Network reconstruction and prediction of the transition to synchrony of coupled oscillators directly from data

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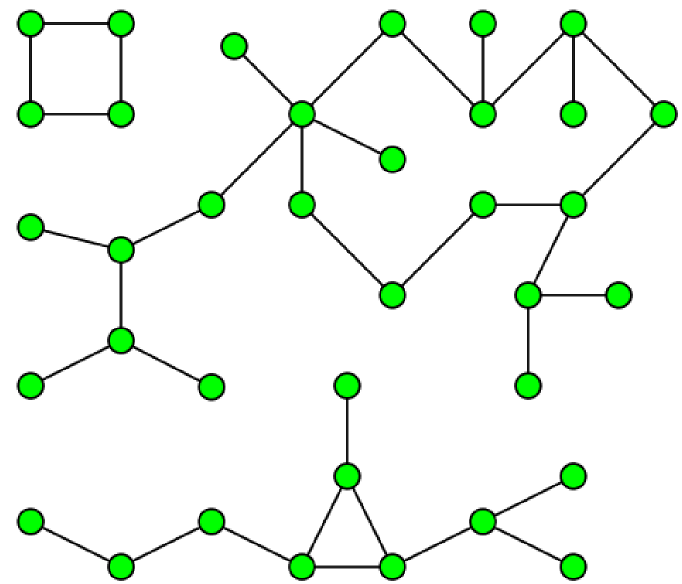


26/8/2021, DynamicsDays2021 Nice

In a system composed by *N* oscillators

From the analysis of the evolution of *an observed variable "u"* in each oscillator: $u_1(t)$, $u_2(t)$, … $u_N(t)$, *can we:*

Infer the existing links between pairs of oscillators?

Predict the transition to synchrony when the coupling increases?

Methods for "network inference" require:

- **A good model of the system (the coupling strengths are fitted parameters)**

  Or

- **The possibility of perturbing the oscillators, to analyze how the perturbations propagate.**

Model-free, non-invasive technique for network inference

- **Bivariate time series analysis: analyze the recorded signals, $u_i(t)$, $u_j(t)$, in each pair of oscillators**

- **Define a statistical similarity measure (SSM: cross correlation, mutual information, Granger causality, etc.)**

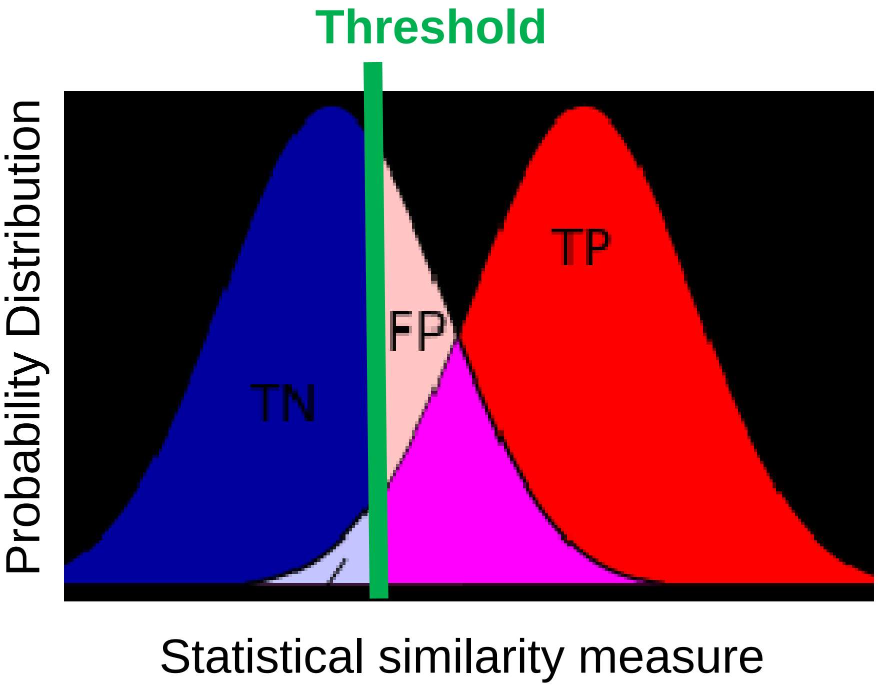
- **Define a "significance" threshold, then**
  - **Two oscillators are connected (Aij=1) if**

    SSM(i,j) > threshold

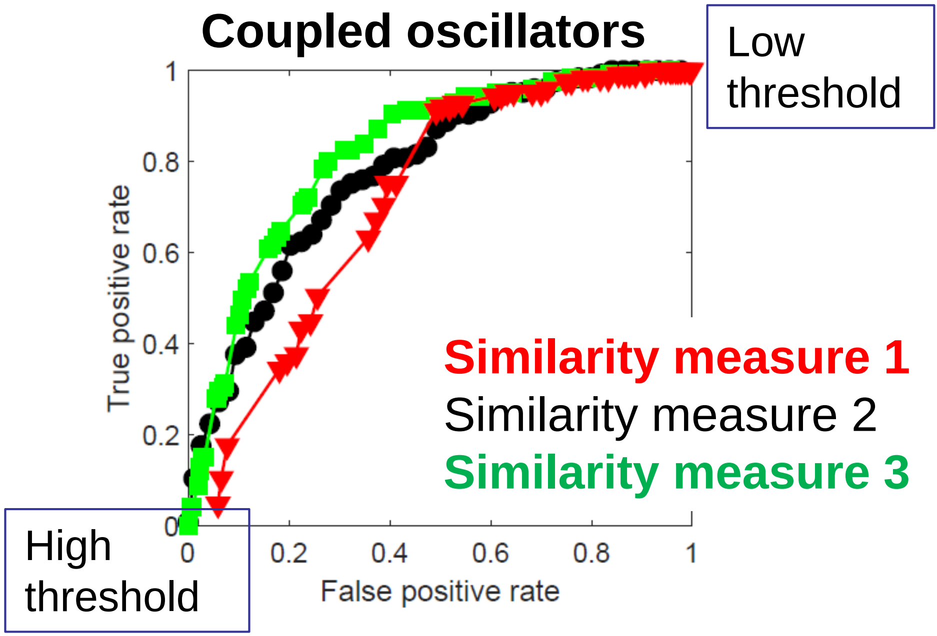
  else
  - **They are disconnected (Aij=0).**

