Identifying and characterizing complex dynamical regimes with nonlinear time series analysis tools

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- Analysis tools (Ordinal & Hilbert)
- Applications:
  - Lasers
  - Brain
  - Climate

  (light brain storming)

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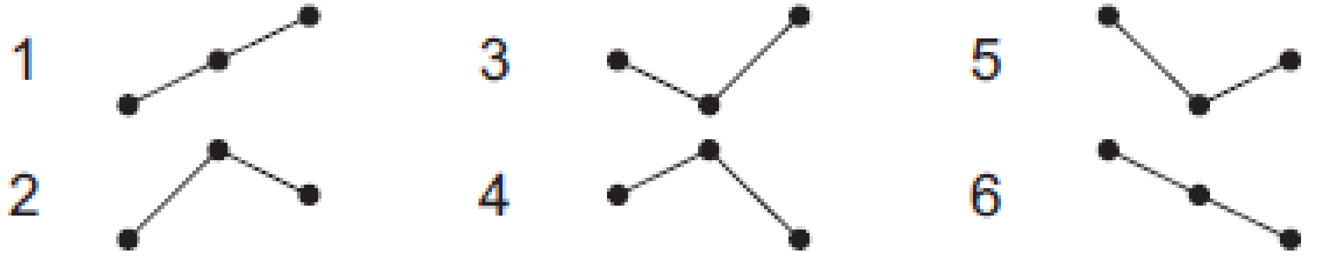
How to extract information from complex signals?

First analysis tools: ordinal symbolic analysis

It allows to identify patterns in data

Ordinal analysis: a tool to look for patterns in data

- Consider a time series $x(t)=\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$
- Which are the possible order relations among three data points?

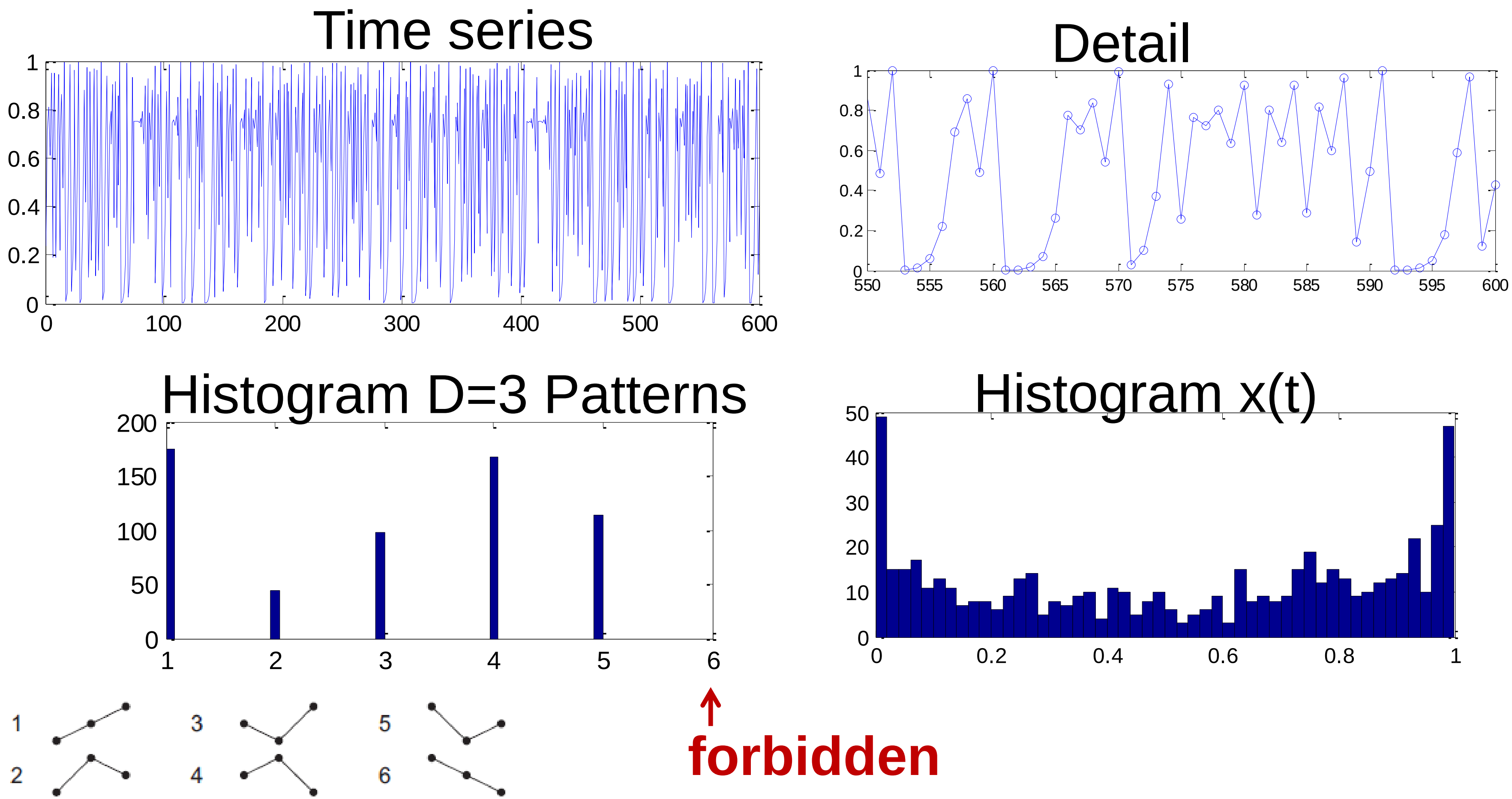


- Count how many times each "ordinal pattern" appears.
- Advantages: allows to identify temporal structures & is robust to noise.
- Drawback: information about actual data values is lost.

Bandt and Pompe PRL (2002)

Example: Logistic map

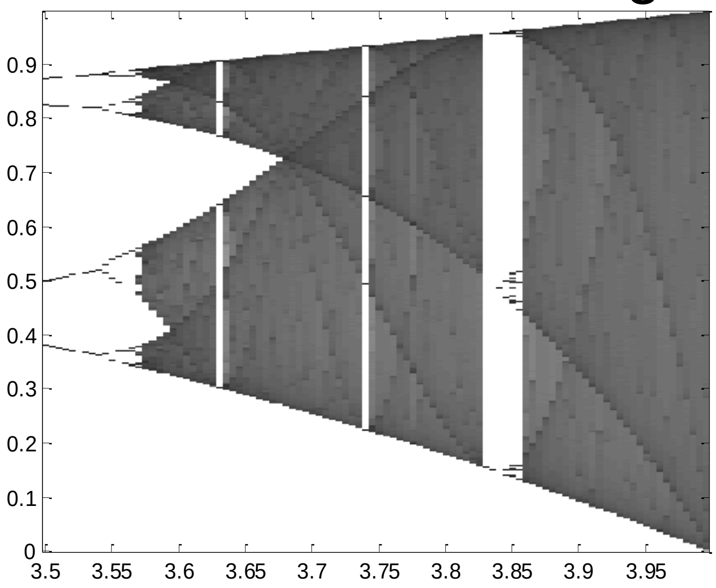
$$x(i+1) = r\, x(i)[1 - x(i)]$$



Ordinal analysis yields information about more expressed and less expressed oscillation patterns in the data.

Ordinal analysis yields complementary information

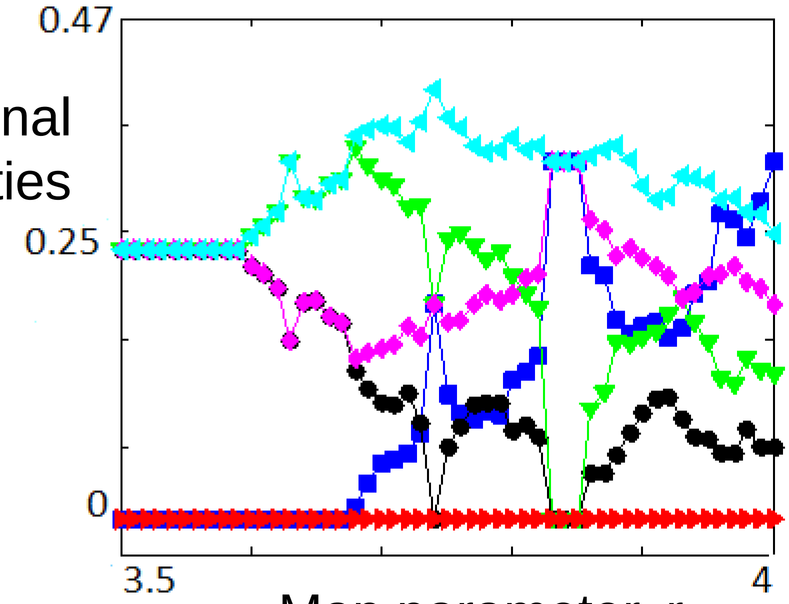
Normal bifurcation diagram



X_i

Map parameter

Ordinal bifurcation diagram

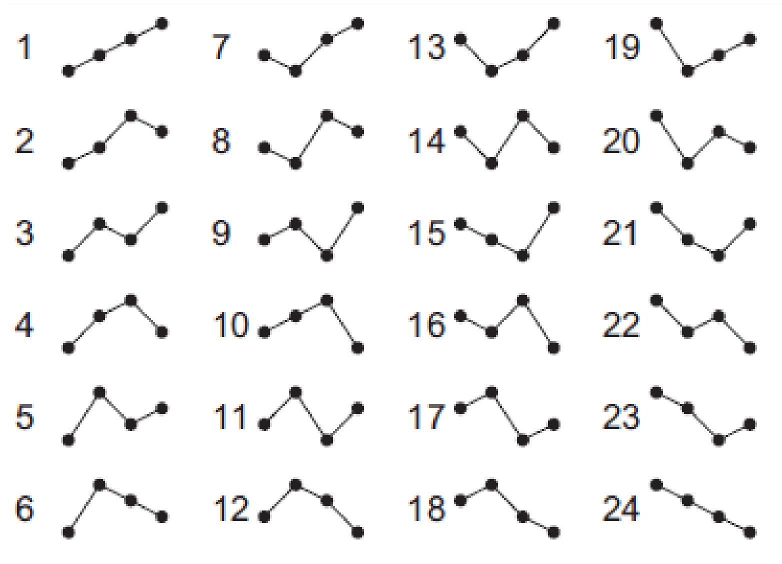


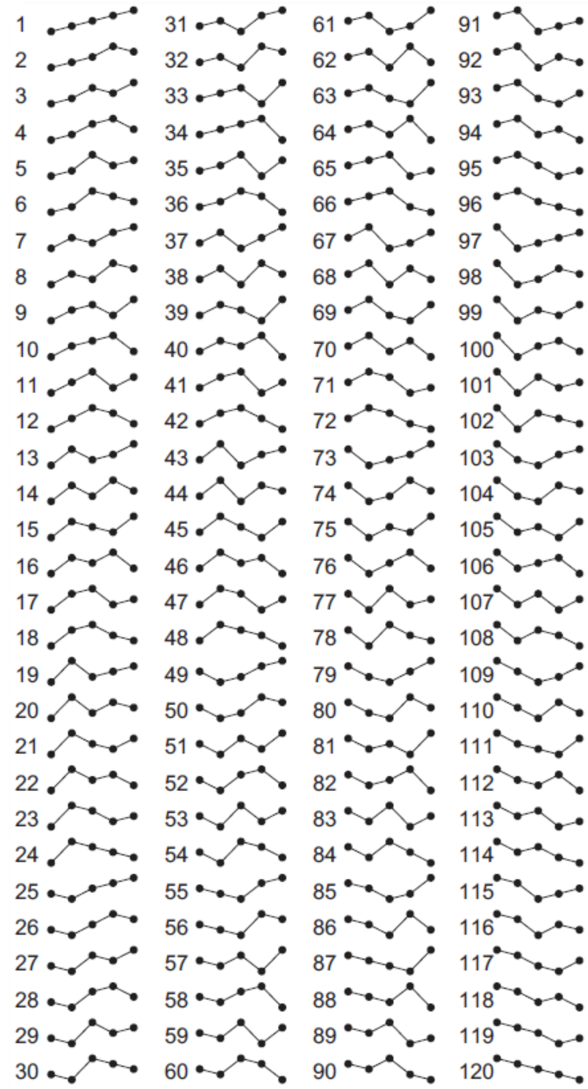
Ordinal Probabilities

Map parameter, r

Pattern **6 (210)** is always forbidden; pattern **1 (012)** is more frequently expressed as r increases

