Are the spikes emitted by a semiconductor laser with feedback similar to neuronal spikes?

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Computational Neuroscience and Optical Dynamics (CNOD)
Nice, May 2017

ICREA

GOBIERNO DE ESPAÑA
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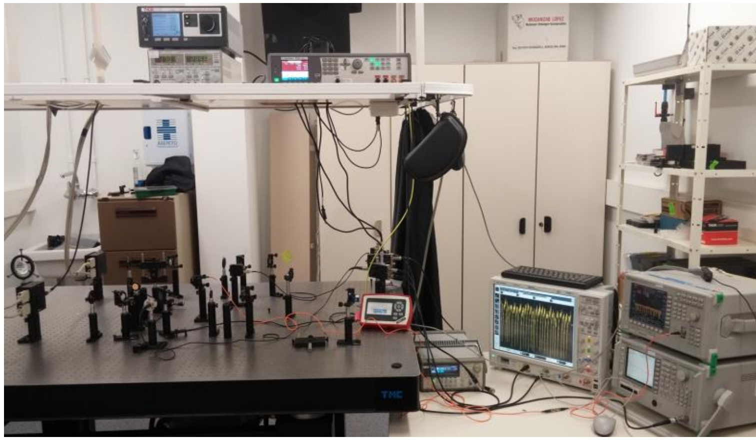
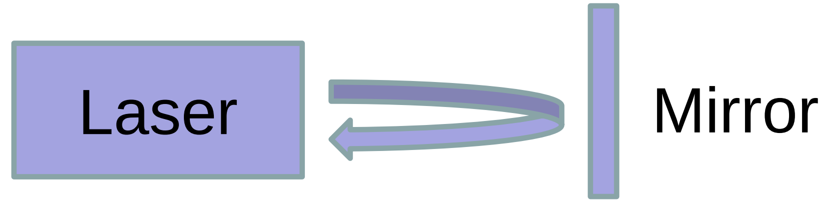
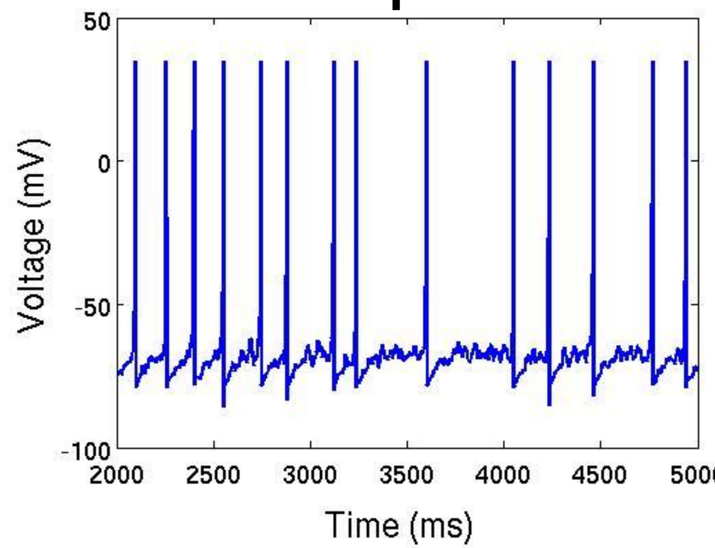


In our lab: experiments with semiconductor lasers



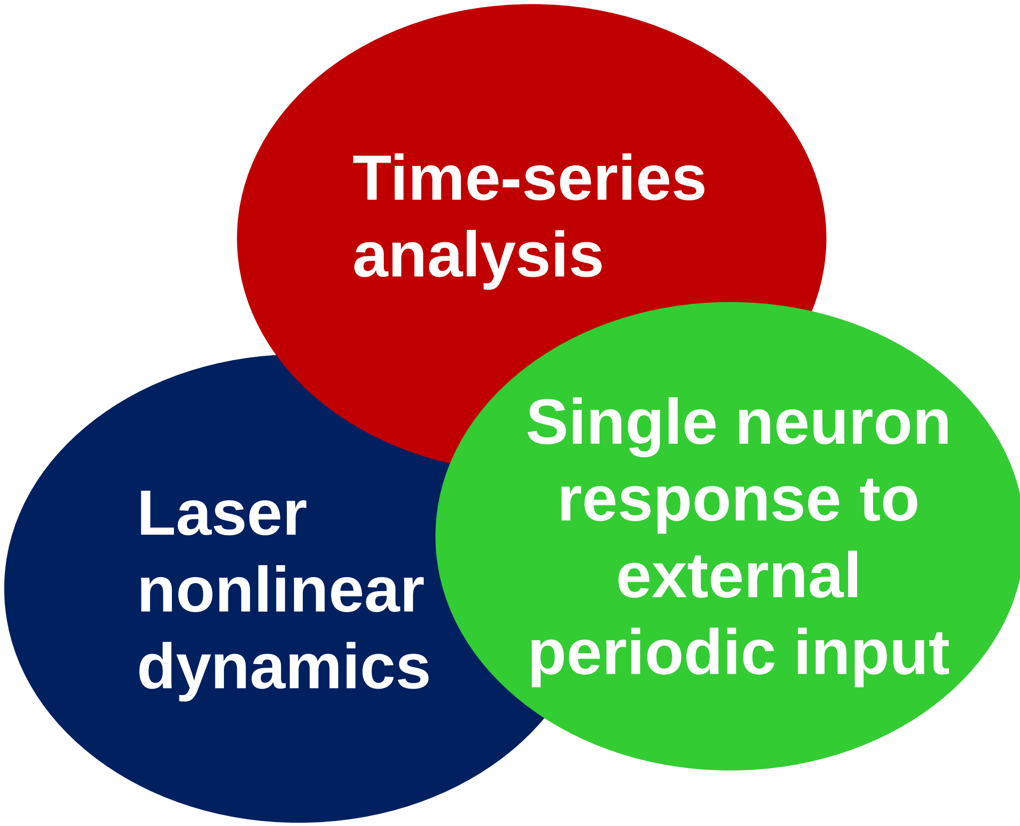
Optical spikes

Neuronal spikes



Similar processes generate these output signals?
Similar statistics of inter-spike intervals?

Outline



- Introduction
  - Motivation: spiking lasers that mimic neuronal behavior
  - Symbolic method of time-series analysis
- Results:
  - Analysis of optical spikes
  - Contrasting optical and neuronal spikes
  - Analysis of neuronal spikes
- Summary

MOTIVATION

Science 345, 668 (2014)

"a computer that is **inspired** by the brain."

Neuro-synaptic architecture allows to do things like image classification at a very low power consumption.

- Spiking lasers: photonic neurons?
- potential building blocks of brain-inspired computers.
- Ultra fast ! (micro-sec vs. mili-sec)

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Comparison of empirical data: neuron & optical spikes

Neuron inter-spike interval (ISI) distribution

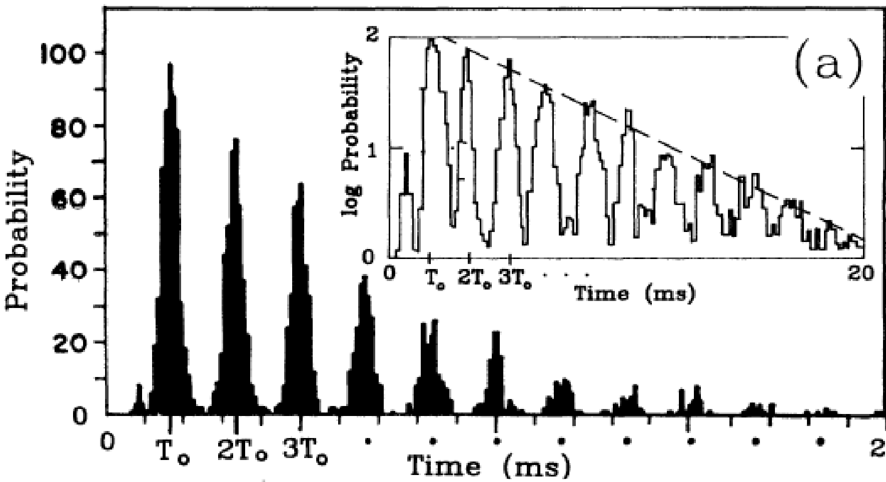
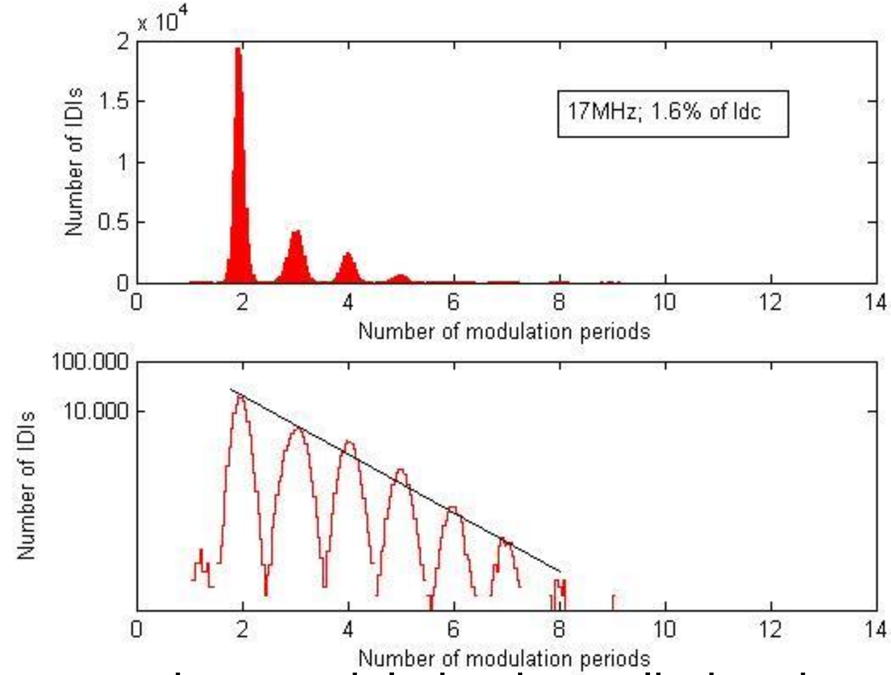


FIG. 1. (a) An experimental ISIH obtained from a single auditory nerve fiber of a squirrel monkey with a sinusoidal 80-dB sound-pressure-level stimulus of period $T_0 = 1.66$ ms applied at the ear. Note the modes at integer multiples of T_0. Inset:

A. Longtin et al, PRL 67 (1991) 656

Optical ISI distribution, data collected in our lab



when modulation is applied to the laser current

HOW SIMILAR NEURONAL AND OPTICAL SPIKES ARE?

