Signal encoding and transmission by noisy coupled neurons

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How neurons encode information?

- In the spike rate?
- In the relative timing of the spikes?
- Single neuron encoding or ensemble encoding?
- How can temporal correlations be detected and quantified?

Our goal: try to understand how neurons encode, in the sequence of spikes, an external weak (subthreshold) signal, in the presence of noise.
Cracking the neural code is an important problem in neuroscience, with huge potential applications in information processing systems.

Can lasers mimic real neurons? How the laser encodes a weak external signal?
Photonic neurons

- Excitable lasers could be the building blocks of ultra-fast, energy-efficient information processing systems.
- Inexpensive laser diodes (perturbed by optical feedback).
The laser dynamics: excitability, tonic spikes and bursting. Similar to real neurons?


Coherence resonance

- FitzHugh–Nagumo model ($\varepsilon=0.01$, $a=1.05$)

\[
e \frac{dx}{dt} = x - \frac{x^3}{3} - y,
\]

\[
\frac{dy}{dt} = x + a + D\xi(t)
\]

Quantification of coherence resonance

\[ R = \frac{\sqrt{\langle I^2 \rangle - \langle I \rangle^2}}{\langle I \rangle} \]

Correlation length (solid) and Coef. of variation R (dashed)

FHG model

Laser spikes

Pikovsky and Kurths, PRL (1997)

With an external signal, are there statistical similarities between neuronal and laser spikes?

FIG. 1. (a) An experimental ISIH obtained from a single auditory nerve fiber of a squirrel monkey with a sinusoidal 80-dB sound-pressure-level stimulus of period $T_0=1.66$ ms applied at the ear. Note the modes at integer multiples of $T_0$. Inset:


Experimental data when the laser current is modulated with a sinusoidal signal of period $T_0$.

A. Aragoneses et al. *Optics Express* (2014)
Return maps of inter-spike-intervals

**Neuronal ISIs**

\[ \Delta T_{i+1} \]

\[ \Delta T_i \]

A. Longtin  
*Int. J. Bif. Chaos (1993)*

**Laser ISIs**

M. Giudici et al *PRE* (1997)  
A. Aragoneses et al *Optics Express* (2014)

**HOW TO IDENTIFY TEMPORAL ORDER?**

**SIMILAR MORE/LESS EXPRESSED PATTERNS?**
Outline

- Symbolic method of analysis of ISI sequences
- Single neuron
- Two coupled neurons
- Neuronal ensemble

\[ \{ \ldots I_{i-1}, I_i, I_{i+1} \ldots \} \]

\[ l_i = t_{i+1} - t_i \]

inter-spike-intervals

\text{Output of Neuron 1}

\text{Output of Neuron 1 and Neuron 2}

\text{Weak signal}
Symbolic method of time-series analysis
Relative order of three consecutive intervals

\[ l_i = t_{i+1} - t_i \]

\{ ...l_i, l_{i+1}, l_{i+2}, ... \}

Example: (5, 1, 7) gives “102” because 1 < 5 < 7

Brandt & Pompe, PRL 88, 174102 (2002)
The number of ordinal patterns increases as $D!$

- How to select the optimal $D$?
  - it depends on:
    - The length of the data
    - The length of the correlations
Are the $D!$ ordinal patterns equally probable?

- **Null hypothesis:**
  
  $$p_i = p = 1/D! \quad \text{for all } i = 1 \ldots D!$$

- If at least one probability is not in the interval $p \pm 3\sigma$ with $\sigma = \sqrt{p(1-p)/N}$ and $N$ the number of ordinal patterns:
  
  We **reject** the NH with 99.74% confidence level.

- Else
  
  We **fail to reject** the NH.
Individual neuron
- more / less expressed patterns in spike sequences encode information about subthreshold signal?
FitzHugh-Nagumo model

\[
\begin{align*}
\epsilon \frac{dx}{dt} &= x - \frac{x^3}{3} - y, \\
\frac{dy}{dt} &= x + a + a_0 \cos \left( \frac{2\pi t}{T} \right) + D \delta(t),
\end{align*}
\]

- Gaussian white noise and subthreshold signal: \(a_0\) and \(T\) such that spikes are noise-induced.
- Time series with \(M=100,000\) spikes simulated (\(a=1.05, \varepsilon=0.01\)).

Longtin and Chialvo, PRL 1998
Results

- Gray region: $3\sigma$ confidence level.

Data requirements

With signal

Without signal

J. M. Aparicio-Reinoso et al. PRE 94, 032218 (2016)
Comparison with the laser spikes, when a small sinusoidal modulation is applied to the laser pump current

Role of the level of noise

$a_0 = 0$

No signal $\Rightarrow$ no temporal ordering
With external signal

The signal induces preferred and infrequent patterns.
They depend on the period and on the noise strength.
Resonant-like behavior.

