Quantification of network dissimilarities and application to modeling the Power Grid network

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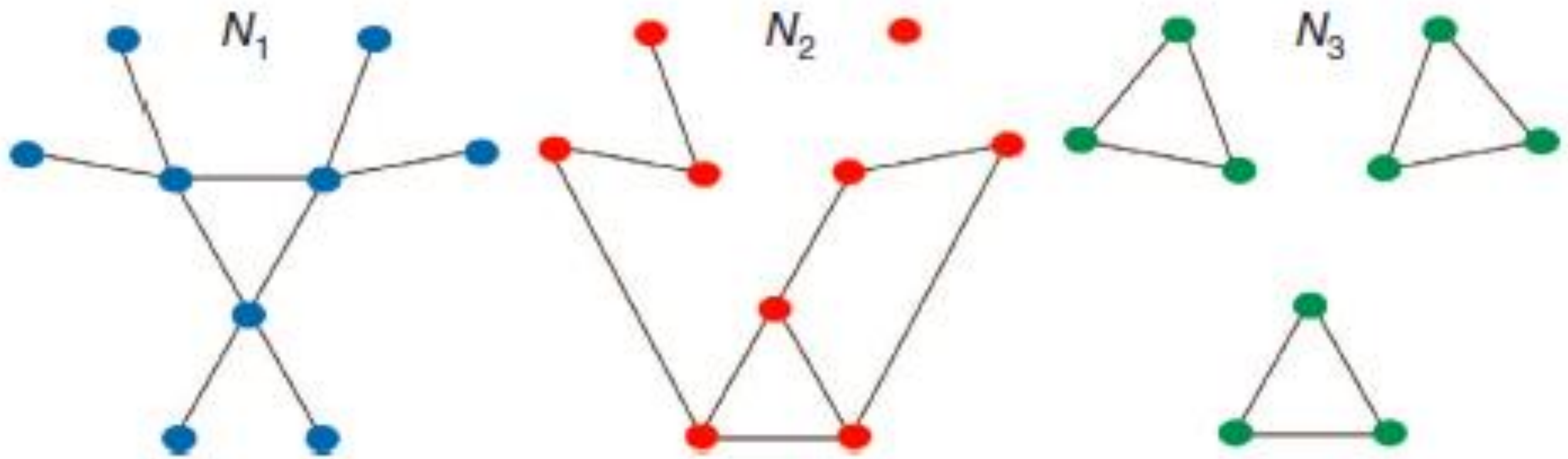
Same number of nodes and links



How to measure distances between networks?

Complex network measures

- Degree distribution, closeness centrality, betweenness centrality, average path length, etc.

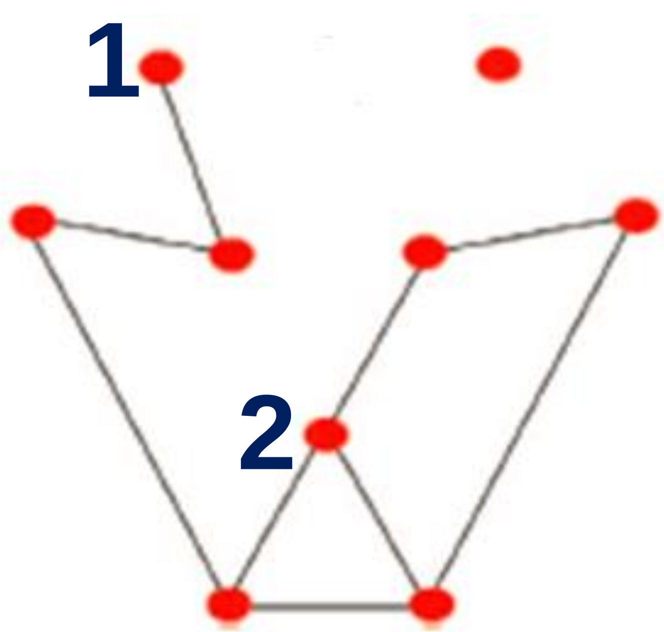
- Provide *partial* information.

- How to define a measure that contains detailed information about the *global topology* of a network, in a *compact* way?