Neuronal ISIs

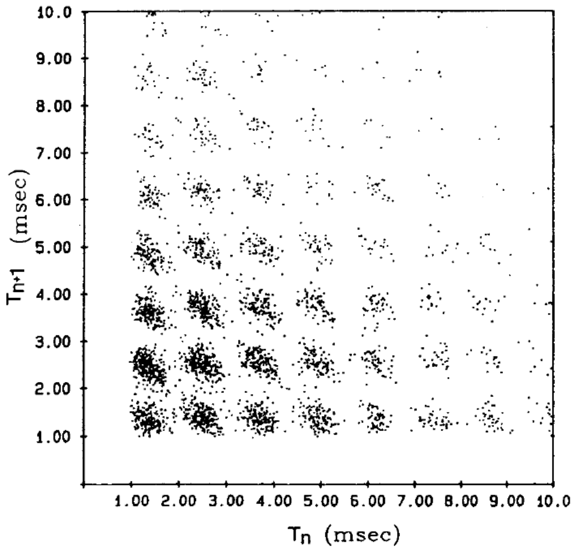
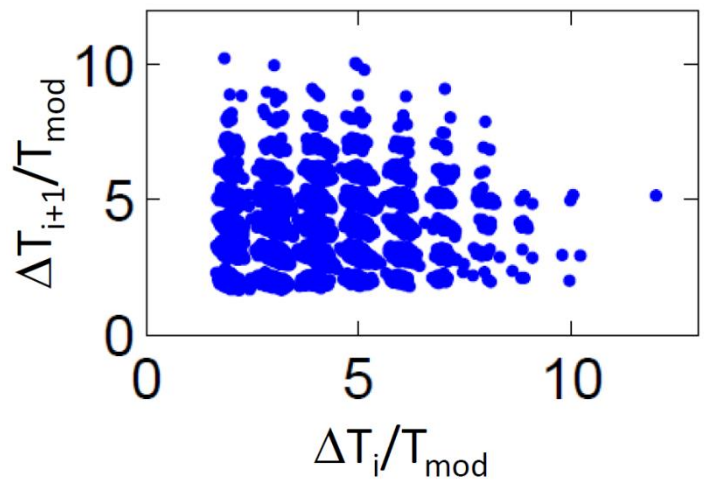
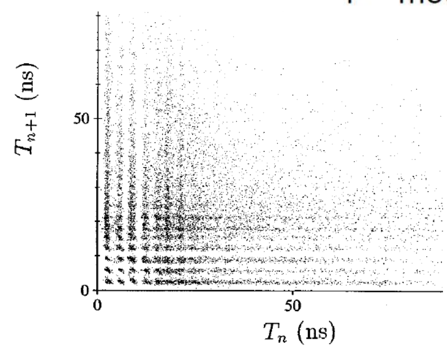


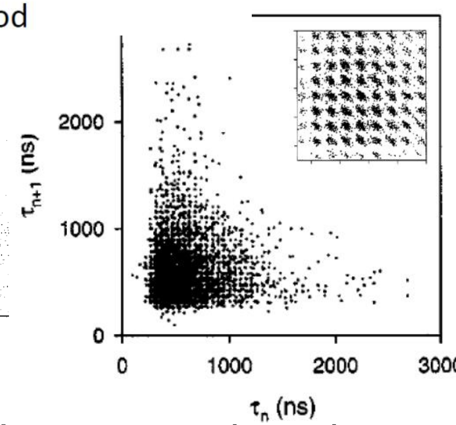
Fig. 4. Scatter plot of spike train data obtained from extracellular measurements of cat auditory fiber activity in response to an 800 Hz 60 dB sound pressure level pure tone presented to the outer ear. The stimulus is discontinuous (see

A. Longtin IJBC 3 (1993) 651

Optical ISIs

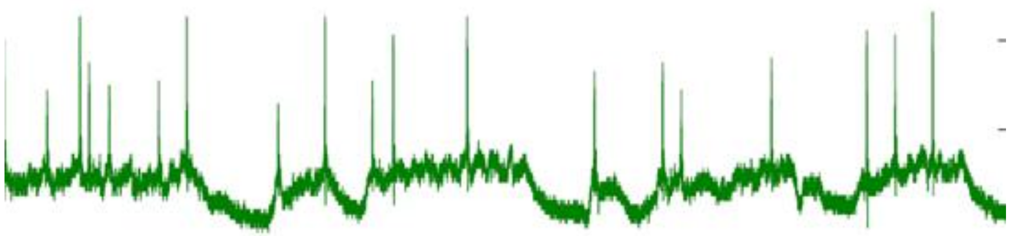






A. Aragoneses et al, Opt. Exp. (2014)
M. Giudici et al, PRE 55, 6414 (1997)
D. Sukow and D. Gautheir, JQE (2000)

How neurons encode information?



- In the spike rate?

- In the relative **timing** of the spikes?

- How temporal correlations can be detected and quantified?

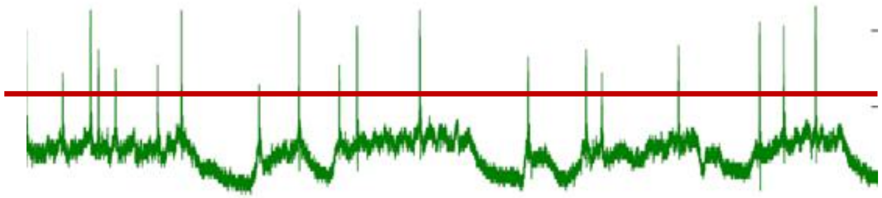
- Our goal: try to understand how a single neuron encodes a weak (subthreshold) periodic input.

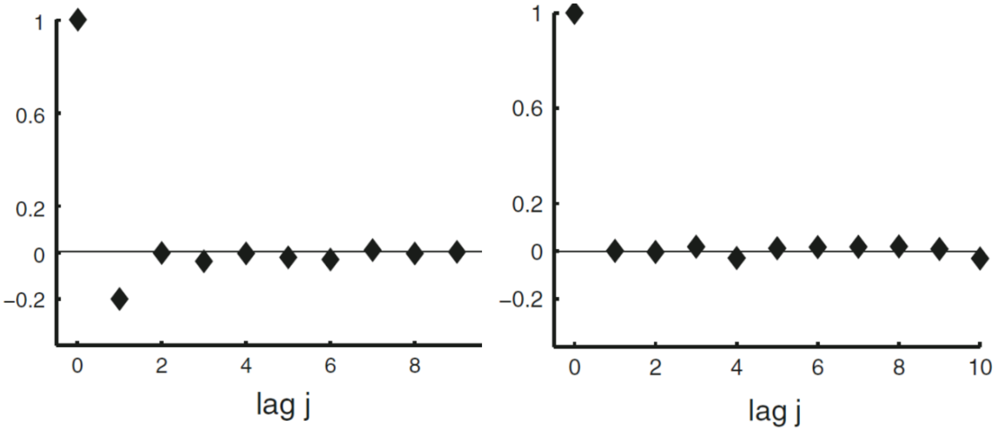
Inter-spike-intervals serial correlation coefficients

$$\{\ldots I_{i-1}, I_i, I_{i+1}\ldots\}$$

$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

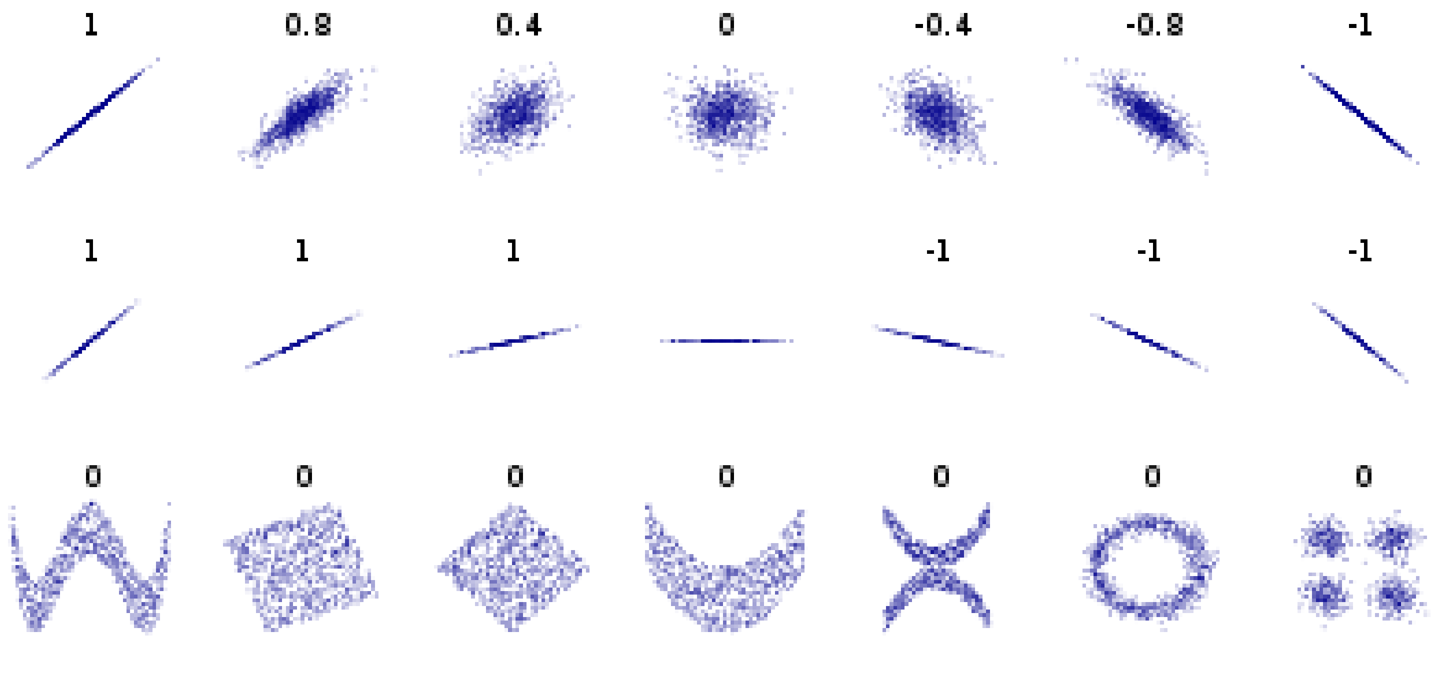
inter-spike-intervals $I_i = t_{i+1} - t_i$





