Reconstructing the State Space of Dynamical Systems from Data:

Comparison Between 1D Spatially Extended and Time Delayed Systems

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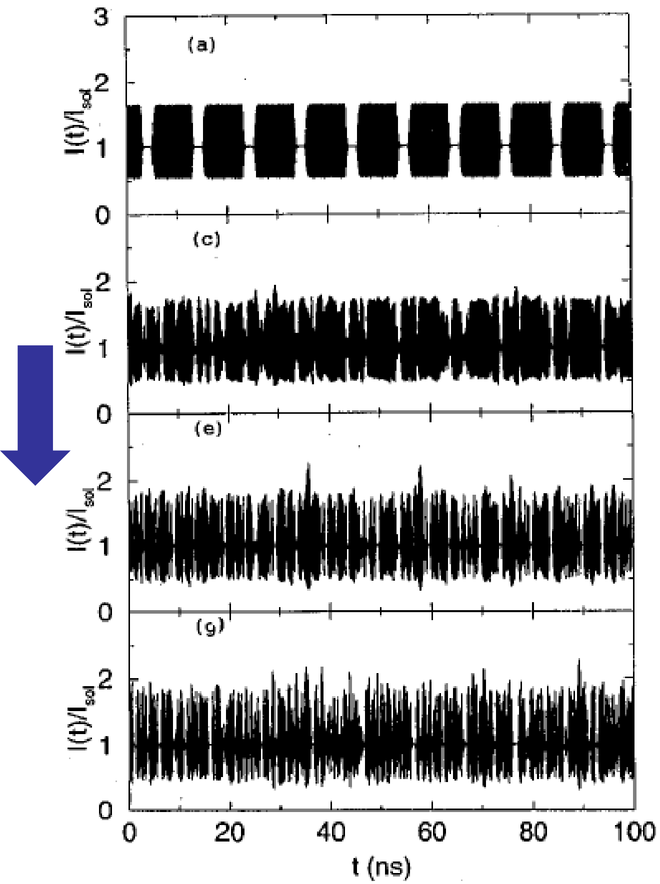
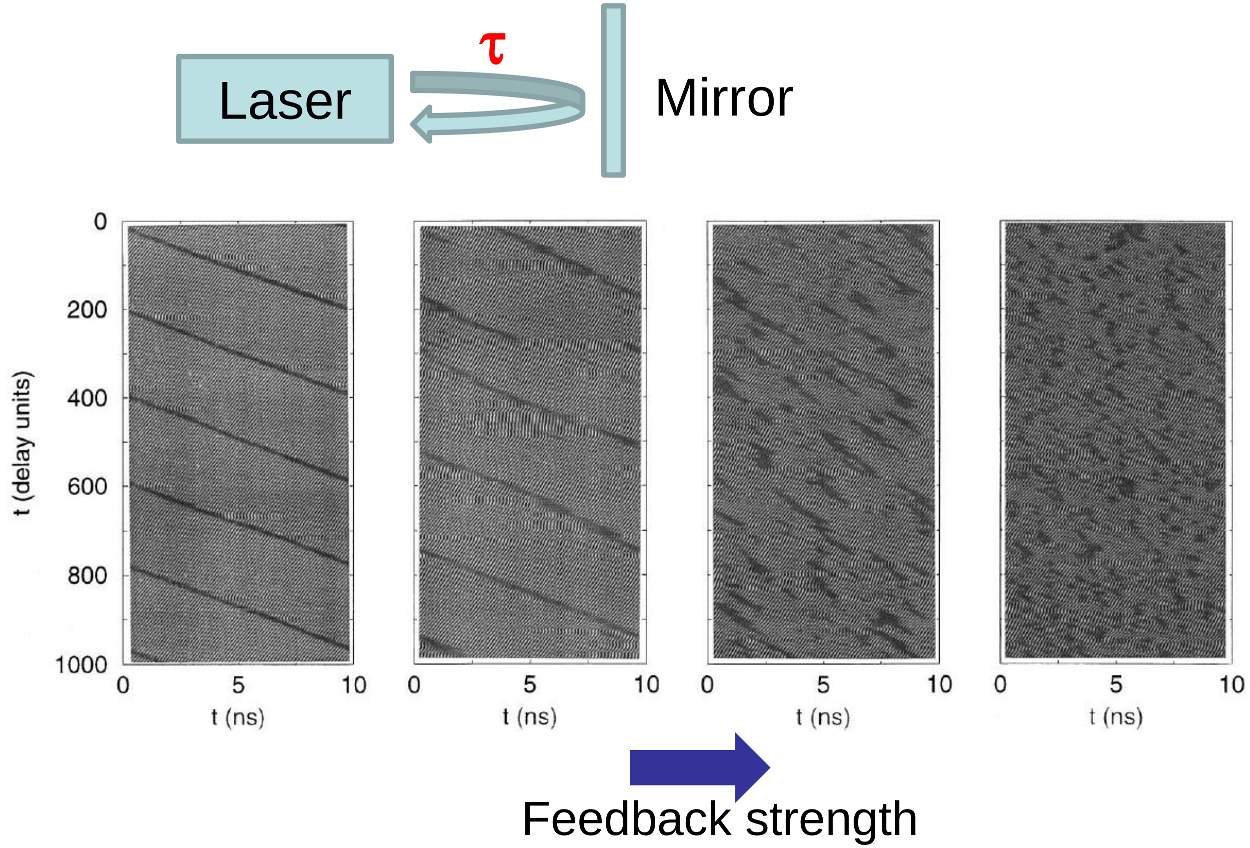
NOLTA 2018, Tarragona, Spain

