Experimental Observation of Highly Regular Entrained Oscillations in the Output of a Weakly Modulated Semiconductor Laser with Optical Feedback Near Threshold

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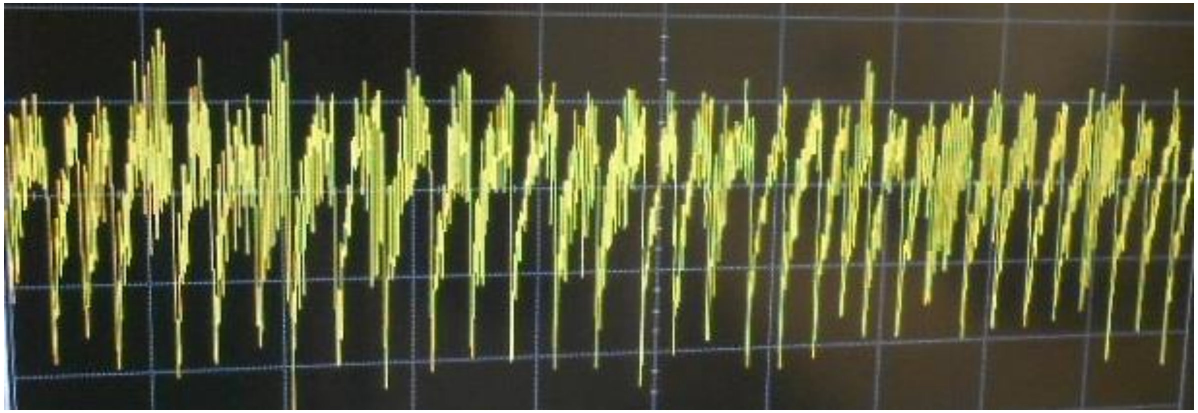
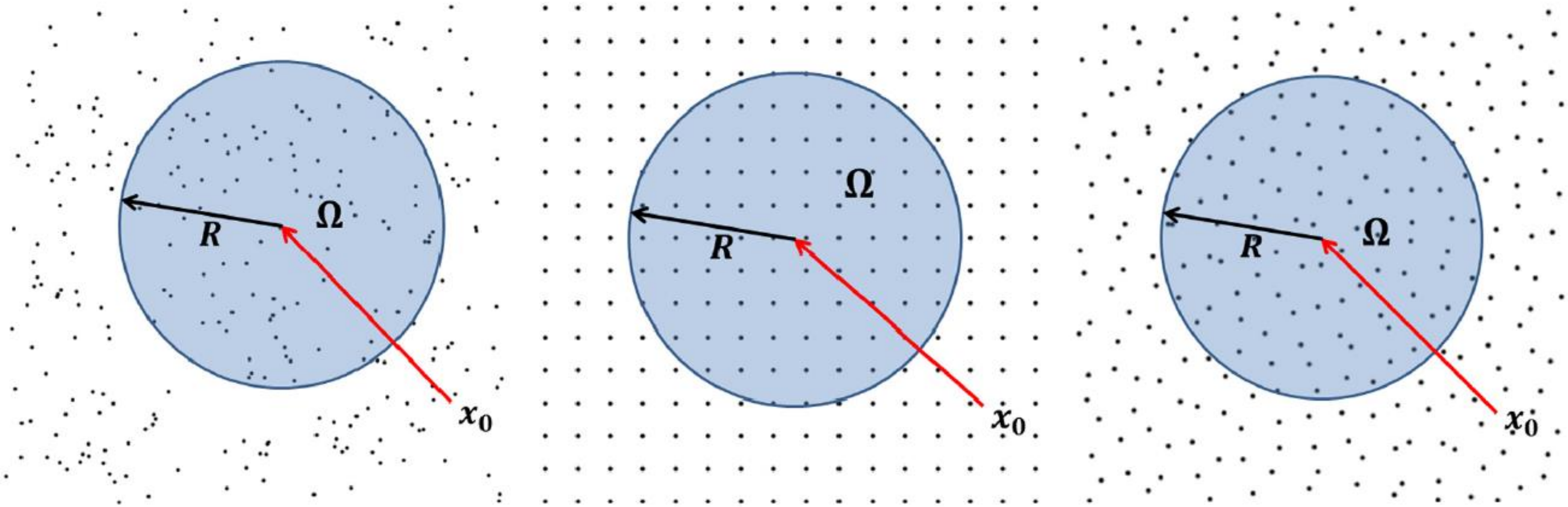
Outline



- Motivation: hyper-regular and time-crystal states
- Analogy between time-delayed and spatially-extended systems
- Overview of the dynamics of external cavity diode lasers (EELs) with direct pump current modulation
  - Near threshold: low frequency fluctuations
  - Role of periodic current modulation: amplitude, frequency and waveform
- Quantification of the temporal regularity of LFF dynamics
- Conclusions and open questions

Not all “disordered systems” are equally disordered



Non-hyper-uniform

Trivial hyper-uniform

Non-trivial hyper-uniform

How does the number of particles inside the circle varies with the radius of the circle?

$$\sigma_N^2(R) \sim R^d$$

Some disordered system are *special*: they show a slower grow of density fluctuations (asymptotic scaling between surface-area and volume growth).

S. Torquato / Physics Reports 745 (2018) 1–95

Time-crystal and hyper-uniform states: special states of disordered systems

- Hyper-uniform states exhibit an anomalously long-ranged suppression of density fluctuations. Many observations.
- Time-crystal states in *periodically driven systems* are characterized by:
  - regular oscillations in space and in time that are *stable over very long time intervals*,
  - *rigidly* persist under small variations of the initial conditions or parameters,
  - *time-translation symmetry breaking* because the oscillation period differs from the period of the driving signal.

So far, TC states have been observed in many-particle quantum systems, as discussed in Alejandro Giacomotti's invited talk.

S. Torquato, Phys. Rep. 745, 1 (2018).
F. Wilczek, Phys. Rev. Lett. 109, 160401 (2012).
N. Y. Yao, and C. Nayak, Physics Today 71, 9, 40 (2018).

Motivation

- Can we use the **“space-time” analogy** of time-delayed and spatially-extended systems, to find a “disordered” time-delayed system that exhibits anomalous long-range regularity?

- Can we find states that are analogous to hyper-uniform and/or time-crystal states?

What is the "space-time" analogy?

RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 45, NUMBER 7 1 APRIL 1992

Two-dimensional representation of a delayed dynamical system

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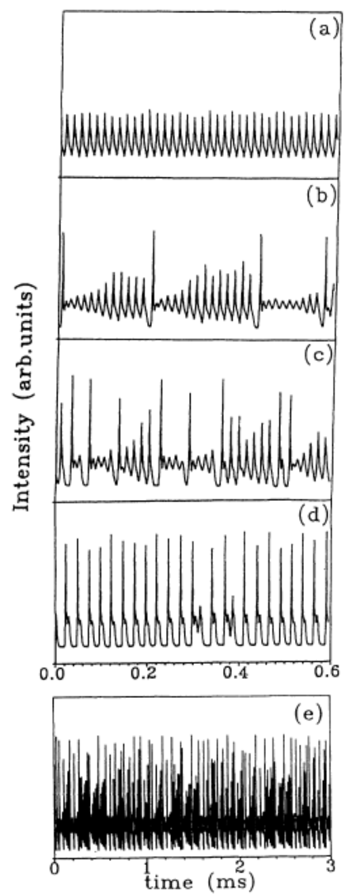
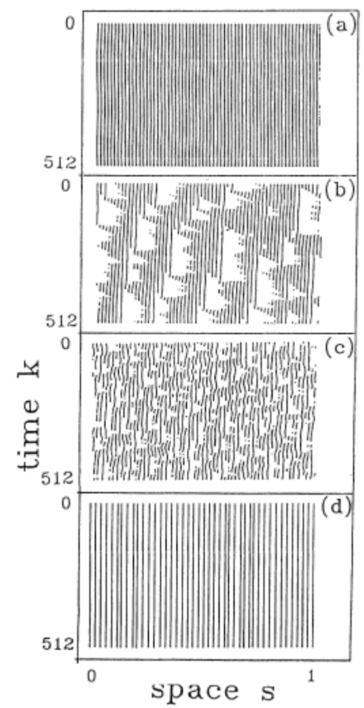
(Received 31 July 1991; revised manuscript received 10 December 1991)





Spatiotemporal dynamics in the coherence collapsed regime of semiconductor lasers with optical feedback

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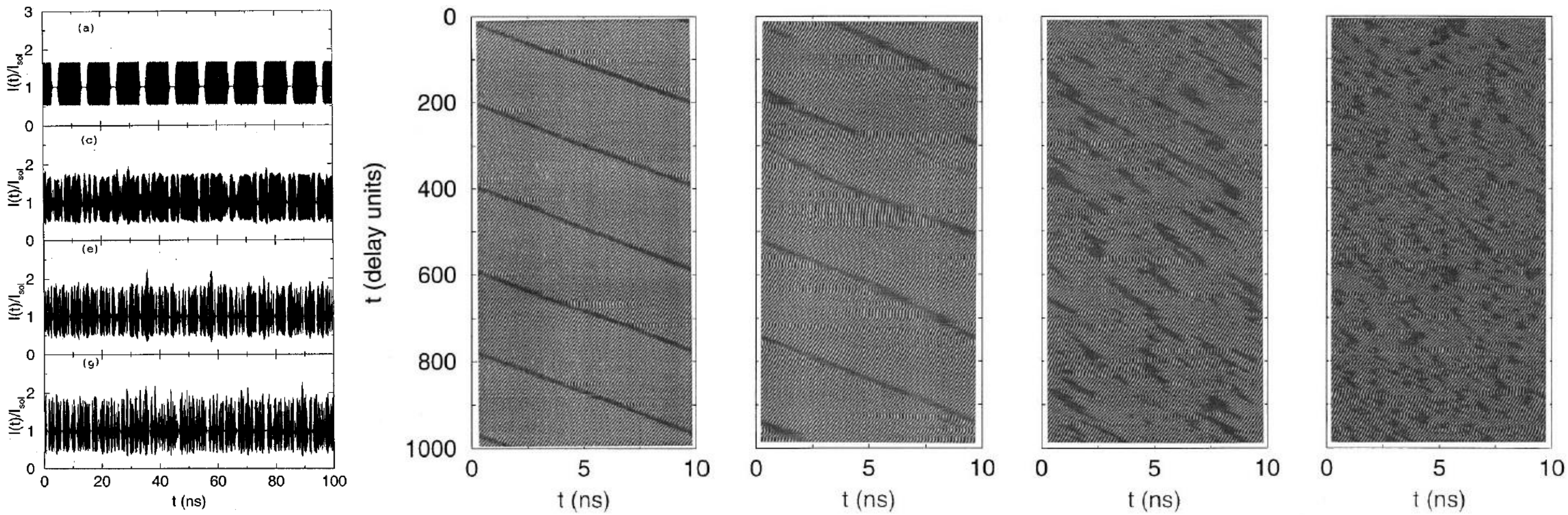
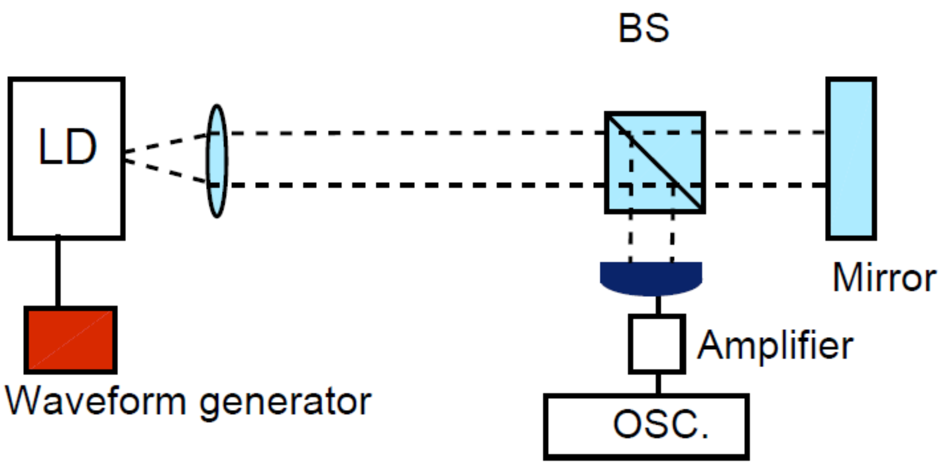
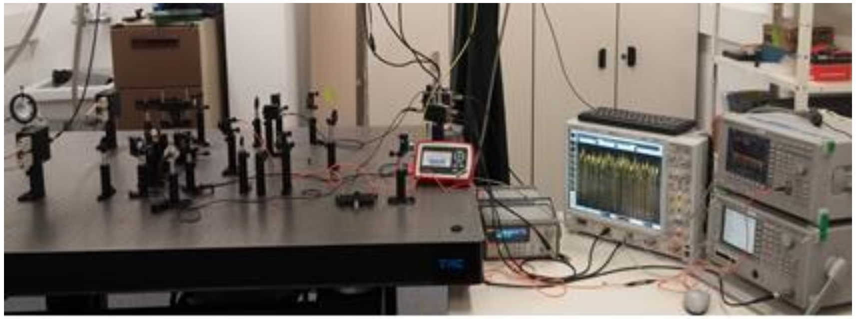
Chaos **7** (3), 1997