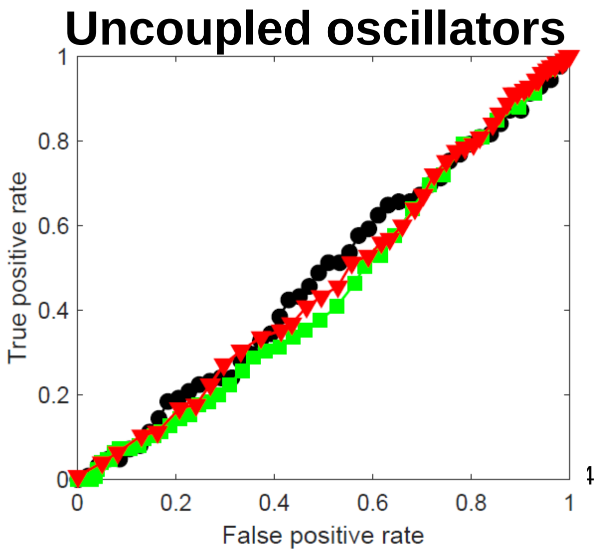
How to quantify the quality of the network reconstruction?

Receiver operating characteristic (ROC curve)



TP	FP
FN	TN





Source: Wikipedia

First numerical example: 12 Kuramoto oscillators randomly connected

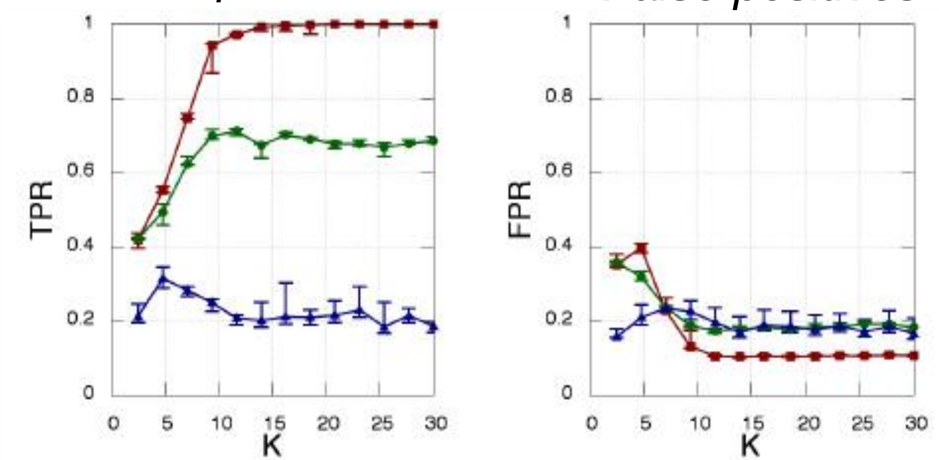
$$d\theta_i = \omega_i dt + \frac{K}{N}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i)dt + D\, dW_t^i$$

A_{ij} is a symmetric random matrix;
N=12 time-series, each with 10^4 data points.

CC MI MIOP

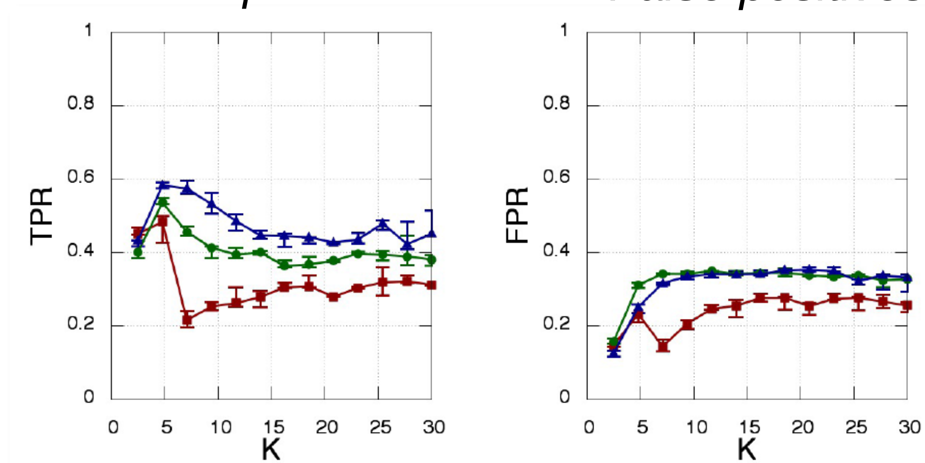
Phases (θ)

