

# Nonlinear tools to extract information from complex signals

(examples from climate, brain, neurons & lasers)

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DE CATALUNYA  
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*Campus d'Excel·lència Internacional*

**VIII Jornada de Complexitat, May 21 2019**



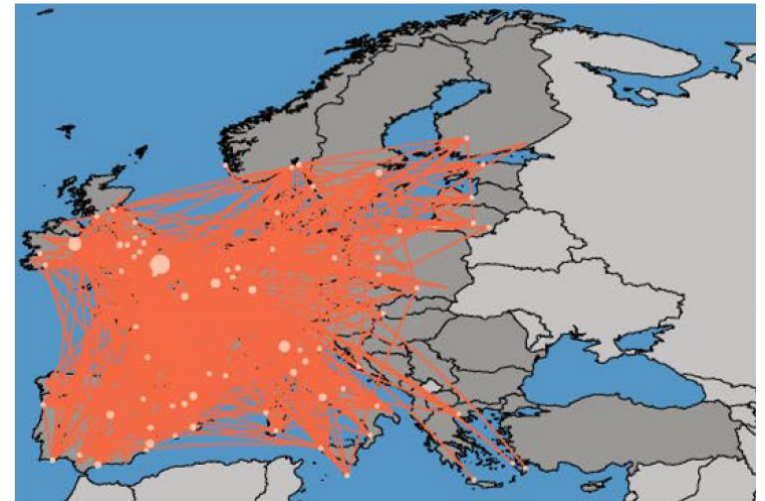
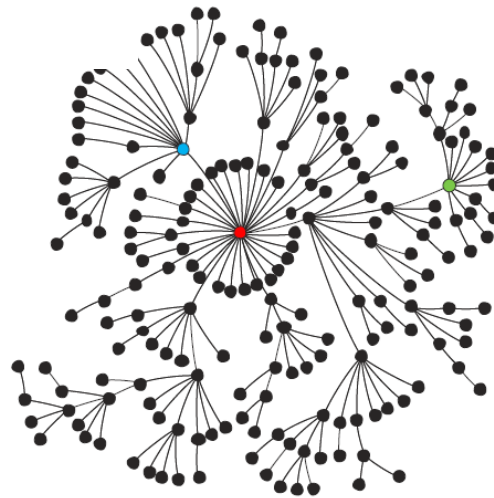
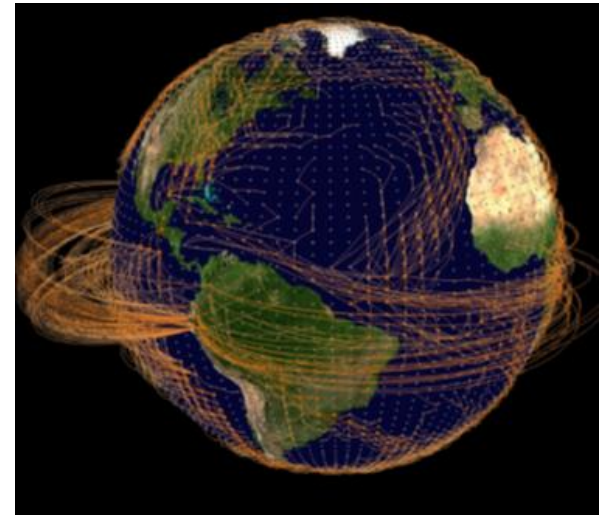
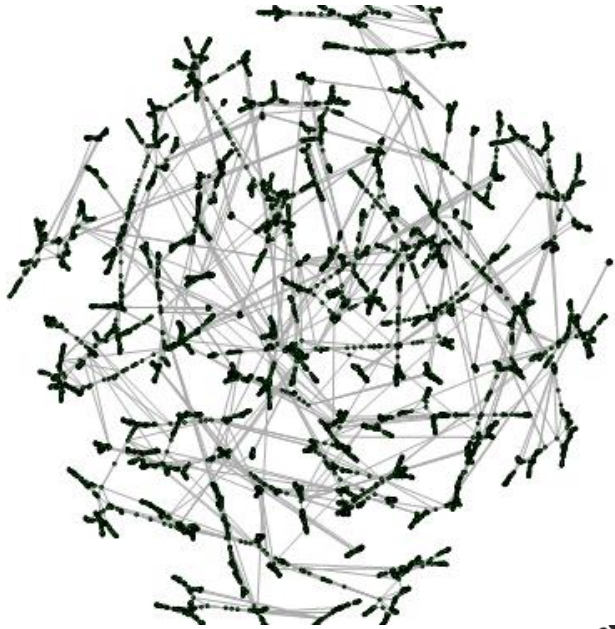
# Presentation

- Originally from Montevideo, Uruguay
- PhD in physics (lasers, Bryn Mawr College, USA)
- Since 2004 @ *Universitat Politecnica de Catalunya*.
- Member of the research group on *Dynamics, Nonlinear Optics and Lasers*.

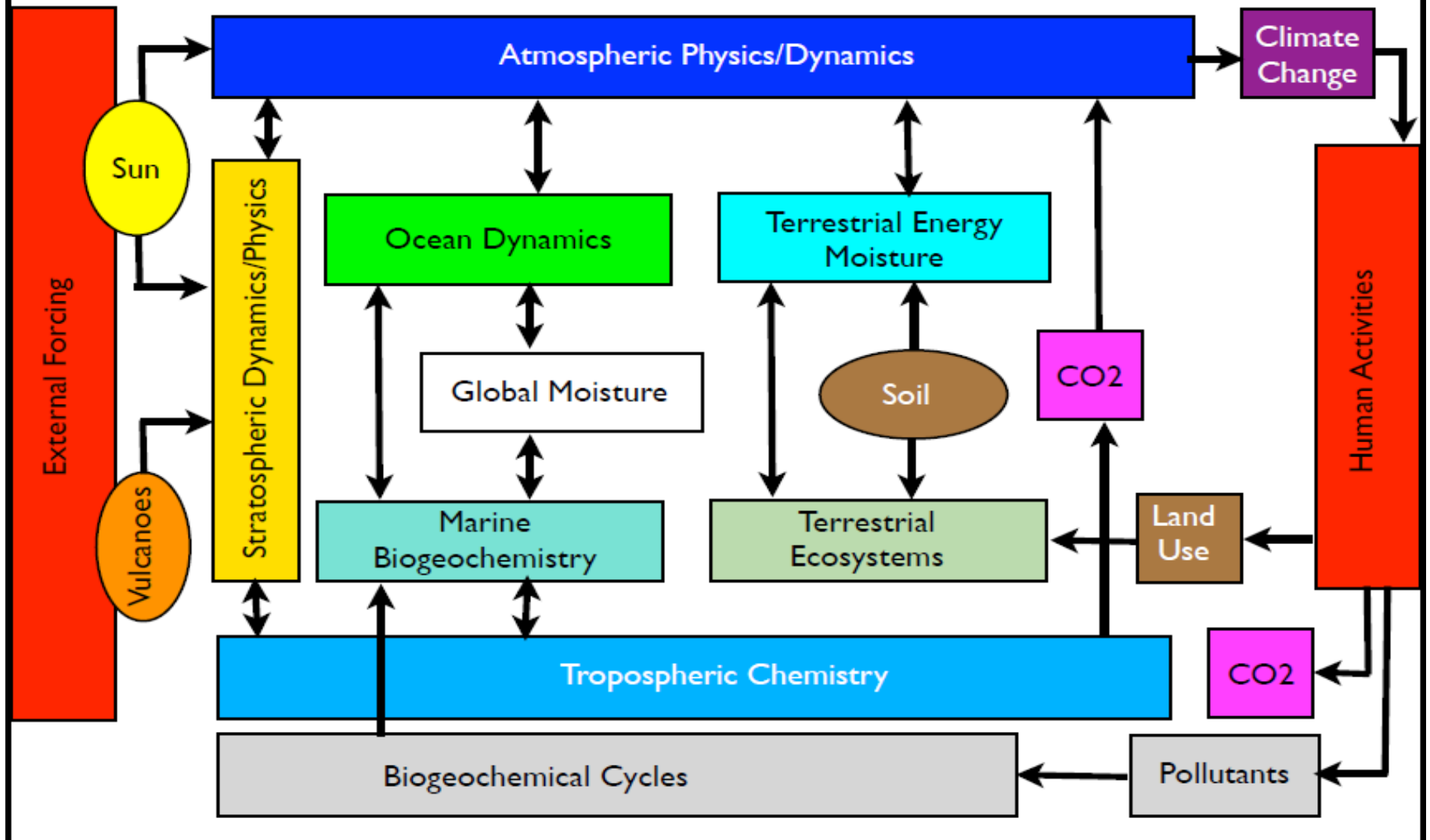




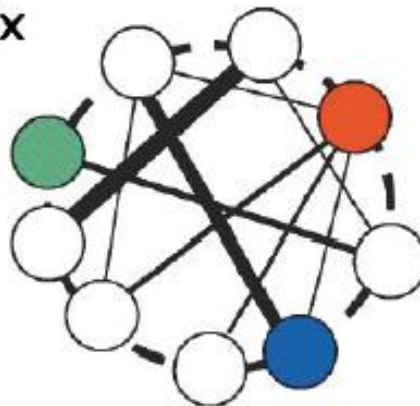
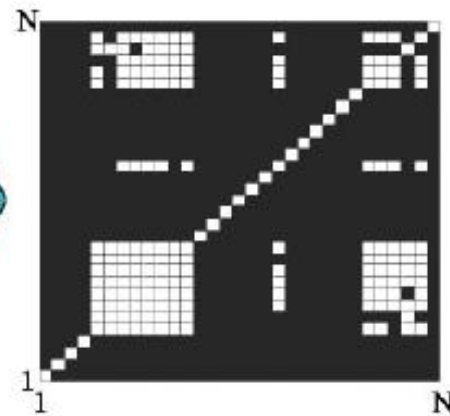
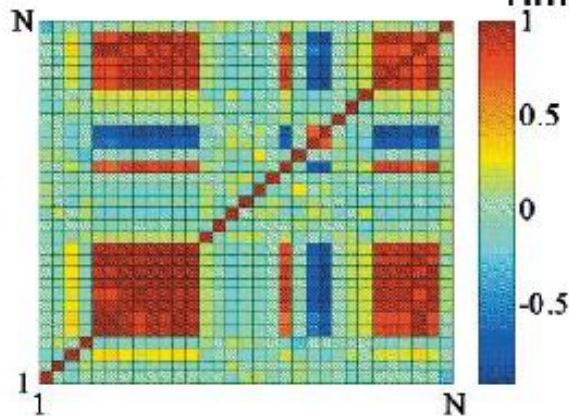
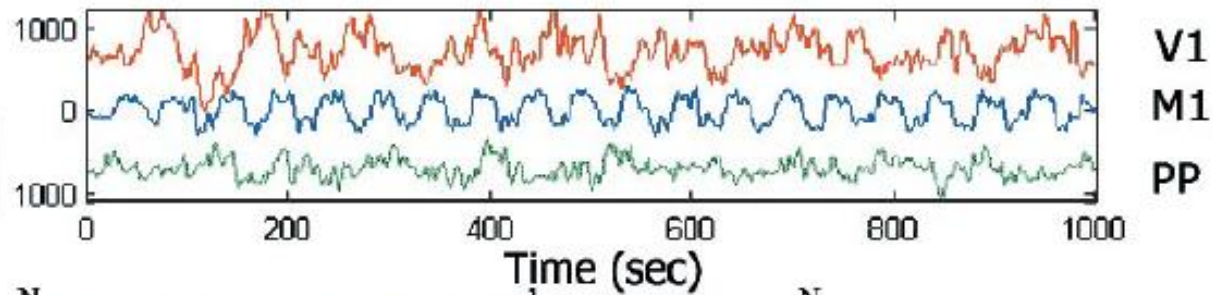
# Complex systems



