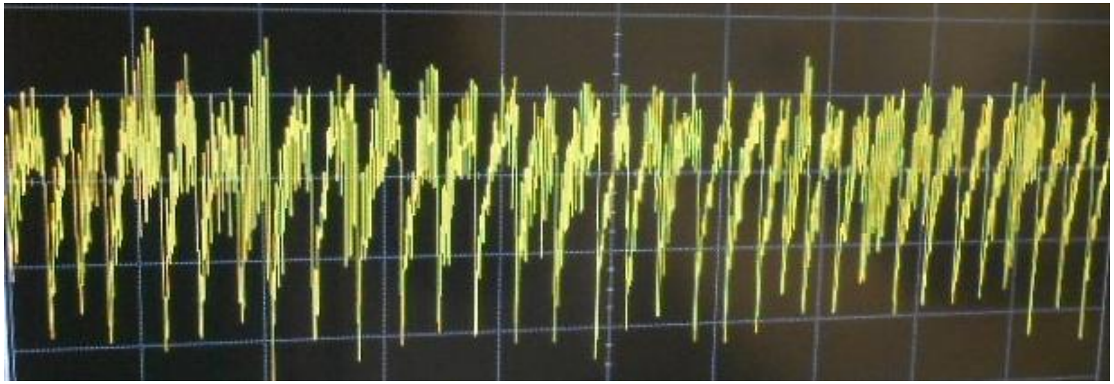
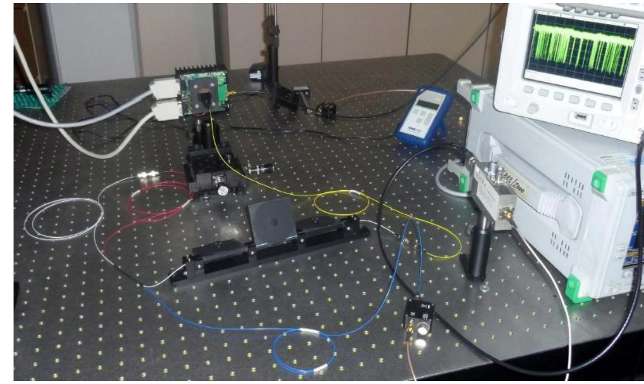


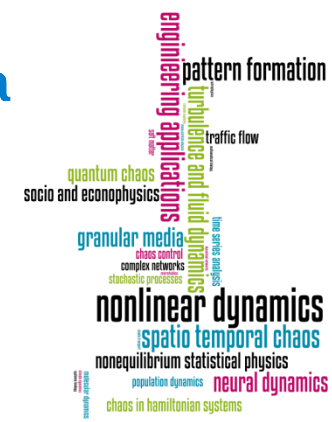


Distinguishing Signatures of Determinism and Stochasticity in Spiking Complex Systems

Cristina Masoller

Universitat Politecnica de Catalunya, Terrassa, Barcelona

www.fisica.edu.uy/~cris



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XXXIII Dynamics Days Europe
Madrid, Spain, June 2013

People involved

- Andres Aragoneses

- Nicolas Rubido (U. Aberdeen)

- Sandro Perrone

- Taciano Sorrentino

- Jordi Tiana

- Carme Torrent

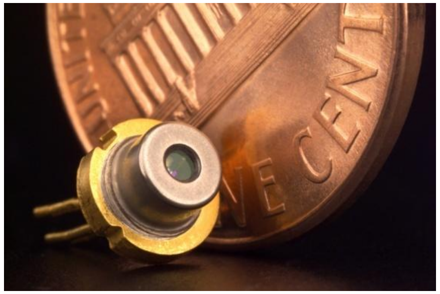
Event level description of dynamical complex systems

- Analysis of sequences of events generated by complex systems:
  - Intervals between threshold crossings and barrier crossings,
  - Neurons: inter-spike intervals (ISIs),
  - Human communication: inter-event user times (sms, emails, Twitters).
  - Earth and climate: earthquakes, extreme events (tornados, rainfalls), etc.
- Interplay of
  - Nonlinearity, memory, stochastic effects
  - Processes with different time scales
  - High dimensionality
- The identification of patterns in the sequence of events can allow for
  - Model verification, parameter estimation
  - Classification of different types of dynamical behaviors
  - Improving predictability and forecasting

Outline

- Semiconductor lasers with feedback as stochastic spiking high-dimensional complex systems
- Method of time-series analysis and experimental setup
- Experimental and model observations: signatures of determinism in the sequence of optical spikes + response to periodic forcing
- Conclusions and take home message

Why semiconductor lasers?

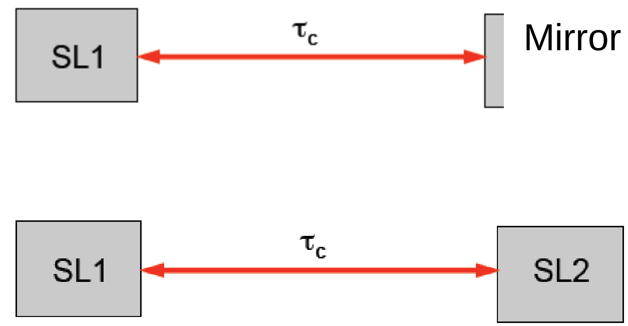


- SLs have many advantages:
  – compact, fast, reliable, inexpensive
  – wide range of wavelengths

- Used in
  – Telecommunications
  – Data storage (CDs, DVDs, Blu rays)
  – Barcode scanners, printers, mouse
  – Material processing
  – Biomedical applications (imaging, sensing, etc)

Nonlinear dynamics

- "solitary" semiconductor lasers emit a stable output intensity.
- With optical feedback or injection: nonlinear oscillator.
- Complex interplay of:
  - Time delay
  - noise
  - nonlinearity

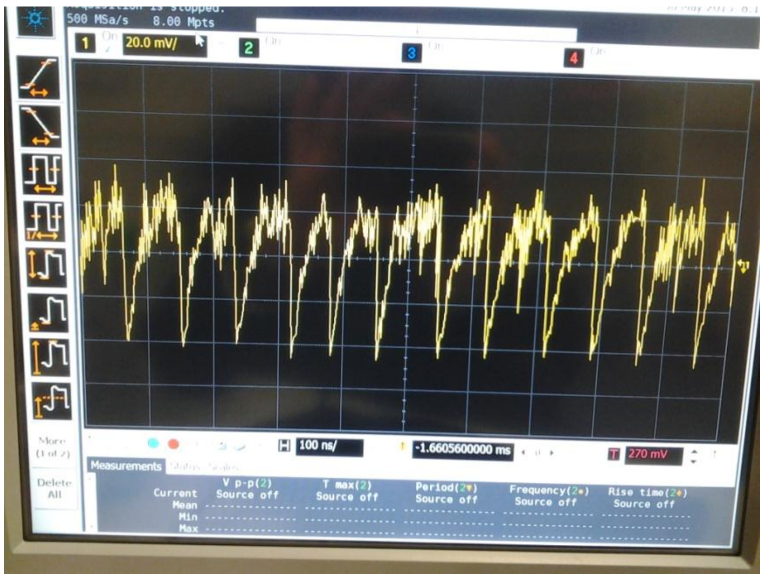


that can be exploited for applications.

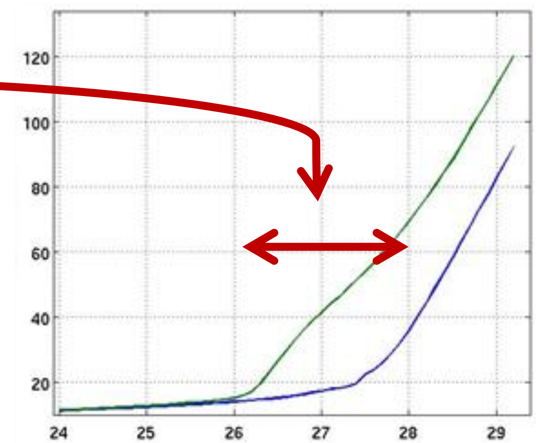
Kathy Ludge: "Nonlinear Laser Dynamics: From Quantum Dots to Cryptography", Wiley-VCH Verlag GmbH & Co. KGaA. (2012). ISBN: 3527411003

Optical spikes

- Close to threshold, with optical feedback (self coupling) or with optical coupling (to another laser) the laser intensity displays optical spikes that can resemble neuronal spikes.



Laser output intensity



Pump current (mA)

with feedback

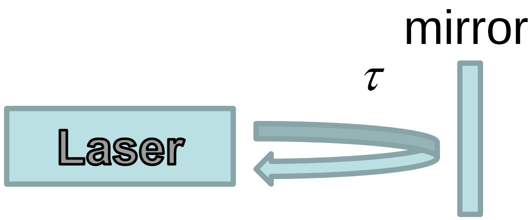
without feedback

Goals

- to develop a method of time-series analysis that allows inferring signatures of determinism in the sequence of optical spikes;
- to extract new information;
- to compare model predictions with observations;
- to explore potential for building optical neurons.

Governing equations

R. Lang and K. Kobayashi, IEEE J. Quantum Electron. 16, 347 (1980)



$|E|^2$ ~ photon number (output intensity)

N ~ number of carriers (electron-holes)

$$\frac{dE}{dt} = \frac{1}{2\tau_p}(1+i\alpha)(G-1)E + \underbrace{\eta E(t-\tau)e^{-i\omega_0\tau}}_{\text{feedback}} + \underbrace{\sqrt{2\beta_{sp}}\,\xi}_{\text{noise}}$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - G|E|^2\right)$$

η = feedback strength

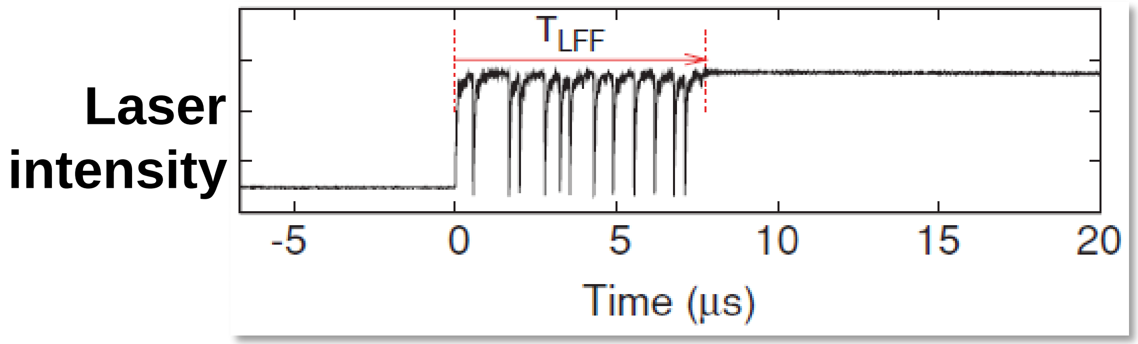
μ = pump current parameter

Gain: $G = N/(1+\varepsilon|E|^2)$

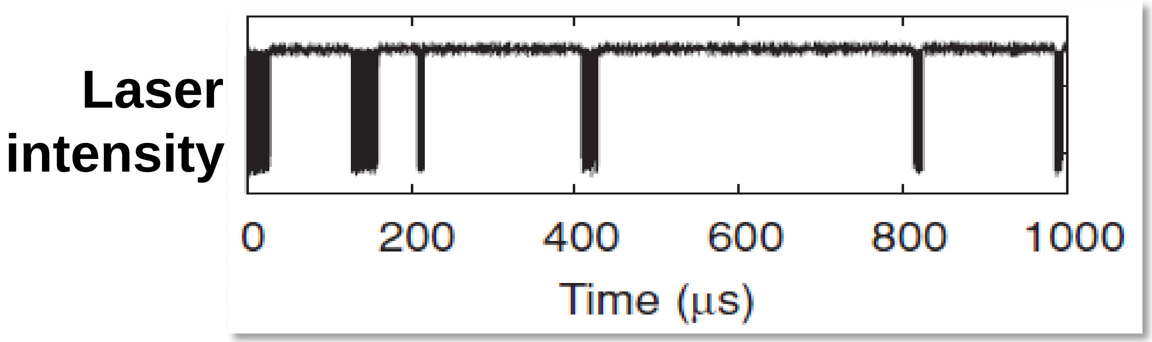
Stochastic and high dimensional dynamical system

Model predictions

- Depending on the parameters the dropouts can be a **transient** dynamics.



- Burst of dropouts can be triggered by **noise**.

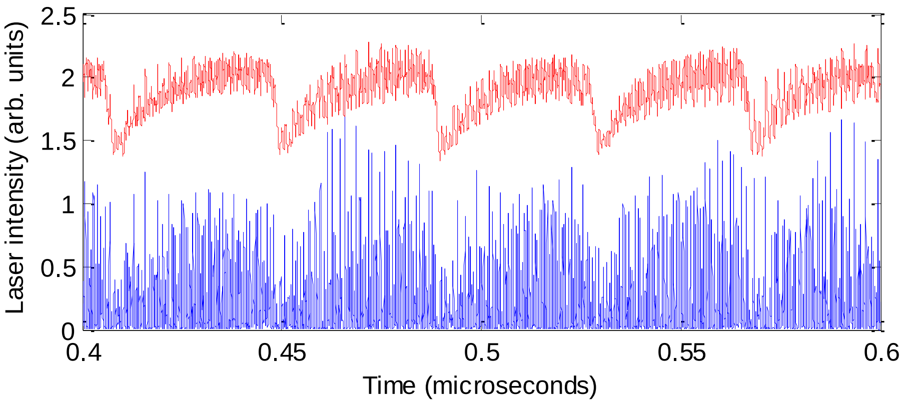


- In experimental sequences of optical spikes: which ones are deterministic and which ones are triggered by noise?

A. Torcini et al, Phys. Rev. A 74, 063801 (2006)
J. Zamora-Munt et al, Phys Rev A 81, 033820 (2010)

Problems

- Main problem: we can measure only one "output" variable (the laser output intensity)
- Also a problem: the measure system (photodiode, oscilloscope) has a finite *bandwidth* that gives a limited temporal resolution.



