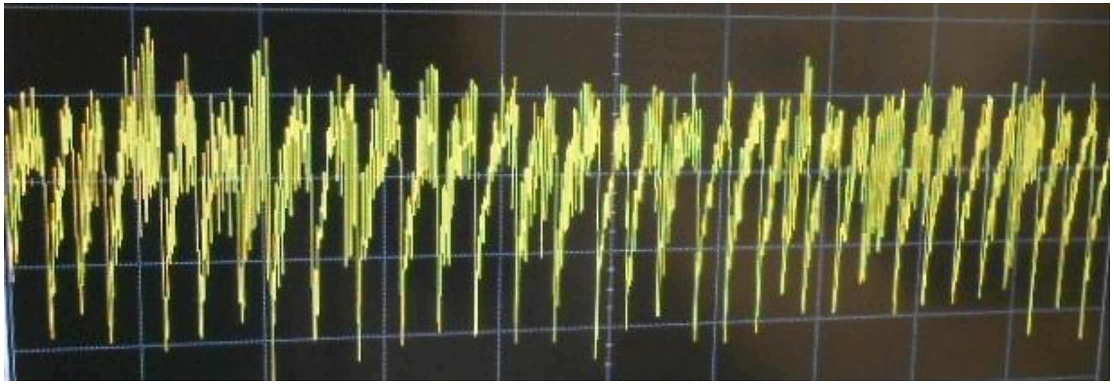

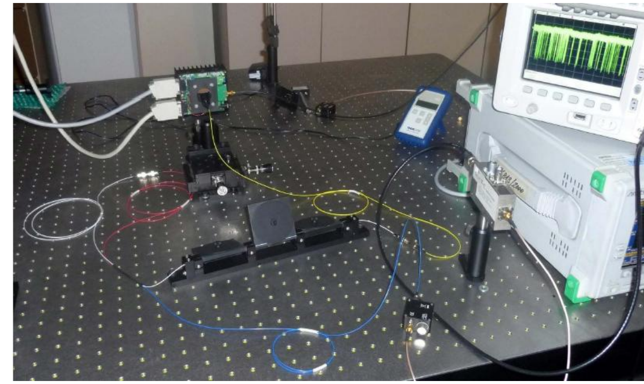

Inferring signatures of determinism in stochastic complex systems

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XIII Latin American Workshop on Nonlinear Phenomena, LAWNP 2013, Villa Carlos Paz

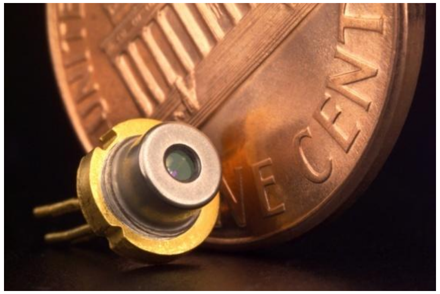
Event level description of dynamical complex systems

- Sequences of events generated by complex systems
  - Intervals between threshold crossings and barrier crossings,
  - Neurons: inter-spike intervals (ISIs),
  - Human communication: inter-event user times (SMS, emails, Twitters).
  - Earth and climate: earthquakes, extreme events (tornados, rainfalls), etc.
- Interplay of
  - Different time scales, memory
  - Nonlinear, high dimensional & stochastic effects
- The identification of patterns in the sequence of events can allow for
  - Model verification, parameter estimation
  - Classification of different types of dynamical behaviors
  - Improving predictability and forecasting

Outline

- Introduction: semiconductor lasers with feedback as high-dimensional & stochastic dynamical systems
- Method of time-series analysis and experimental setup
- Results. Experimental and model observations: inferring signatures of determinism + response to periodic forcing
- Conclusions and take home message

Why semiconductor lasers?

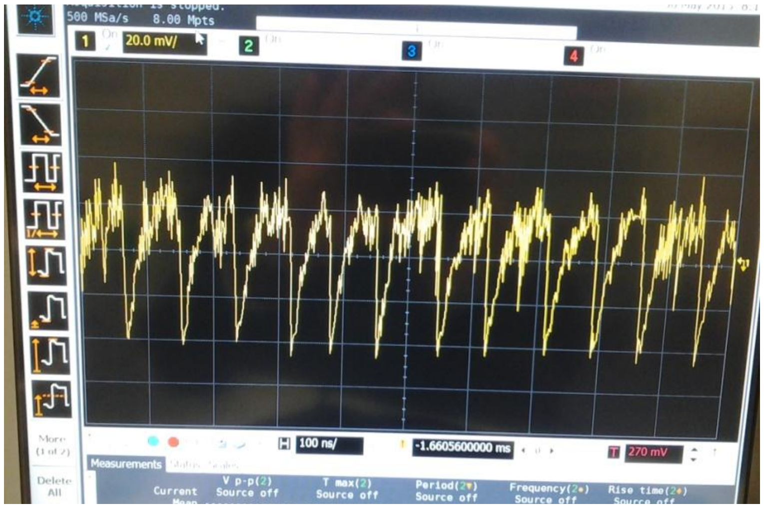


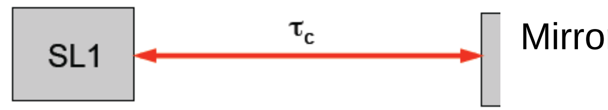
- SLs have many advantages:
  - compact, fast, reliable, inexpensive
  - wide range of wavelengths

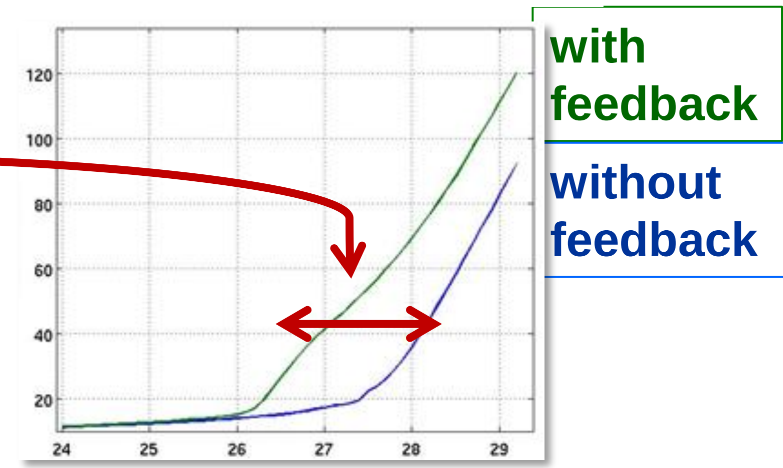
- Used in
  - Telecommunications
  - Data storage (CDs, DVDs, Blu rays)
  - Barcode scanners, printers, mouse
  - Material processing
  - Biomedical applications (imaging, sensing, etc)

Nonlinear oscillator: optical spikes

- With optical feedback the laser **intensity** displays **dropouts** similar to neuronal **spikes**.







Pump current (mA)
(CONTROL PARAMETER)

- Complex interplay of:
  - Feedback delay time
  - Noise
  - Nonlineary

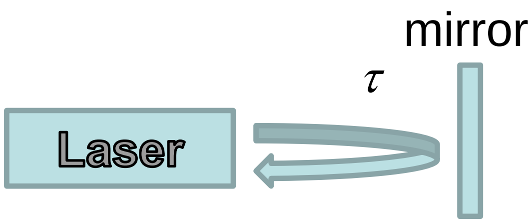
→ Stochastic and high-dimensional system

Goals

- to develop a method of time-series analysis that allows inferring signatures of determinism in the sequence of optical spikes;
- to extract new information;
- to compare model predictions with observations;
- to explore potential for building optical neurons.

Governing equations

R. Lang and K. Kobayashi, IEEE J. Quantum Electron. 16, 347 (1980)



$|E|^2$ ~ photon number (output intensity)

N ~ number of carriers (electron-holes)

$$\frac{dE}{dt} = \frac{1}{2\tau_p}(1+i\alpha)(G-1)E + \underbrace{\eta E(t-\tau)e^{-i\omega_0\tau}}_{\text{feedback}} + \underbrace{\sqrt{2\beta_{sp}}\,\xi}_{\text{noise}}$$

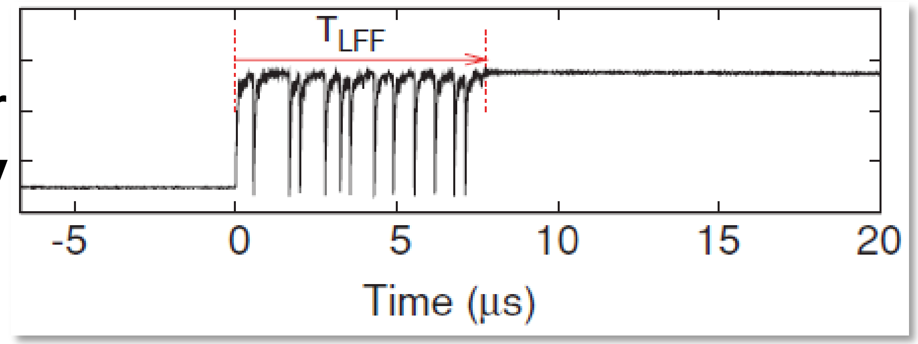
$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - G|E|^2\right)$$

Gain: $G = N/(1+\varepsilon|E|^2)$

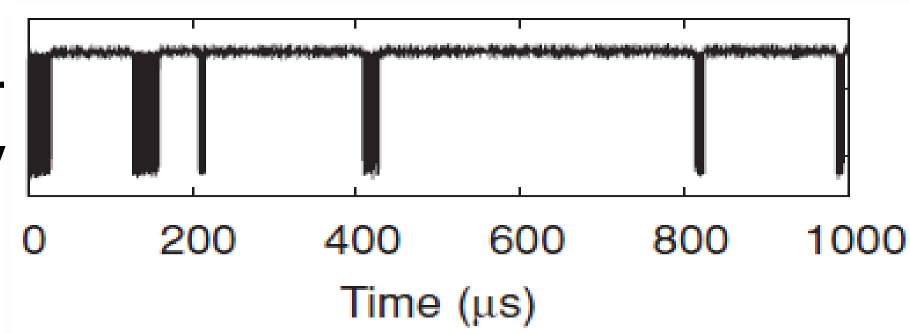
η = feedback strength
τ = feedback delay time
μ = pump current
(control parameter)

Model predictions

- The dropouts are a **transient** dynamics.