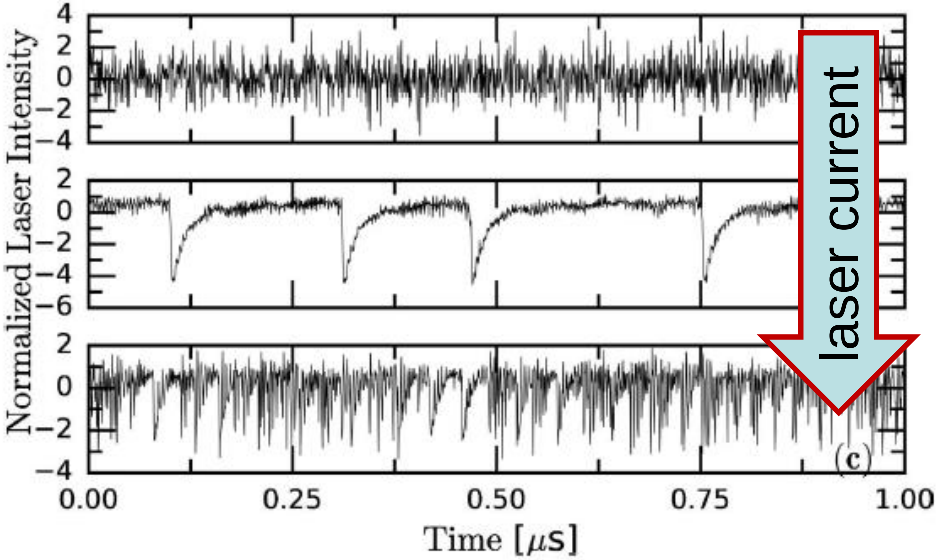


Semiconductor lasers with time-delayed optical feedback are good candidates to search for "highly regular states" generated by a small-amplitude modulation of the laser current.

Dynamics of a semiconductor laser with optical feedback





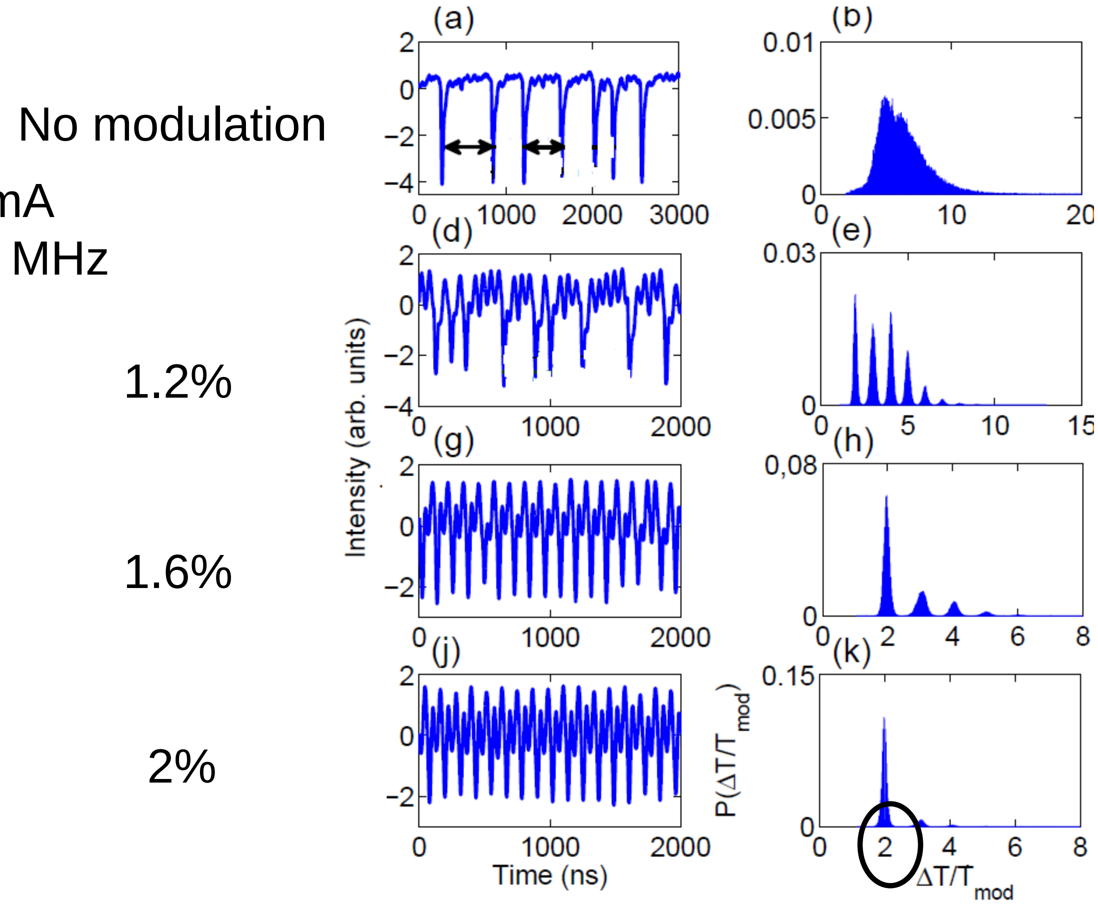


Questions

- Can we entrain the optical spikes ("low frequency fluctuations") to a small-amplitude electric periodic signal that modulates the laser current?
- Which modulation waveform is best for obtaining highly regular spikes?
- How can we detect and quantify the regularity of the spike timing?

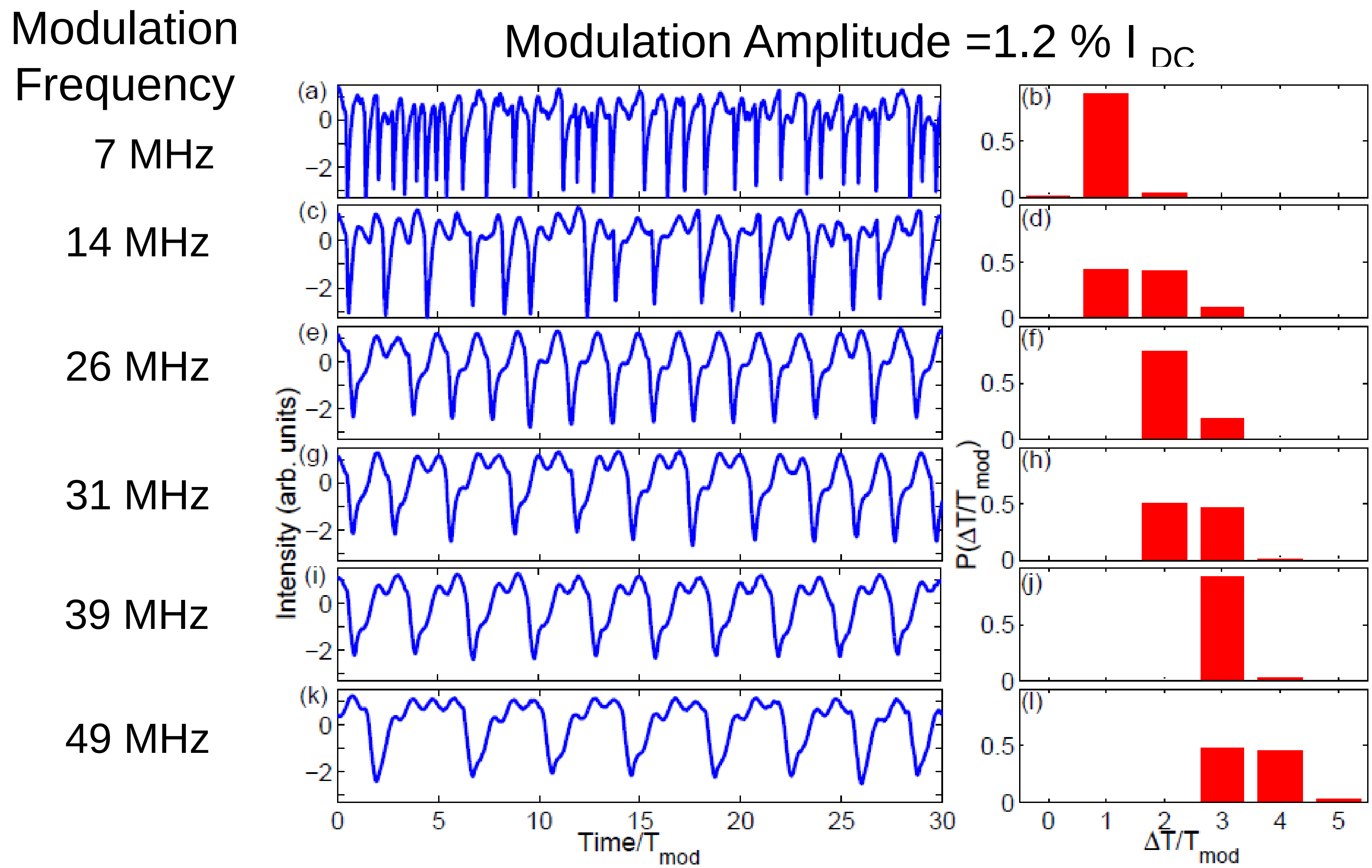
Small-amplitude sinusoidal current modulation: the distribution of the intervals between spikes (inter-spike-intervals) shows noisy locking

I_{dc} = 39 mA
f_{mod} = 17 MHz



A. Aragoneses, T. Sorrentino, S. Perrone, D. J. Gauthier, M. C. Torrent, C. Masoller, "*Experimental and numerical study of the symbolic dynamics of a modulated external-cavity semiconductor laser*", Optics Express **22**, 4705 (2014).