- Event-level description: we study the sequence of **inter-dropout-intervals**: $\Delta T_i = t_{i+1} - t_i$

Outline

- Semiconductor lasers with feedback as stochastic spiking high-dimensional complex systems
- **Method of time-series analysis** and experimental setup
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Nonlinear time-series analysis

- Many methods have been developed to test for determinism, nonlinearity and correlations in data generated from complex systems (climate, brain EEGs, financial data, social systems, etc).

- The appropriateness of the method depends on the characteristics of the time series.

- Different methods can provide complementary ***new*** information.

Symbolic time-series analysis

- The time series of Inter-Dropout-Intervals $\{\Delta T_1, \Delta T_2, \ldots\}$ is transformed (using an appropriated **rule**) into a sequence of symbols $\{s_1, s_2, \ldots\}$
- taken from an "**alphabet**" of possible **symbols** $\{a_1, a_2, \ldots\}$.
- Then we consider "blocks" of D symbols ("patterns" or "**words**").
- All the possible words form the "**dictionary**".
- Then analyze the "language" of the sequence of words
  - the probabilities of the words,
  - missing/forbidden words,
  - transition probabilities, etc

Ordinal analysis

- Proposed by Christoph Bandt and Bernd Pompe in 2002

- It has been used to analyze data generated from complex systems
  - Financial, economical
  - Biological, life sciences
  - Geosciences, climate (**Advertisement: Giulio Tirabassi's talk**, Friday, 11 hs, Room 1, MS 26)
  - Physics, chemistry, etc

- It has been shown to be able to:
  - Distinguish stochasticity and determinism
  - Classify different types of dynamical behaviors (pathological, healthy)
  - Quantify complexity
  - Identify coupling and directionality.

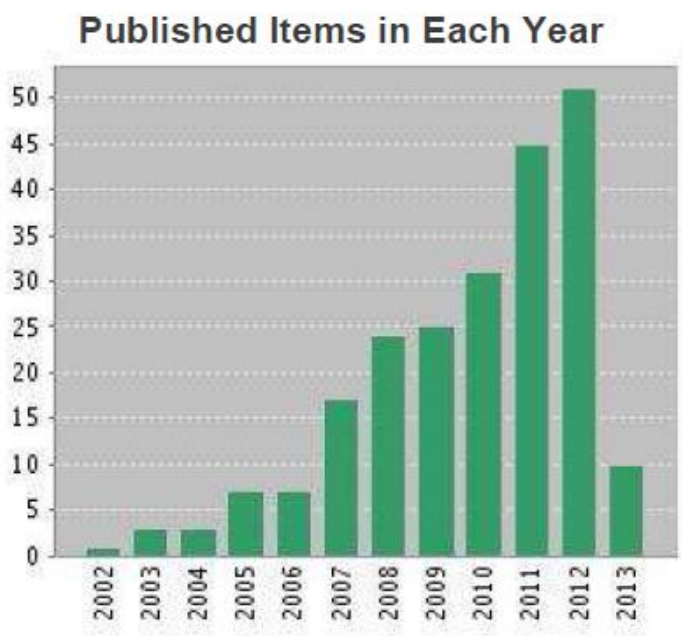
Brandt & Pompe, Phys. Rev. Lett. 88, 174102, (2002).

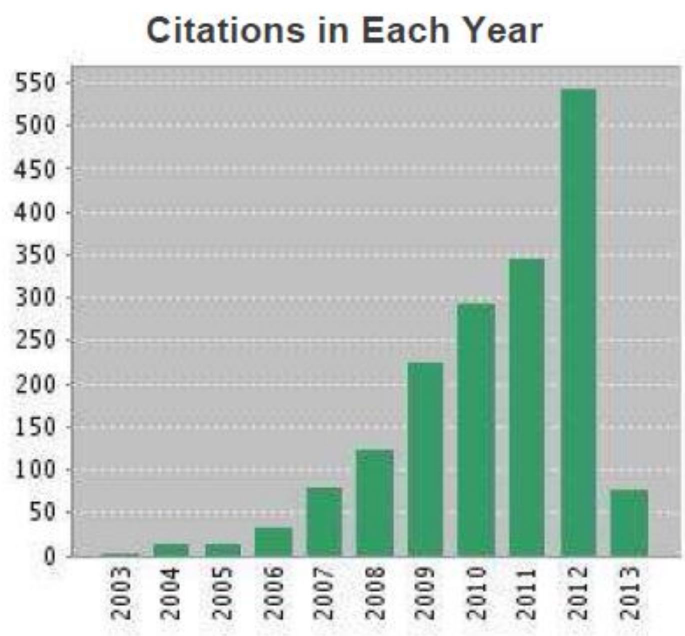
Ordinal analysis is becoming increasingly popular

Citation Report Title: **Permutation entropy: A natural complexity measure for time series**
Author(s): **Bandt**, **C** ; **Pompe**, **B**
Source: **PHYSICAL REVIEW LETTERS** Volume: **88** Issue: **17** Article Number: **174102**
10.1103/PhysRevLett.88.174102 Published: **APR 29 2002**

Timespan=All Years. Databases=SCI-EXPANDED, A&HCI, SSCI, CPCI-SSH, CPCI-S.

This report reflects citations to source items indexed within Web of Science. Perform a Cited Reference Search to include c
indexed within Web of Science.





J. M. Amigo, Permutation Complexity, Springer Series in Synergetics (2010)
M. Zanin et al, Entropy 14, 1553 (2012)
EPJST topical issue on permutation complexity (2013)

Ordinal transformation

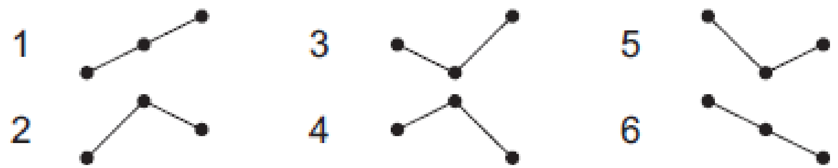
- A time series can be transformed into a sequence of 0s and 1s using the rule:

$$\text{if } x_i > x_{i-1} \Rightarrow s_i = 0; \text{ else } s_i = 1$$

- "words" of **D letters** can be formed by considering the **order relation** between sets of D values $\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$.

Ordinal Patterns (or "words")

- For **D=3**, $\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$ there are 6 possible orders
  012, 021, 102, 120, 201, 210



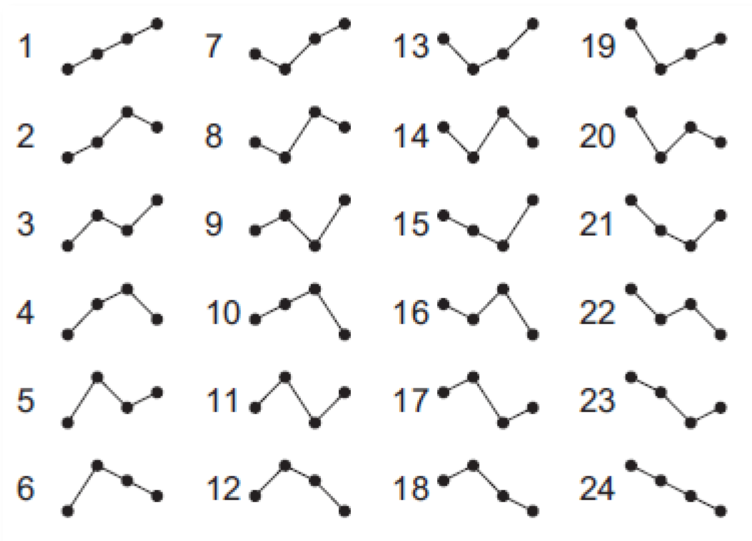
Example: the set (5, 1, 7) gives "102" because 1 < 5 < 7

– Advantage: the transformation keeps information about correlations in the sequence & does not need a threshold
– Drawback: does not keep information about the values (the set (5,1,100) also gives word "102")

Number of possible ordinal patterns

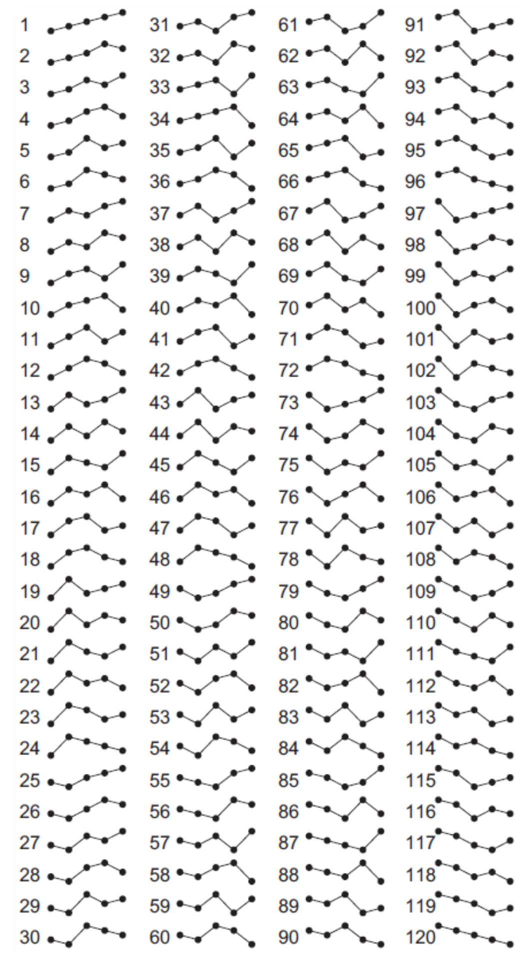
- **D!** possible words in the dictionary.

D=4



U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

D=5



- **D! x D!** possible transitions (pairs of consecutive words) in the language.

How to select D?

- The optimal length of the pattern depends on

  – The length of the time series (to compute words and transition probabilities with good statistics).

  – The correlation time-scale of the system.

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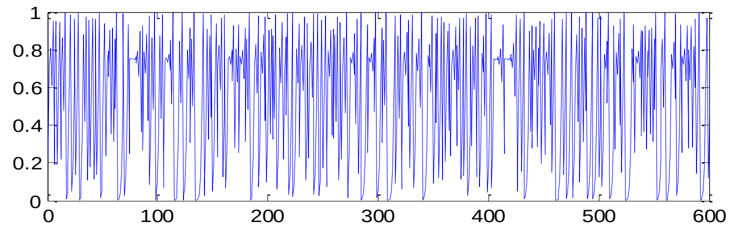
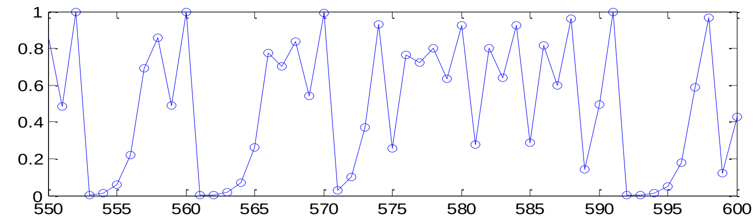
What can we learn using ordinal analysis?

- Example: Logistic map, x(i+1)=4 x(i) [1-x(i)]

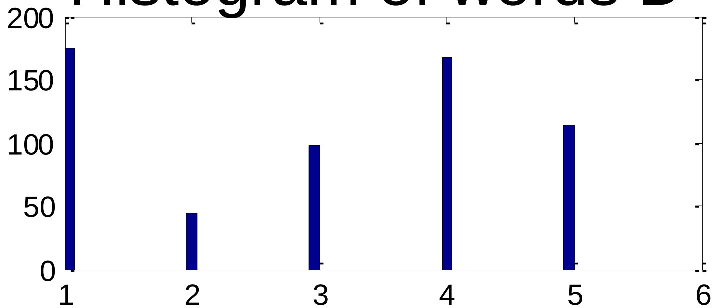
Time series

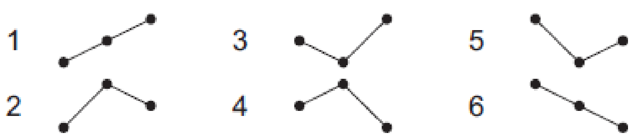


Detail



Histogram of words D=3

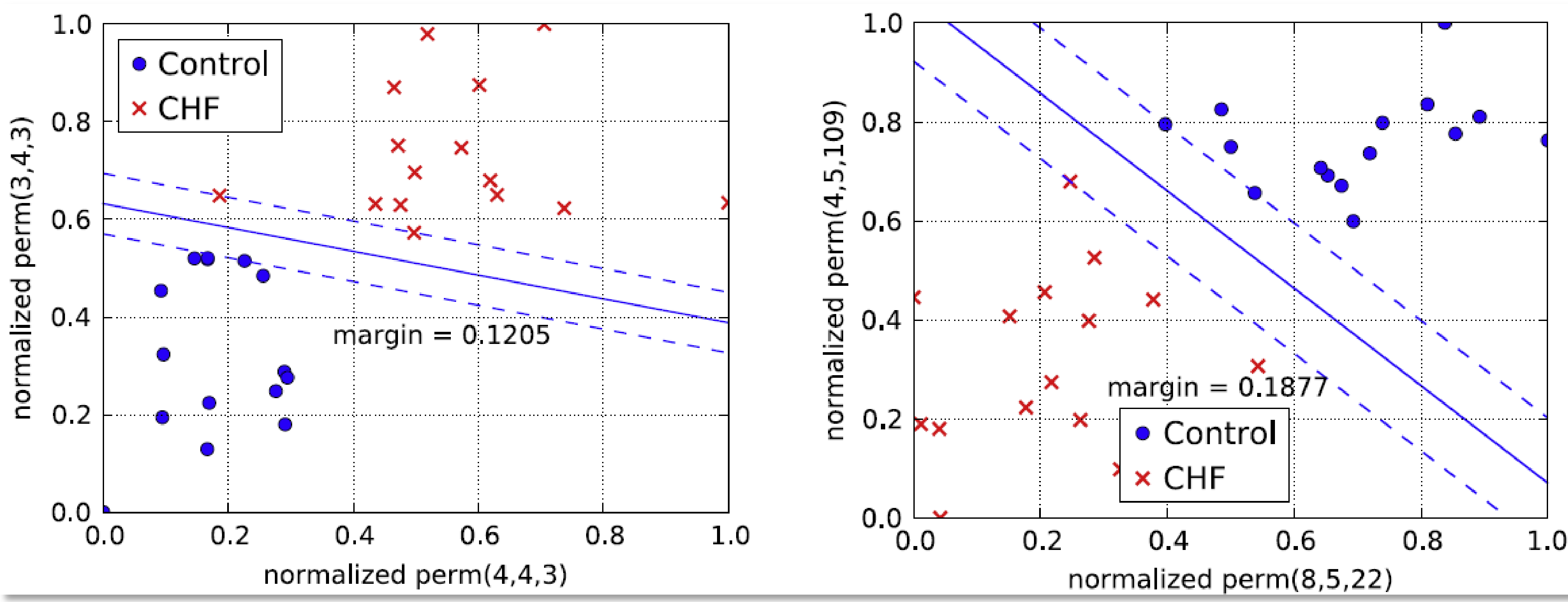




forbidden

- The word distribution has been used to classify time-series, and to estimate model parameters.