# The Climate System



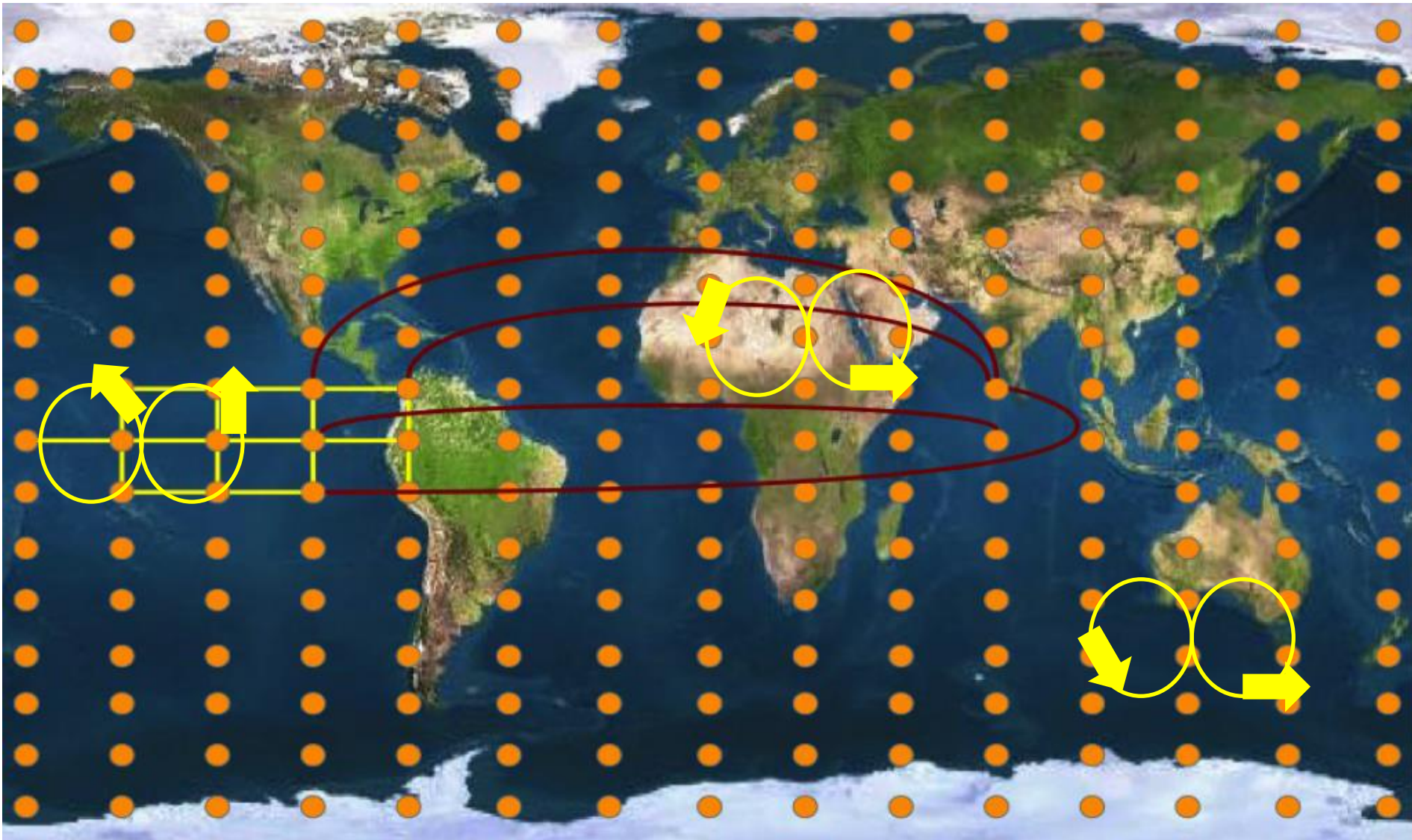
# Functional brain network

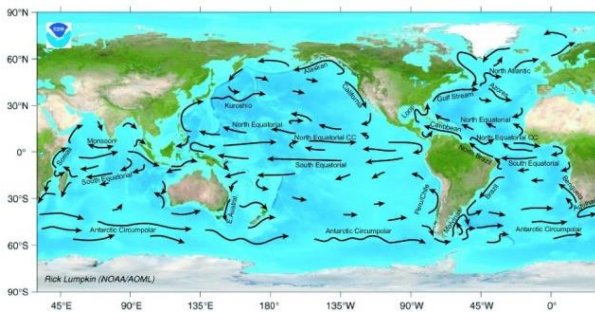


*Eguiluz et al, PRL 2005*



**We can use the same method to construct a climate network**





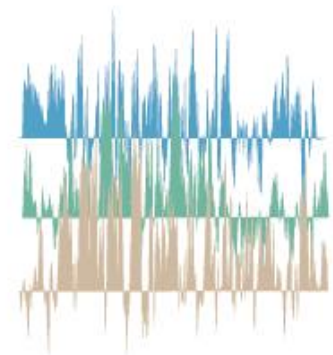
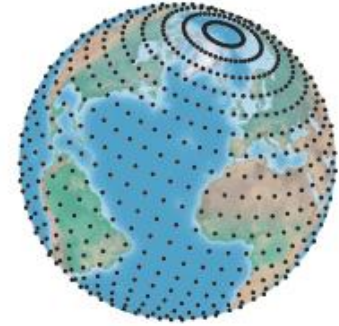
Earth system



(Data  
assimilation)



Grid points / observation sites



Time series data

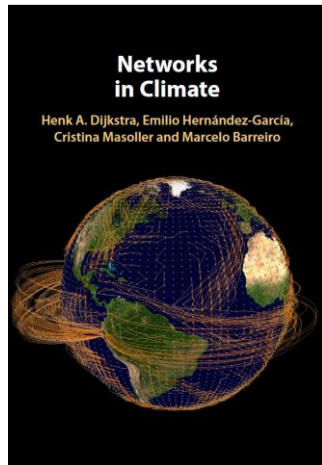


Network analysis

$$b_v^* = \frac{1}{W^2} \sum_{\substack{i,j \in V \\ i,j \neq v}} w_i w_j \frac{\sigma_{ij}^*(v)}{\sigma_{ij}^*}$$



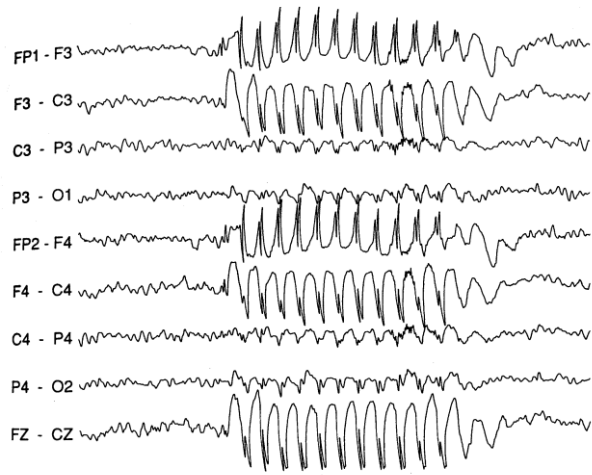
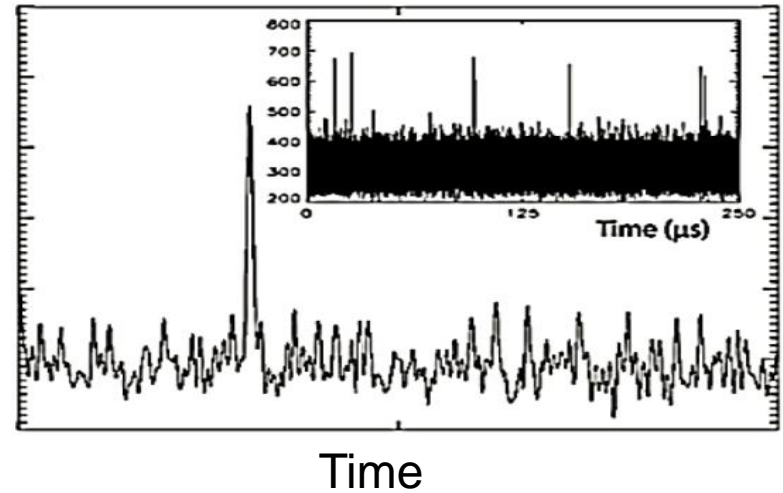
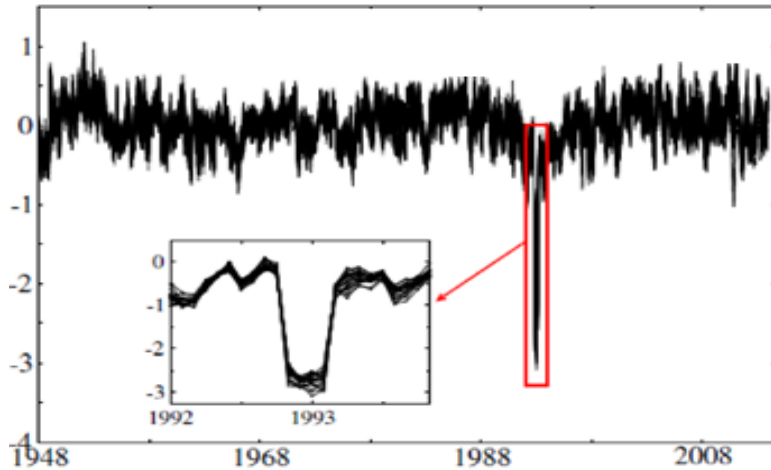
Functional climate network



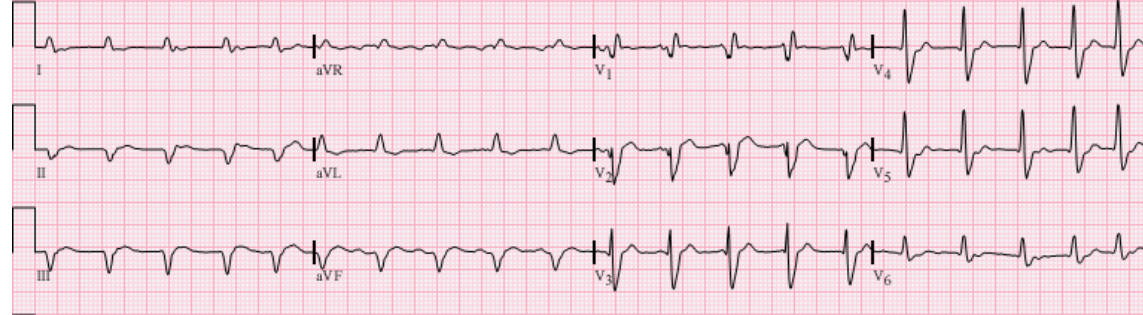
Cambridge  
University  
Press 2019

*Donges et al, Chaos 2015*

# Complex systems often display dynamical transitions and/or large fluctuations



Courtesy of Jason E. Roediger, CCT, CRAT



Source: Wikipedia

Source: P. Suffczynski

1 SEC. 200  $\mu$ V



A main goal is to **predict**, and also, to try to **understand**, the dynamical behavior.



# Are extreme fluctuations becoming more frequent? More extreme?



ECMWF



Credit: Richard Williams, North Wales, UK



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# Rogue waves occurring less but 'becoming more extreme'

By Rebecca Morelle  
Science Correspondent, BBC News

🕒 21 March 2019



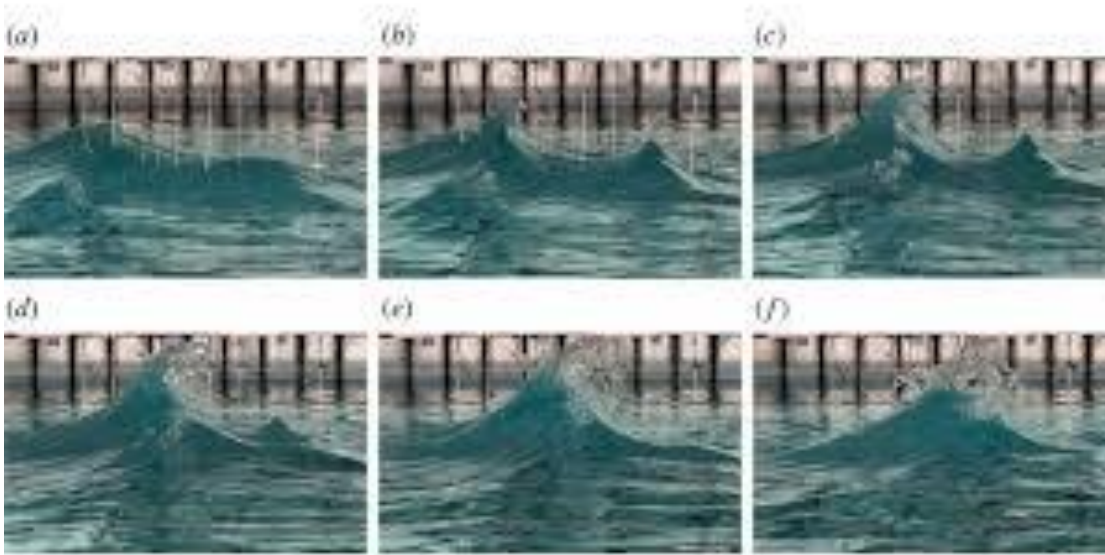
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Rogue waves are a growing threat for the global shipping industry