$\Rightarrow$ Node Distance Distributions (NDDs)

- $p_i(j)$ of node i = fraction of nodes connected to i at distance j

Same number of nodes and links



Node 1:

j	# nodes at distance j
1	1
2	1
3	1
4	2
5	2
∞	1

Node 2:

j	# nodes at distance j
1	3
2	2
3	1
4	1
∞	1

- With *N* nodes, the Node Distance Distributions is a vector of N pdfs $\{p_1, p_2, \ldots, p_N\}$
- If two networks have the same set of NDDs $\Rightarrow$ they have the same diameter, average path length, etc.

How to condense the information contained in the node-distance distributions?

- The *Network Node Dispersion (NND*) measures the heterogeneity of the N pdfs $\{p_1, p_2, \ldots, p_N\}$
- Quantifies the heterogeneity of connectivity distances.

$$\mathrm{NND}(G) = \frac{\mathcal{J}(\mathbf{P}_1, \ldots, \mathbf{P}_\mathrm{N})}{\log(d+1)}$$

d = diameter

$$\mathcal{J}(\mathbf{P}_1, \ldots, \mathbf{P}_\mathrm{N}) = \frac{1}{N}\sum_{\mathrm{i,j}} p_\mathrm{i}(j)\log\left(\frac{p_\mathrm{i}(j)}{\mu_\mathrm{j}}\right)$$

$$\mu_\mathrm{j} = \left(\sum_{\mathrm{i}=1}^{N} p_\mathrm{i}(j)\right)/N$$

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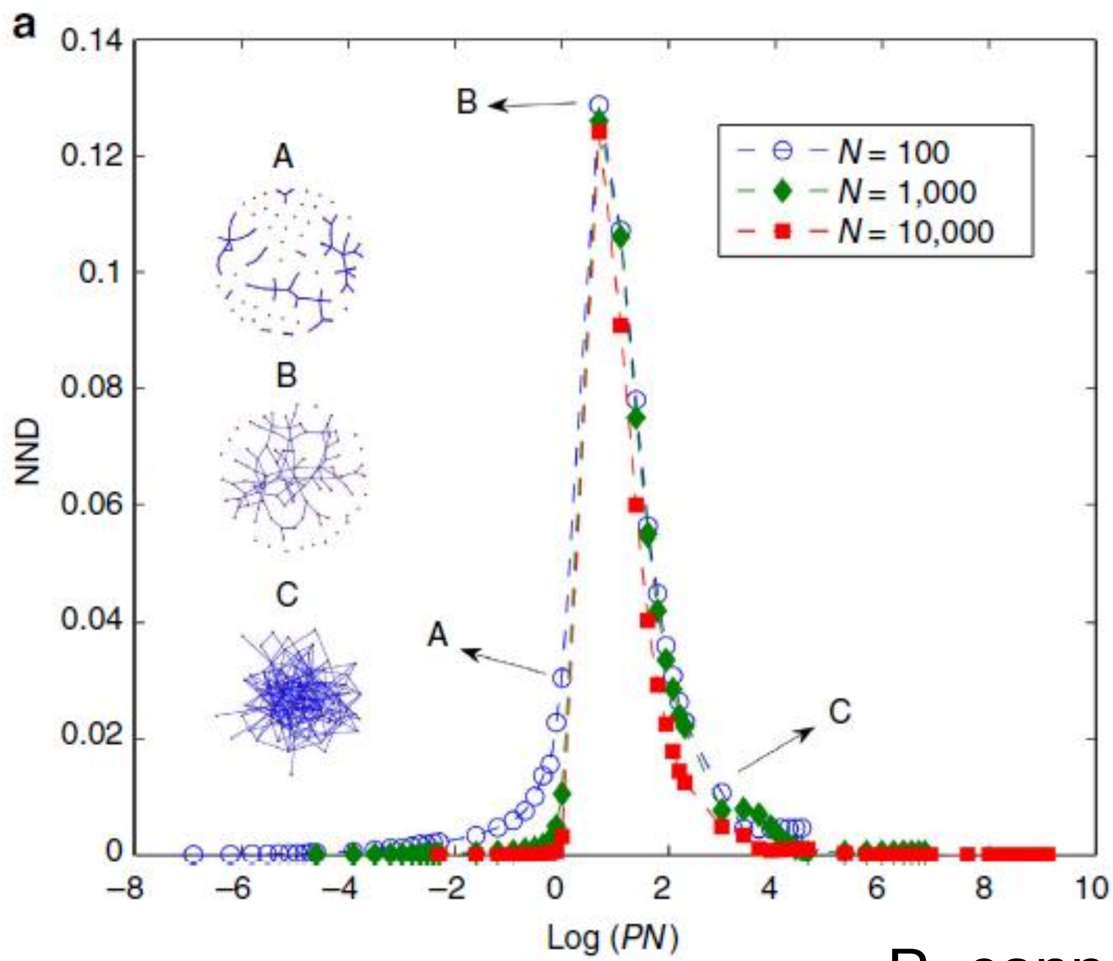
Example of application: percolation transition in a random network