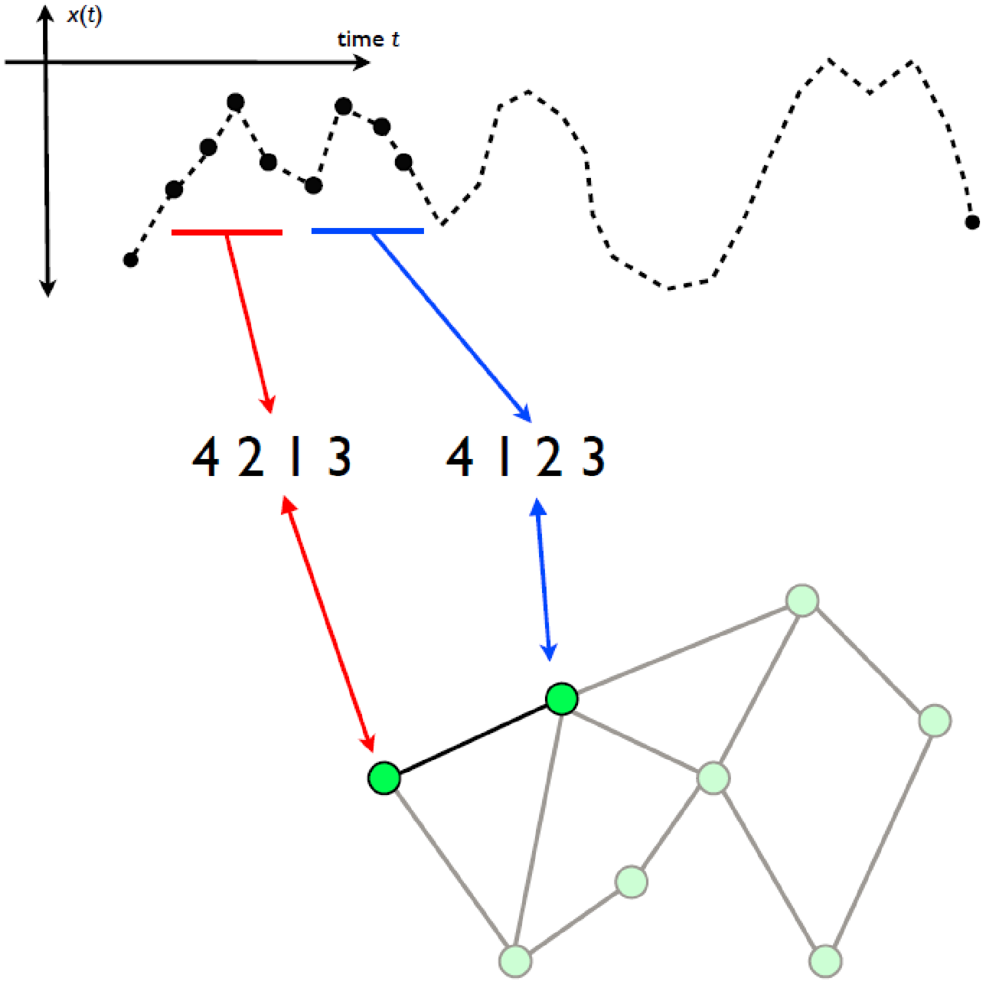
The number of patterns increases as D!





U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Ordinal analysis can be used to transform a time series into a graph (weighted and directed)



- D! nodes
- Weigh of node i: the probability of pattern i
  ($\sum_i p_i = 1$)
- Weight of the link i→j: probability of transition i→j (for each *i*: $\sum_j w_{ij} = 1$)

Adapted from M. Small
(The University of Western Australia)

Measures to characterize the graph

- Entropy computed from node weights (**permutation entropy**)

$$s_p = -\sum p_i \log p_i$$

- Average node entropy (entropy of the link weights)

$$s_n = \frac{1}{M}\sum_{i=1}^{M} s_i \qquad s_i = -\sum_{j=1}^{M} w_{ij} \log w_{ij}$$

- Asymmetry coefficient: normalized difference of transition probabilities, P('01'→ '10') - P('10'→ '01'), etc.

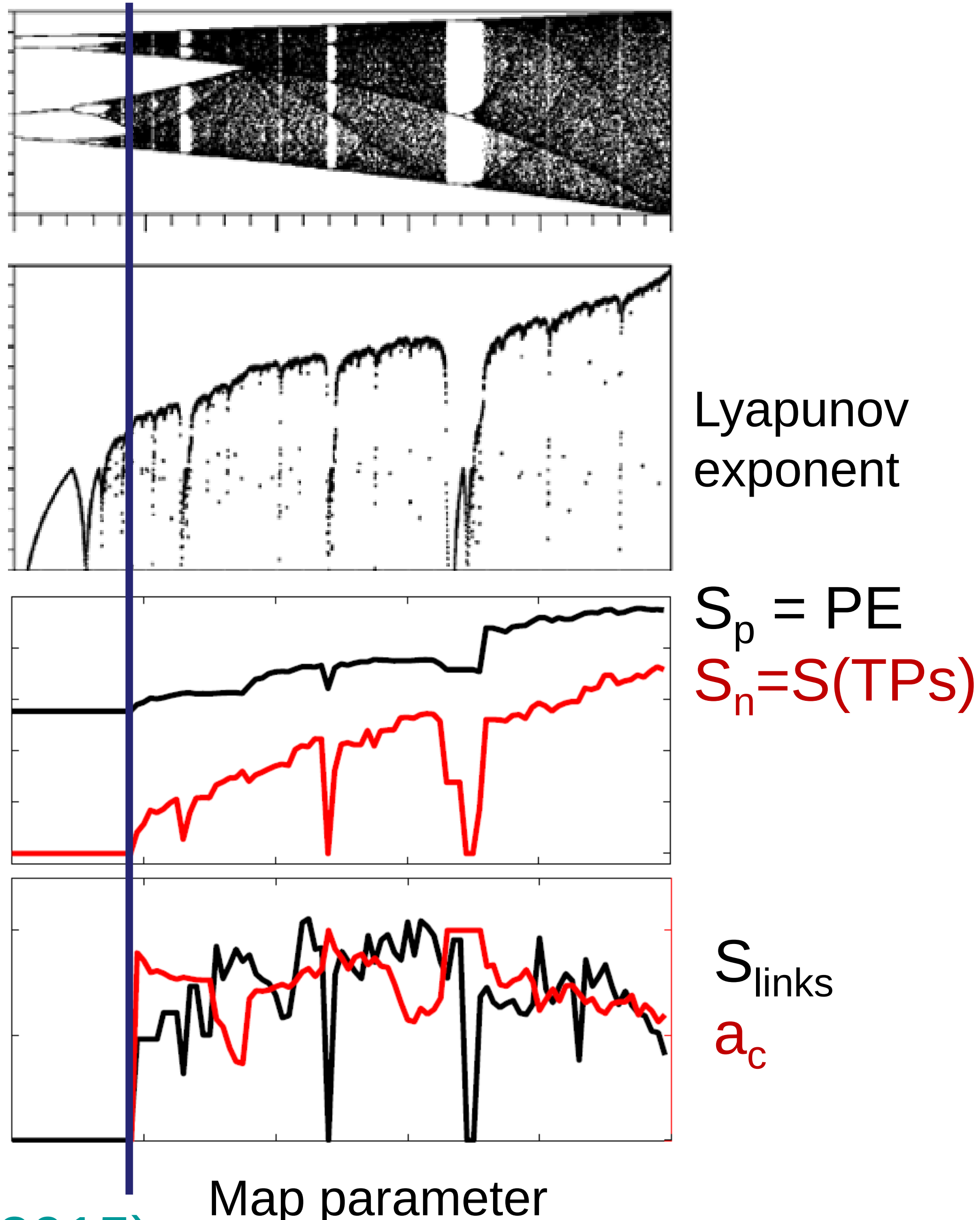
$$a_c = \frac{\sum_i \sum_{j \neq i} \left| w_{ij} - w_{ji} \right|}{\sum_i \sum_{j \neq i} \left(w_{ij} + w_{ji} \right)}$$

(0 in a fully symmetric network;
1 in a fully directed network)

A first test with synthetic data.

Ordinal patterns of length D=4.

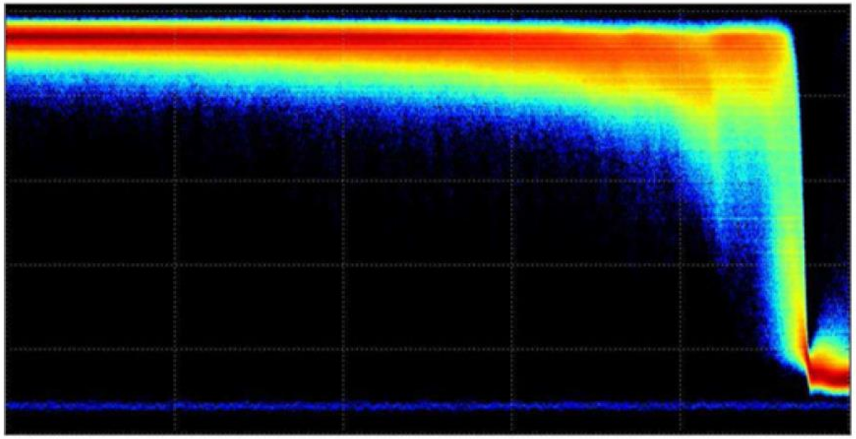
The measures detect the merging of four branches, not detected by the Lyapunov exponent.



C. Masoller et al, NJP (2015)

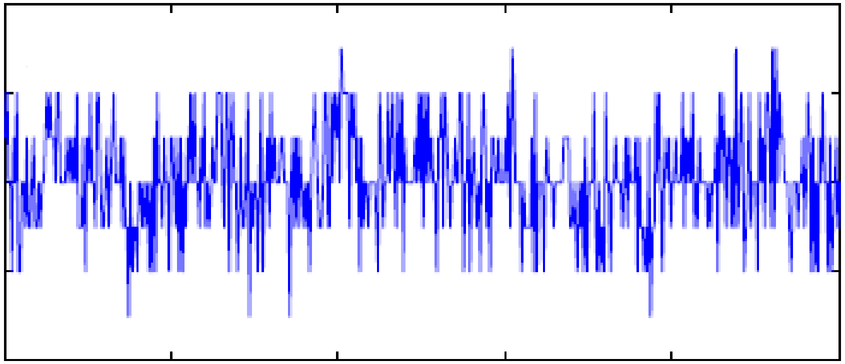
Analysis of laser empirical data: can we use the ordinal method to anticipate the polarization switching point?

As the laser current increases



Time

Intensity @ constant current

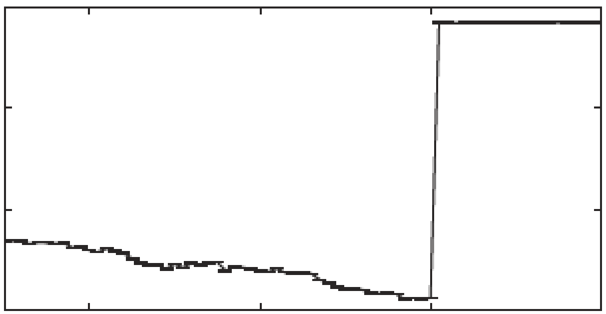


Time

No “early warning” should be possible if the mechanism that triggers the switching is fully stochastic.