Classifying ECG-signals according to the appearance of words



U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

(the probabilities are normalized with respect to the smallest and the largest value occurring in the data set)

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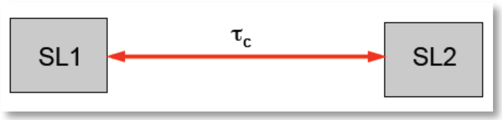
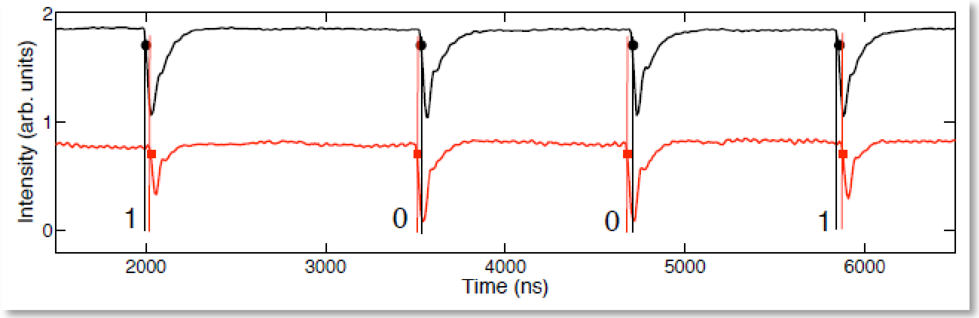
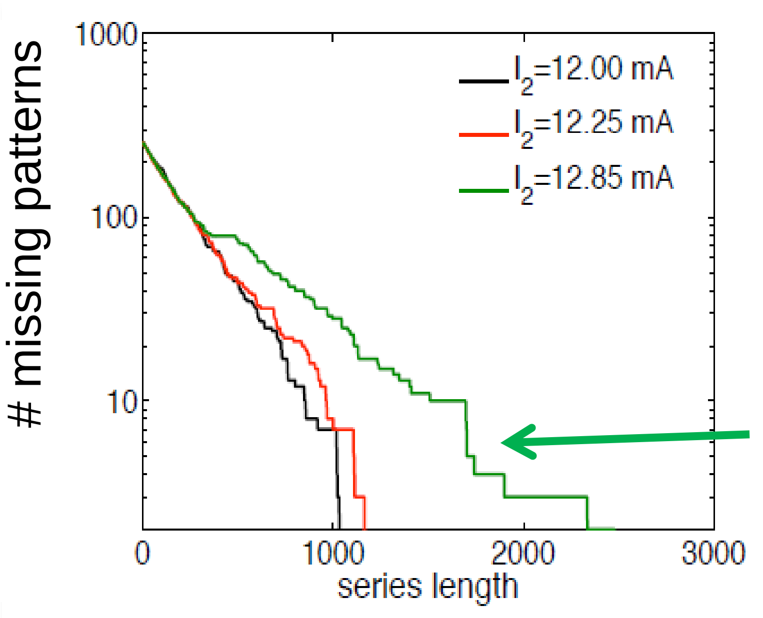
Missing patterns: signature of determinism







Less stochastic

J. Tiana-Alsina et al, Phil. Trans. Royal Soc. A 368, 367 (2010)

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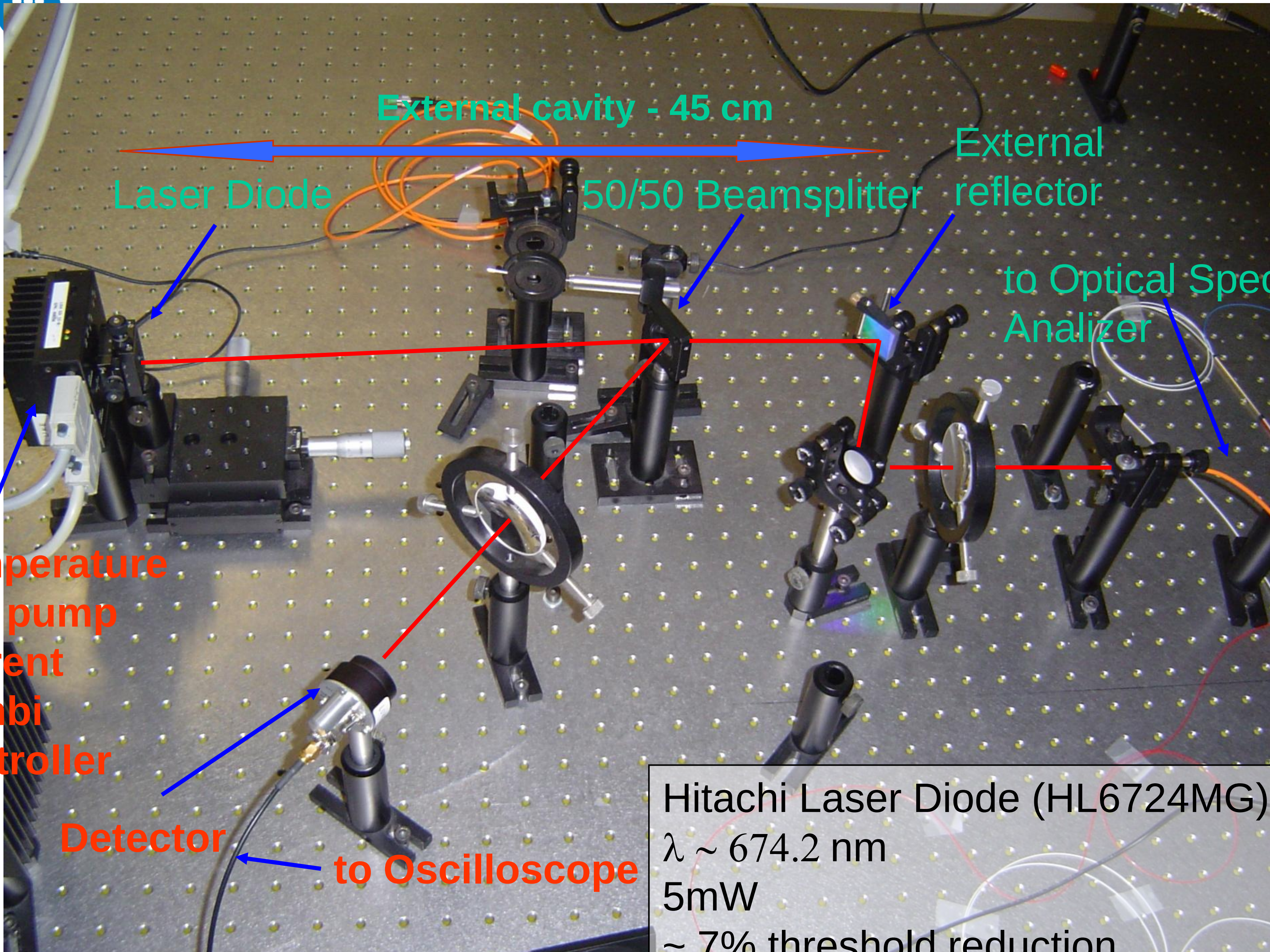

Hitachi Laser Diode (HL6724MG)
λ ~ 674.2 nm
5mW
~ 7% threshold reduction

Ordinal analysis of time-series of inter-dropout intervals (IDIs)

Laser output
(1 GHz oscilloscope)

<ΔT> = 100-200 ns
τ~ 5 ns

# of IDIs recorded
45,000 - 220000

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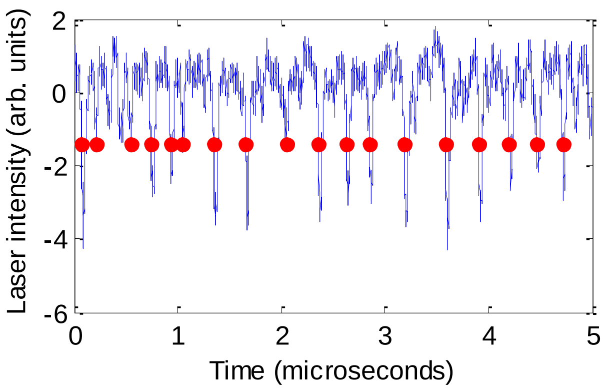
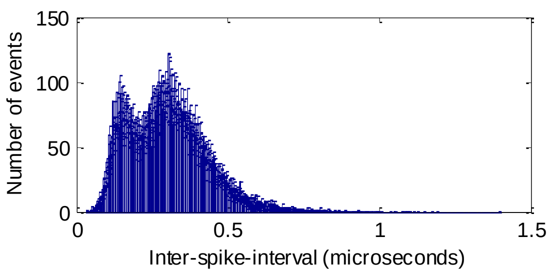
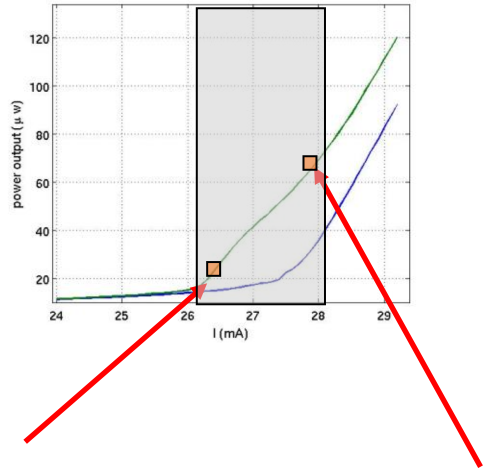
Inter-spike-interval distributions

I =26.2 mA

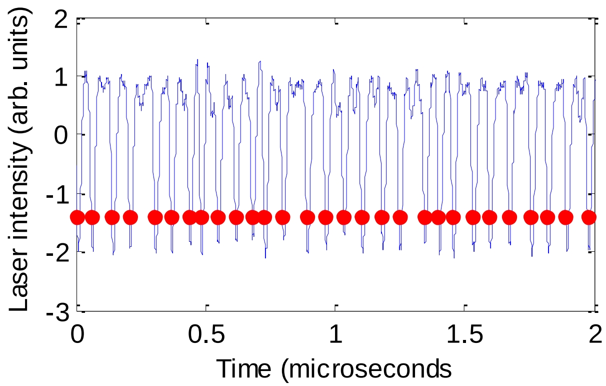


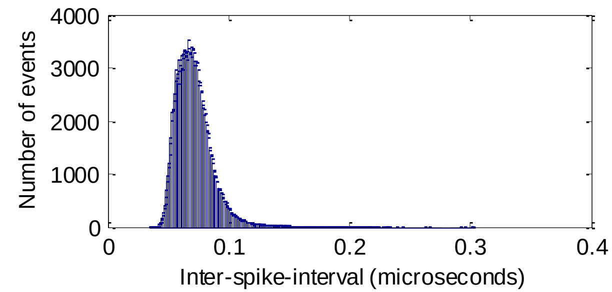


(~ 45000 dropouts)



I = 28 mA





(~ 225000 dropouts)

Is there any information in the sequence of optical spikes?

- What **new information** can we obtain from the sequence of ordinal patterns (OPs) or "words" formed with consecutive IDIs?

- Analogous to deciphering a foreign text.