⇒ the Network Node Dispersion detects the percolation transition

P=connection probability

T. A. Schieber, L. Carpi, A. Diaz-Guilera, P. M. Pardalos, C. Masoller and M. G. Ravetti, Nat. Comm. 8:13928 (2017).

Dissimilarity between two networks

$$D(G,G') = w_1 \sqrt{\frac{\mathcal{J}(\mu_G, \mu_{G'})}{\log 2}} + w_2 \left| \sqrt{\mathrm{NND}(G)} - \sqrt{\mathrm{NND}(G')} \right|$$

$w_1=w_2=0.5$

compares the averaged connectivity

compares the heterogeneity of the connectivity distances

- Extensive numerical experiments demonstrate that isomorphic graphs return ***D=0***
- Can be applied to networks of different sizes.
- *D* time complexity is polynomial because it relies on the computation of all shortest paths length which is a polynomial problem.

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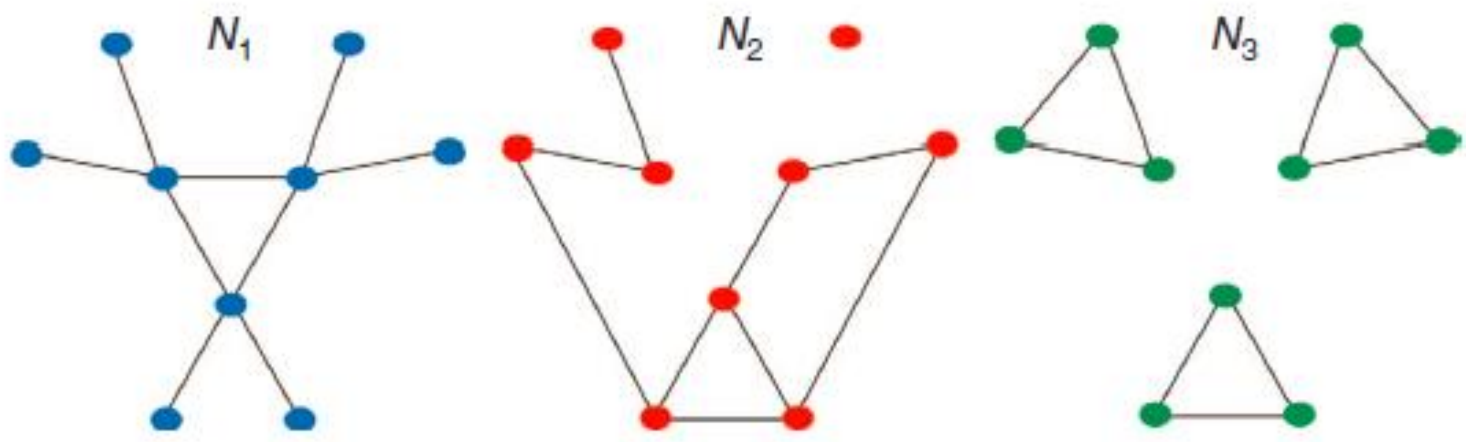
Comparing three networks with the same number of nodes and links



	D	Hamming	Graph Edit Distance
N_1,N_2	0.25	12	6
N_1,N_3	0.56	12	6
N_2,N_3	0.47	12	6

