Big data approach to dynamical optical complexity

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Workshop on Pattern Dynamics in Nonlinear Optical Cavities
MPIKPS, Dresden, Germany, August 2016

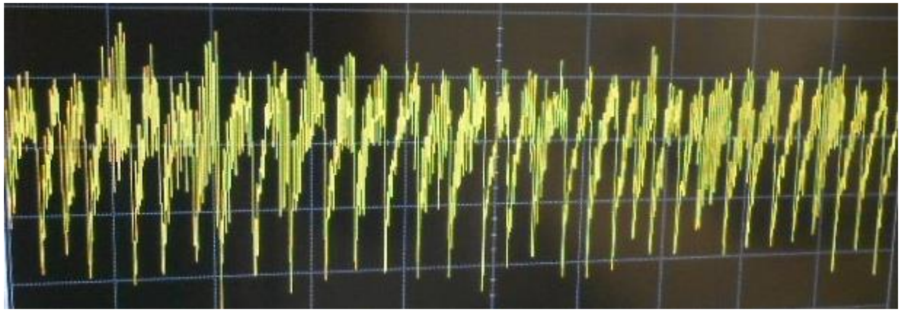
ICREA

GOBIERNO DE ESPAÑA MINISTERIO DE CIENCIA E INNOVACIÓN

Dynamical complexity

- **Low frequency fluctuations –LFFs**

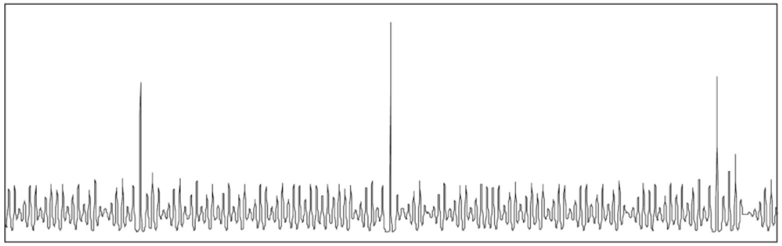
Semiconductor laser with optical feedback



Time

- **Extreme pulses (rogue waves)**

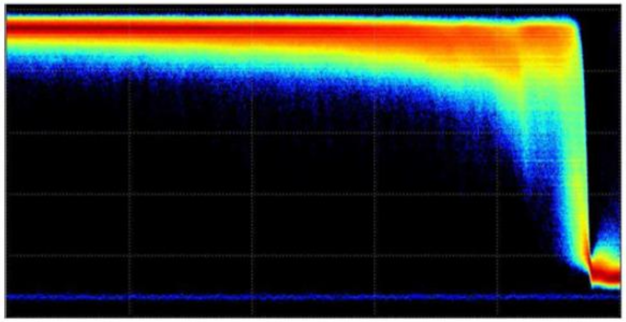
Semiconductor laser with continuous-wave optical injection



Time

- **Polarization switching**

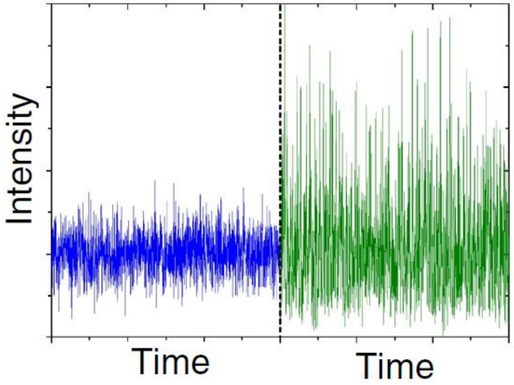
VCSELs with time-varying pump current



Time

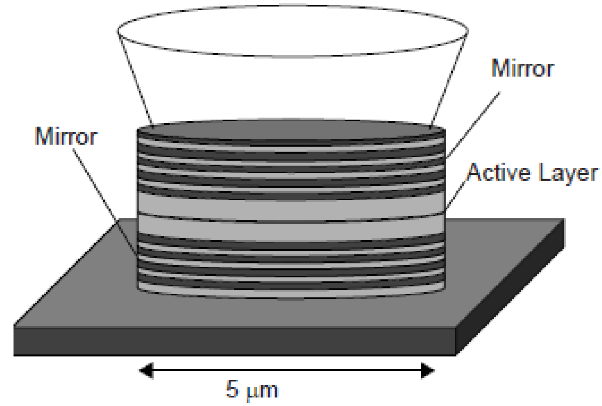
- **Transition to turbulence**

Raman fiber laser with time-varying pump power



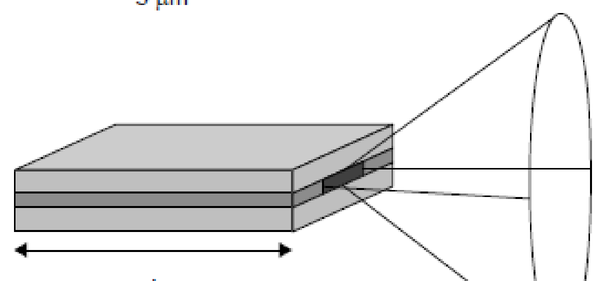
Laser types

- Semiconductor lasers
  - Vertical-cavity (VCSELs)



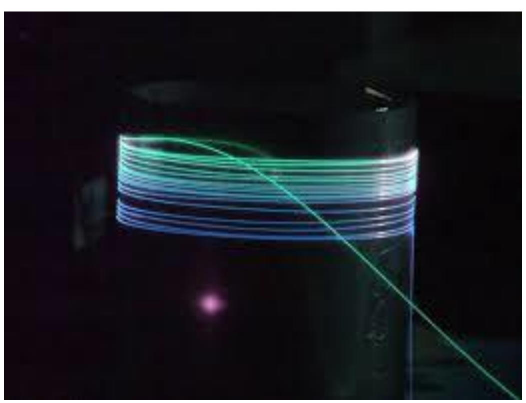
L=1-10 µm, single longitudinal mode

  - Edge-emitting lasers (EELs)



L~300 µm, several longitudinal modes

- Raman fiber laser



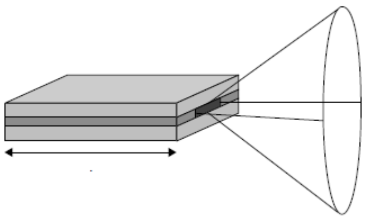
L=1 km, millions of longitudinal modes

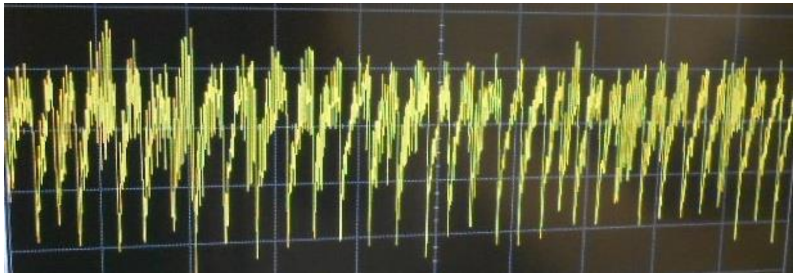
normal dispersion fiber; cavity formed by two fiber Bragg gratings

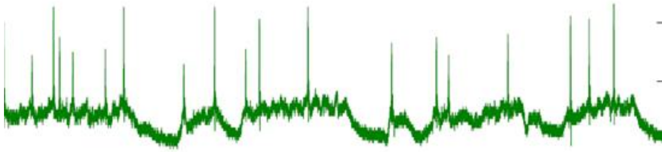
Scope and outline

- Scope:
  - controlled experiments generate output signals that are useful for testing novel analysis tools.
  - novel analysis tools can yield new insights on the underlying dynamics.

- Outline
  - Temporal correlations of LFF dropouts
  - Early warning indicators of the PS transition
  - Characterization of the laminar-turbulence transition
  - Oscillatory patterns anticipate extreme pulses
  - Summary







Temporal correlations of LFF dropouts

- similar to neuronal spikes?

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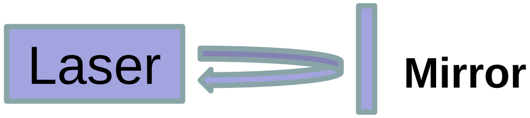
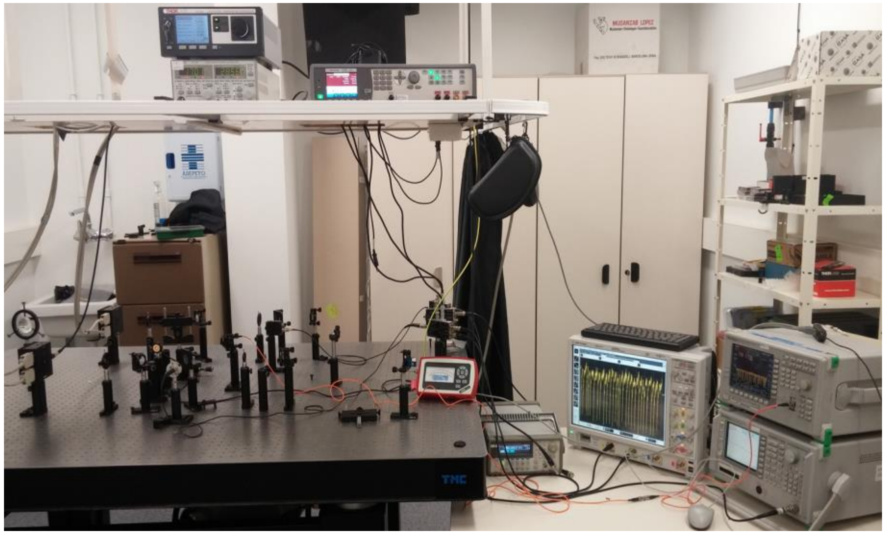
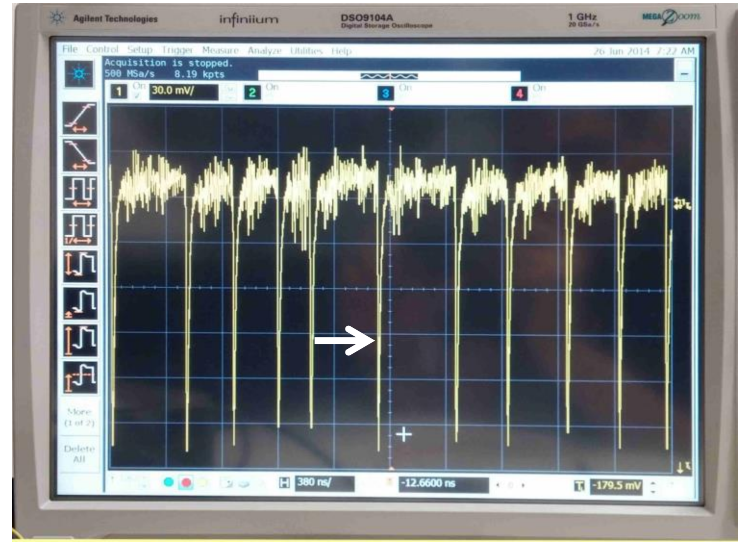


Semiconductor lasers with optical feedback: neuron-like behavior?





WHAT DO LASERS
HAVE TO DO WITH NEURONS?

Similar statistics of inter-spike intervals?

MOTIVATION

Science 345, 668 (2014)

"a computer that is **inspired** by the brain."

Neuro-synaptic architecture allows to do things like image classification at a very low power consumption.

- Spiking lasers: photonic neurons?
- potential building blocks of brain-inspired computers.
- Ultra fast ! (micro-sec vs. mili-sec)

Comparison of empirical data: neuron & optical spikes

Neuron inter-spike interval (ISI) distribution

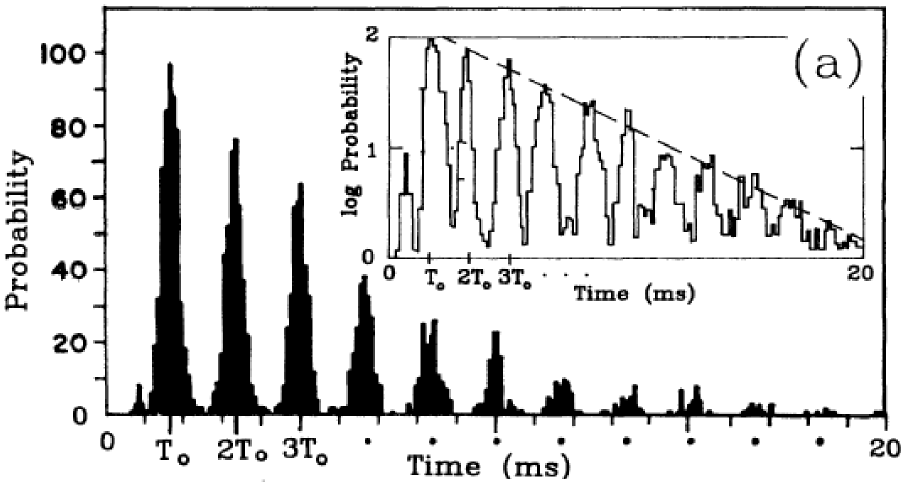
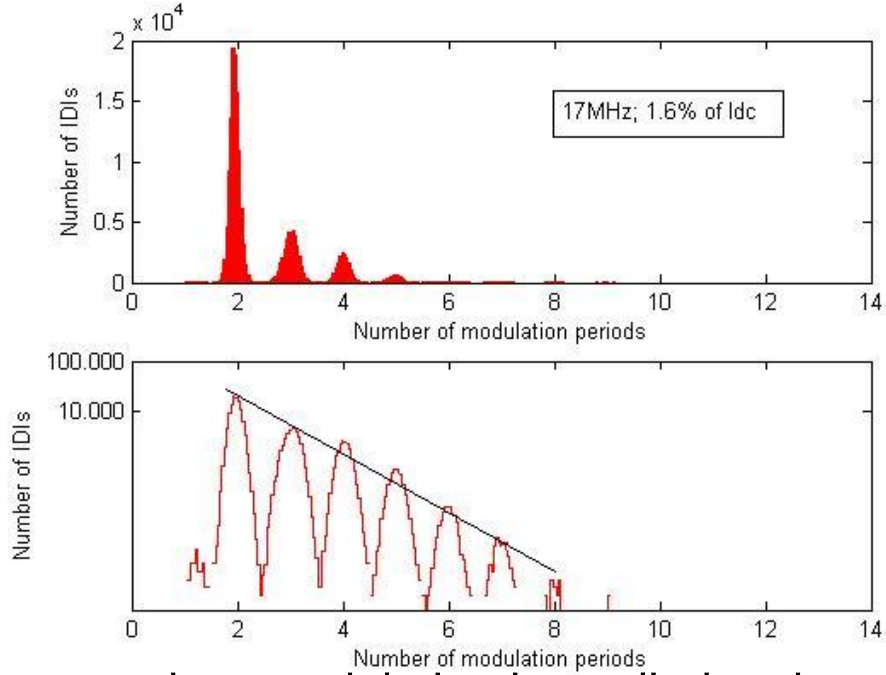


FIG. 1. (a) An experimental ISIH obtained from a single auditory nerve fiber of a squirrel monkey with a sinusoidal 80-dB sound-pressure-level stimulus of period $T_0 = 1.66$ ms applied at the ear. Note the modes at integer multiples of T_0. Inset:

A. Longtin et al, PRL 67 (1991) 656

Optical ISI distribution, data collected in our lab



when modulation is applied to the laser current

HOW SIMILAR NEURONAL AND OPTICAL SPIKES ARE?

