Inferring signatures of determinism in stochastic complex systems

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Event level description of dynamical complex systems

- Sequences of events generated by complex systems
  - Intervals between threshold crossings and barrier crossings,
  - Neurons: inter-spike intervals (ISIs),
  - Human communication: inter-event user times (SMS, emails, Twitters).
  - Earth and climate: earthquakes, extreme events (tornados, rainfalls), etc.

- Interplay of
  - Different time scales, memory
  - Nonlinear, high dimensional & stochastic effects

- The identification of patterns in the sequence of events can allow for
  - Model verification, parameter estimation
  - Classification of different types of dynamical behaviors
  - Improving predictability and forecasting
Introduction: semiconductor lasers with feedback as high-dimensional & stochastic dynamical systems

Method of time-series analysis and experimental setup

Results. Experimental and model observations: inferring signatures of determinism + response to periodic forcing

Conclusions and take home message
Why semiconductor lasers?

- SLs have many advantages:
  - compact, fast, reliable, inexpensive
  - wide range of wavelengths

- Used in
  - Telecommunications
  - Data storage (CDs, DVDs, Blu rays)
  - Barcode scanners, printers, mouse
  - Material processing
  - Biomedical applications (imaging, sensing, etc)
With optical feedback the laser intensity displays dropouts similar to neuronal spikes.

- Feedback delay time
- Noise
- Nonlinearity

Stochastic and high-dimensional system
— to develop a method of time-series analysis that allows inferring signatures of determinism in the sequence of optical spikes;

— to extract new information;

— to compare model predictions with observations;

— to explore potential for building optical neurons.
Governing equations


$|E|^2 \sim$ photon number (output intensity)

$N \sim$ number of carriers (electron-holes)

\[
\frac{dE}{dt} = \frac{1}{2\tau_p} (1 + i\alpha)(G - 1)E + \eta E(t - \tau)e^{-i\omega_0 \tau} + \sqrt{2\beta_{sp}} \xi
\]

\[
\frac{dN}{dt} = \frac{1}{\tau_N} \left( \mu - N - G|E|^2 \right)
\]

Gain: $G = N / (1 + \varepsilon|E|^2)$

feedback

noise

$\eta =$ feedback strength

$\tau =$ feedback delay time

$\mu =$ pump current

(control parameter)

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The dropouts are a **transient** dynamics.

Burst of dropouts are triggered by **noise**.

In experimental sequences of dropouts: which ones are **deterministic** and which ones are **stochastic**?

Main problem: we can measure only one relevant variable (the laser intensity)

Also a problem: the measure system (photodiode, oscilloscope) has a finite *bandwidth* that gives a limited temporal resolution.

Approach: event-level description. We study the sequence of *inter-dropout-intervals*: $\Delta T_i = t_{i+1} - t_i$
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Symbolic ordinal analysis

- It has been used to analyze data generated from complex systems
  - Financial, economical
  - Biological, life sciences
  - Geosciences, climate
  - Physics, chemistry, etc

- It has been shown to be able to:
  - Distinguish stochasticity and determinism
  - Classify different types of dynamical behaviors (pathological, healthy)
  - Quantify complexity
  - Identify coupling and directionality.


“words” of **D letters** can be formed by considering the **order relation** between sets of D values \{..., x_i, x_{i+1}, x_{i+2}, ...\}.

For **D=3** there are 6 possible orders:

- 012
- 021
- 102
- 120
- 201
- 210

Example: the set (5, 1, 7) gives “102” because 1 < 5 < 7

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**Advantage:** the transformation keeps information about correlations in the time-series & does not need a threshold

**Drawback:** it does not keep information about the values, the set (5,1,100) also gives word “102”.
How to select $D$? Optimal $D$ depends on:

- The length of the time series.
- The time scale of correlations.