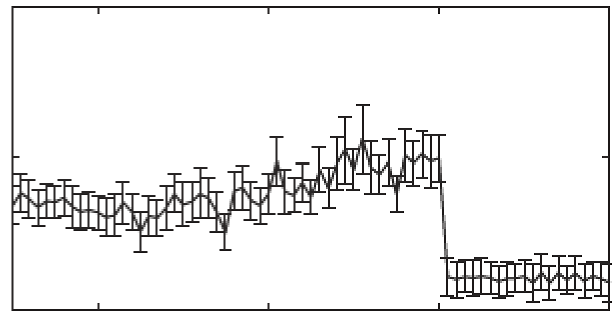
Results

$\langle I \rangle$



Laser current

σ_I

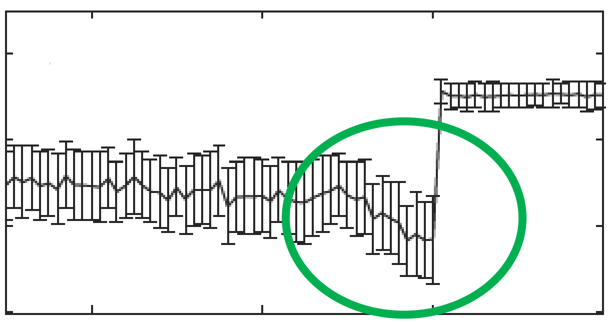


Laser current

L=1000
100 windows

$\Rightarrow$ **No warning**

Node entropy s_n (D=3)



Laser current

Early warning

$\Rightarrow$ Deterministic mechanisms must be involved.

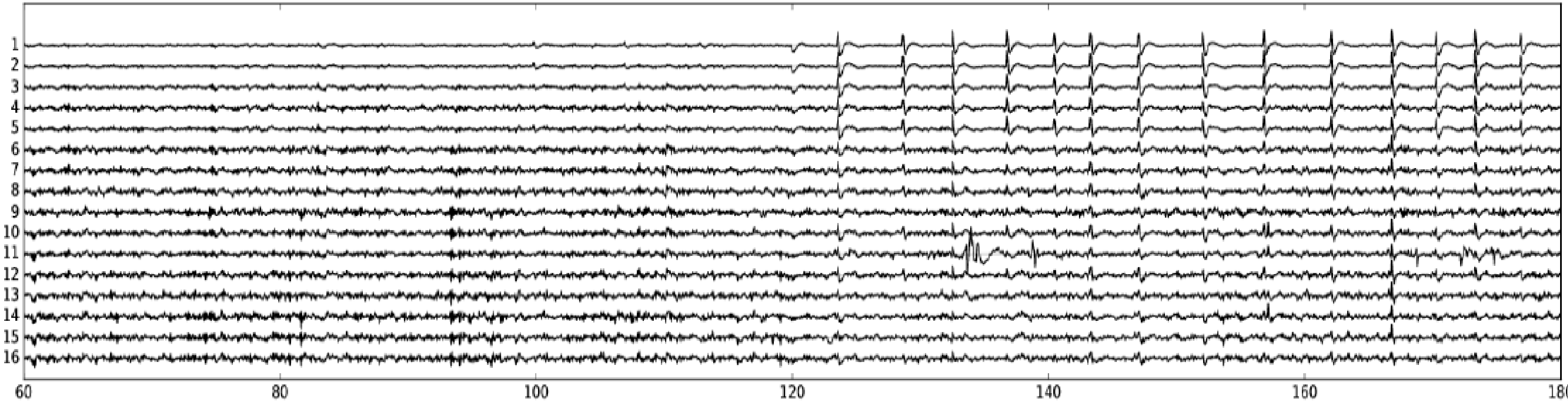
C. Masoller et al, NJP (2015)

Application: distinguishing eyes closed (EC) and eyes open (EO) brain states

	BitBrain DTS1	PhysioNet DTS2
Sampling rate(Hz)	256	160
Time task(seg)	120	60
Total points	30720	9600
Number of electrodes	16	64
Number of subjects	70	109

Eye closed

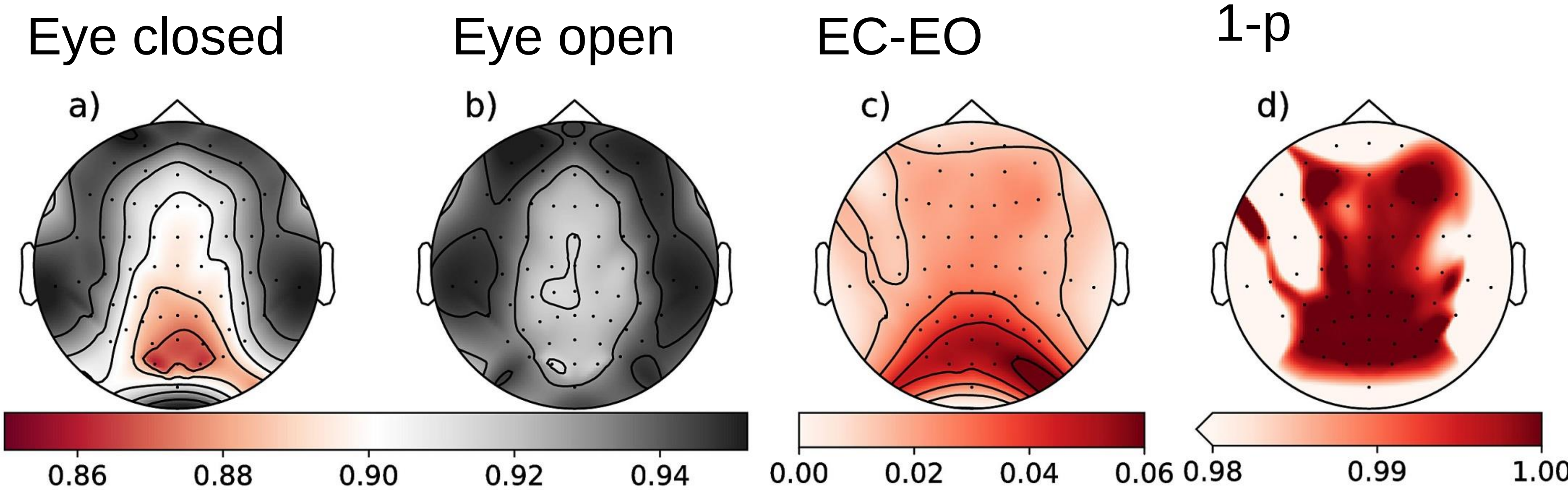
Eye open

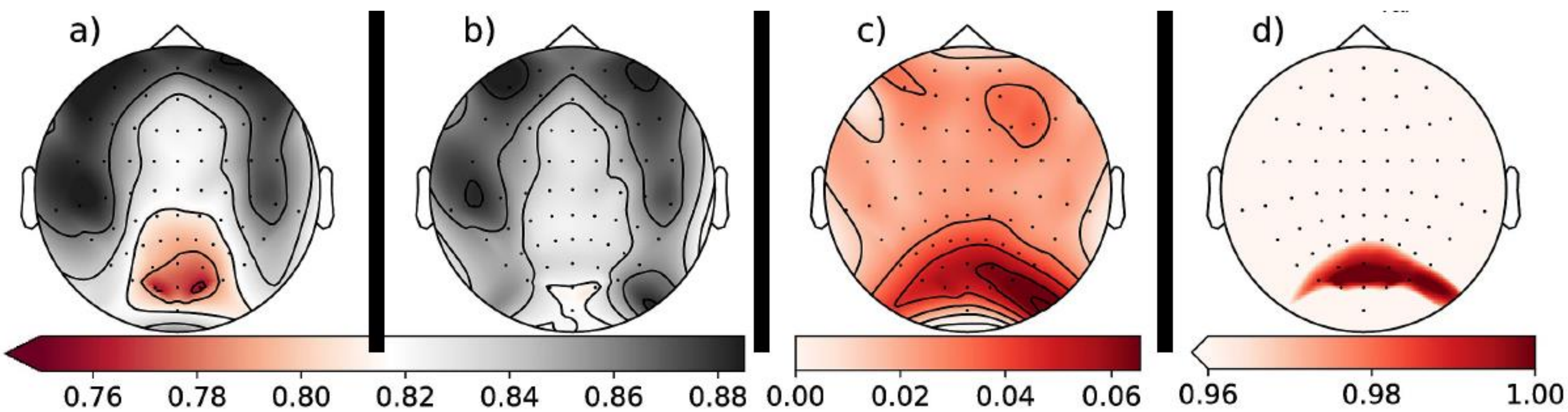


- Symbolic analysis is applied to the raw data; similar results were found with filtered data using independent component analysis.

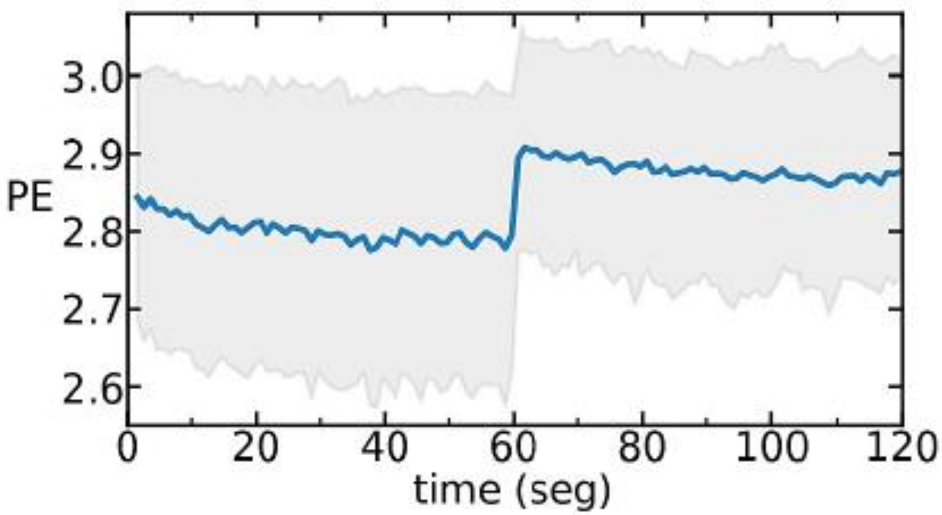
Permutation entropy (top) and node entropy (bottom)