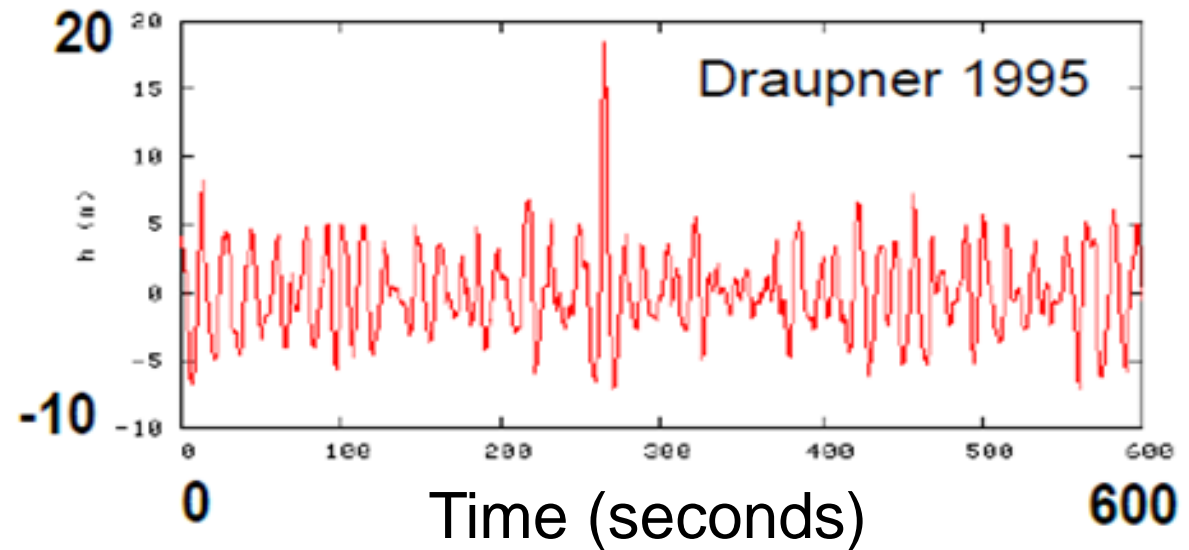




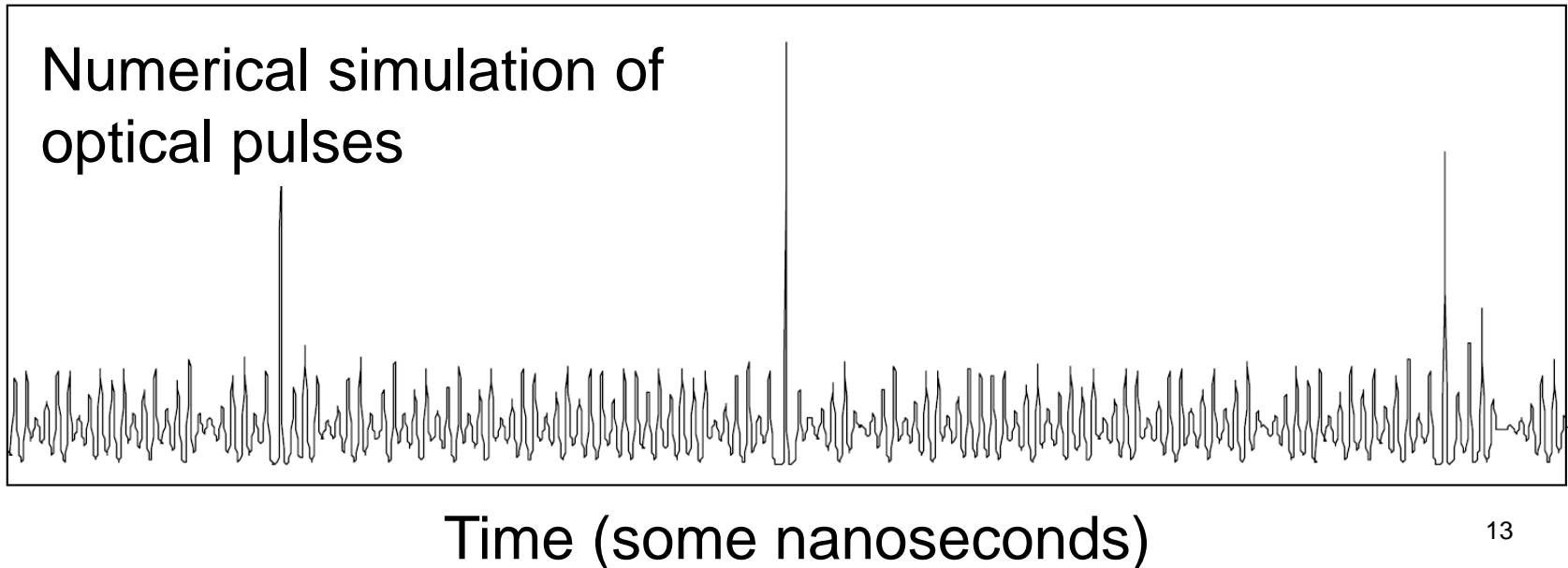
Rogue waves are a huge risk for the global shipping industry.

Are rogue waves in the ocean similar to those in a water tank, or in an optical system?

# Ocean rogue waves and optical rogue waves



Numerical simulation of  
optical pulses



# Time series analysis can unveil underlying statistical similarities in very different systems

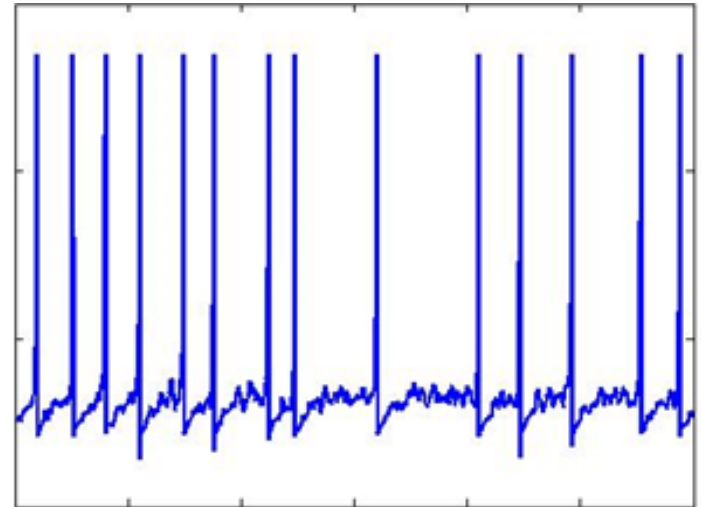
## Laser intensity



Time

$10^{-9}$  s

## Neuron model

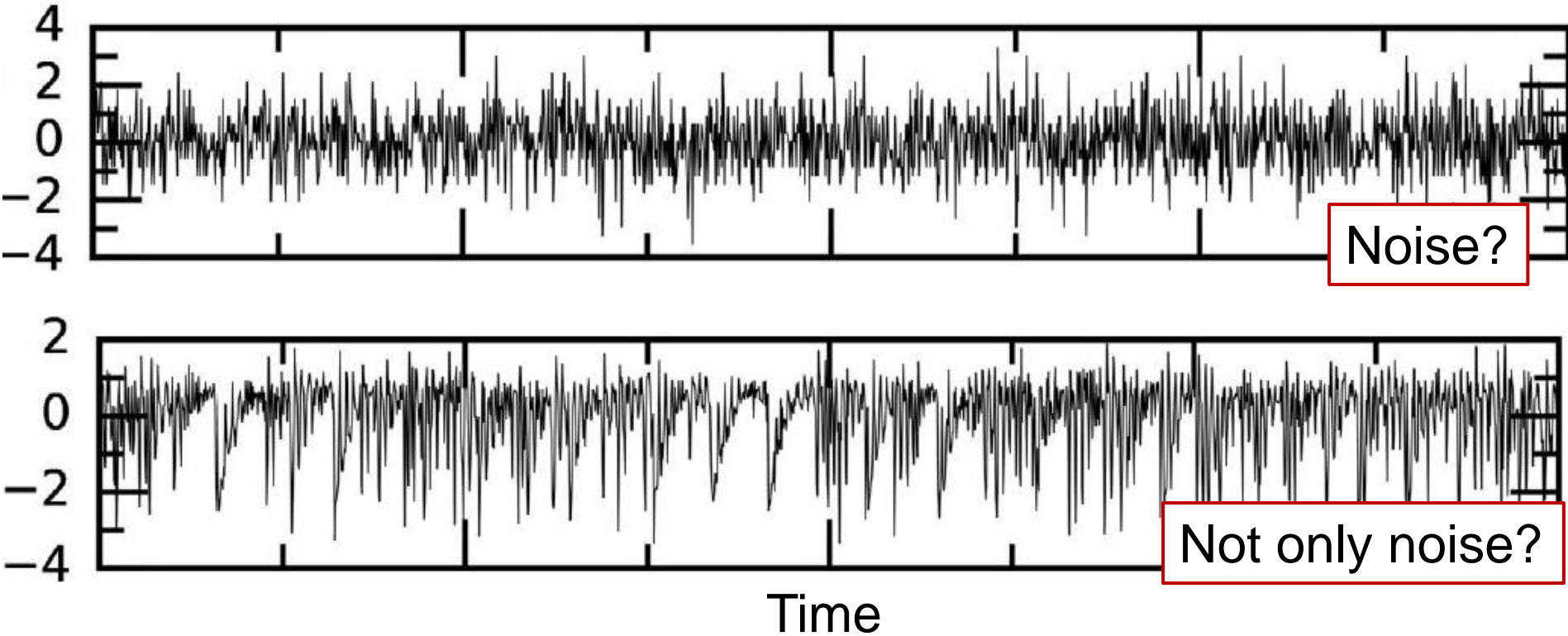


Time

$10^{-3}$  s



# Methods of time-series analysis allow to detect and to quantify differences

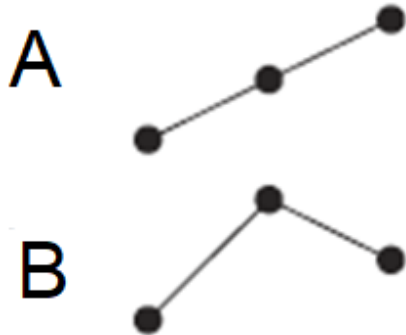


# First analysis method: ordinal analysis

$$\{\dots X_i, X_{i+1}, X_{i+2}, \dots\}$$

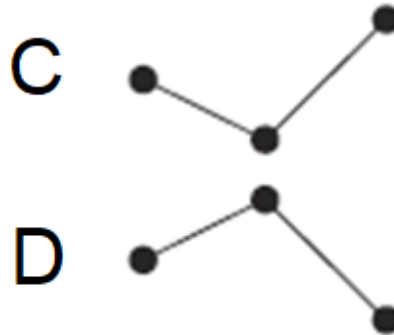
Possible order relations among three numbers (e.g., 2, 5, 7)