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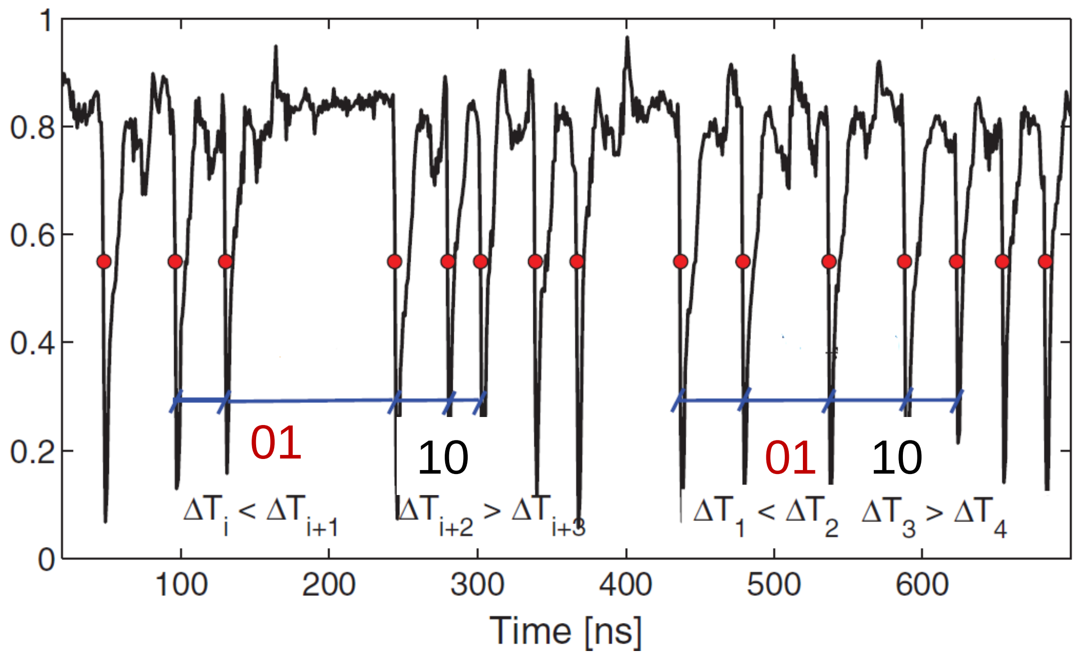
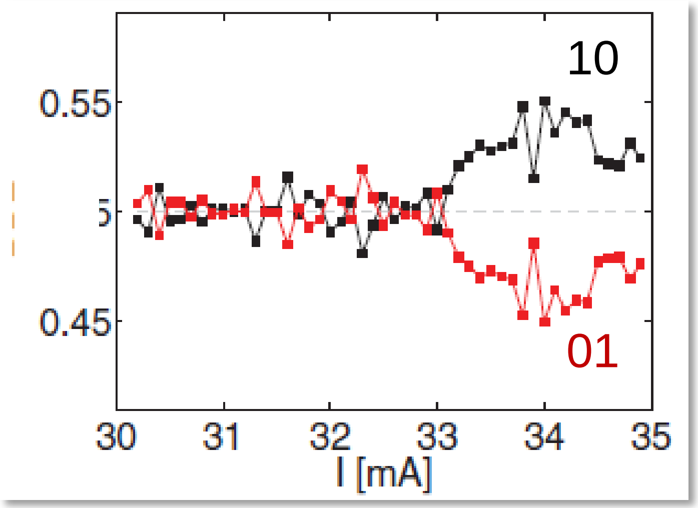
"language" analysis: "ordinal" experimental bifurcation diagram



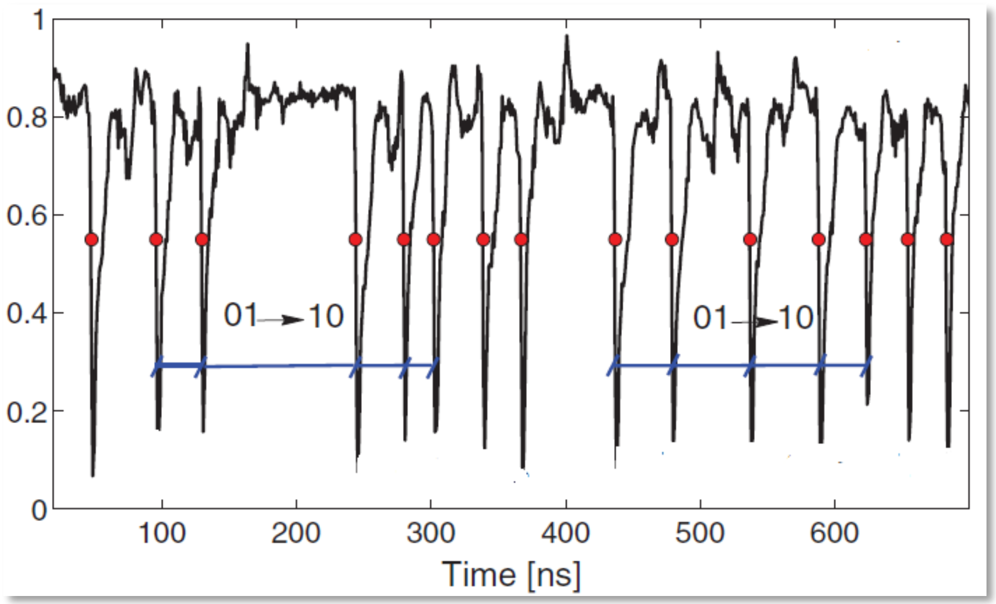
Word Probabilities:

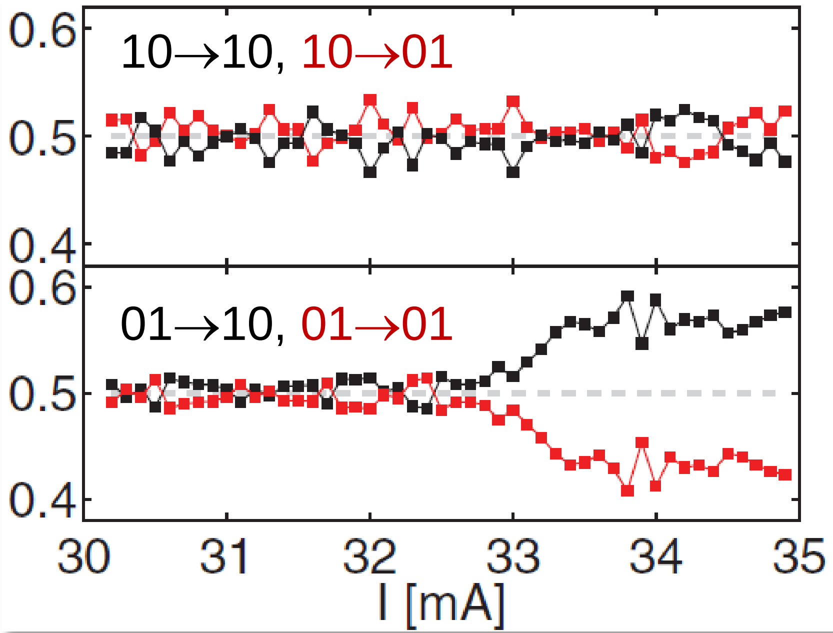


Consistent with stochastic dynamics at low pump current, but signatures of determinism at high pump current.

N. Rubido et al, Phys. Rev. E 84, 026202 (2011)

Transition probabilities





4 possible transitions:

10 → 10		p
10 → 01		1-p

01 → 10		q
01 → 01		1-q

Consistent with stochastic dynamics at low pump current, signatures of determinism at high pump current.

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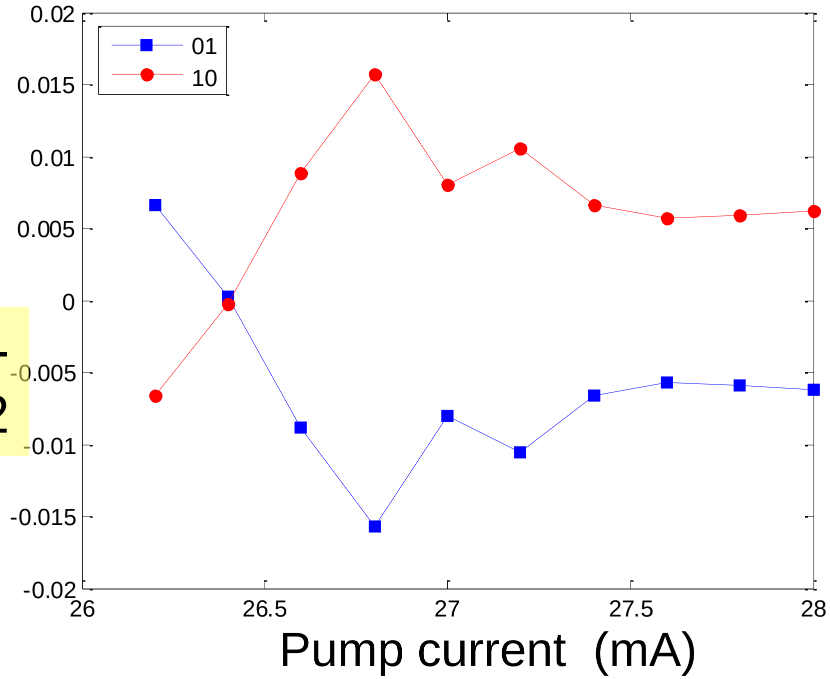
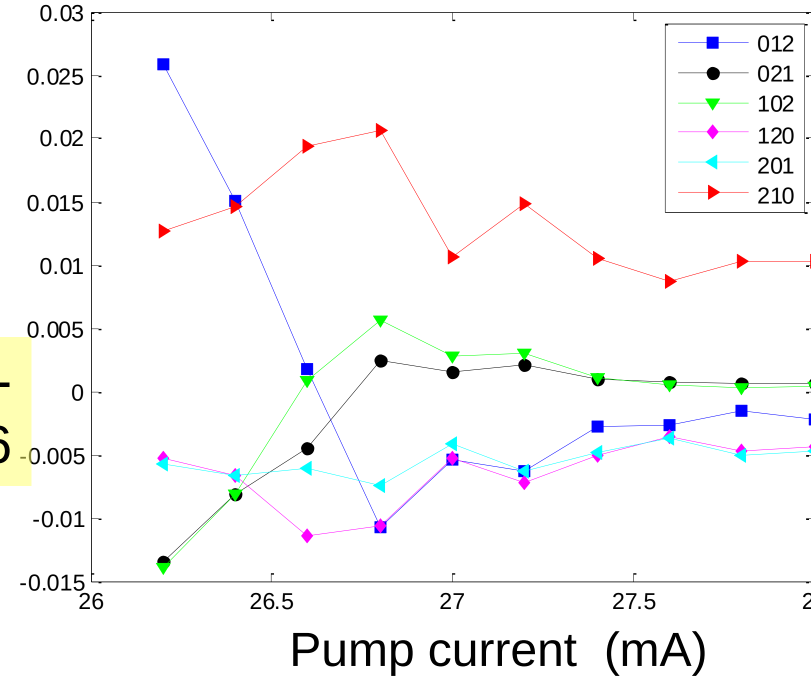
At low pump current: are the spikes fully random?

P-1/2

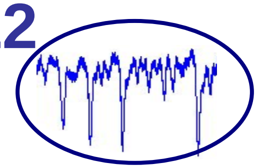


P-1/6



Data set recorded a **T=18 C**, 45000 - 220000 IDIs

012



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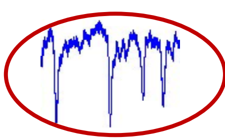
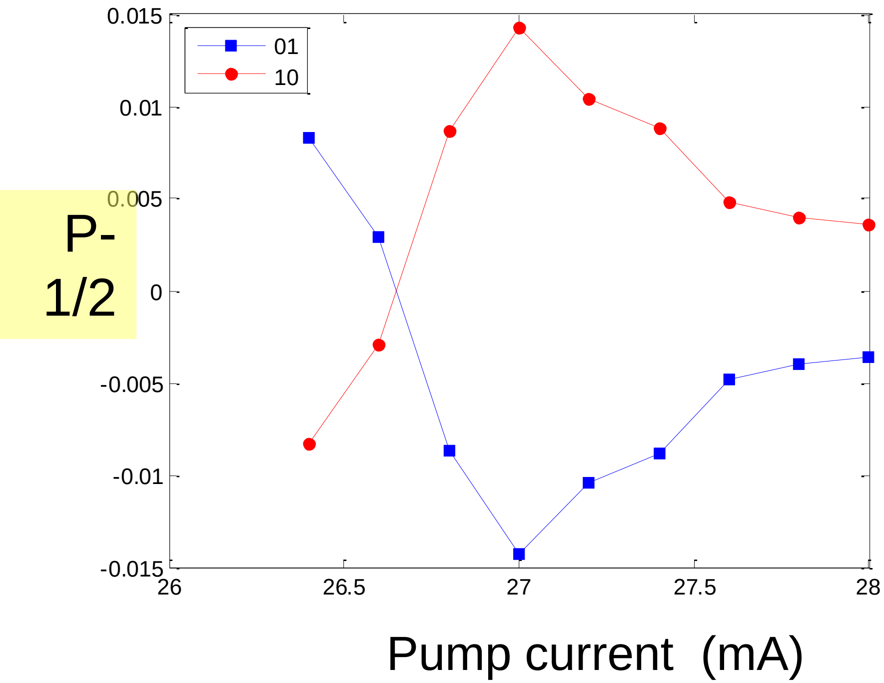
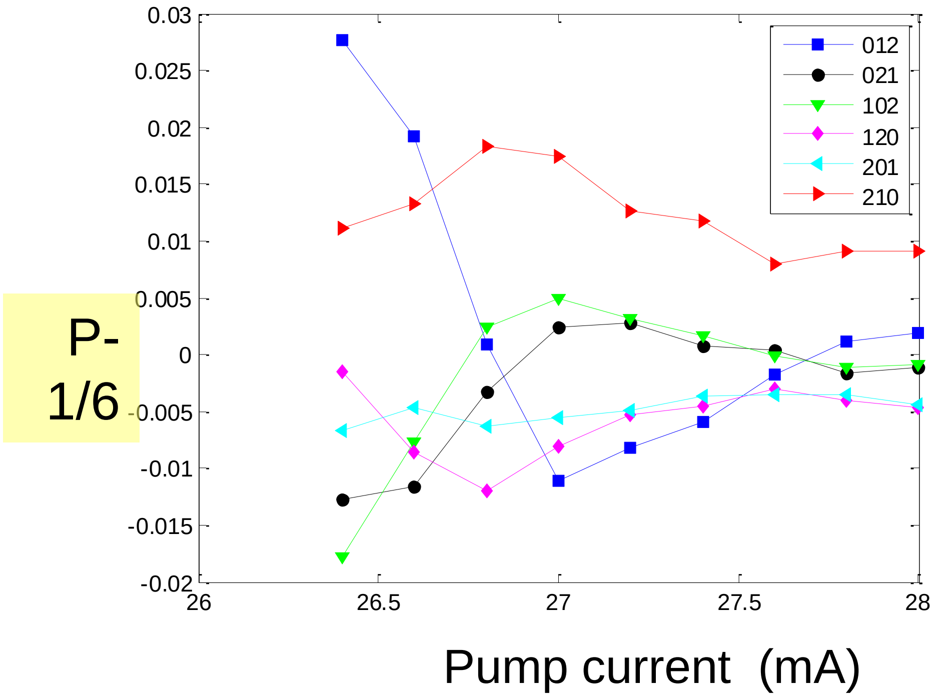