- Burst of dropouts are triggered by **noise**.

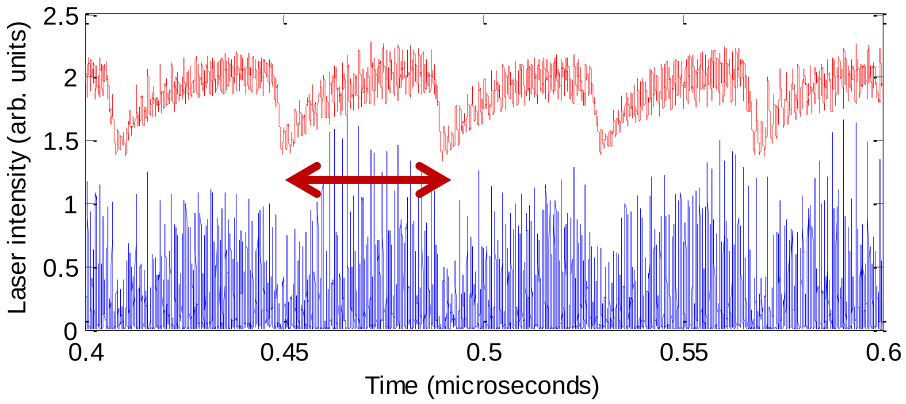


- In experimental sequences of dropouts: which ones are **deterministic** and which ones are **stochastic**?

A. Torcini et al, Phys. Rev. A 74, 063801 (2006)
J. Zamora-Munt et al, Phys Rev A 81, 033820 (2010)

Problems

- Main problem: we can measure only one relevant variable (the laser intensity)
- Also a problem: the measure system (photodiode, oscilloscope) has a finite *bandwidth* that gives a limited temporal resolution.



- Approach: event-level description. We study the sequence of **inter-dropout-intervals**: $\Delta T_i = t_{i+1} - t_i$

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Symbolic ordinal analysis

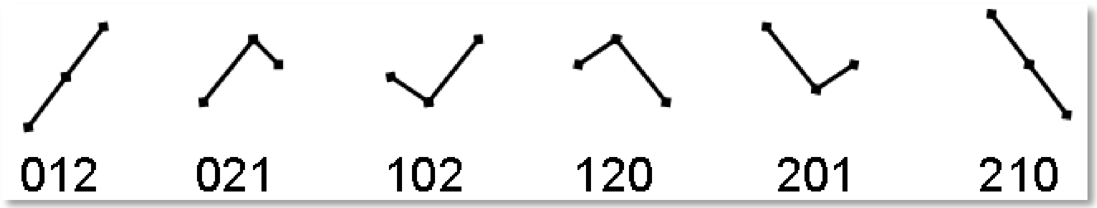
- It has been used to analyze data generated from complex systems
  - Financial, economical
  - Biological, life sciences
  - Geosciences, climate
  - Physics, chemistry, etc
- It has been shown to be able to:
  - Distinguish stochasticity and determinism
  - Classify different types of dynamical behaviors (pathological, healthy)
  - Quantify complexity
  - Identify coupling and directionality.

Brandt & Pompe, Phys. Rev. Lett. 88, 174102, (2002).

Review by O. A. Rosso and co-workers, *Permutation Entropy and Its Main Biomedical and Econophysics Applications*, Entropy 14, 1553 (2012)

Ordinal Patterns (or "words")

- "words" of **D letters** can be formed by considering the **order relation** between sets of D values $\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$.
- For **D=3** there are 6 possible orders



Example: the set (5, 1, 7) gives "102" because 1 < 5 < 7

- Advantage: the transformation keeps information about correlations in the time-series & does not need a threshold
- Drawback: it does not keep information about the values, the set (5,1,100) also gives word "102".

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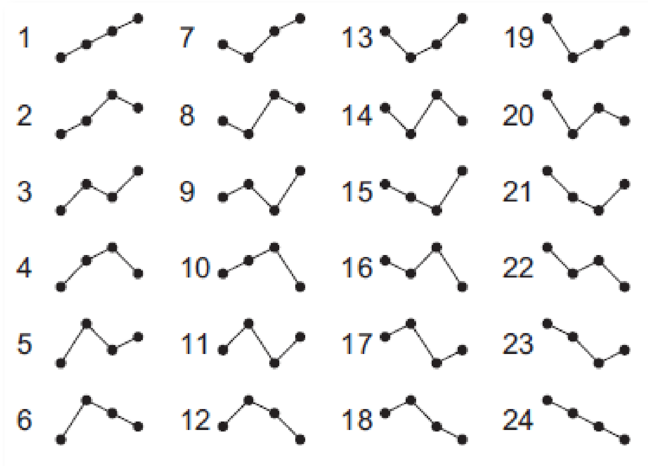
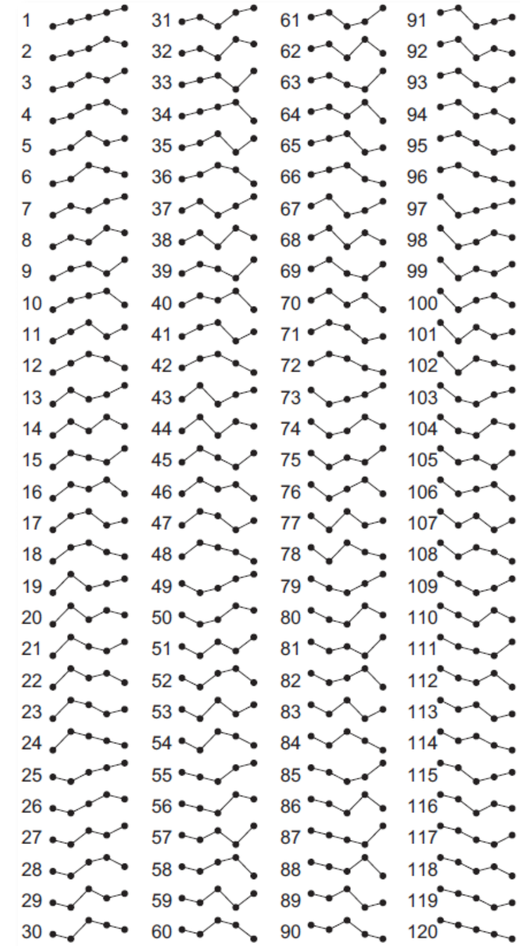
Number of possible ordinal patterns: D!

D=4



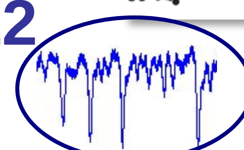
U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

D=5

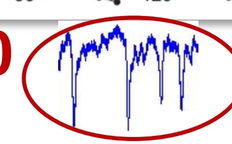


- How to select D? Optimal D depends on:
  - The length of the time series.
  - The time scale of correlations.
- For optical spikes: D=2 (D=3) unveil correlations of 3 (4) spikes

012



210



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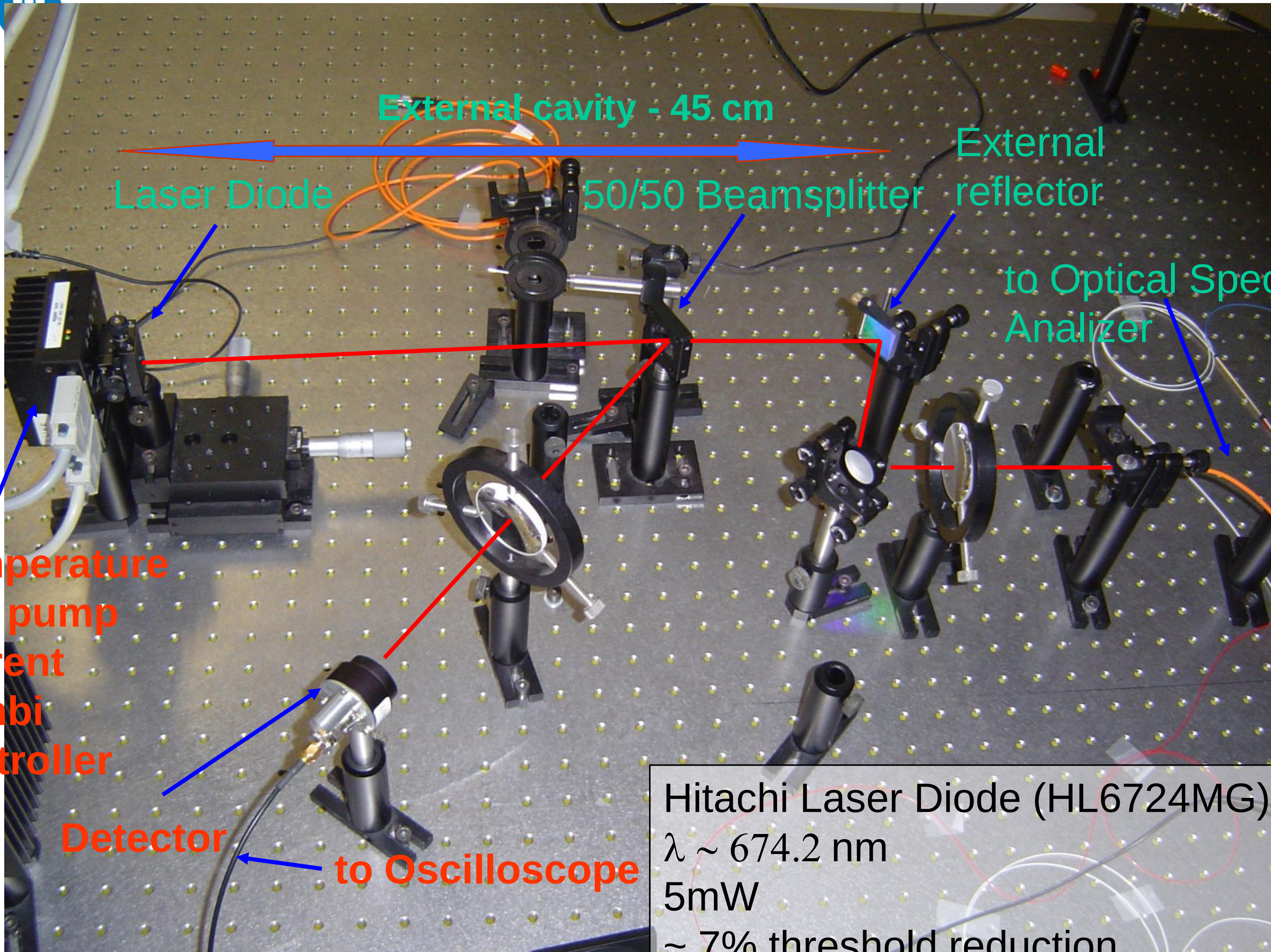