$\{\dots 2, 5, 7 \dots\}$



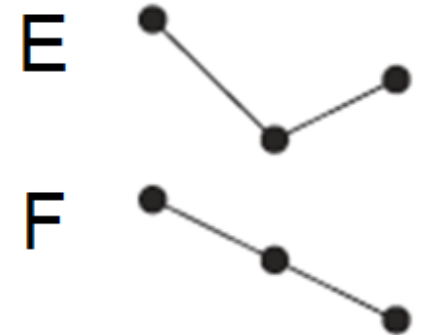
$\{\dots 2, 7, 5 \dots\}$

$\{\dots 5, 2, 7 \dots\}$



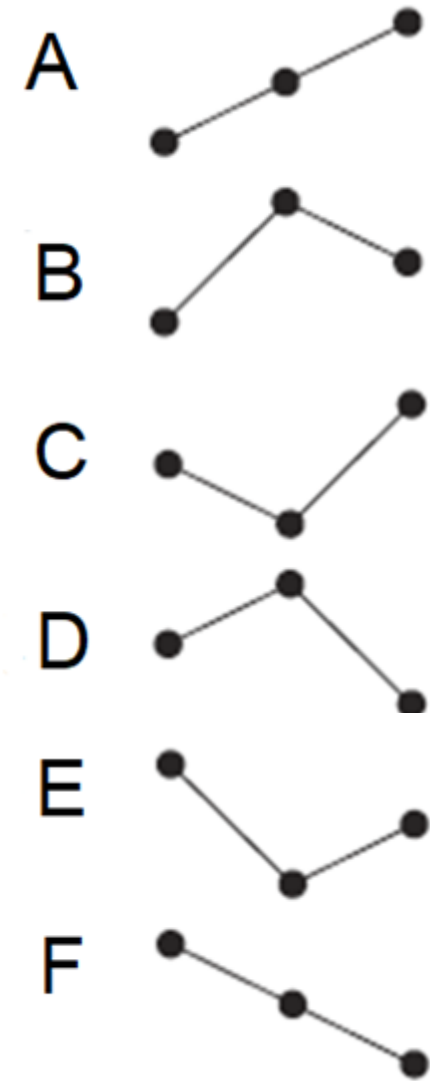
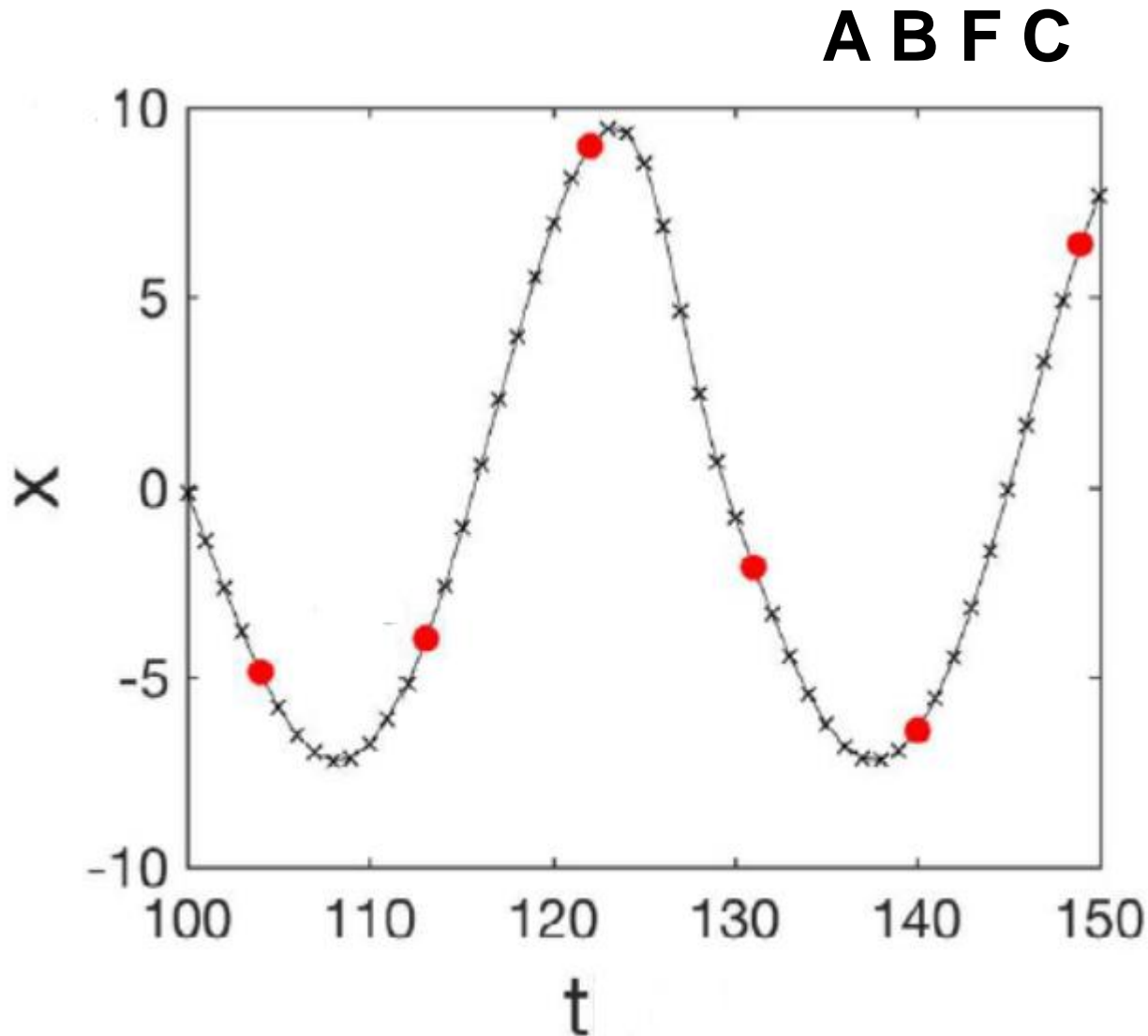
$\{\dots 5, 7, 2 \dots\}$

$\{\dots 7, 2, 5 \dots\}$



$\{\dots 7, 5, 2 \dots\}$

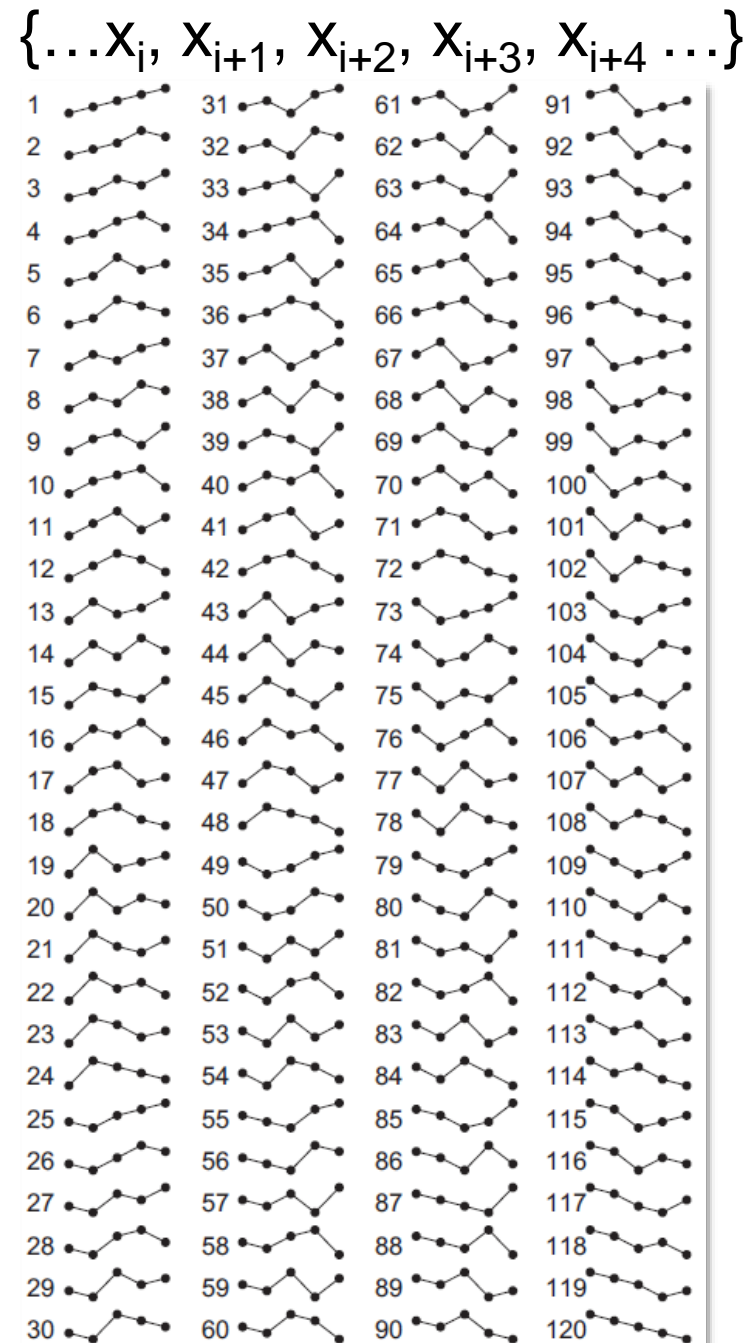
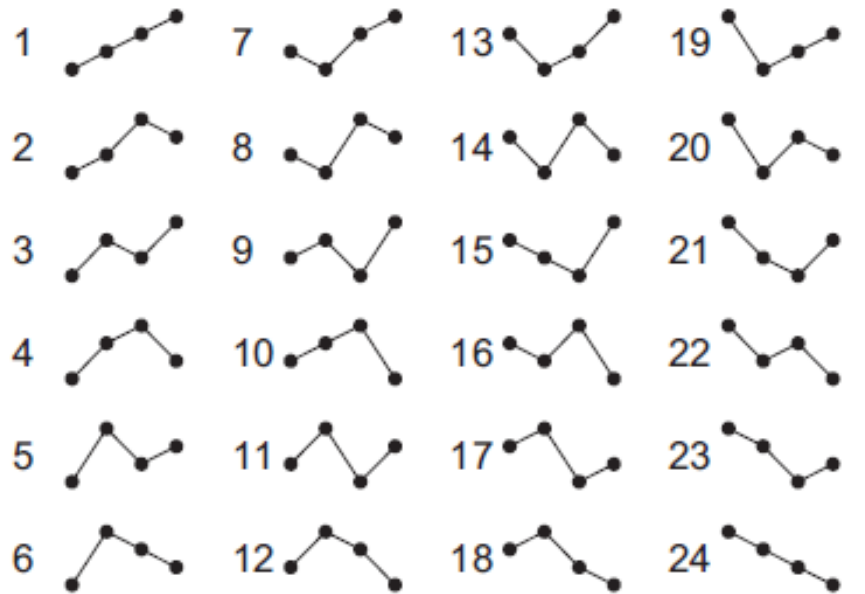
Which is the sequence of “letters” (patterns) defined by the red dots?



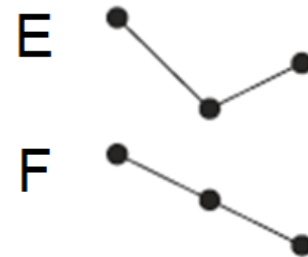
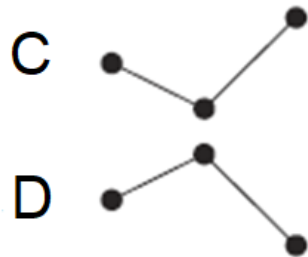
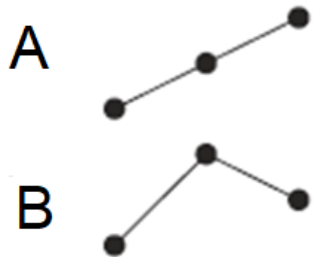
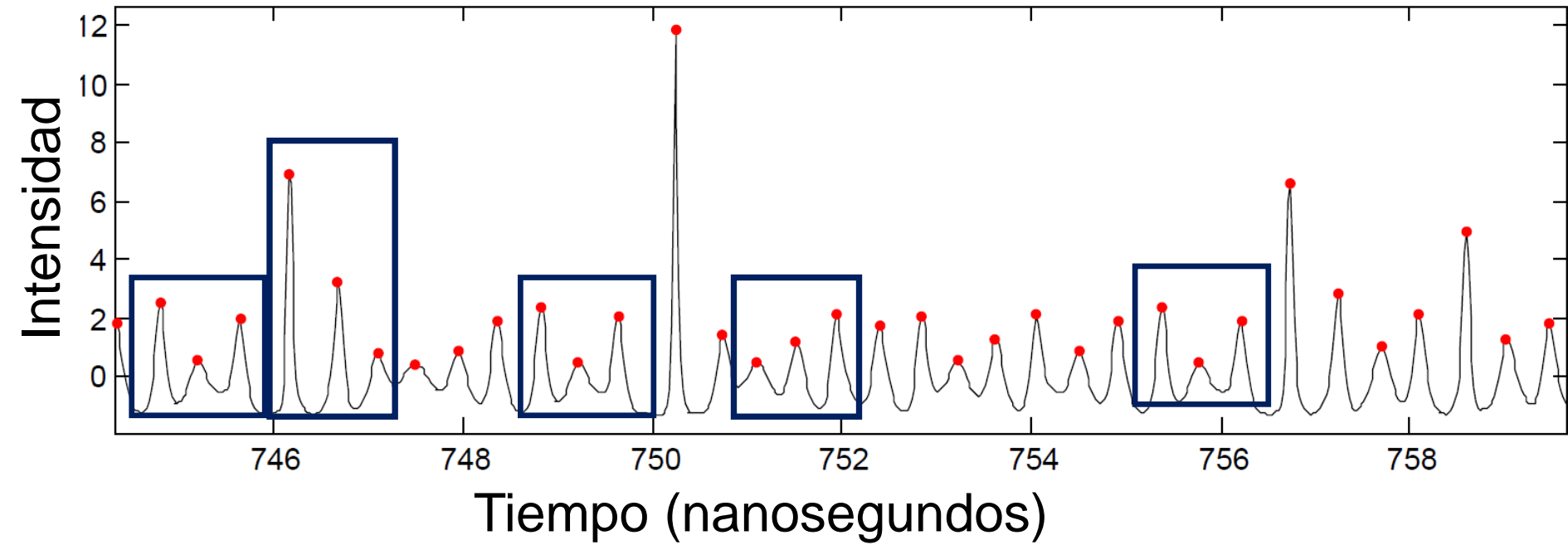


# Possible number of patterns increases as D! (D=size of pattern)

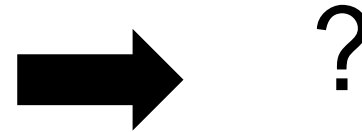
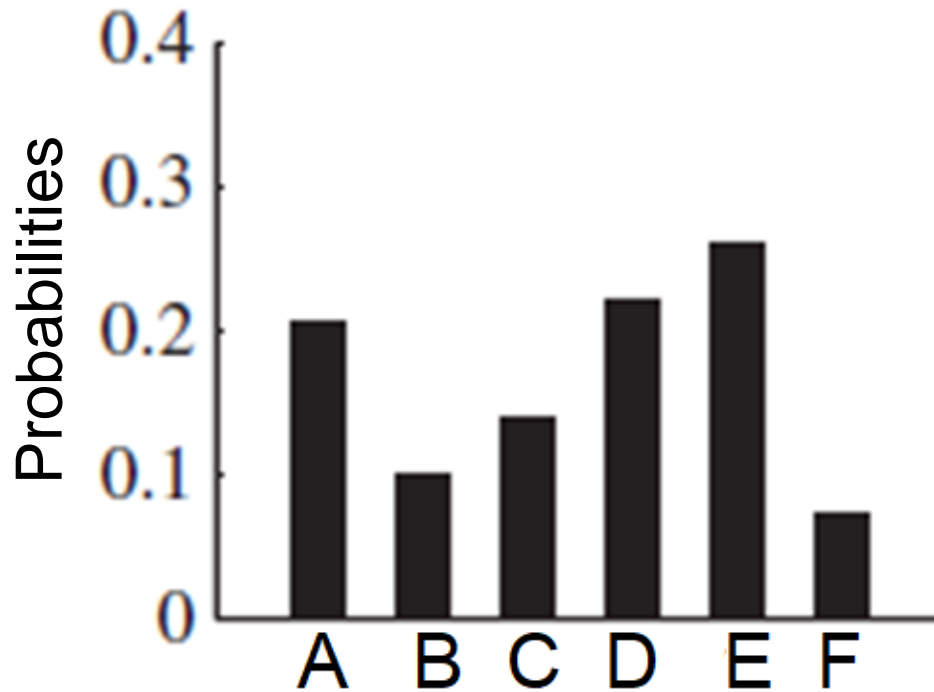
$$\{\dots X_i, X_{i+1}, X_{i+2}, X_{i+3} \dots\}$$