Percolation on real networks

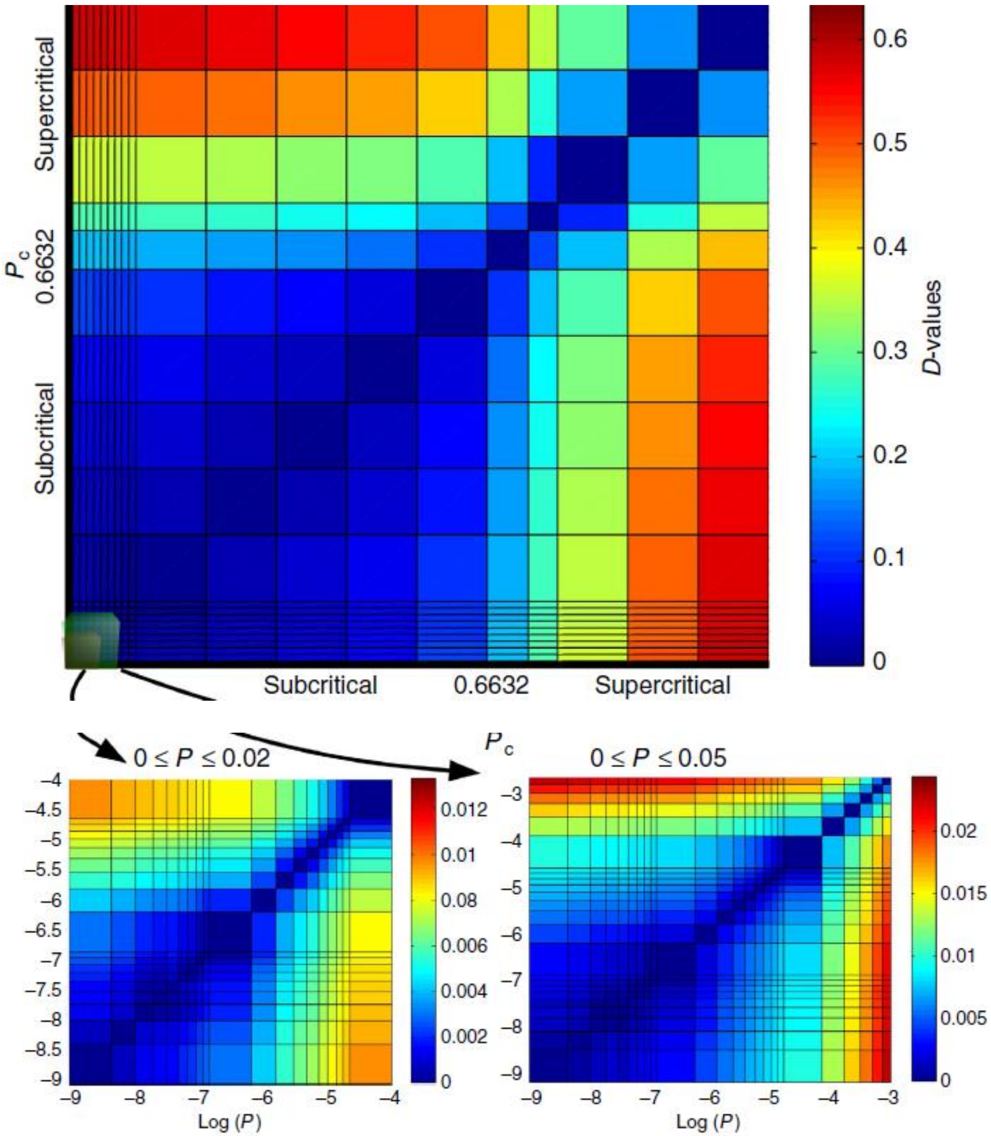
- Main idea: when looking for the percolation transition, two graphs in the same phase (subcritical or supercritical) present smaller D-values than a pair of graphs in different phases.