Hitachi Laser Diode (HL6724MG)
λ ~ 674.2 nm
5mW
~ 7% threshold reduction

Experimental inter-dropout-intervals (IDIs in lasers – ISIs neurons)

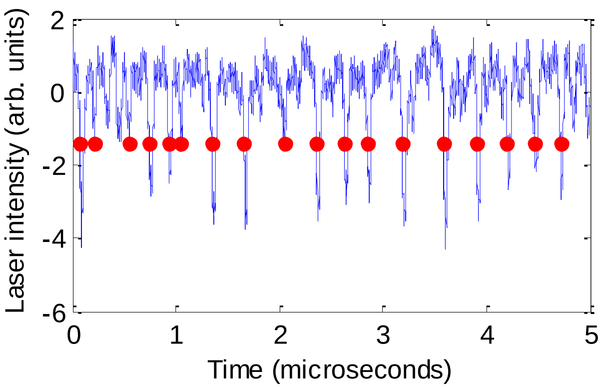
Laser output
(1 GHz oscilloscope)

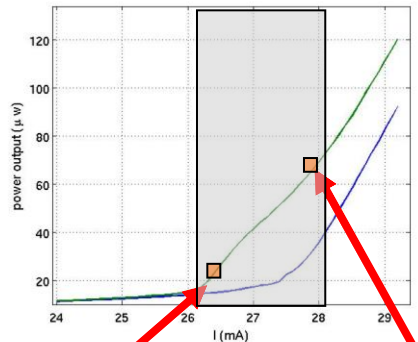
<ΔT> = 100-200 ns

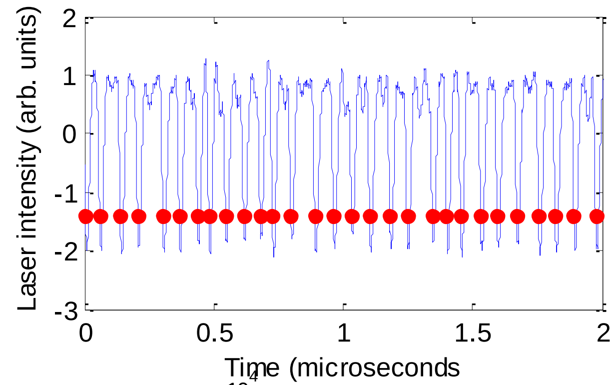
τ~ 5 ns

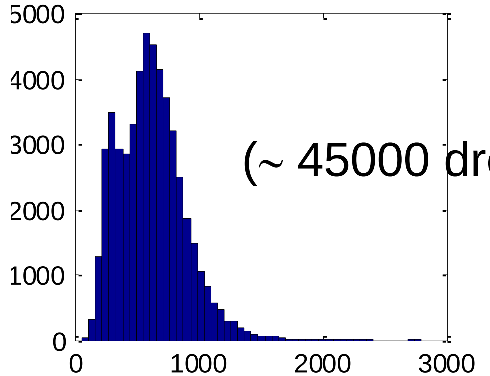
# of IDIs recorded
45,000 - 220000

IDI distributions



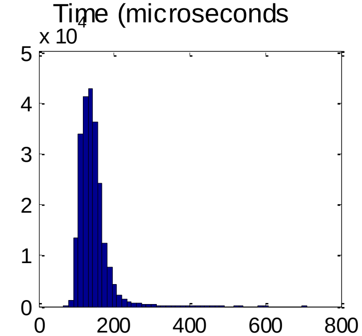






(~ 45000 dropouts)

(~ 225000 dropouts)



Is there any **information** in the 'spike' sequence?

Analogous to deciphering a foreign text.

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Correlations between 3 consecutive spikes: probabilities of 01 & 10

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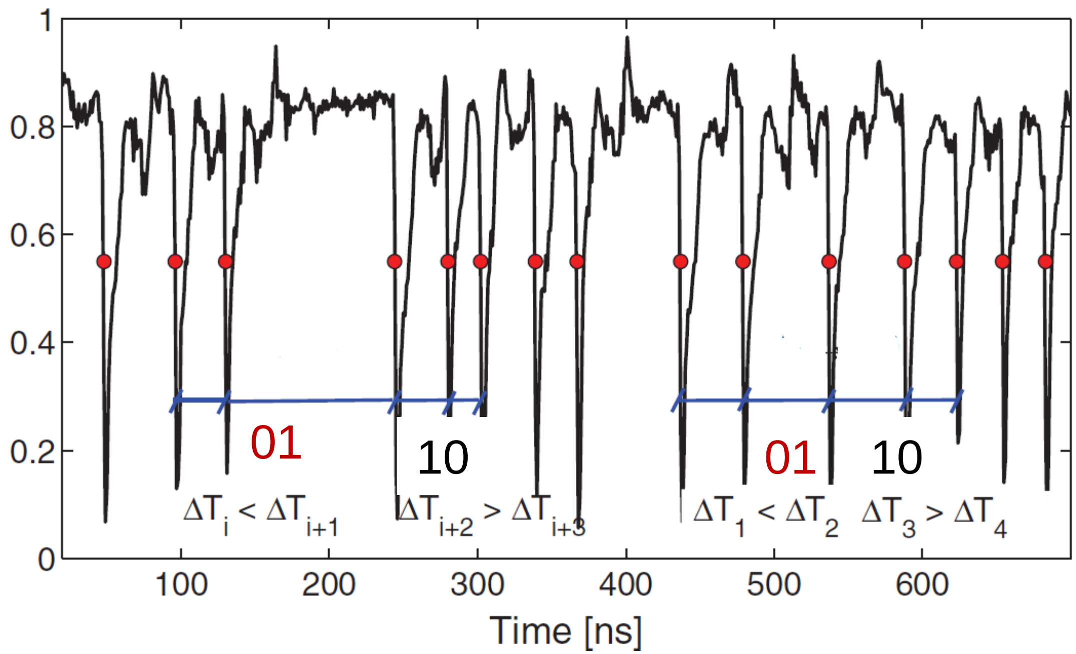
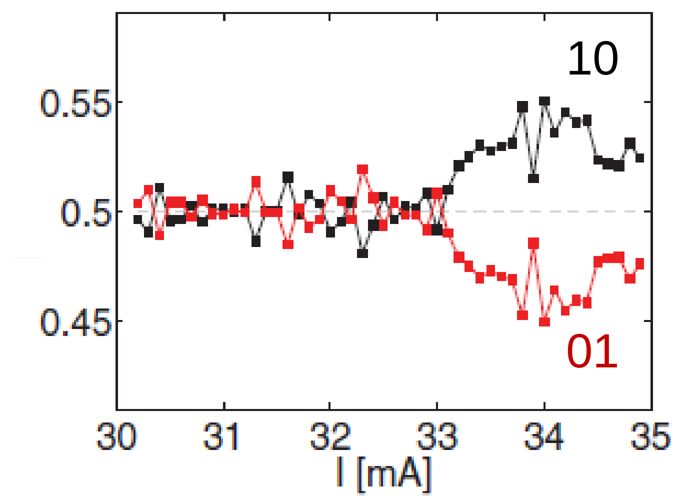


Probabilities:



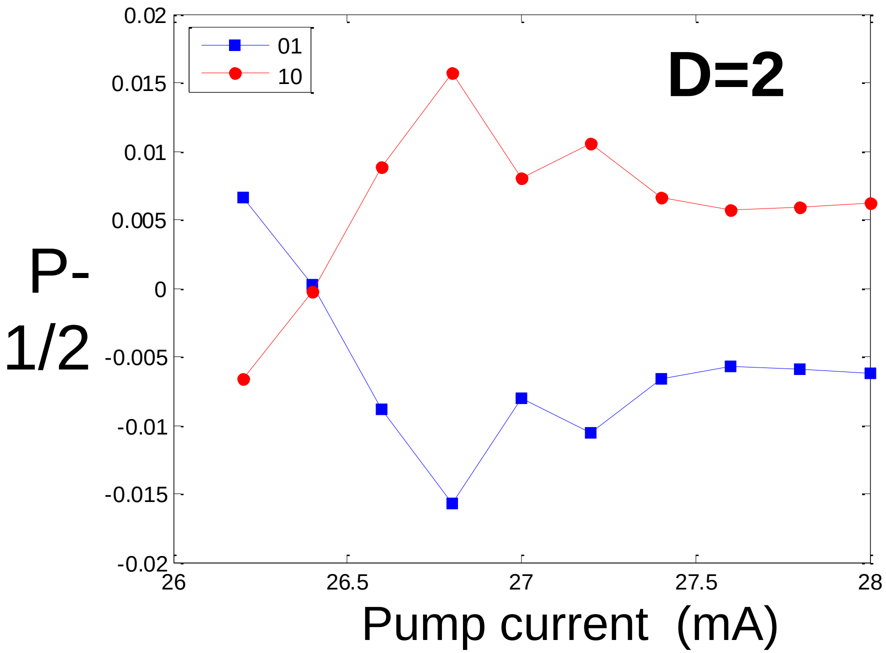
Consistent with stochastic at low pump current, but signatures of determinism at high pump current.

