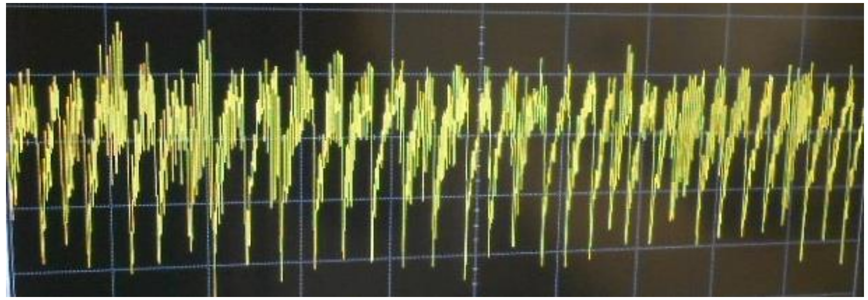

Time crystal like oscillations in a weakly modulated stochastic time delayed system

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slides: www.fisica.edu.uy/~cris/ndopp22.pdf

17th SICC INTERNATIONAL TUTORIAL WORKSHOP
Nonlinear Dynamics in Optics: Present and New Perspectives (NDOPP22), July 20, 2022

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Outline

- Motivation: hyper-regular and time-crystal states
- Analogy between time-delayed systems and spatially-extended systems
- Dynamics of diode lasers with time-delayed feedback
  - Without modulation: irregular optical spikes
  - With weak periodic modulation of the laser current
- Quantification of the temporal regularity of the timing of the spikes using the Fano Factor
- Conclusions and open questions

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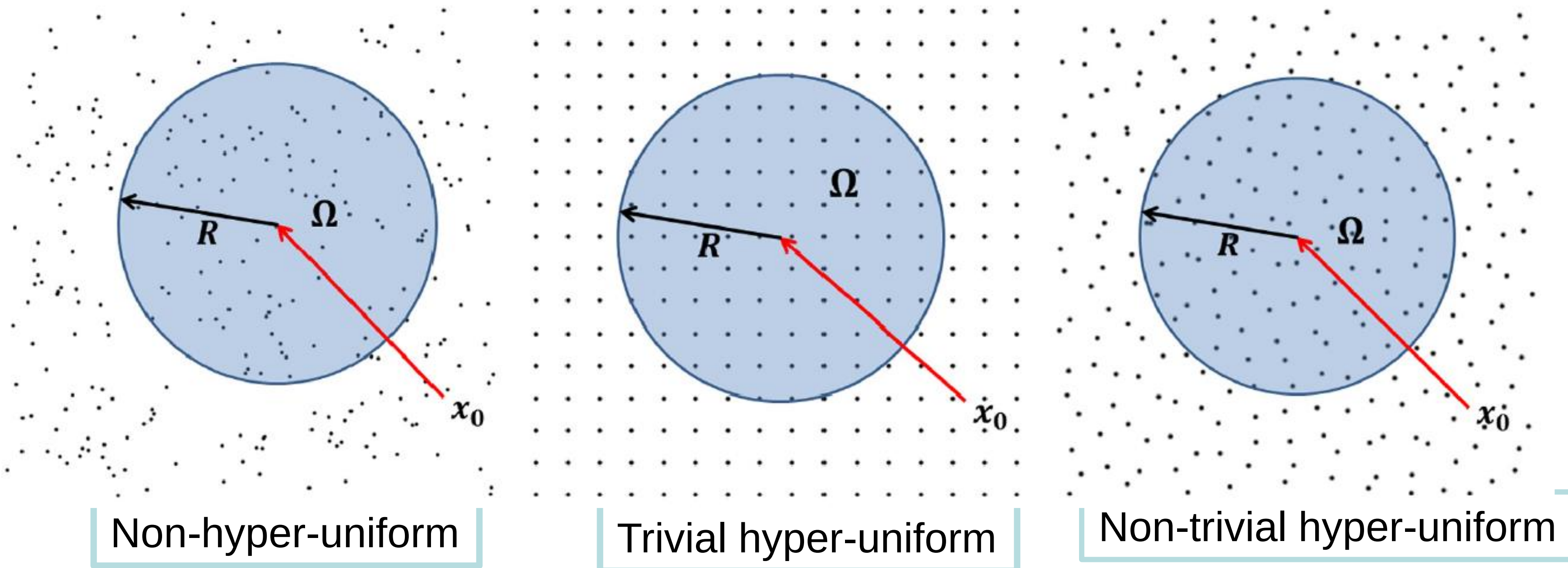
Not all "disordered systems" are equally disordered



How does the number of particles inside the circle varies with the radius of the circle?

$$\sigma_N^2(R) \sim R^d$$

But some disordered systems show a slower grow of density fluctuations (asymptotic scaling between surface and volume growth).

S. Torquato / Physics Reports 745 (2018) 1–95

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Time-crystal and hyper-uniform states: peculiar states of some disordered systems

- **Hyper-uniform states** exhibit an anomalously long-ranged suppression of density fluctuations. Many observations.
- **Time-crystal states** in ***periodically driven*** systems exhibit:
  - **highly regular oscillations** (in space and in time) that persist over long time intervals,
  - these oscillations are robust under small variations of the initial conditions or parameters ("**rigidity**"),
  - **break time-translation symmetry** because the period of the oscillations differs from the period of the driving signal: subharmonic locking but **no harmonic locking**.
  - Observed in many-particle quantum systems.

S. Torquato, Phys. Rep. 745, 1 (2018).
F. Wilczek, Phys. Rev. Lett. 109, 160401 (2012).
N. Y. Yao, and C. Nayak, Physics Today 71, 9, 40 (2018).

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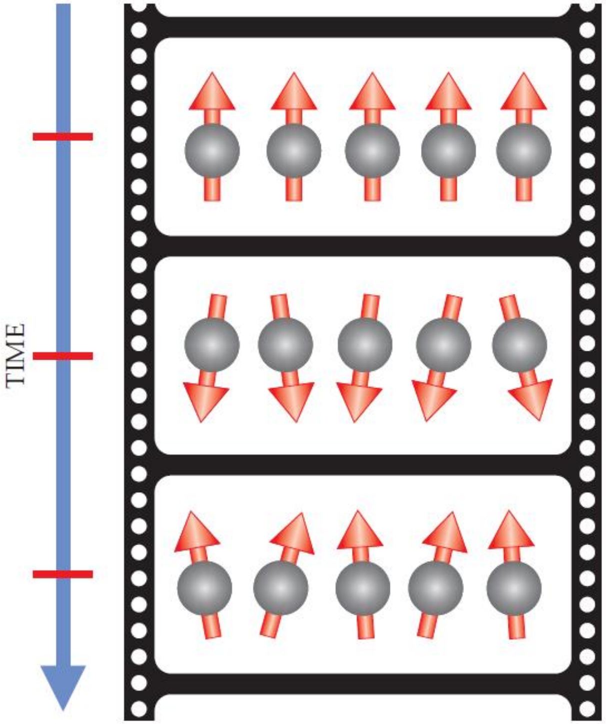

- The spin orientation flips during each driving period, so it takes two periods for the spins to return to something resembling their initial state.
- But to someone viewing the system at fixed intervals (that is, stroboscopically), the system appears to be in equilibrium.

Time crystals' defining traits

	Time crystal	Period-doubled nonlinear dynamical system	Mode-locked laser	Parametric down-conversion	NMR spin echo	Belousov–Zhabotinsky reaction	Convection cells	AC Josephson effect
Many-body interactions	✓	X	✓	✓	X	X	✓	✓
Long-range order	✓	X	X	X	X	X	X	X
Crypto-equilibrium	✓	X	X	X	X	X	X	X

N. Y. Yao and C. Nayak, Physics Today Sep. 2018

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Can we find time crystal behavior in classical high-dimensional dynamical systems?