Neuronal ISIs

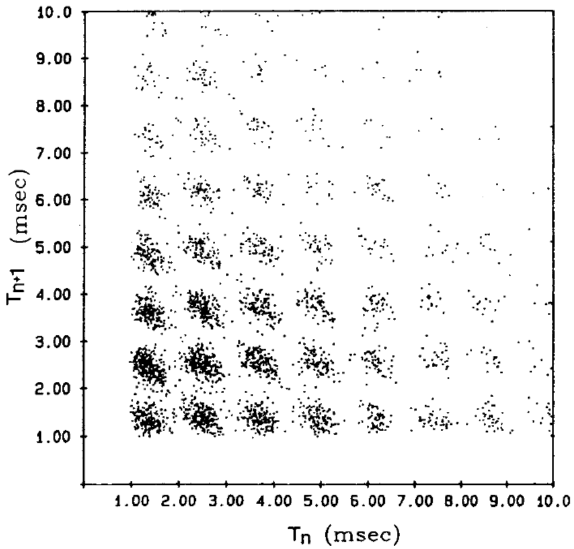
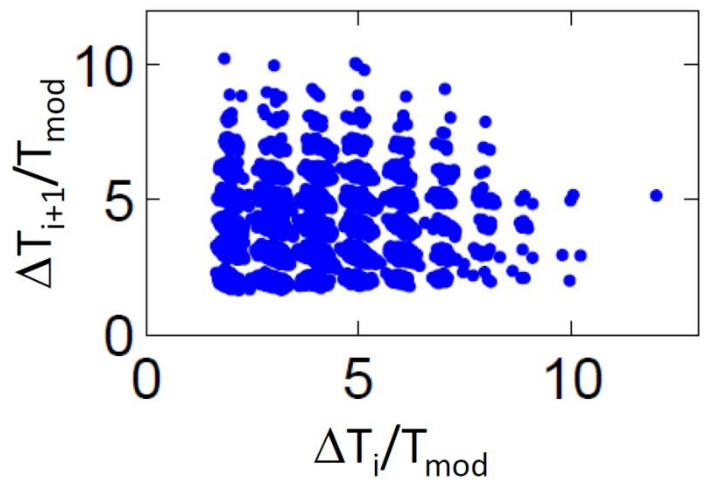
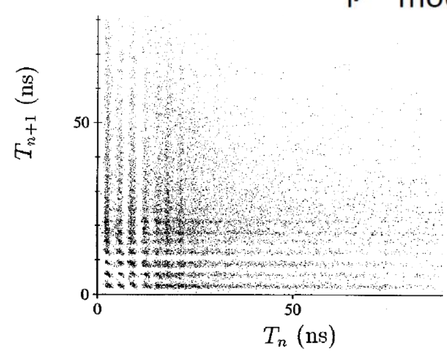


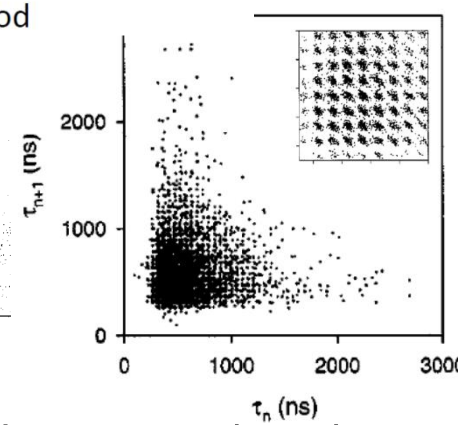
Fig. 4. Scatter plot of spike train data obtained from extracellular measurements of cat auditory fiber activity in response to an 800 Hz 60 dB sound pressure level pure tone presented to the outer ear. The stimulus is discontinuous (see

A. Longtin IJBC 3 (1993) 651

Optical ISIs

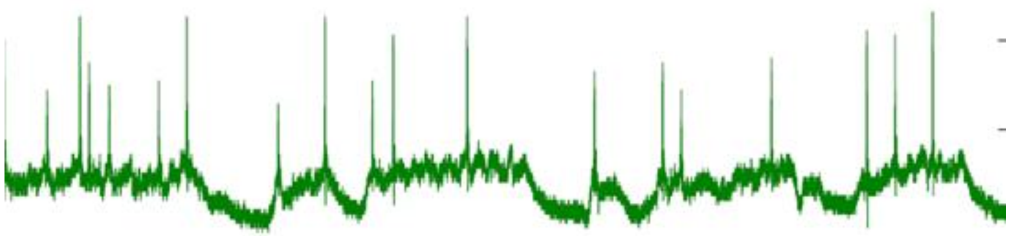






A. Aragoneses et al, Opt. Exp. (2014)
M. Giudici et al, PRE 55, 6414 (1997)
D. Sukow and D. Gautheir, JQE (2000)

How neurons encode information?



- In the spike rate?

- Is the **relative timing** of the spikes relevant?
  - Rate-based information encoding is slow.
  - Temporal codes transmit more information.

ARE THERE TEMPORAL PATTERNS IN THE SPIKE SEQUENCE?

Symbolic method of time-series analysis

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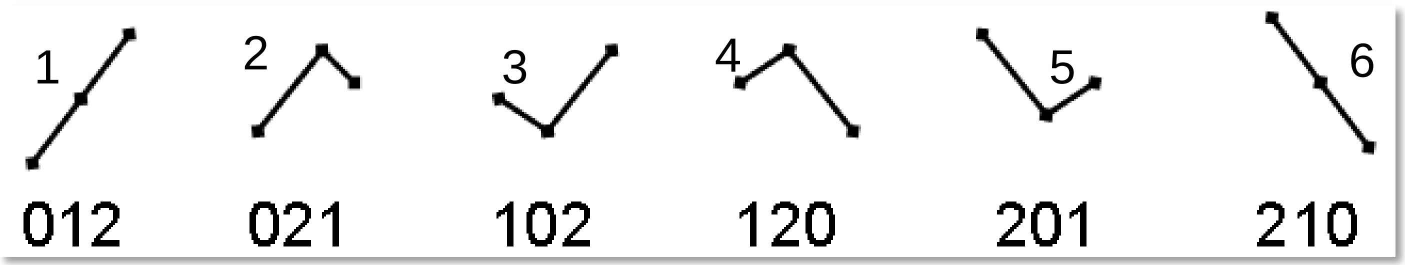
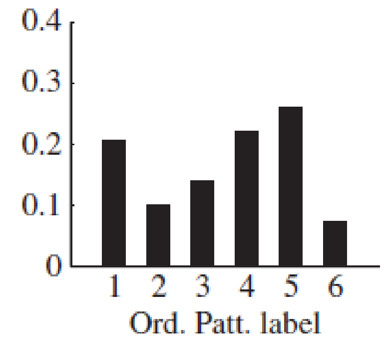
Method of symbolic time-series analysis: ordinal patterns

- $X = \{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$

Brandt & Pompe, PRL 88, 174102 (2002)





The OP probabilities allow to identify frequent patterns in the *ordering* of the data points

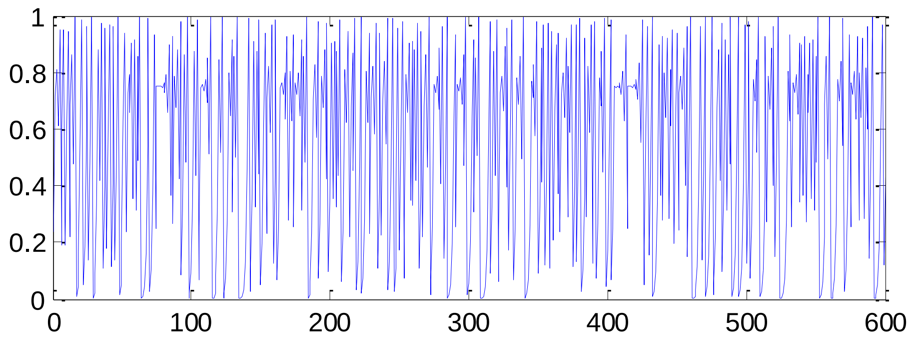
Random data
⇒ OPs are equally probable

– Advantage: the probabilities uncover temporal correlations.

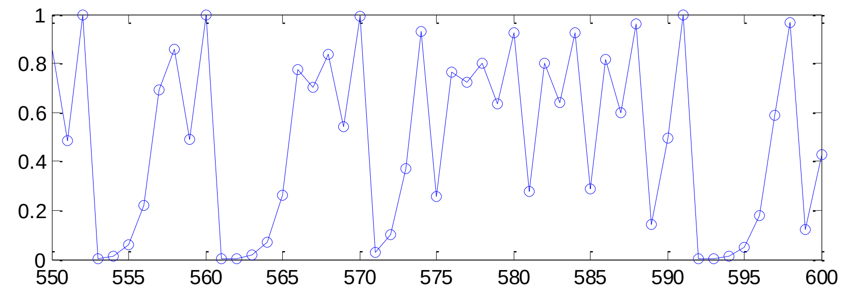
– Drawback: we lose information about the actual values.

Example1: the logistic map $x(i+1)=4x(i)[1-x(i)]$

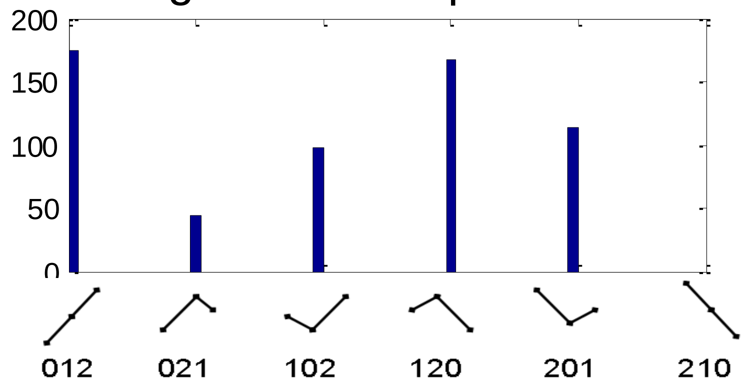
Time series



Detail

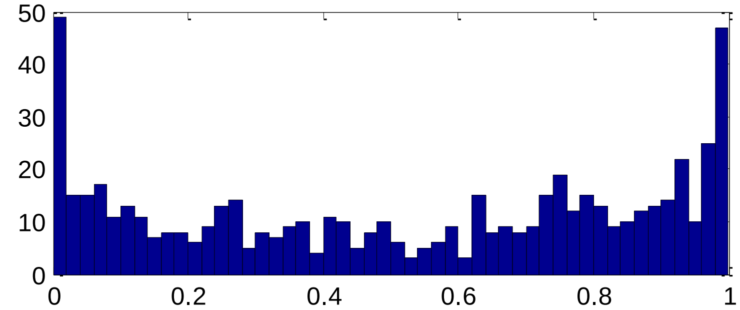


Histogram ordinal patterns D=3



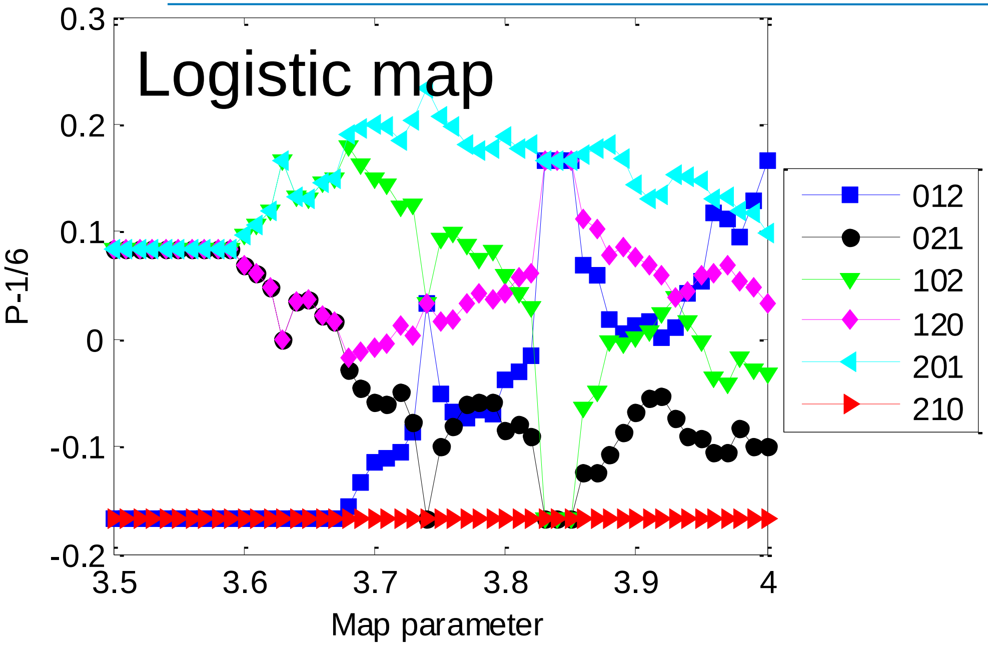
Forbidden pattern

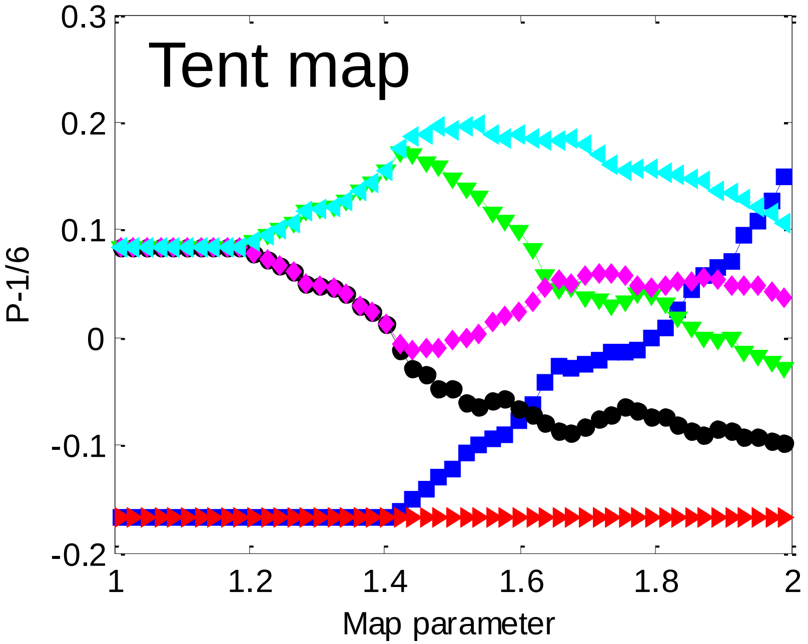
Histogram x(i)

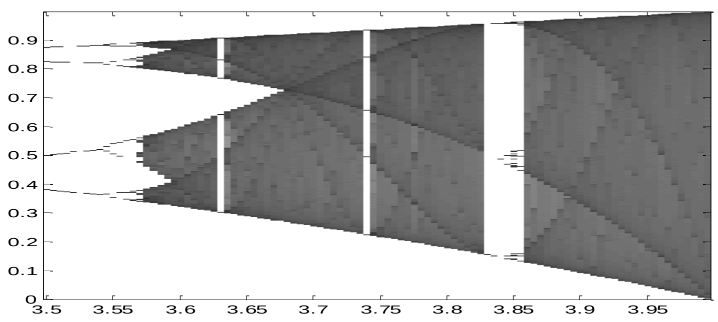


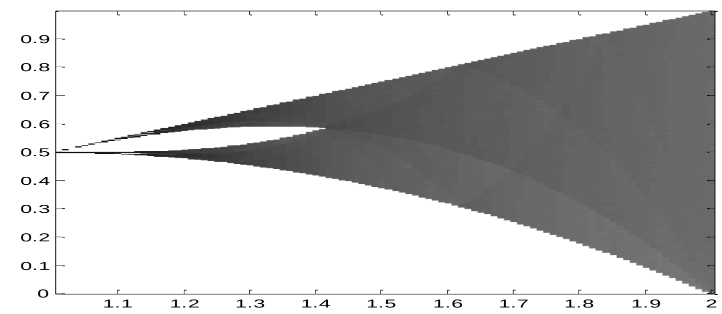
$\Rightarrow$ Ordinal analysis provides complementary information