Exp Brain Res (2011) 210:353–371

However, correlation analysis detects linear relationships only



HOW TO INDENTIFY TEMPORAL STRUCTURES?
RECURRENT / INFREQUENT PATTERNS?

Source: wikipedia

Symbolic method of time-series analysis

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Symbolic analysis

- The time series $\{x_1, x_2, x_3, \ldots\}$ is transformed (using an appropriated rule) into a sequence of symbols $\{s_1, s_2, \ldots\}$
- taken from an "alphabet" of possible symbols $\{a_1, a_2, \ldots\}$.
- Then consider "blocks" of D symbols ("patterns" or "words").
- All the possible words form the "dictionary".

- Then analyze the "language" of the sequence of words
  - the probabilities of the words,
  - missing/forbidden words,
  - transition probabilities,
  - information measures (entropy, mutual information, etc).

Threshold transformation (phase space partition)

- if $x_i > x_{th} \Rightarrow s_i = 0$; else $s_i = 1$

  transforms a time series into a sequence of 0s and 1s, e.g., {011100001011111…}

- Considering "blocks" of D letters gives the sequence of words. Example, with D=3:

  {011 100 001 011 111 …}

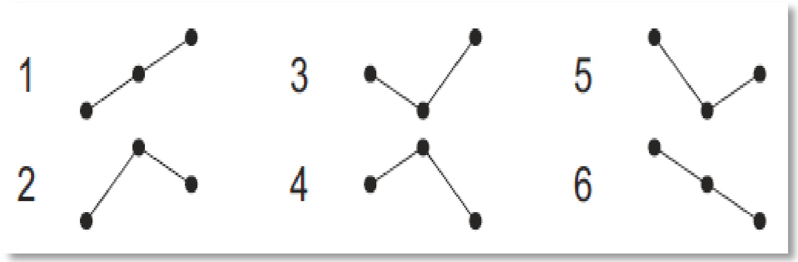
- The number of words (patterns) grows as **2^D**

Alternative symbolic rule

- Ordinal transformation (Bandt and Pompe PRL 88, 174102):
  if $x_i > x_{i-1} \Rightarrow s_i = 0$; else $s_i = 1$
  also transforms a time-series into a sequence of 0s and 1s <u>without</u> using a predefined threshold.

- "words" (*Ordinal Patterns*) are formed by considering the order relation between sets of D values.

- D=3: $\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$

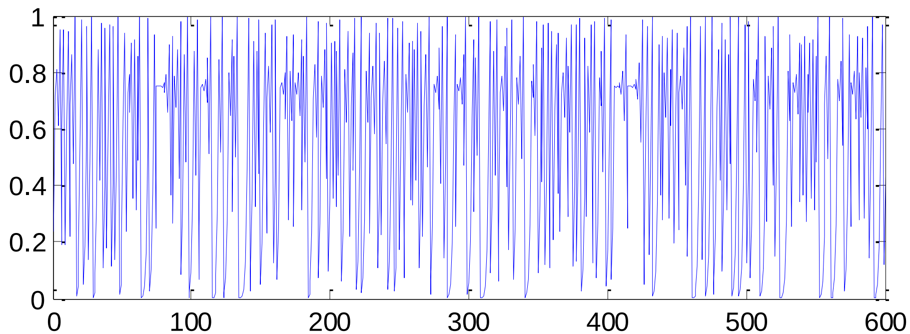


- The number of patterns grows as D!

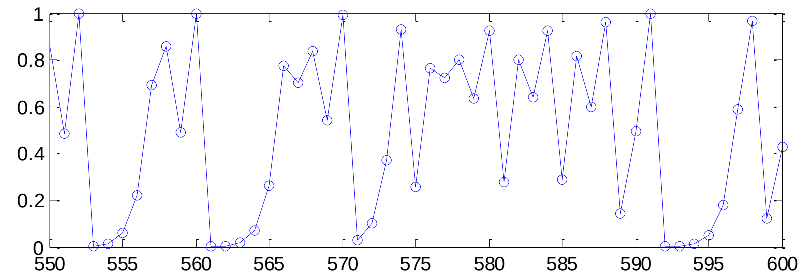
Example: the logistic map

$x(i+1)=r\,x(i)[1-x(i)]$

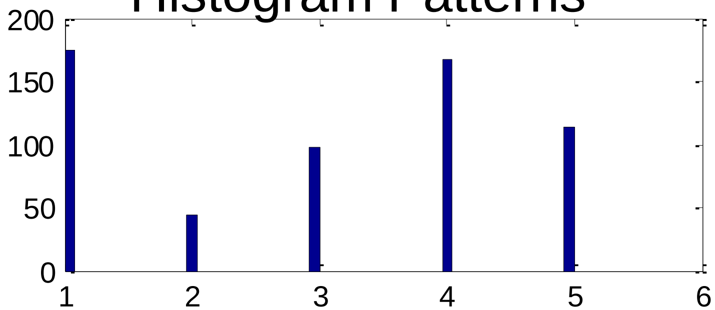
Time series

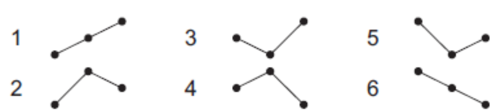


Detail



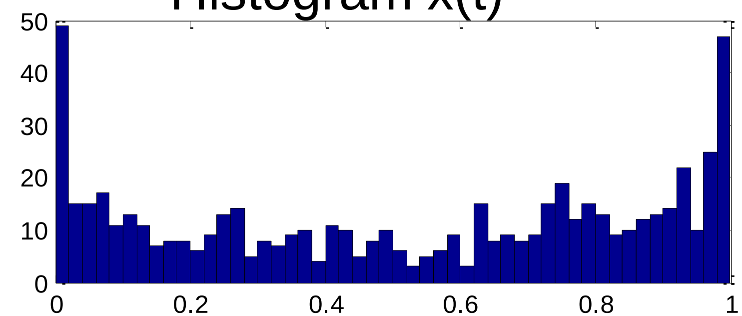
Histogram Patterns





forbidden

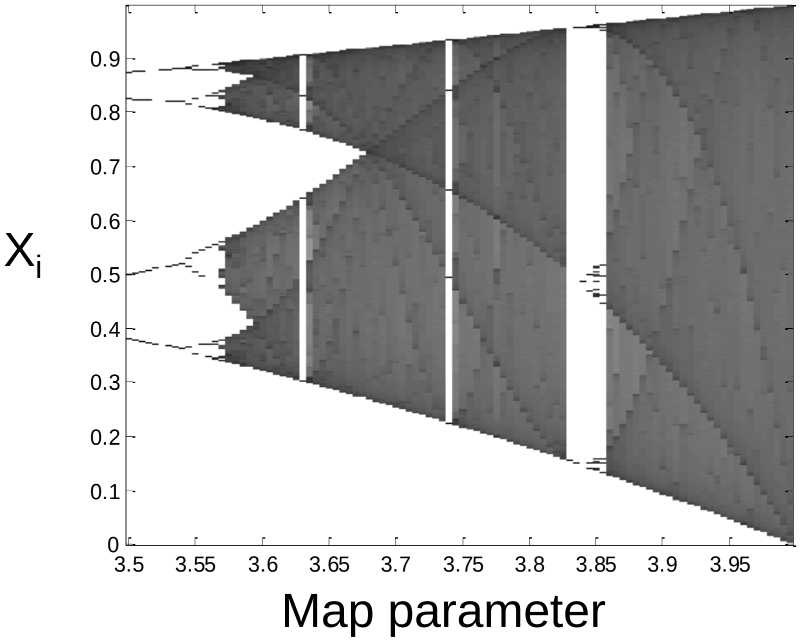
Histogram x(t)



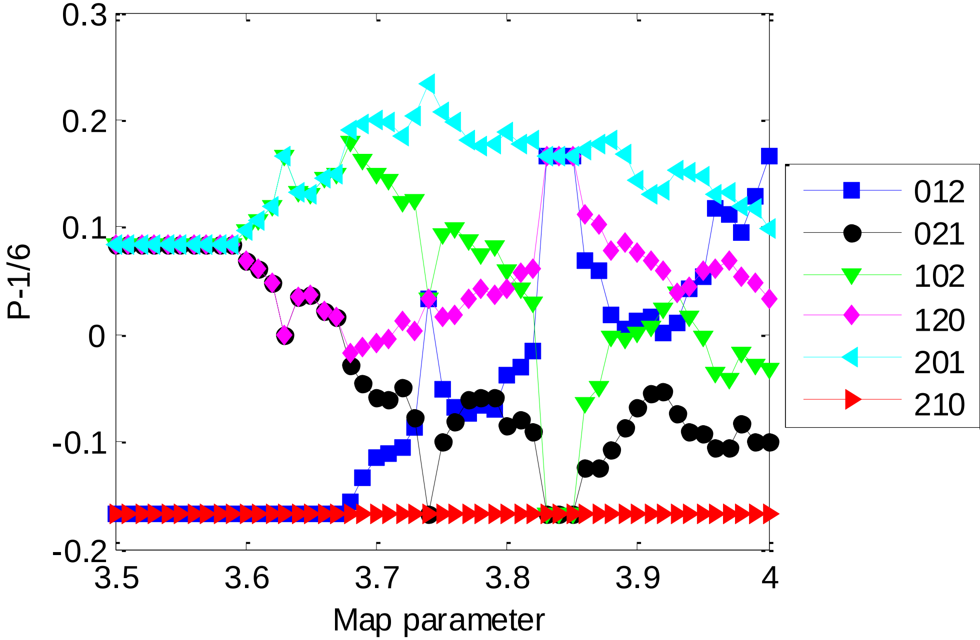
Take home message: ordinal analysis can yield information about more expressed (and/or missing) patterns in the data.