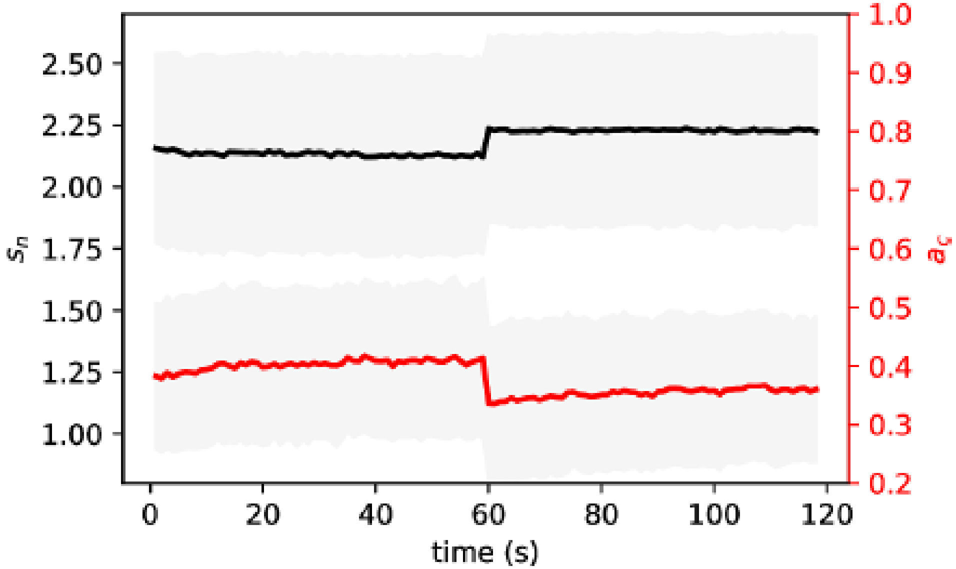
PhysioNet dataset





“Randomization”: the entropies increase and the asymmetry coefficient decreases

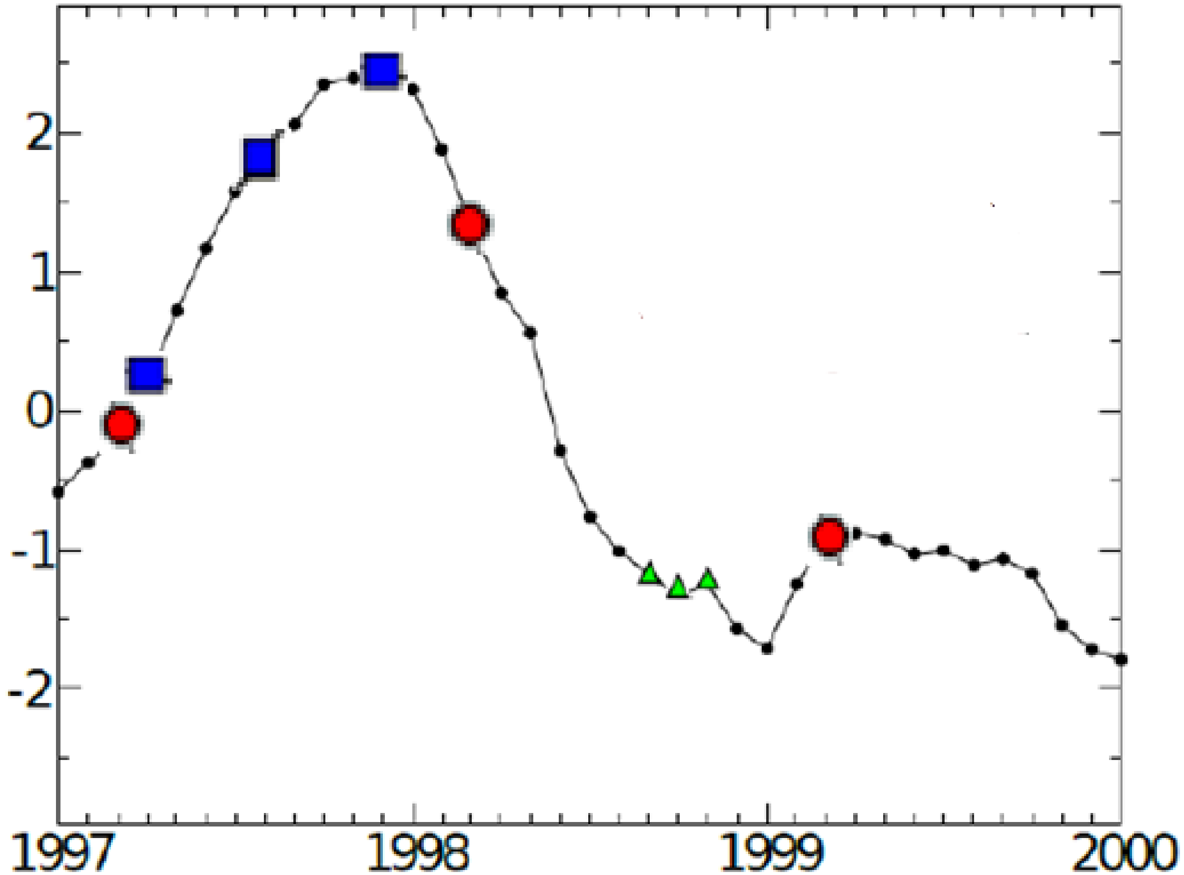




Time window = 1 s
(160 data points)

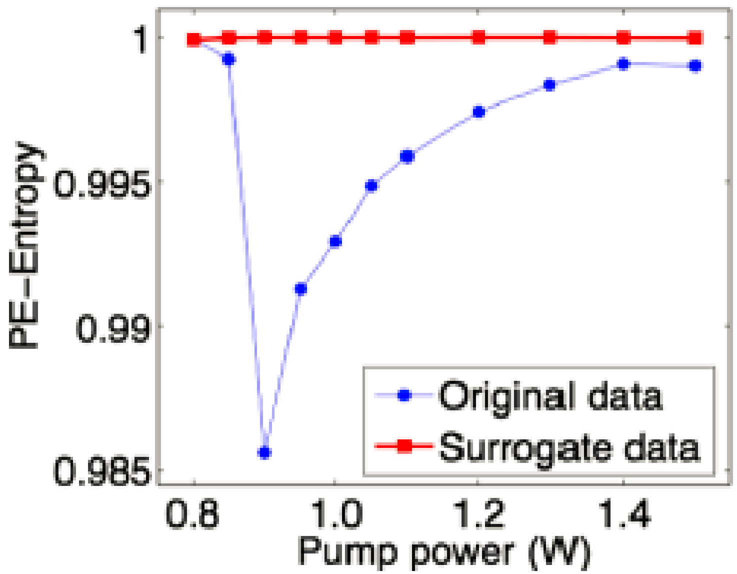
Ordinal patterns can be defined using a lag time between the data points (varying the effective “sampling time”)

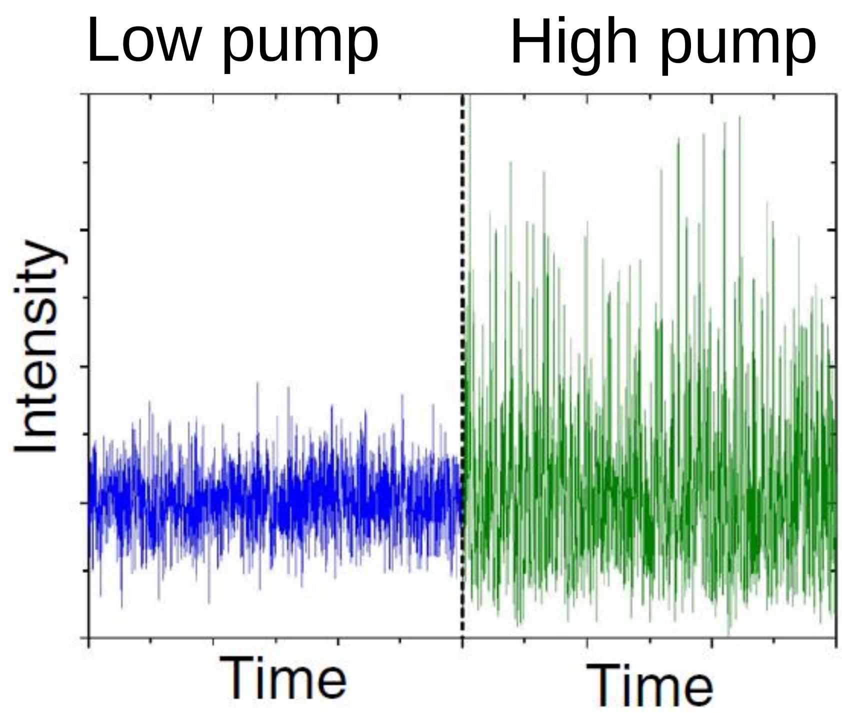
Example: el Niño index, monthly sampled



Analysis of the transition to optical turbulence in a quasi-cw Raman fiber laser

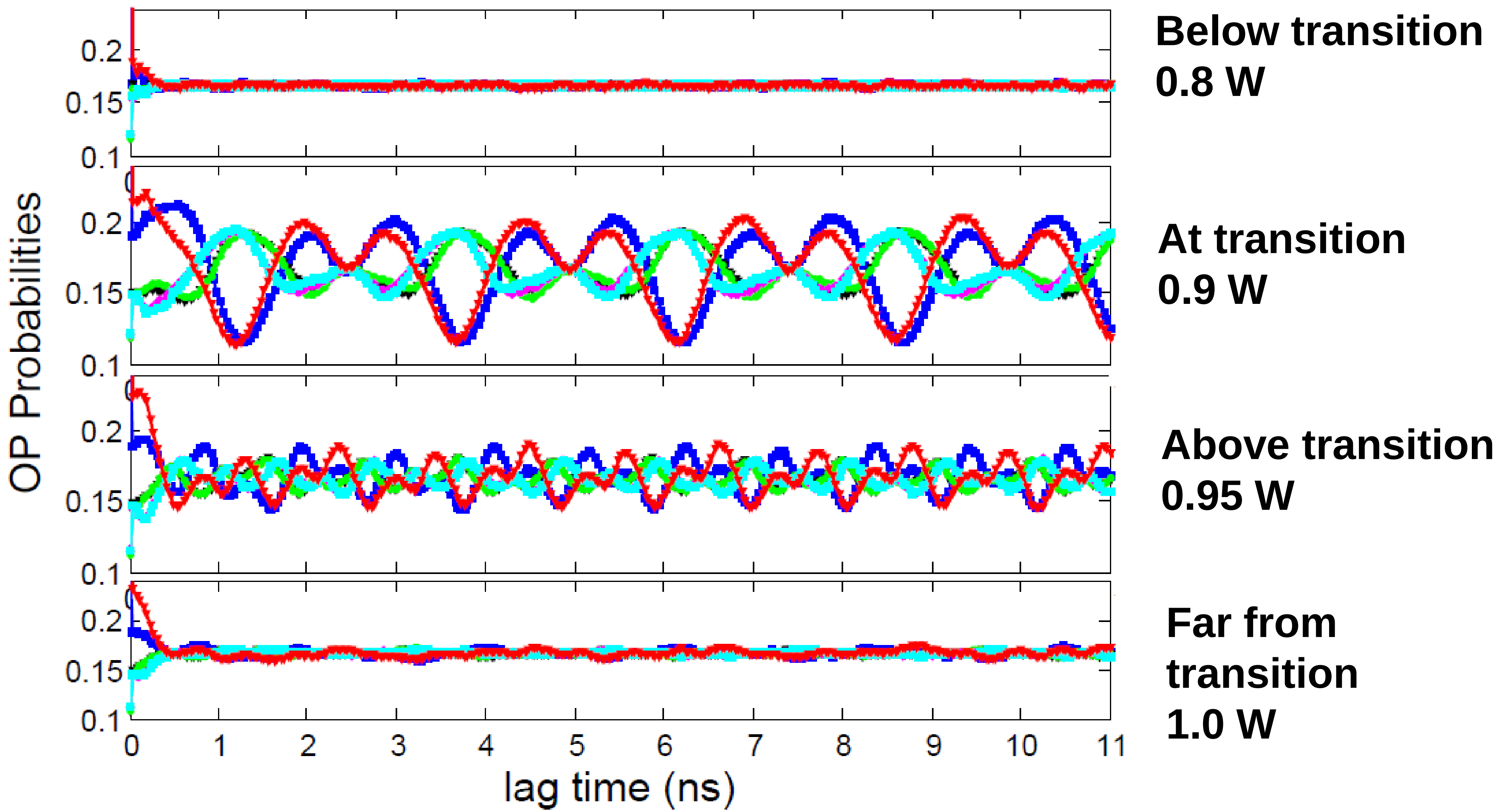
- 1 km of normal dispersion fiber placed between two fiber Bragg gratings acting as cavity mirrors.