Small-amplitude sinusoidal current modulation: the distribution of the intervals between spikes (inter-spike-intervals) shows noisy locking

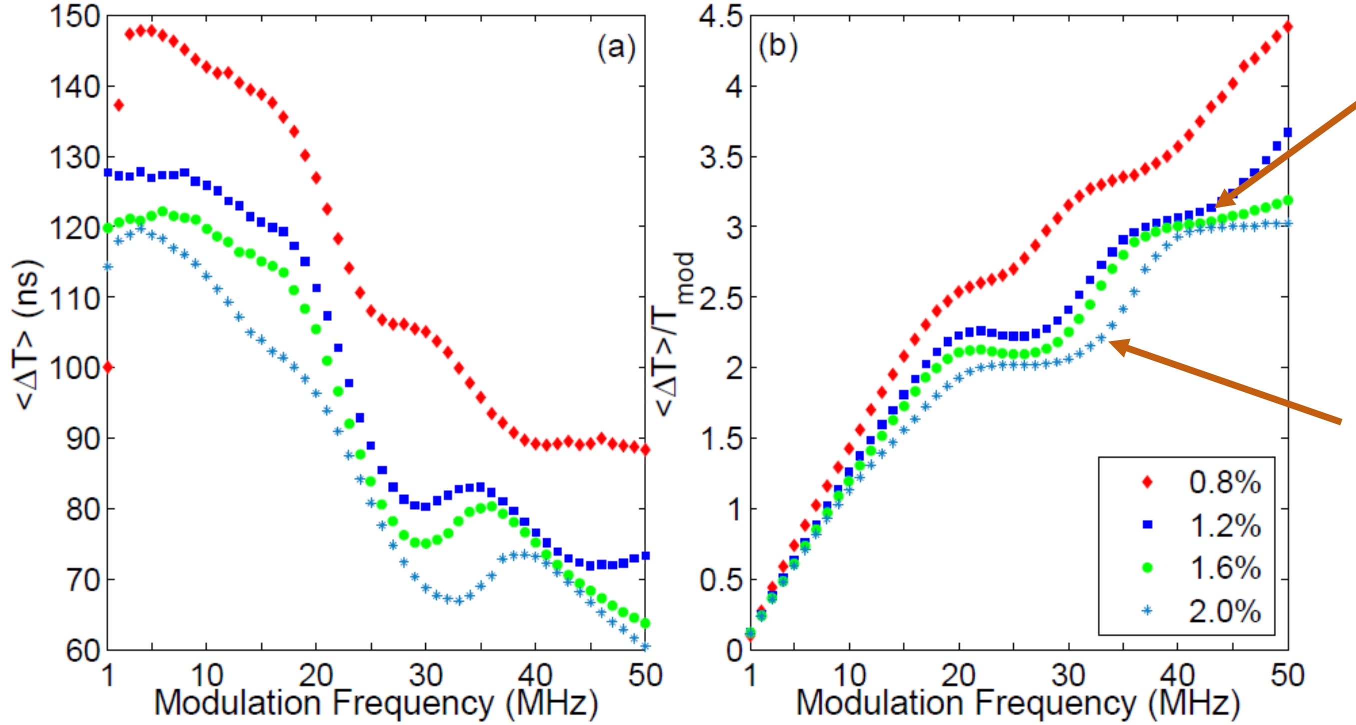


T. Sorrentino, C. Quintero-Quiroz, A. Aragoneses, M. C. Torrent, <u>C. Masoller</u>, "*Effects of periodic forcing on the temporally correlated spikes of a semiconductor laser with feedback*", Optics Express **23**, 5571 (2015).

Locking "plateaus"

Mean inter-spike-interval

Mean ISI / modulation period



1 spike every 3 modulation cycles

1 spike every 2 modulation cycles

Why no 1:1 locking plateau?

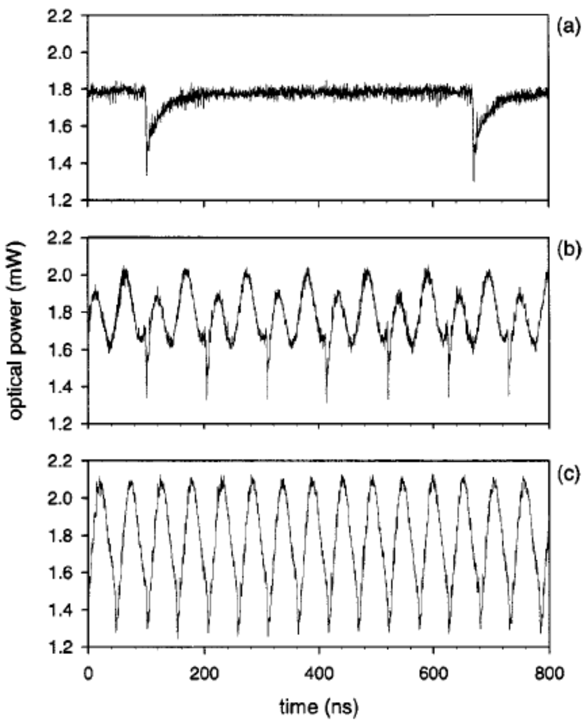
T. Sorrentino et al., Optics Express **23**, 5571 (2015).

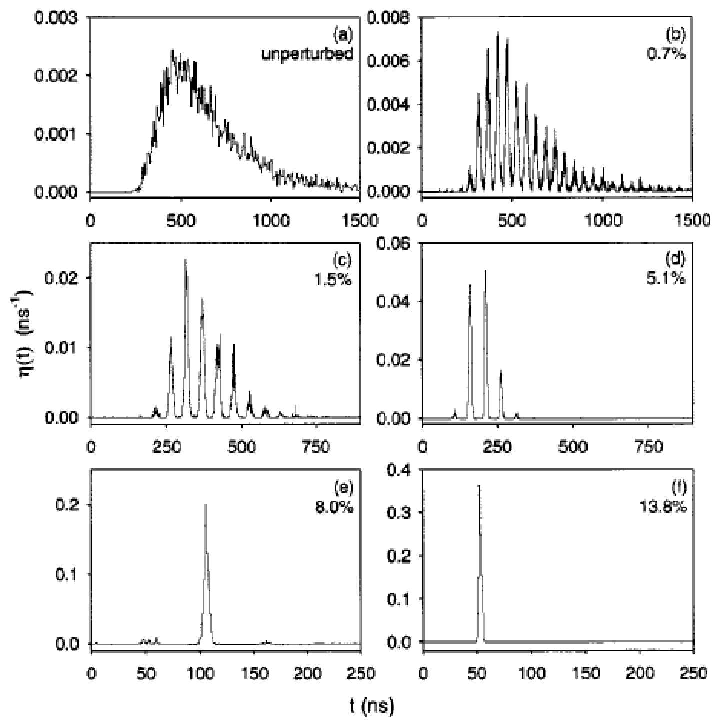
Earlier work

IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. 36, NO. 2, FEBRUARY 2000

David W. Sukow and Daniel J. Gauthier

Entraining Power-Dropout Events in an External-Cavity Semiconductor Laser Using Weak Modulation of the Injection Current