Bifurcation diagrams

Standard diagram



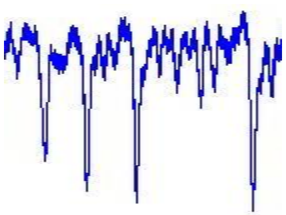
Ordinal diagram



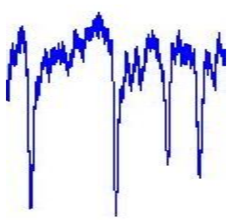
Applications

- Analysis of complex signals
  - Financial, Biomedical, Geosciences, Climate, etc
- Able to:
  - Distinguish stochasticity and determinism
  - Classify different types of dynamical behaviors (pathological, healthy)
  - Quantify complexity, identify coupling and directionality, etc.

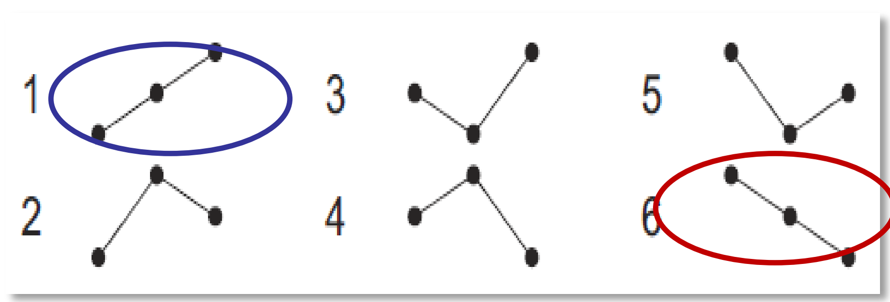
- Here: correlations among 3 ***inter-spike-intervals*** (ISIs).



⇒012



⇒210



Ordinal analysis of optical spikes

- temporal ISI correlations?
- how do they vary with the control parameters?

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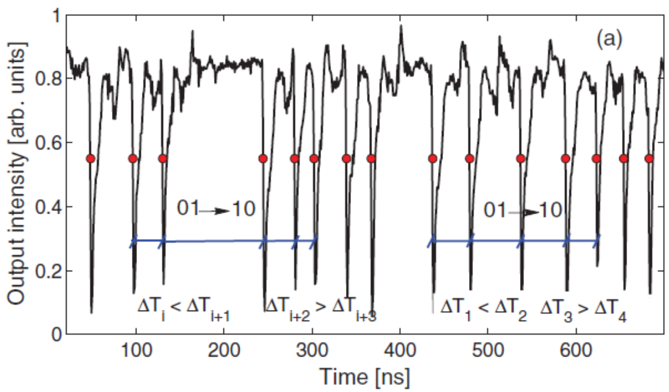
Transition to optical complexity

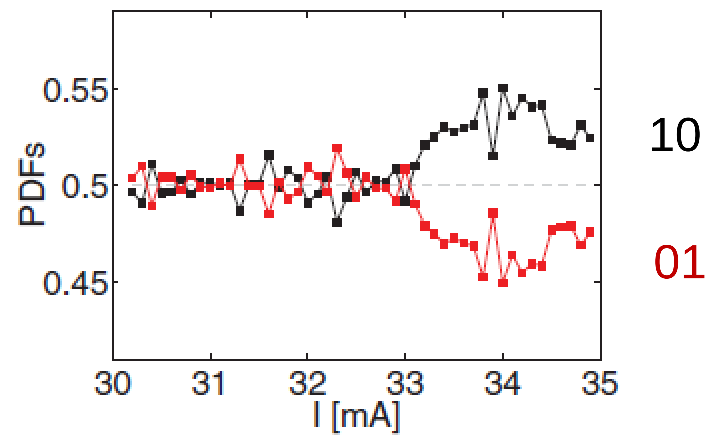
PHYSICAL REVIEW E **84**, 026202 (2011)

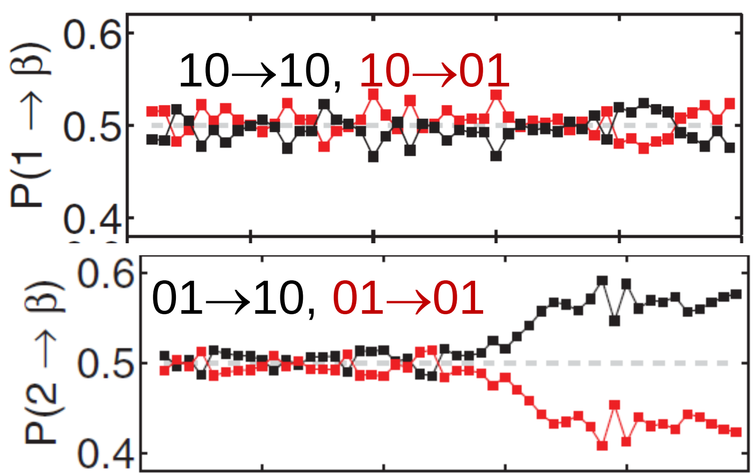
video

Language organization and temporal correlations in the spiking activity of an excitable laser: Experiments and model comparison

Nicolas Rubido,[1] Jordi Tiana-Alsina,[2] M. C. Torrent,[2] Jordi Garcia-Ojalvo,[2] and Cristina Masoller[2]







Consistent with stochastic dynamics at low pump current, signatures of determinism at higher pump currents.

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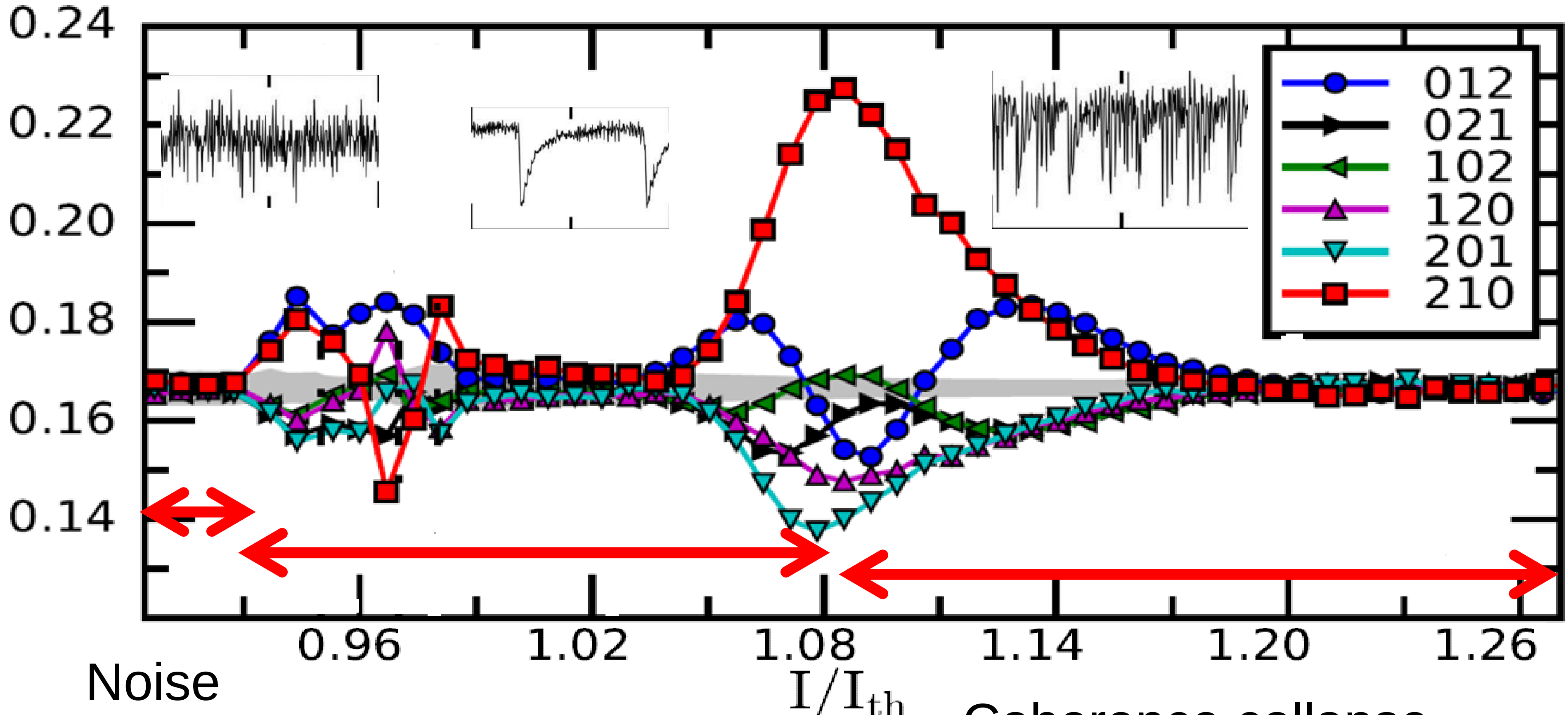
Ordinal analysis allows to quantify the onset of different dynamical regimes