# Which pattern occurs before high pulses?



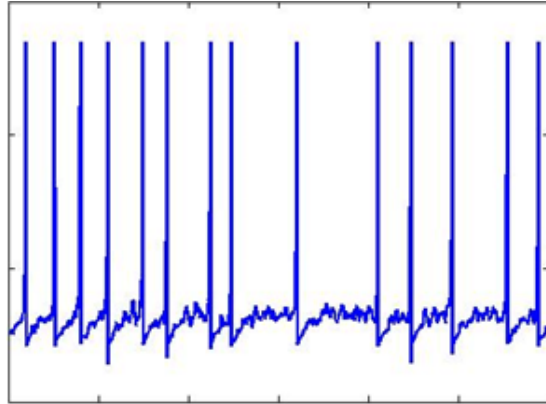
**For each time series we compute the probabilities of the different patterns**



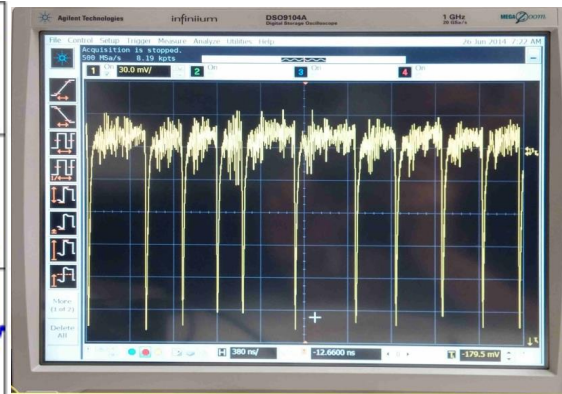
- A. Analyze probabilities.
- B. Compute information theory measures (entropy, complexity).

# A. By analyzing the ordinal probabilities (from inter-spike intervals, ISIs) we uncover laser-neuron similarities

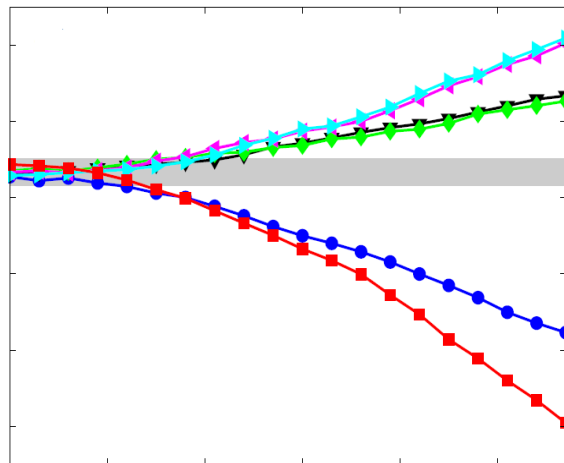
Neuron model



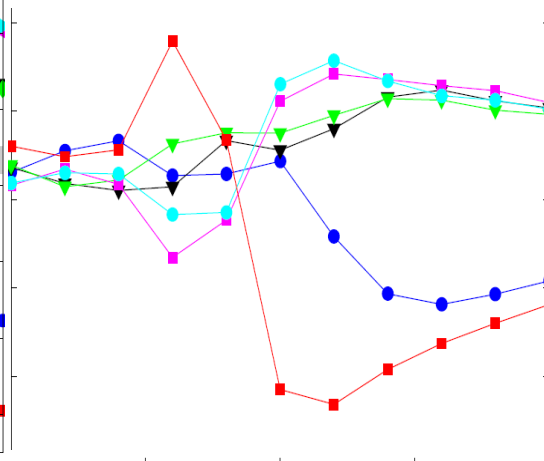
Diode laser with feedback



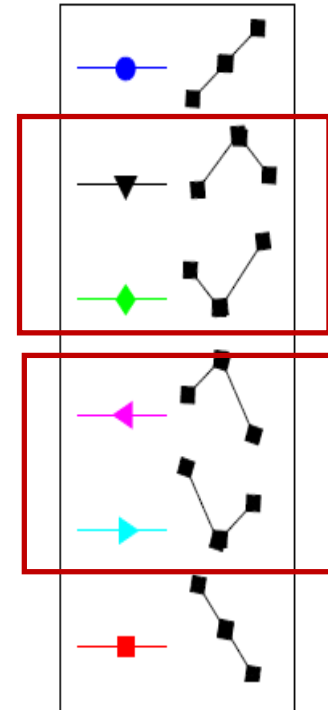
Ordinal probabilities



Forcing amplitude



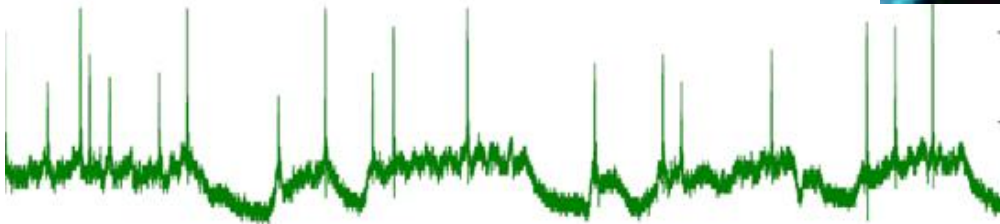
Forcing amplitude





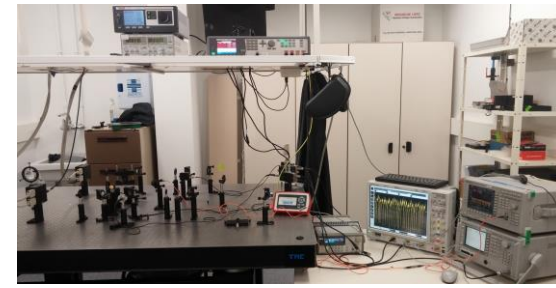
# Interesting but useful? Maybe...

- Photonic neurons.
- Diode lasers: very low cost, highly energy efficient.
- Very very fast.
- Main challenge: understand how neurons receive, encode and process information.

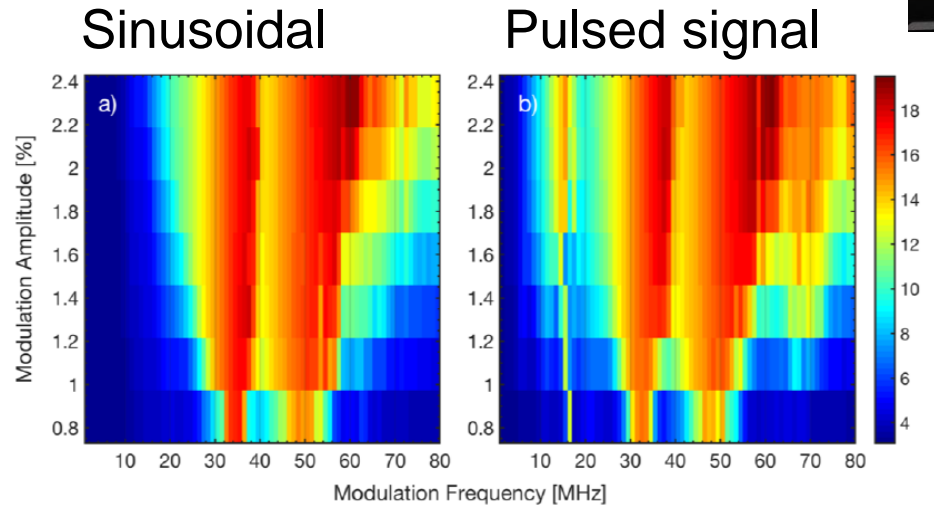


# How a diode laser and a neuron encode a weak periodic signal?

## spike rate code?

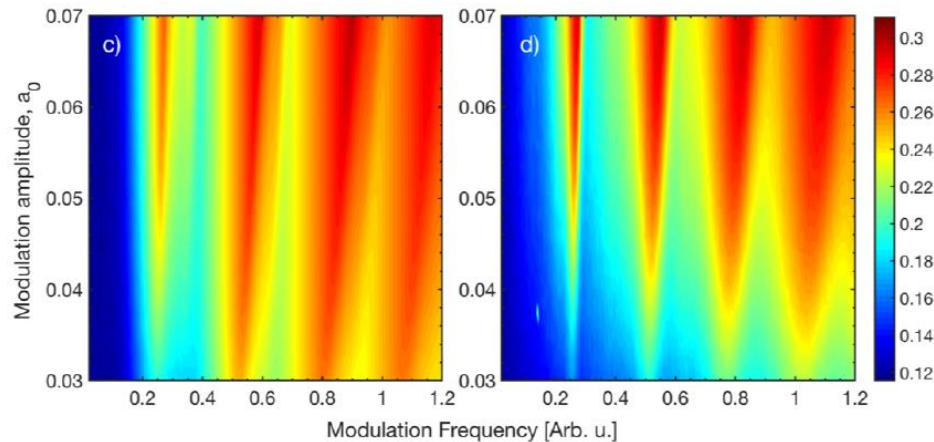


Diode laser with optical feedback (experiments modulating the laser current)



**Spike rate in color code**

FitzHugh-Nagumo neuron model (with the same input signal)



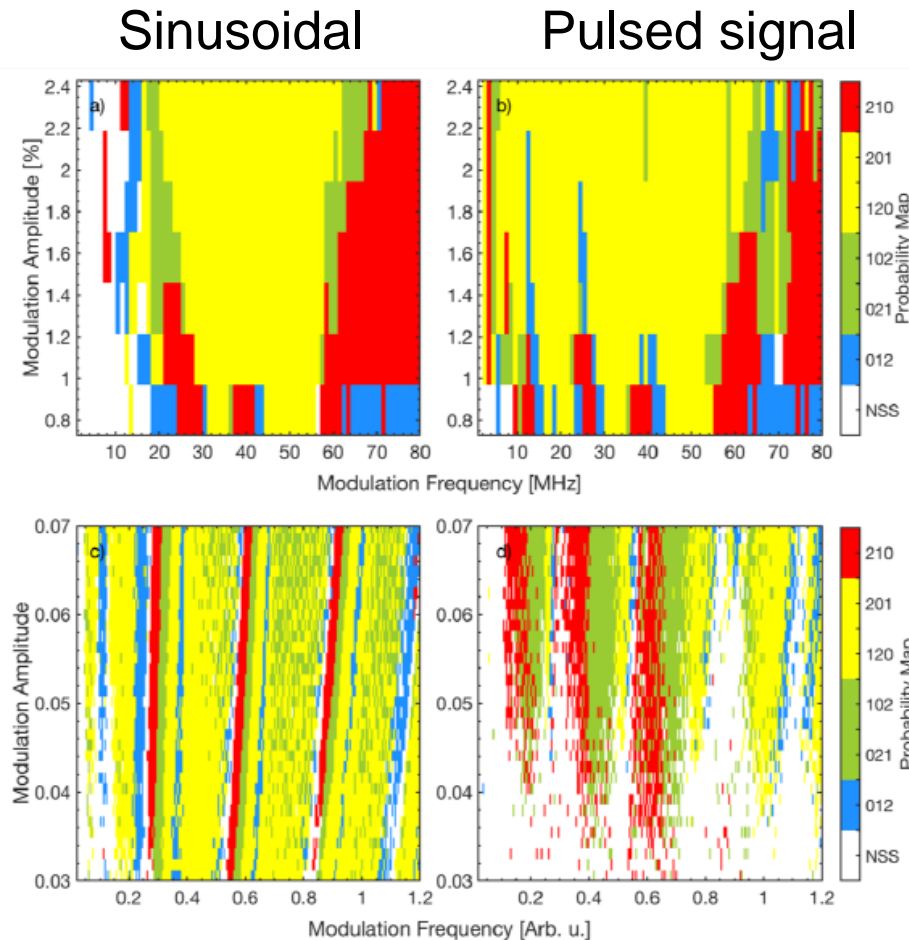
J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, “*Comparing the dynamics of periodically forced lasers and neurons*”, submitted (2019).

# Temporal code?

Ordinal analysis unveils some differences in spike timing.

Diode  
laser with  
optical  
feedback

FitzHugh-  
Nagumo  
model



**Most  
probable  
pattern in  
color  
code**

J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, “*Comparing the dynamics of periodically forced lasers and neurons*”, submitted (2019).