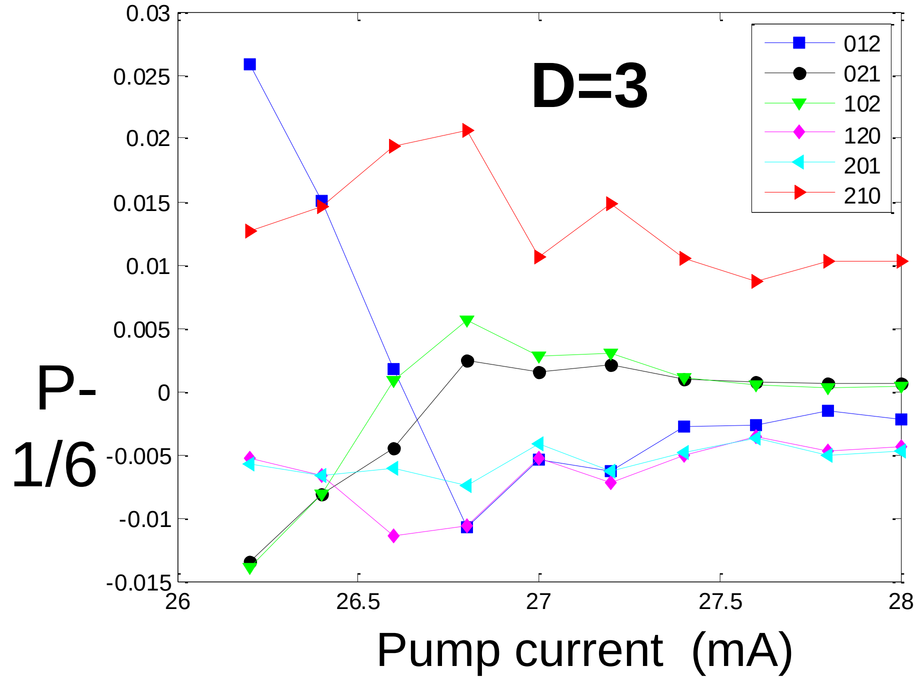
D=2: 3-spike correlations?

Null hypothesis: fully random sequence of spikes ⇒ P(01) = P(10)

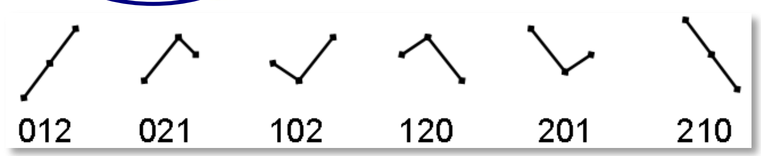
N. Rubido et al, Phys. Rev. E 84, 026202 (2011)

At low pump current: are the spikes fully random? New experiment



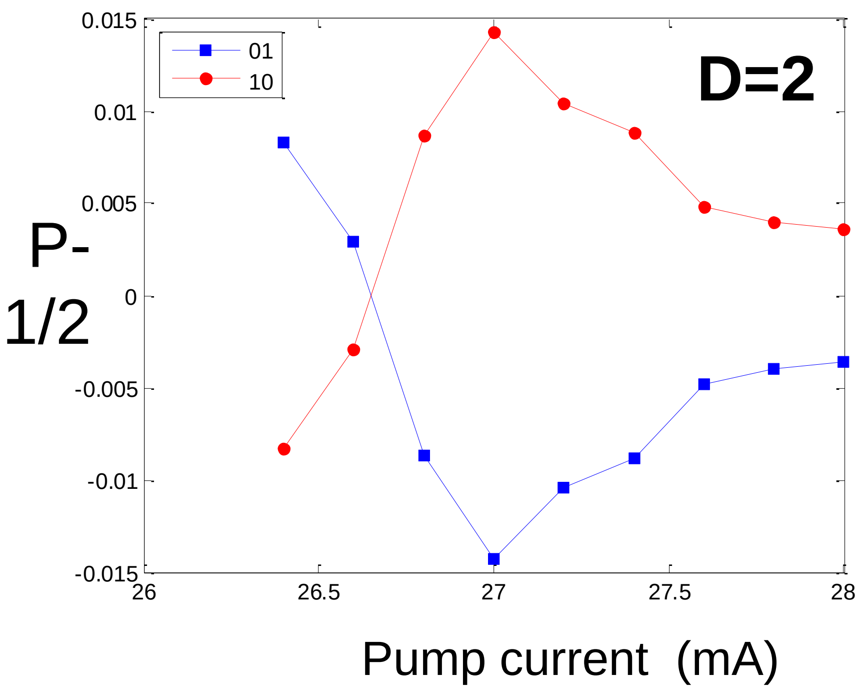


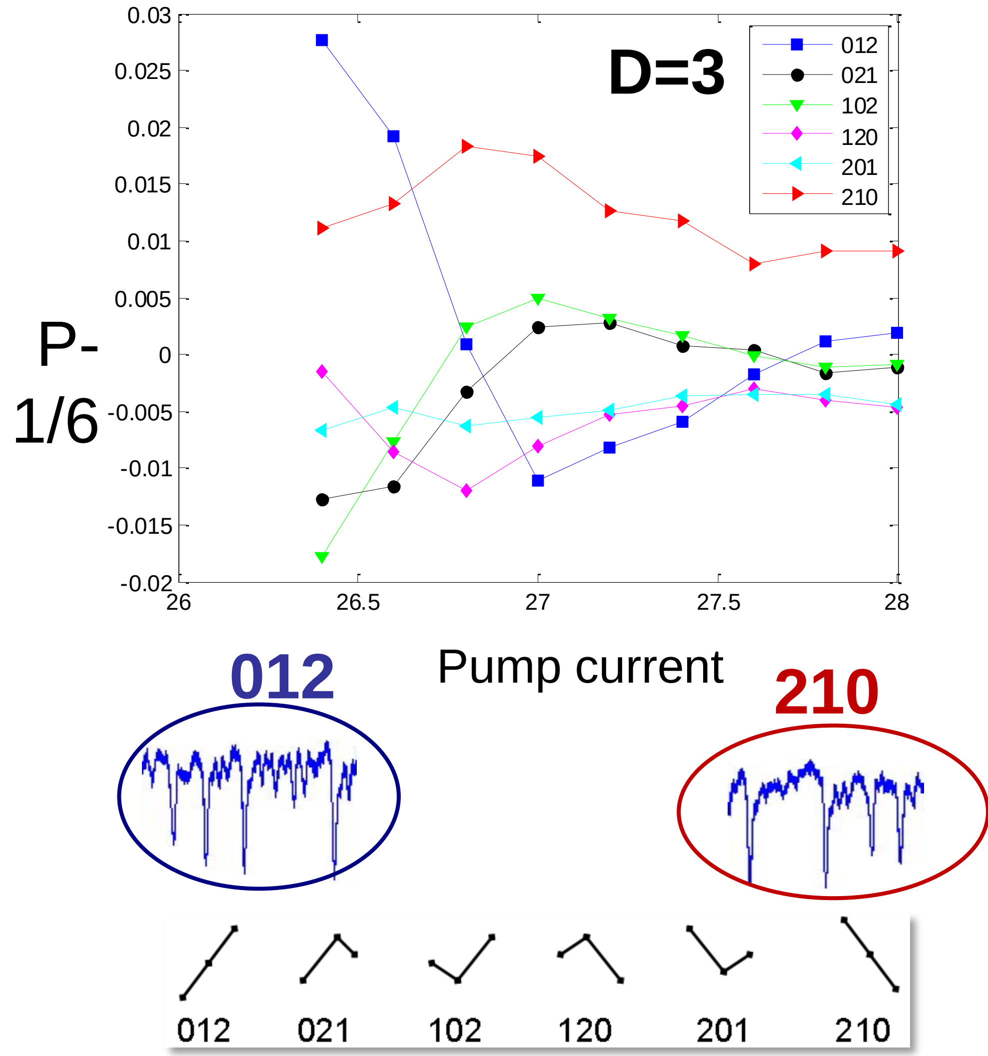
45000 - 220000 IDIs



A. Aragoneses, N. Rubido, J. Tiana, M. C. Torrent and C. Masoller, Scientific Reports (2013)

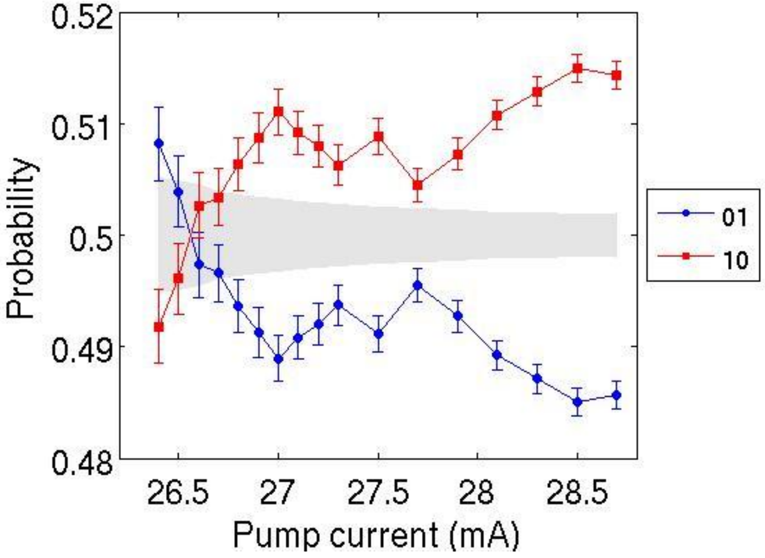
Also in another data set recorded at a different temperature (T=20 C)



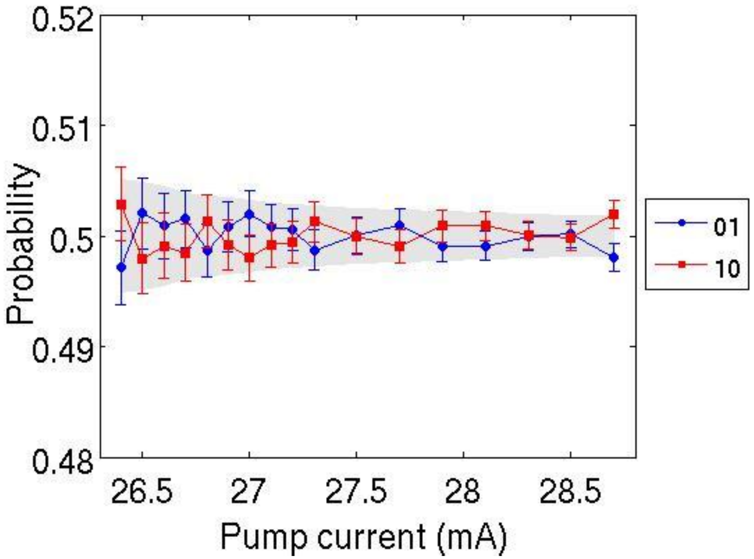


Are the deviations from the null hypothesis significant?