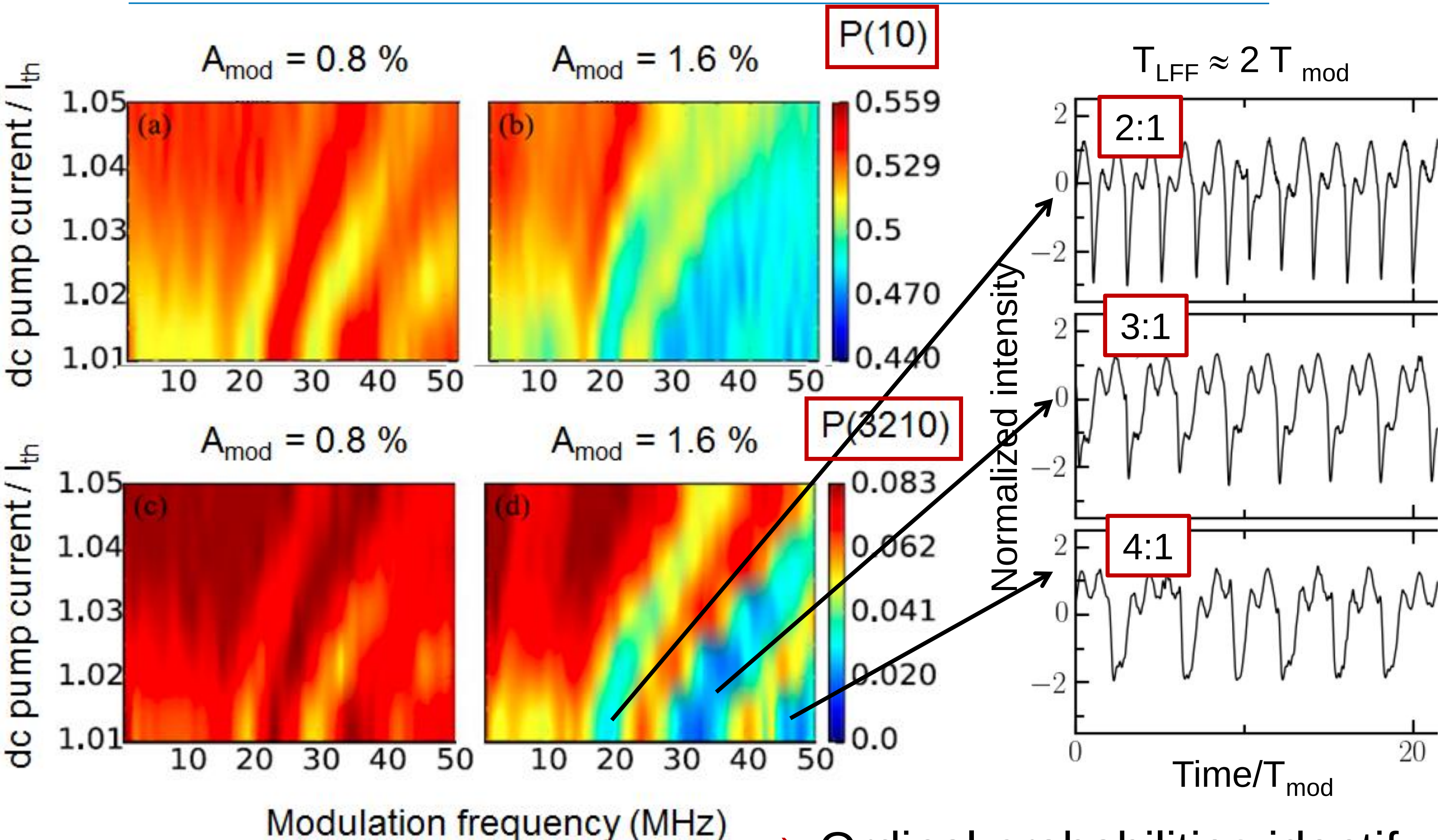
Noise

Low frequency fluctuations

Coherence collapse

C. Quintero-Quiroz et al, "*Characterizing how complex optical signals emerge from noisy intensity fluctuations*", Sci. Rep. 6 37510 (2016)

Ordinal analysis allows to identify noisy entrainment to sinusoidal modulation



$\Rightarrow$ Ordinal probabilities identify regions of noisy locking

T. Sorrentino et al, JSTQE 21, 1801107 (2015)

Contrasting empirical optical spikes with synthetic neuronal spikes

- do they have similar ordinal statistics?
- are there more/less frequent patterns?

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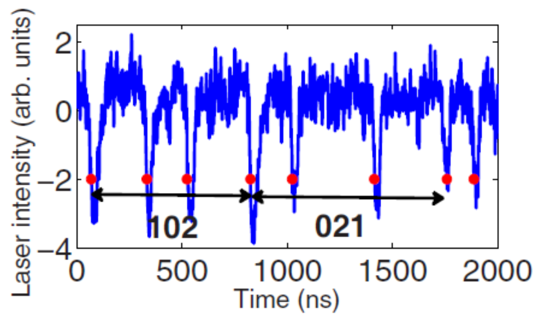
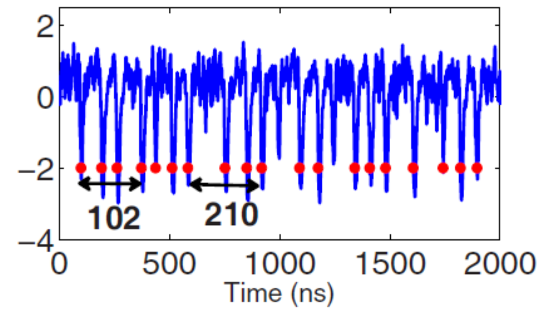
Ordinal analysis of ISI correlations in the region of low-frequency fluctuations

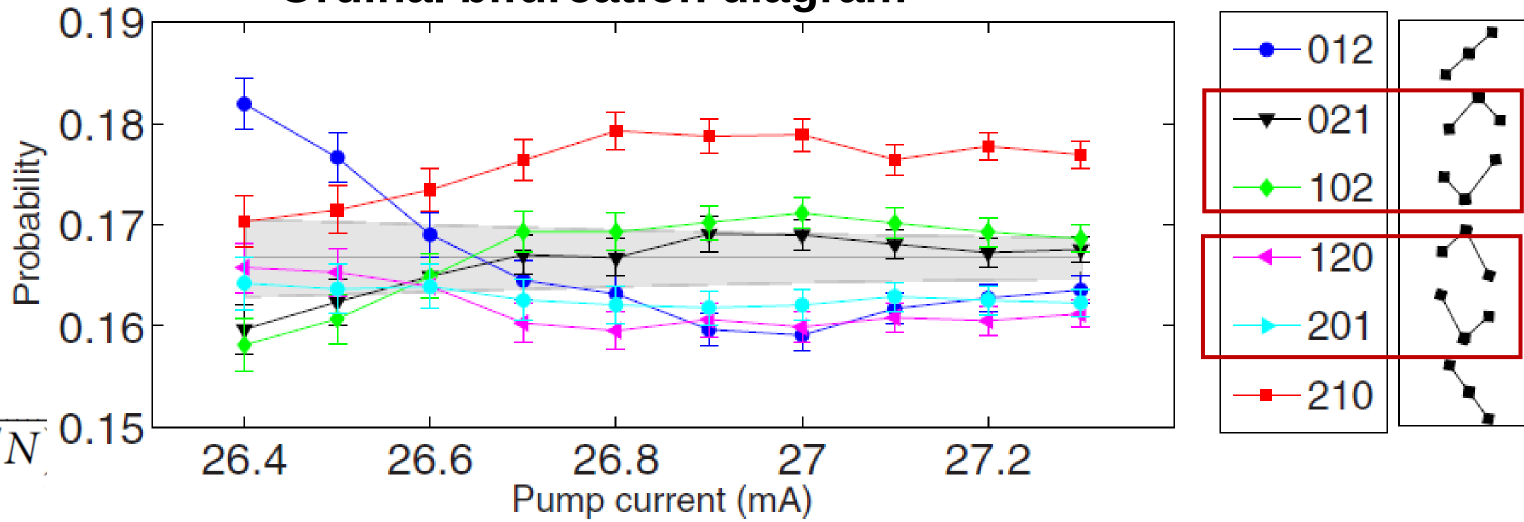
Close to threshold



Higher pump current



Ordinal bifurcation diagram



Grey region 99.7% confidence level.