Ordinal bifurcation diagrams

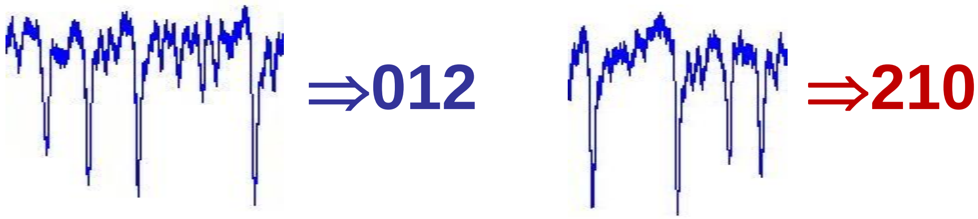


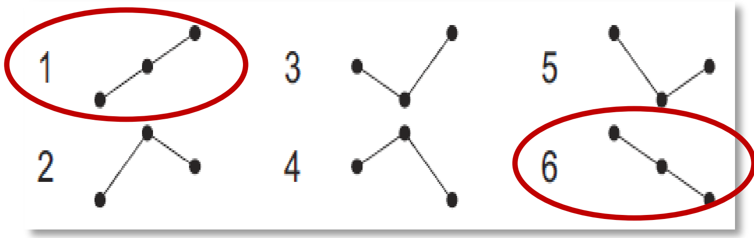




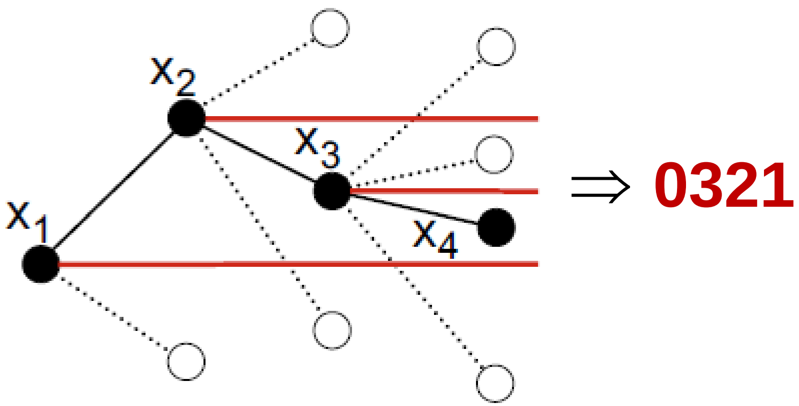


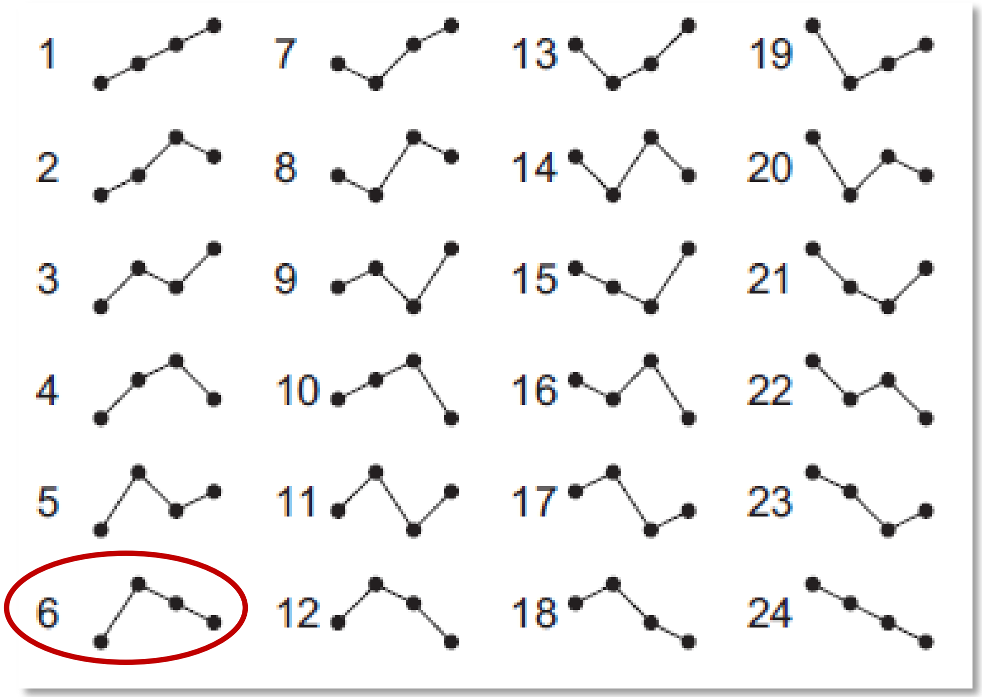
- With **D=3** we can study correlations among 4 spikes.





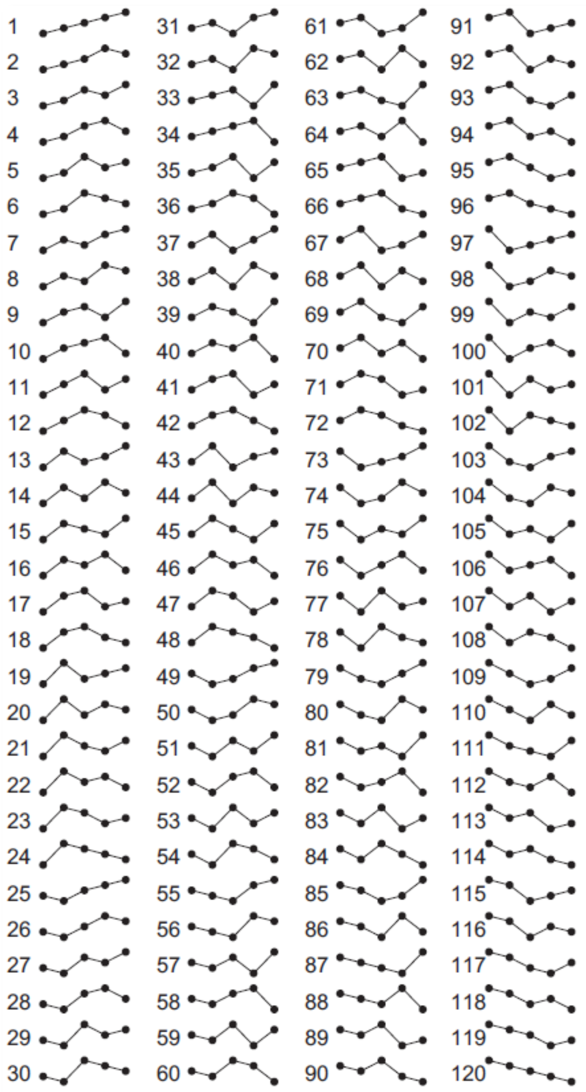
- With **D=4**





- The number of patterns grows with the length of the pattern as **D!**

U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327



D=5: 5!=120 patterns

- How to quantify the information?
  - Permutation entropy

$$s_p = -\sum p_i \log p_i$$

- How to select optimal D? depends on:
  - The length of the data.
  - The length of the correlations.

U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Contrasting empirical optical spikes with synthetic neuronal spikes

- do they have similar ordinal statistics?
- are there more/less frequent patterns?

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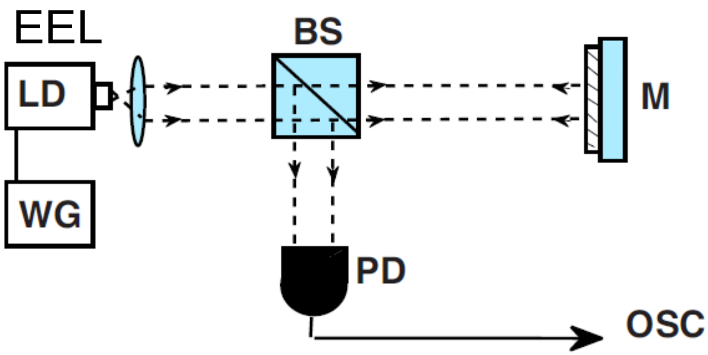
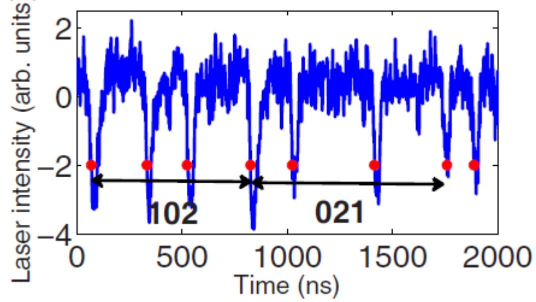
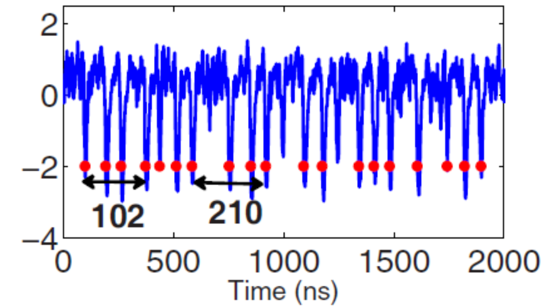
Ordinal analysis of ISI correlations



Close to threshold



Higher pump current



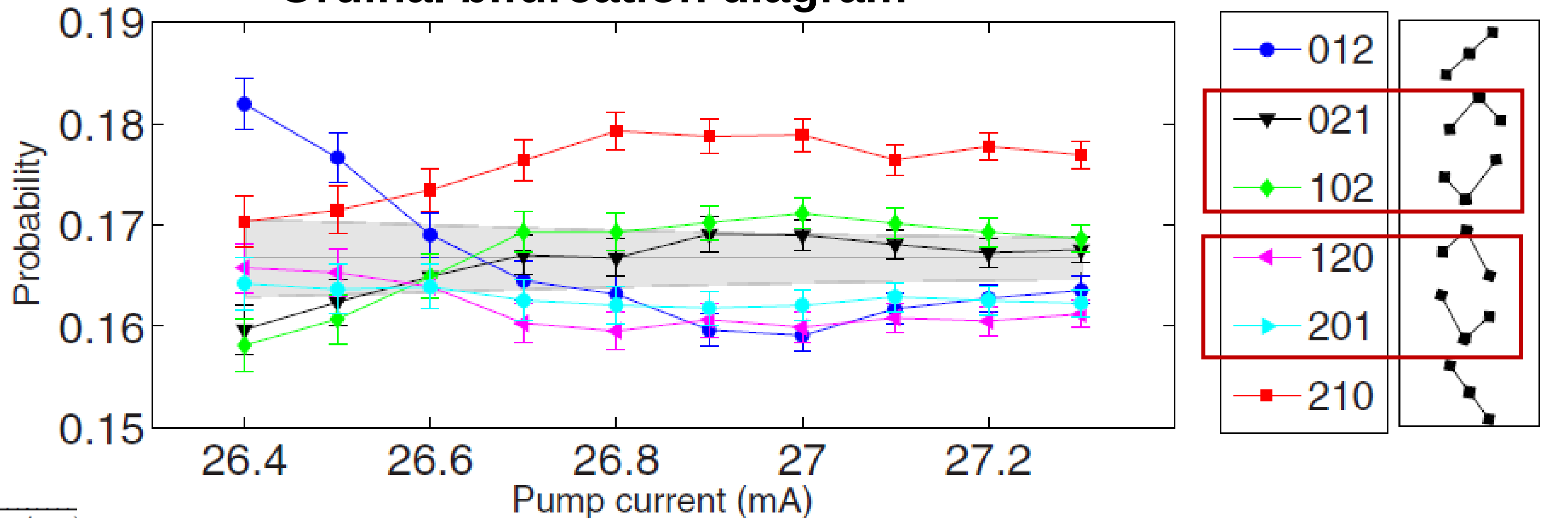
Grey region computed with binomial test: probabilities consistent with uniform distribution with 95% confidence level.

$$p \pm 3\sigma_p$$

$$\sigma_p = \sqrt{(p(1-p)/N)}$$

P =1 /6; ***N* > 10,000 ISIs**

Ordinal bifurcation diagram



A. Aragoneses, S. Perrone, T. Sorrentino, M. C. Torrent and C. Masoller, Sci. Rep. **4**, 4696 (2014)

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Minimal model of ISI correlations: modified circle map

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha_c \sin(4\pi\varphi_i)\right] + D\zeta$$

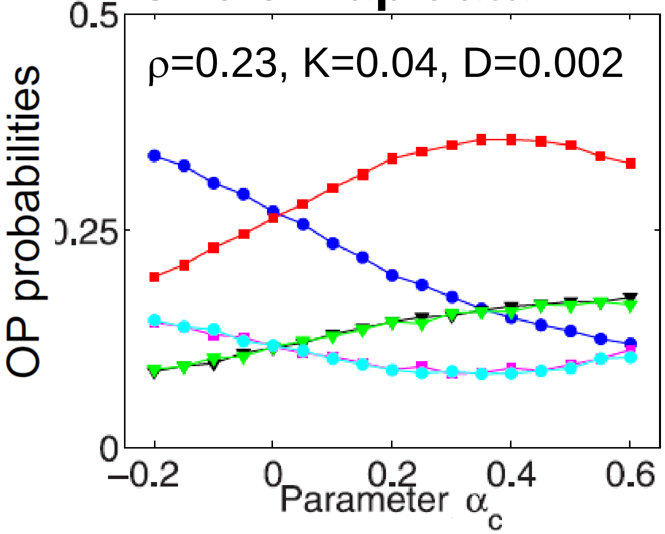
$$X_i = \varphi_{i+1} - \varphi_i$$

$\rho = \dfrac{\text{natural frequency}}{\text{forcing frequency}}$

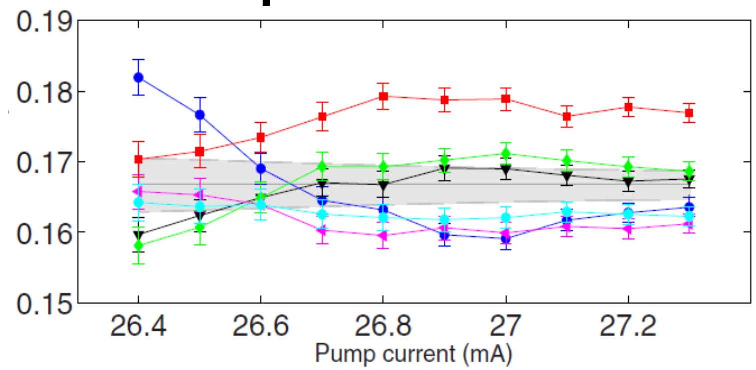
K = forcing amplitude

D = noise strength

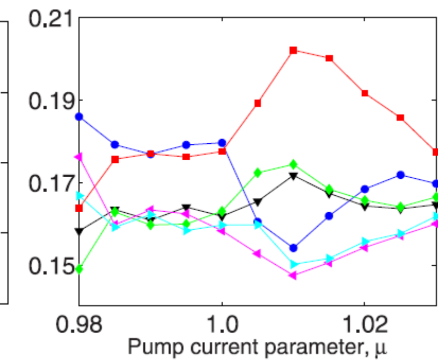
Circle map data

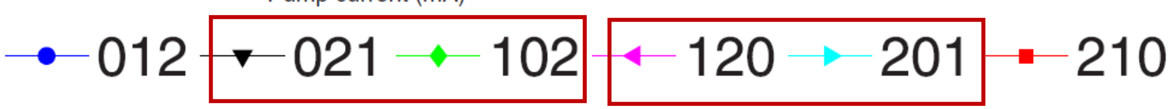


Empirical laser data



Lang-Kobayashi time-delay model

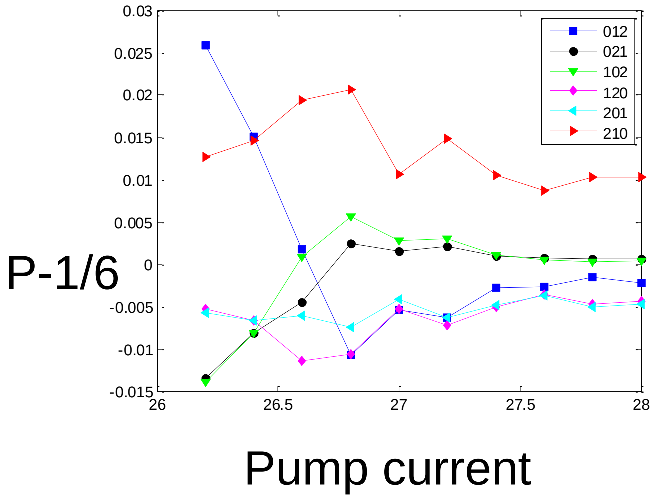


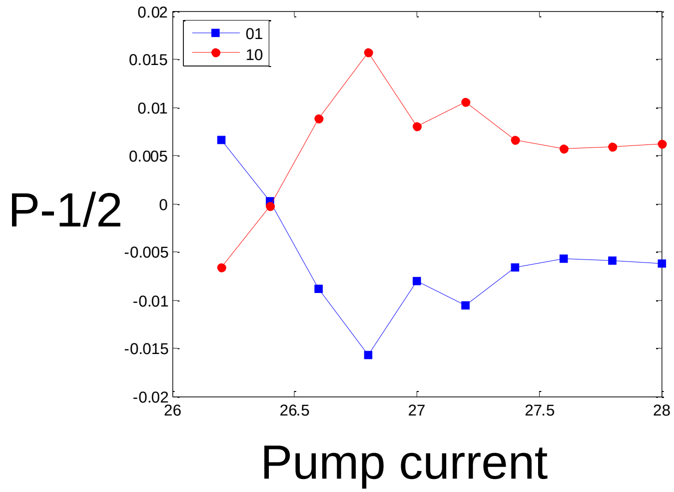


- Same "clusters" & same hierarchical structure.
- Modified circle map: minimal model for ordinal correlations.
- Same qualitative behavior found with other lasers & feedback conditions.

