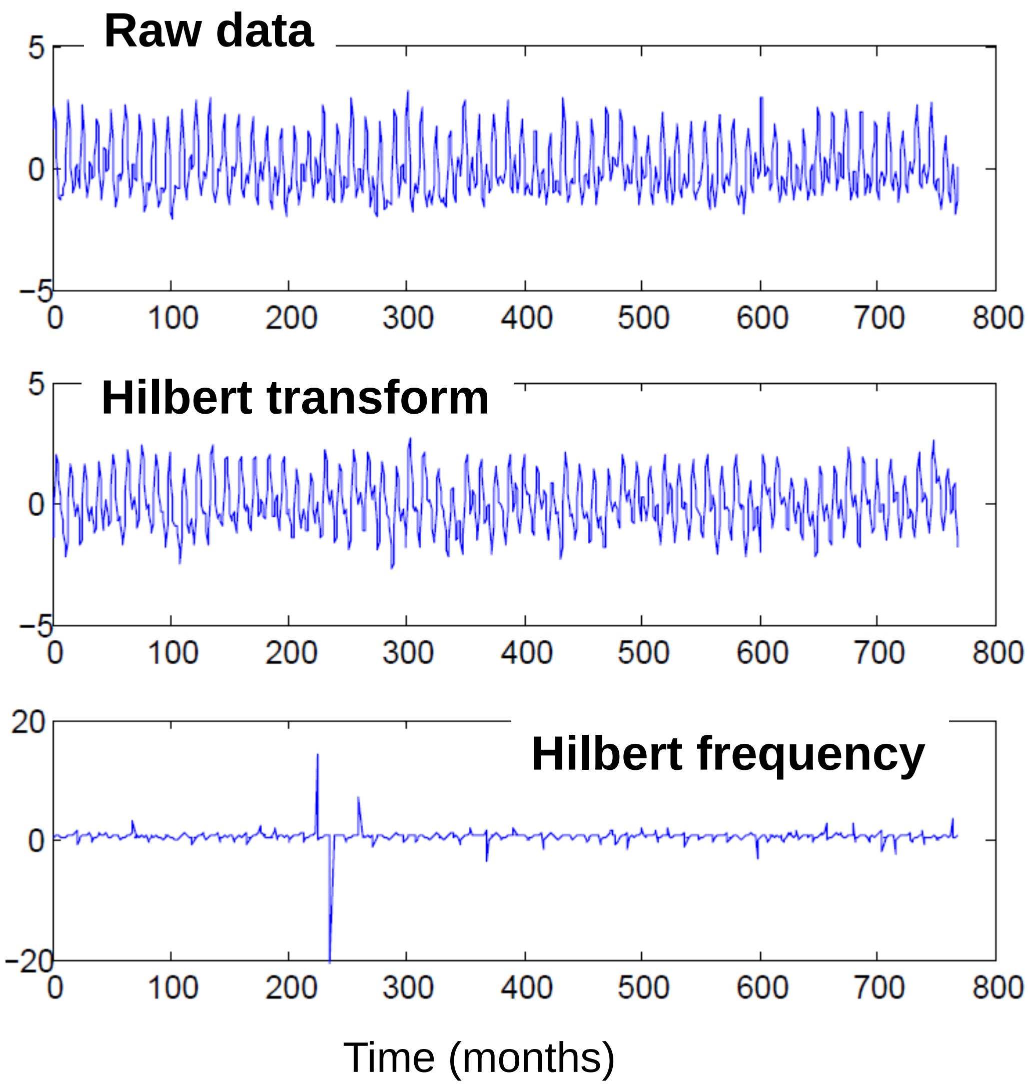
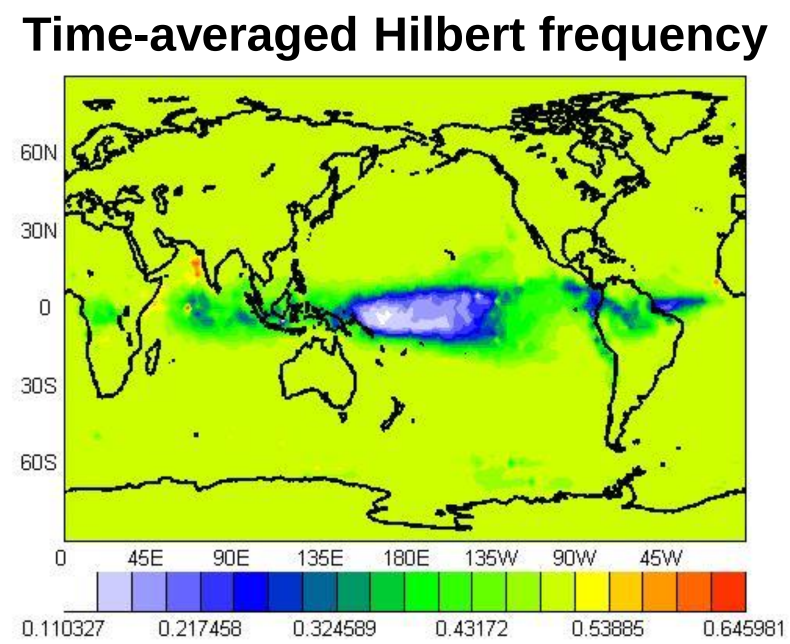
Ongoing work: Hilbert transform to extract frequencies from observed Surface Air Temperature time-series