Time delayed systems represented by

$$du(t)/dt = f(u(t), t) + K\, u(t-\tau)$$

are infinite dimensional because the initial condition is the function u(t) **defined in [-τ,0].**

The dynamics of some TDSs has similarities to the dynamics of some one-dimensional **spatially extended systems**

$$\partial u(x,t)/\partial t = f(u,x,t) + D\, \partial^2 u/\partial x^2 \quad \text{with } x(t) \text{ in } [0, L]$$

C. Quintero-Quiroz, M. C. Torrent, and C. Masoller, Chaos 28, 075504 (2018)

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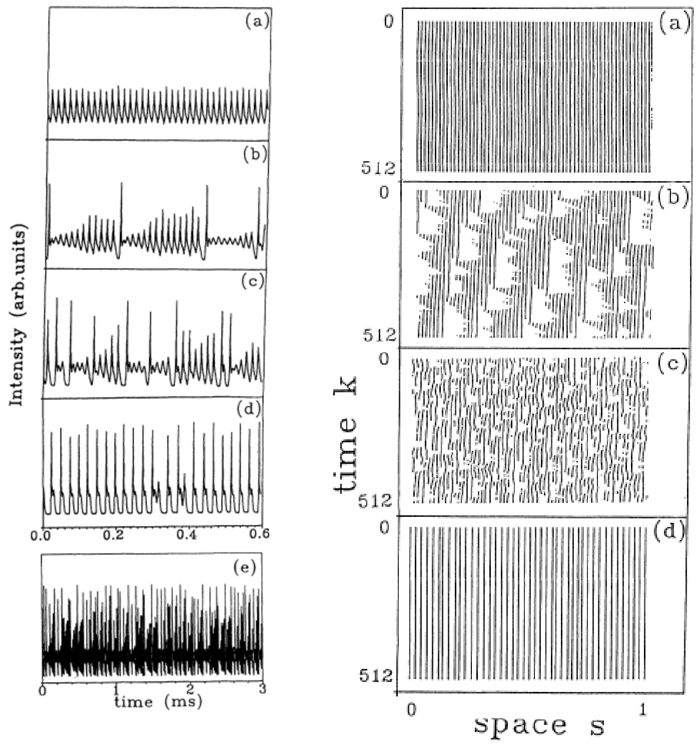
Similarities between time-delayed and spatially-extended systems (patterns, wave propagation) are unveiled with 2D representation

RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 45, NUMBER 7 1 APRIL 1992

Two-dimensional representation of a delayed dynamical system

F. T. Arecchi,* G. Giacomelli, A. Lapucci, and R. Meucci
Istituto Nazionale di Ottica, Largo E. Fermi 6, 50125 Firenze, Italy
(Received 31 July 1991; revised manuscript received 10 December 1991)



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Main question

Can we find a **stochastic time delayed system** that has peculiar states that are analogous to hyper-uniform or time-crystal states?

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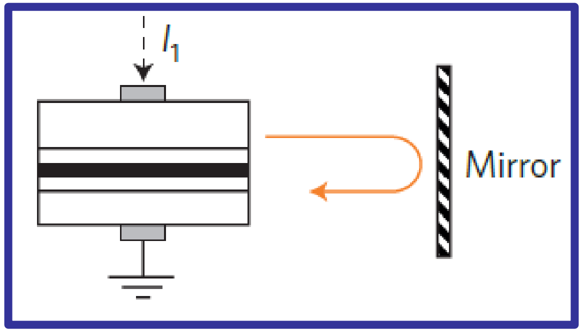
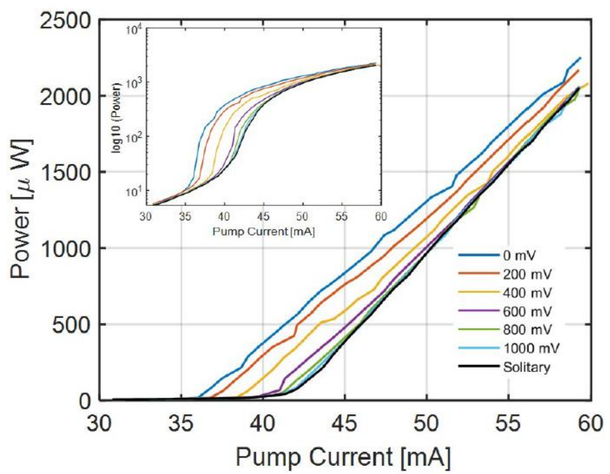
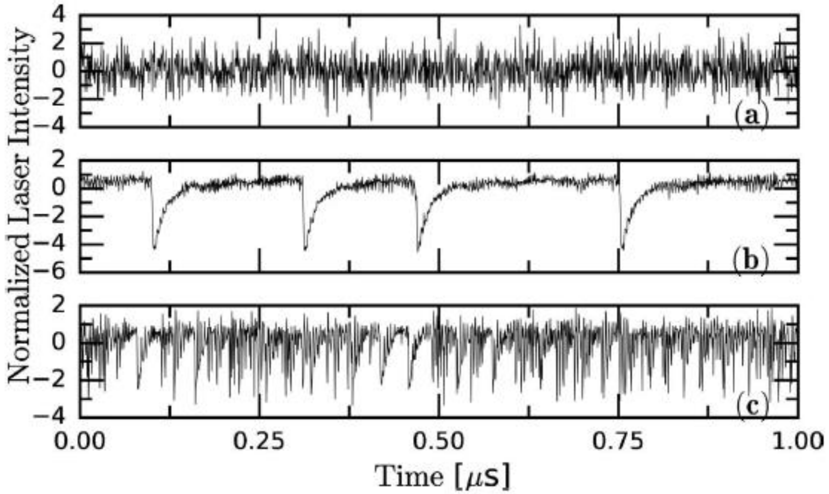
Semiconductor laser with optical feedback: a well-known stochastic time delayed system

- Time-delay due to propagation time (ns)
- Near threshold: stochastic dynamics (quantum spontaneous emission).







Pump current ↓

Figure courtesy of M. Duque, UPC.

M. Sciamanna, K. A. Shore, *Physics and applications of laser diode chaos,* Nat. Phot. 9, 151 (2015).

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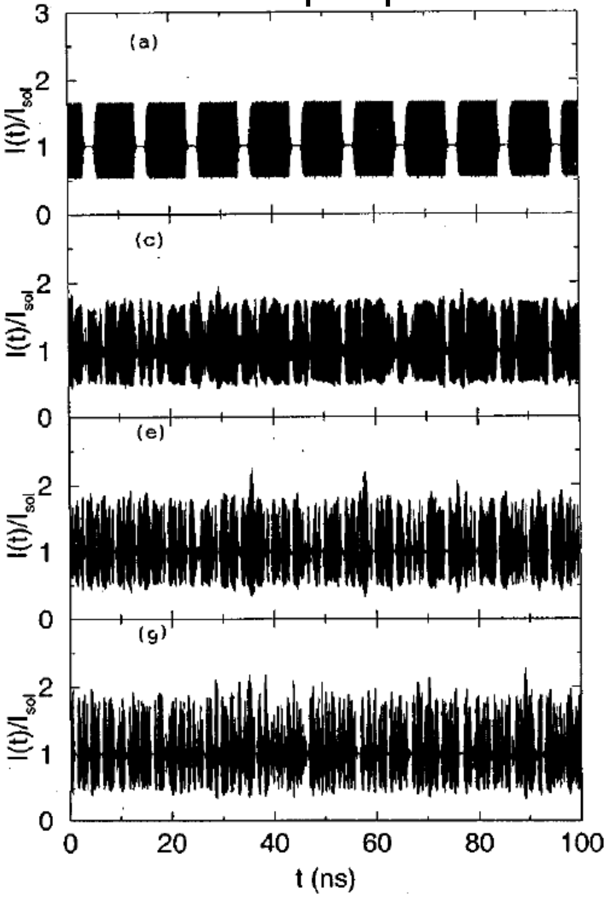
Model simulations of feedback-induced complex dynamics