Turitsyna et al., Nat. Phot. (2013)
Aragoneses et al., PRL (2016)

Ordinal probabilities vs. lag (“sampling time”)



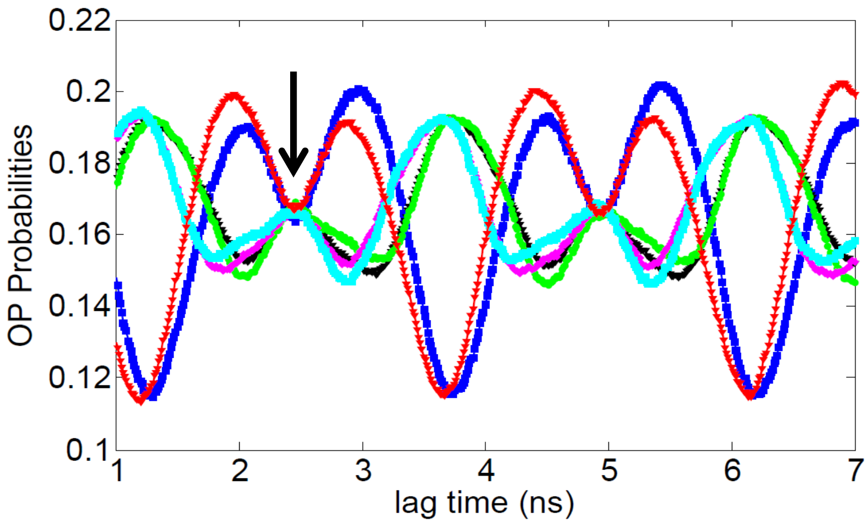
Can we find a minimal model that describes the ordinal probabilities at the transition?

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\sin(2\pi\varphi_i) + D\xi$$

$$\{X_i, X_{i+1}, X_{i+2}, \ldots\}$$
$$X_i = \varphi_{i+1} - \varphi_i$$

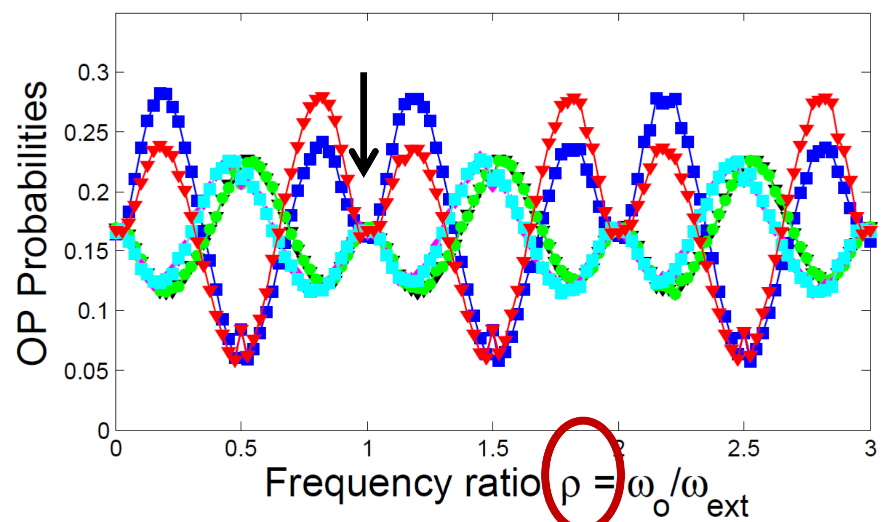
Fiber laser data $\{I_i, I_{i+\tau}, I_{i+2\tau}, \ldots\}$

(at the transition, pump power 0.9 W)



Synthetic data (phase increments)

(K=0.23 & D=0.02)



$\tau = 2.5$ ns $\Leftrightarrow \rho = 1 = \nu_0\,\tau \Rightarrow \nu_0 = 1/\tau = 0.4$ GHz

The Fourier spectrum is broadband but has a peak at ~0.4 GHz

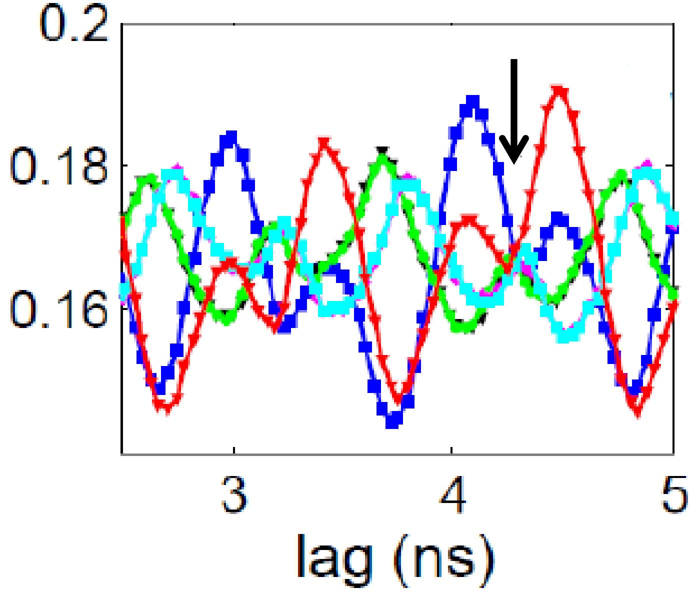
L. Carpi and C. Masoller, Phys. Rev. A 97, 023842 (2018)

Good agreement also at higher pump power

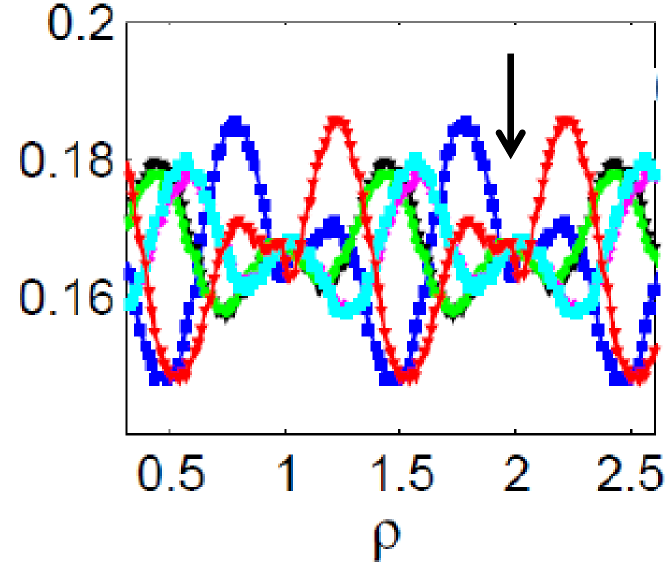
$$\phi_{i+1} = \phi_i + \epsilon\rho + (K/2\pi)\sin(2\pi\phi_i) + D\xi_i$$

Fiber laser data

(above the transition, 0.95 W)



$K = 0.25,\ D = 0.075,\ \epsilon = -1$



τ = 4.3 ns $\Leftrightarrow$ $\rho = 2 = \nu_0\,\tau$ $\Rightarrow$ $\nu_0 = 1/(2\tau)$ = 0.46 GHz

The spectrum has a peak at ~0.93 GHz, consistent with $2\nu_0$.

L. Carpi and C. Masoller, Phys. Rev. A 97, 023842 (2018)

Second analysis tool: Hilbert analysis

It provides an instantaneous phase, amplitude and frequency for each data point of a scalar oscillatory time series

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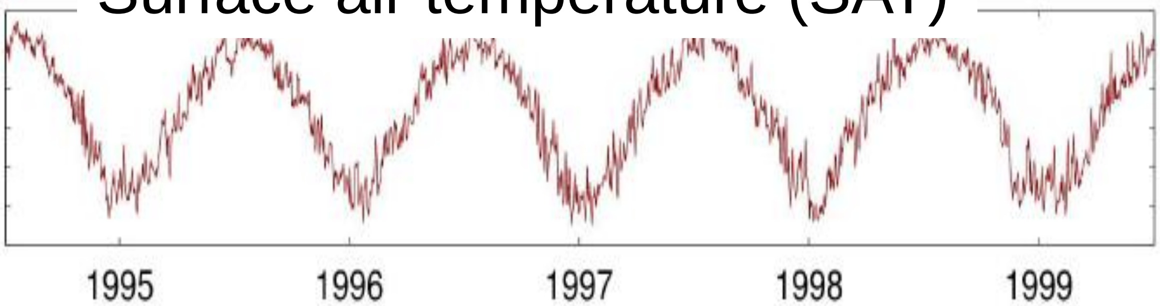
The Hilbert transform

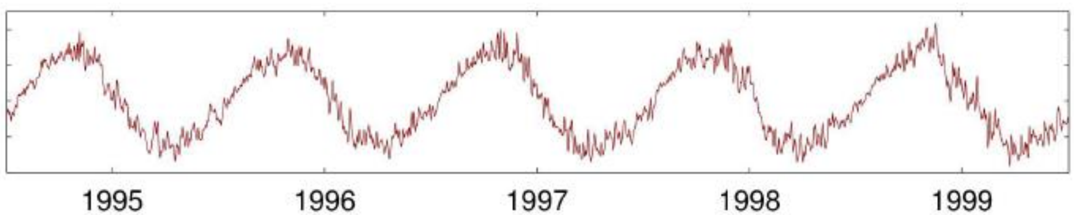
Surface air temperature (SAT)

x

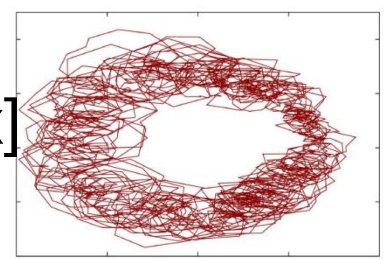


- HT[sin(ωt)]=cos(ωt)

HT[x]



y=HT[x]



x

$$a(t) = \sqrt{[x(t)]^2 + [y(t)]^2}$$

$$\varphi(t) = \arctan[y(t)/x(t)]$$

Can we use the Hilbert amplitude, phase, frequency, to identify and quantify regional climate change?

- <u>A word of warning</u>: only if $x(t)$ is a narrow-band signal $a(t)$ and $\omega(t) = d\varphi/dt$ have clear physical meaning
  - ***a(t)*** is the envelope of $x(t)$
  - ***ω(t)*** is the main frequency in the Fourier spectrum
- Problem: climate time series are not narrow-band
- Usual solution (e.g. brain signals): isolate a narrow frequency band
- However, HT directly applied to surface air temperature uncovers the “hot spots” where changes in atmospheric dynamics are more pronounced.

The data: surface air temperature (SAT)

- Spatial resolution $2.5^0 \times 2.5^0 \Rightarrow$ 10226 time series
- Daily resolution 1979 – 2016 $\Rightarrow$ 13700 data points

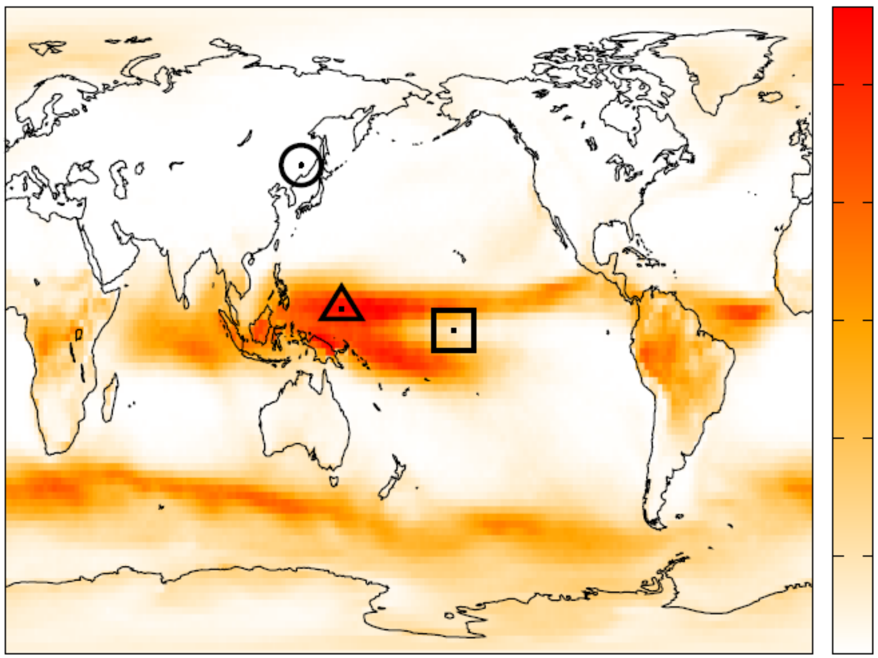
Where does the data come from?