A. Aragoneses, N. Rubido, J. Tiana, M. C. Torrent and C. Masoller, Scientific Reports (2013)

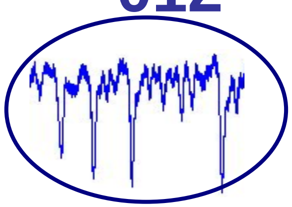
Also in another data set





Data set recorded a **T=20 C**

012



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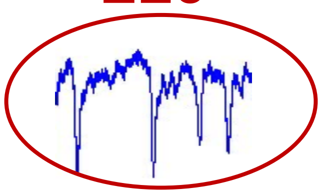
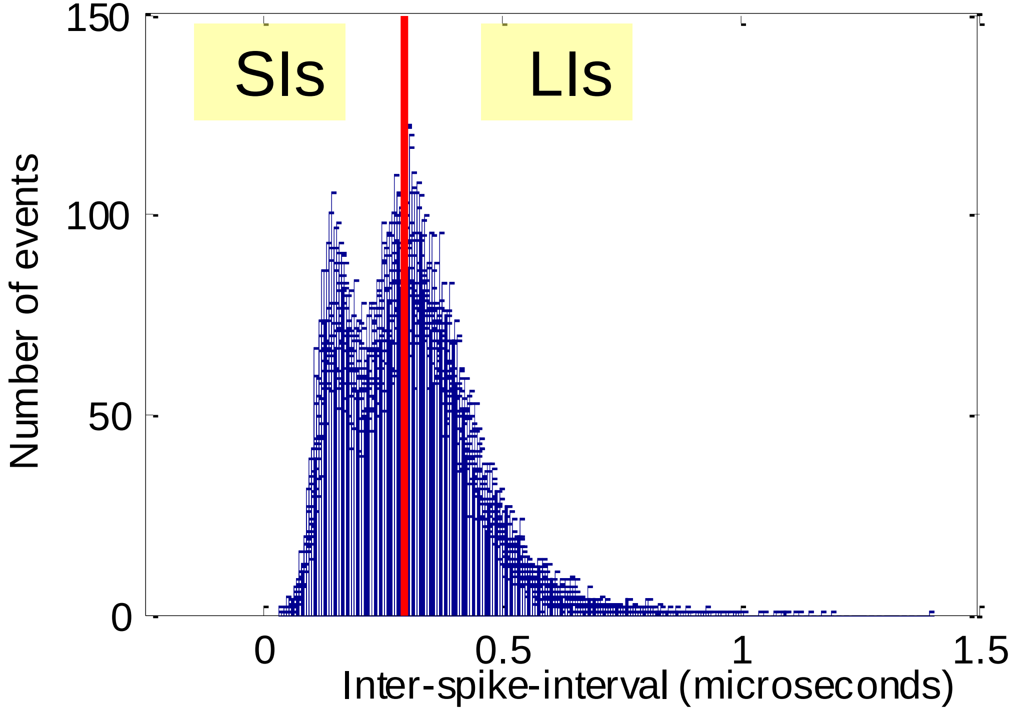


Short and long inter-spike-intervals

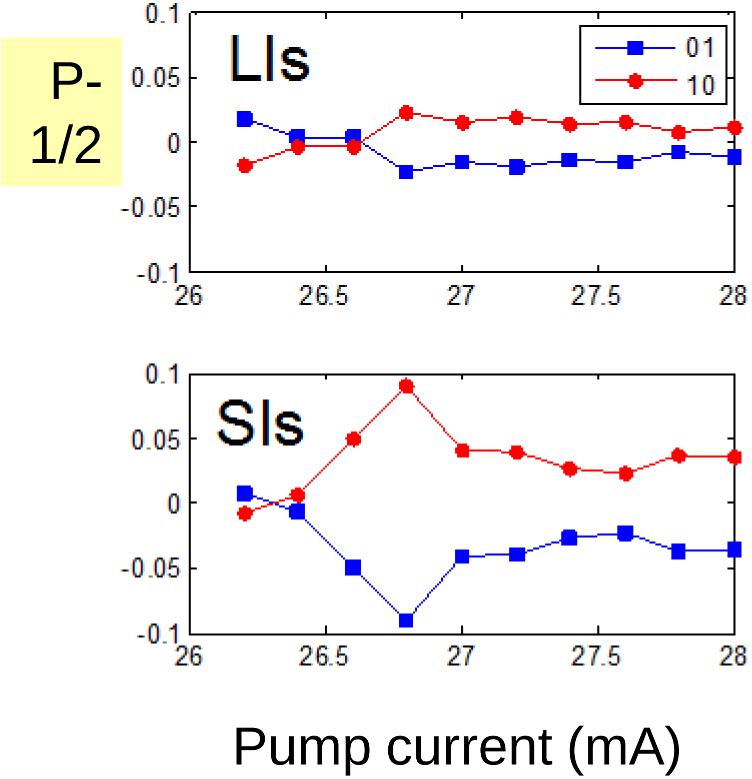


Exponential decay

Words constructed with 2 consecutive LIs or SIs only

– Significant differences at high pump currents

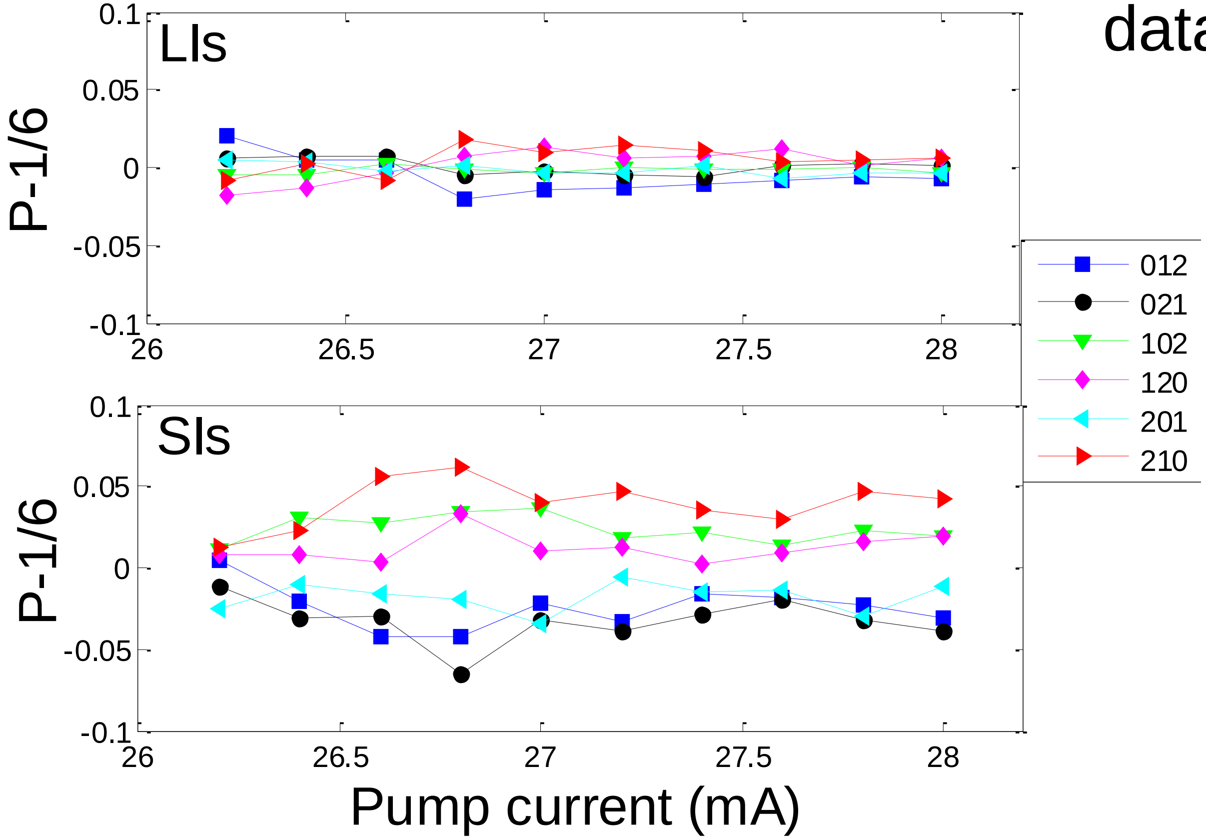
– But at low pump currents, the events can not be classified in two types with significant differences.



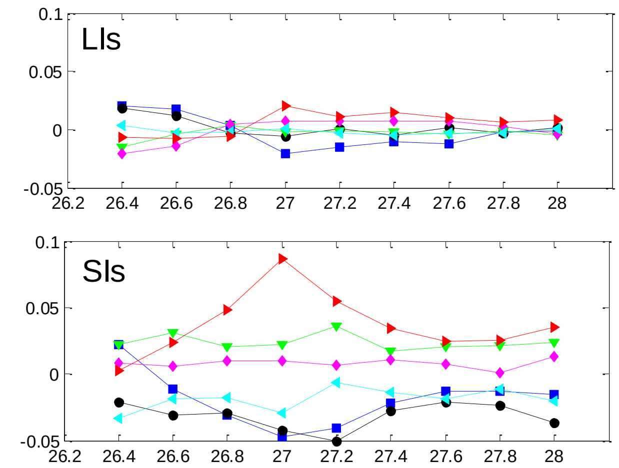
A. Aragoneses, N. Rubido, J. Tiana, M. C. Torrent and C. Masoller, Scientific Reports (2013)

Constructing the words with 3 consecutive SIs or LIs

T=18 C



Similar results in the other dataset (T=20 C)



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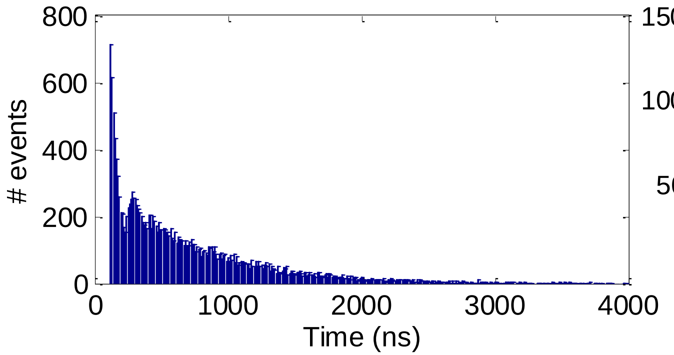
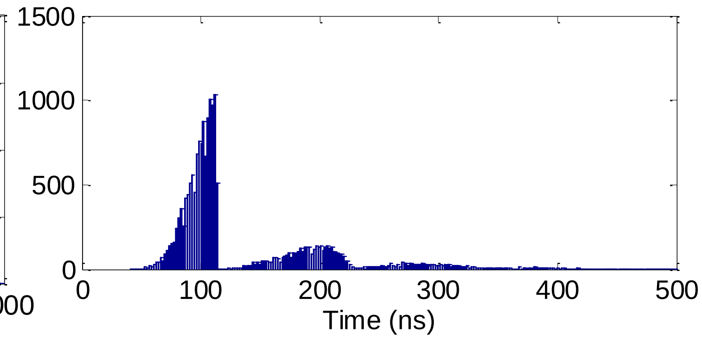
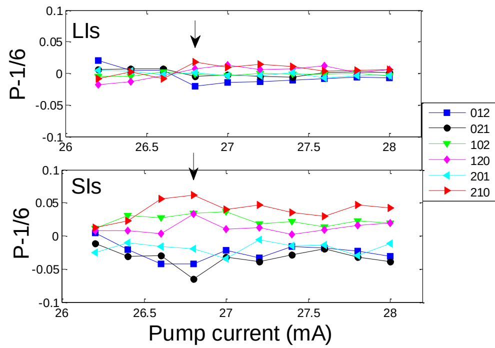
Statistics of the sum of consecutive SIs and LIs