True positives *False positives*



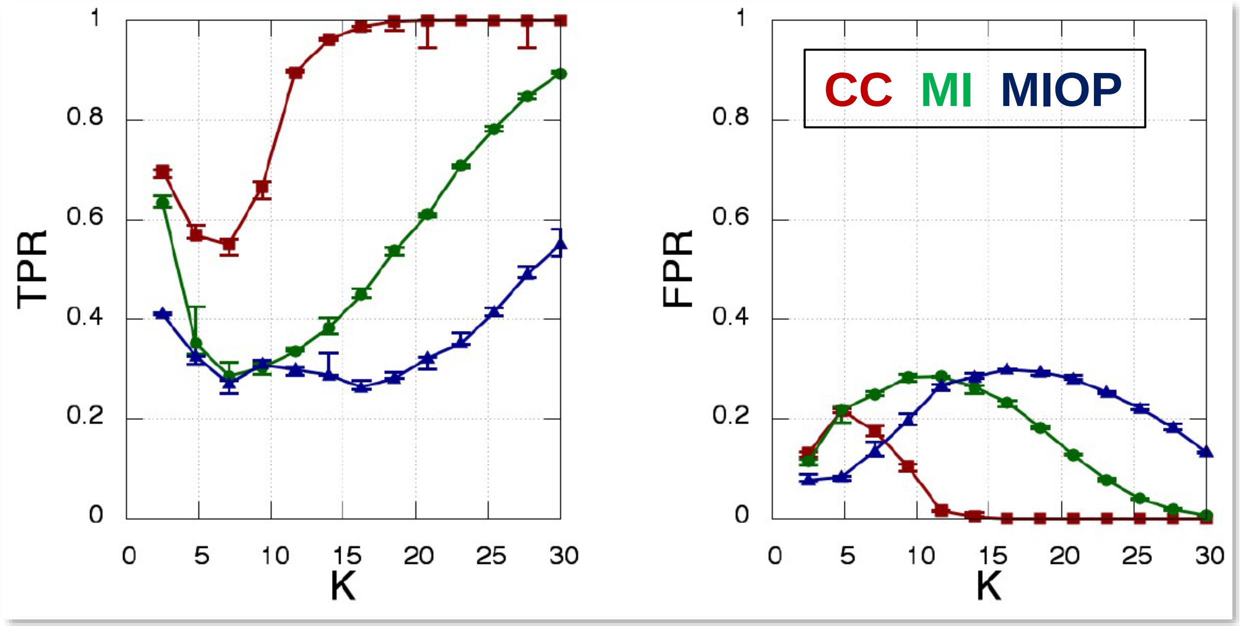
"Observable" Y=sin(θ)

True positives *False positives*



Results of a 100 simulations with different oscillators' frequencies, random matrices, noise realizations and initial conditions.
For each K, the threshold was varied to obtain optimal reconstruction.

The analysis of the instantaneous frequencies ($d\theta/dt$) allows perfect reconstruction

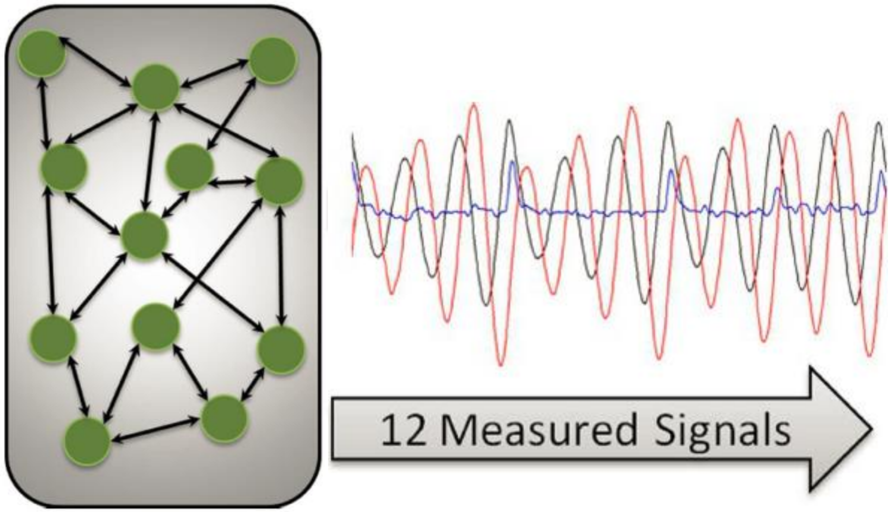


BUT

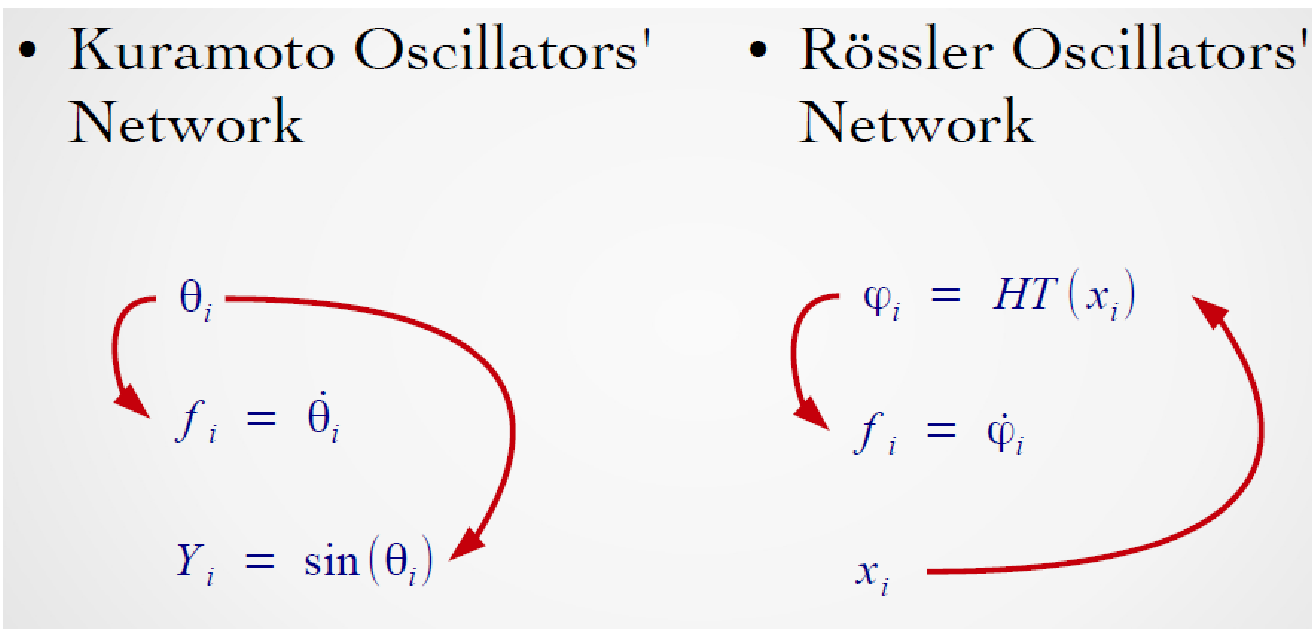
- the number of oscillators is small (12),
- the coupling is symmetric ($\Rightarrow$ only 66 possible links) and
- the time series are long (10^4 points)

G. Tirabassi et al, Sci. Rep. 5 10829 (2015).

First experimental example: 12 chaotic Rössler electronic oscillators, symmetrically and randomly coupled



In each node only one variable (voltage) measured. How to compute phases and frequencies?