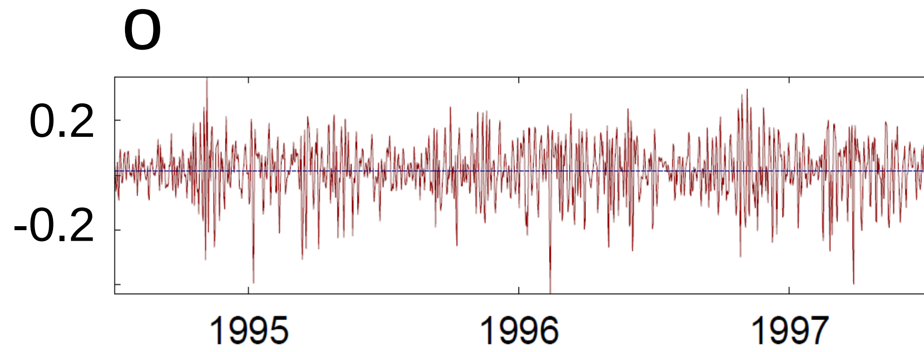
- European Centre for Medium-Range Weather Forecasts (ECMWF, ERA-Interim).
- Freely available.
- <u>Reanalysis</u> = general atmospheric circulation model feed with empirical data, where and when available (data assimilation).

Features extracted from each SAT time series

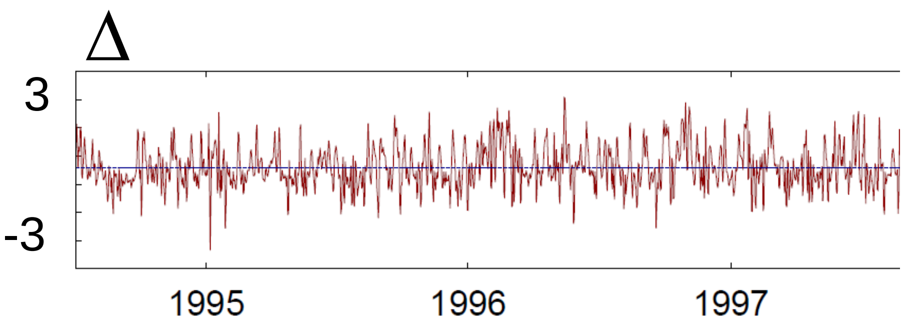
- Time averaged amplitude, $\langle a \rangle$
- Time averaged frequency, $\langle \omega \rangle$
- Standard deviations, σ_a, σ_ω

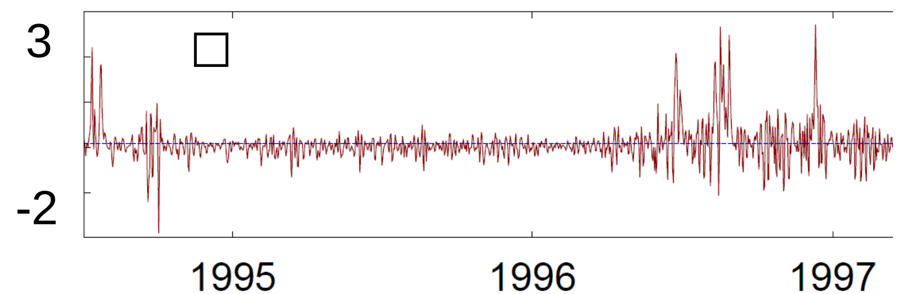
The map of time average frequency uncovers regions of fast frequency dynamics





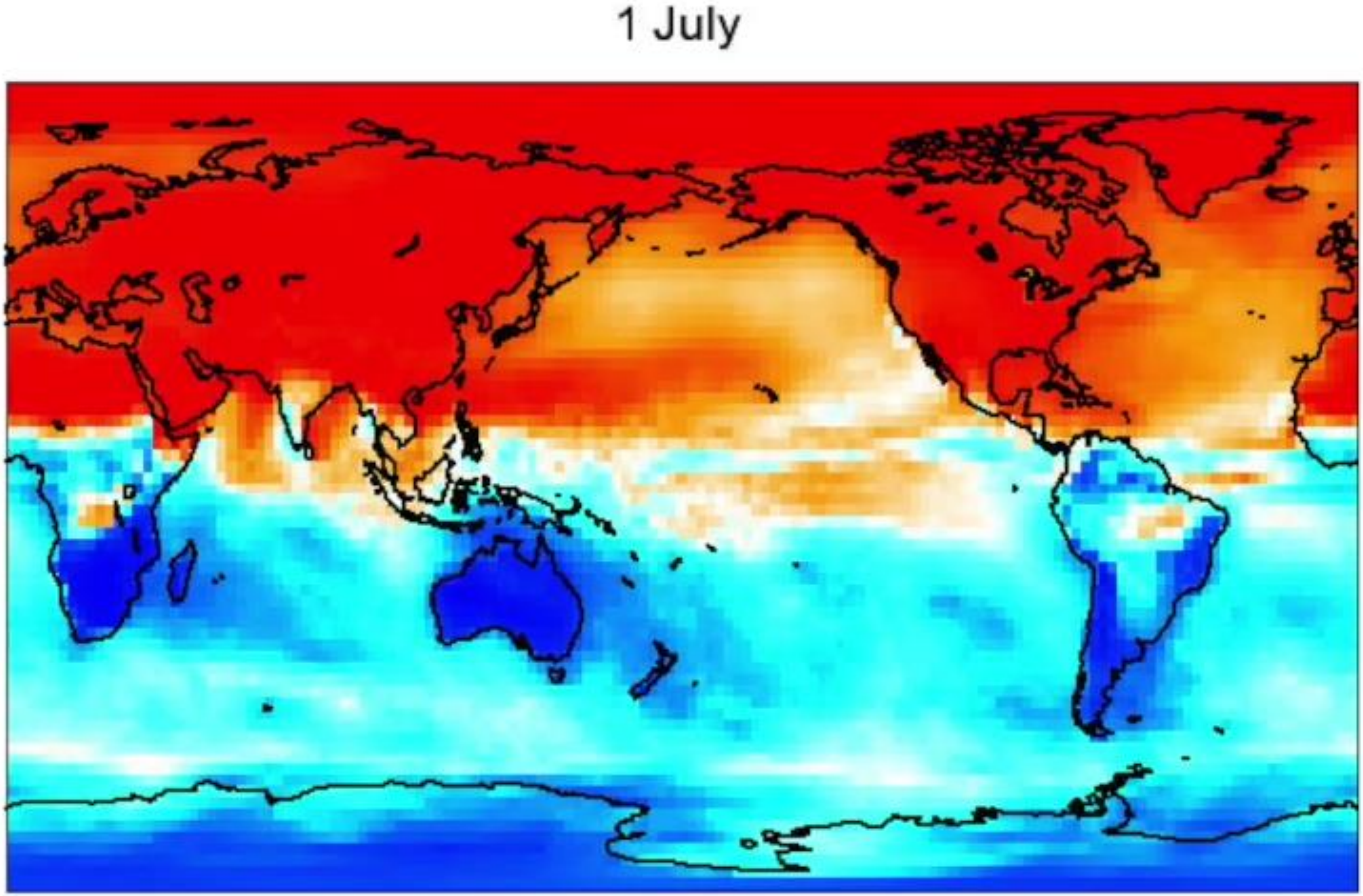
White = 0.017 rad/day
= one cycle per year





Zappala, Barreiro and Masoller, Earth Syst. Dynamics (2018)

Phase dynamics: temporal evolution of the cosine of the Hilbert phase on a typical year



Relative decadal variations

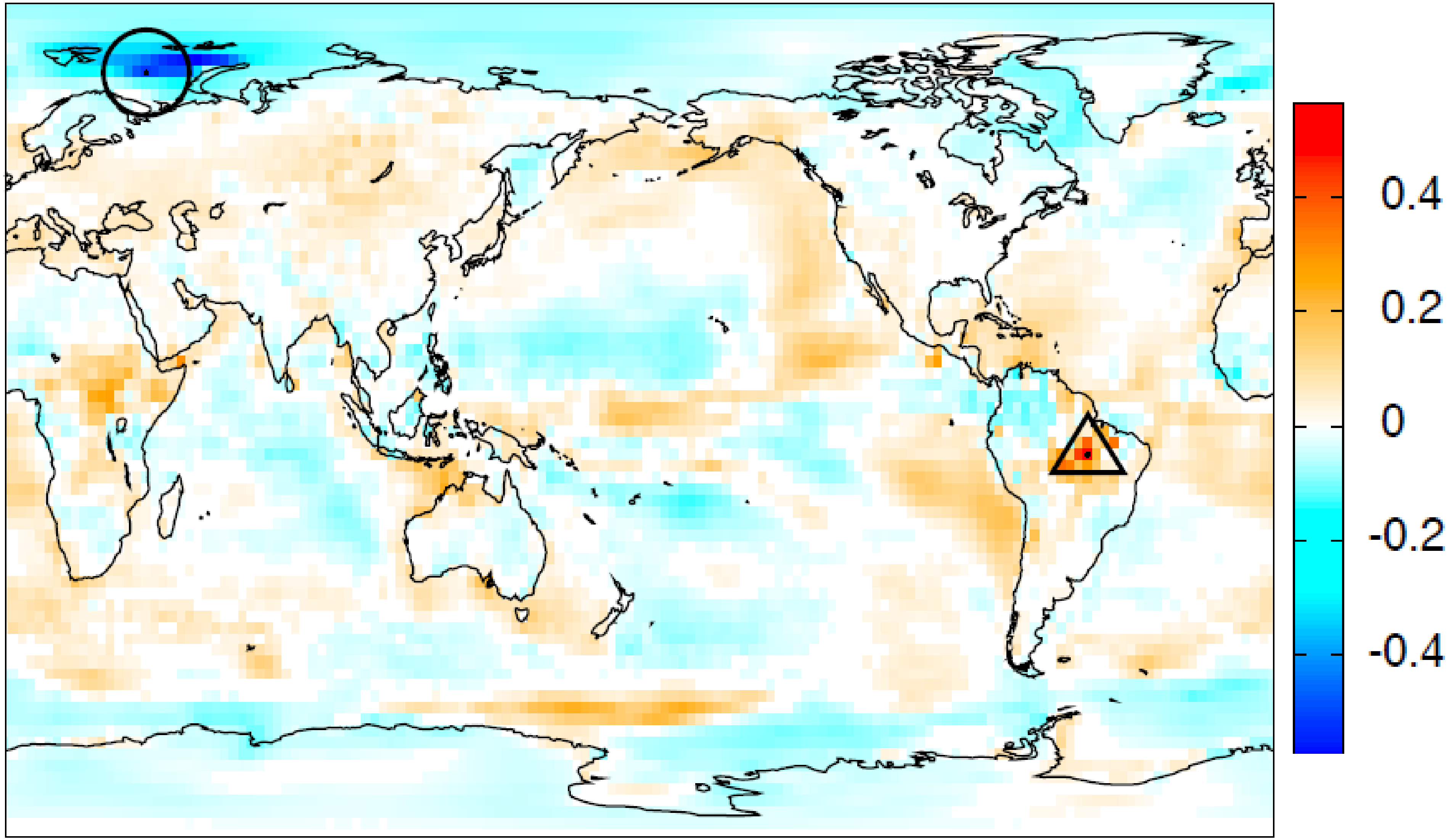
$$\Delta a = \langle a \rangle_{2016-2007} - \langle a \rangle_{1988-1979}$$