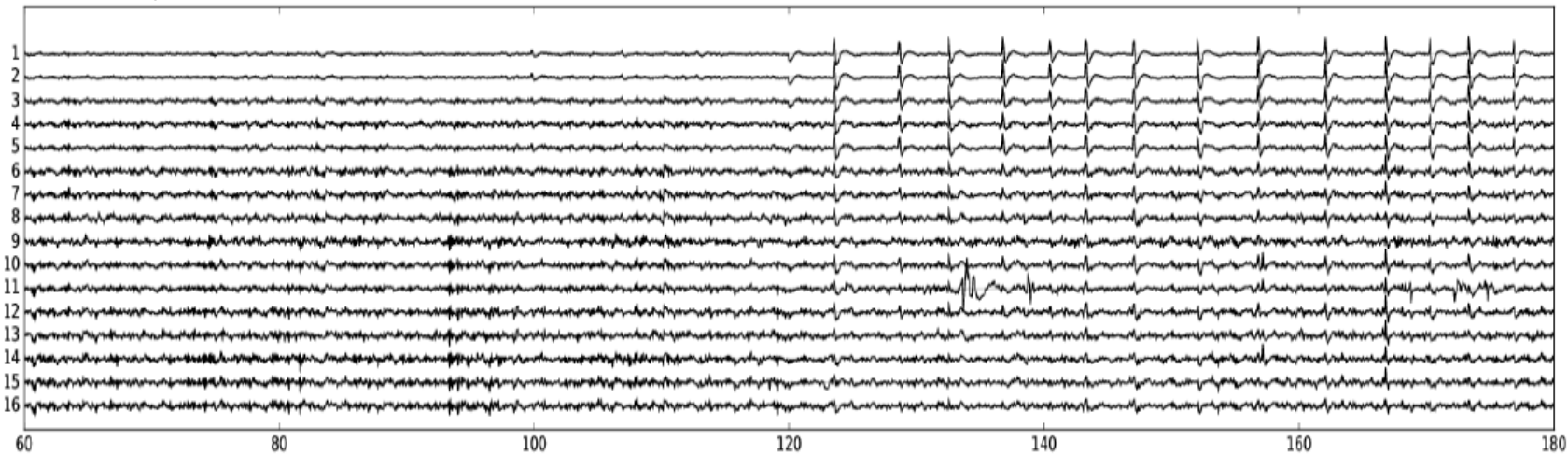
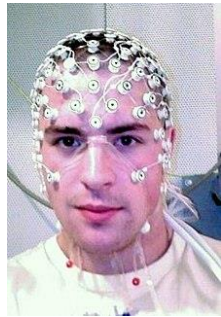
## B. By computing information theory measures (entropy, complexity) we can characterize dynamical transitions

- The time-series is described by a set of probabilities  $\sum_{i=1}^N p_i = 1$
- **Shannon** entropy:  $H = -\sum_i p_i \log_2 p_i$
- Interpretation: “*quantity of **surprise** one should feel upon reading the result of a measurement*”  
K. Hlavackova-Schindler et al, Physics Reports 441 (2007)
- The entropy computed from the ordinal probabilities is known as **permutation entropy**.

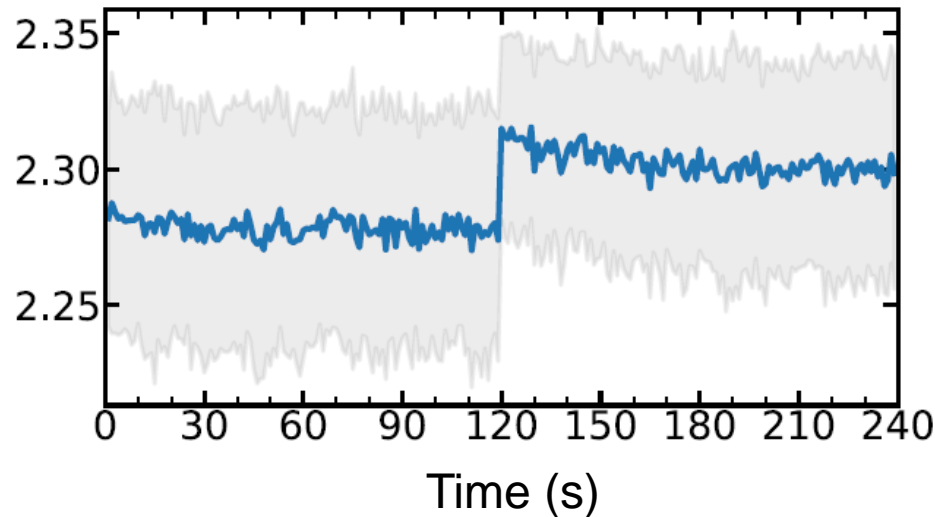


Eyes closed

Eyes open

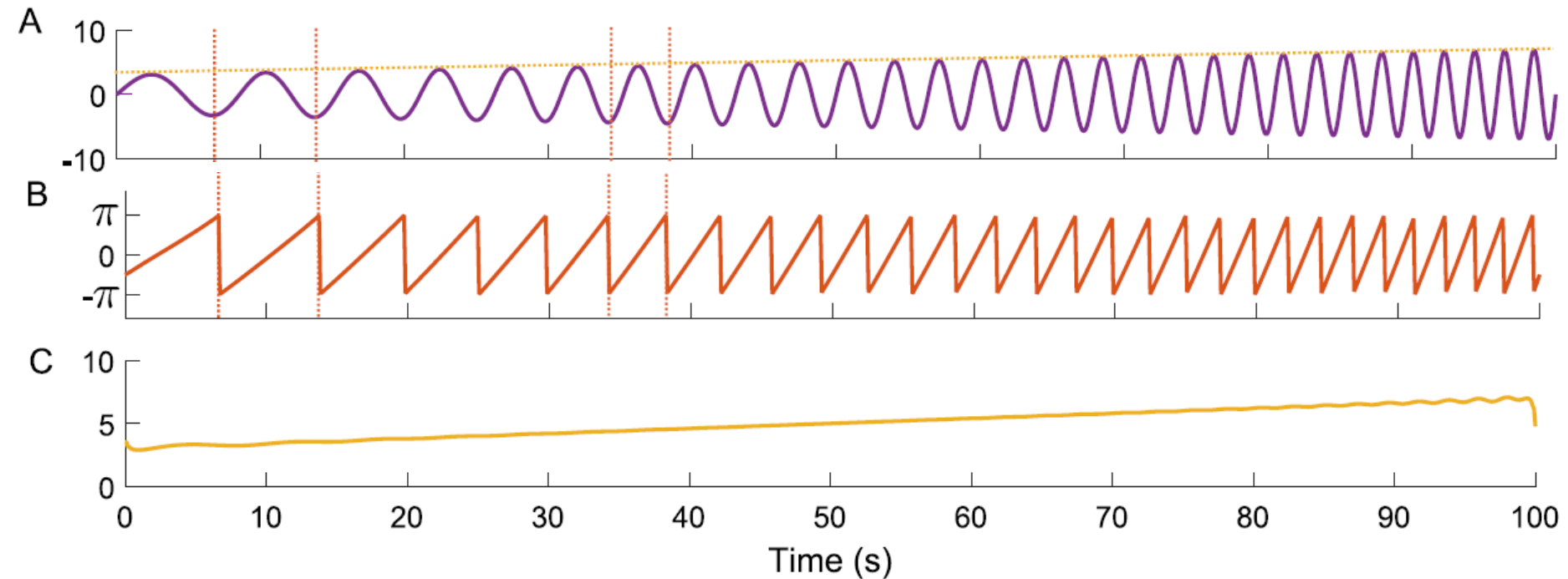


$\langle \text{Permutation entropy (D=4)} \rangle$

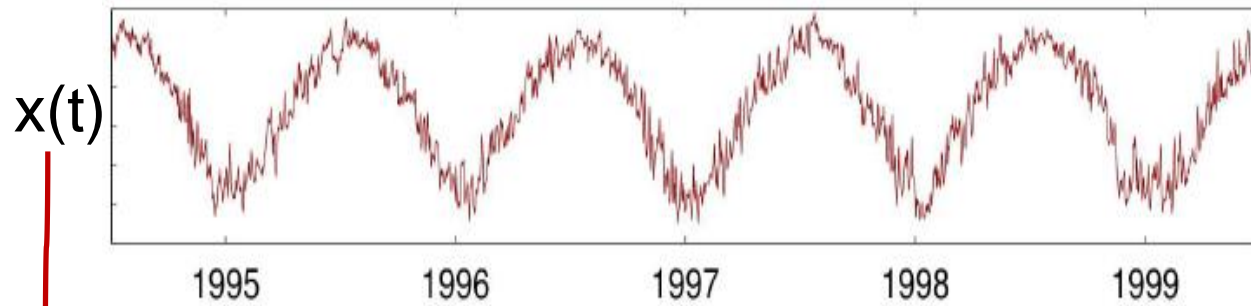


C. Quintero-Quiroz et al, “*Differentiating resting brain states using ordinal symbolic analysis*”, Chaos 28, 106307 (2018).

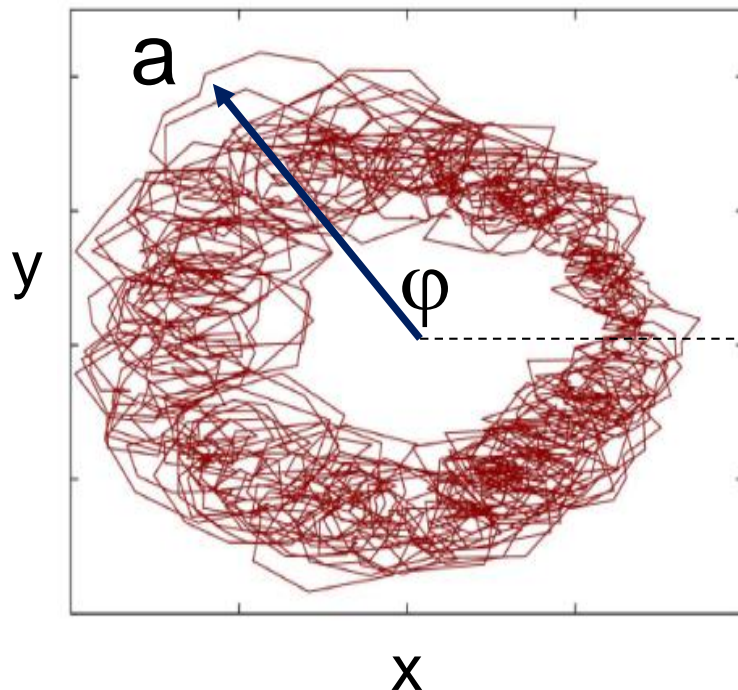
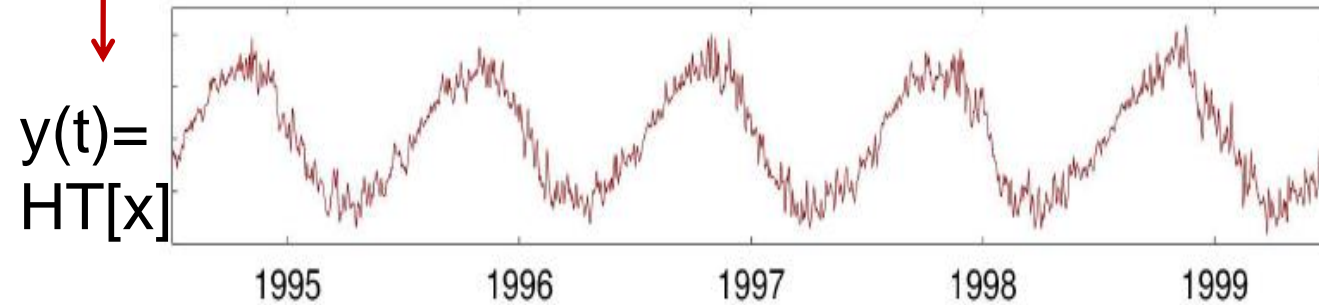
## Second analysis method: The Hilbert transform



- (A) The original signal. (B) The instantaneous phase extracted using the Hilbert transform. (C) The instantaneous amplitude.
- $A = C \cos(B)$ .
- How to calculate the instantaneous amplitude and phase?



Surface air temperature



$$a_j(t) = \sqrt{x_j^2(t) + y_j^2(t)}$$

$$\varphi_j(t) = \arctan[y_j(t)/x_j(t)]$$

The phase has clear physical meaning of rotation only if the signal is “narrow band”.