In a diode laser, time-delay feedback induces chaos





Similar to pattern formation in spatio-temporal systems

F. T. Arecchi et al, Phys. Rev. A 45, R4225 (1992).
C. Masoller, “*Spatiotemporal dynamics of semiconductor lasers with feedback*”, Chaos 7, 455 (1997).

The space-time representation: a convenient way to uncover underlying structure in data

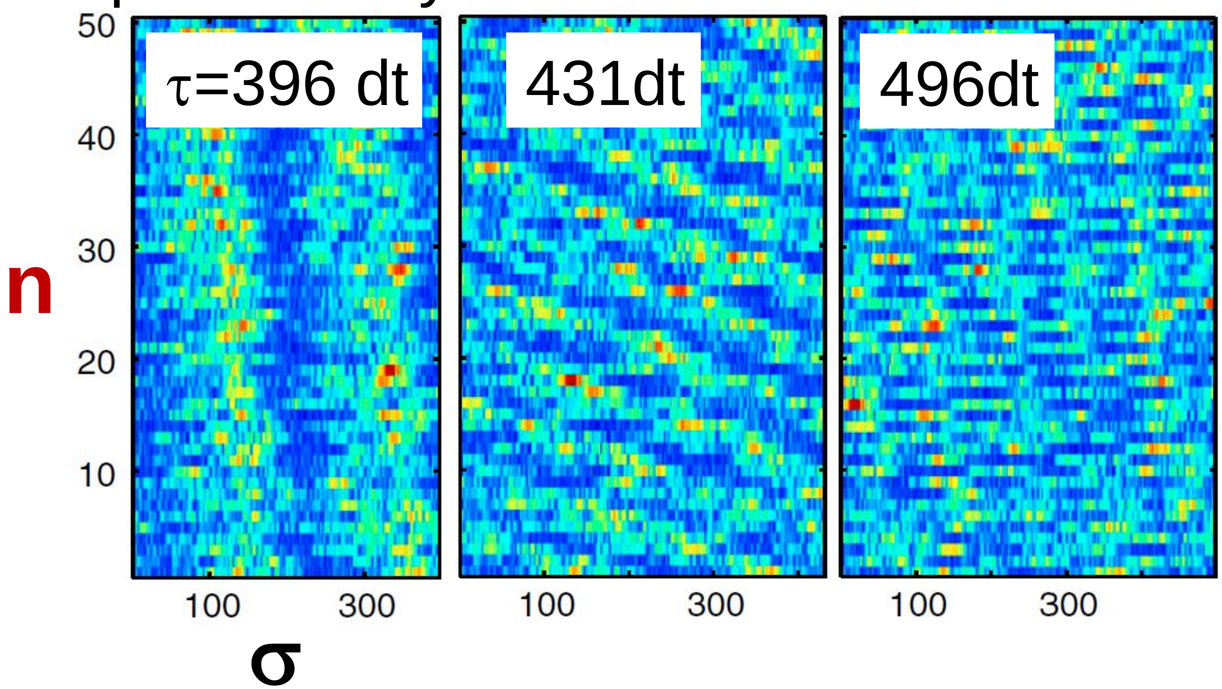
$$\{I_1, I_2, \ldots I_\tau, I_{\tau+1}, \ldots\} \Rightarrow \begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ I_{2\tau+1} & I_{2\tau+2} & \cdots & I_{3\tau} \\ I_{\tau+1} & I_{\tau+2} & \cdots & I_{2\tau} \\ I_1 & I_2 & \cdots & I_\tau \end{bmatrix}$$

Color scale: I_i

$\uparrow$ **n**

σ $\rightarrow$

Output intensity of a fiber laser



Aragoneses et al, PRL (2016)

Space-time analogy

- Space-time dynamical systems and time-delayed systems both live in an infinite phase space: an initial function –in space or in time– needs to be specified in order to obtain the system's evolution.