$$\frac{\Delta a}{\langle a \rangle_{2016-1979}}$$

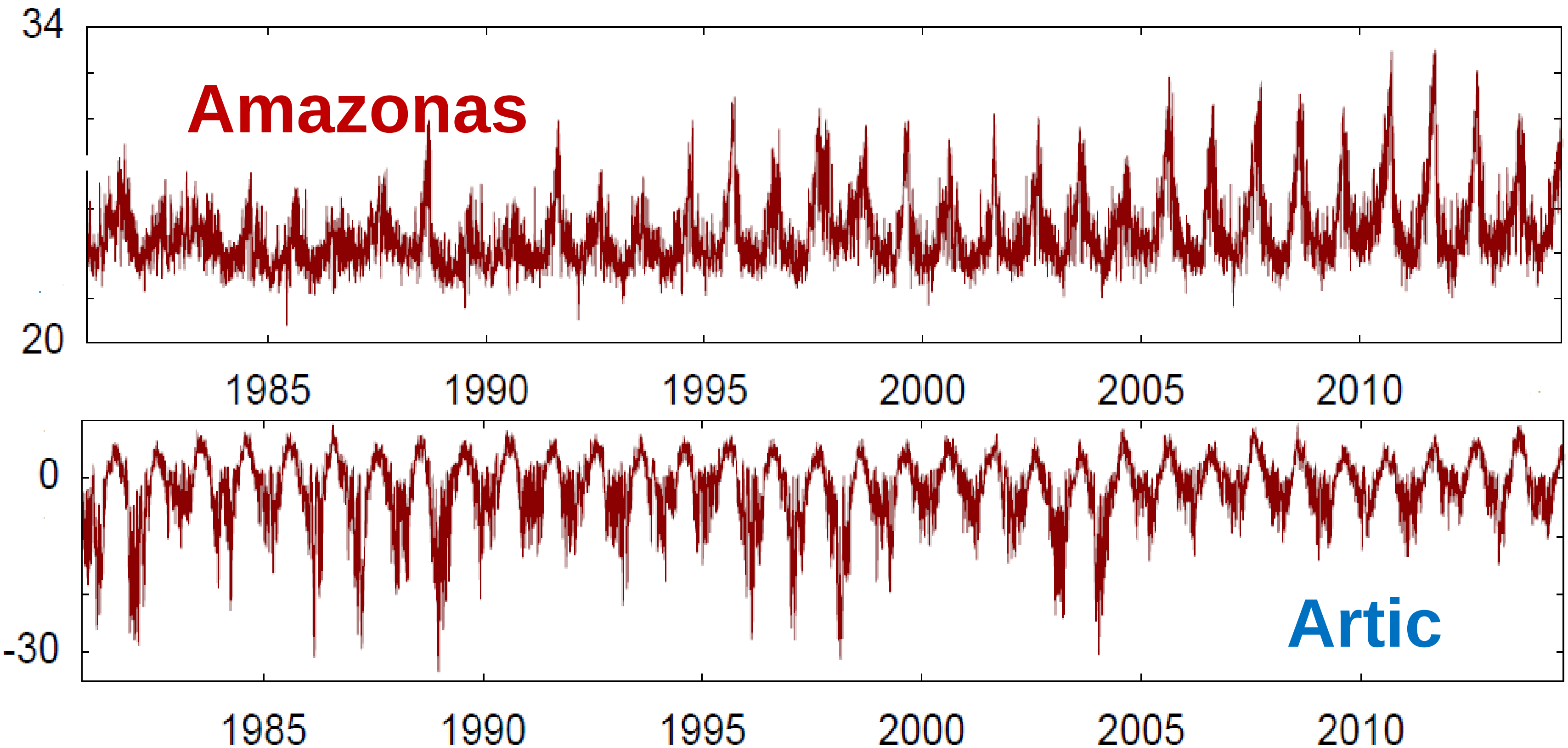
Relative variation is considered significant if:

$$\frac{\Delta a}{\langle a \rangle} \geq \langle . \rangle_s + 2\sigma_s \quad \text{or} \quad \frac{\Delta a}{\langle a \rangle} \leq \langle . \rangle_s - 2\sigma_s$$

100 surrogates

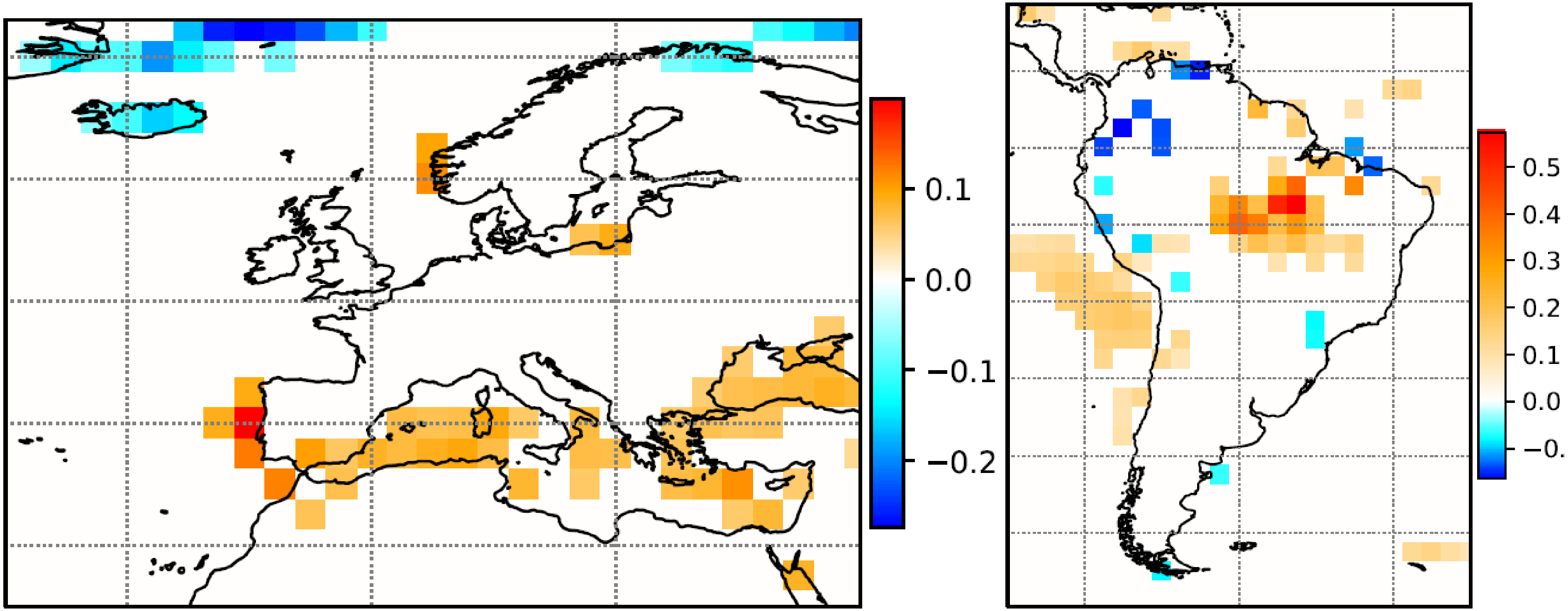


0.4
0.2
0
-0.2
-0.4

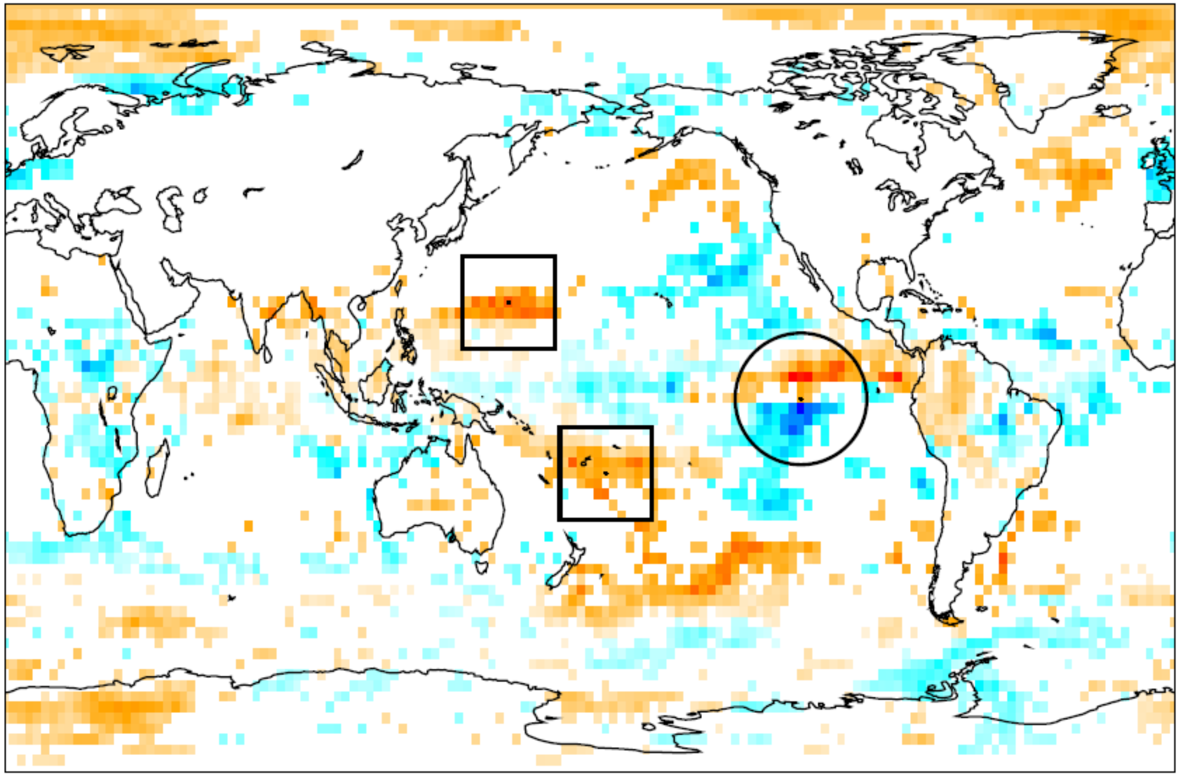


- **Decrease of precipitation**: the solar radiation that is not used for evaporation is used to heat the ground.

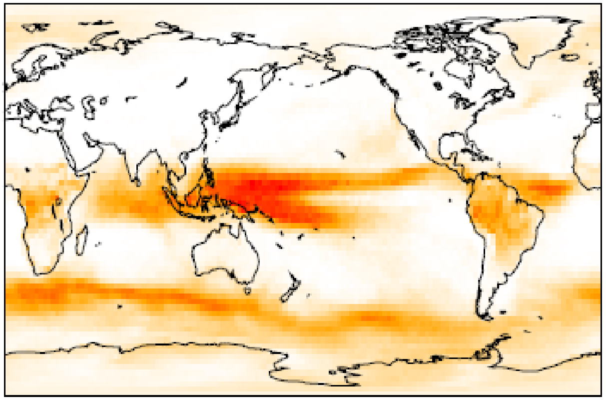
- **Melting of sea ice**: during winter the air temperature is mitigated by the sea and tends to be more moderated.