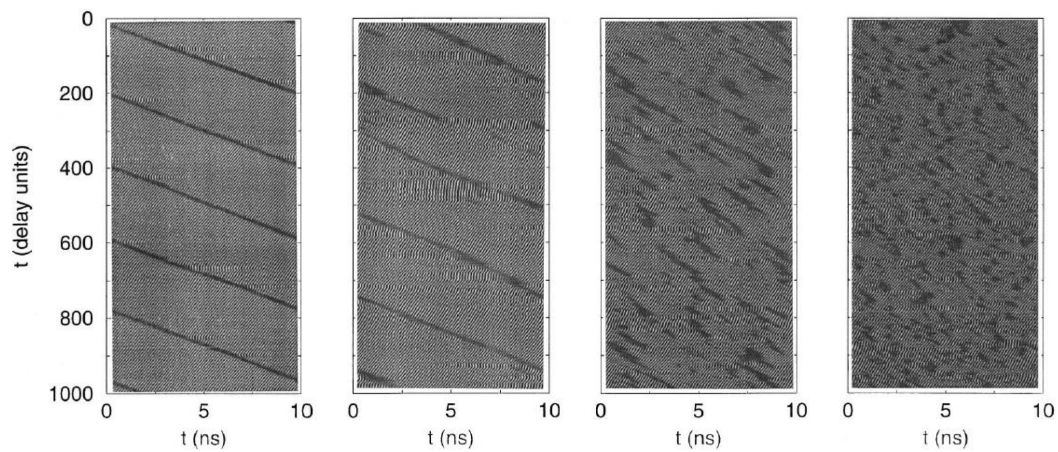
Feedback strength ↓

Time-series of the emitted output power



C. Masoller, Chaos 7, 455 (1997)

Spatio-temporal representation
Feedback strength →



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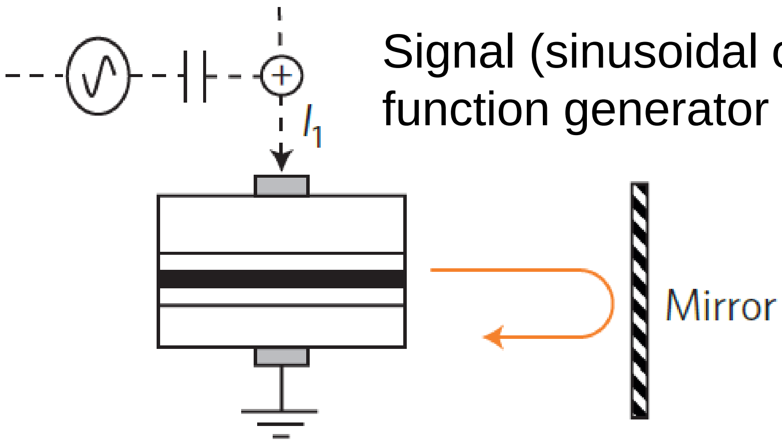
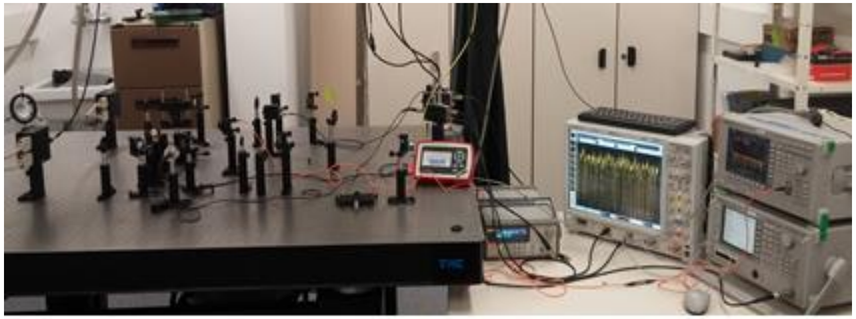
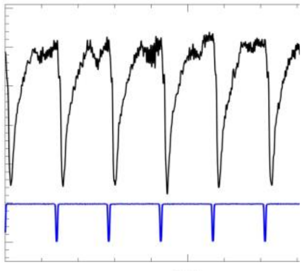
Semiconductor laser with feedback and current modulation

Main advantage: experiments can be done with precise control of three modulation parameters (the dc value of the laser current, modulation amplitude and frequency) keeping constant the feedback parameters (feedback strength and delay).



Signal (sinusoidal or pulsed) from a wave function generator is applied to the laser current.





Time-series of the emitted output power

Time-series of the input electric current

Note: **regular** spike timing, but **irregular** fluctuations in-between spikes

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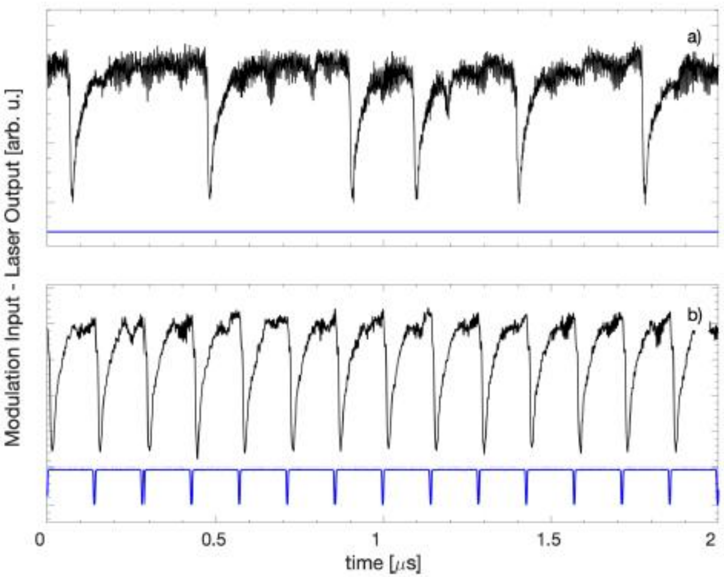
We focus on the region near threshold, where the laser emits "optical spikes"



Questions:

- Can we **lock the spikes** to a **weak** periodic signal?
- Which waveform is best?
- How regular can the spikes be?
- How can we quantify the (long-range or short-range) regularity of the spike timing?

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Early experiments with sinusoidal modulation

I_{dc} = 39 mA
f_{mod} = 17 MHz

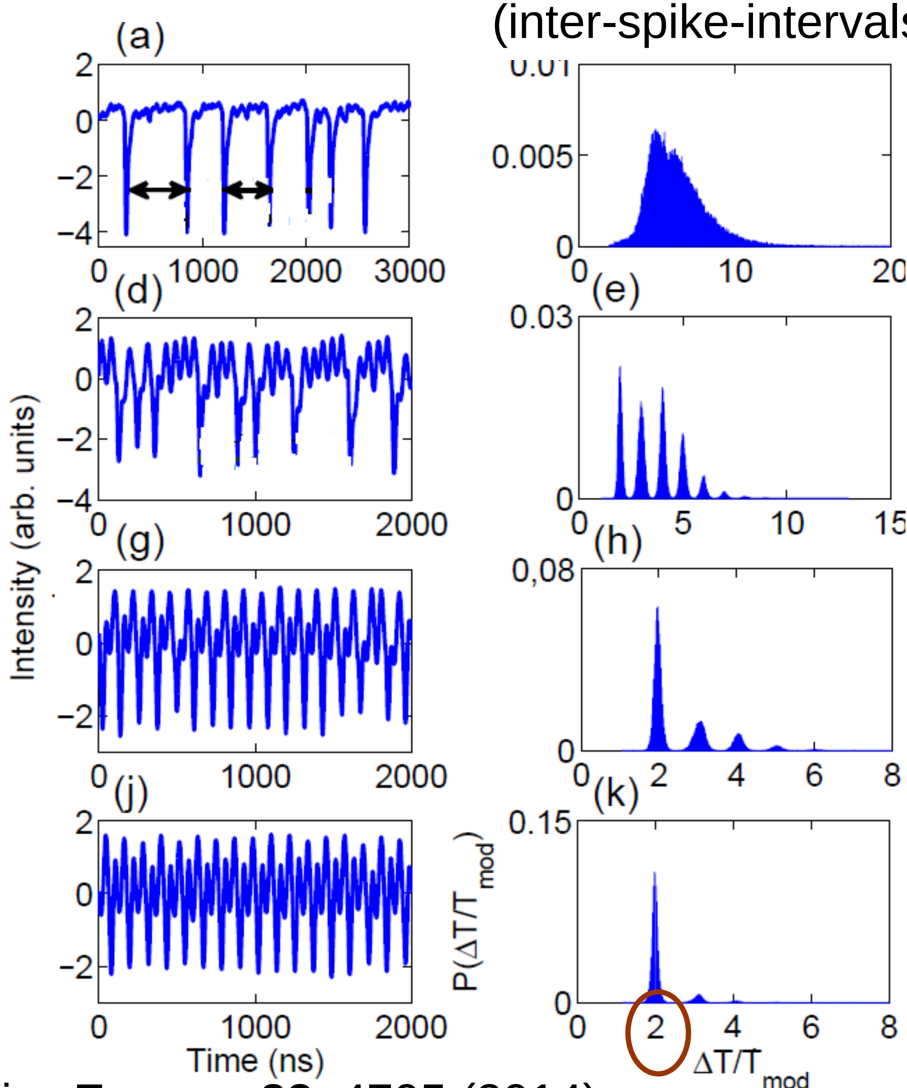
Distribution of intervals between spikes (inter-spike-intervals)