Time series with different $P(012)$

- **Low noise**
  - $P(012) > 1/6$
  - ~40 spikes in 20 $T$

- **Stronger noise**
  - $P(012) < 1/6$
Role of the signal amplitude

\[ a_0 = 0.02 \]
\[ T = 20 \]

Ordinal probabilities

The amplitude of the (subthreshold) signal does not modify the preferred or the infrequent patterns. The values of the probabilities encode information about the amplitude of the signal.
Role of the signal period

$a_0 = 0.02$

$T = 20$

Ordinal probabilities

$D = 0.015$
More probable patterns depend on period and noise strength.

Which is the underlying mechanism?  
A change of the mean inter-spike-interval?

⇒ No direct relation.
The amplitude and the period of the signal might be encoded in more and less expressed patterns.

**Single-neuron encoding:** very **slow** because long spike sequences are needed to estimate the probabilities.

**Ensemble encoding:** can be **fast** because few spikes per neuron are enough to estimate the probabilities.
Coupling to a second neuron

- how does it affect signal encoding?
Identical neurons.

Linear & instantaneous & asymmetric coupling

Signal, coupling and noise in the fast variable.

\( a=1.05 \) and \( \varepsilon=0.01 \); parameters: \( a_0, T, D, \sigma_1, \sigma_2 \)

M. Masoliver and C. Masoller, Scientific Reports 8, 8276 (2018)
Output of neuron 1 (that perceives the signal)

No signal, no noise, no coupling

\[ u_1 \]

With signal

Signal + noise

Signal + noise + coupling
Identification of the subthreshold region

$D=0$

\[
\sigma_1 = \sigma_2 = 0 \quad \sigma_1 = \sigma_2 = 0.05
\]

The signal is subthreshold if the amplitude is small and/or the period is long.

No Spikes

\[\text{Spike rate} = \frac{1}{\text{ISI}}\]
With noise

\[ \sigma_1 = \sigma_2 = 0 \]

\[ \sigma_1 = \sigma_2 = 0.05 \]

Coupling increases the spike rate

Spike rate = 1/\langle ISI \rangle
Effect of noise

\[ \sigma_1 = \sigma_2 = 0 \]

\[ \sigma_1 = \sigma_2 = 0.05 \]

The spike rate \((=1/\langle I \rangle)\) does not encode the period of the signal.
Coherence and stochastic resonances

\[ R = \frac{\sqrt{\langle I^2 \rangle - \langle I \rangle^2}}{\langle I \rangle} \]

\[ \sigma_1 = \sigma_2 = 0 \]

\[ \sigma_1 = \sigma_2 = 0.05 \]
No signal $\Rightarrow$ no spike correlations

$a_0=0$
The signal induces spike correlations

\[ a_0 = 0.05 \quad T = 10 \]

M. Masoliver and C. Masoller, Scientific Reports 8, 8276 (2018)
Spike correlations depend on the amplitude of the signal

⇒ With coupling the signal is still encoded in the ordinal probabilities.
⇒ Coupling changes the more and less expressed patterns.
The probabilities also depend on the period of the signal.

\[ \sigma_1 = \sigma_2 = 0 \]

\[ \sigma_1 = \sigma_2 = 0.05 \]

\[ \Rightarrow \text{Coupling changes the preferred patterns.} \]
Are the spike correlations captured by linear analysis?

$$a_0 = \sigma = 0.05, \ T=8$$

$$C_j = \frac{\langle (I_i - \langle I \rangle) (I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

⇒ For strong noise, correlation coefficients at lag 1 and 2 vanish but ordinal analysis detects more / less expressed patterns.
Neuronal ensemble?
Model

\[
\epsilon \dot{u}_i = u_i - \frac{u_i^3}{3} - \nu_i + a_0 \cos(2\pi t/T) + \frac{\sigma_i}{\langle k \rangle} \sum_{j=0}^{N} A_{ij} (u_j - u_i) + \sqrt{2D} \xi_i \quad j \neq i
\]

\[
\dot{\nu}_i = u_i + a
\]

The signal is applied to each neuron.

The signal is subthreshold for each individual neuron:

with \( D = 0 \) and \( \sigma_i = 0 \), no spikes.
Spike rate and spike sequence regularity

$A_{ij} = 1 \ \forall \ i \neq j$
Influence of the signal amplitude

1 neuron

50 neurons
Influence of the signal period

1 neuron

50 neurons
Influence of noise

1 neuron

50 neurons

P(012) of each neuron
Influence of the number of neurons

![Graph showing the influence of the number of neurons on OP probabilities.]
In the model the coupling is normalized to the number of links: it does not increase with the network size.

Synchronization?
Conclusions
What did we learn?

- **Take home message:**
  - Ordinal time-series analysis uncovers patterns in data.
  - It detects correlations that might not be captured by linear analysis.

- **Main conclusions:**
  - Neuron fires at lower noise level when coupled.
  - The ordinal probabilities carry information about the signal (amplitude and period) with or without coupling.
  - Coupling changes the preferred/infrequent patterns.
  - In neuronal ensembles the encoding of the signal can be more pronounced.

- **Ongoing work:**
  - Similar results with other models and types of coupling.
  - Synchronization, network structure?
THANK YOU FOR YOUR ATTENTION!

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Emergence of spike correlations in periodically forced excitable systems

Subthreshold signal encoding in coupled FitzHugh-Nagumo neurons

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