PHYSICAL REVIEW E, VOLUME 63, 066218

Dynamics of periodically forced semiconductor laser with optical feedback

Jorge Manuel Mendez,[1] R. Laje,[1] M. Giudici,[2], J. Aliaga,[1] and G. B. Mindlin[1]
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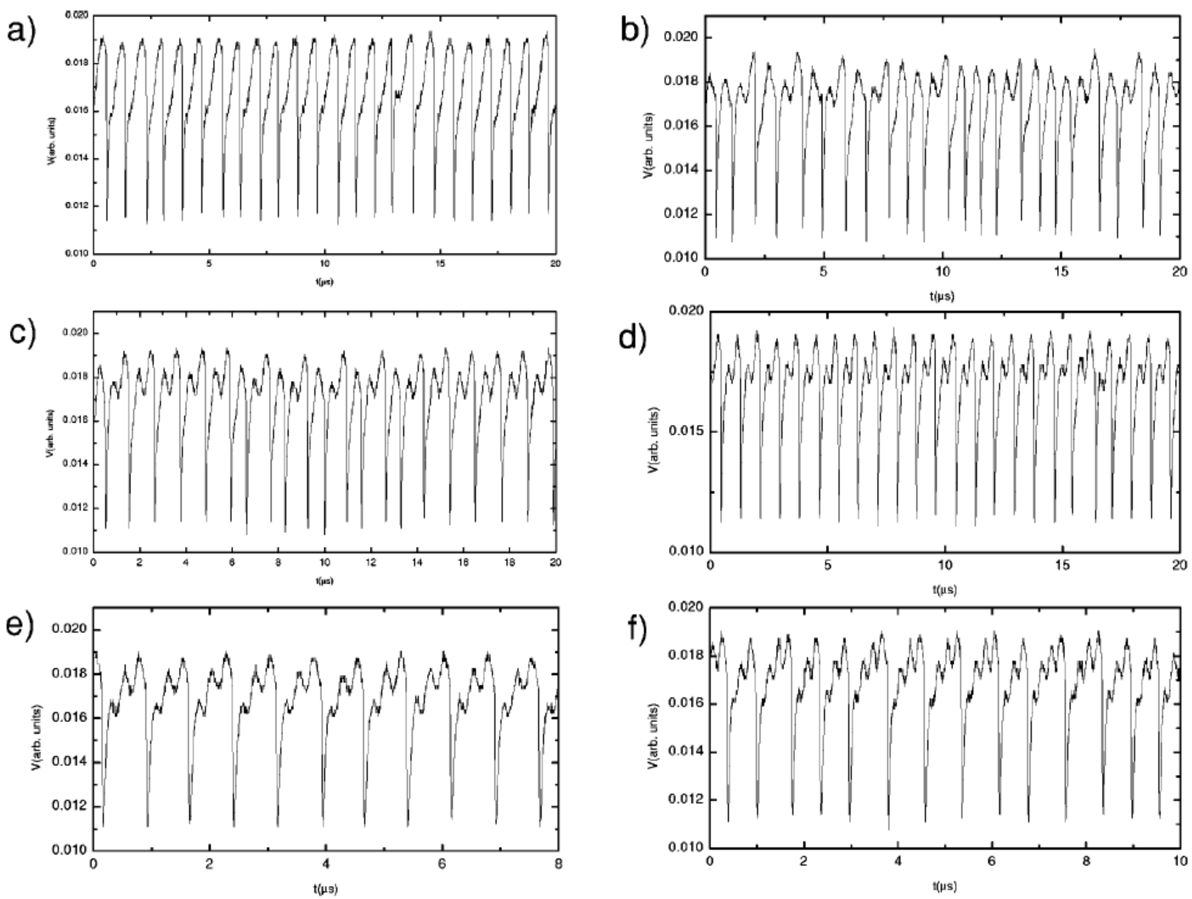
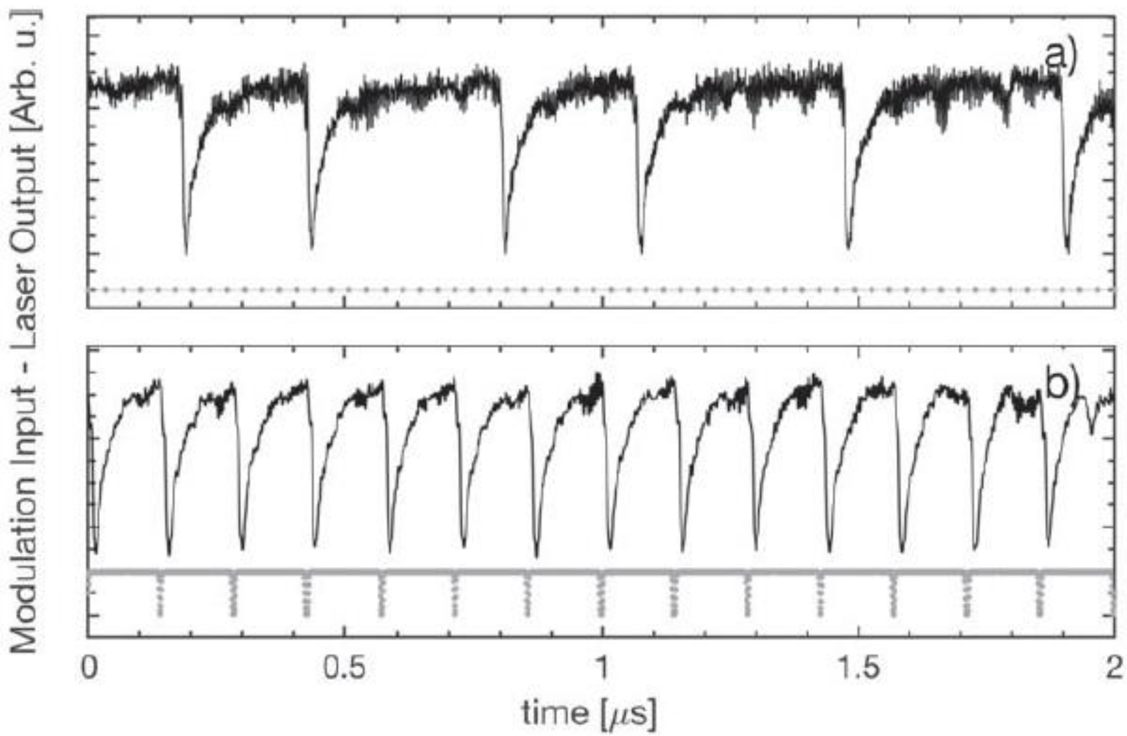
(Received 16 June 2000; published 25 May 2001)



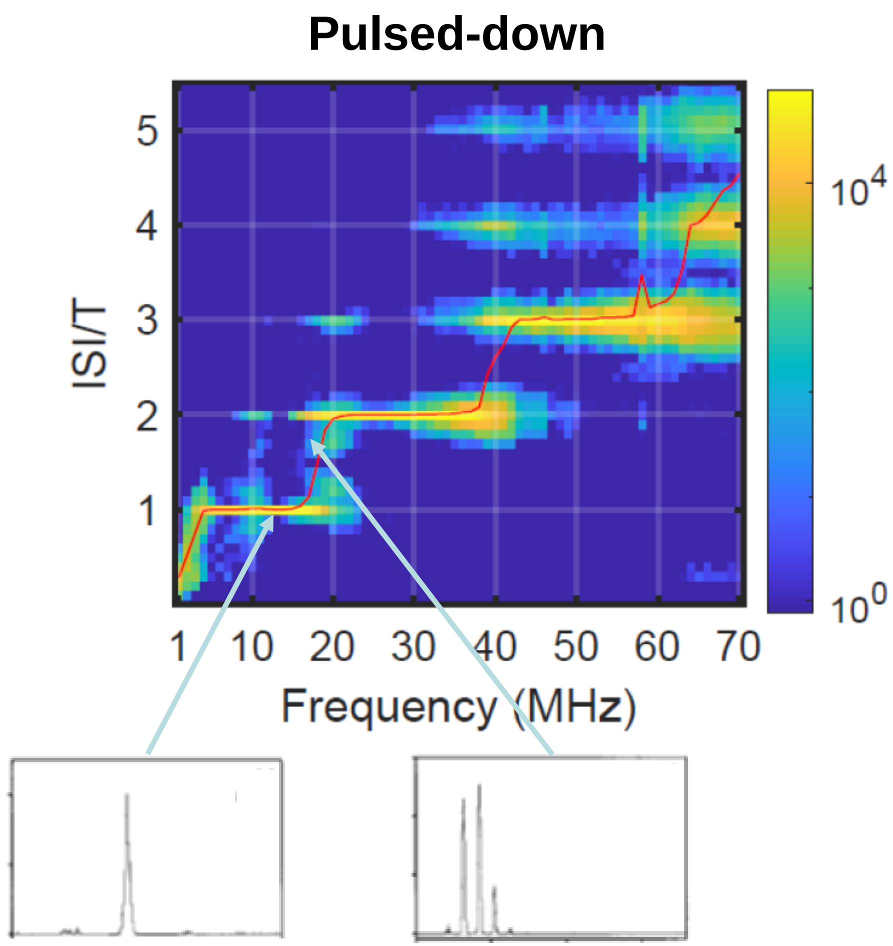
How regular can the *timing* of the optical spikes be?

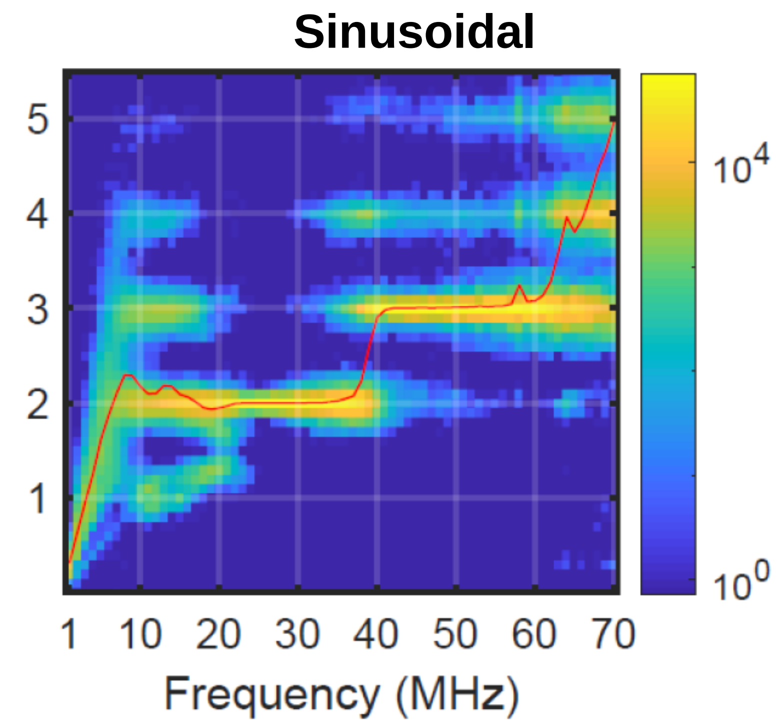
Another set of experiments using different modulation waveforms.



J. Tiana-Alsina, C. Quintero-Quiroz, M. Panozzo, M. C. Torrent, C. Masoller, "*Experimental study of modulation waveforms for entraining the spikes emitted by a semiconductor laser with optical feedback*", Opt. Express **26**, 9298 (2018).

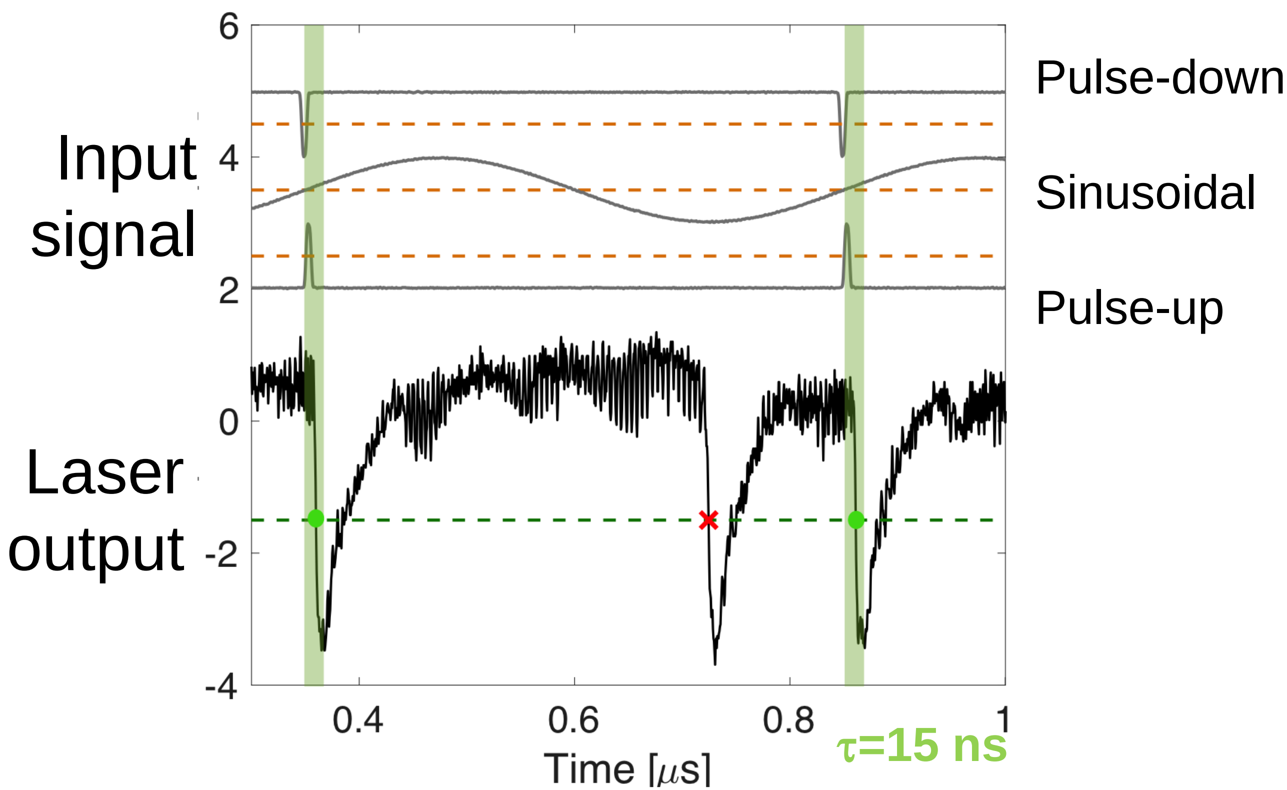
In log color code the ISI distribution; T is the period of the modulation





With sinusoidal modulation
no 1:1 locking plateau.