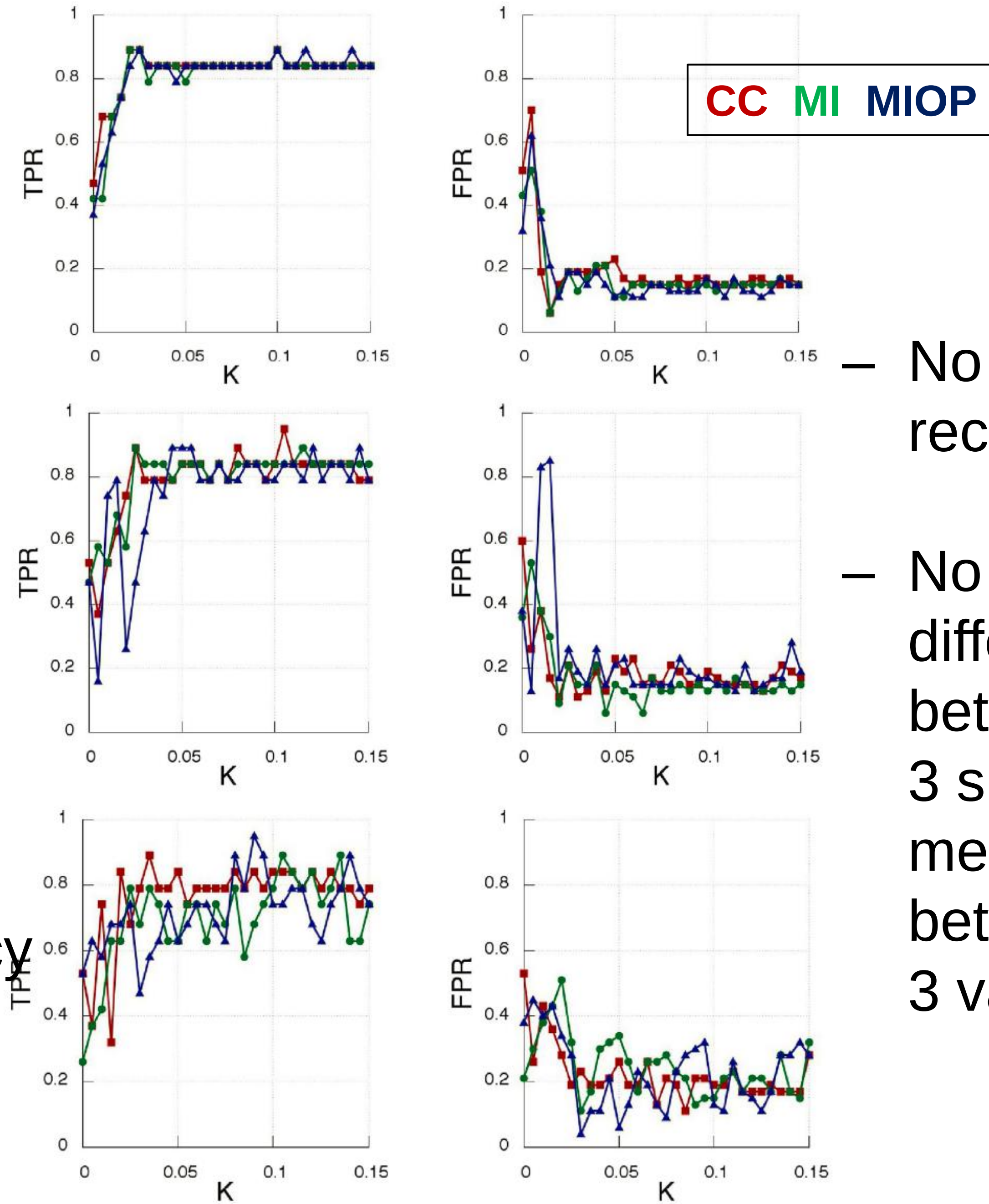


Results

Observed variable (voltage)

Hilbert phase

Hilbert frequency

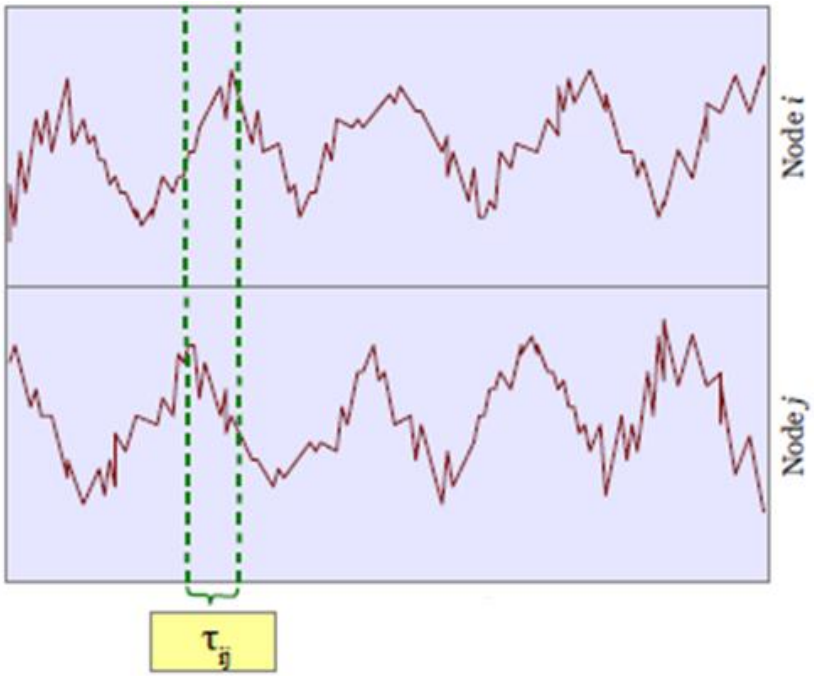


- No perfect reconstruction
- No significant difference between the 3 similarity measures, nor between the 3 variables.