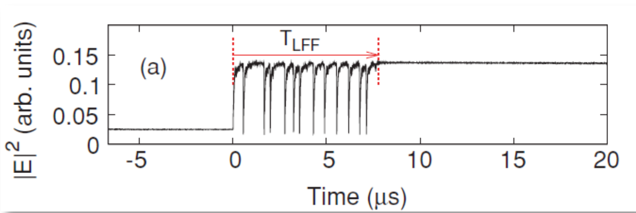
Sum LIs

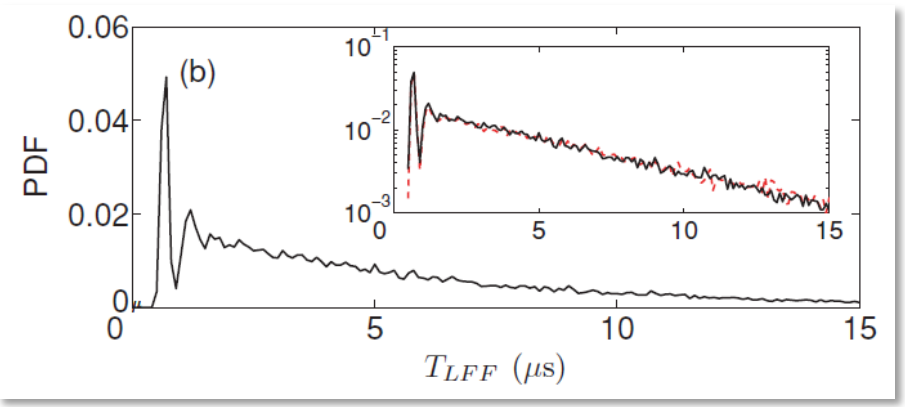


Sum SIs



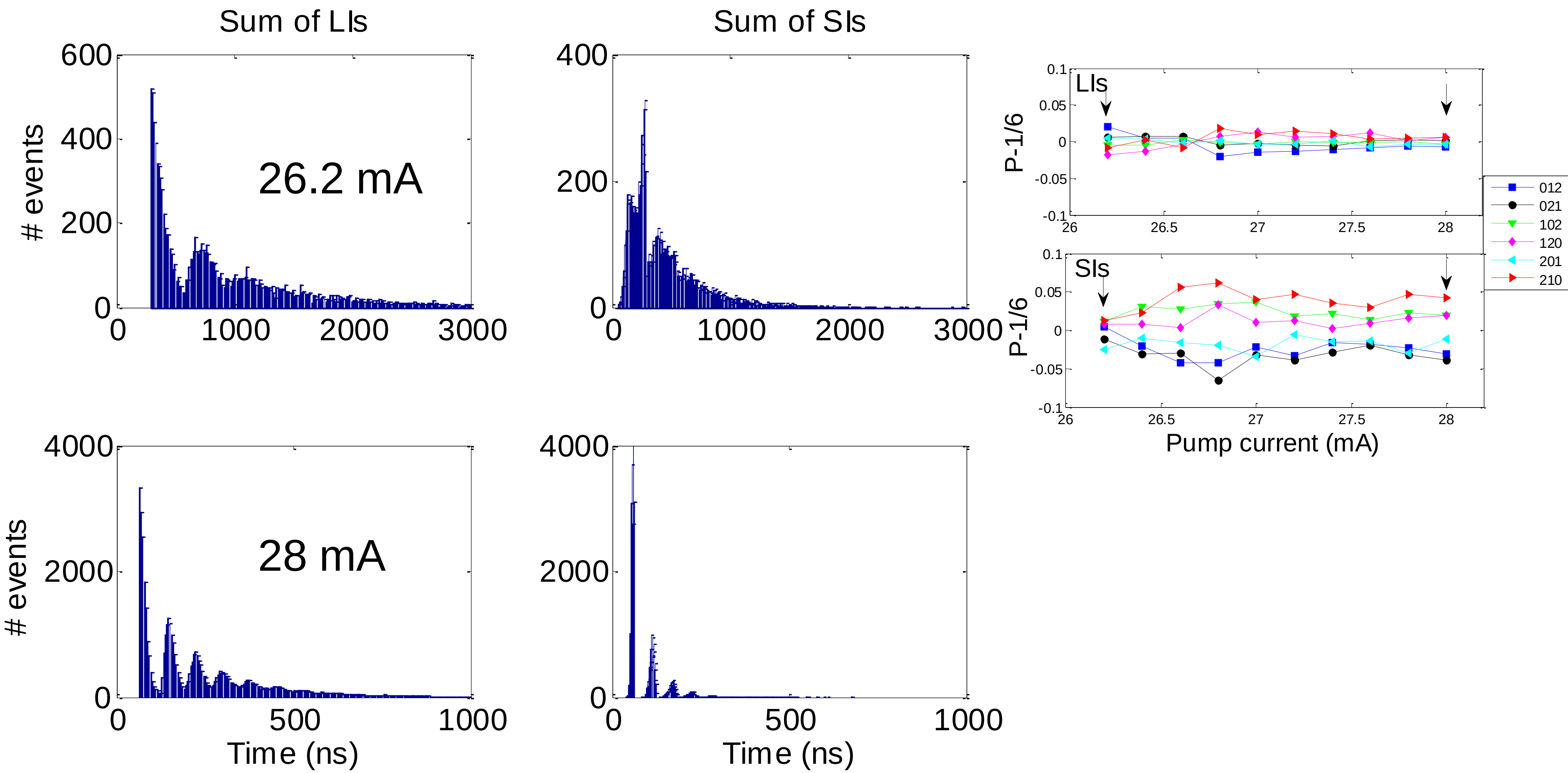






J. Zamora-Munt et al, PRA 2010

But at lower or at higher pump current: similar distributions of ∑ LIs & ∑ SIs

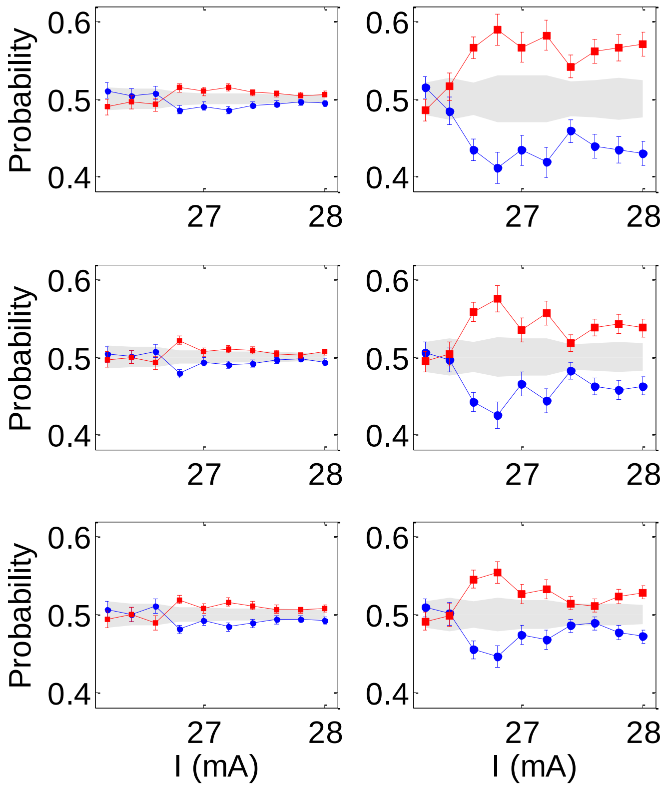


Influence of the threshold used to classify IDIs as LIs and SIs

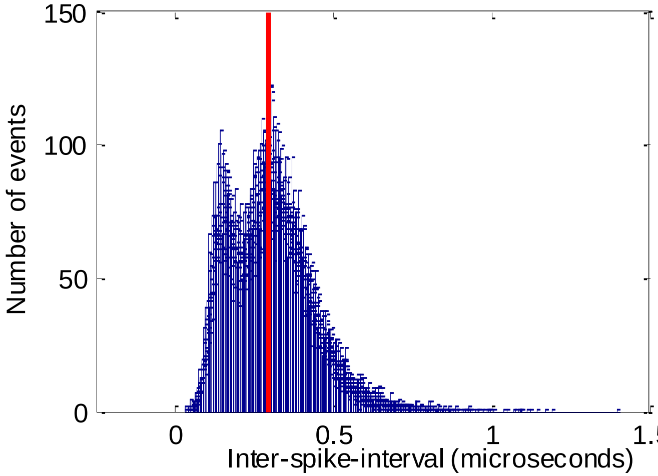
0.85 ΔT*

0.90 ΔT*

0.95 ΔT*



ΔT*= most probable value



Error bars computed with a binomial test, gray region is consistent with null hypothesis

A. Aragoneses, N. Rubido, J. Tiana, M. C. Torrent and C. Masoller, Scientific Reports (2013)

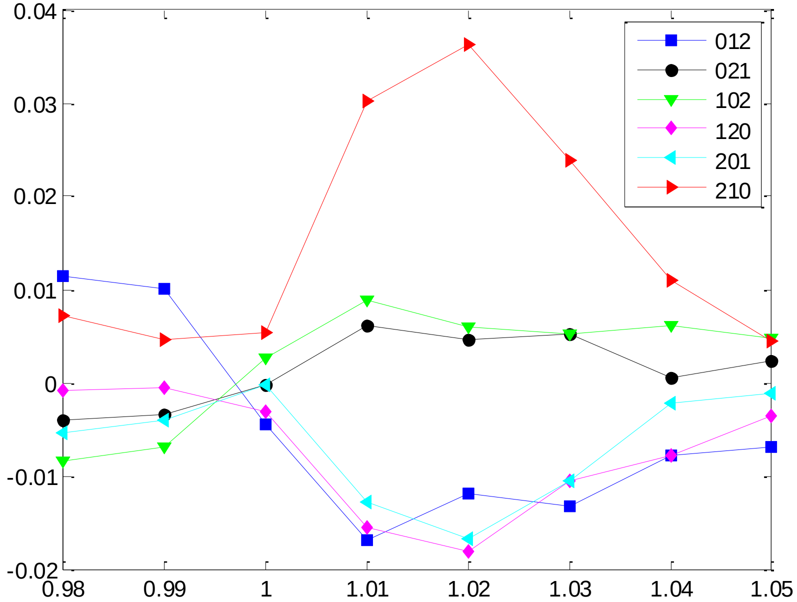
So how to chose the threshold?

- LIs have statistical features as close as possible to random, noise-triggered events:
  - the distribution of values decays exponentially
  - the distribution of "words" is uniform.
- There are enough consecutive LIs and SIs to compute the probabilities of the words with good statistics
  - The null hypothesis (NH) region is narrow
  - For the LIs, the error bars are in the NH region
  - For the SIs, the error bars are out of the NH region.
- For low pump currents we did not find a threshold that allowed to classify the dropouts in two significantly different categories.

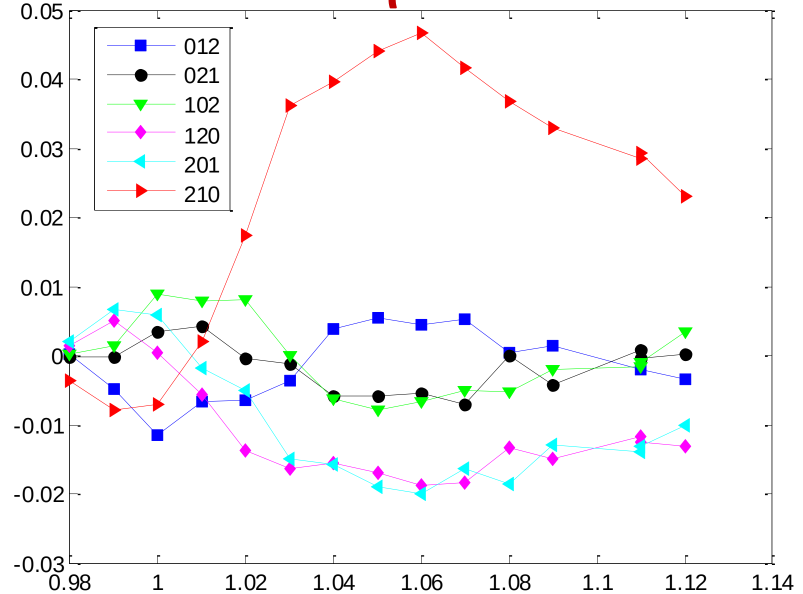
Model observations: "ordinal" bifurcation diagram

P-
1/6

Feedback η=10 ns^{-1}



Feedback η=20 ns^{-1}



Pump current parameter (arb. units)

Two clusters of words: (102 - 021) and (120 - 201)

12,000 – 40,000 dropouts

Model parameters (adjusted to fit mean IDI): k=300 ns^{-1}, γ_n= 1 ns^{-1}, α=4, ε=0.01, τ=4.7 ns, β_{sp}=10^{-4} ns^{-1}

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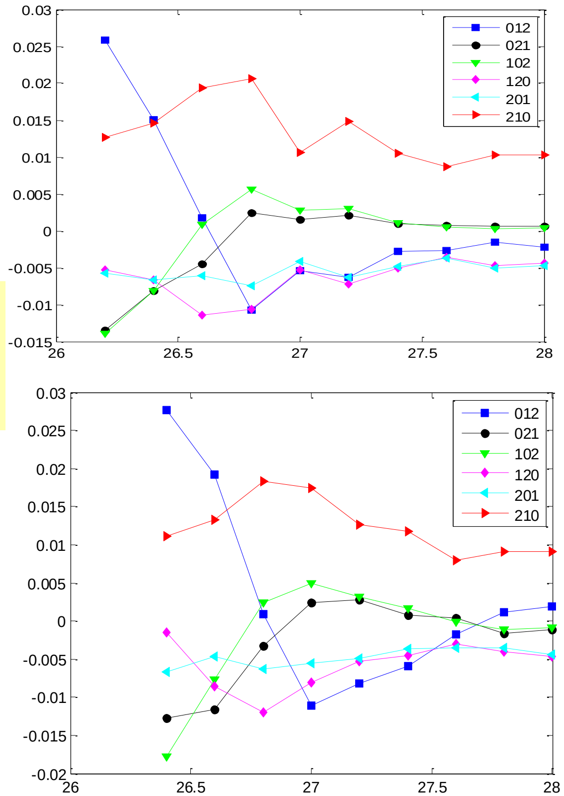
Comparing experiments with simulations

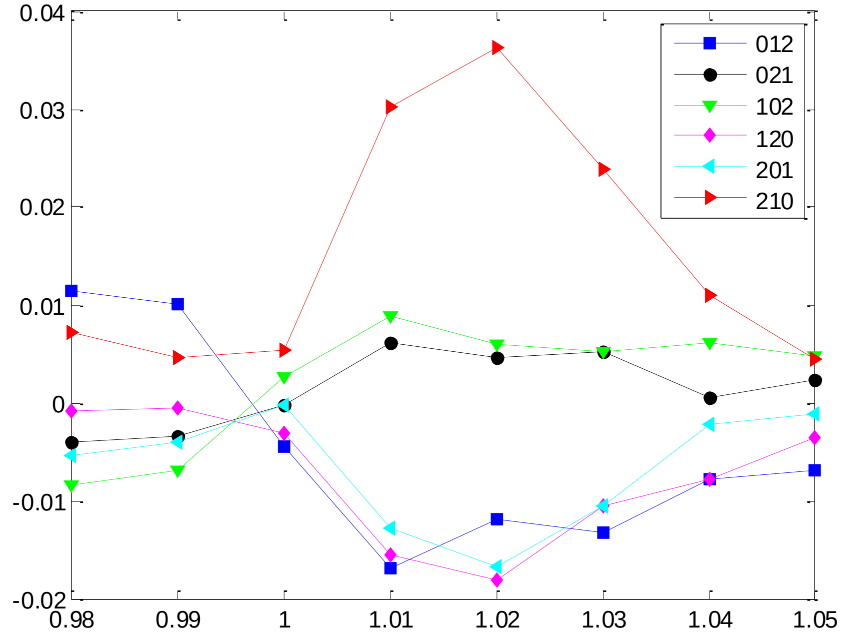
Experiments

P-1/6



Pump current (mA)