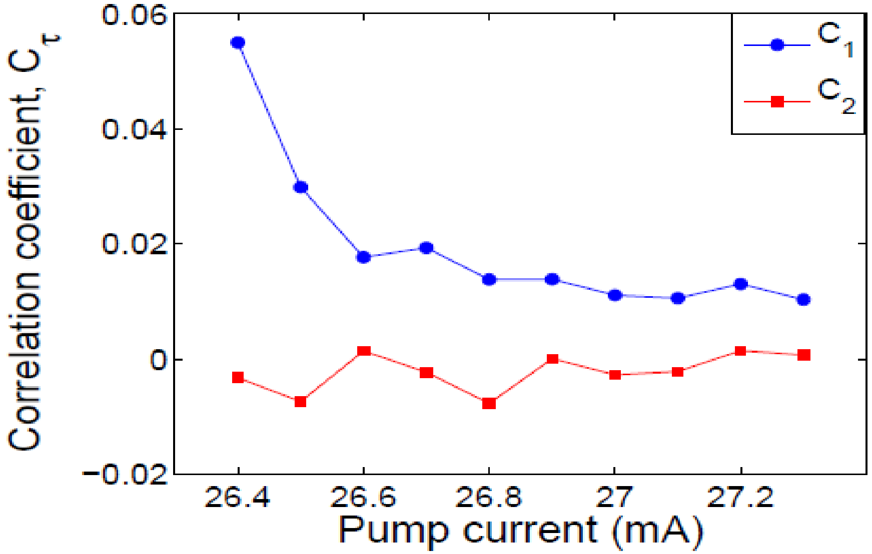
$$p \pm 3\sigma_p$$

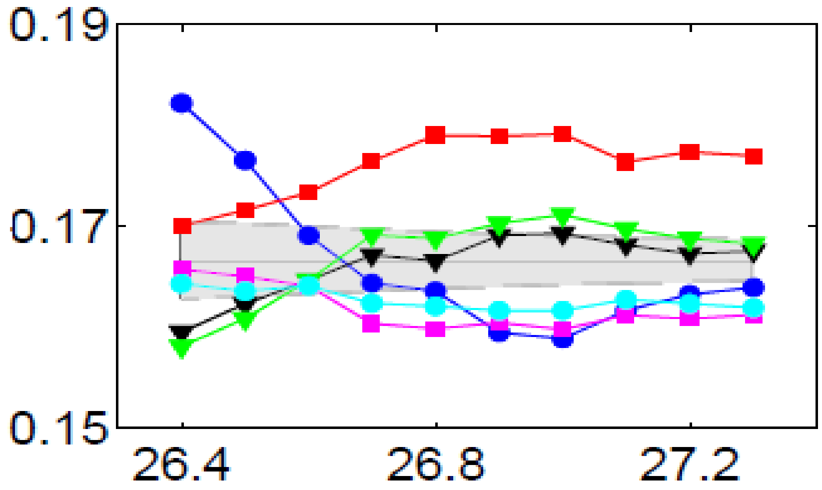
$$\sigma_p = \sqrt{(p(1-p)/N)}$$

P =1 /6; ***N* > 10,000 ISIs**

A. Aragoneses, S. Perrone, T. Sorrentino, M. C. Torrent and C. Masoller, Sci. Rep. **4**, 4696 (2014)

Comparison of linear and nonlinear ISI correlations





$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

- C_1 is always positive $\Rightarrow$ cross over (012 more/less probable than 210) is not detected.
- C_2 is very small, indicating no linear correlation between I_i and I_{i+2} but ordinal probabilities are not consistent with equally probable patterns.

A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)

Minimal model of ISI nonlinear correlations: modified circle map

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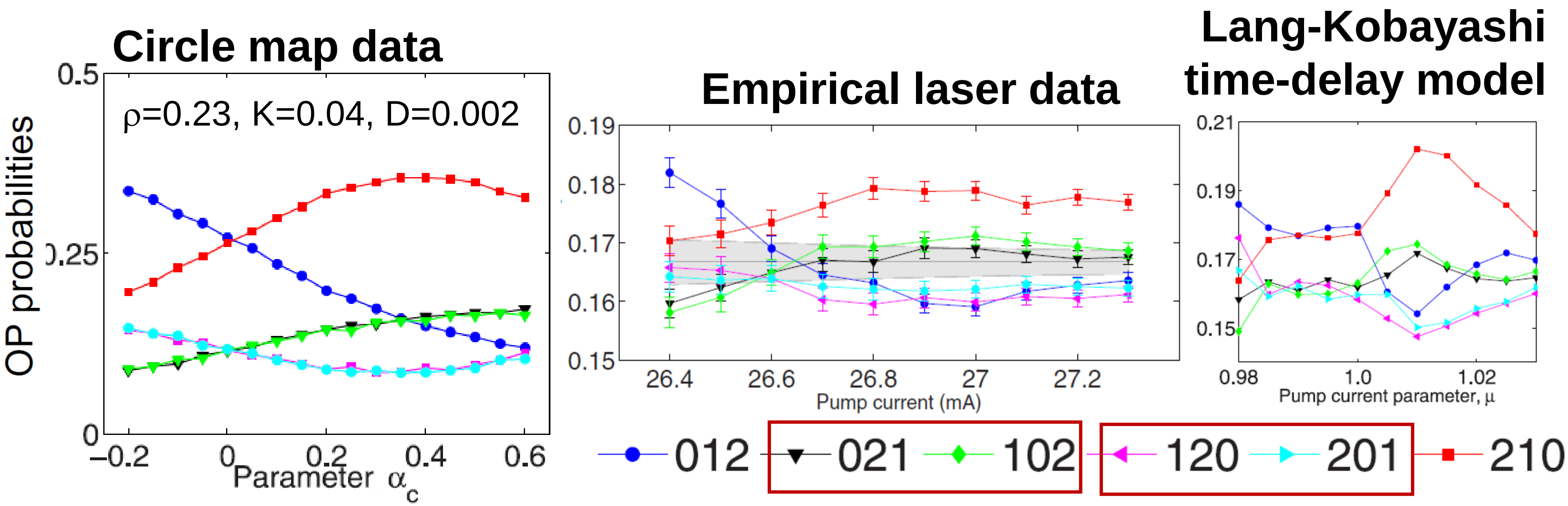
$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha_c \sin(4\pi\varphi_i)\right] + D\zeta$$

$$X_i = \varphi_{i+1} - \varphi_i$$

ρ = $\frac{\text{natural frequency}}{\text{forcing frequency}}$

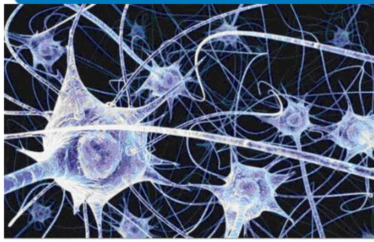
K = forcing amplitude

D = noise strength



- Same "clusters" & same hierarchical structure.
- Modified circle map: minimal model for ordinal correlations.
- Same qualitative behavior found with other lasers & feedback conditions.

Model equations and parameters: A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)

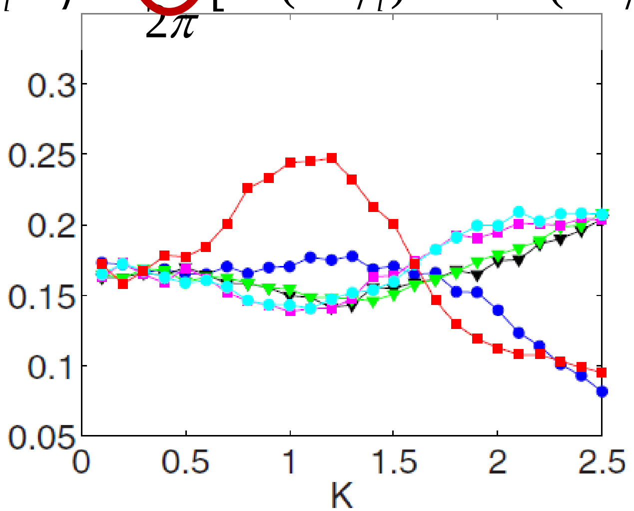


Connection with neurons

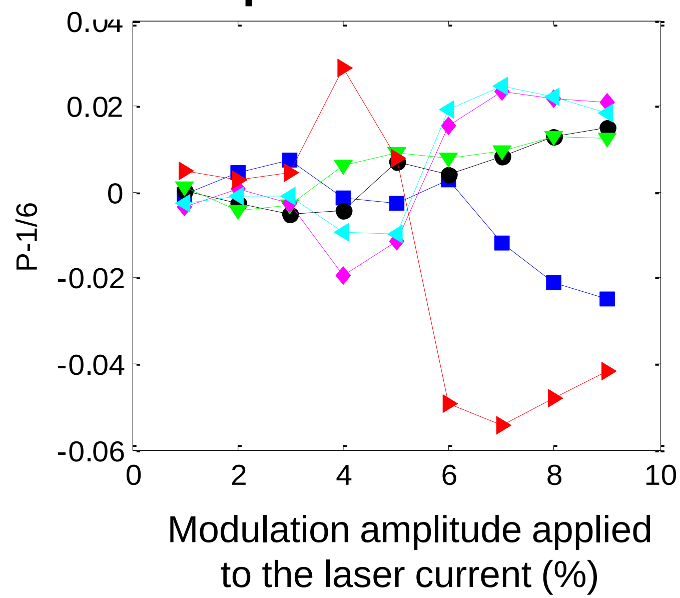
- The circle map describes many excitable systems.
- The modified circle map has been used to describe spike correlations in biological neurons.

  A. B. Neiman and D. F. Russell, *Models of stochastic biperiodic oscillations and extended serial correlations in electroreceptors of paddlefish*, PRE 71, 061915 (2005)

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha\sin(4\pi\varphi_i)\right]$$