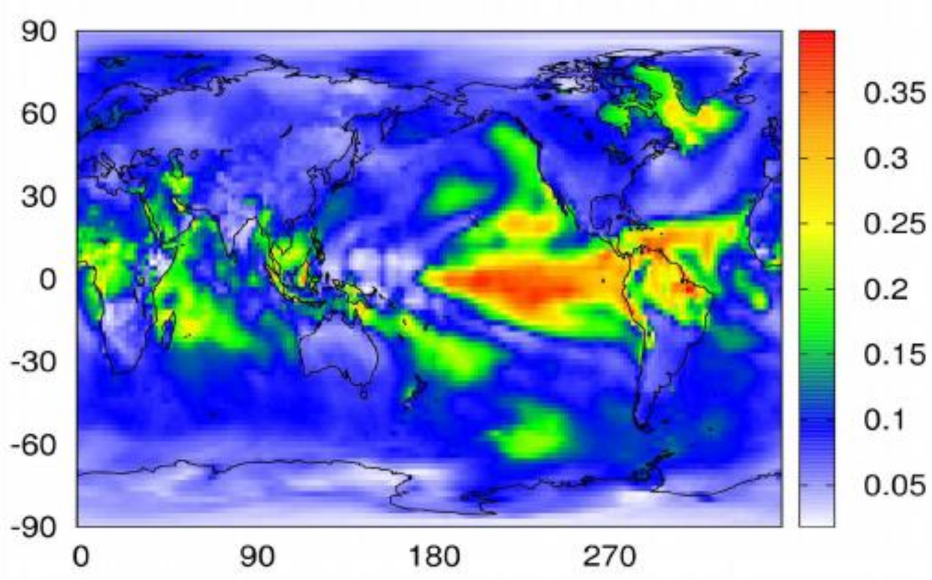




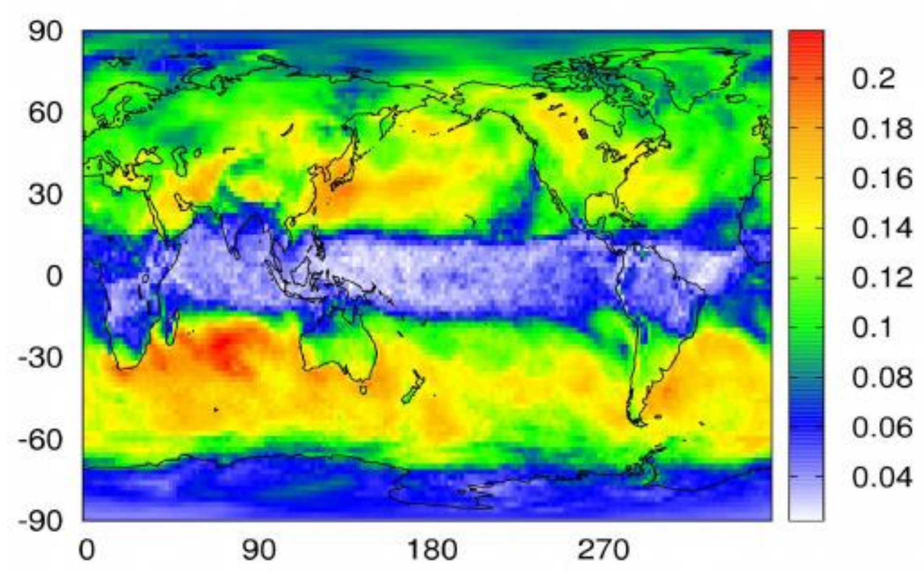
10226 nodes, 700 data points (60 years x 12 months). Reanalysis from National Center for Environmental Prediction, National Center for Atmospheric Research (NCEP-NCAR)

Contrasting two methods for constructing the climate network

Network constructed from correlation analysis of SAT anomalies



Network constructed from correlation analysis of Hilbert frequencies



In each node we keep the strongest links (10%)

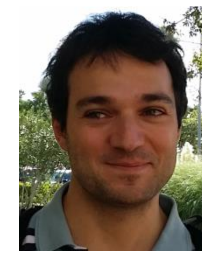
Low connectivity in the tropics is perhaps due to the properties of the annual solar cycle in the region

What did we learn?

- In small synthetic networks, under appropriate conditions, perfect network inference is possible.
- The similarity method to be used and the variable to be analyzed, for optimal network reconstruction depends on the specific system.
- The challenge: the applicability to real-world data (finite and highly stochastic, such as climate data) is an open question.

THANKS!

Collaborators: **Giulio Tirabassi** and **Dario Zappala** (UPC)



Experiments with chaotic electronic circuits:
Javier Buldu (Technical University of Madrid), **Ricardo Sevilla-Escoboza** (Universidad de Guadalajara, Mexico)

G. Tirabassi et al, *Inferring the connectivity of coupled oscillators from time-series statistical similarity analysis*. Sci. Rep. 5, 10829 (2015).

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