No modulation

1.2%

1.6%

2%



A. Aragoneses et al., Optics Express **22**, 4705 (2014).

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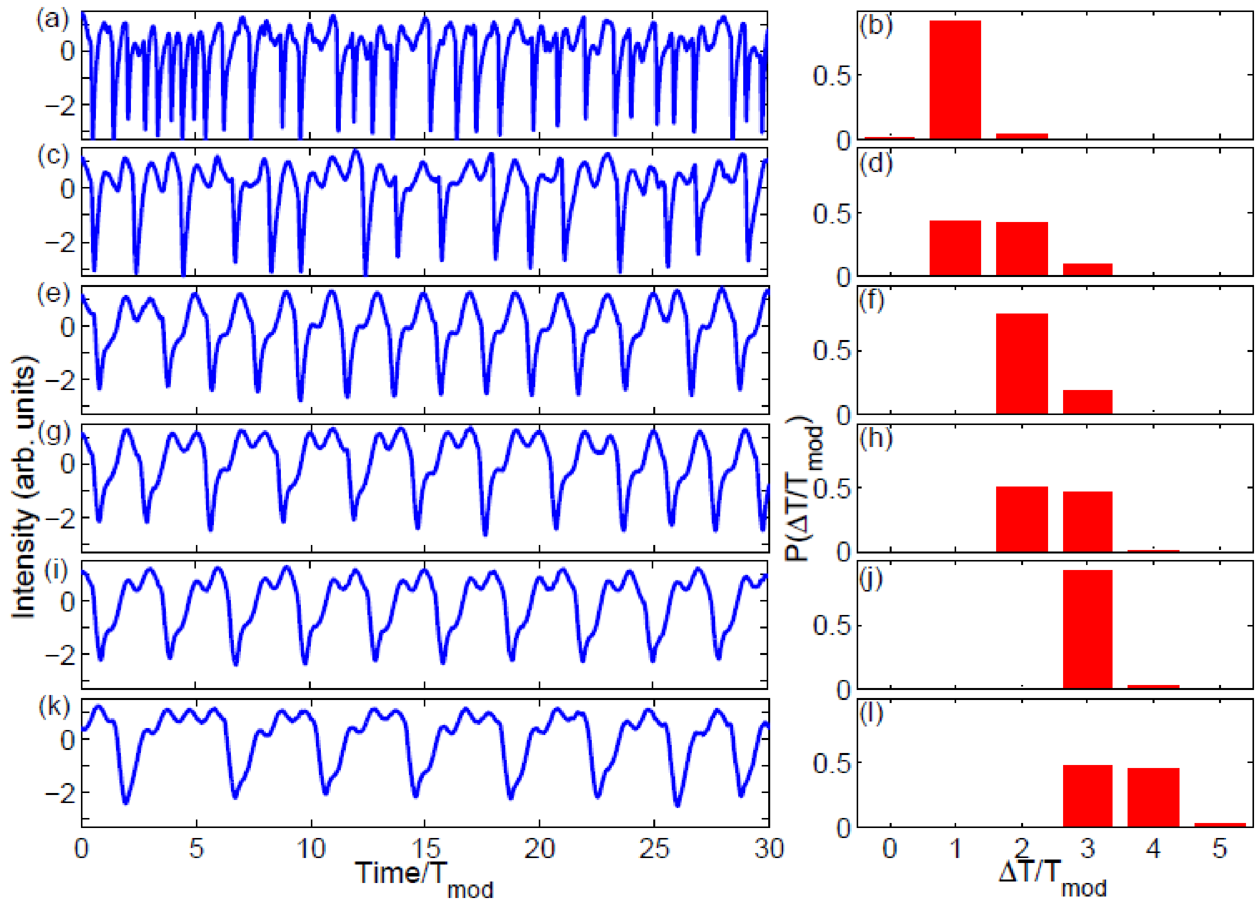
Sinusoidal modulation: varying the modulation frequency while keeping constant the modulation amplitude (1.2 % I_{DC})

7 MHz
14 MHz
26 MHz
31 MHz
39 MHz
49 MHz



T. Sorrentino et al., Optics Express **23**, 5571 (2015).

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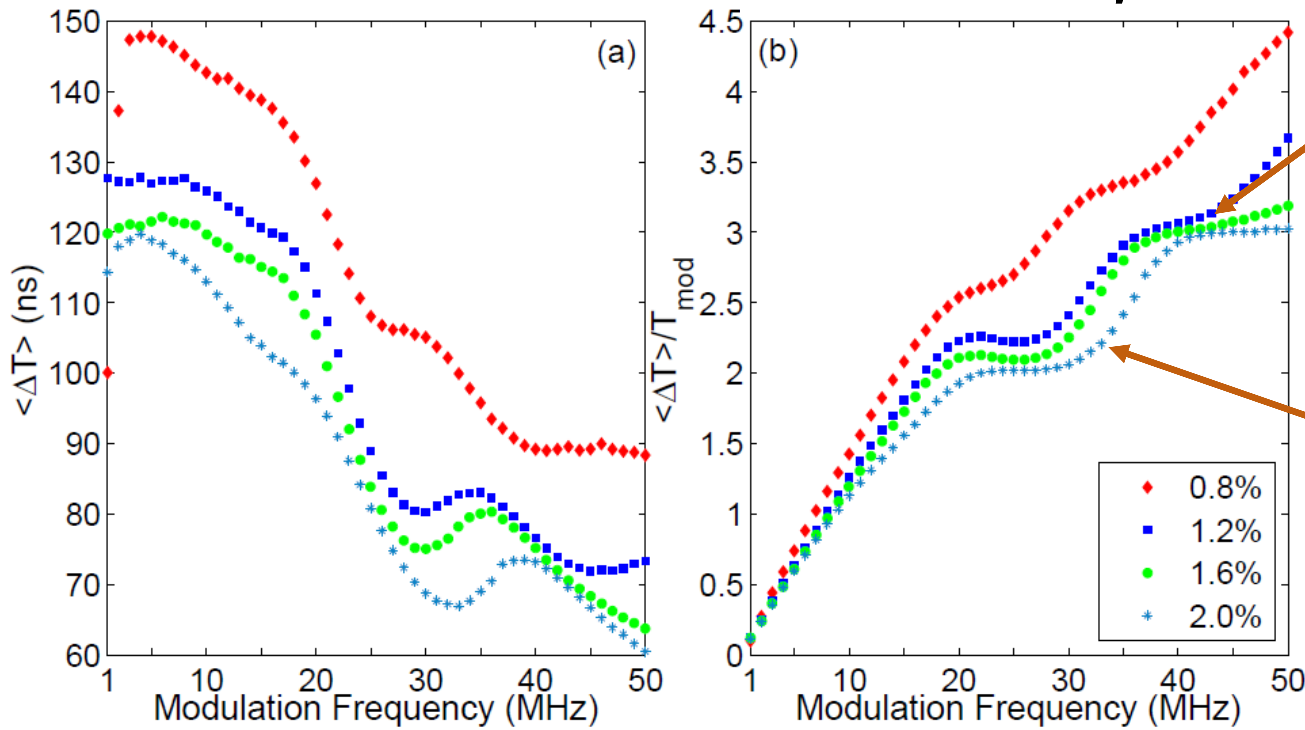
Locking "plateaus"

Average time interval between consecutive spikes.

Average time interval between consecutive spikes, *normalized to the modulation period*.



1 spike every 3 modulation cycles

1 spike every 2 modulation cycles

Why no 1:1 locking plateau?