0.1
0.0
−0.1
−0.2
0.5
0.4
0.3
0.2
0.1
0.0
−0.1

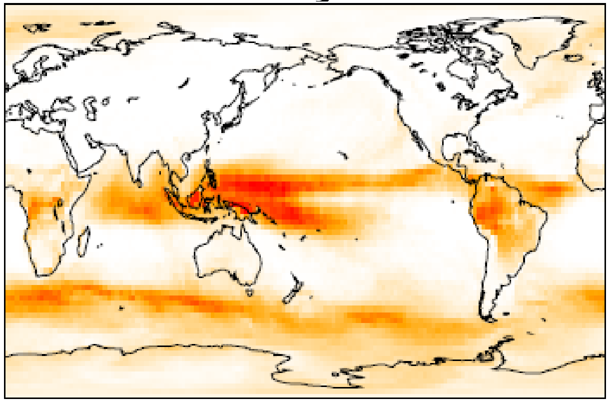
Relative change of time-averaged Hilbert frequency consistent with a north shift and enlargement of the InterTropical Convergence Zone (ITCZ)



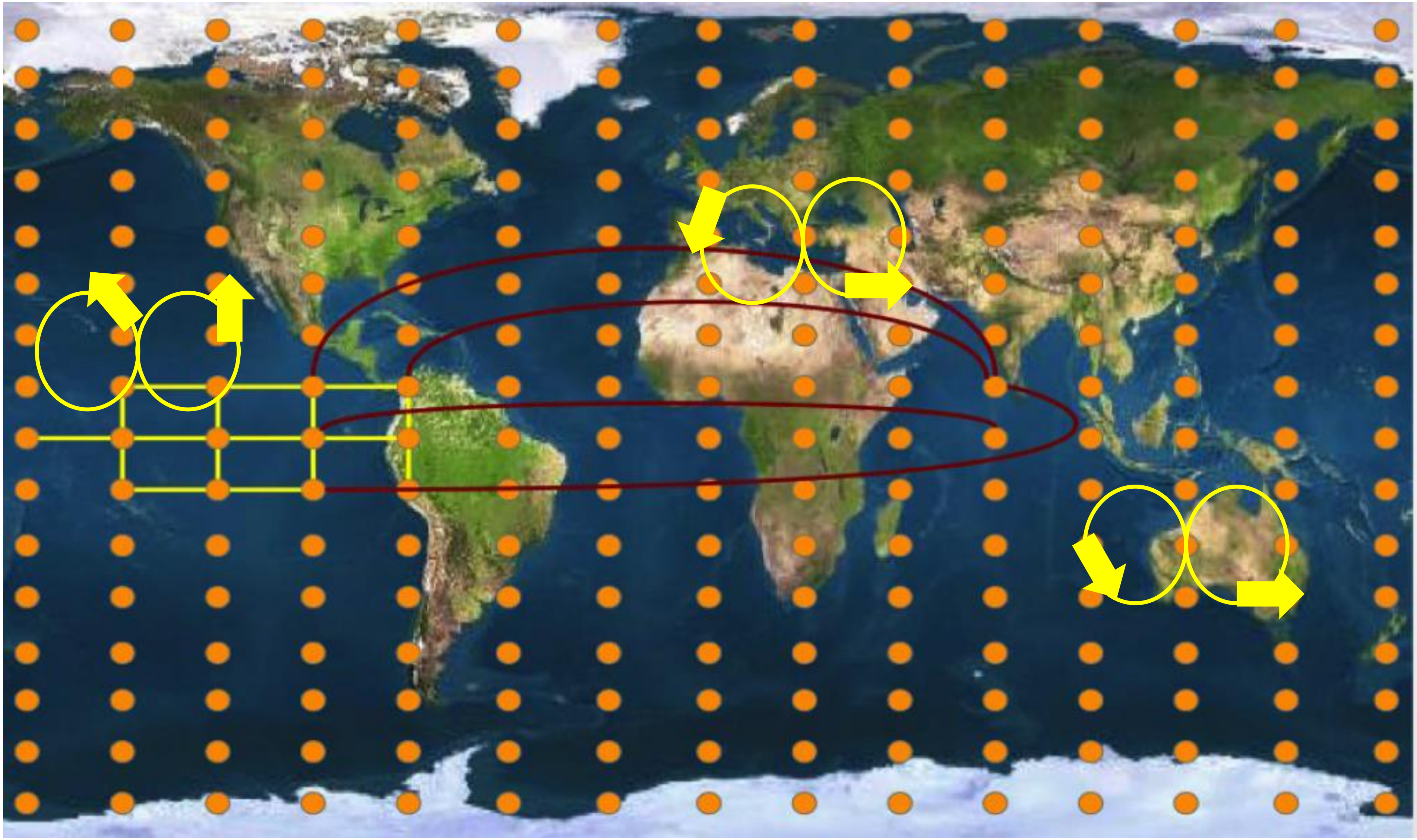
First ten years



Last ten years



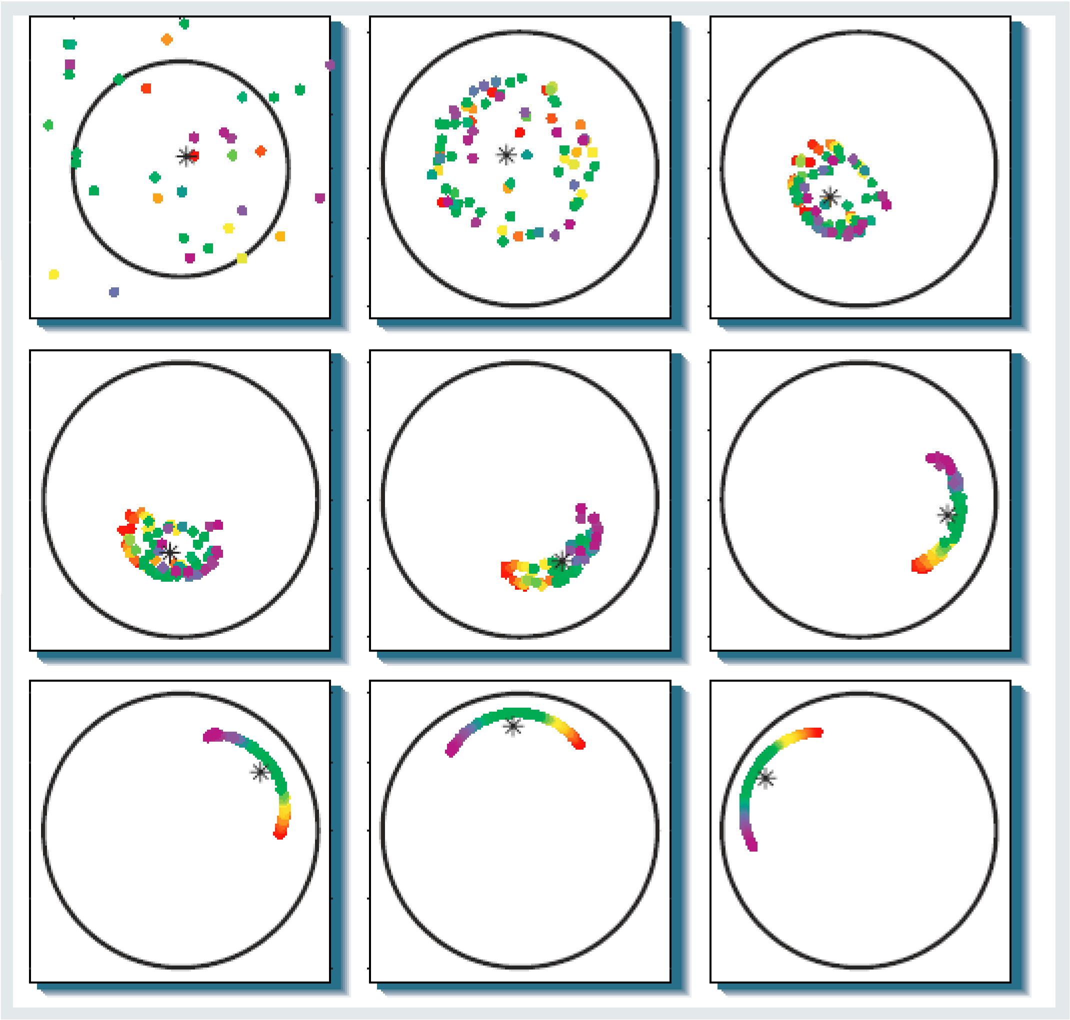
Network of individual oscillators



Quantifying phase synchronization

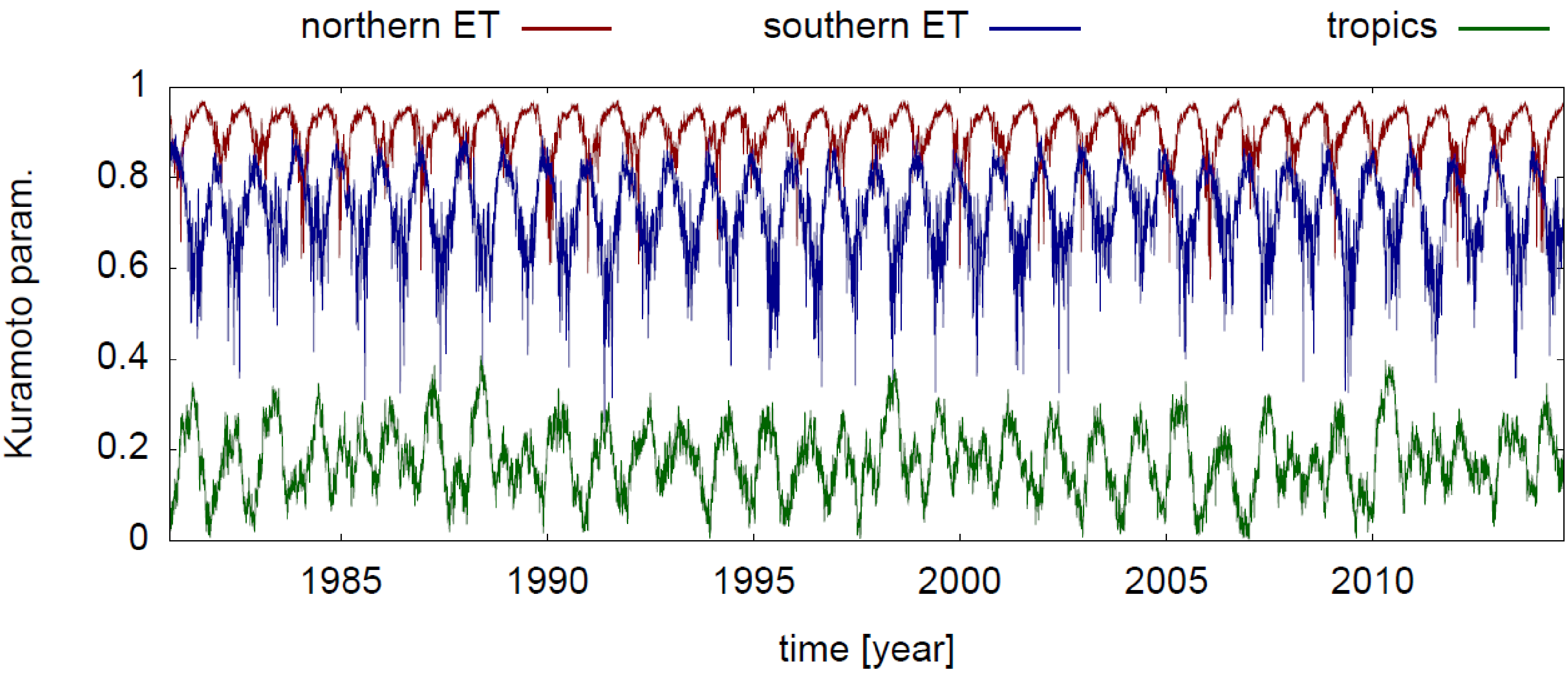
- Kuramoto order parameter

$$r(t) = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j(t)} \right|$$



Steven H. Strogatz, Nature 2001

Quantifying synchronization in atmospheric data



Take home message

- Symbolic analysis, Hilbert analysis and information theory measures are useful tools to understand complex signals.
- They provide *complementary* information.
- Ordinal analysis was used to identify regime transitions in laser data (polarization switching, optical turbulence) and in EEG data (eyes closed – eyes open)
- Hilbert analysis allowed us to identify changes (in the last three decades) in atmospheric data.
- Both analysis tools were applied directly to the raw data.

Dynamic Days LAC, Punta del Este, Uruguay

26--30 November 2018 (MS Time Series Analysis)

https://ddayslac2018.org/

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Obrigada !

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- Laura Carpi (*laser data*)
- Carlos Quintero, Carme Torrent (*brain data*)
- Dario Zappala, Marcelo Barreiro (*climate data*)

C. Masoller et al., New J. Phys. 17, 023068 (2015)
L. Carpi and C. Masoller, Phys. Rev. A 97, 023842 (2018)
D. A. Zappala et al., Earth Syst. Dynamics 9, 383 (2018)

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