G. Tirabassi et al, Sci. Rep. 5 10829 (2015).

Can the analysis of the lag times between the oscillators improve the inference of the network?

Observed time series in oscillators i and j: $u_i(t)$, $u_j(t)$, $t=1,\ldots,T$ *(normalized to zero mean and unit variance)*

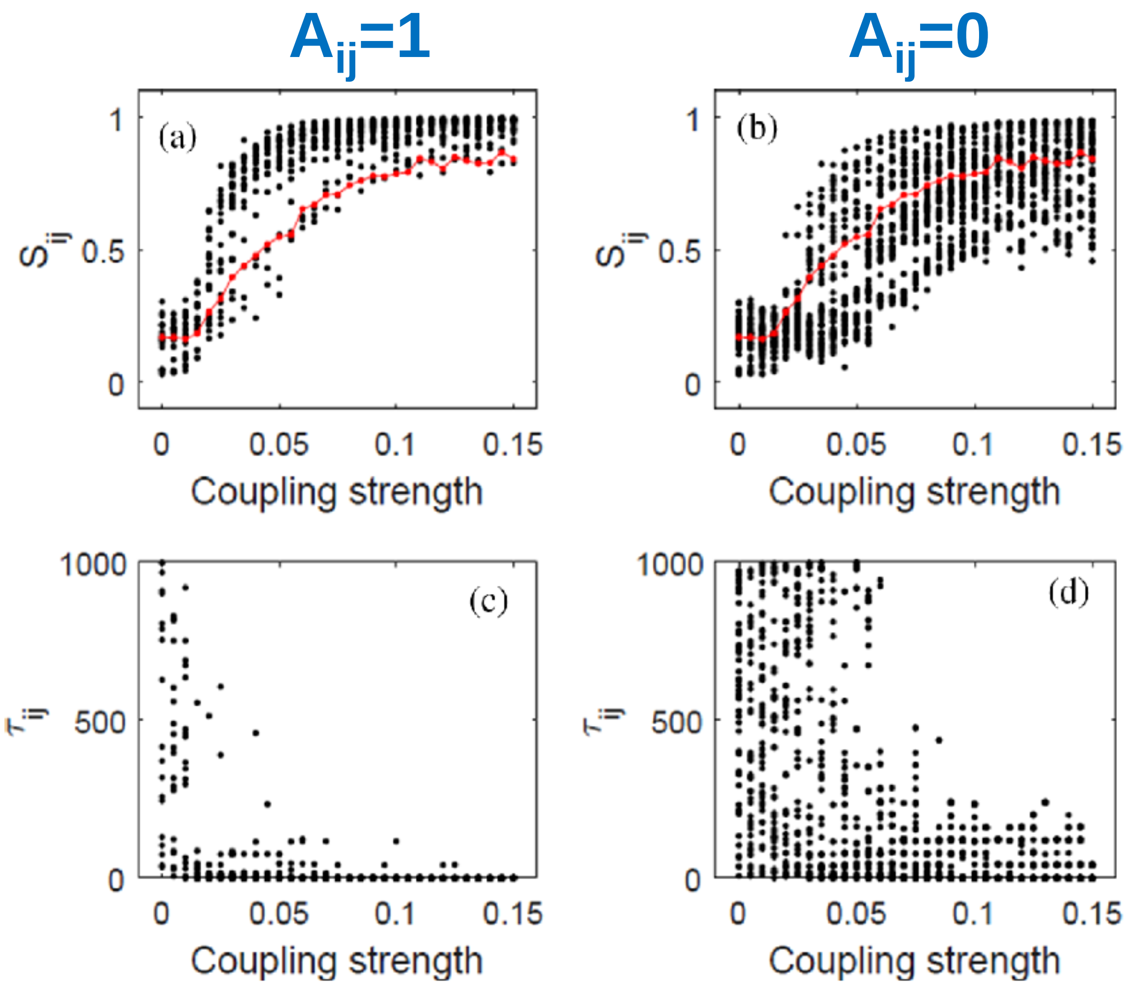


Cross-correlation

$$\tau_{ij} = \underset{1\le\tau\le\tau_{\max}}{\operatorname{argmax}_\tau}\left(\left|\sum_t u_i(t)u_j(t+\tau)\right|\right)$$

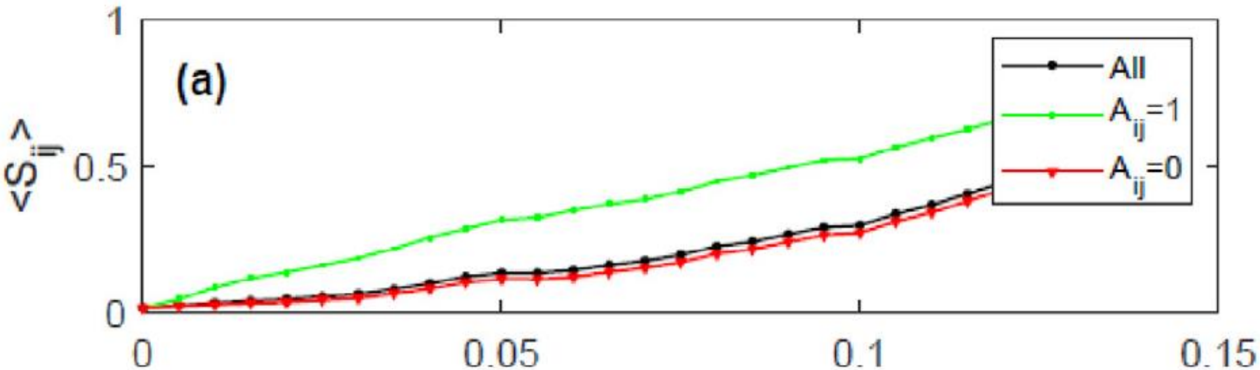
Coupled oscillators tend to have smaller τ_{ij} than uncoupled ones?