T. Sorrentino et al., Optics Express **23**, 5571 (2015).

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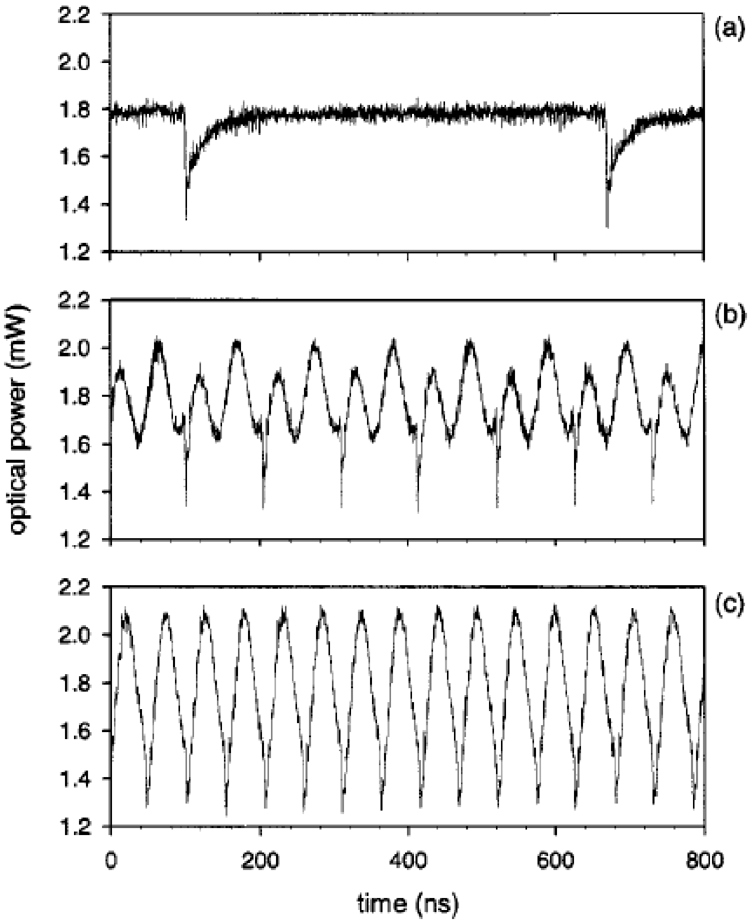
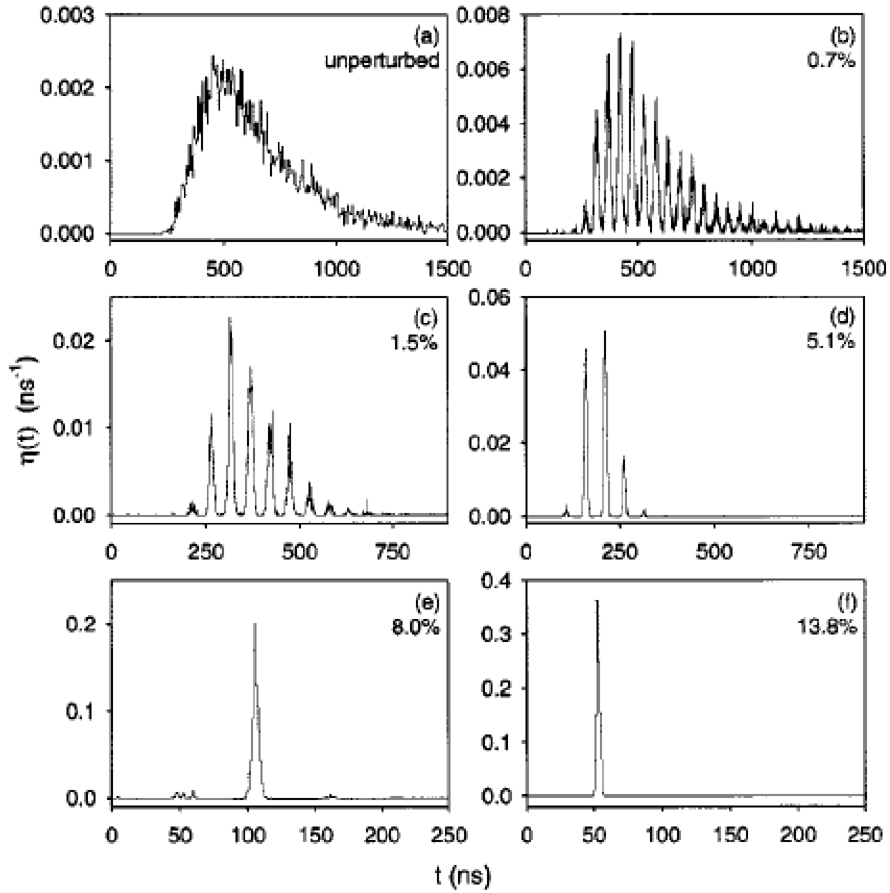
Earlier work





1:1 locking found with large modulation amplitude (~14%)

D. W. Sukow and D. J. Gauthier, IEEE J. Quantum Electron 36 (2000).

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How regular can the *timing* of the spikes be?

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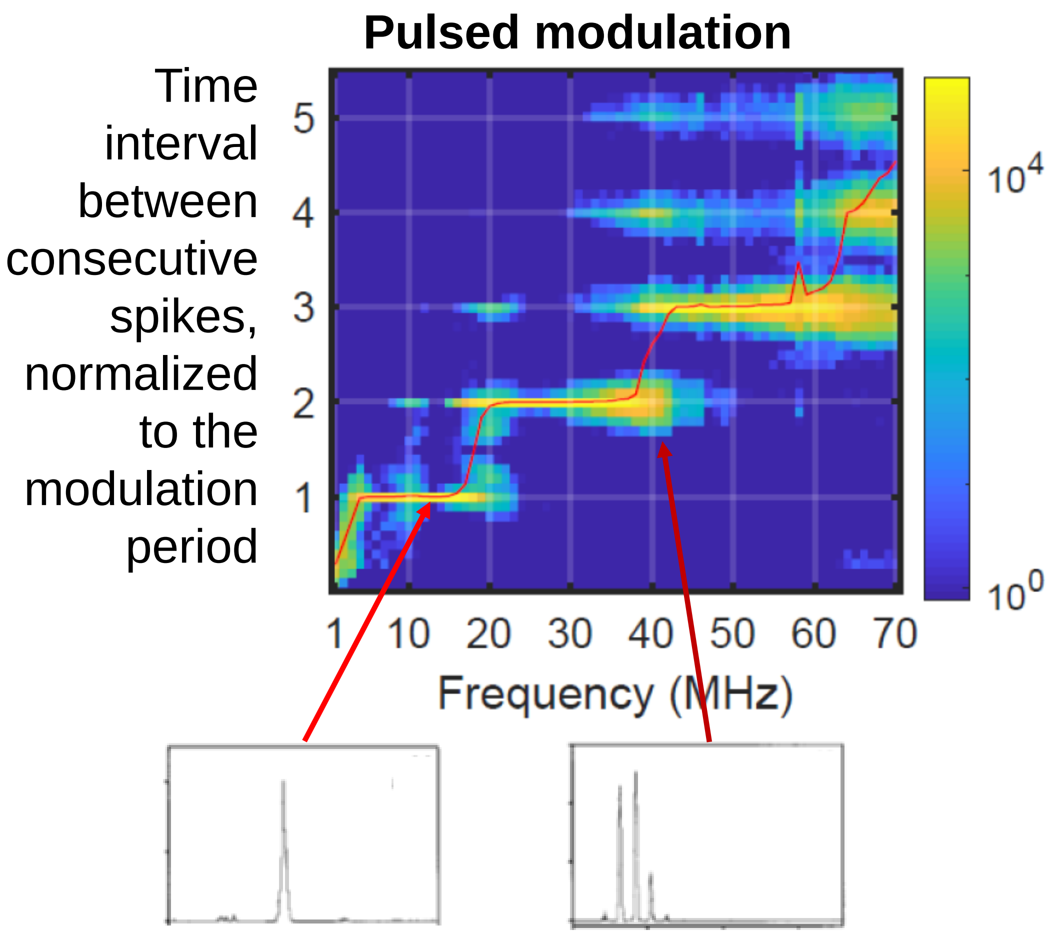
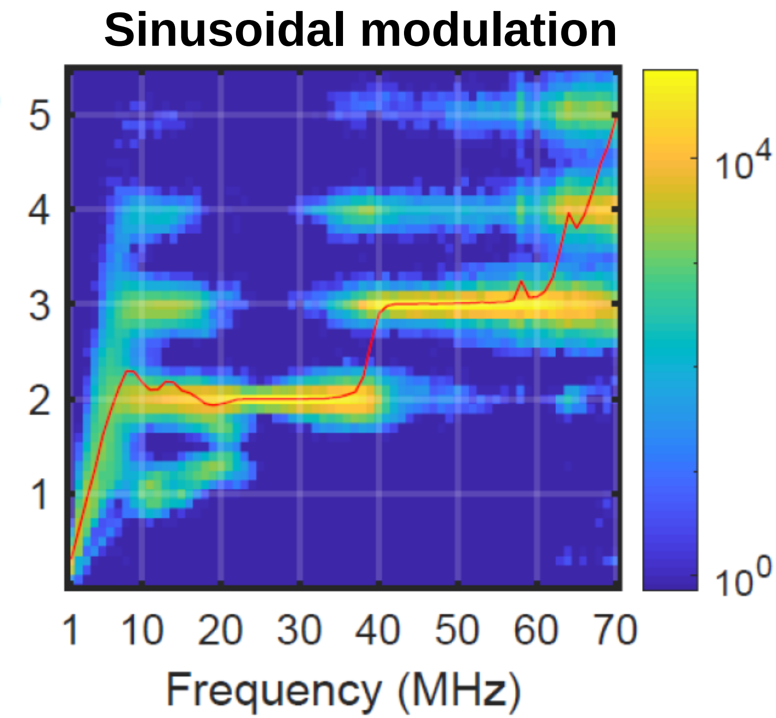
Distribution of inter-spike-intervals (log color code) for different modulation frequencies (I_{dc}=26 mA, mod. amplitude=0.631 mA ≈ **2.4%**)





With weak sinusoidal modulation: no 1:1 locking plateau (consistent with earlier experiments).