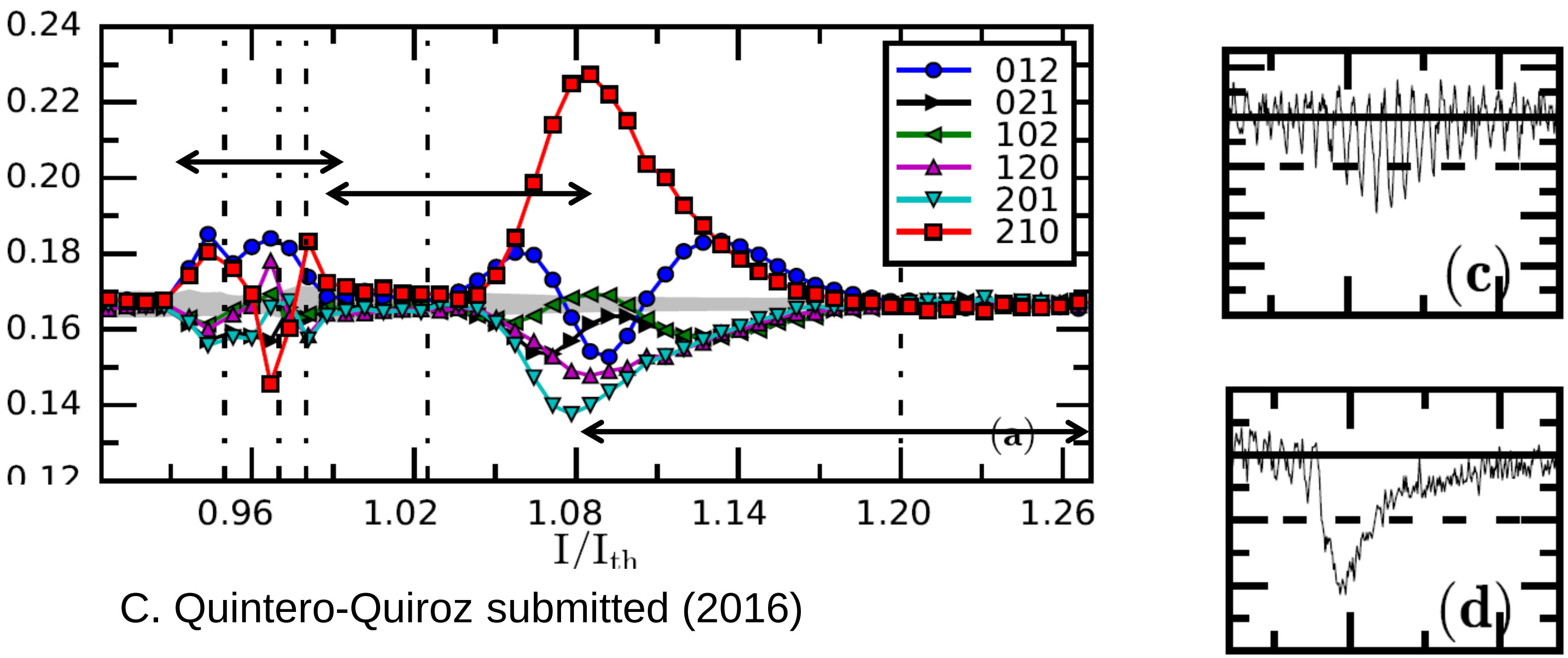
Model equations and parameters: A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)

Other experiments: influence of OP size and LFF threshold

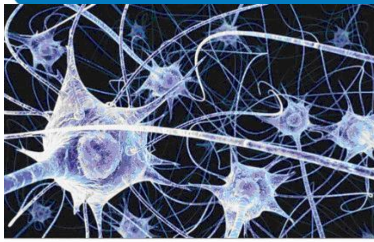




A. Aragoneses, et al, Sci. Rep. 3 1778 (2013)



C. Quintero-Quiroz submitted (2016)

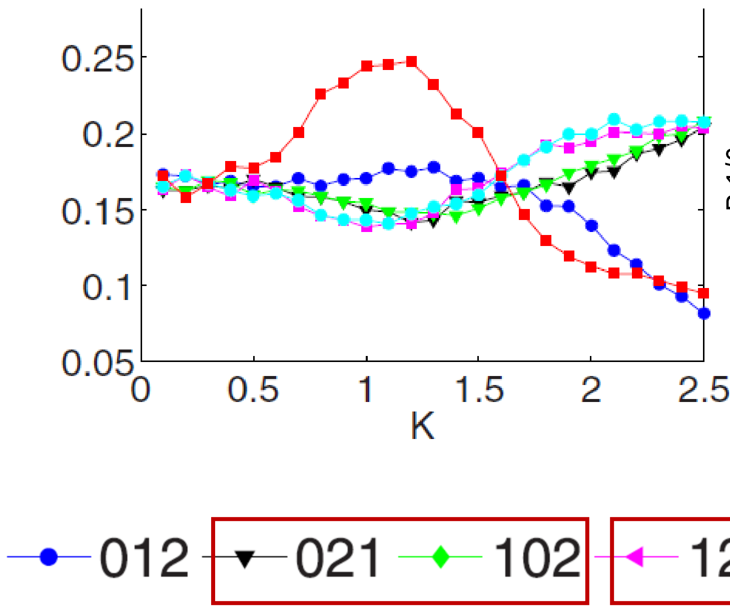


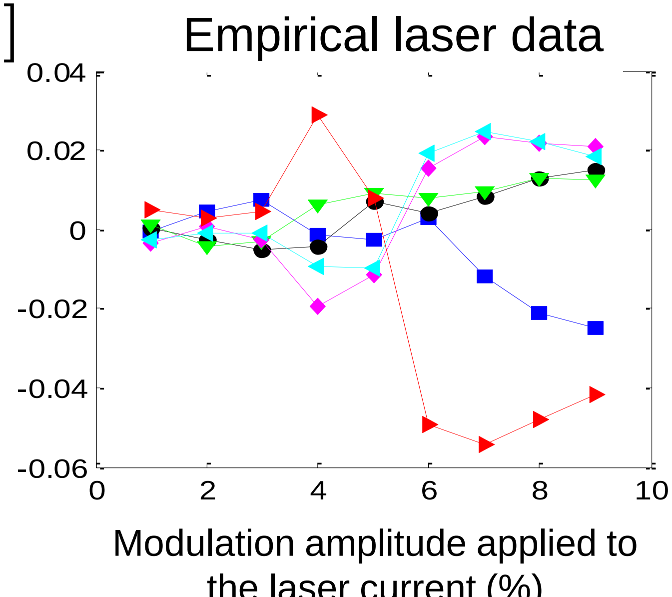
Connection with neurons

- The circle map describes many excitable systems.
- The modified circle map has been used to describe spike correlations in biological neurons.

A. B. Neiman and D. F. Russell, PRE 71, 061915 (2005)

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha\sin(4\pi\varphi_i)\right]$$



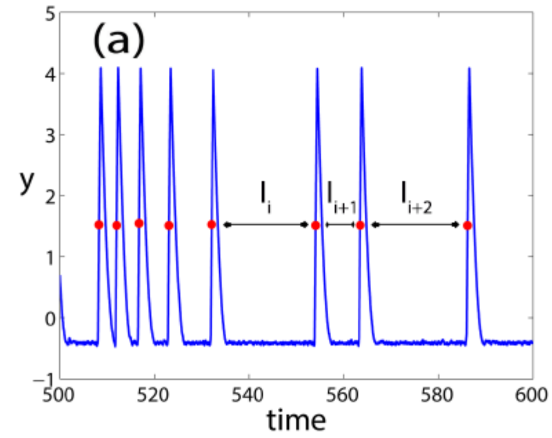


$\Rightarrow$ Good qualitative agreement.

A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)

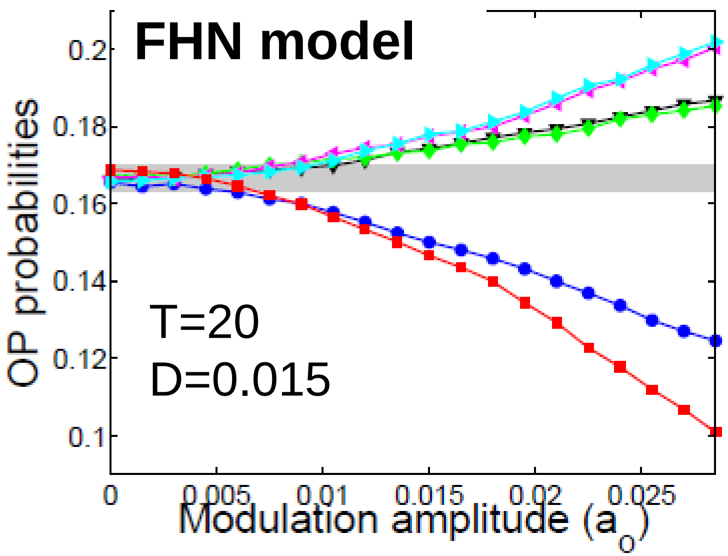
FHN neuron model

$$\epsilon\frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o\cos(2\pi t/T)} + D\xi(t),$$

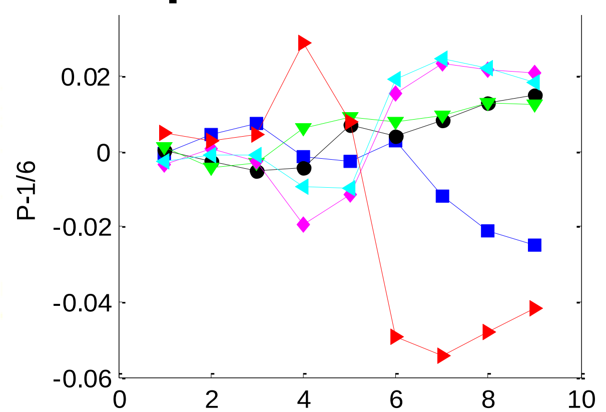


Gaussian white noise and <u>subthreshold</u> (weak) modulation: **a_0** and **T** such that spikes are only noise-induced.

Time series with 100,000 ISIs simulated.



Empirical laser data



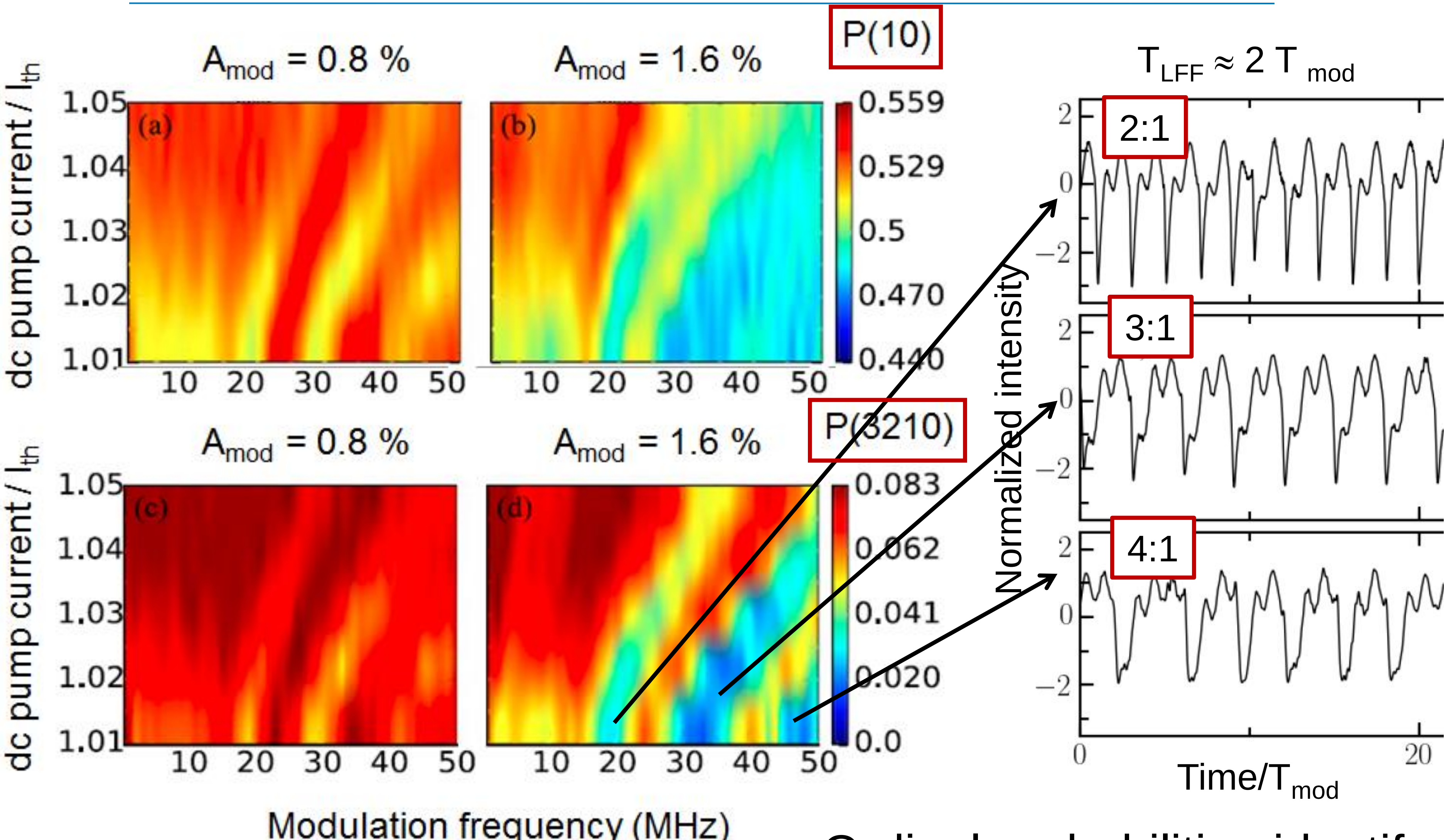
Modulation amplitude applied to the laser current (%)

$\Rightarrow$ Good qualitative agreement.

012 021 102 120 201 210

J. M. Aparicio-Reinoso et al, PRE in press (2016)

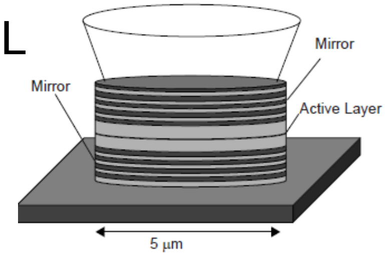
Noisy entrainment to sinusoidal current modulation

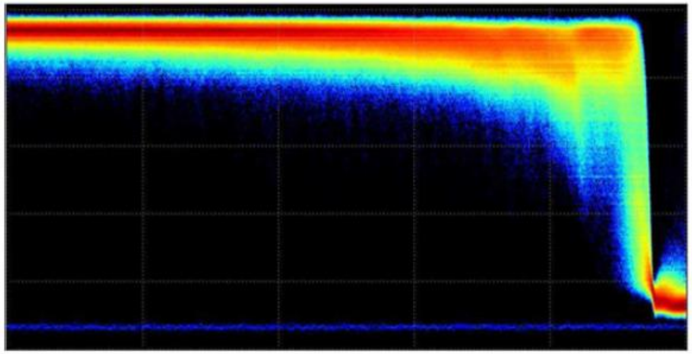


⇒ Ordinal probabilities identify regions of noisy locking

T. Sorrentino et al, JSTQE 21, 1801107 (2015)

VCSEL





Early-warning indicators of abrupt transitions

- controlled experiments generate “optical big data” that is useful for testing novel diagnostic tools

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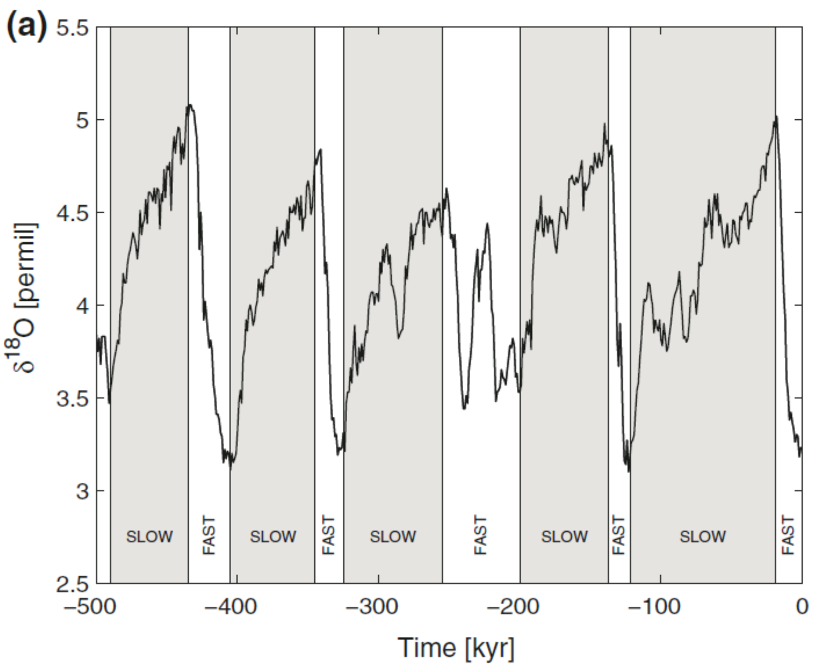
Experimental data from INLN & Bangor University (S. Barland & Y. Hong)

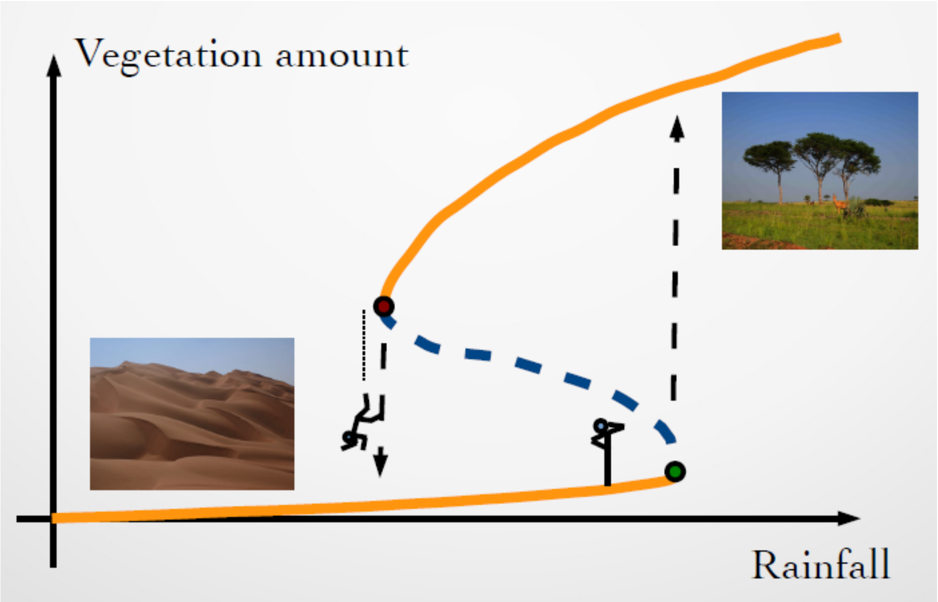
Motivation

A lot of work is aimed at developing reliable diagnostic tools for "early warning" of abrupt transitions.



long-term climatic signals reveal a slow-fast dynamics.