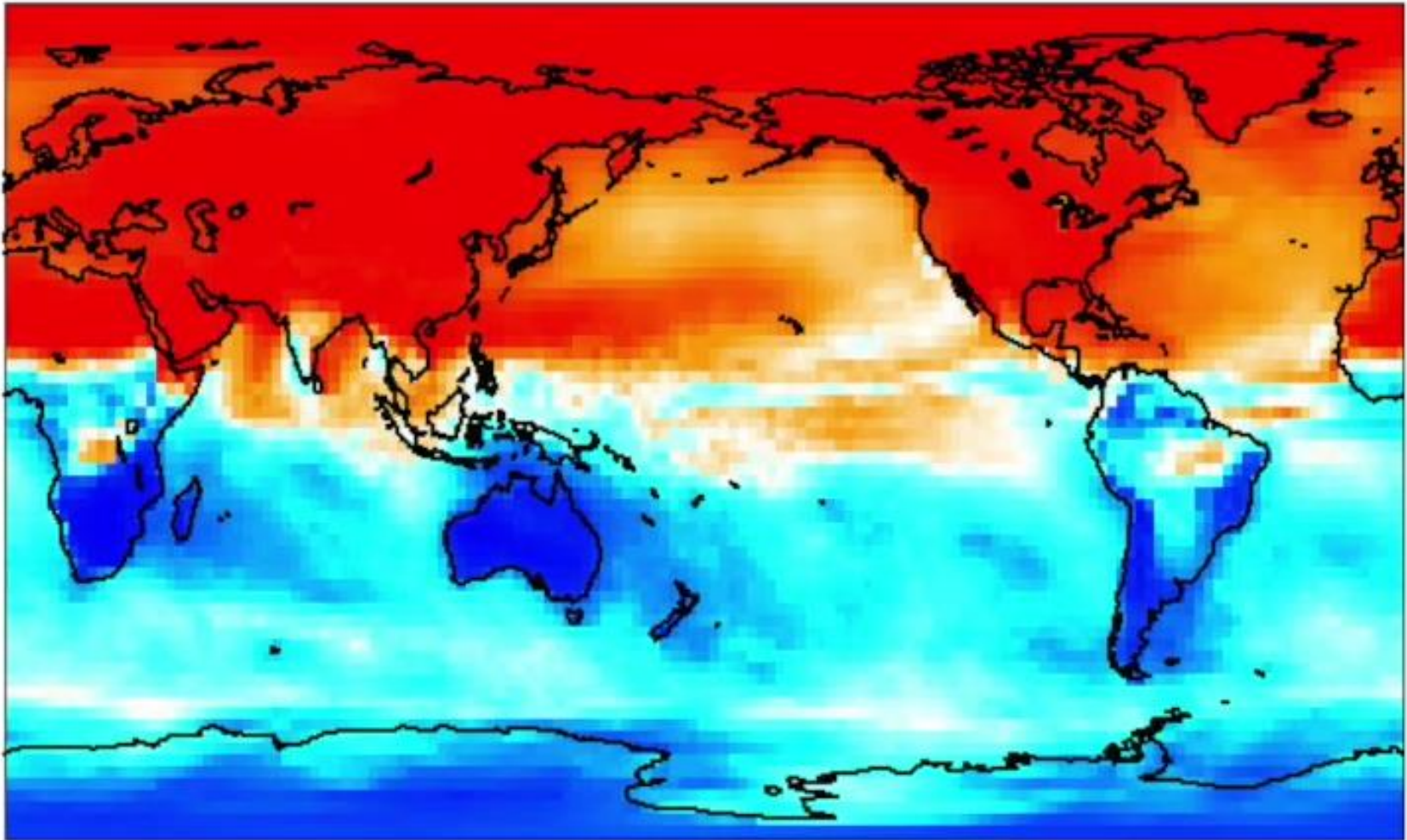
Problem: climate dynamics involves many time scales (days – decades).



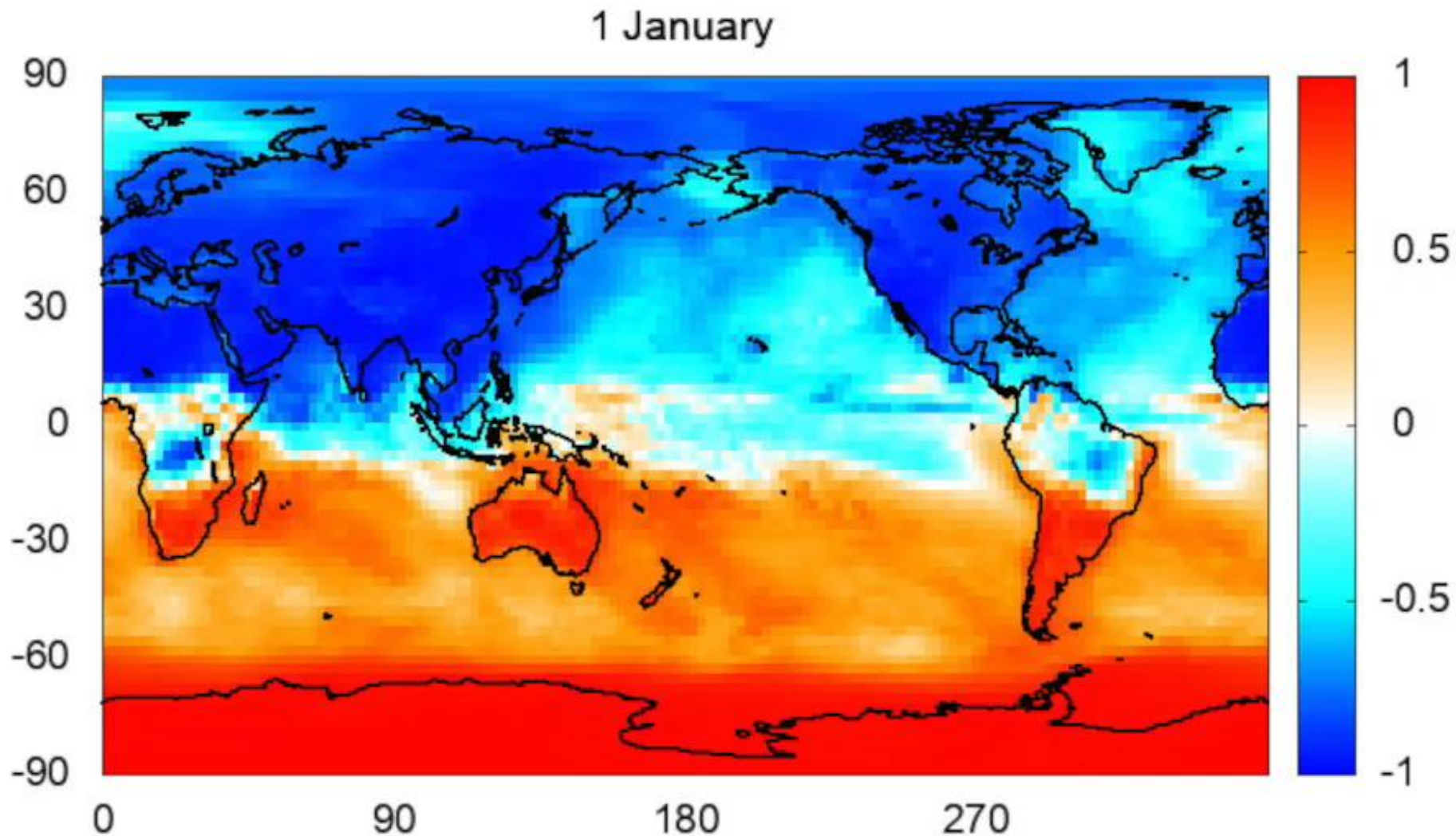
**Cosine of the Hilbert phase in color code.**

**Where the data comes from? Daily reanalysis, 1979 to 2017**

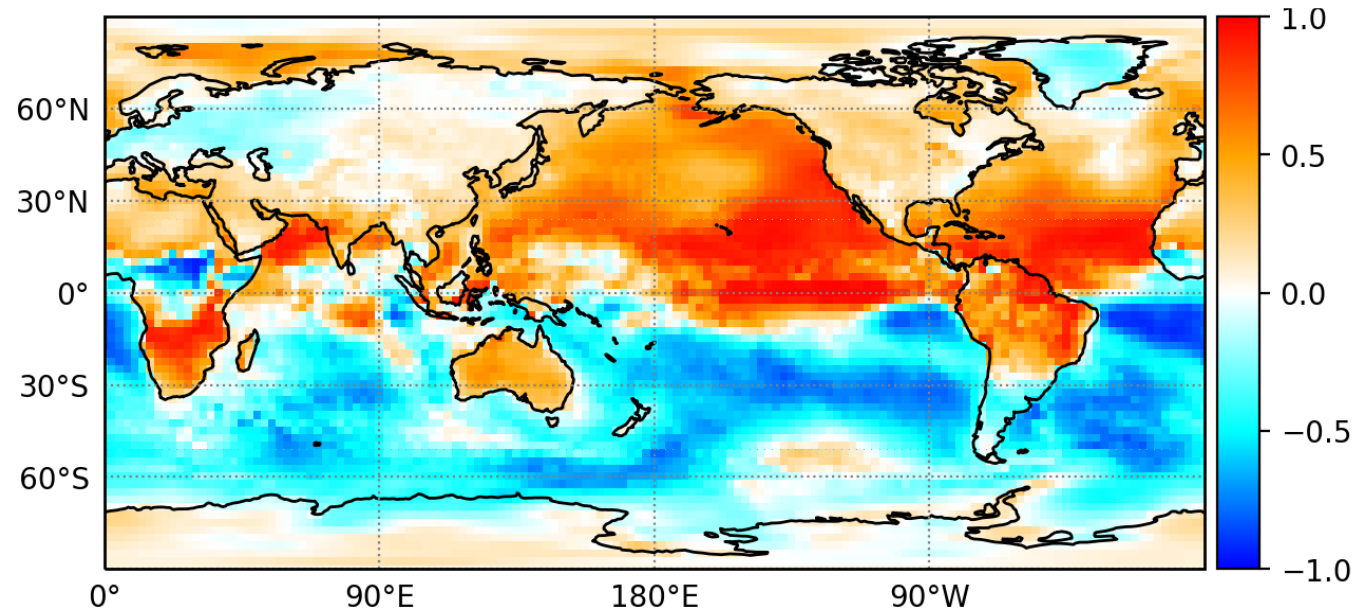
1 July



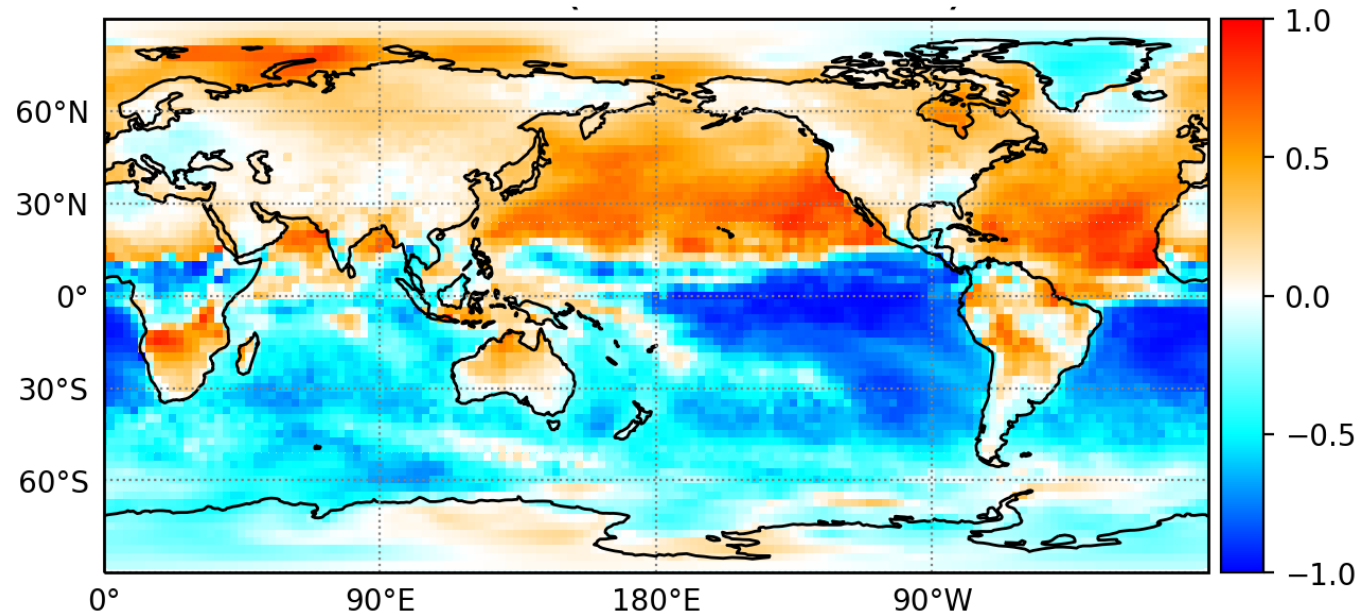
# Hilbert visualization of the seasons: temporal evolution of the cosine of the phase



El Niño period  
(October 2015)



La Niña period  
(Octubre 2011)