- Do they have similar "attractors"?

- We analyze bi-stable systems, described by a state variable *u*:
  - *Space time system: u(space, time)*
  - *Time delay system: u(time)*.

1D Spatially extended system

1D Time delayed system

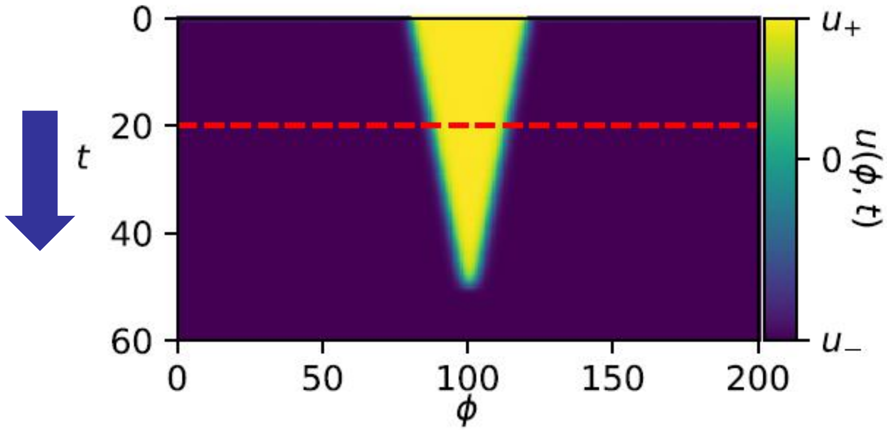
ϕ space variable

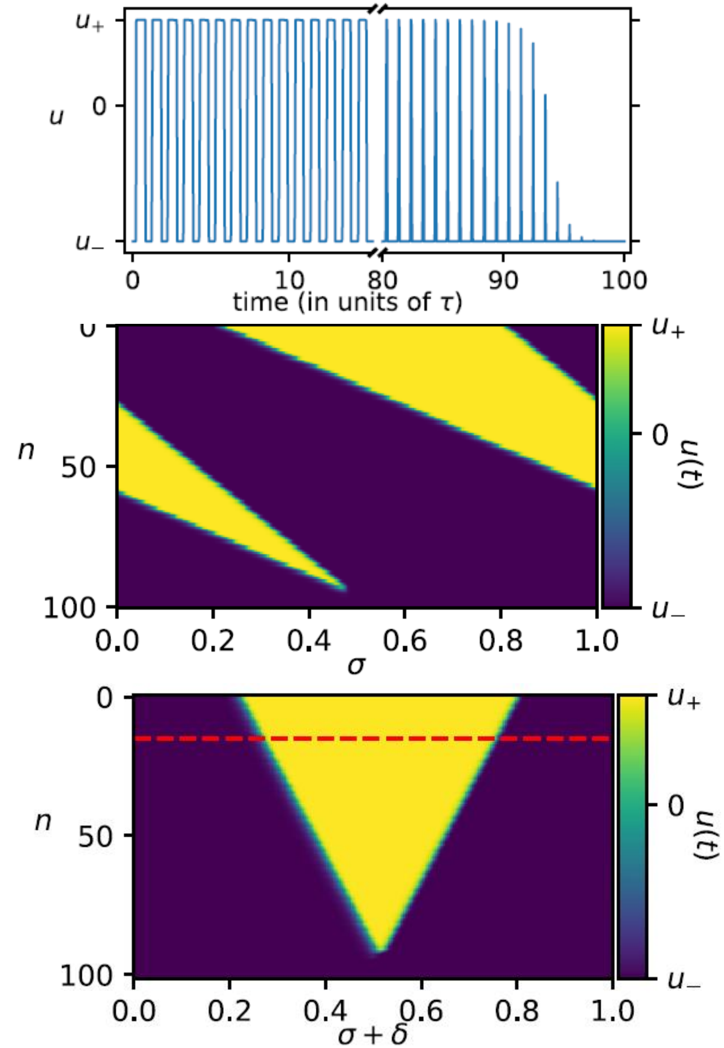
$$\partial_t u = F(u) + D\partial_\phi^2 u$$