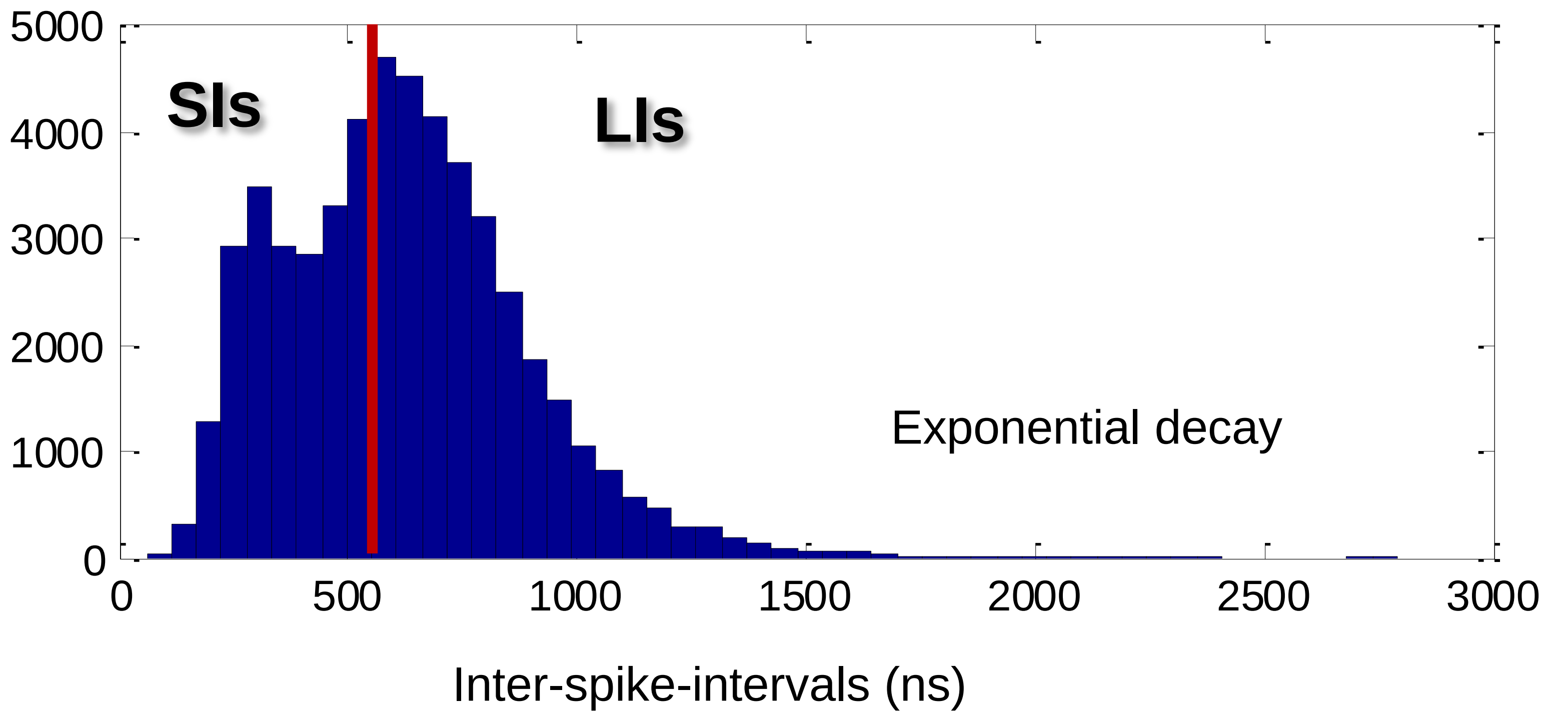
Recorded data



Surrogated data



Which dropouts are noise-induced and which ones are deterministic?



We use a **threshold** to classify the inter-dropout-intervals as **short** and **long** intervals

Constructing the words with 2 consecutive SIs or LIs

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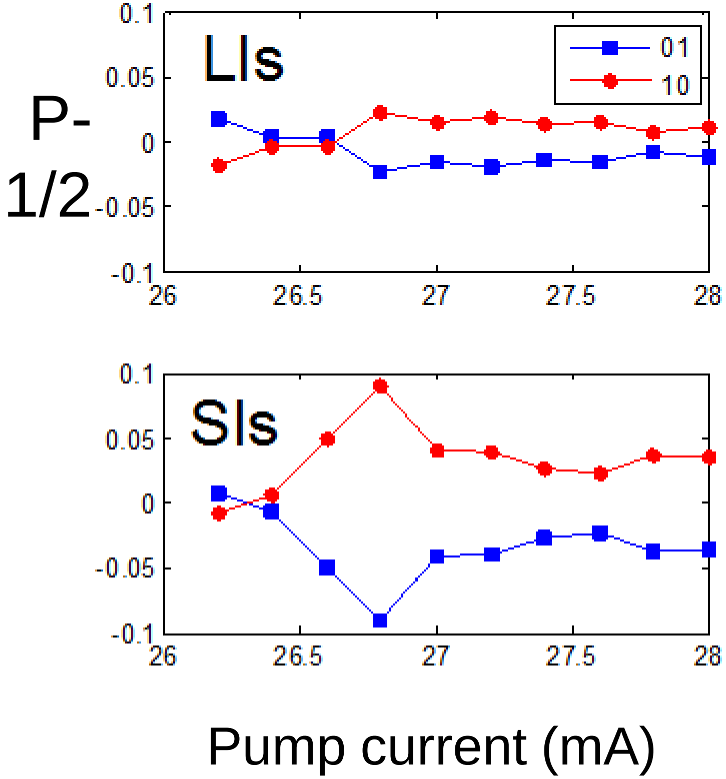
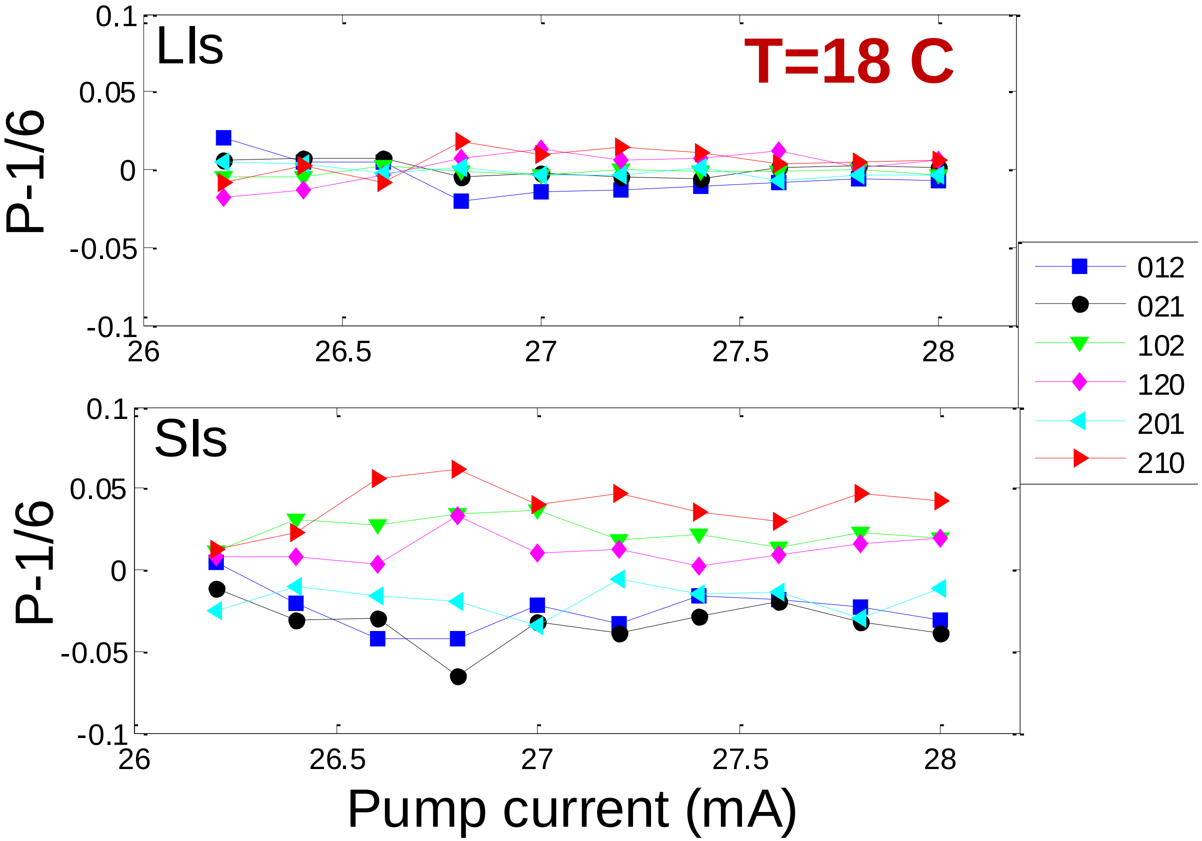
- At high currents: significant differences
  - LIs consistent with random events
  - SIs more deterministic.