Simulations



Pump current parameter (arb. units)

Same hierarchy and clustering of words

In another data set: also the same hierarchy and clustering of words

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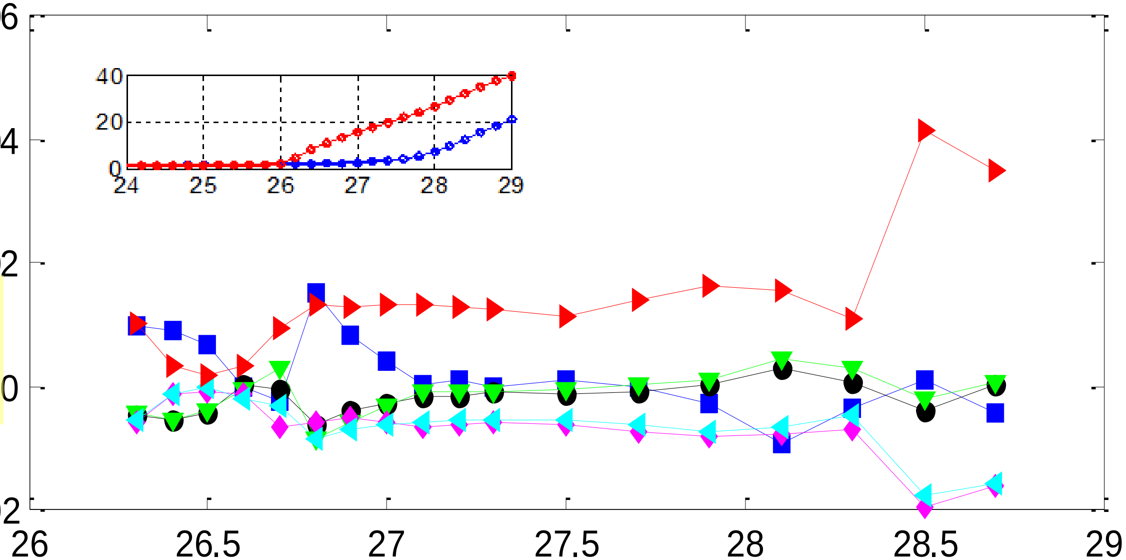
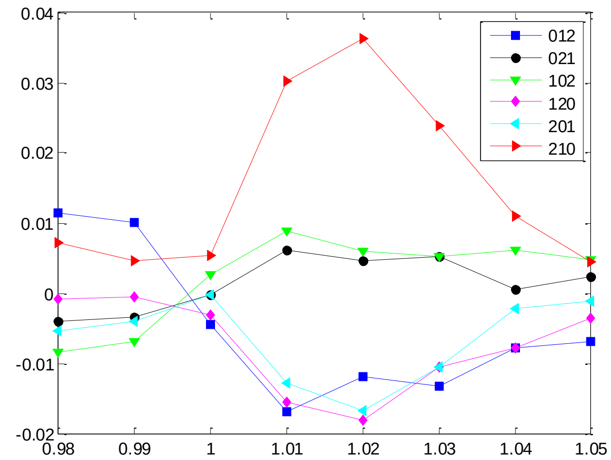
Experiments

Simulations



P-1/6

Pump current (mA)

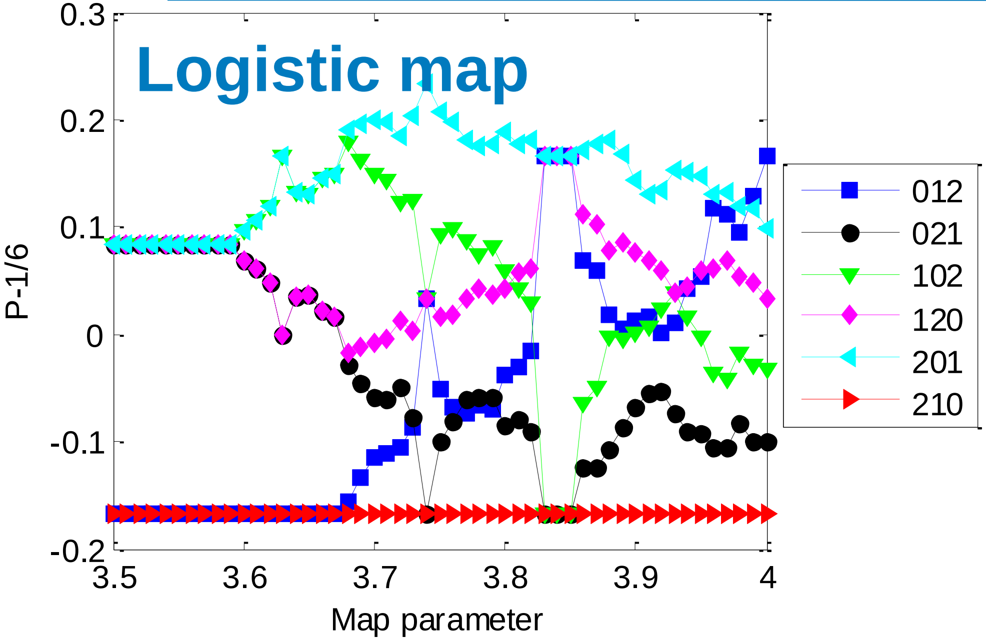


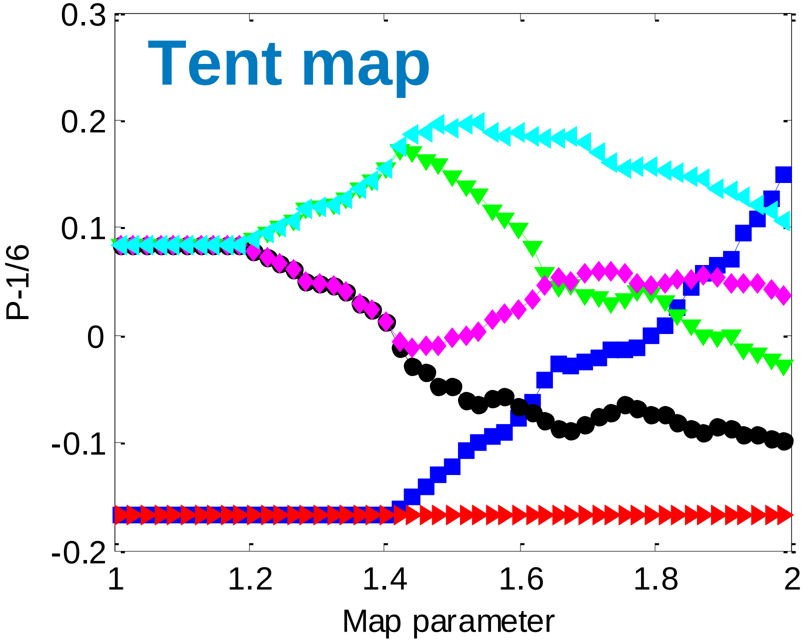
Pump current (arb. units)

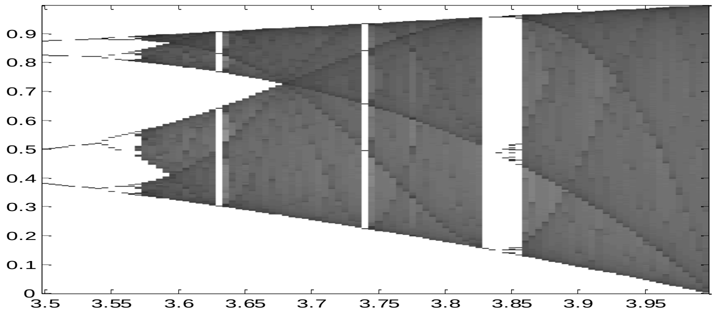
75,000 – 880,000 dropouts

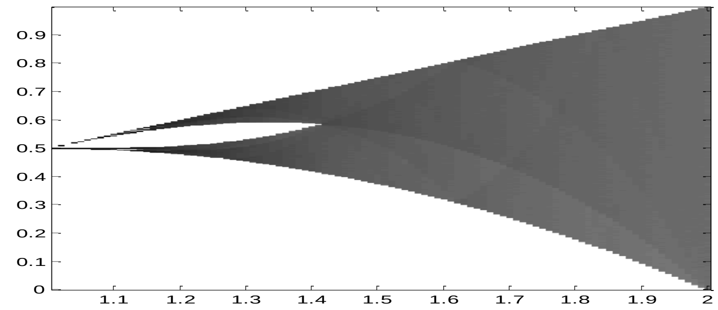
(different laser, new oscilloscope)

Can we find a minimal model displaying the same hierarchy and clustering of words?









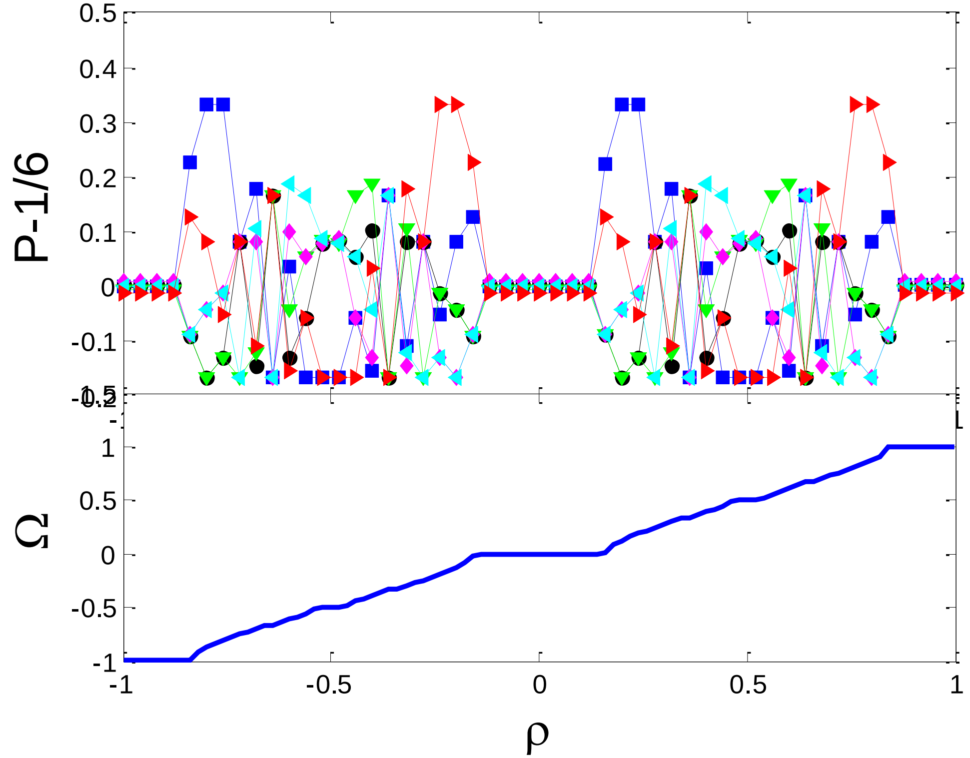
Circle map

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\sin(2\pi\varphi_i)$$

$$X_i = \varphi_{i+1} - \varphi_i$$

forcing strength: K=1

Rotation number $$\Omega = \lim_{i\to\infty} \frac{\varphi_i - \varphi_0}{i}$$

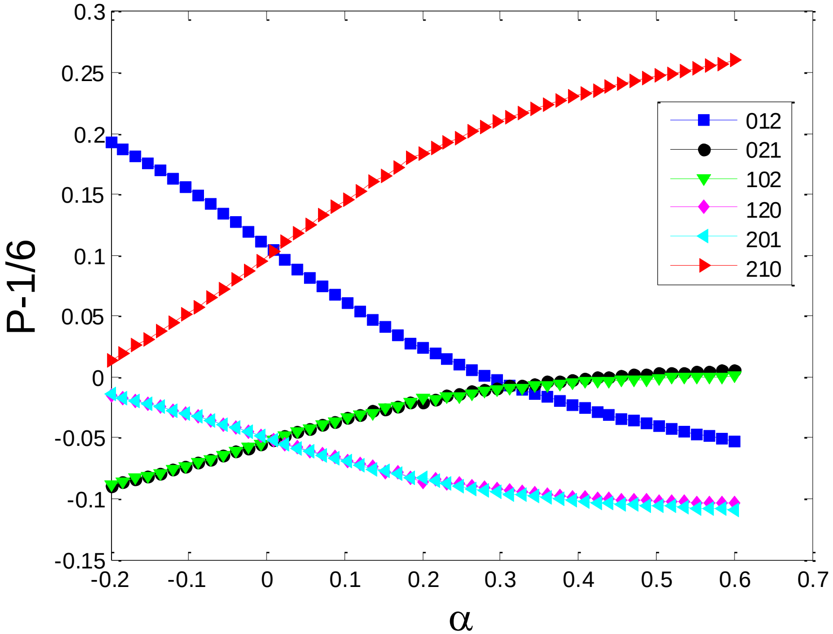


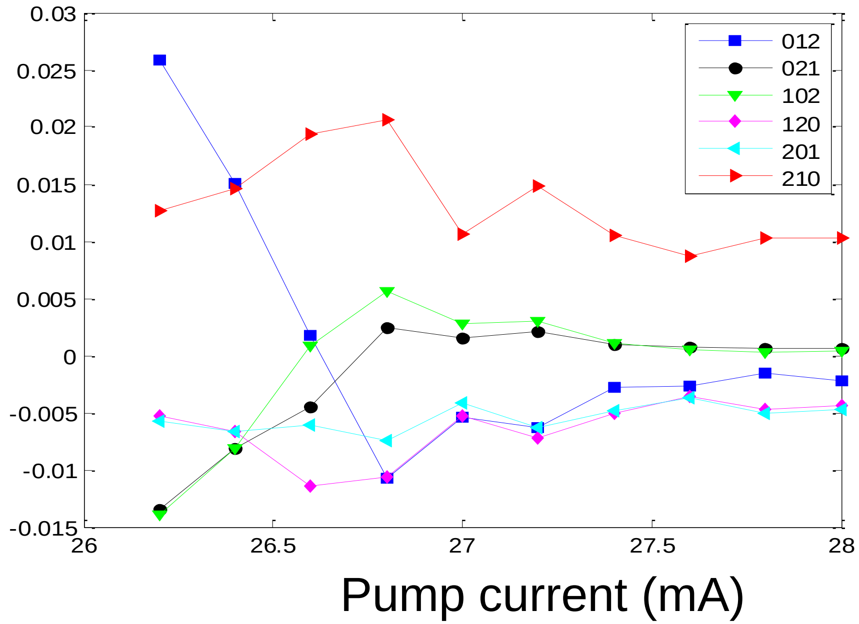
ρ =ratio of the forcing period and the oscillator period