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The Fano Factor: a precise measure of spike timing regularity

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How to calculate the Fano Factor?

- Divide the intensity time trace in N_{int} non-overlapping segments of duration T_{int}.
- <u>Count</u> the number of spikes in each segment, $\{N_1, N_2, \ldots N_{Nint}\}$.
- Calculate the mean and the variance, $\langle N_i \rangle$, σ^2
- Calculate the Fano factor as

$$F = \sigma^2(N_i) / \langle N_i \rangle$$

- *F* depends on the duration of the counting interval, T_{int}.
- If T_{int} is very small, *F=1* because the sequence of counts is a sequence of 0s and 1s.
- If the process that triggers the spikes is fully random, $F=1 \; \forall T_{int}$.
- To test the presence of correlations in the timing of the spikes:
  - Shuffle the inter-spike intervals
  - Recalculate the spike times
  - Recalculate *F*
  - Compare the *F* values of the original and shuffled spike times.

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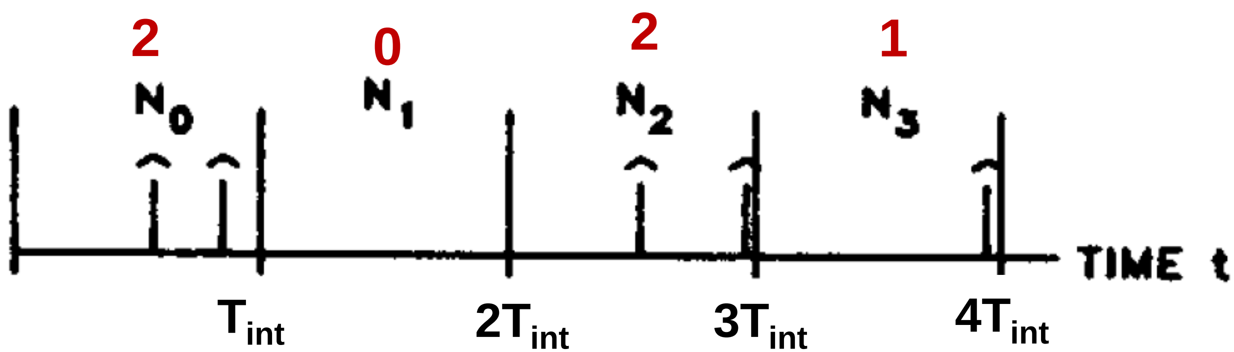
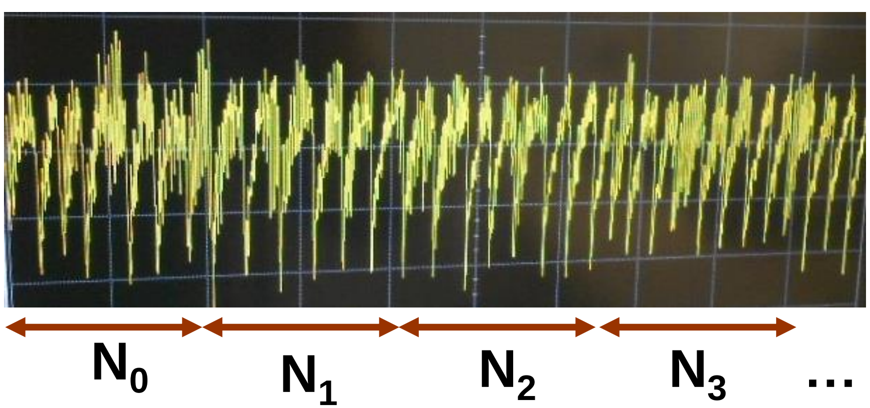
The Fano Factor has been widely used to analyze the timing of neural spike trains

$N_{int} = 4$



Sequence of counts: $\{N_i\} = \{2, 0, 2, 1\}$

$$F = \sigma^2(N_i) / \langle N_i \rangle$$



M. C. Teich et al, J. Opt. Soc. Am. A 14, 529 (1997)

In our experiments the total recorded time, **5 ms**, contains 9000-120000 spikes, depending on the parameters.

Number of intervals: N_{int} = **1000**,

Duration of each interval: T_{int} = **5 μs**

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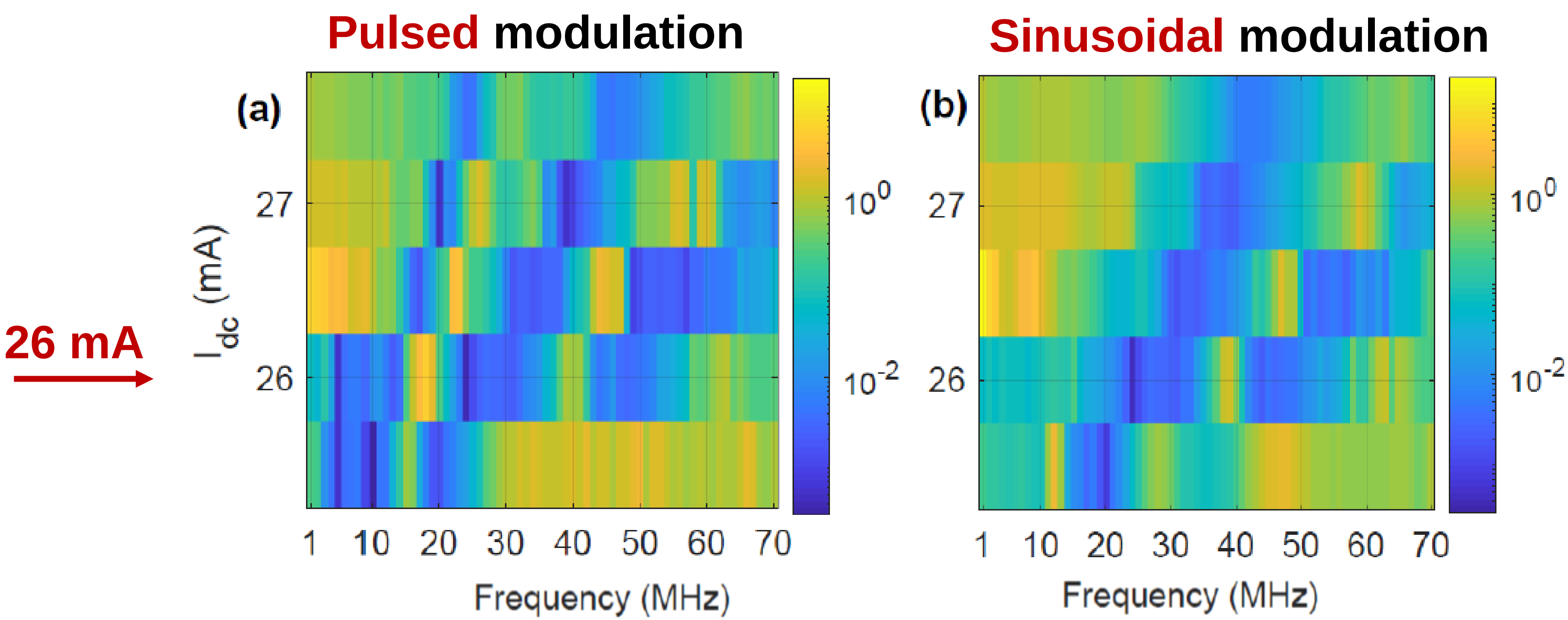
Fano Factor (color code) of sequences of optical spikes

recorded for different I_{dc} and f_{mod}, keeping fixed A_{mod} (≈1-2.5 % of I_{dc})



- **Blue** regions: small $F \Rightarrow$ small $\sigma \Rightarrow$ regular sequence of counts $\Rightarrow$ regular spikes.
- Yellow regions: large $F \Rightarrow$ large $\sigma \Rightarrow$ high variability in the sequence of counts.
- For the pulsed signal there are three blue regions; for the sinusoidal signal, only two.