- But at low pump currents, the inter-spike-intervals can not be classified in two types with significant differences.

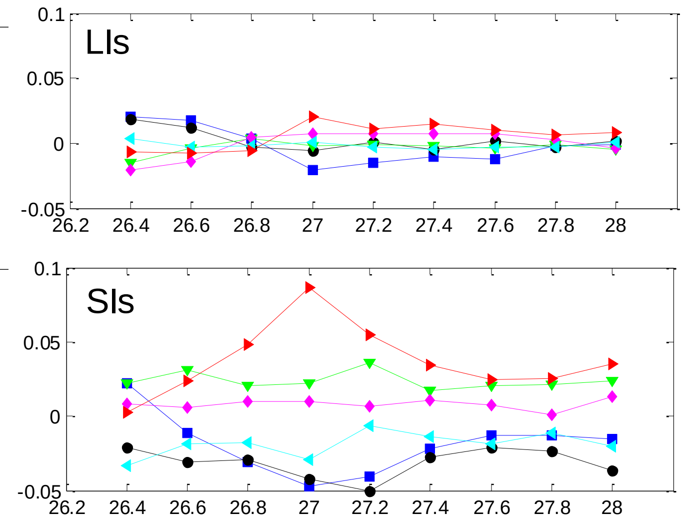


A. Aragoneses, N. Rubido, J. Tiana, M. C. Torrent and C. Masoller, Scientific Reports (2013)

Constructing the words with 3 consecutive SIs or LIs



Similar results in the other dataset (**T=20 C**)

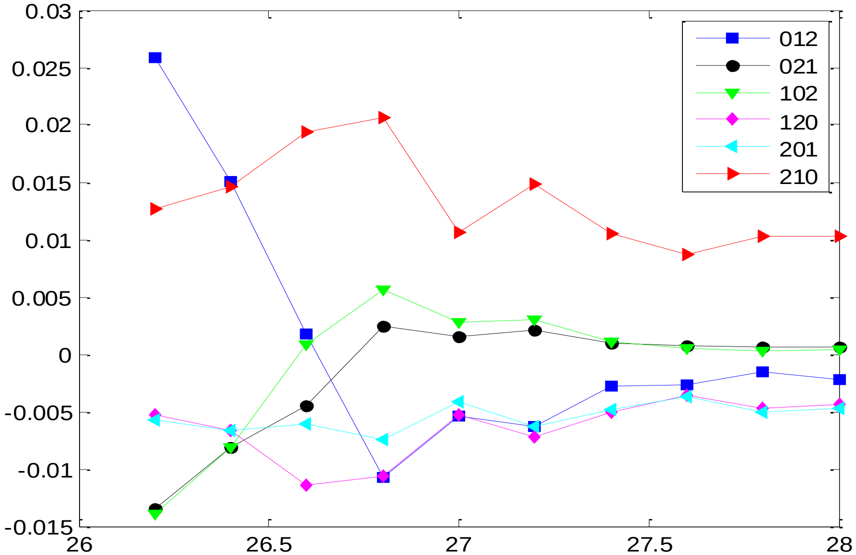


At high pump currents an adequate threshold allows classifying the events in two distinct categories

Ordinal analysis unveils new information

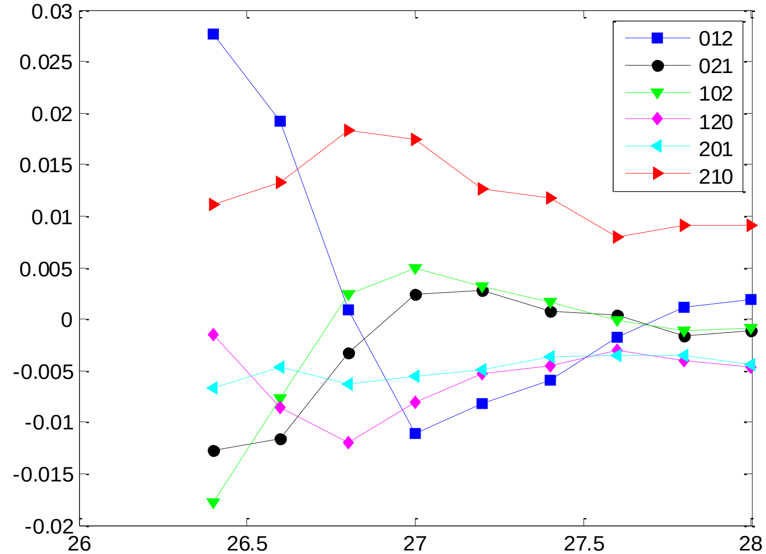
T=18 C

P-1/6



Pump current (mA)

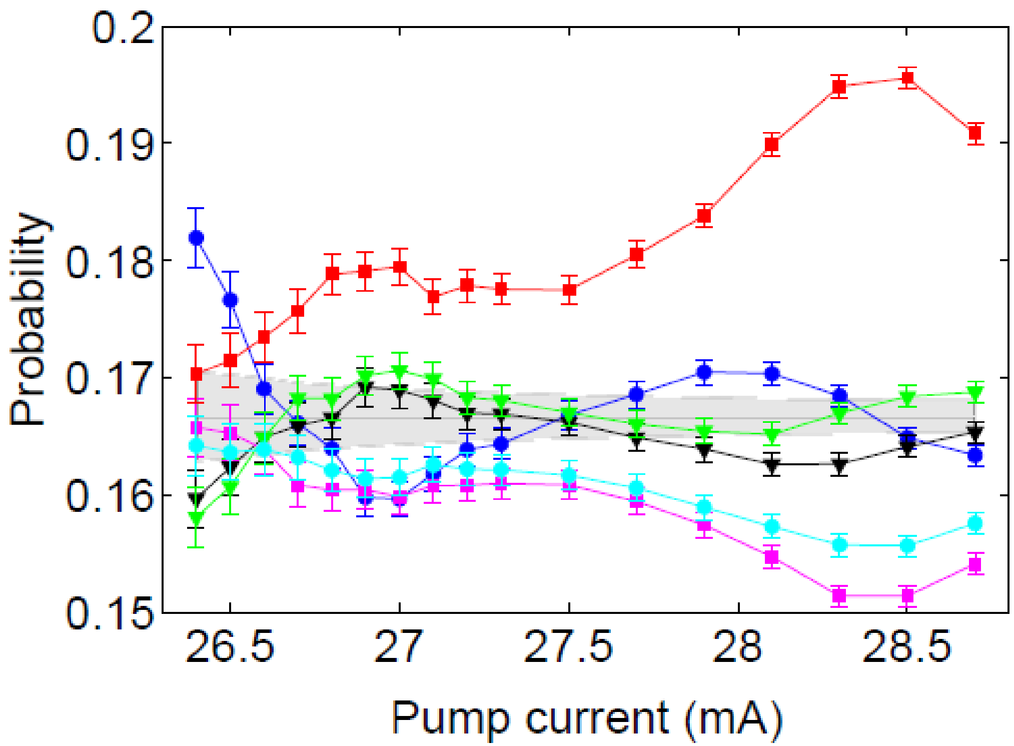
T=20 C

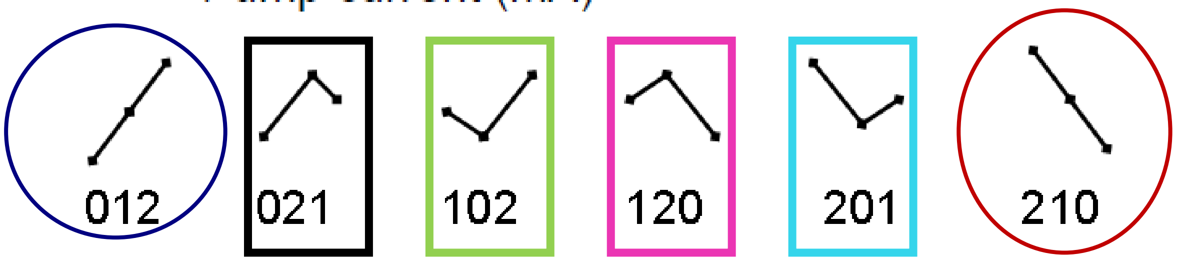


Pump current (mA)

There is a hierarchical and clustered organization of the probabilities of the words