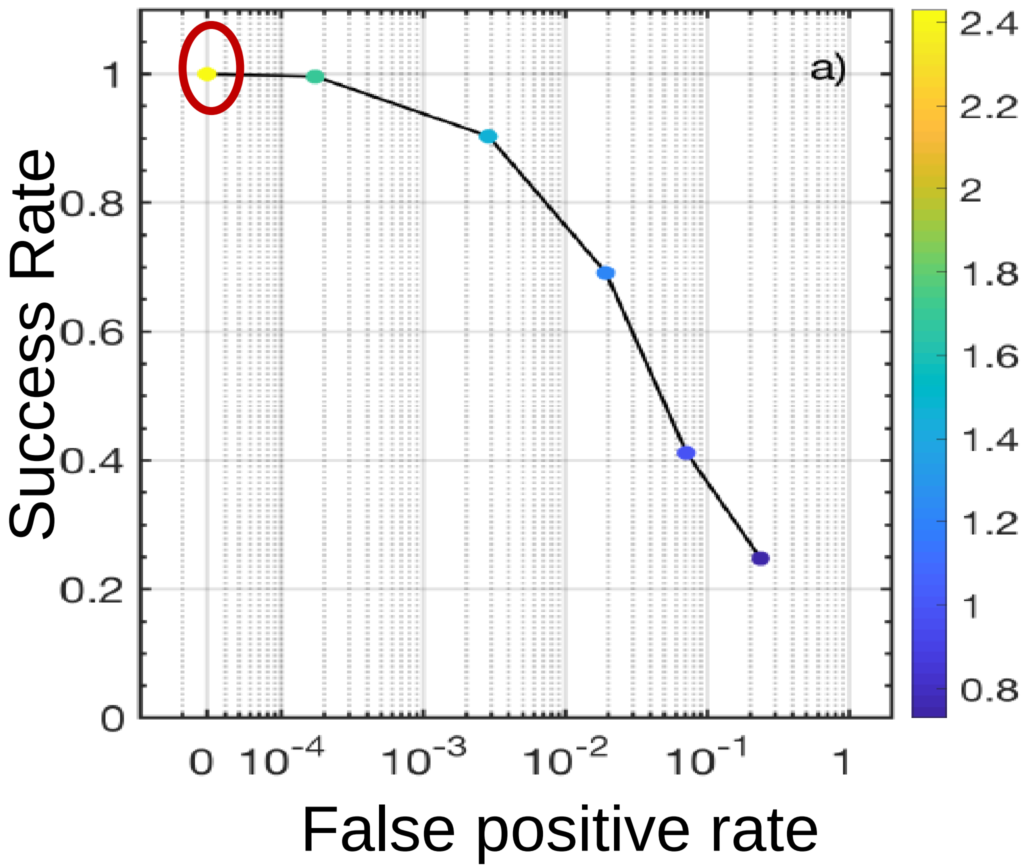
First attempt to quantify the regularity of the *timing* of the laser spikes: success rate and false positive rate



$$SR(\tau) = \frac{\text{\# of spikes emitted in the interval } \tau}{\text{\# of modulation cycles}}$$

$$FPR(\tau) = \frac{\text{\# spikes that are not emitted in the time interval } \tau}{\text{Total \# of spikes}}$$

Receiver operating characteristic (ROC) curve

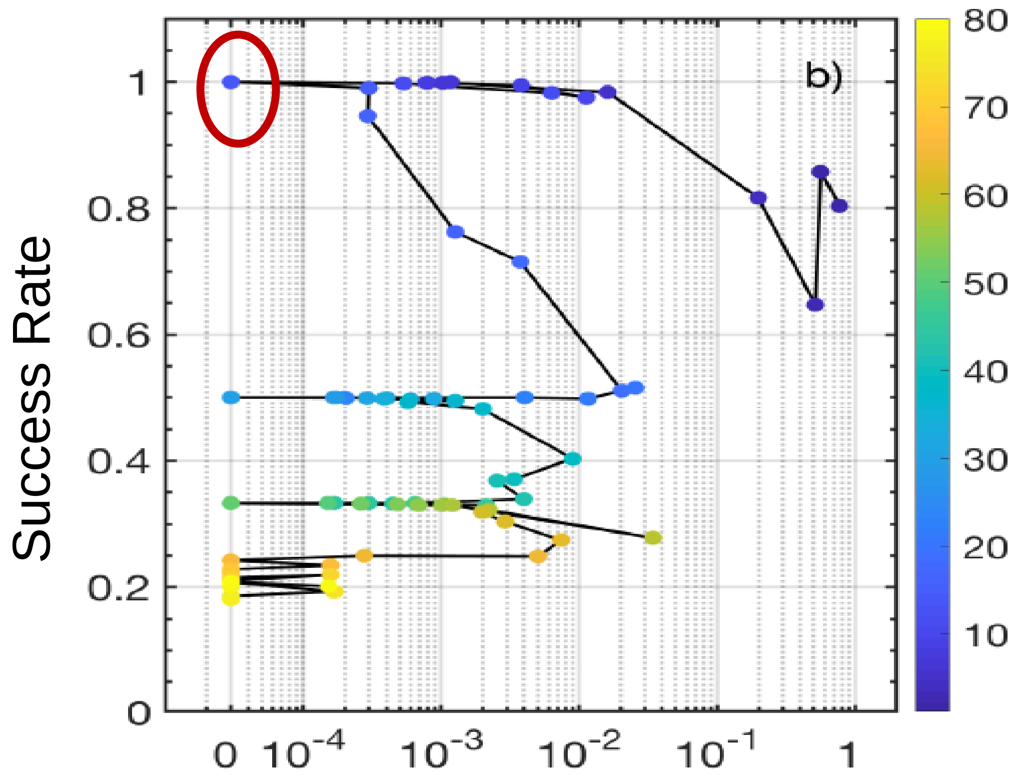


Pulse-down waveform modulation amplitude %

f_{mod} = 14 MHz

J. Tiana-Alsina, C. Quintero-Quiroz, M. C. Torrent, C. Masoller, "*Quantifying the degree of locking in weakly forced stochastic systems*", Phys. Rev. E 99, 022207 (2019).

Locked-unlocked transitions when the modulation frequency increases



Pulse-down waveform modulation frequency (MHz)

$A_{mod} = 2.4\%$

One and only one spike per modulation cycle. However, the SR and FPR measures do not capture fluctuations in the timing of the spikes within the detection interval **(τ=15 ns)**

J. Tiana-Alsina et al., Phys. Rev. E 99, 022207 (2019).

The Fano Factor: a precise measure of spike timing regularity

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BARCELONATECH
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How to calculate the Fano Factor?

- Divide the intensity time trace in N_{int} non-overlapping segments of duration T_{int}.
- Count the number of spikes in each segment, $\{N_1, N_2, \dots N_{Nint}\}$.
- Calculate the mean and the variance, $\langle N_i \rangle$, σ^2
- Calculate the Fano factor as $F = \sigma^2(N_i) / \langle N_i \rangle$
- *F* depends on the duration of the counting interval, T_{int}.
- If T_{int} is very small, *F=1* because the sequence of counts is a sequence of 0s and 1s.
- If the process that triggers the spikes is fully random, $F=1\ \forall T_{int}$.
- To test the presence of “temporal correlations” between spikes:
  - Shuffle the spike times
  - Recalculate *F*
  - Compare with *F* of the original spike sequence.

The Fano Factor is a well-known measure to quantify the regularity of neural spike trains

Vol. 14, No. 3/March 1997/J. Opt. Soc. Am. A 529

Fractal character of the neural spike train in the visual system of the cat

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Conor Heneghan

Department of Electrical and Computer Engineering, Boston University, Boston, Massachusetts 02215, and New York Eye and Ear Infirmary, New York, New York 10003

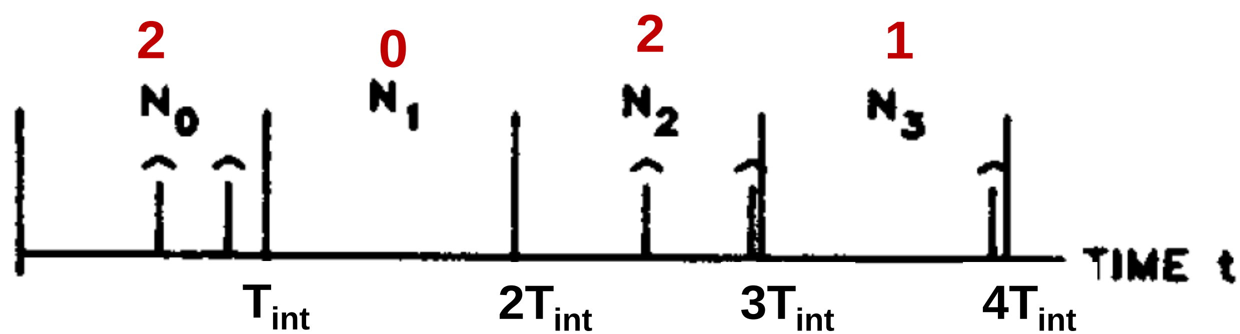
Steven B. Lowen

Department of Electrical and Computer Engineering, Boston University, Boston, Massachusetts 02215

Tsuyoshi Ozaki and Ehud Kaplan

The Mount Sinai School of Medicine, New York, New York 10029, and The Rockefeller University, New York, New York 10021