Minimal phenomenological model

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha \sin(4\pi\varphi_i)\right]$$

$$X_i = \varphi_{i+1} - \varphi_i$$





ρ=0.23

K=0.04

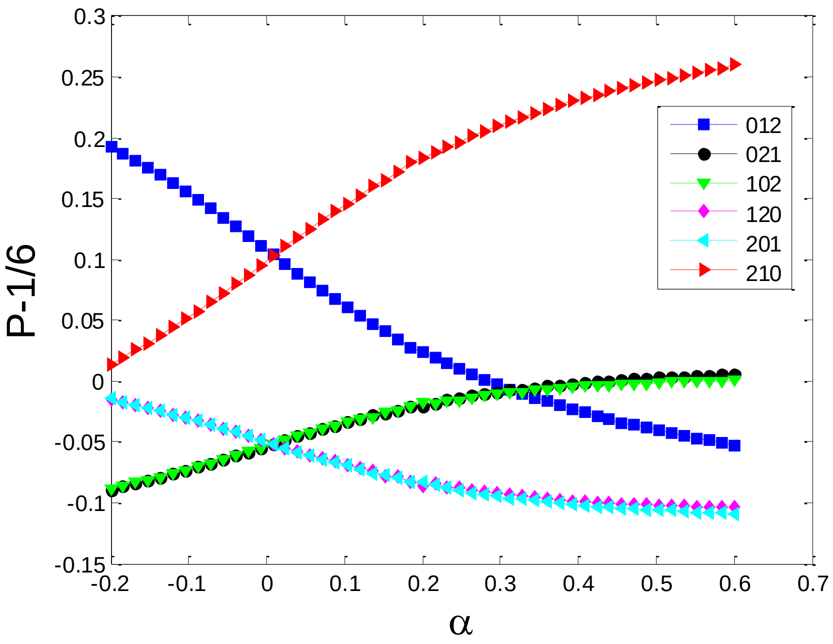
Minimal model for electroreceptors of paddlefish: A. B. Neiman and D. F. Russell, PRE 71, 061915 (2005)

Influence of noise

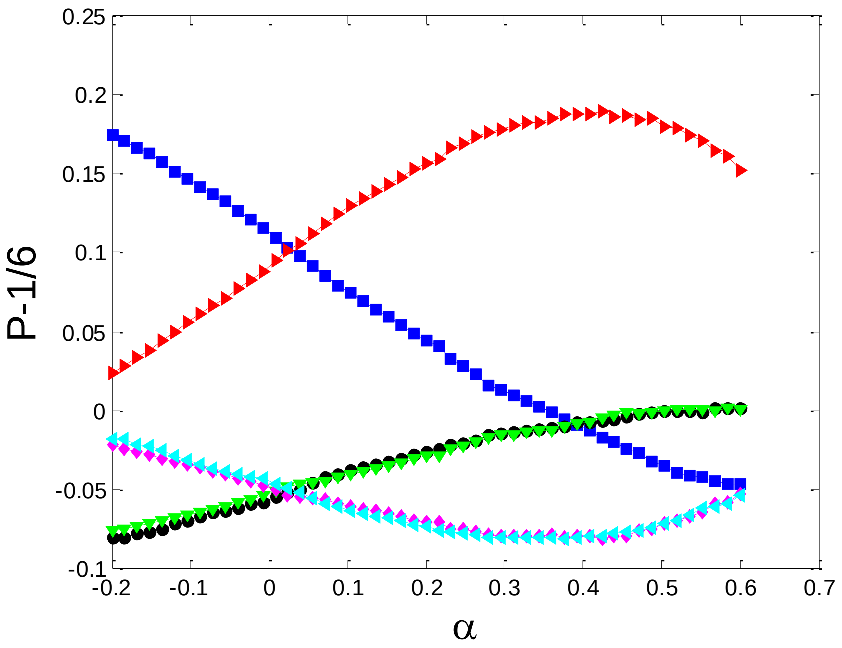
$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha\sin(4\pi\varphi_i)\right] + D\xi$$

$$X_i = \varphi_{i+1} - \varphi_i$$

D=0



D=0.002

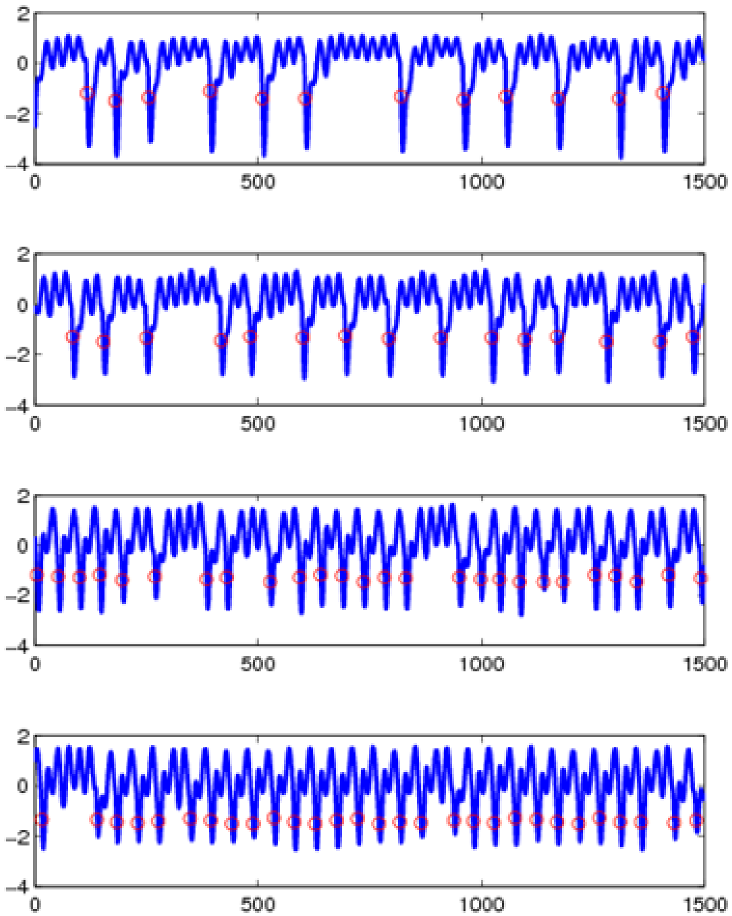


Outline

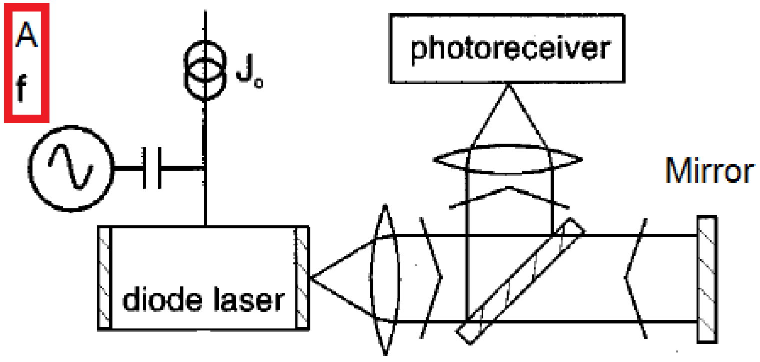
- Semiconductor lasers with feedback as stochastic spiking high-dimensional complex systems
- Method of time-series analysis (ordinal patterns) and experimental setup
- Experimental and model observations: signatures of determinism in the sequence of optical spikes + response to periodic forcing
- Conclusions and take home message

Response to periodic forcing

Laser output intensity (arb. units)



Time (ns)



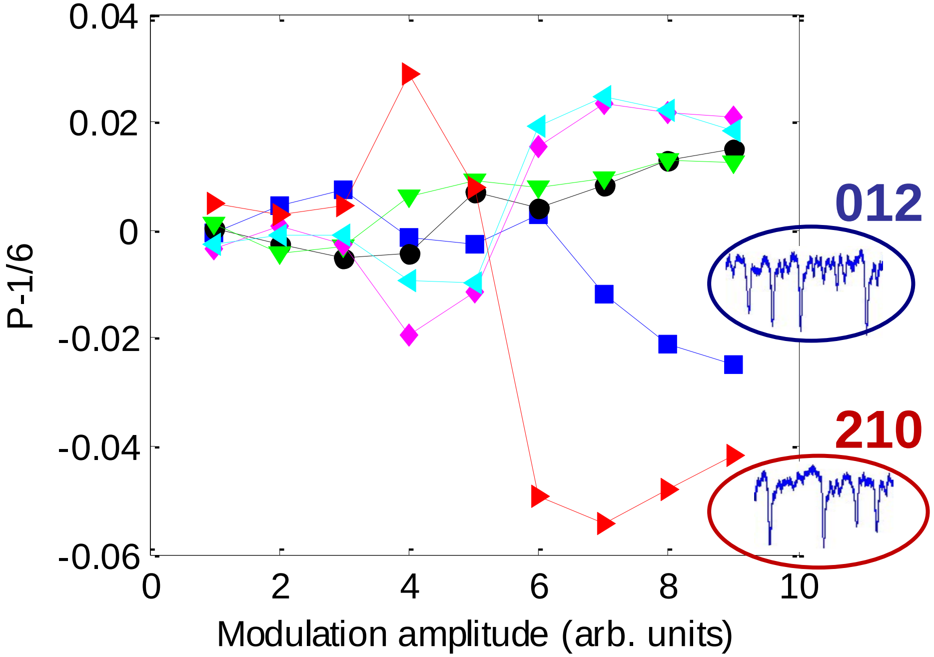
J. M. Buldú et al, Phys. Rev. E, **66**, 021106 (2002)

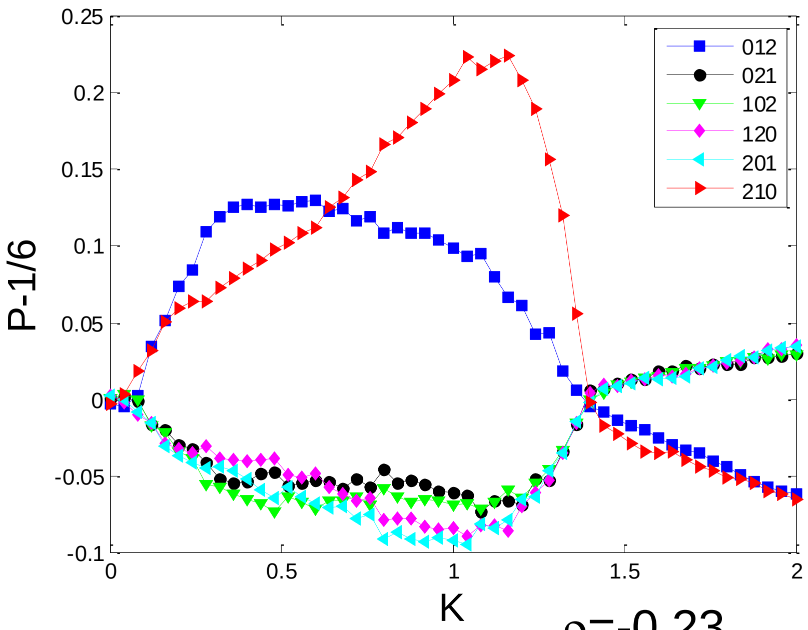
Periodic forcing: ordinal bifurcation diagram

Experiments

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha\sin(4\pi\varphi_i)\right] + D\zeta$$

(68,000 - 200,000 dropouts)





ρ=-0.23
α=0.2
D=0.02

Advertising: Andres Aragoneses' talk, today afternoon, CT13, 14:50, Room 3

Outline

- Semiconductor lasers with feedback as stochastic spiking high-dimensional complex systems
- Method of time-series analysis (ordinal patterns) and experimental setup
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Conclusions

- We proposed a method to infer signatures of determinism in sequences of events in dynamical complex system.
- The method is suitable for the analysis of high-dimensional stochastic systems displaying noise or deterministically induced events.
- With an adequate threshold, events display significant different statistical features.
- We found new symbolic states with an hierarchical and clustered organization of patterns.
- We found a good connection model-experimental data.
- We also identified a minimal phenomenological model.

Perspectives

- On going work is focusing on
  - characterizing and classifying optical spikes (single and coupled lasers)
  - comparing with biological neurons (via ordinal analysis of ISIs)
- For the future:
  - Excitable spikes in optically injected lasers
  - Strong and weak chaos in feedback lasers
- Potential breakthrough: optical neurons for neuro-inspired information processing.

Take home message

- Ordinal analysis is a powerful technique for the event-level description of complex systems
  - useful for data understanding and uncovering patterns in the sequence of events,
  - useful for improving system modeling, model comparison and parameter estimation,
  - useful for classifying different types of behaviors,
  - potential for improving event predictability and forecasting.

Thanks to

You for your attention!

- Andres Aragoneses

- Nicolas Rubido

- Sandro Perrone

- Taciano Sorrentino

- Jordi Tiana

- Carme Torrent

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