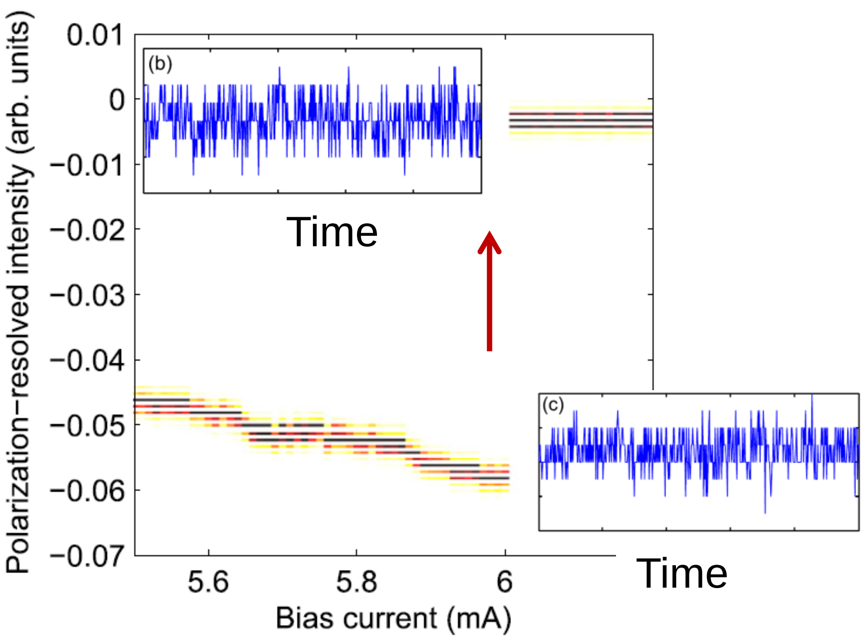
De Saedeleer, Crucifix and Wieczorek, Clim. Dyn. 2013



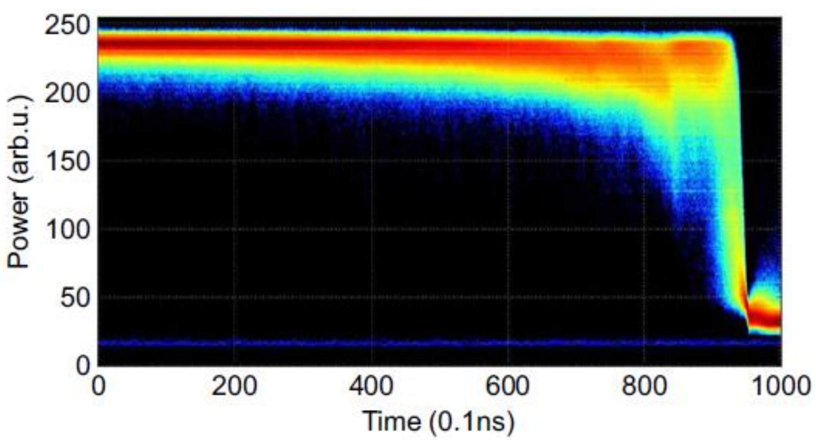
Bangladesh, Nature 2014

VCSEL polarization-resolved intensity: two sets of experiments

- Time series recorded with pump current constant in time.
- Record the turn-on of the orthogonal mode.



- Time series recorded with pump current varying in time.
- Record the turn-off of the fundamental mode.



Is it possible to anticipate the PS?

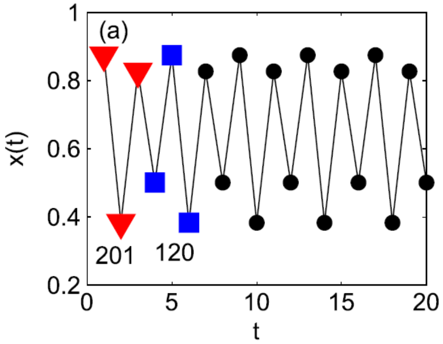
No if the mechanisms that trigger the PS are fully stochastic.

Three diagnostic tools

- Entropy computed from ordinal probabilities (**permutation entropy**) $$s_p = -\sum p_i \log p_i$$

- Entropy computed from **transition probabilities** ('01'→ '01', '01'→ '10', etc.)

$$w_{ij} = \frac{\sum_{t=1}^{L-1} n\left[s(t) = i,\, s(t+1) = j\right]}{\sum_{t=1}^{L-1} n\left[s(t) = i\right]}$$



- Asymmetry coefficient: normalized difference of transition probabilities, P('01'→ '10') - P('10'→ '01'), etc.

$$a_c = \frac{\sum_i \sum_{j \neq i} \left| w_{ij} - w_{ji} \right|}{\sum_i \sum_{j \neq i} \left(w_{ij} + w_{ji} \right)}$$

Results for constant pump current & turn-on of the orthogonal mode

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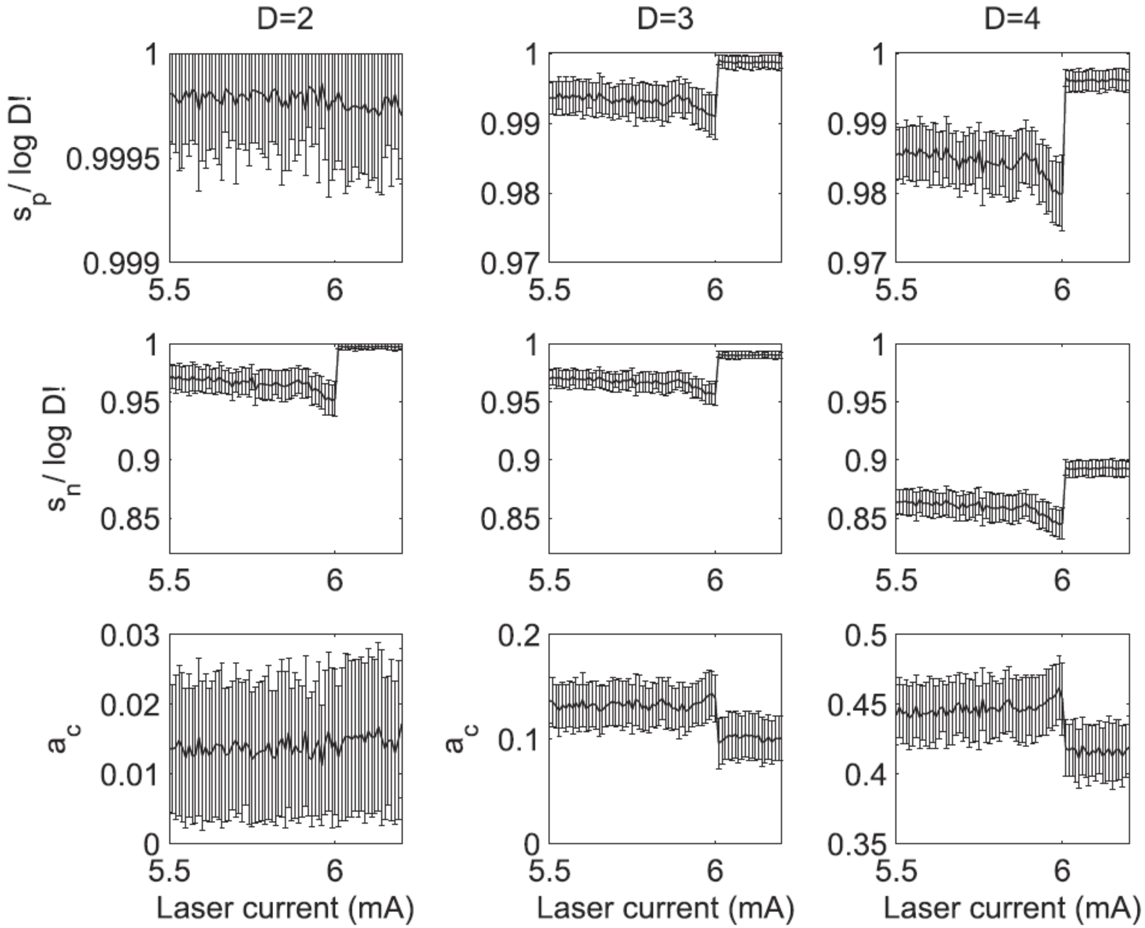


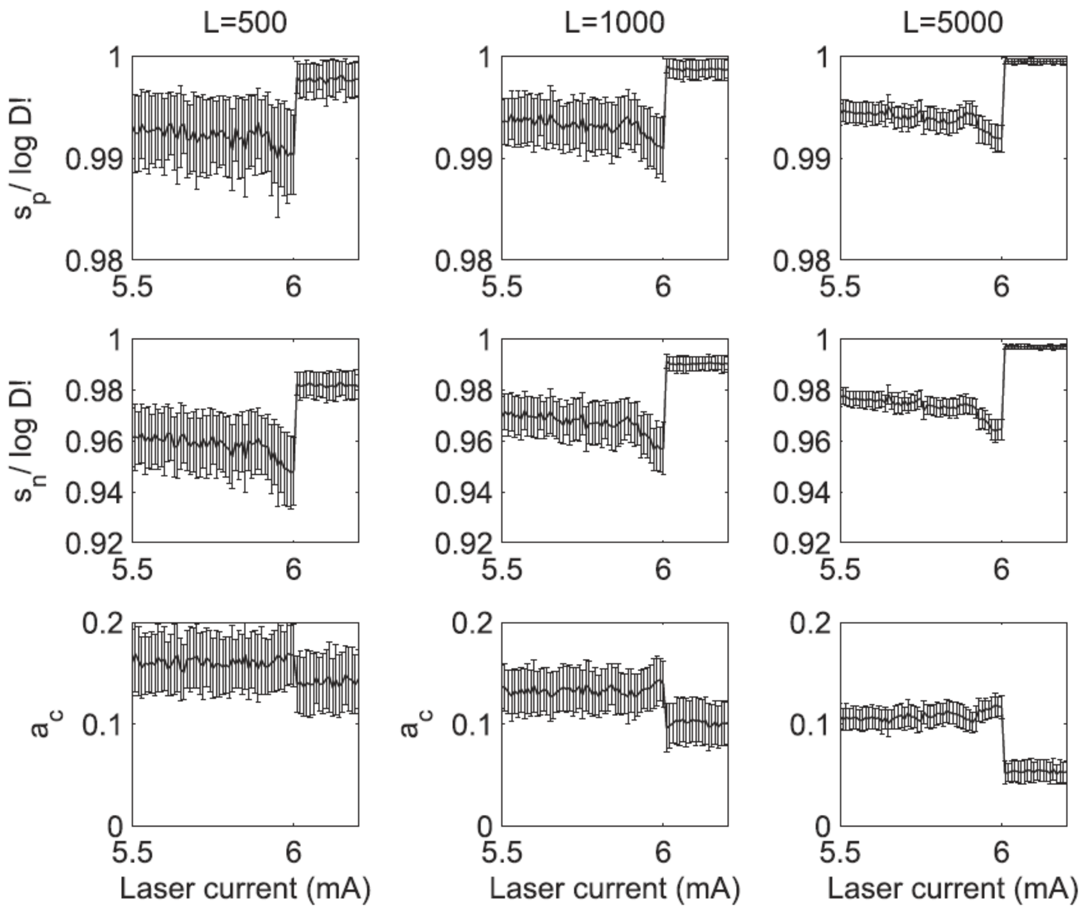
⇒ Despite of the stochasticity of the time-series, the measures "anticipate" the PS.

⇒ Deterministic mechanisms involved.

Error bars computed from 100 non-overlapping windows with L=1000 data points each.

C. Masoller et al, New J. Phys. 17 (2015) 023068

Influence of the length of the dataset



$\Rightarrow$ if L is too short the measures fail to provide an early warning

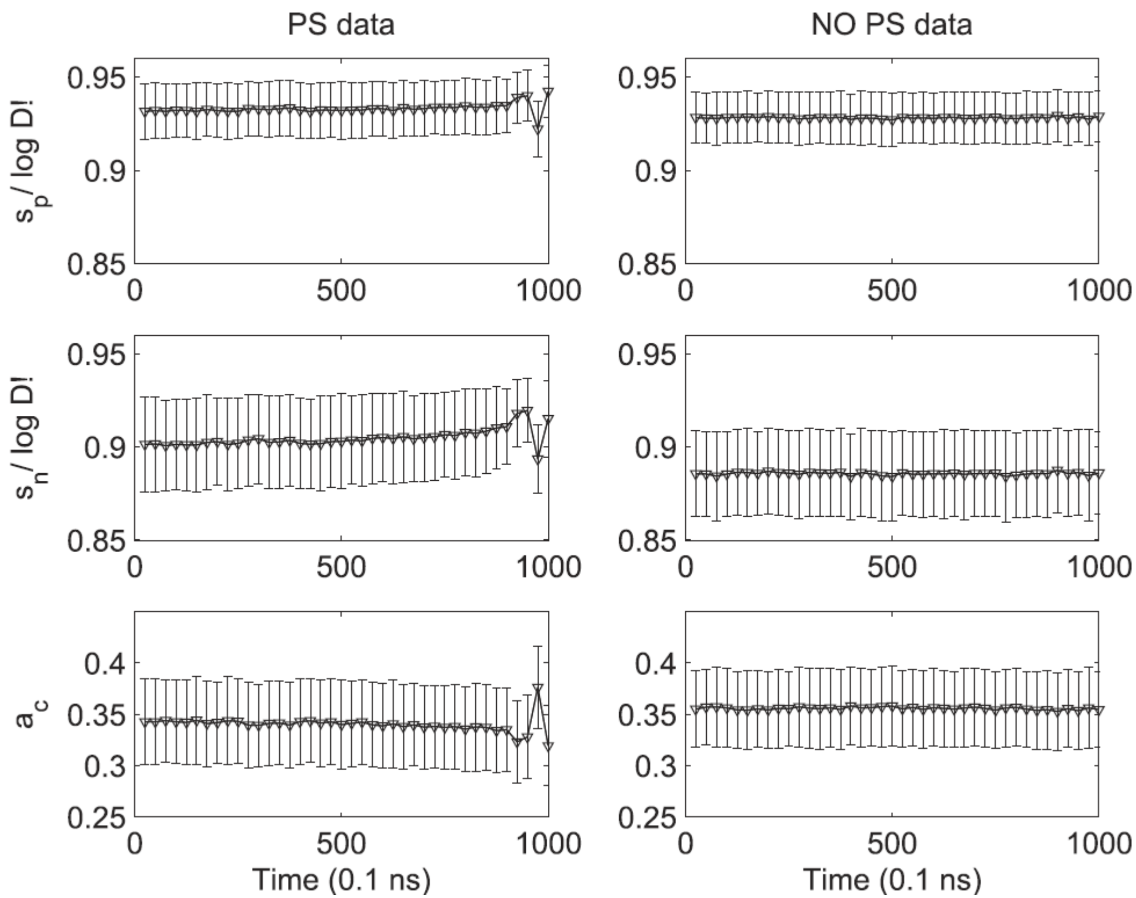
Analysis done with D=3, error bars computed from 100 non-overlapping windows with L data points each.