$N_{int} = 4$



Fano Factor (color code) of the feedback-induced optical spikes

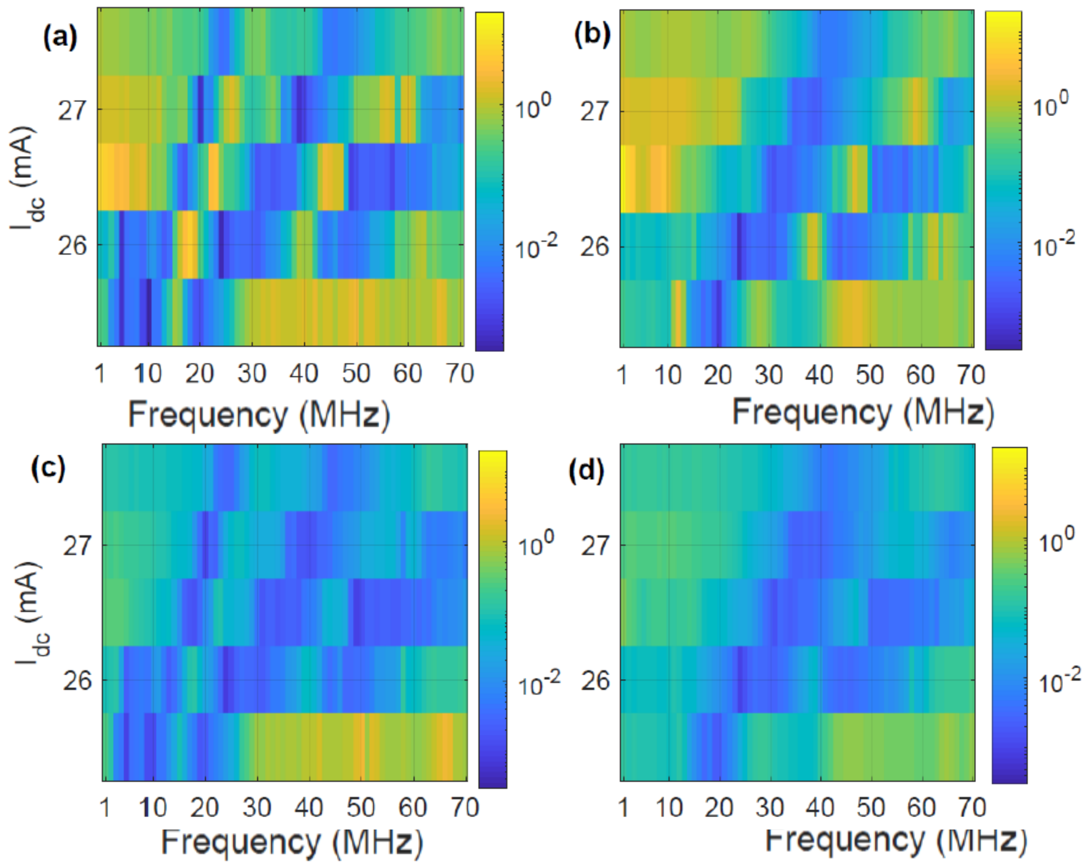
$N_{int} = 1000$, $T_{int} = 5\ \mu s$

(Total recorded time, 5 ms, contains 9000-120000 spikes, depending on I_{dc} and f_{mod})

Pulsed-down **Sinusoidal**

Original

Shuffled

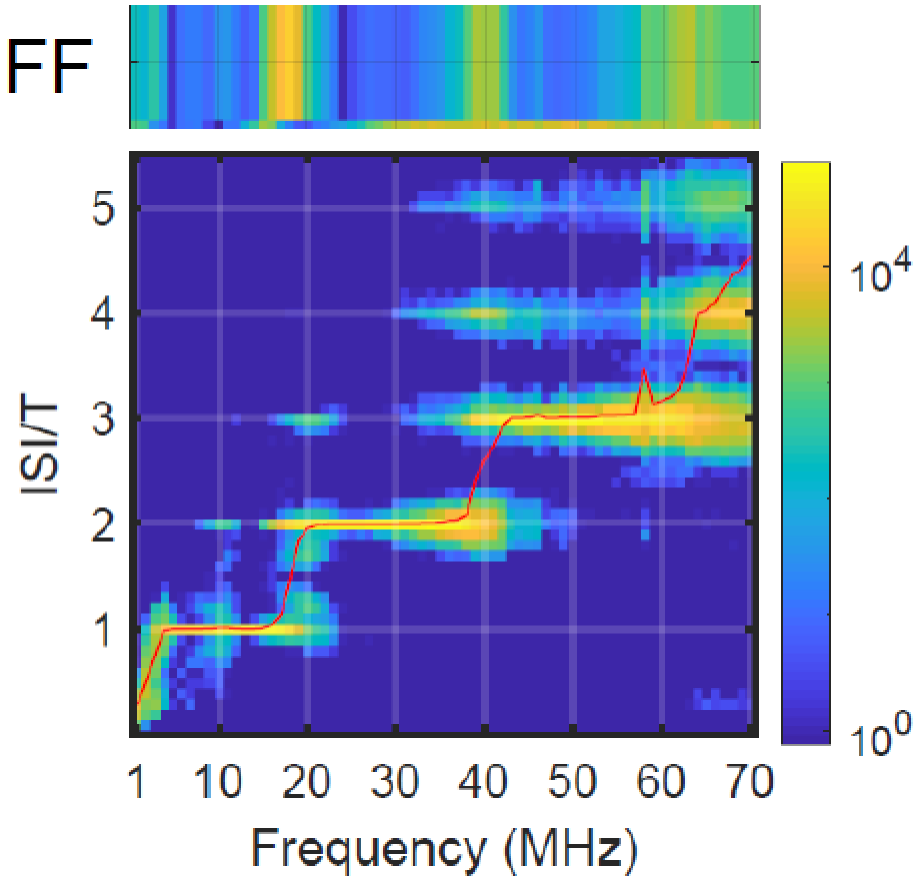


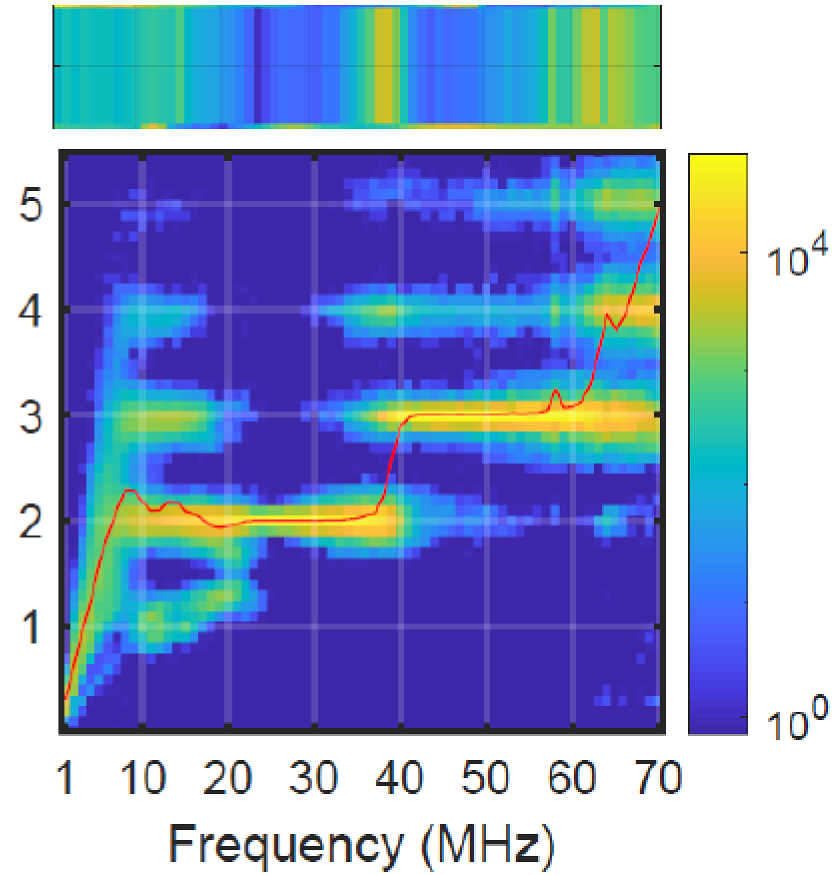
Next: to try to understand why we find blue and yellow regions, I inspect the ISI distribution. I_{dc}=26 mA.

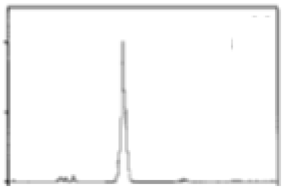
- Blue regions: very low $F \Rightarrow$ small $\sigma \Rightarrow$ the sequence of counts is highly regular.
- Yellow regions: large $F \Rightarrow$ large $\sigma \Rightarrow$ high variability in the sequence of counts.
- In the pulsed-down plot there are three blue regions; in the sinusoidal plot, only two.
- Differences between original and shuffled data reveal spike temporal correlations.

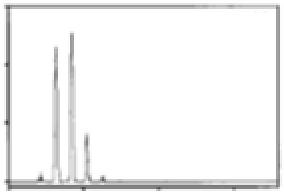
Large F : wide ISI distribution (unlocking spikes)
Small F : narrow ISI distribution (locked spikes)

Pulsed I_{dc}=26 mA **Sinusoidal**





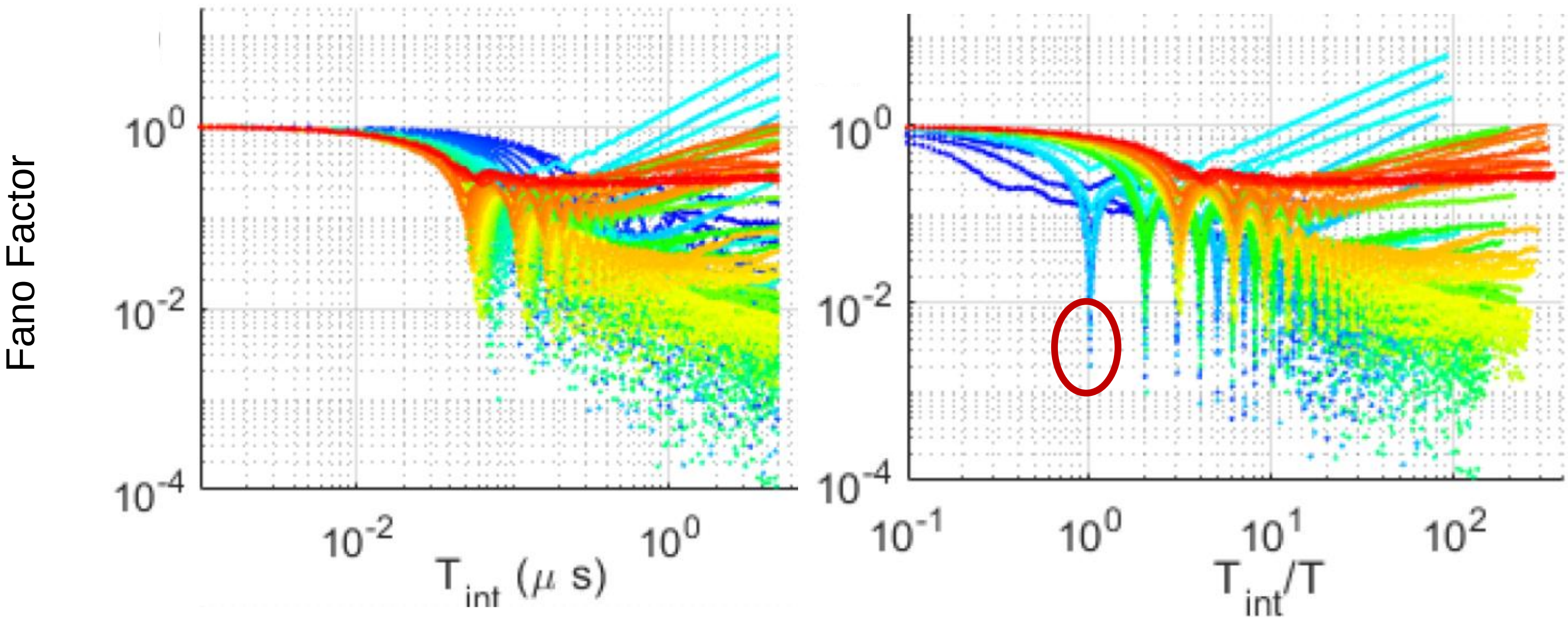


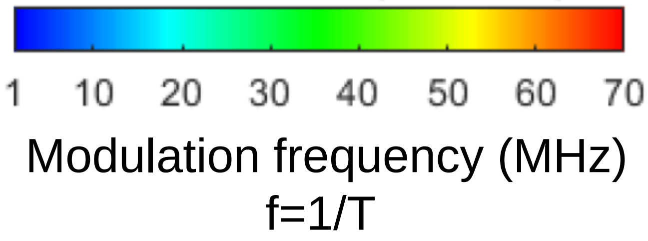


Next: I will analyze how the Fano Factor depends on the duration of the counting interval.