First experimental example: 12 chaotic Rössler electronic oscillators, symmetrically and randomly coupled



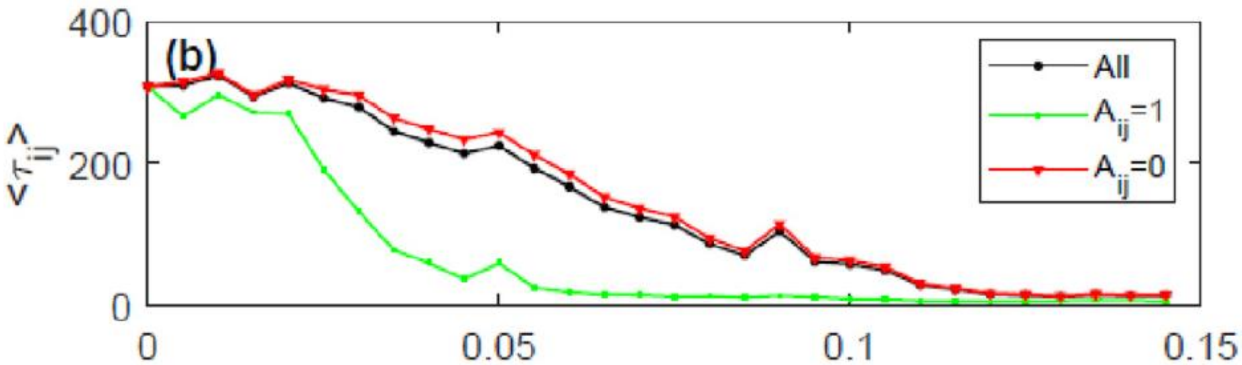
N. Rubido, C. Masoller, Eur. Phys. J. Special Topics 227, 1243-1250 (2018).

Second numerical example: 50 Kuramoto oscillators randomly connected (125 mutual links, 1100 links do not exist)



Coupling strength

The average similarity value is larger for pairs of linked oscillators.



Coupling strength

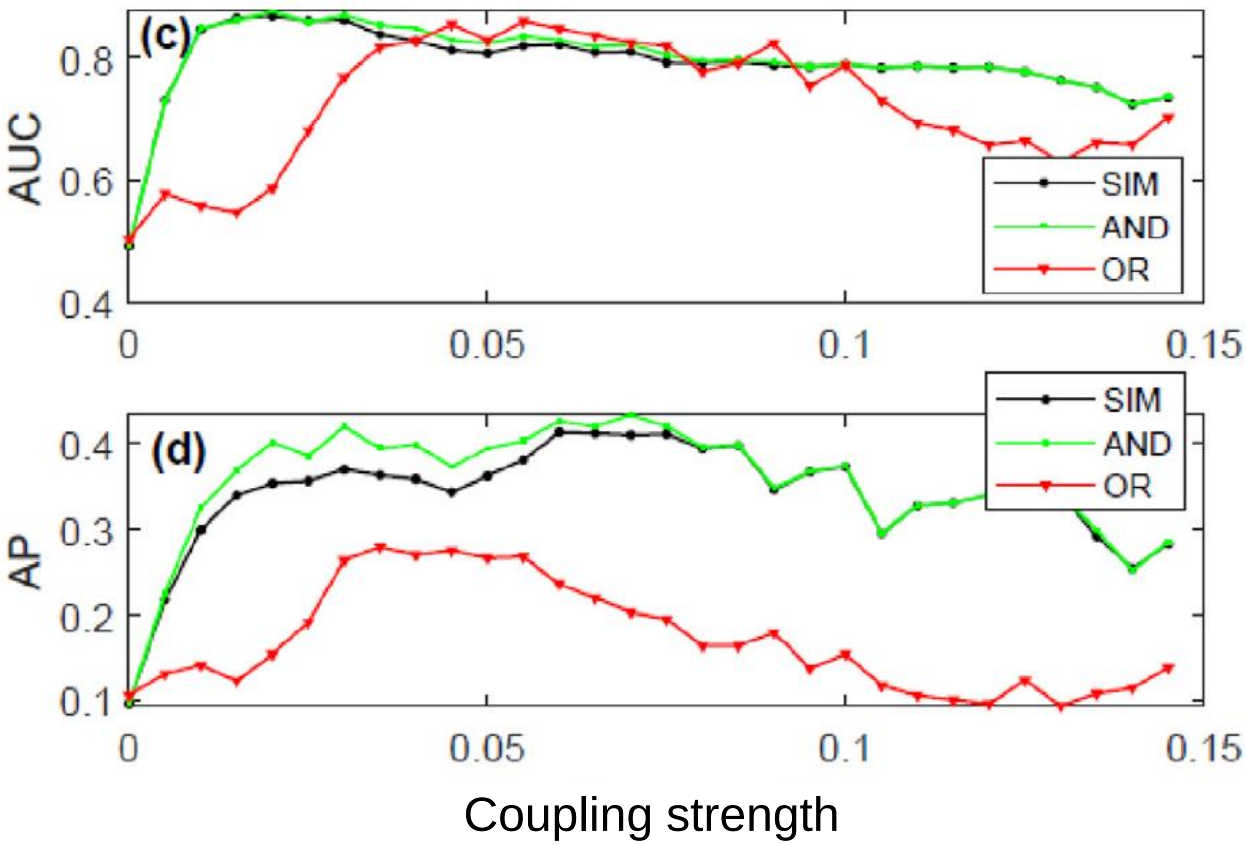
- The average lag between pairs of linked oscillators tends to be smaller than the average lag between unlinked oscillators.
- Also, it goes to zero faster as the coupling strength increases, thus providing **"early warning"** of sync. transition.