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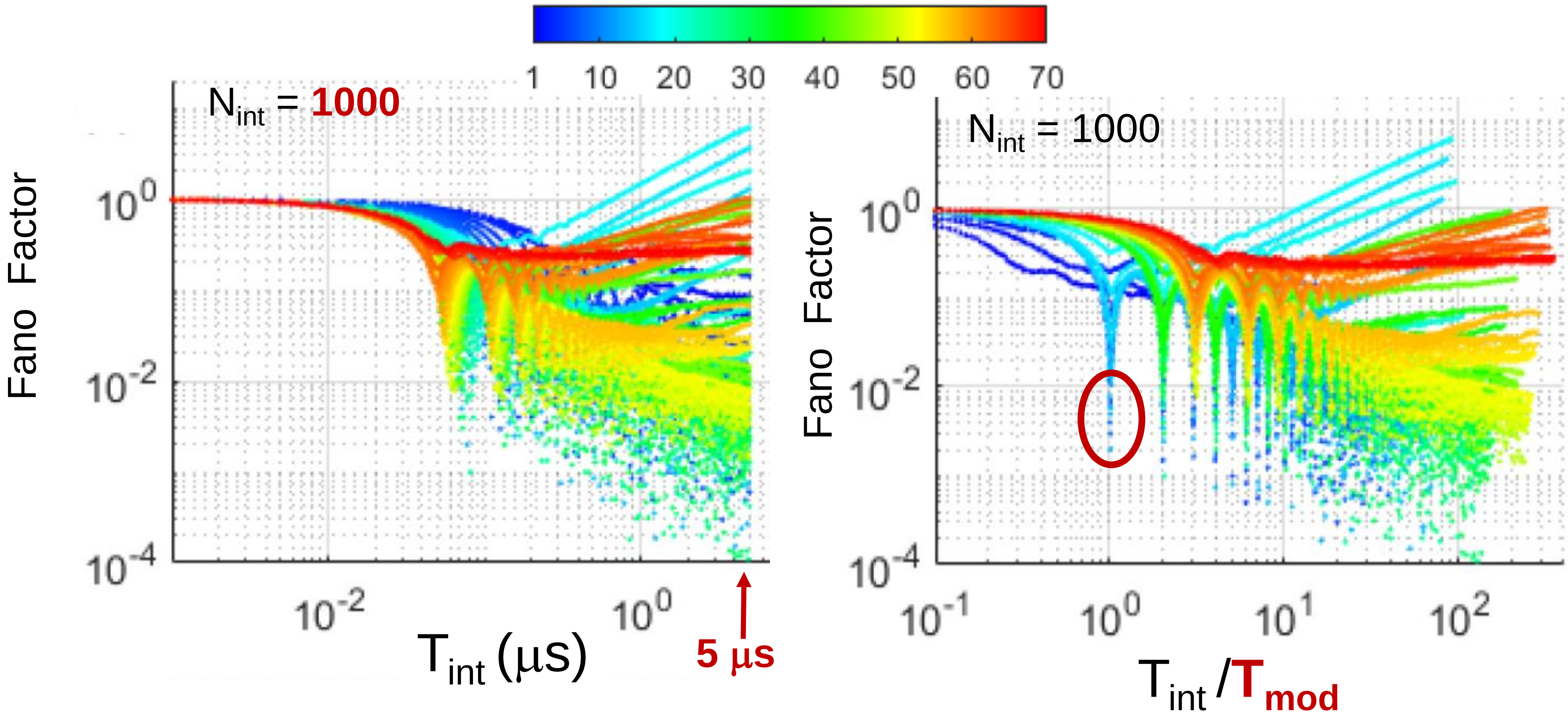
How the Fano Factor depends on the duration of the counting interval?

Here: **pulsed** modulation, I_{dc}=26 mA, the color represents the mod. frequency (MHz).



- Sharp minima reveal that the sequence of counts is very regular when the counting interval contains an integer number of modulation periods.
- $T_{int} = T_{mod}$: the sequence of counts is (1,1,…,1), i.e., 1000 intervals with one spike in each interval.

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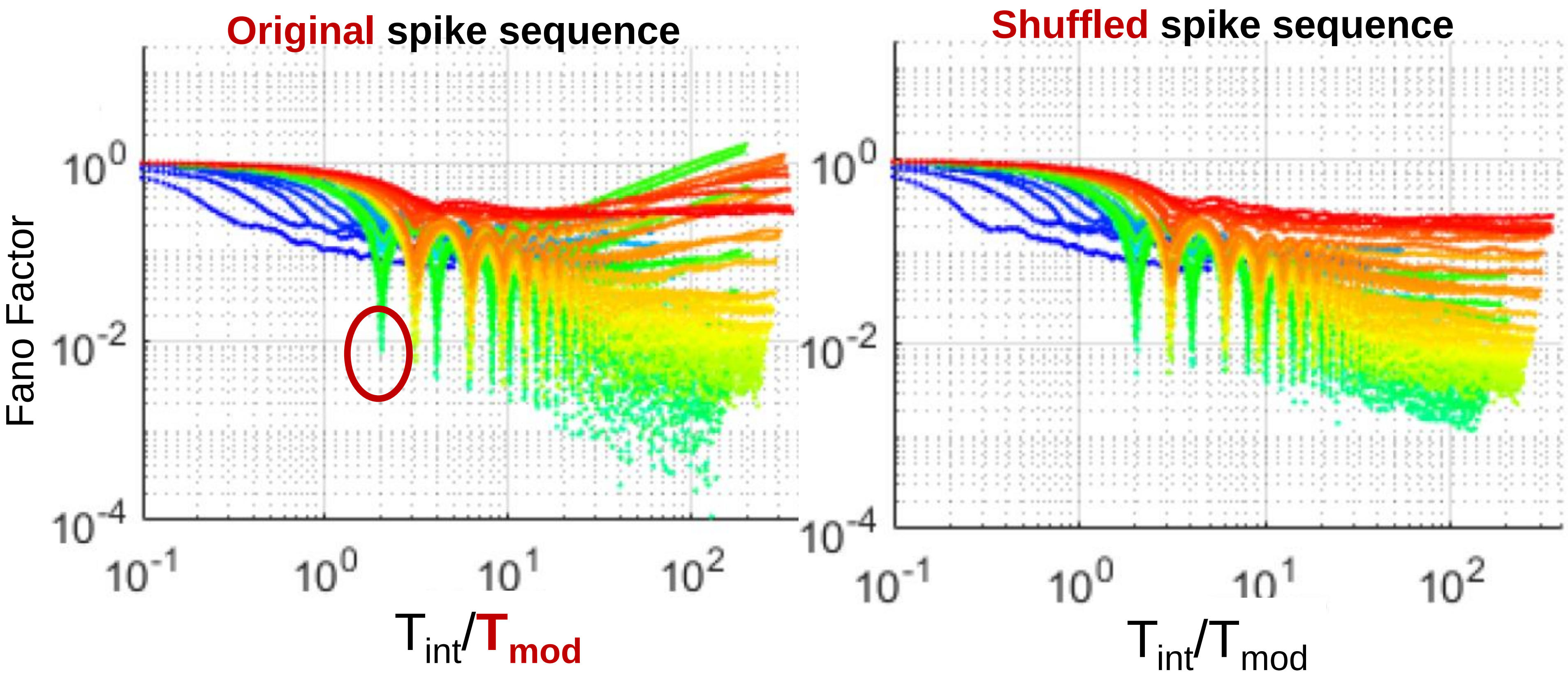
With sinusoidal modulation:



- First minima at T_{int}=2 T_{mod}
- Minima are more pronounced in the original spike sequence than in the shuffled one (temporal correlations are removed when shuffling the spikes).