Variation of the Fano Factor with the duration of the counting interval for different modulation frequencies (in color code)

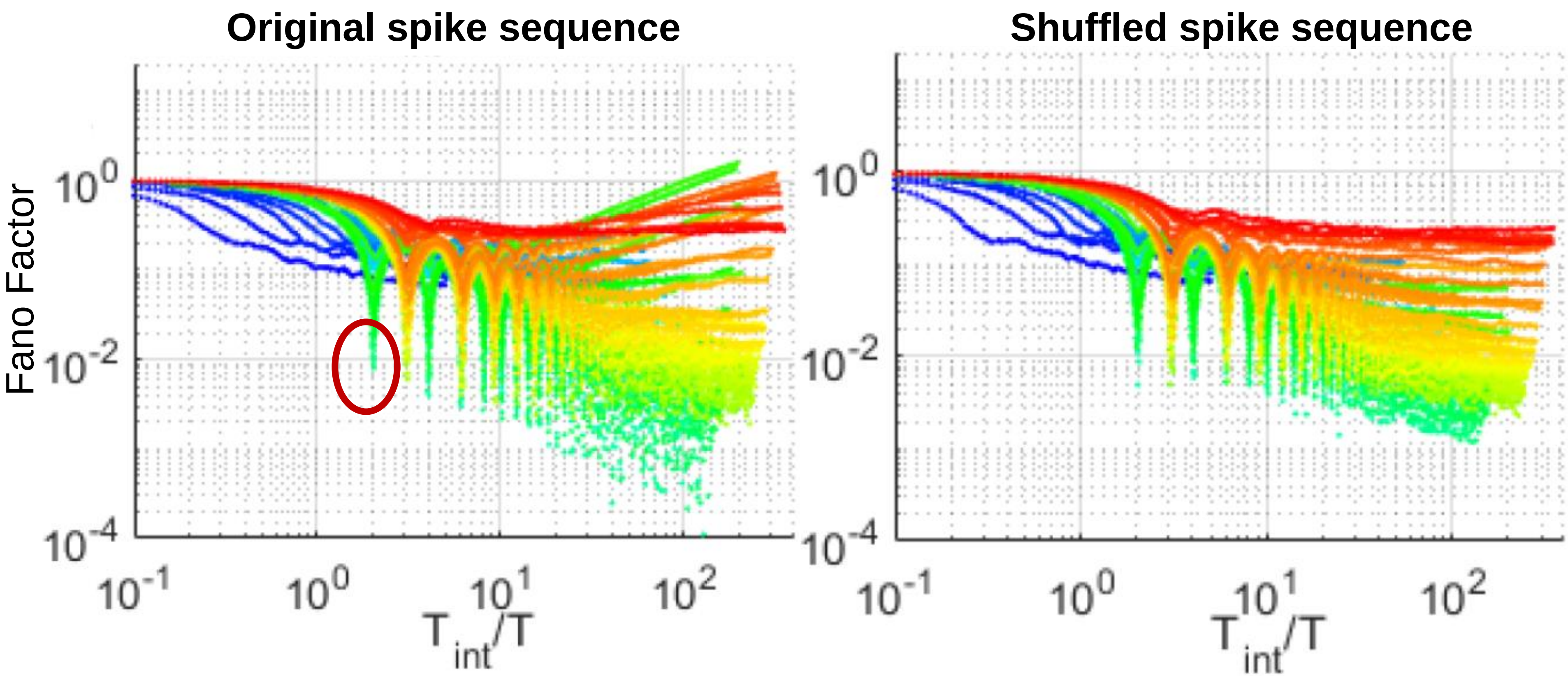
Pulsed waveform of period T





- Power law grow reveals a fractal nature of the spike train (spike clustering).
- Sharp minima reveal high regularity of the number of spikes when the counting window includes an integer number of modulation periods.

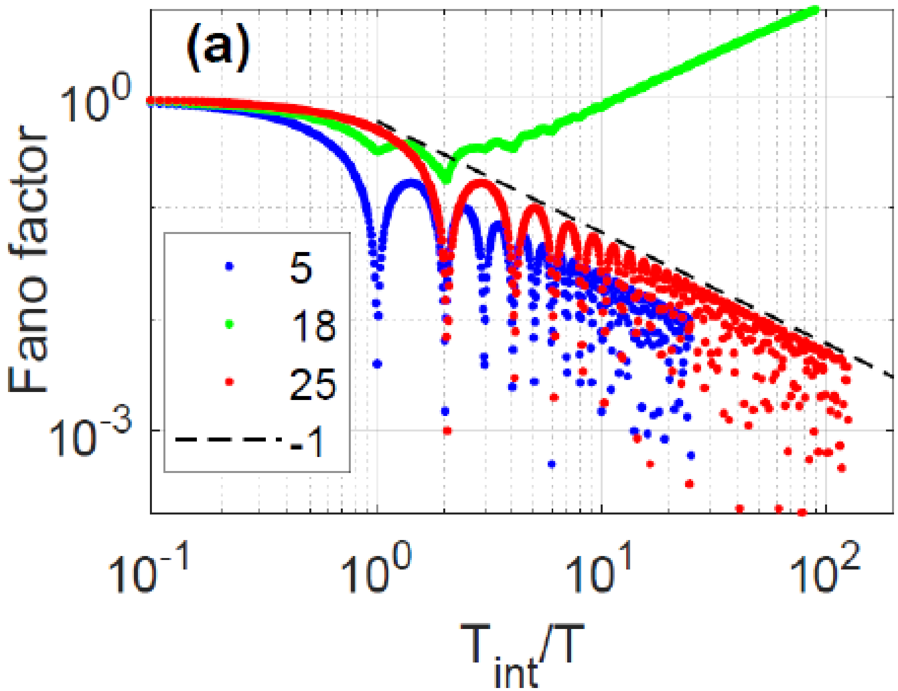
However, when the signal that modulates the laser current is sinusoidal:



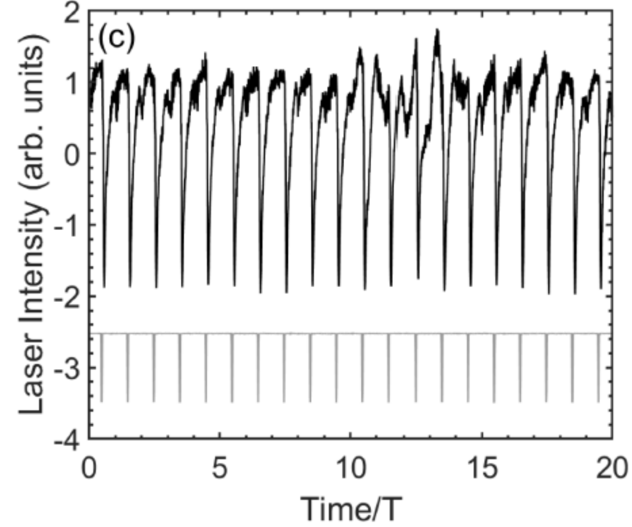
- No minimum at $T_{int}/T = 1$ (Reminder: T_{int} is the length of the "counting interval"; T is the period of the modulation).
- Minima are significantly less pronounced in the shuffled data

Looking in detail, for some modulation frequencies, FF *decreases* with the counting window size as a power law