- Start with two probabilities, β and α, on the supercritical and subcritical phases. $Pm=(\alpha + \beta)/2$

- If $D(Gm, G\alpha)>D(Gm, G\beta)$ then β =Pm else α =Pm

- Stop when $|\beta - \alpha| <$ precision ε

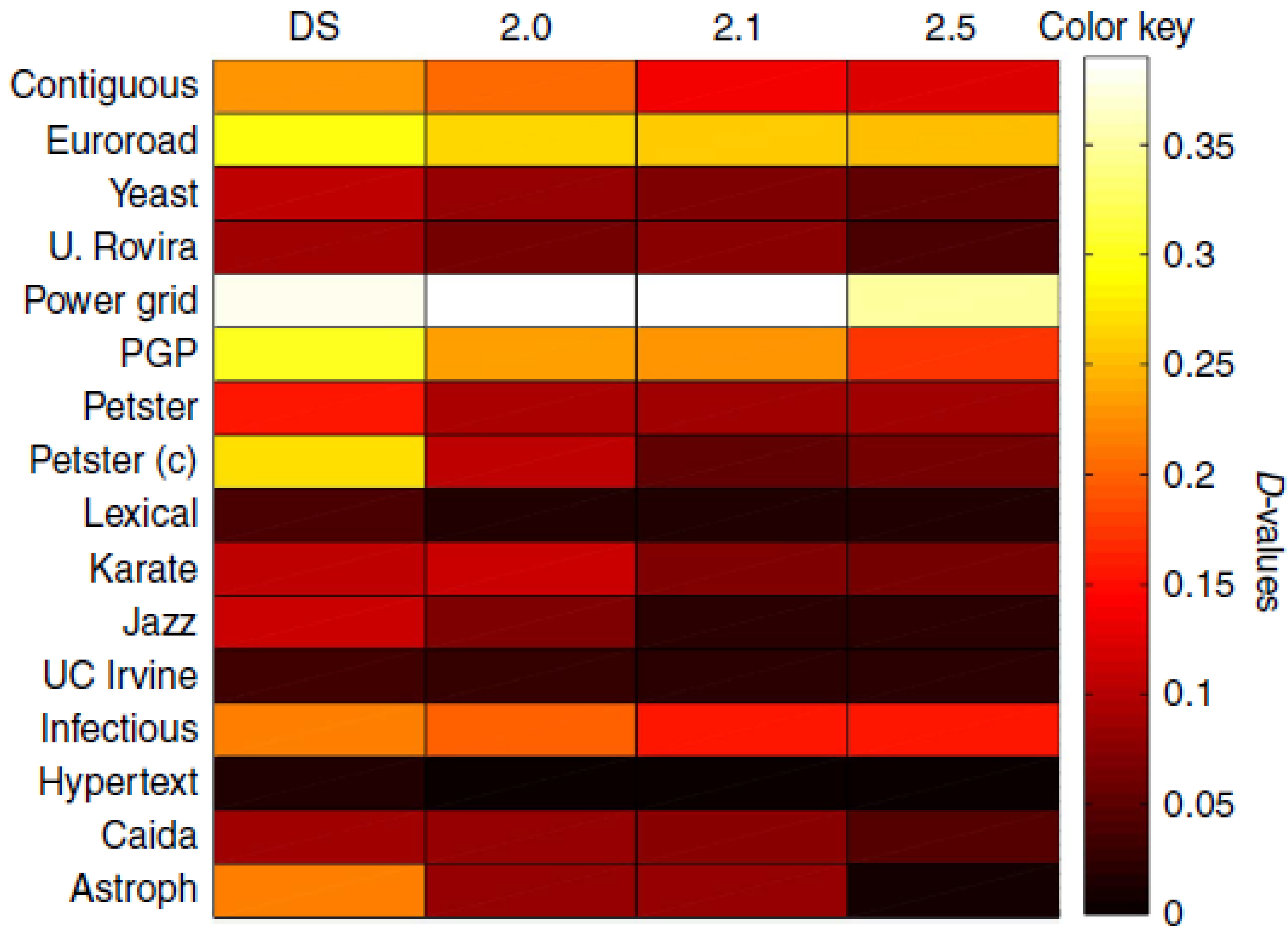
Percolation on the Power Grid network



	Pc	Comp. time(s)
D	0.6826	5,500
MC	0.6632	35,000

Comparing real networks to null models

DS preserves the degree sequence;
2.0 also preserves the degree correlation;
2.1 also the clustering coefficient;
2.5 also the clustering spectrum