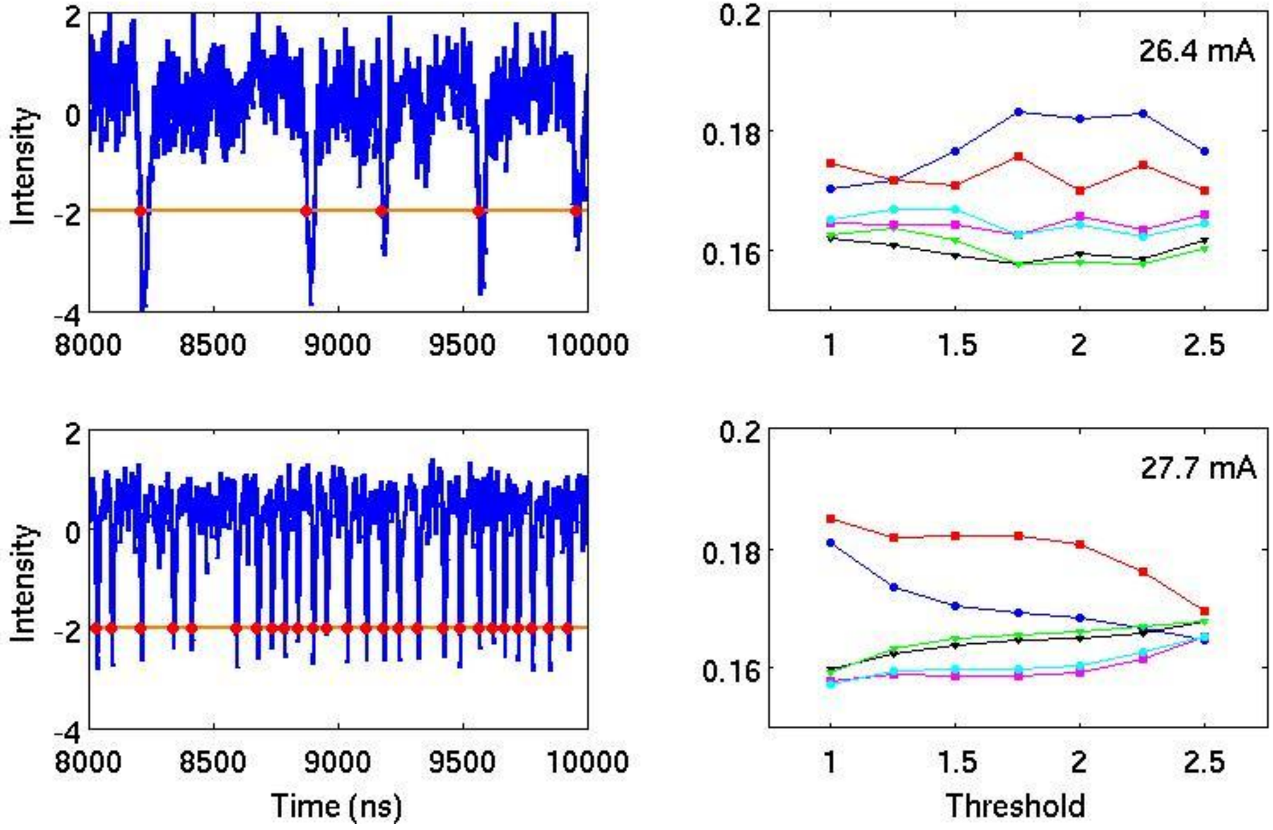
In another experiment: also the same hierarchy and the same 2 clusters





75,000 – 880,000 dropouts
(different laser, new oscilloscope)

Sensitivity to the threshold that defines the event times?



The hierarchy and the clusters are robust to the threshold chosen to define the spike times

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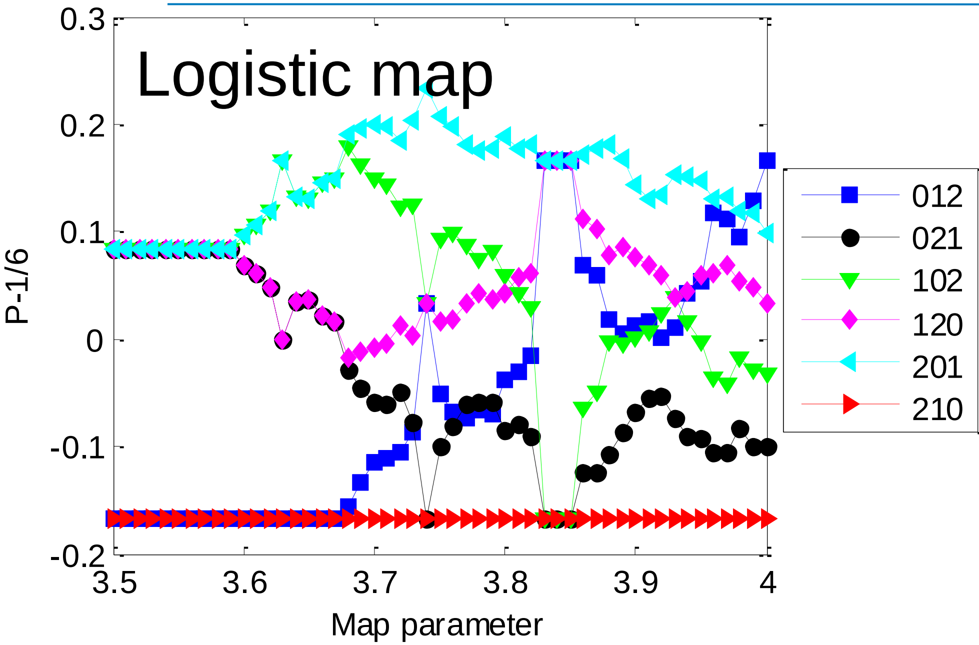
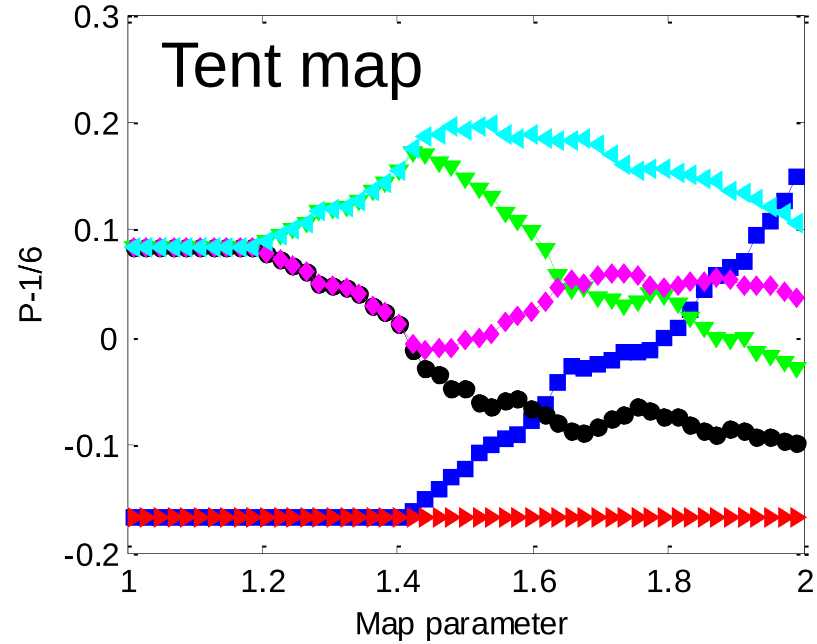
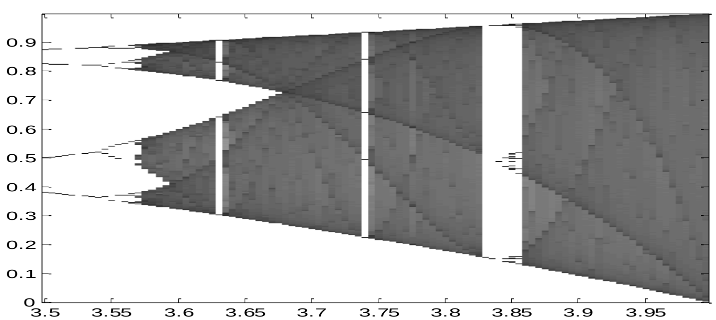
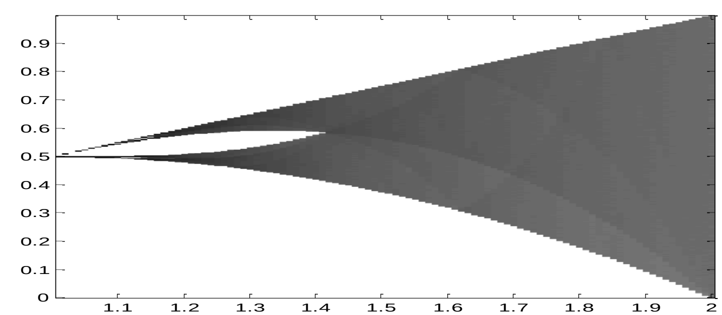
Can we find a minimal model that displays these features?







Map parameter

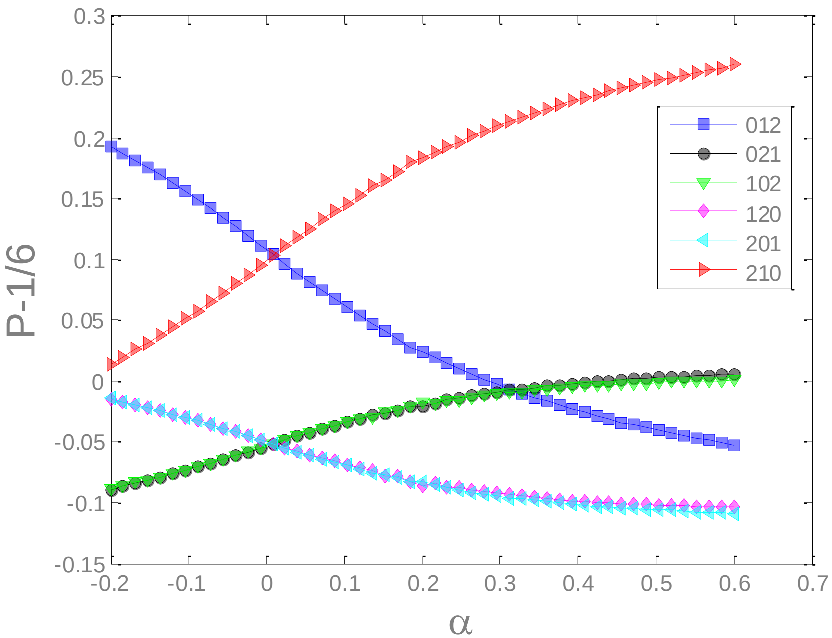


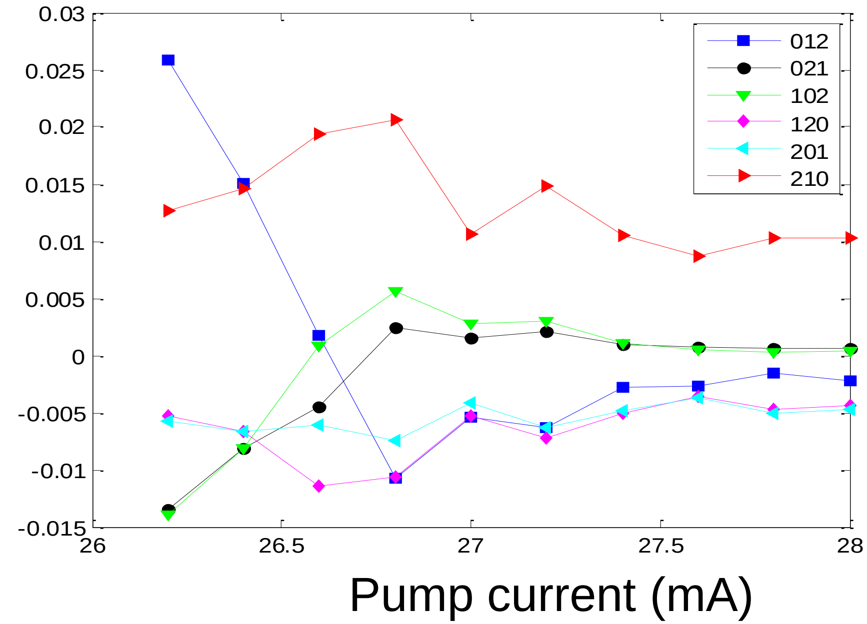
Map parameter

A modified circle map: minimal phenomenological model

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha \sin(4\pi\varphi_i)\right]$$

$$X_i = \varphi_{i+1} - \varphi_i$$





ρ=0.23
K=0.04

Minimal model for electroreceptors of paddlefish: A. B. Neiman and D. F. Russell, PRE 71, 061915 (2005)