C. Masoller et al, New J. Phys. 17 (2015) 023068

Results for time-varying pump current & turn-off of the fundamental mode

Slightly different optical feedback conditions result in PS or no PS as the pump current is increased.

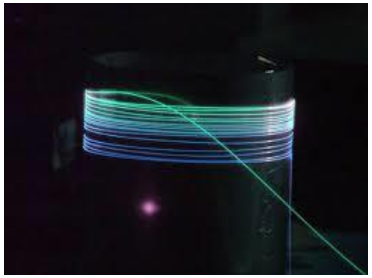
Analysis done with D=3, error bars computed with 1000 time series, L=500 data points.



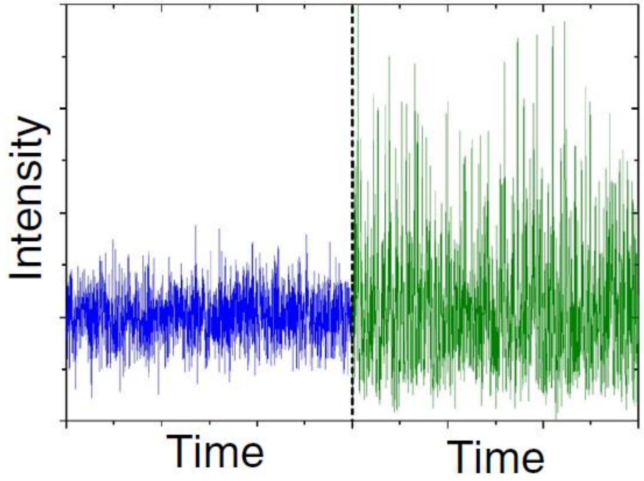
Open issues: quantification of the anticipation time and comparison with other diagnostic tools

C. Masoller et al, New J. Phys. 17 (2015) 023068

Raman fiber laser



Low -- High pump power



Characterizing the laminar-turbulence transition in a fiber laser

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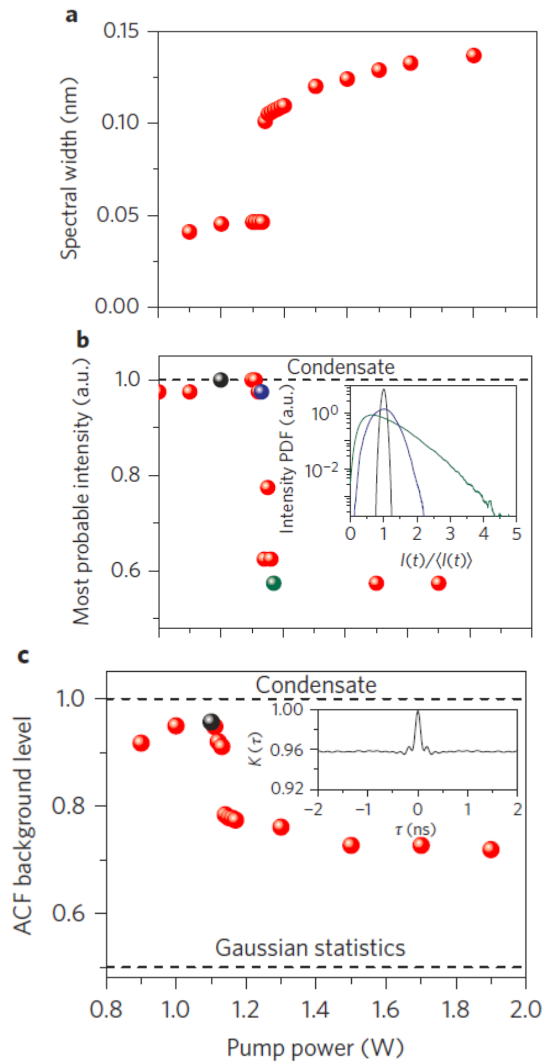
Experimental data from Aston University, UK
(Prof. Turitsyn' group; details with **NikitaTarasov**)

Motivation

E. G. Turitsyna et al Nat. Phot. 7, 783 (2013)

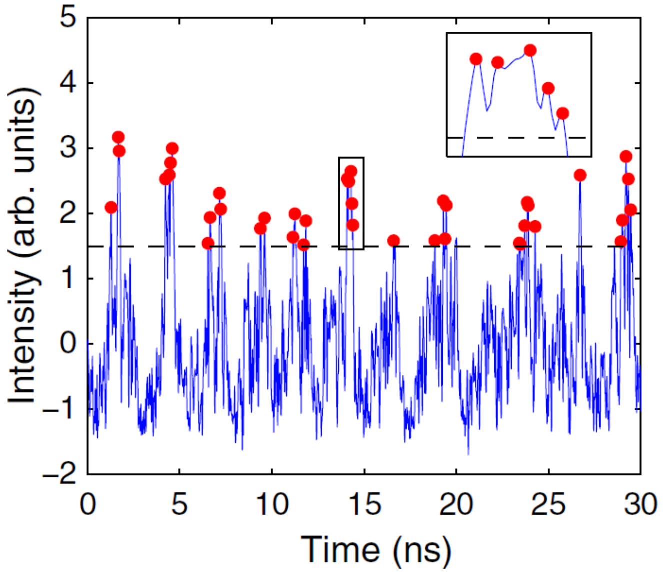
- Width of optical spectrum: abrupt increase of the number of excited modes.
- Most probable intensity: abrupt change in PDF.
- Background level of the intensity autocorrelation function at long lag
  - for a coherent state →1
  - for stochastic radiation with Gaussian statistics → 0.5.



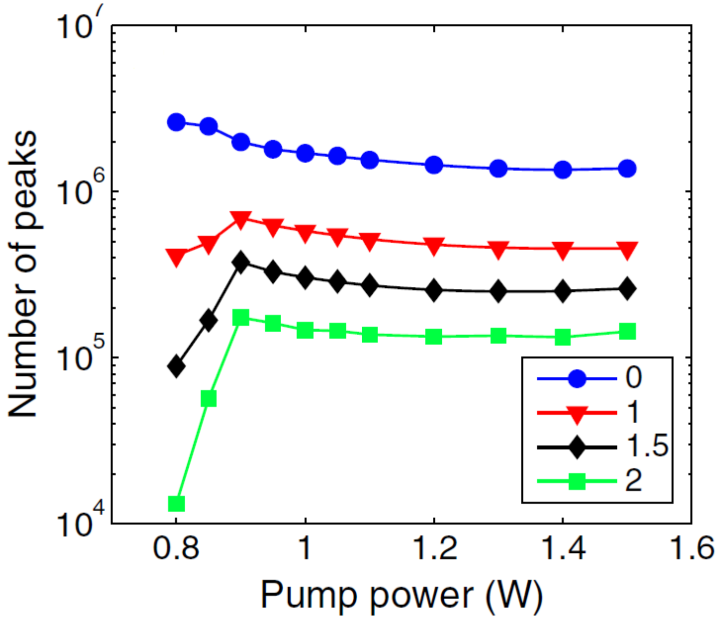
TEMPORAL CORRELATIONS?

Analysis of the intensity peaks higher than a threshold

Each time series is first normalized to ⟨I⟩=0 and σ=1



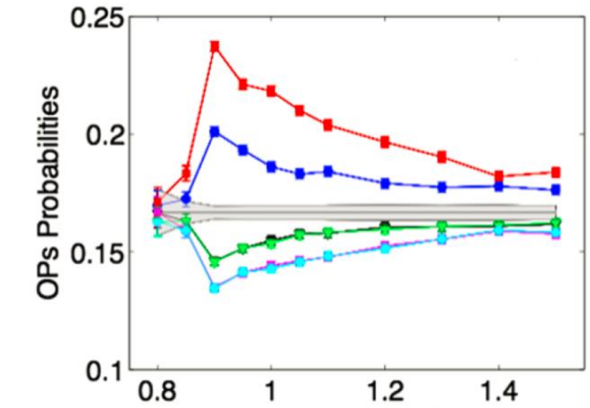
$$\{I_{\max,i}\}$$

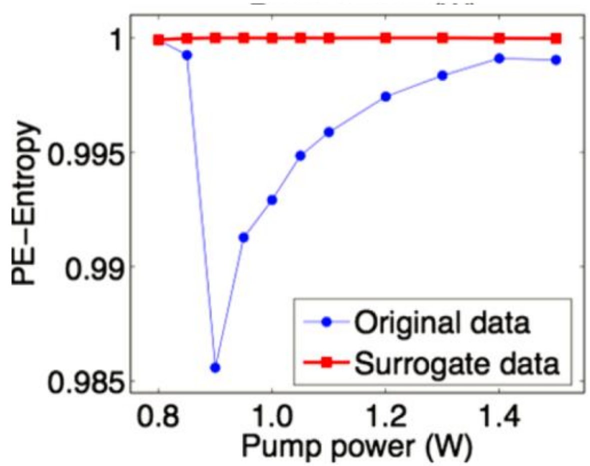


In each time series 5×10^7 data points.
Sampling time dt = 12.5 ps

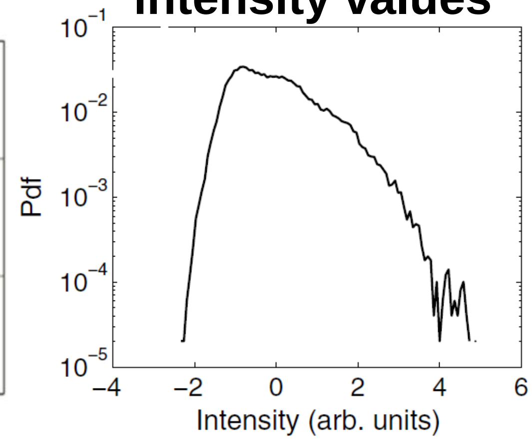
Th = 2: number of peaks $>10^4$ for all values of the pump power

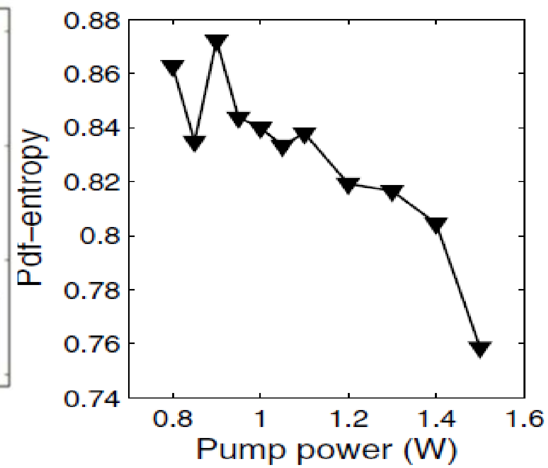
Ordinal analysis of the intensity peaks above Th = 2



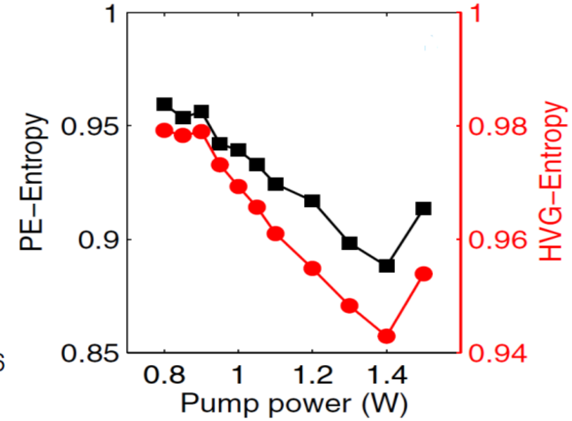


Histogram of raw intensity values





Ordinal analysis of the raw data



- Sharp transition <u>not captured</u> by analysis of raw data.
- Different entropy behavior.

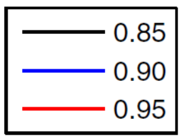
Can we gain more information from the raw data?

Aragoneses et al, PRL 116, 033902 (2016)

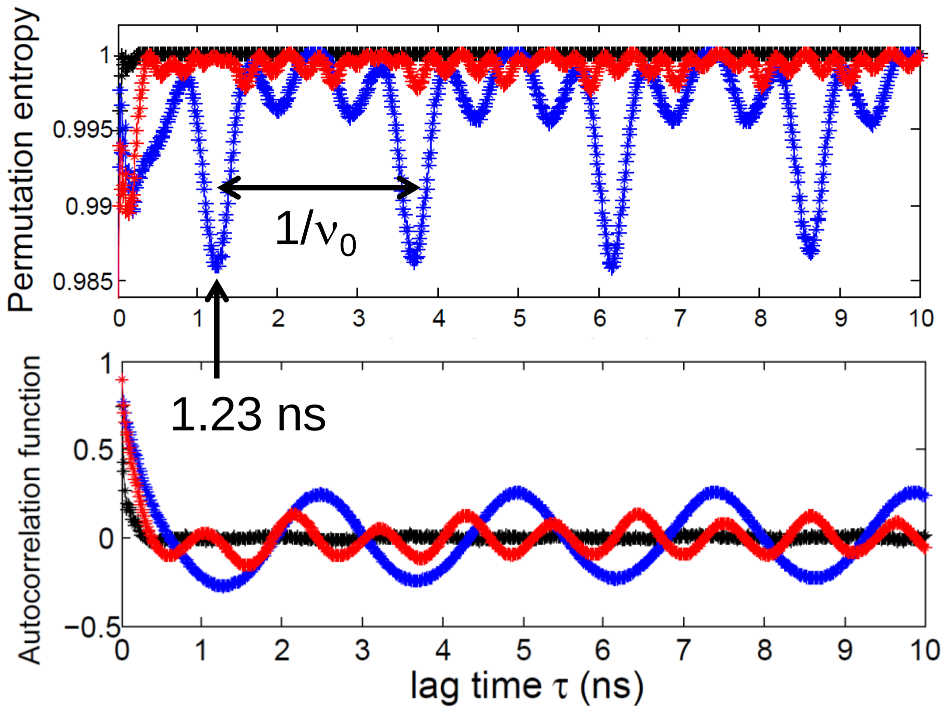
Ordinal analysis of lagged raw data

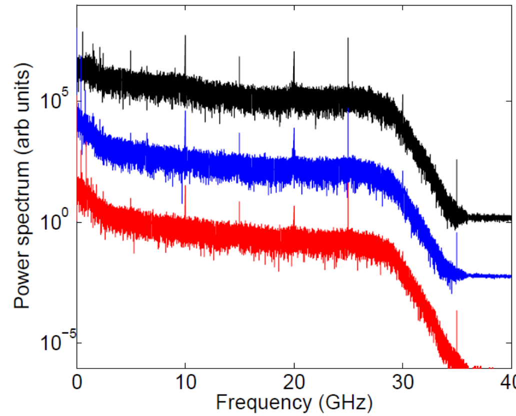
$$\{I_i, I_{i+\tau}, I_{i+2\tau}, \dots\}$$

all data points, no threshold used



Pump power below, at, and above the transition.