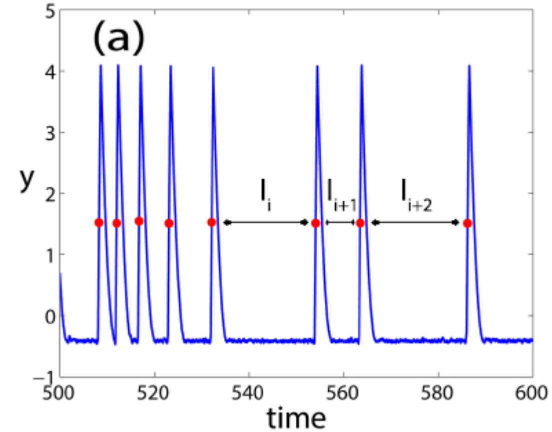


Empirical laser data



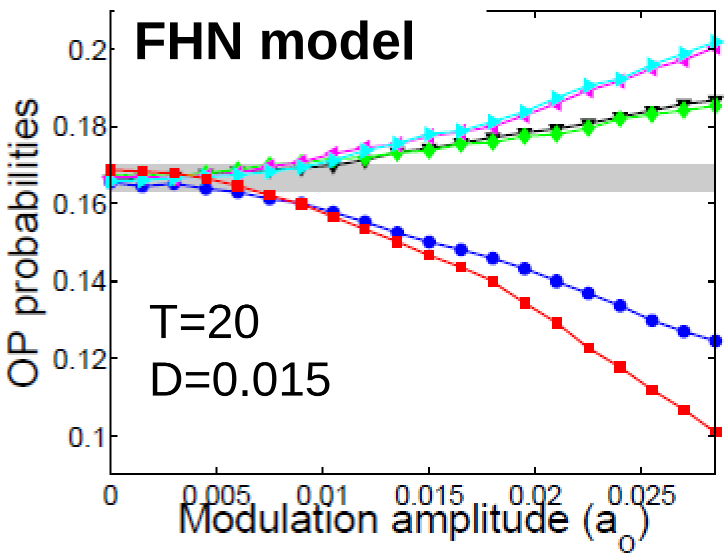
FHN neuron model

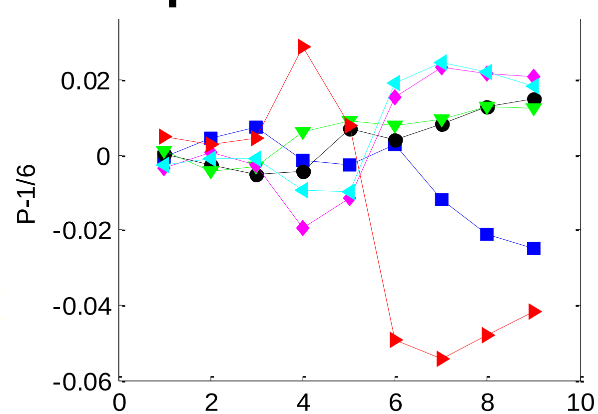
$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o \cos(2\pi t/T)} + D\xi(t),$$



Gaussian white noise and <u>subthreshold</u> (weak) modulation: **a_0** and **T** such that spikes are only noise-induced.

Time series with 100,000 ISIs simulated.





012 021 102 120 201 210

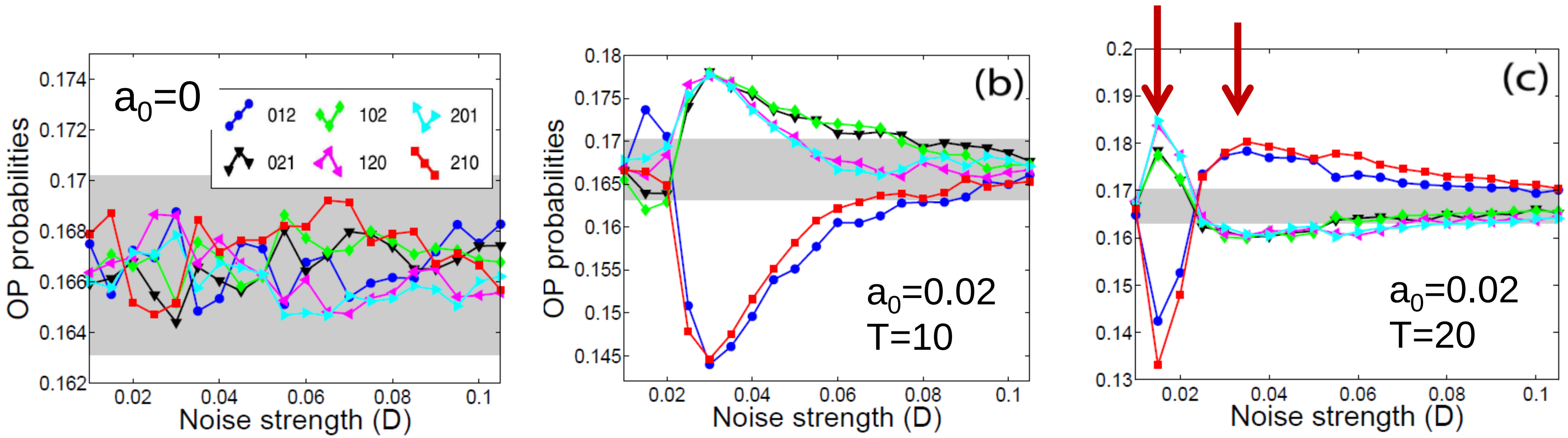
⇒ Good qualitative agreement.

J. M. Aparicio-Reinoso et al, PRE 94, 032218 (2016).

Analysis of ISI sequences generated by FHN model

- more/less frequent patterns encode information about subthreshold signal?

FHN model: role of the noise strength



- No signal ⇒ no noise-induced temporal ordering.
- Subthreshold periodic input induces temporal ordering.
- Preferred ordinal patterns depend on the period and on the noise strength.
- Resonant-like behavior.

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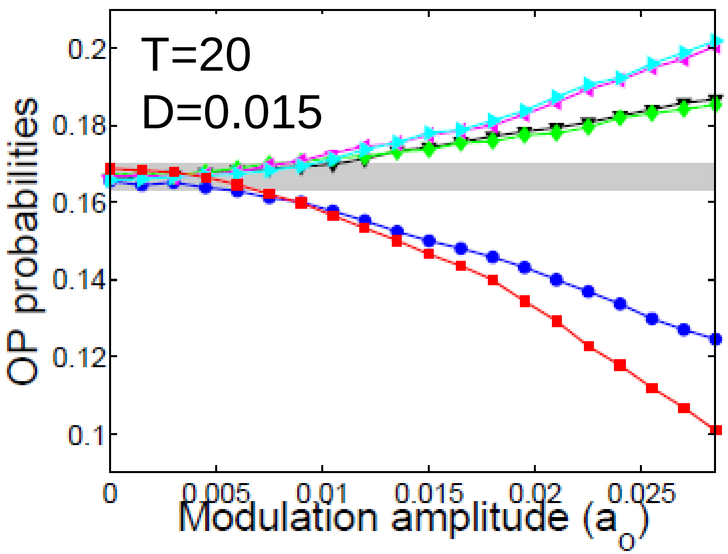
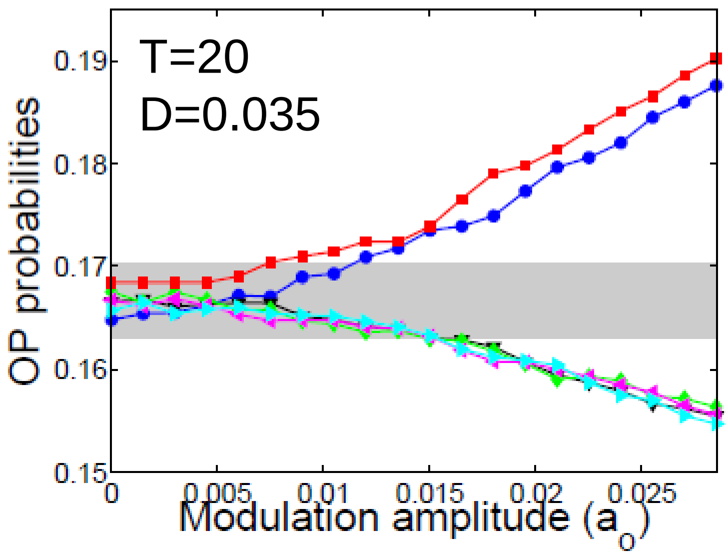
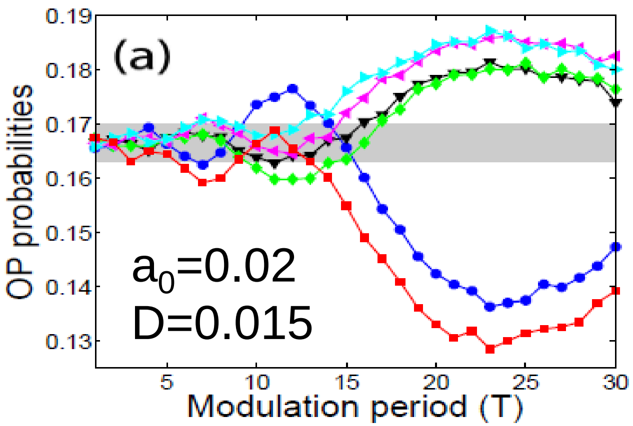
Role of the modulation amplitude

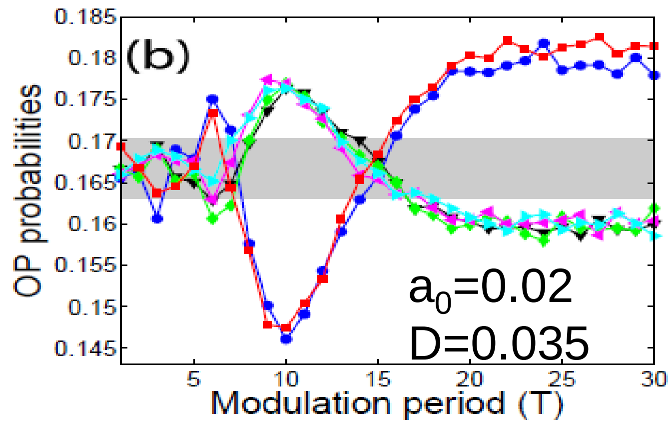




- The amplitude of the (weak) modulation does not modify the preferred and the infrequent patterns.

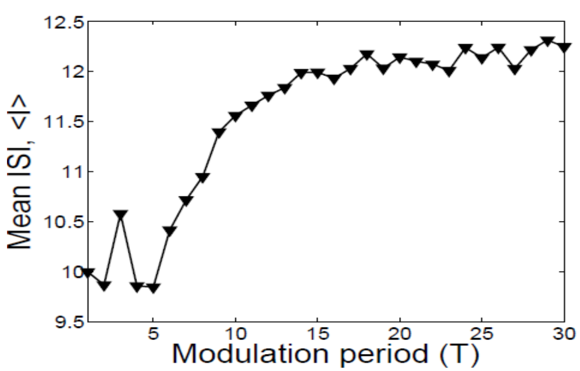
Role of the modulation period

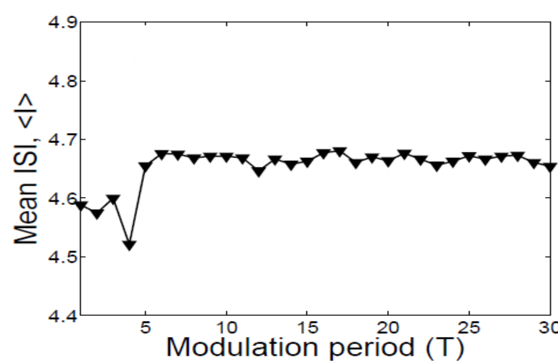




- More probable patterns depend on the period of the external input and on the noise strength.