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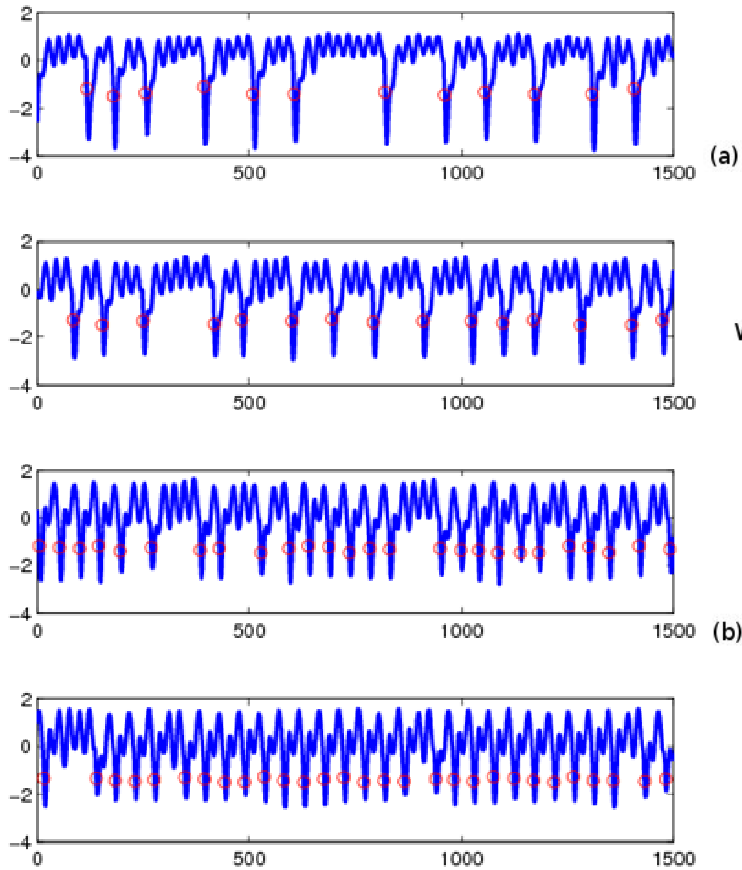
Periodic modulation of the laser current

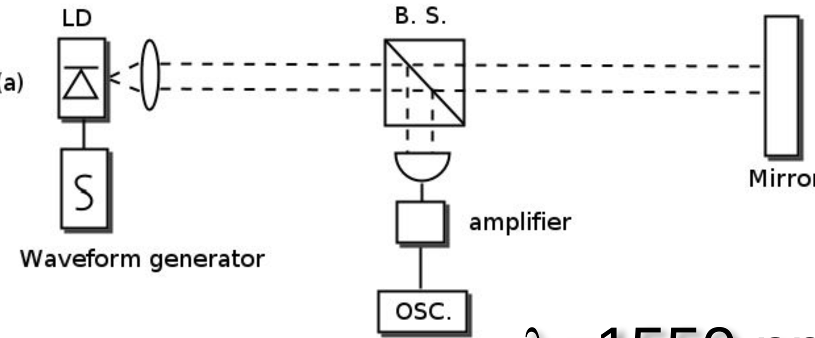
Laser intensity:

Increasing the modulation amplitude

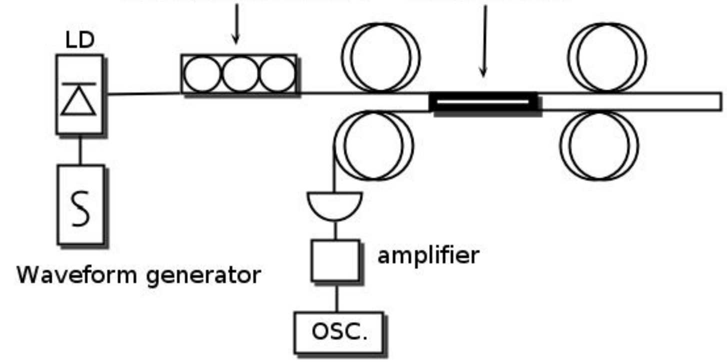


Time (ns)

λ=660 nm



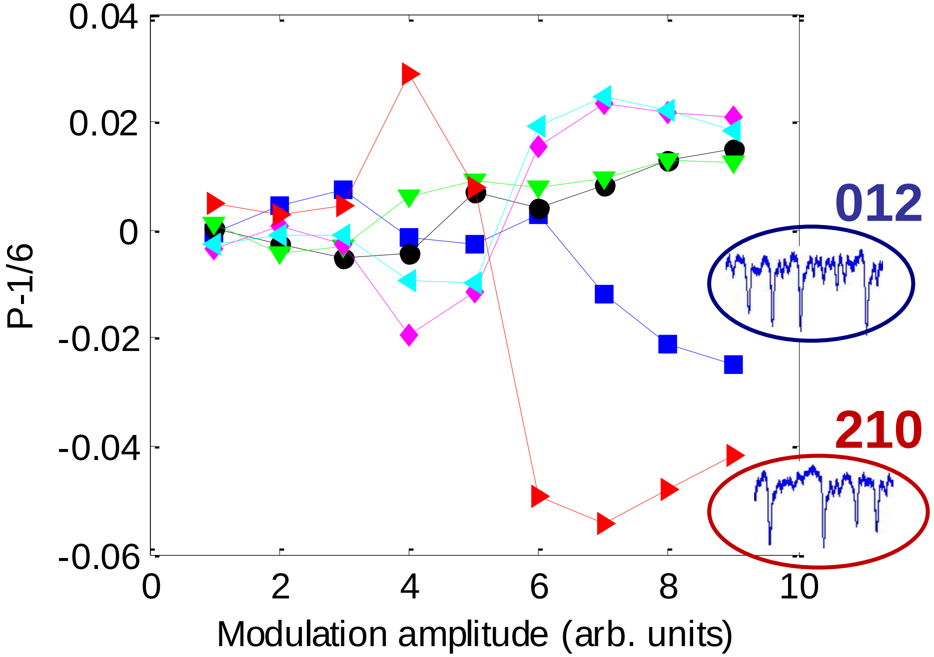
λ=1550 nm



Experiment-model comparison

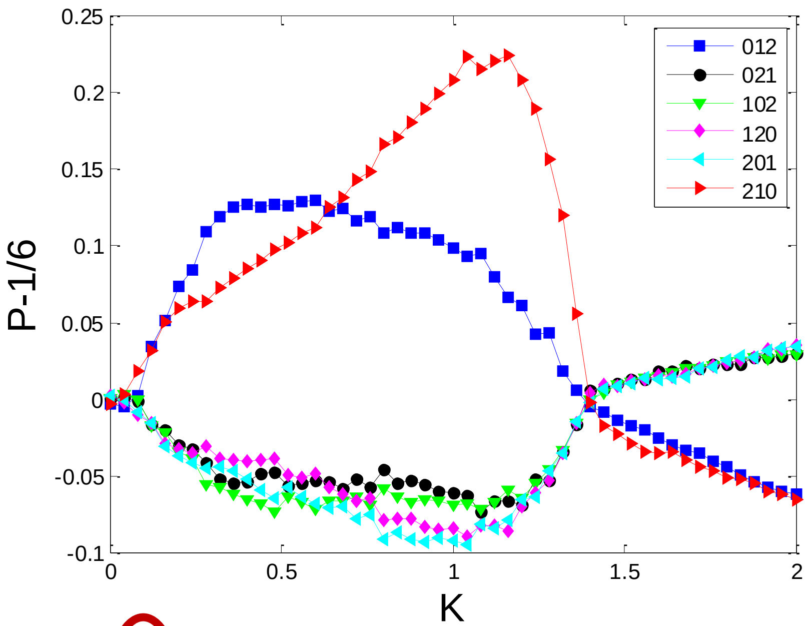
Experiments @ 660 nm

(68,000 - 200,000 dropouts)



Similar observations @ 1550 nm

Minimal circle-map model



$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha \sin(4\pi\varphi_i)\right] + D\zeta$$

ρ=-0.23
α=0.2
D=0.02

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Conclusions

- We proposed a novel method to infer signatures of determinism in sequences of events in dynamical complex systems.
- Adequate for high-dimensional & stochastic systems displaying noise or deterministically induced events.

- We found new symbolic states with an hierarchical and clustered organization of the probabilities of the patterns.
- We identified a minimal phenomenological model.
- LK model is in good agreement with observations (not shown because lack of time)
- Potential breakthrough: optical neurons for neuro-inspired information processing.

Take home message

- Ordinal analysis is a powerful technique for the event-level description of complex systems

– useful for data understanding and uncovering patterns in the sequence of events,

– useful for improving system modeling, model comparison and parameter estimation,

– useful for classifying different types of behaviors,

– potential for improving event predictability and forecasting.

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Thanks to

You for your attention!

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