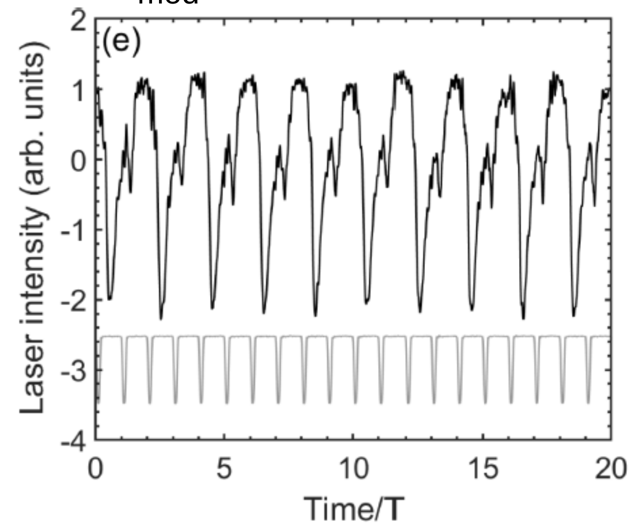
Pulsed modulation



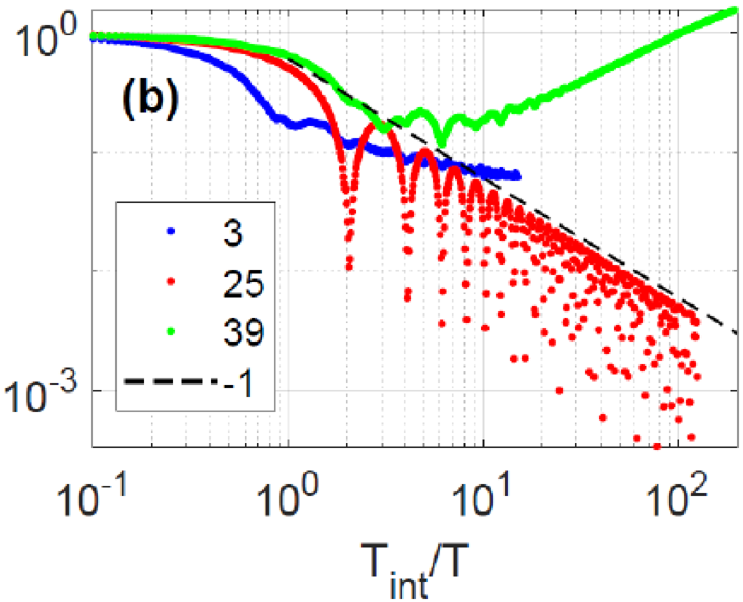
f_{mod}=5 MHz



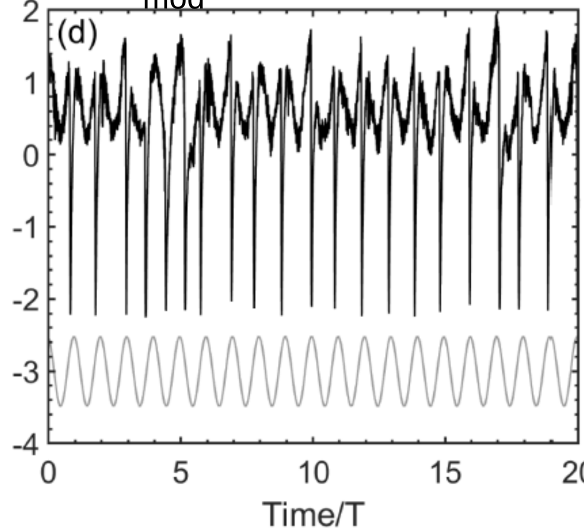
f_{mod}=25 MHz



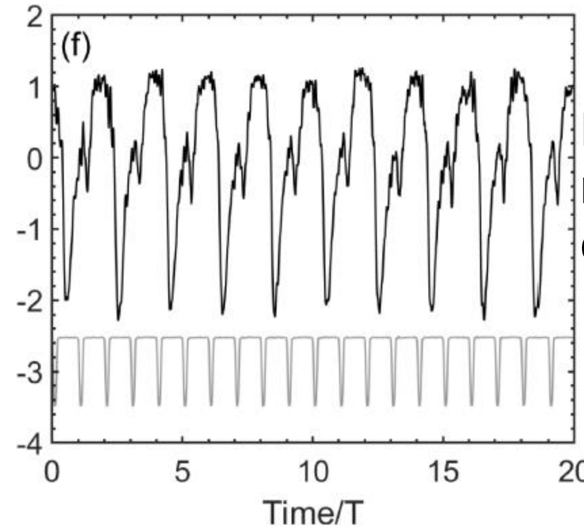
In the case of sinusoidal modulation: no 1:1 locking, but yes, it produces highly regular 2:1 locking



f_{mod}=5 MHz

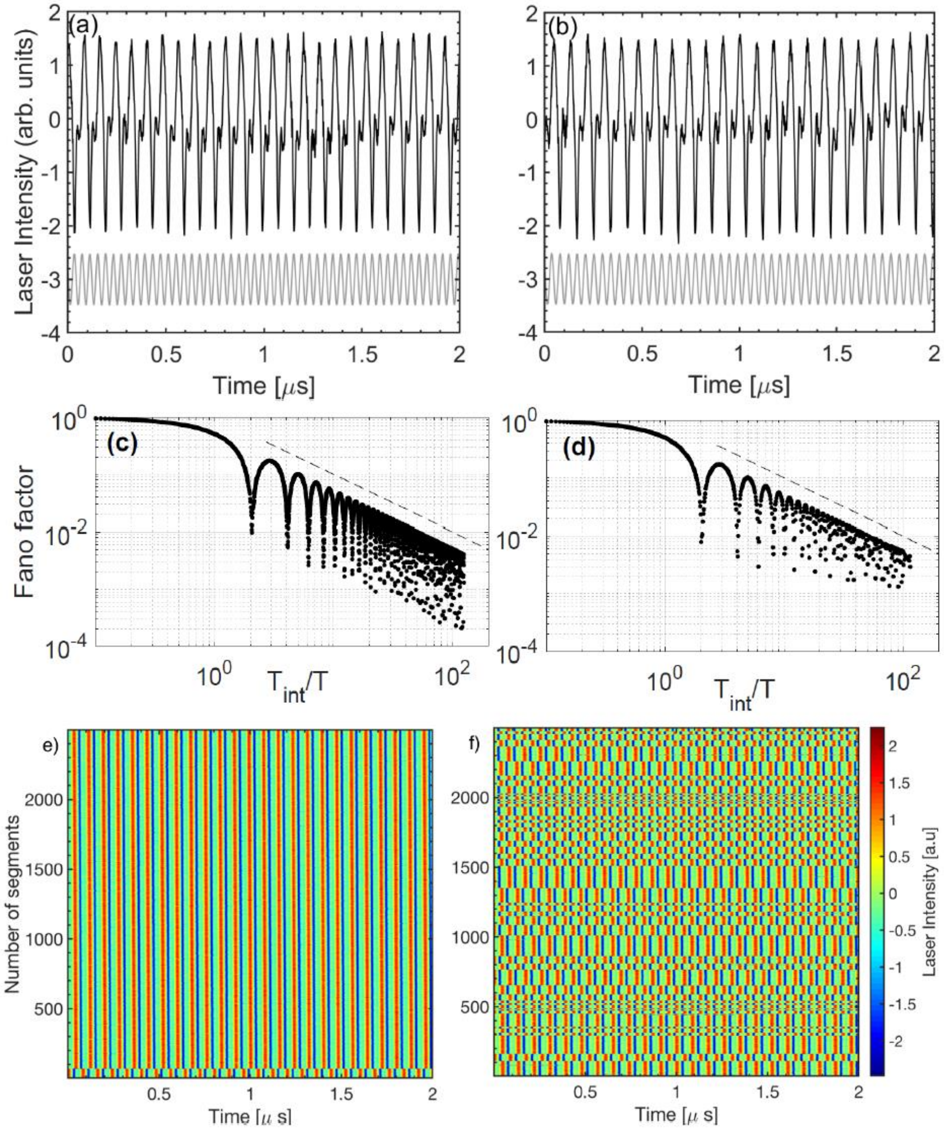


Noisy 1:1 entrainment



Highly regular 2:1 entrainment

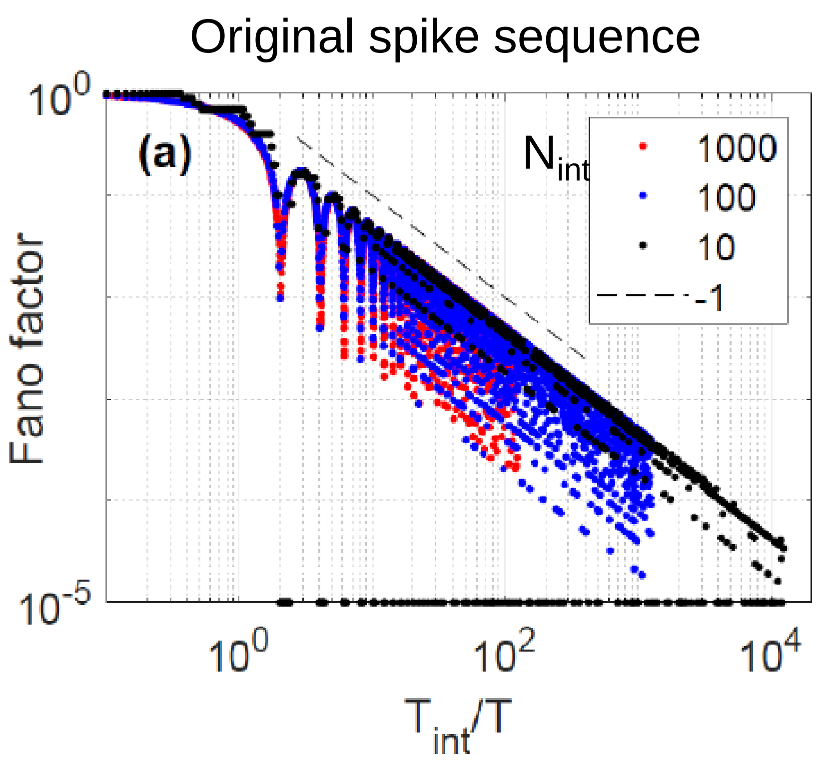
Comparison: highly regular locking vs normal locking

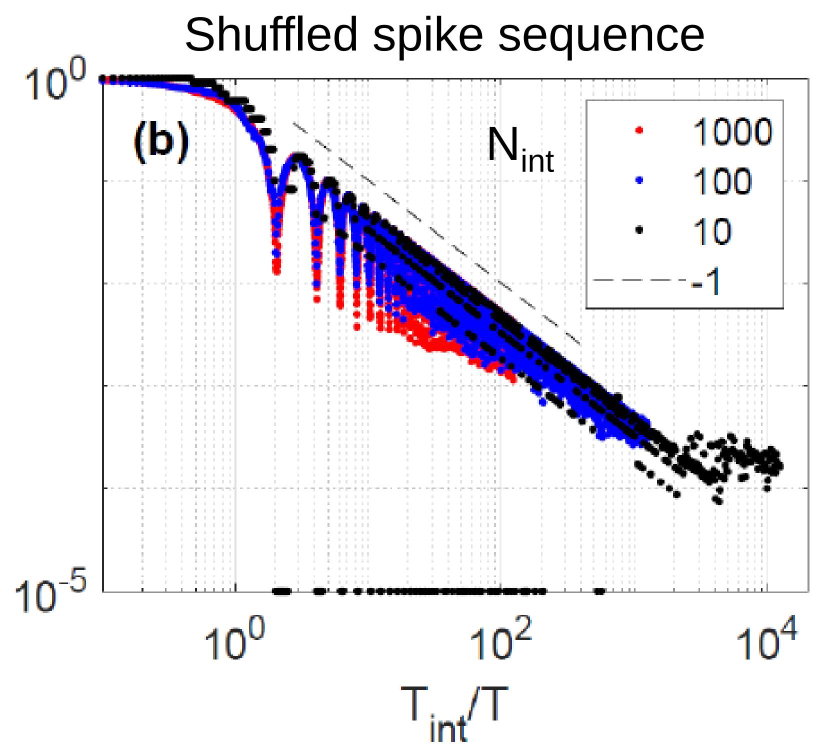


Differences are not appreciated when inspecting a short segment of the time series.

Differences are clearly seen in the behavior of the Fano Factor and in the space-time plots.

So far, I have analyzed 1000 non-superposing "counting windows", each of 5 μs. How about longer windows?





The Fano Factor continues decreasing as power law.
Very regular spikes.

The Fano Factor saturates, due to the fact that temporal correlations are removed when shuffling the spike times.

Conclusions and open questions

- Pulsed periodic current modulation generates locked spikes (1:1 and 2:1) with long-range regularity ("hyper-uniform" in the analogy between time-delay and spatially-extended systems).
- Sinusoidal modulation generates long-range regularity <u>with a period different from the period of the drive</u> (no 1:1 locking, so "time-crystal" behavior in the space-time analogy).
- The laser dynamics under sinusoidal modulation appears to be different from typical nonlinear oscillators (that show 1:1 locking).

Ongoing work: Which mechanisms are responsible for the long-ranged correlations? Why the sinusoidal signal does not generate 1:1 locking with long range regularity?

Thanks to former and present collaborators

- Andres Aragoneses
- Taciano Sorrentino
- Carlos Quintero-Quiroz
- Mattias Panozzo
- Carme Torrent
- Jordi Tiana-Alsina

Thank you for your attention!
Cristina.masoller@upc.edu

J. Tiana-Alsina et al., Opt. Express 26, 9298 (2018)
J. Tiana-Alsina et al., Phys. Rev. E 99, 022207 (2019)