# How to detect significant changes in the last 30 years?

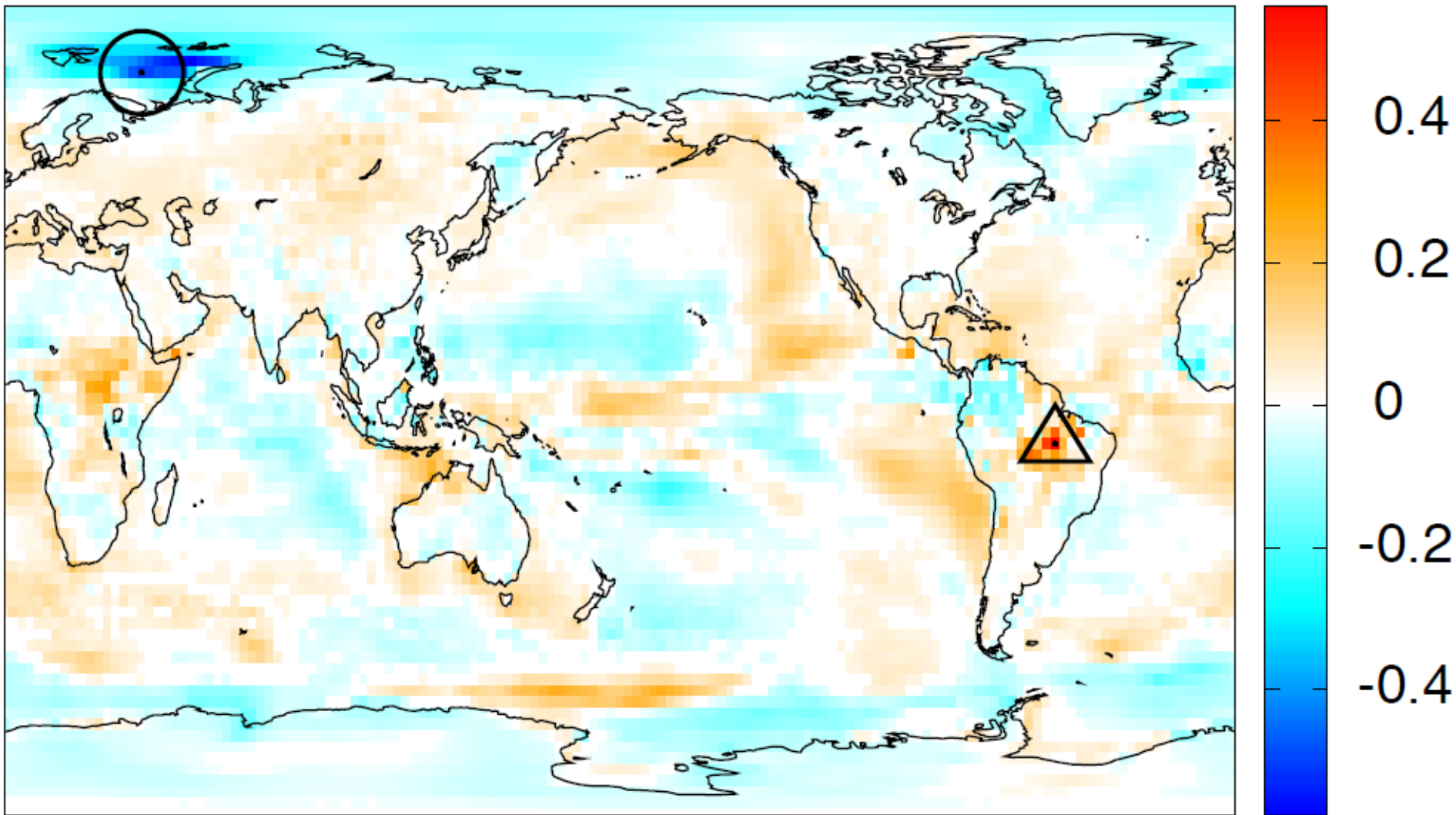
$$\Delta a = \langle a \rangle_{2016-2007} - \langle a \rangle_{1988-1979}$$

$$\frac{\Delta a}{\langle a \rangle_{2016-1979}}$$

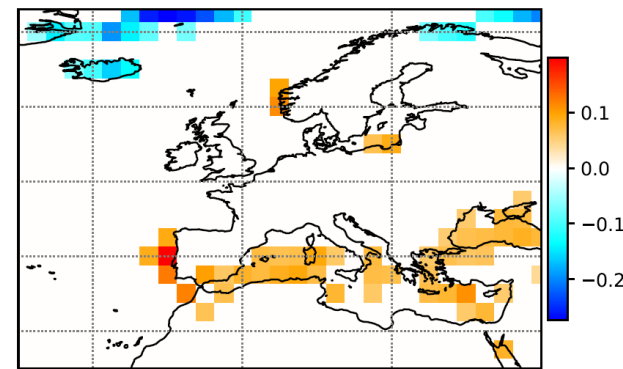
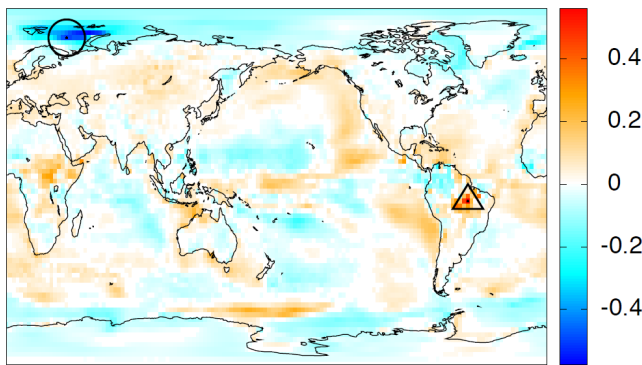
Significant if  $\frac{\Delta a}{\langle a \rangle} \geq \langle . \rangle_s + 2\sigma_s$  or  $\frac{\Delta a}{\langle a \rangle} \leq \langle . \rangle_s - 2\sigma_s$

Average over 100 “surrogates”





D. A. Zappala, M. Barreiro, and C. Masoller, “*Quantifying changes in spatial patterns of surface air temperature dynamics over several decades*”, *Earth Syst. Dynam.* **9**, 383–391 (2018).



34

**Amazonas**

20

0

**Artic**

1985

1990

1995

2000

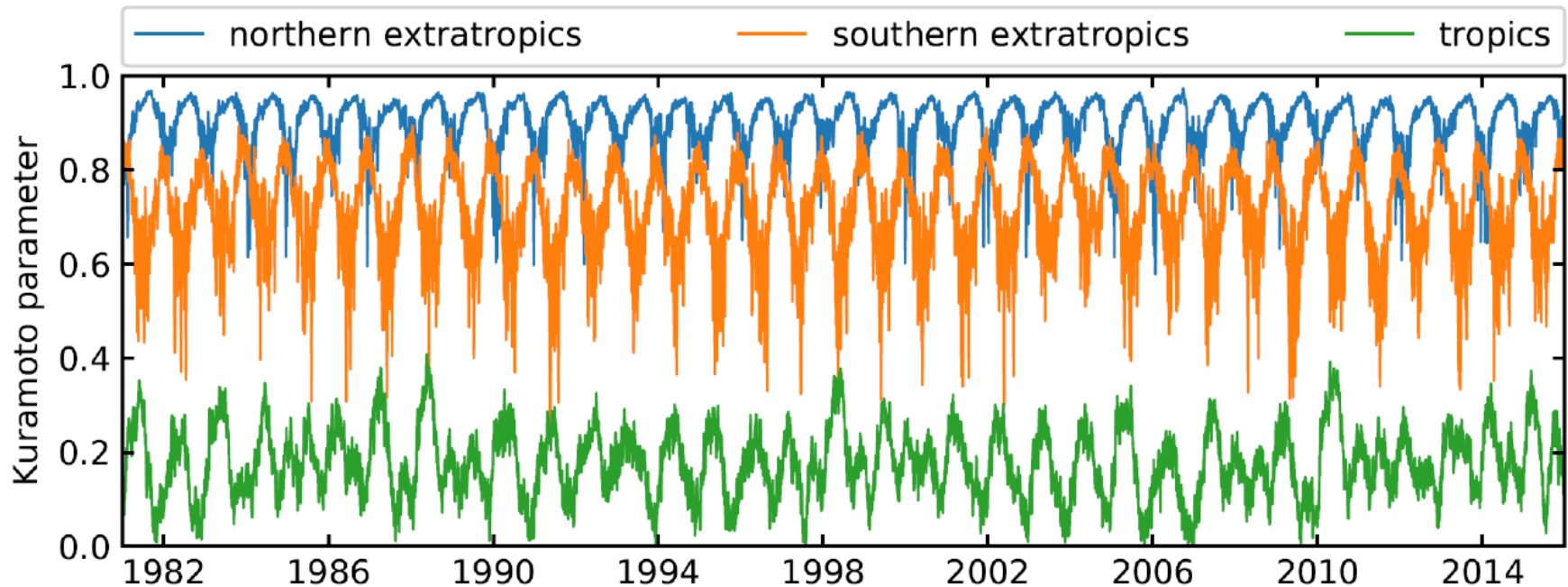
2005

2010

D. A. Zappala, M. Barreiro, and C. Masoller, “Quantifying changes in spatial patterns of surface air temperature dynamics over several decades”, *Earth Syst. Dynam.* **9**, 383–391 (2018).

# Quantification of phase synchronization

area-weighted Kuramoto parameter:  $r(t) = \left| \frac{\sum_{j \in S} w_j \exp[i\varphi_j(t)]}{\sum_{j \in S} w_j} \right|$

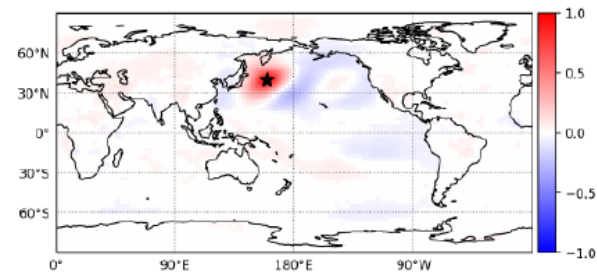


D. A. Zappala, M. Barreiro and C. Masoller, “*Quantifying phase synchronization and unveiling Rossby wave patterns in surface air temperature dynamics*”, in preparation.

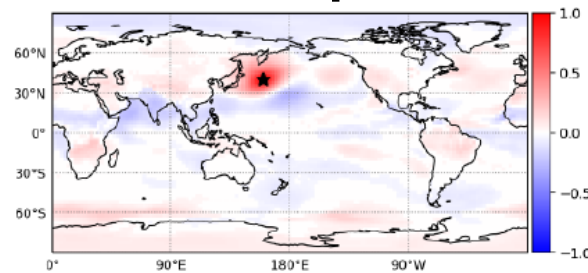


# Identification of Rossby waves

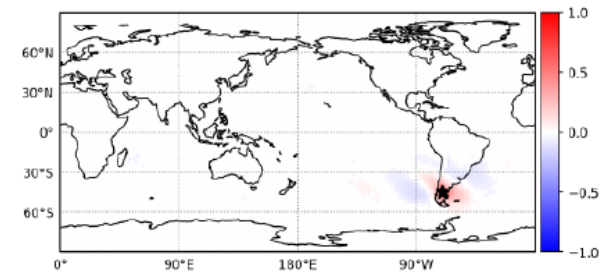
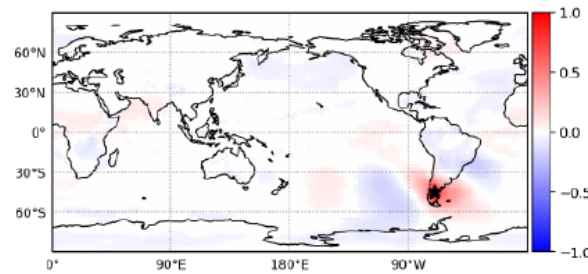
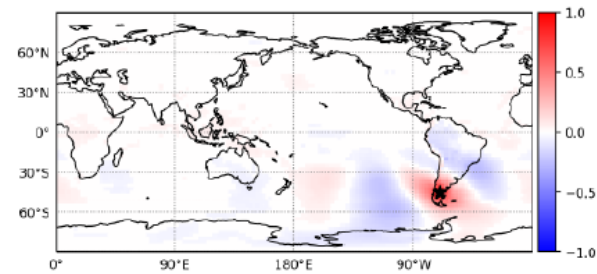
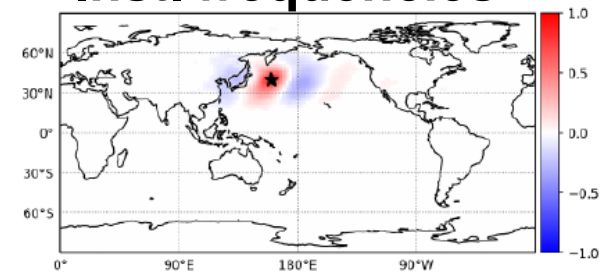
## Anomalies



## Inst. amplitudes



## Inst. frequencies



**Cross-correlation in color code.**

D. A. Zappala, M. Barreiro and C. Masoller, “*Quantifying phase synchronization and unveiling Rossby wave patterns in surface air temperature dynamics*”, in preparation.

# Take home message

- Time series analysis allows to understand, predict, or classify dynamical behaviors of complex systems.
- Even when the data does not meet the mathematical or algorithmic requirements, the results can give useful insights.
- Many interdisciplinary applications.



Thanks to:

Jordi Tiana (UPC)

Carlos Quintero (UPC)

Dario Zappala (UPC)

Marcelo Barreiro (Uruguay)

**Thank you for your attention!**

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<http://www.fisica.edu.uy/~cris/>



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