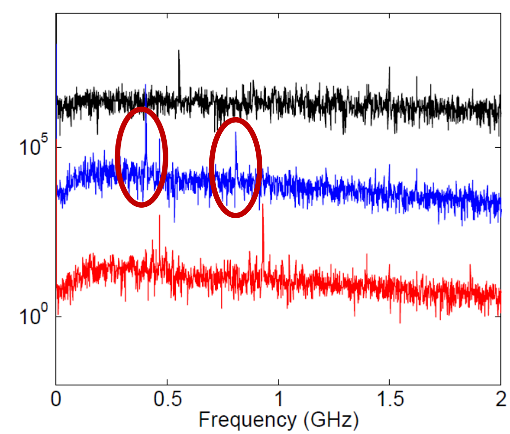




L=770 m

τ_{ext} = 7.7μs

ν_{ext} = 0.13 MHz



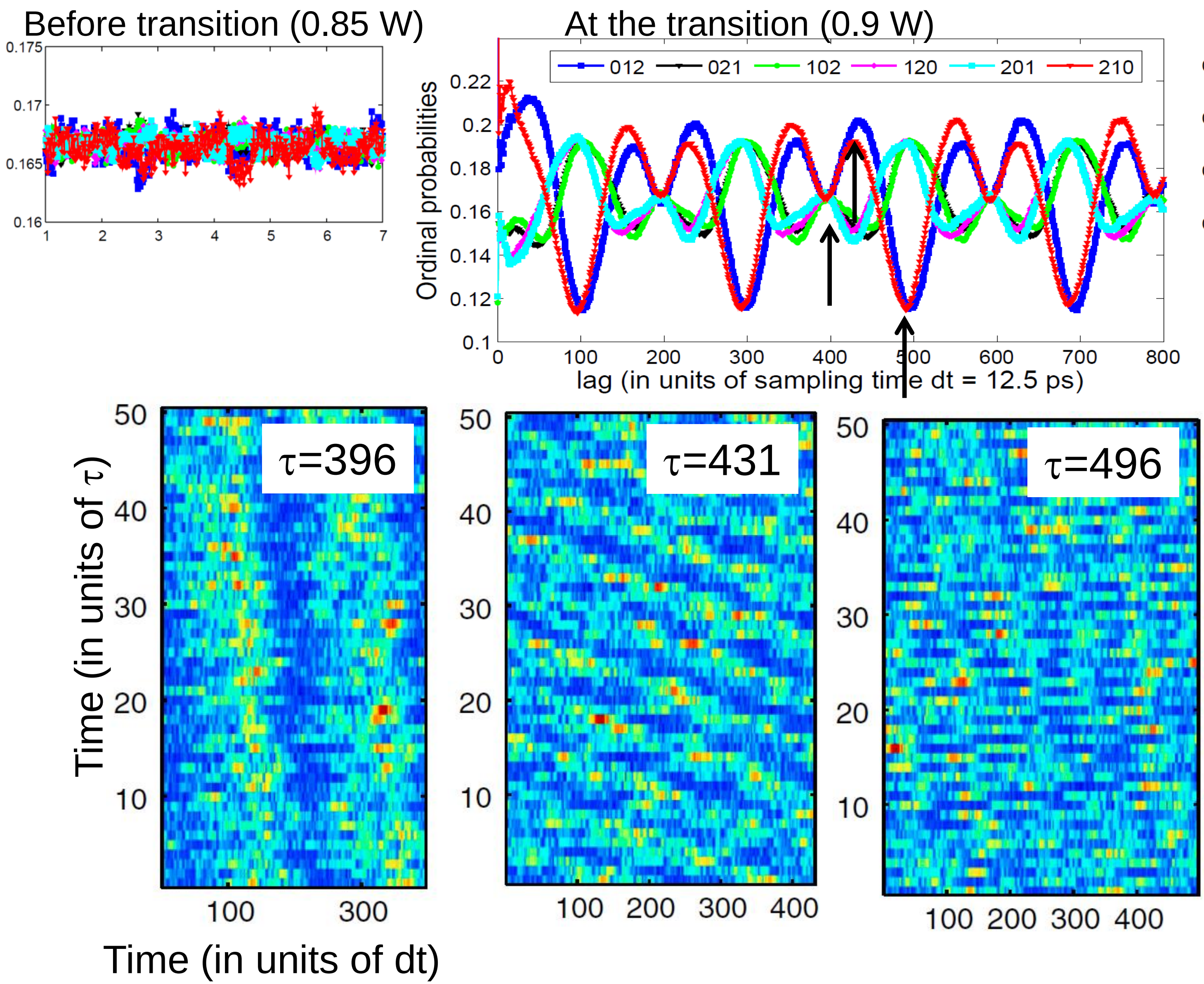
$\Rightarrow$ Sharp variation not captured by correlation or Fourier analysis.

0.81 GHz →1.23 ns

ν_0 = 0.4 GHz

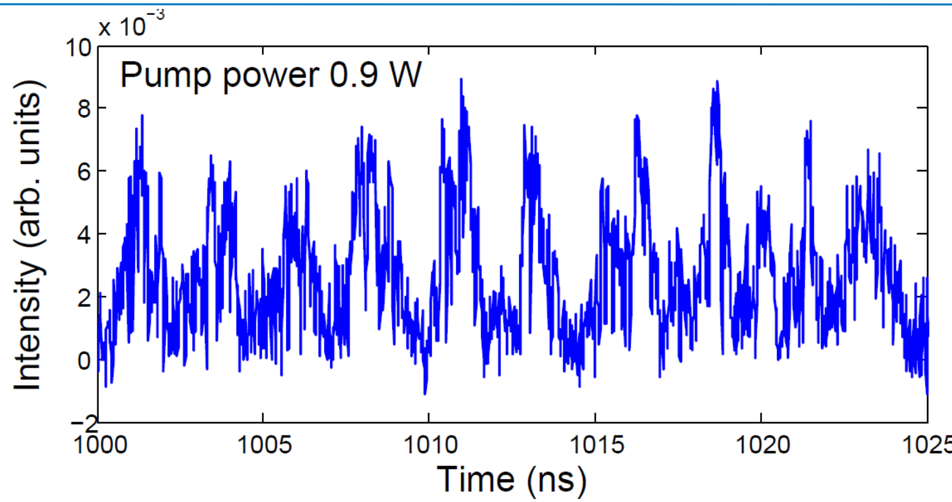
Aragoneses et al, PRL 116, 033902 (2016)

Ordinal probabilities vs. lag



⇒ Different coherent structures unveiled by using different lags (sampling times).

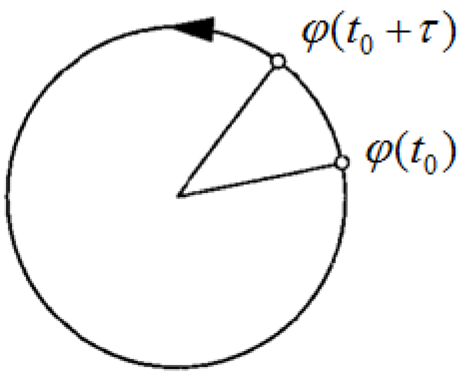
Minimal model of intensity correlations at the transition



Stochastic limit cycle: phase equation

$$\frac{d\varphi}{dt} = \omega_0 + f(\varphi, t) + \xi$$

Stroboscopic sampling at time-interval τ



$$\varphi(t_0+\tau) = \varphi(t_0) + \omega_0\tau + F(\varphi(t_0)) + \xi$$

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\sin(2\pi\varphi_i) + D\xi$$

$$\rho = \nu_0\tau = \nu_0 / \nu_{ext}$$

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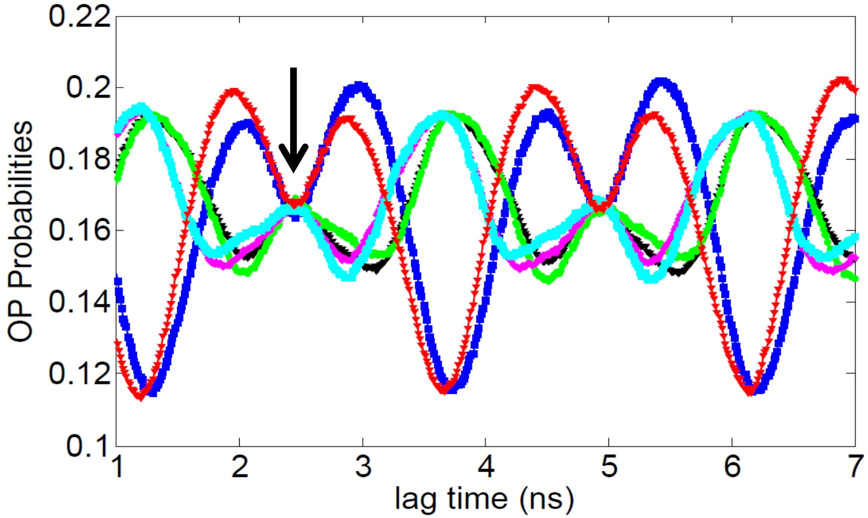
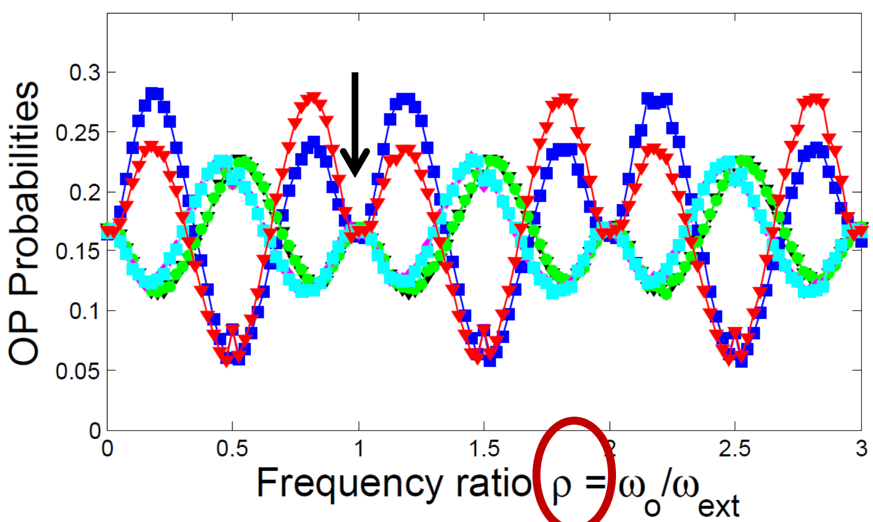
Circle map: minimal model of ordinal probabilities at the transition

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\sin(2\pi\varphi_i) + D\xi$$

Fiber laser data $\{I_i, I_{i+\tau}, I_{i+2\tau}, \ldots\}$
(at the transition, pump power 0.9 W)



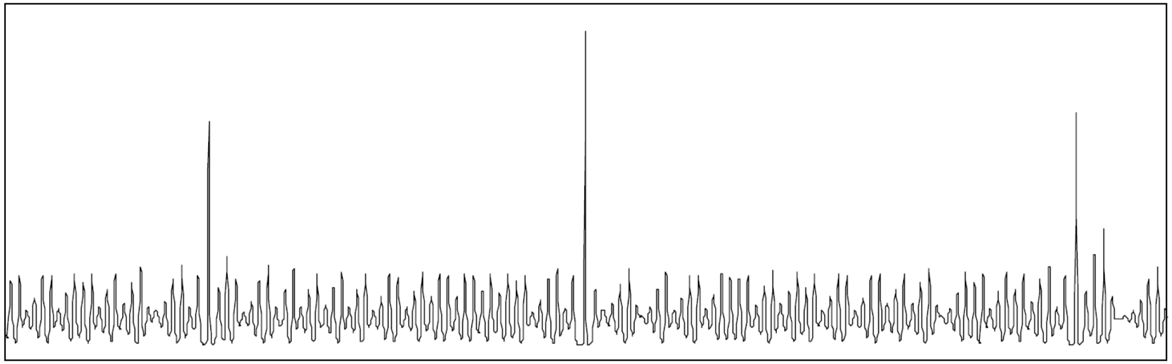
Synthetic data $\{X_i, X_{i+1}, X_{i+2}, \ldots\}$
(K=0.23 & D=0.02) $X_i = \varphi_{i+1} - \varphi_i$



τ = 2.5 ns $\Leftrightarrow$ $\rho = 1 = \nu_0\,\tau$ $\Rightarrow$ $\nu_0 = 1/\tau$ = 0.4 GHz

Present work is aimed at gaining further understanding from these observations.

Diode laser



Time

Anticipating extreme optical pulses

- can we identify patterns likely to occur (or not to occur) before an extreme pulse?

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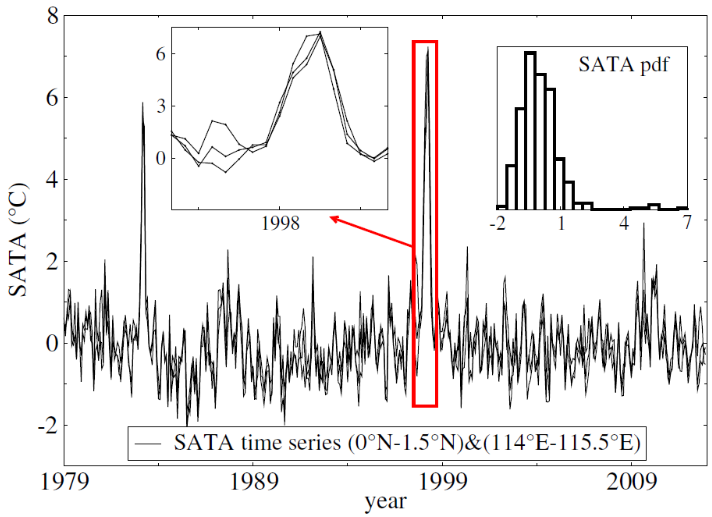
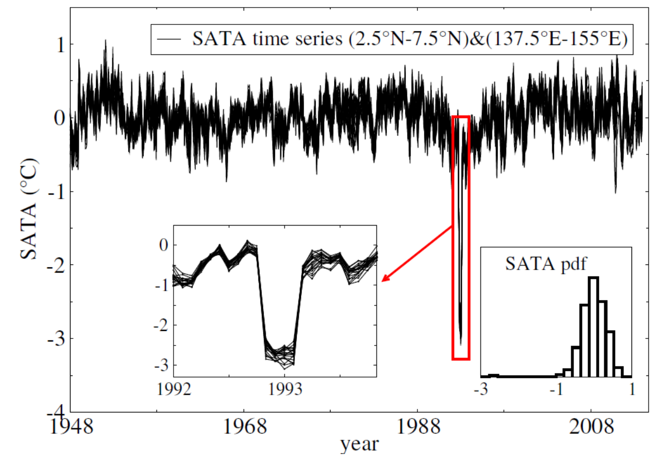
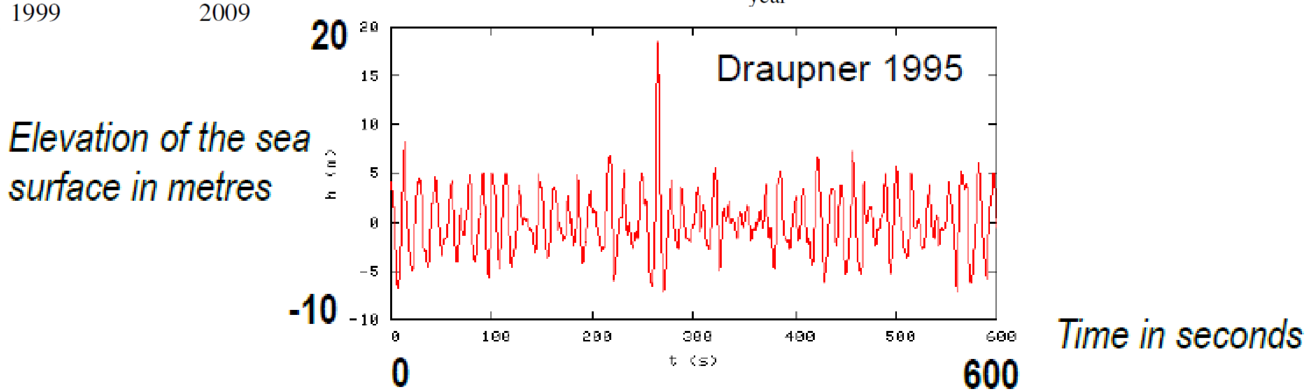
Rare and extreme events: a characteristic feature of the dynamics of complex systems

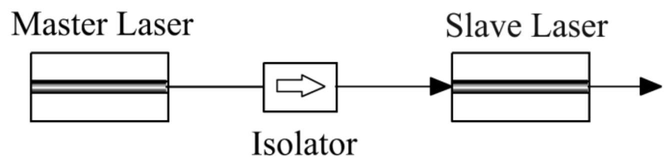






The challenges:

- Controlled experiments can advance the understanding of the mechanisms capable of triggering (or suppressing) extreme events.
- Developing reliable diagnostic tools for "early warning" of extreme events.



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Physics Reports 416 (2005) 1–128
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The dynamical complexity of optically injected semiconductor lasers

S. Wieczorek[a,*], B. Krauskopf[b,d], T.B. Simpson[c], D. Lenstra[d,e]

A lot of work has focused on understanding the dynamics (Amann, Barland, Kelleher, Lüdge, Sciamanna, and many others!)

Also extreme intensity pulses (optical rogue waves)?

What have we learned

In cw optically injected semiconductor lasers, ORWs can be

- **deterministic**, generated by a crisis-like process.
- **controlled** by noise and/or by current modulation.
- **predicted** with a certain anticipation time.

- C. Bonatto et al, *Deterministic optical rogue waves*, PRL 107, 053901 (2011).
- J. Zamora-Munt et al, *Rogue waves in optically injected lasers: origin, predictability and suppression,* PRA 87, 035802 (2013).
- S. Perrone et al, *Controlling the likelihood of RWs in an optically injected semiconductor laser via direct current modulation*, PRA 89, 033804 (2014).
- J. Ahuja et al, *Rogue waves in injected semiconductor lasers with current modulation: role of the modulation phase*, Optics Express 22, 28377 (2014).