I. Leyva and C. Masoller, Chaos, Solitons & Fractals 133, 109604 (2020).

Three possible criteria to infer the presence or absence of links:

1. SIM: the link between i and j exists if $S_{ij} > S_{th}$, else, it does not exist.
2. AND: the link between i and j exists if $\tau_{ij} < \tau_{th}$ and $S_{ij} > S_{th}$, else, it does not exist.
3. OR: the link between i and j exists if $\tau_{ij} < \tau_{th}$ or $S_{ij} > S_{th}$, else, it does not exist.

Inferring the links of 50 Kuramotos randomly connected



The area under the ROC curve and the average precision are the same when we add the lag information.

Thus, **lag info does not improve network inference.**

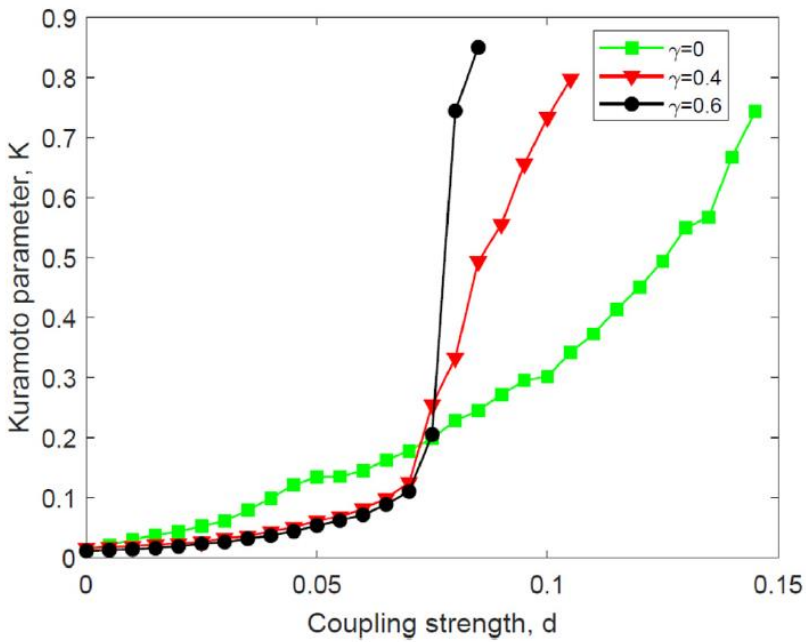
(AP=ratio of correct positive detections over all positive detections)

I. Leyva and C. Masoller, Chaos, Solitons & Fractals 133, 109604 (2020).

What if the oscillators are not randomly connected?

- For particular network topologies, "explosive" synchronization transition occurs.
- When the frequencies of kinked oscillators satisfy $|\omega_i \omega_j| > \gamma$
- This condition avoids that pairs of kinked oscillators that have similar frequencies act as synchronization seeds.
- γ large enough: explosive synchronization.

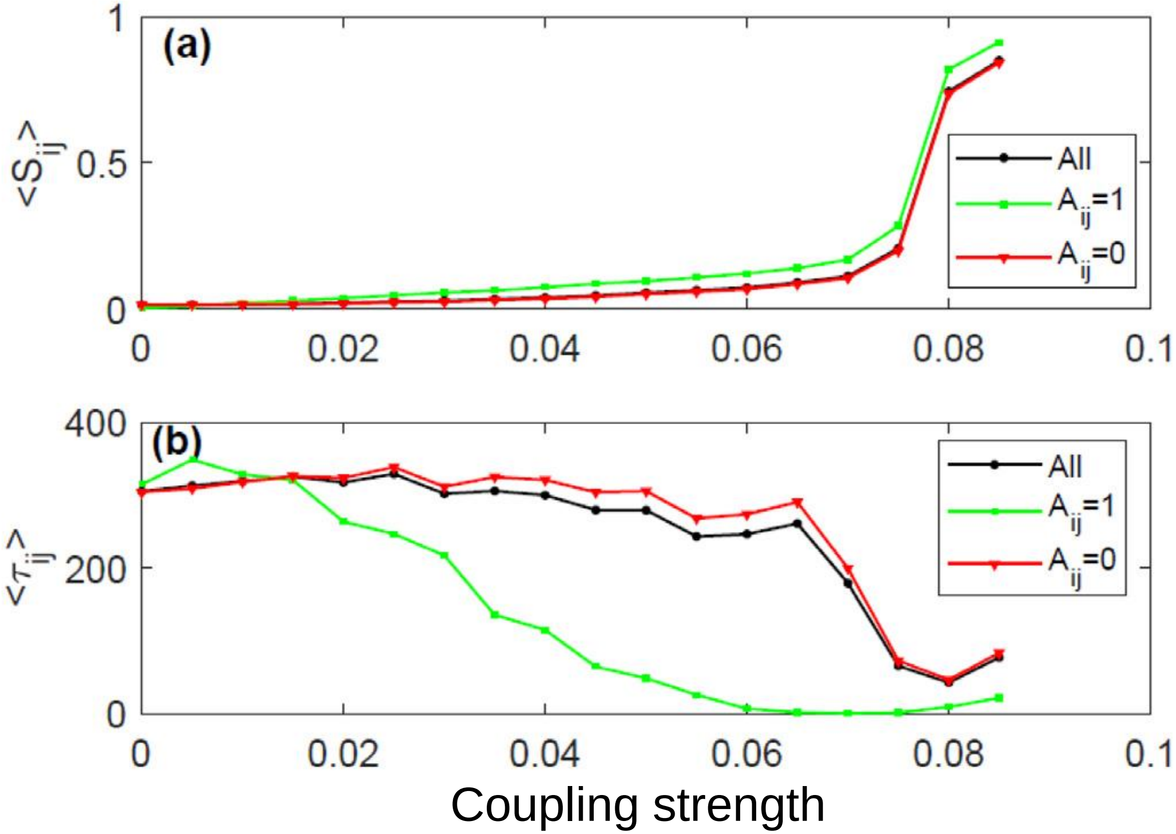
$$K = \left\langle \frac{1}{N} \left| \sum_{i=1}^{N} e^{i\phi_i(t)} \right| \right\rangle_T$$