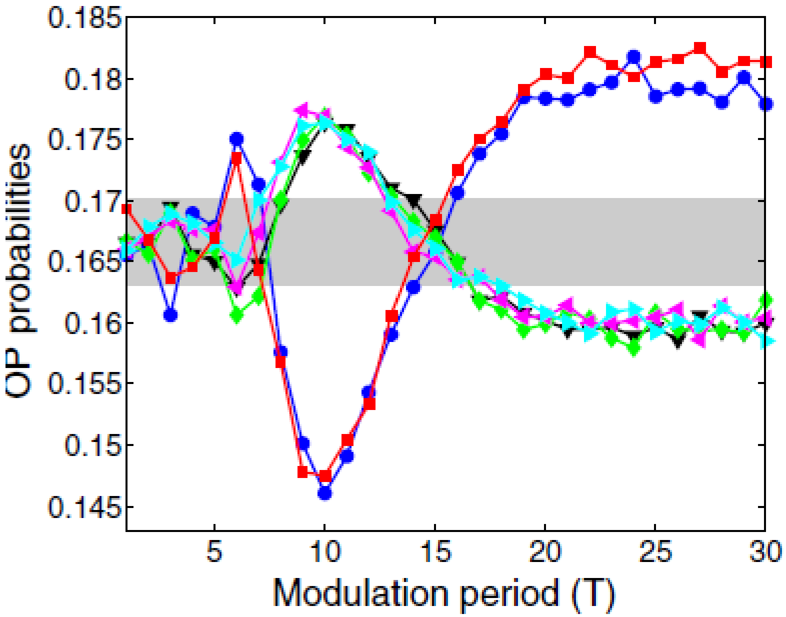
Which is the underlying mechanism? A change of the spike rate?

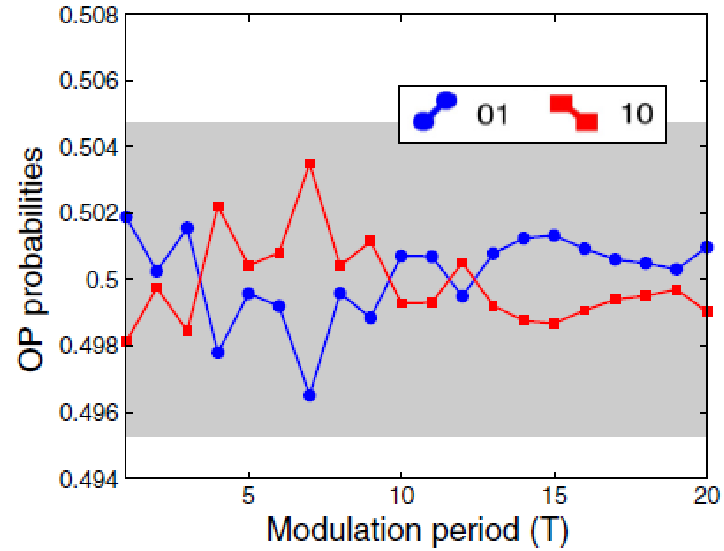


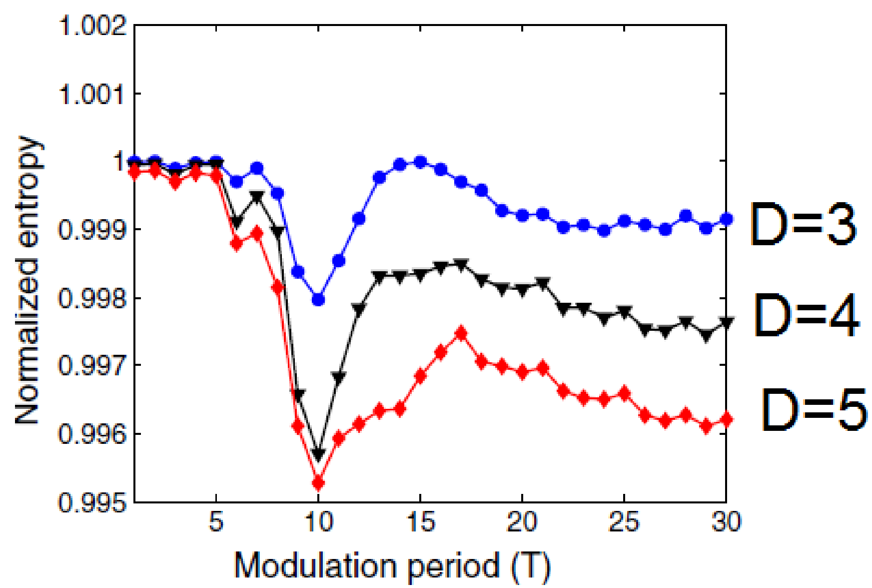


$\Rightarrow$ No direct relation.

Length of ISI correlations



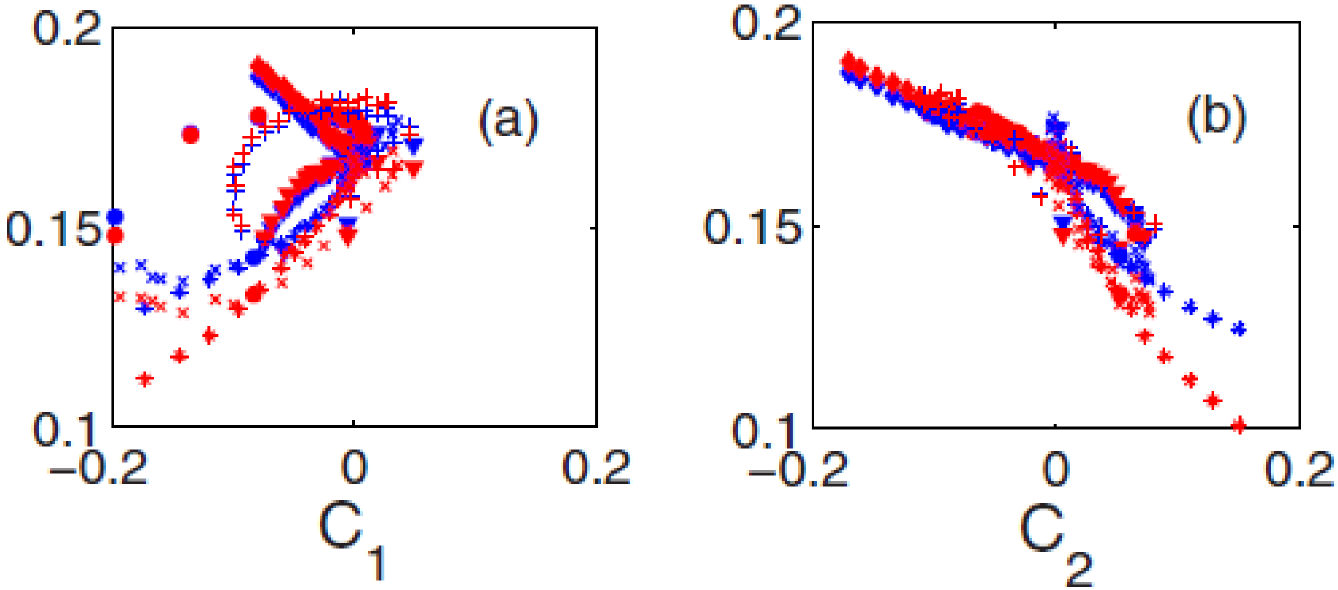




$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

Relation between OPs and correlation coefficients C_1, C_2

P(012)
P(210)



scatter plot with all data-points collapsed (varying noise strength, modulation amplitude and period)

$\Rightarrow$ clear trend with C_2, no trend with C_1

Conclusions

What did we learn?

- Take home message:
  - ordinal analysis is useful for understanding data, uncovering patterns,
  - for model comparison, parameter estimation, classifying events, etc.
  - robust to noise and artifacts in the data.
- Main conclusions
  - Optical & neuronal spikes compared: good qualitative agreement.
  - Minimal model for optical spikes identified: a modified circle map.
  - FHN model with subthreshold modulation and Gaussian white noise
    - There are preferred ordinal patterns which depend on the noise strength and on the period of the input signal, but not on (weak) amplitude of the signal.
    - resonance-like behavior: certain periods and noise levels maximize the probabilities of the preferred patterns, enhancing temporal order.
- Open issues (ongoing and future work):
  - Hierarchical & clustered structure: universal feature of excitable systems?
  - Mathematical insight: can we calculate the probabilities analytically?
  - Role of coupling? induce preferred/infrequent patterns? (Maria's talk)
  - Compare with empirical data (single-neuron ISI sequences)

Collaborators

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THANK YOU FOR YOUR ATTENTION !

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Papers at: **http://www.fisica.edu.uy/~cris/**

- ***Unveiling the complex organization of recurrent patterns in spiking dynamical systems***

  A. Aragoneses et al., Sci. Rep. 4, 4696 (2014).
- ***Analysis of the spike rate and spike correlations in modulated semiconductor lasers with optical feedback***

  T. Sorrentino et al., IEEE J. Sel. Top. Quantum Electron. 21, 1801107 (2015).
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  J. A. Reinoso et al. Phys. Rev. E. 94, 032218 (2016).
- ***Characterizing how complex optical signals emerge from noisy intensity fluctuations***

  C. Quintero-Quiroz et al, Sci. Rep. 6 37510 (2016).

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