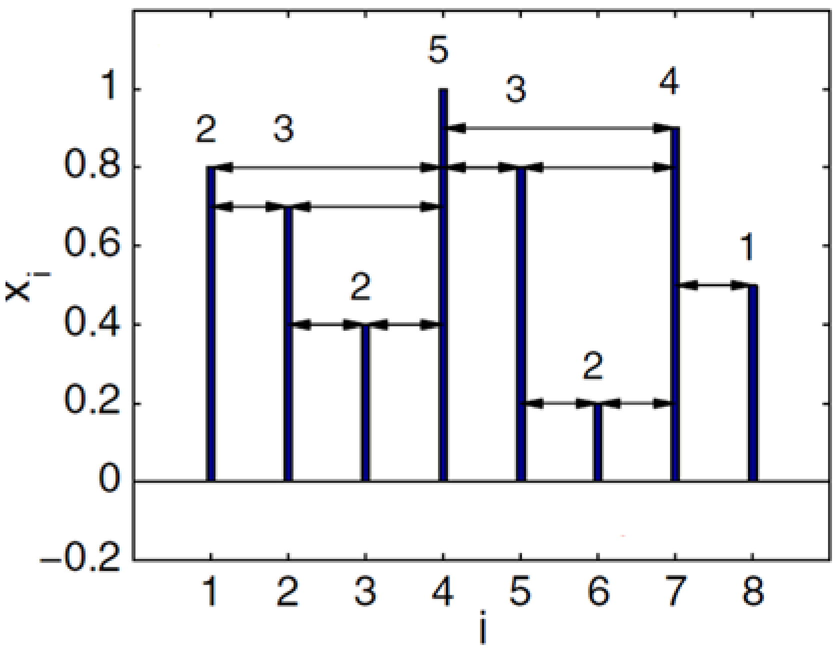
Each model is run 30 times, ⟨D⟩ is plotted in color code

dk model: *Orsini, C. et al. Nat. Commun. 6, 8627 (2015)*

- Synthetic model for Power Grid Network?

Horizontal Visibility Graph: graph representation of a time series

Synthetic time series: fractional Brownian Motion (fBm) with controllable Hurst exponent