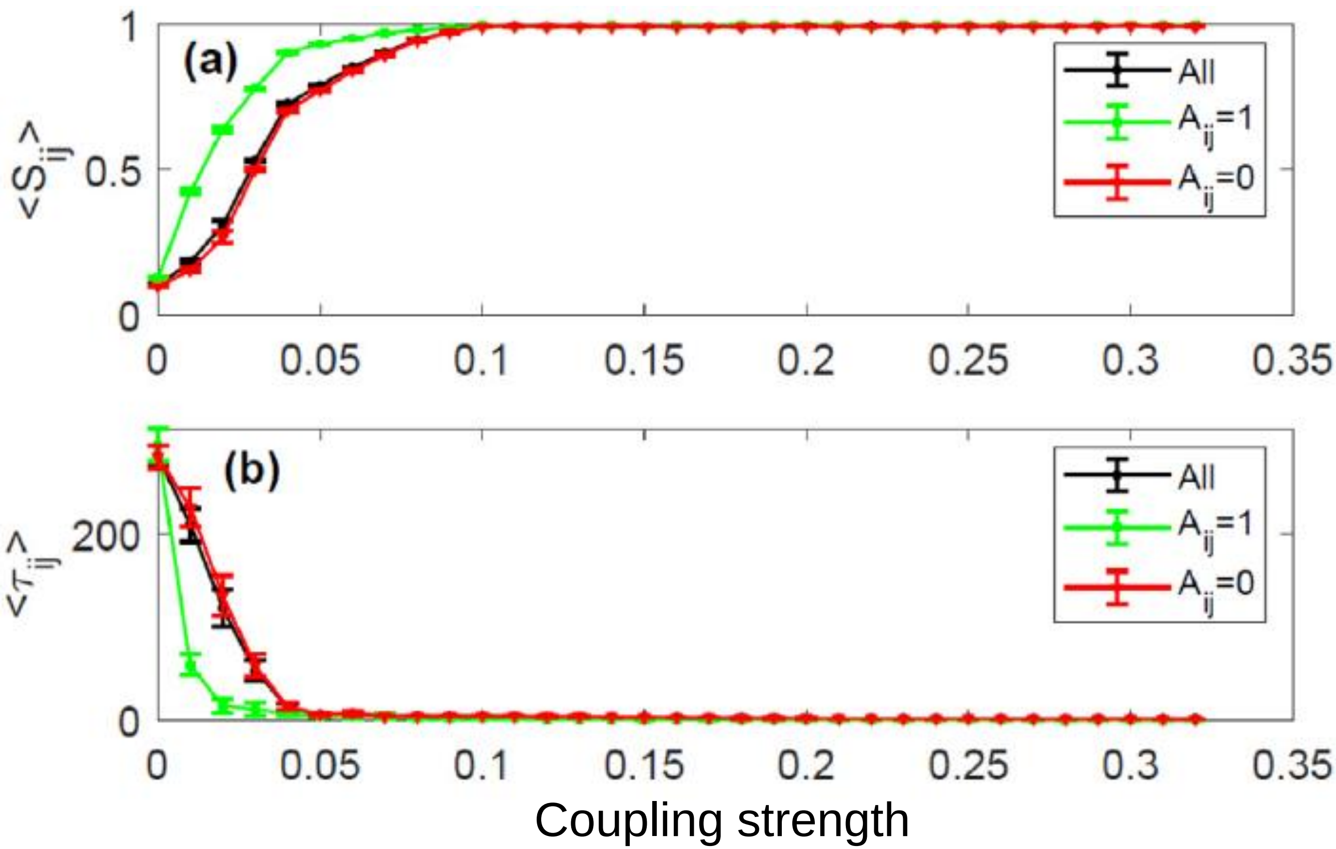
I. Leyva et al. Explosive transitions to synchronization in networked phase oscillators, Sci. Rep. 3, 1281 (2013).

50 Kuramoto oscillators: Even when the synch transition is explosive, the lags between linked oscillators decrease faster and provide an "early warning" of the transition.

$\gamma = 0.6$



Second experimental example: 28 chaotic Rossler circuits randomly coupled with 42 mutual links (336 links do not exist)



Consistent with the numerical results, in the experiments the lags between linked oscillators decrease faster than the lags between non-linked oscillators.

Conclusions

- Lag information does not improve the inference of the topology of a network of coupled oscillators (Kuramoto oscillators, Rossler electronic circuits).
- When the coupling increases the average lag between coupled oscillators decrease faster than the average lag between uncoupled ones.
- *Assuming we know the network topology*, the average lag between coupled oscillators is reliable an “early warning indicator” of the transition to synchrony.
- It will be interesting to test this idea in real-world systems where synchrony is dangerous (eg, EEG signals recorded from Parkinson patients).

THANK YOU FOR YOUR ATTENTION !

References

- ***G. Tirabassi*** *et al, "Inferring the connectivity of coupled oscillators from time-series statistical similarity analysis", Sci. Rep. 5 10829 (2015).*
- ***N. Rubido*** *and C. Masoller, "Impact of lag information on network inference", Eur. Phys. J. Special Topics 227, 1243-1250 (2018).*
- ***I. Leyva*** *and C. Masoller, "Inferring the connectivity of coupled oscillators and anticipating their transition to synchrony through lag-time analysis", Chaos, Solitons & Fractals 133 109604 (2020).*

Experimental data is freely available: *R. Sevilla-Escoboza, J. M. Buldu, Data in Brief 7, 1185 (2016), doi: 10.1016/ j.dib.2016.03.097.*

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