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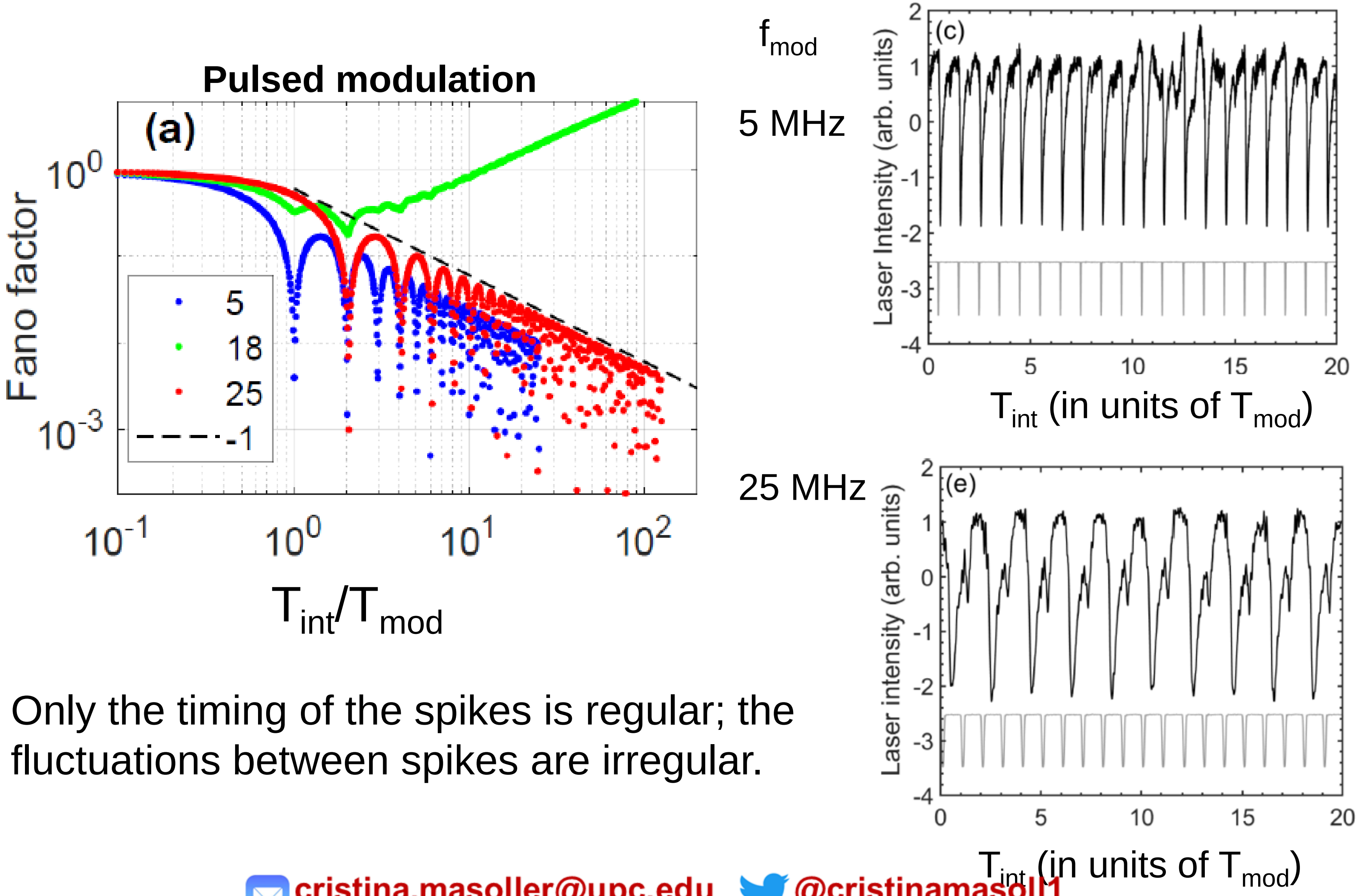
For some modulation frequencies: power law variation of the Fano factor with the size of the counting window



Only the timing of the spikes is regular; the fluctuations between spikes are irregular.

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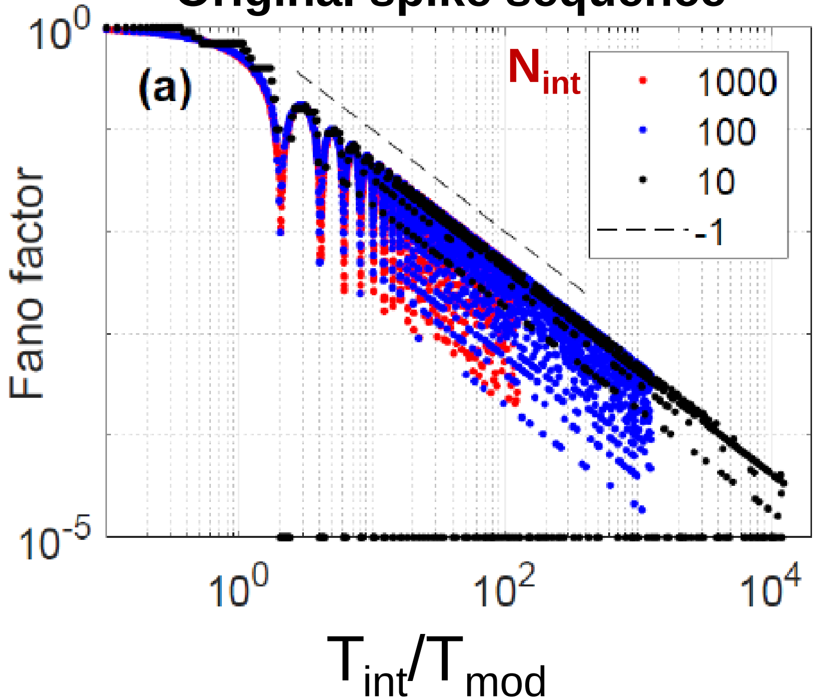
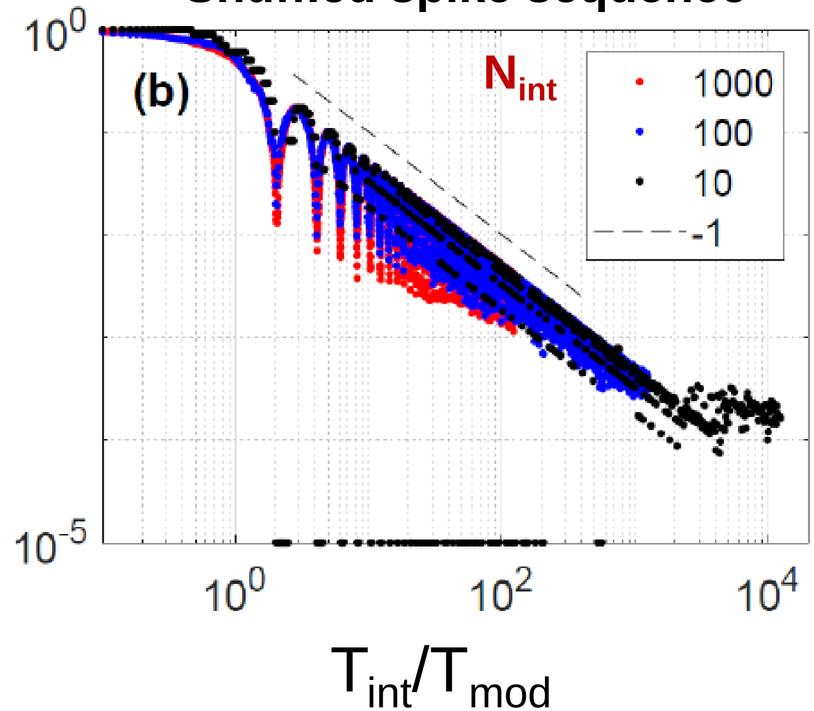
Power law persists for longer counting windows?

Original spike sequence



Shuffled spike sequence



Spike timing is regular over time intervals that contain 10^4 cycles of the modulation.

Power law variation saturates when the sequence of inter-spike intervals is shuffled.

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Conclusions and open questions

- Pulsed modulation generates locked spikes (1:1 and 2:1) with long-range regularity (analogous to “hyper-uniform” states).
- Sinusoidal modulation generates sub-harmonically locked spikes with long-range regularity (no 1:1 locking, analogous to “time-crystal” states).
- This is in contrast with classical nonlinear oscillators that show both, 1:1 and higher order lockings.
- Which mechanisms induce long-range order?
- Why the sinusoidal signal does not produce 1:1 locking?
- Model simulations are in good agreement with the observations

  [J. Tiana-Alsina and C. Masoller, Appl. Sci. 11, 7871 (2021)]
- Generic phenomena that may be observed in other periodically modulated stochastic time delayed systems?
- Influence of the feedback parameters (strength and the delay)?

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Thank you for your attention!

slides: www.fisica.edu.uy/~cris/ndopp22.pdf

J. Tiana-Alsina, C. Masoller, "*Time crystal dynamics in a weakly modulated stochastic time delayed system*", Scientific Reports 12, 4914 (2022)

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