RW predictability

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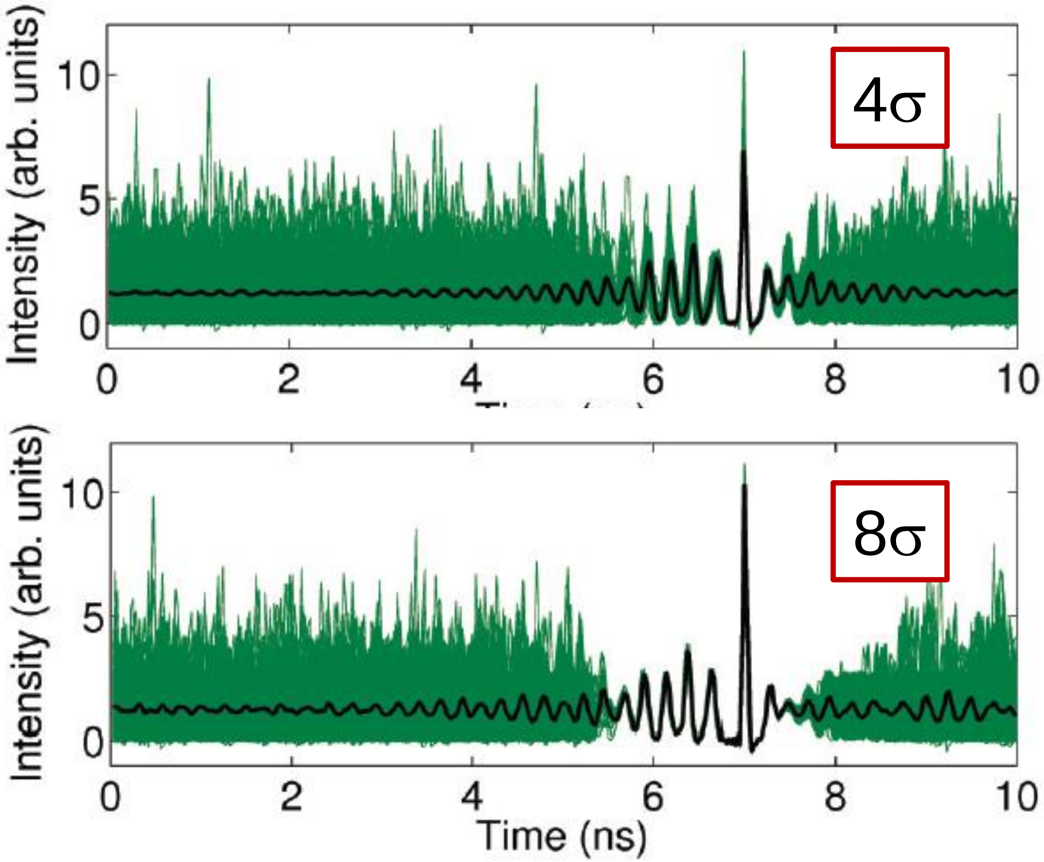
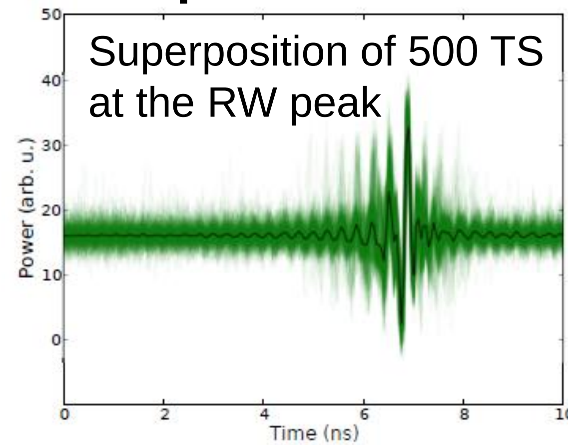
Deterministic simulations



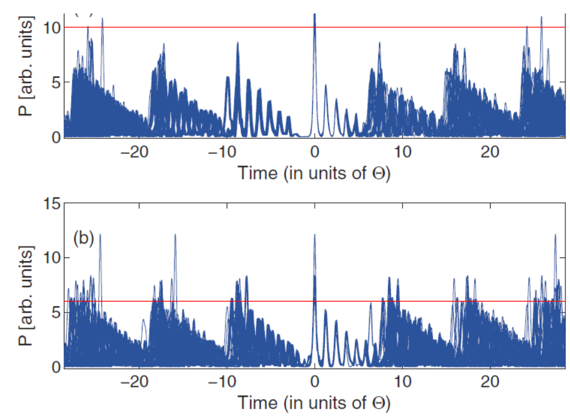
Superposition of 50 time-series at the RW peak

J. Zamora-Munt et al, PRA 87, 035802 (2013)

Experiments



Also with optical feedback (Aparicio Reinoso PRE 87, 062913 2013)

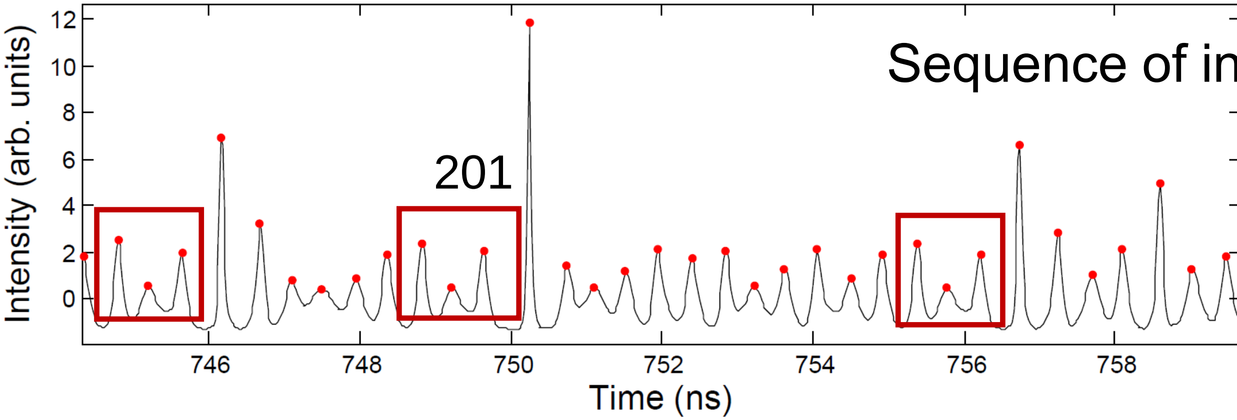


⇒ Well-defined oscillation pattern anticipates extreme pulses.

⇒ How can this be quantified?

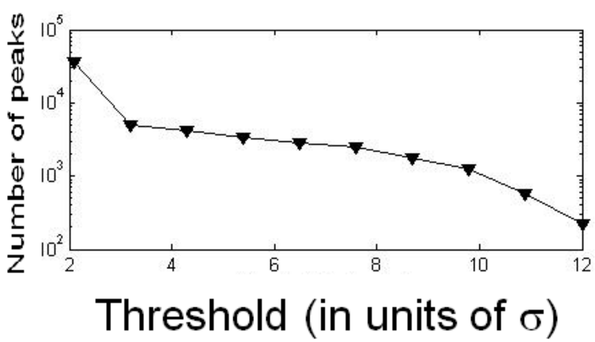
Ordinal analysis applied to three pulse heights before a RW

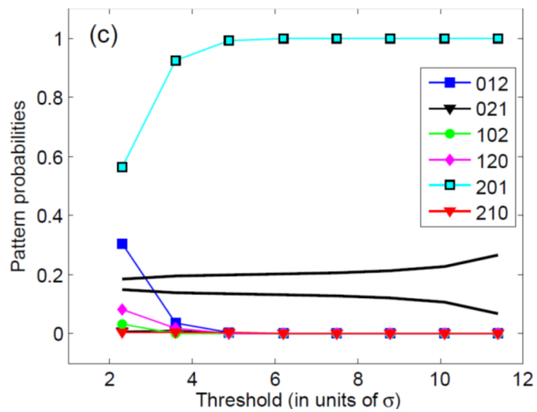
Deterministic simulations: time-series normalized to zero-mean and σ=1



Sequence of intensity peaks: {… I_i, I_{i+1},…}

If I_i > TH, we analyze the pattern defined by (I_{i-3}, I_{i-2}, I_{i-1})





Ordinal probabilities computed from the 3 peaks before a pulse.

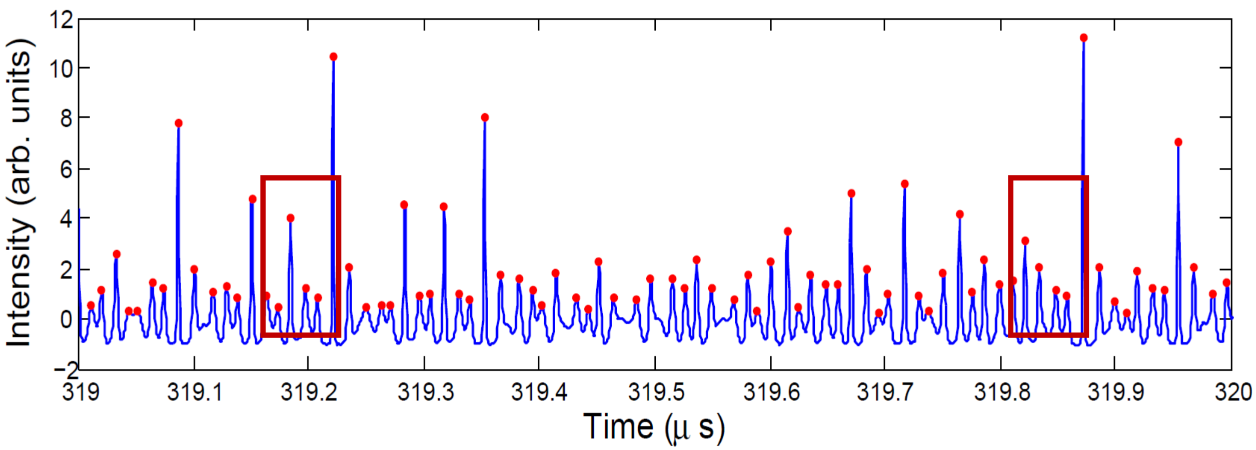
Black lines: 95% confidence p_i=1/6 $\forall$ i

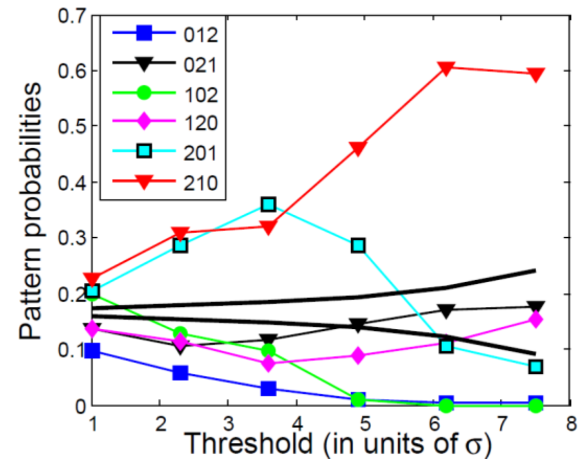
- P(201)=1 if TH >6
- Drawback: P(201)≠0 if TH <6 (pattern 201 also anticipates some small pulses) $\Longrightarrow$ false alarms (false positives)

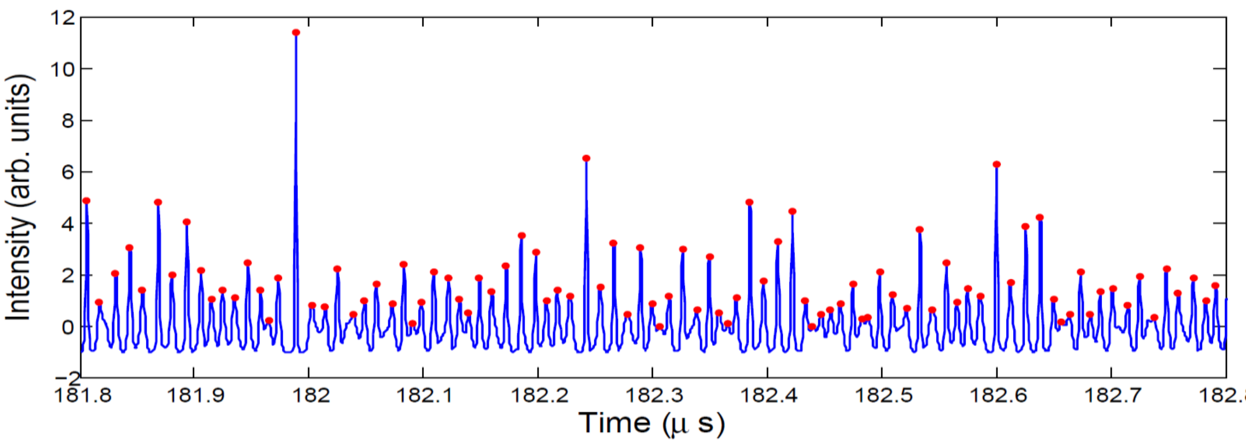
Model and parameters as in S. Perrone et al, PRA 89, 033804 (2014)

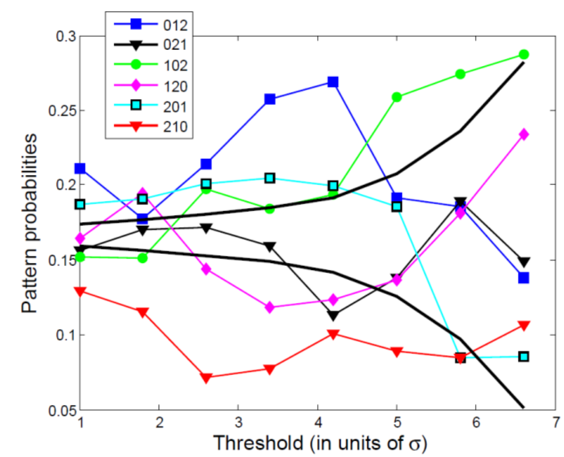
Including spontaneous emission noise and current modulation

- Two different modulation frequencies









"early warning pattern" depends on parameters and is not always well-defined. Next step: analysis of empirical data.

Conclusions

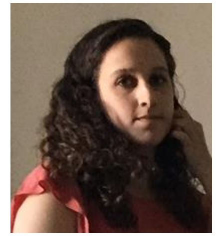
What did we learn?

- Take home message:
  - ordinal analysis is useful for understanding data, uncovering patterns,
  - for model validation, parameter estimation, classifying events, etc.
  - robust to noise and artifacts in the data.
- Main conclusions
  - LFFs: minimal model for ISI correlations identified (a modified circle map); with current modulation good agreement with the FHN model.
  - PS: three early-warning indicators of abrupt transition validated.
  - Laminar-turbulent transition: sharp transition seen in thresholded data but not in raw data; particular time-scales and phenomenological model for intensity correlations at the transition identified.
  - ORWs: some patterns are more (or less) likely to occur before high pulses
- Open issues (ongoing and future work):
  - Hierarchical & clustered structure: universal feature of excitable systems?
  - Mathematical insight: can we calculate the probabilities analytically?
  - Role of coupling? Induce different preferred/infrequent patterns?
  - Quantify the performance of the diagnostic tools.
  - Applicability to real-word time-series (climate, financial, etc)?

Collaborators

At UPC

- Nuria Martinez
- Carlos Quintero-Quiroz
- Dr. Taciano Sorrentino
- Dr. Andres Aragoneses
- Dr. Laura Carpi
- Dr. J. M. Aparicio Reinoso
- Prof. M. C. Torrent







Experimental data: VCSEL PS from INLN (S. Barland) and Bangor University (Y. Hong); fiber laser from Aston University (S. Turitsyn)

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THANK YOU FOR YOUR ATTENTION !

<cristina.masoller@upc.edu>

Papers at **http://www.fisica.edu.uy/~cris/**

- C. Bonatto et al, PRL 107, 053901 (2011).
- J. Zamora-Munt et al, PRA 87, 035802 (2013).
- A. Aragoneses et al, Sci. Rep. 4, 4696 (2014).
- S. Perrone et al, PRA 89, 033804 (2014).
- C. Masoller et al, New J. Phys. 17, 023068 (2015).
- A. Aragoneses et al, PRL 116, 033902 (2016).
- J. M. Aparicio Reinoso et al, PRE in press (http://arxiv.org/abs/1510.09035)

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