For optical spikes: $D=2$ ($D=3$) unveil correlations of 3 (4) spikes.
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Conclusions and take home message
Laser Diode

50/50 Beamsplitter

External reflector

to Optical Spectrum Analizer

External cavity - 45 cm

Detector to Oscilloscope

Temperature and pump current combi controller

Hitachi Laser Diode (HL6724MG)

$\lambda \sim 674.2\,\text{nm}$

5mW

$\sim 7\%$ threshold reduction

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Experimental inter-dropout-intervals (IDIs in lasers – ISIs neurons)

Laser output
(1 GHz oscilloscope)

\[ \langle \Delta T \rangle = 100-200 \text{ ns} \]
\[ \tau \sim 5 \text{ ns} \]

# of IDIs recorded
45,000 - 220,000

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Is there any information in the ‘spike’ sequence? Analogous to deciphering a foreign text.
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Correlations between 3 consecutive spikes: probabilities of 01 & 10

Consistent with stochastic at low pump current, but signatures of determinism at high pump current.

D=2: 3-spike correlations?

**Null hypothesis**: fully random sequence of spikes $\Rightarrow P(01) = P(10)$

At low pump current: are the spikes fully random? New experiment

45000 - 220000 IDIs


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Also in another data set recorded at a different temperature (T=20 C)
Are the deviations from the null hypothesis significant?

Recorded data

Surrogated data
Which dropouts are noise-induced and which ones are deterministic?

We use a threshold to classify the inter-dropout-intervals as short and long intervals.
At high currents: significant differences
   - LIs consistent with random events
   - SIs more deterministic.

But at low pump currents, the inter-spike-intervals can not be classified in two types with significant differences.

Constructing the words with 3 consecutive SIs or LIs

Similar results in the other dataset (T=20 C)

At high pump currents an adequate threshold allows classifying the events in two distinct categories
Ordinal analysis unveils new information

There is a hierarchical and clustered organization of the probabilities of the words
In another experiment: also the same hierarchy and the same 2 clusters

75,000 – 880,000 dropouts (different laser, new oscilloscope)
Sensitivity to the threshold that defines the event times?

The hierarchy and the clusters are robust to the threshold chosen to define the spike times
Can we find a minimal model that displays these features?

**Logistic map**

**Tent map**

Can we find a minimal model that displays these features?
A modified circle map: minimal phenomenological model

\[ \varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi} \left[ \sin(2\pi \varphi_i) + \alpha \sin(4\pi \varphi_i) \right] \]

\[ X_i = \varphi_{i+1} - \varphi_i \]

\( \rho = 0.23 \)
\( K = 0.04 \)

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Periodic modulation of the laser current

Laser intensity:

Increasing the modulation amplitude

\[ \text{Time (ns)} \]

\[ \lambda = 660 \text{ nm} \]

\[ \lambda = 1550 \text{ nm} \]
Experiments @ 660 nm

(68,000 - 200,000 dropouts)

Minimal circle-map model

$$\phi_{i+1} = \phi_i + \rho + \frac{K}{2\pi} \left[ \sin(2\pi \phi_i) + \alpha \sin(4\pi \phi_i) \right] + D \zeta$$

$$\rho = -0.23$$
$$\alpha = 0.2$$
$$D = 0.02$$

Similar observations @ 1550 nm
Outline

- Introduction: semiconductor lasers with feedback as high-dimensional & stochastic dynamical systems

- Method of time-series analysis and experimental setup

- Results. Experimental and model observations: inferring signatures of determinism + response to periodic forcing

- Conclusions and take home message
We proposed a novel method to infer signatures of determinism in sequences of events in dynamical complex systems. Adequate for high-dimensional & stochastic systems displaying noise or deterministically induced events.

We found new symbolic states with an hierarchical and clustered organization of the probabilities of the patterns.

We identified a minimal phenomenological model.

LK model is in good agreement with observations (not shown because lack of time)

Potential breakthrough: optical neurons for neuro-inspired information processing.
Ordinal analysis is a powerful technique for the event-level description of complex systems

— useful for data understanding and uncovering patterns in the sequence of events,

— useful for improving system modeling, model comparison and parameter estimation,

— useful for classifying different types of behaviors,

— potential for improving event predictability and forecasting.
Thanks to Carme Torrent for your attention!

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Taciano Sorrentino
Carme Torrent

Papers (@ www.fisica.edu.uy/~cris)

- J. Zamora-Munt et al, PRA 2010
- N. Rubido et al, PRE 2011