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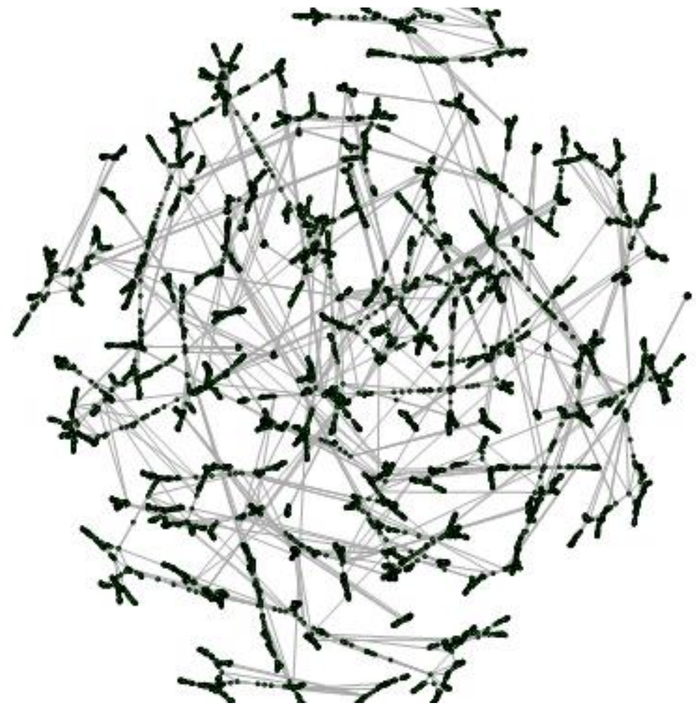
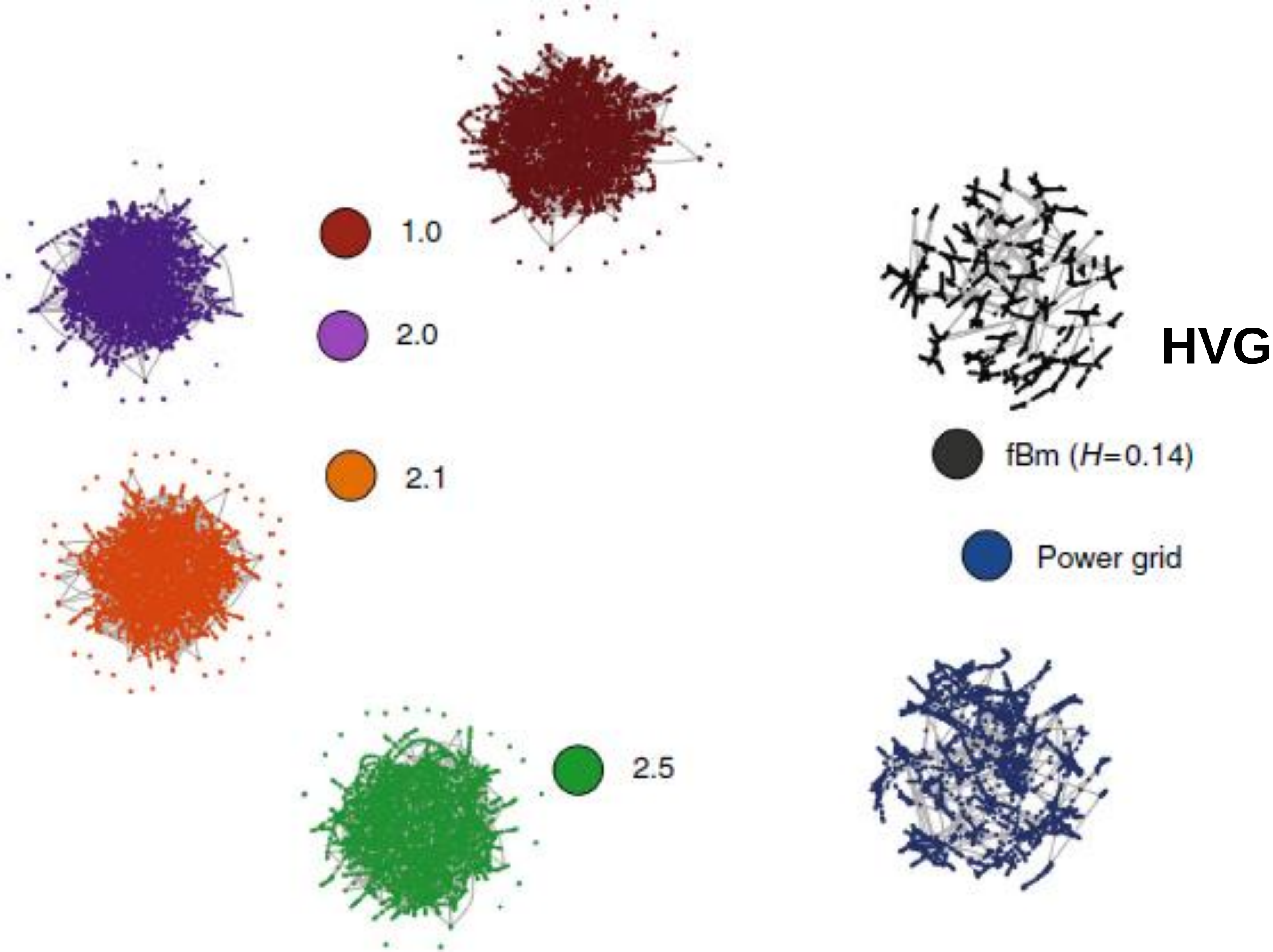


HVG method: Luque et al, Phys. Rev. E 80, 046103 (2009).



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Summary

- New measure to quantify the heterogeneity of the connectivity paths of a single network.
  - It detects the percolation transition in a random network.
- New measure to calculate the distance between two networks
  - Can be applied to networks of different sizes.
  - Returns $D=0$ only if the two networks are isomorphic.
- Many possible applications: characterizing time-evolving networks, classification of biological networks, etc.

THANK YOU FOR YOUR ATTENTION !

T. A. Schieber et al, "*Quantification of network structural dissimilarities*", Nat. Comm. 8:13928 (2017).

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- School on "*Nonlinear Time Series Analysis and Complex Networks in the Big Data Era*", co-organized with Jesus Gomez-Gardenes and Hilda Cerdeira

  ICTP-SAIFR (Sao Paulo): February 19 – March 2, 2018

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