$$du/dt = F(u) + \gamma u_\tau \qquad u_\tau = u(t-\tau)$$

$$F(u) = -dV(u)/du = -u(u+1+\alpha)(u-1)$$

bistable function





τ

$\tau+\delta$

Reconstruction of the attractors

Spatially extended system

$$\partial_t u = F(u) + D\partial_\phi^2 u$$

$$x = u, \quad y = \partial_\phi^2 u, \quad z = \partial_t u$$

Time delayed system

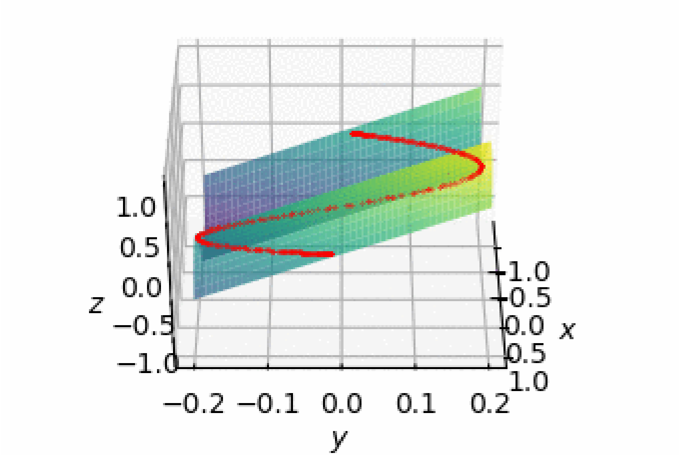
$$du/dt = F(u) + \gamma u_\tau$$

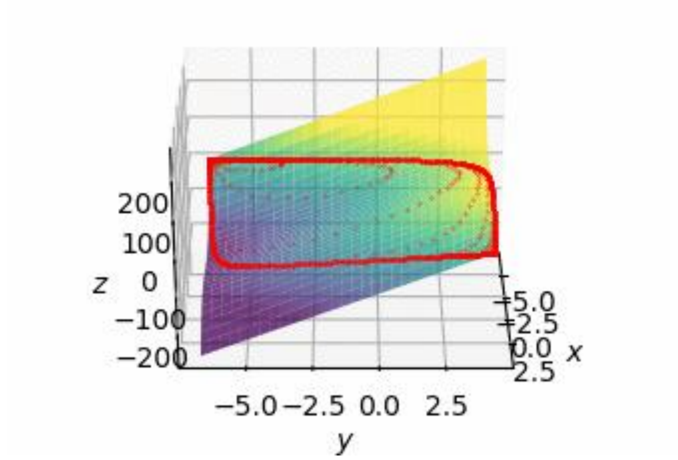
$$x = u, \quad y = u_\tau, \quad z = du/dt$$

$$z = \mathcal{F}(x, y) = F(x) + Dy$$

2D manifold

Bünner et al. PRL 1996





C. Quintero-Quiroz, M. C. Torrent and C. Masoller, "*State space reconstruction of spatially extended systems and of time delayed systems from the time series of a scalar variable*", Chaos 28, 075504 (2018).

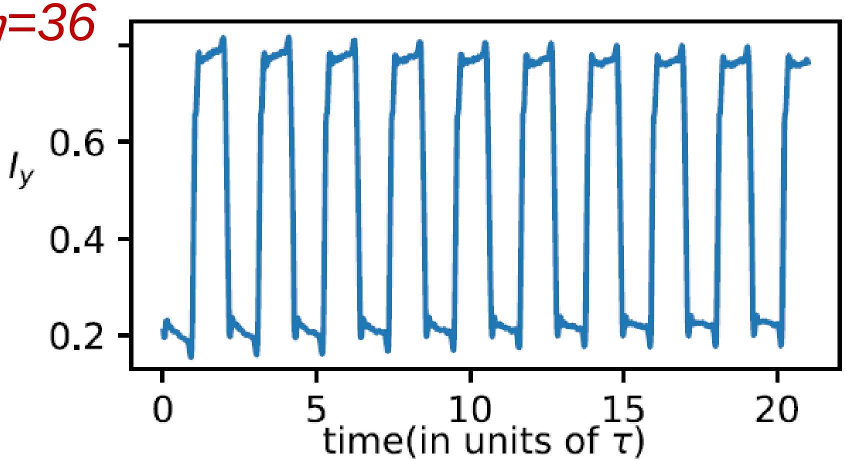
Test: model of delay-coupled lasers

$$\frac{dE_{x,i}}{dt} = k(1 + j\psi)(g_{x,i} - 1)E_{x,i} + \sqrt{\beta_{sp}}\xi_{x,i},$$

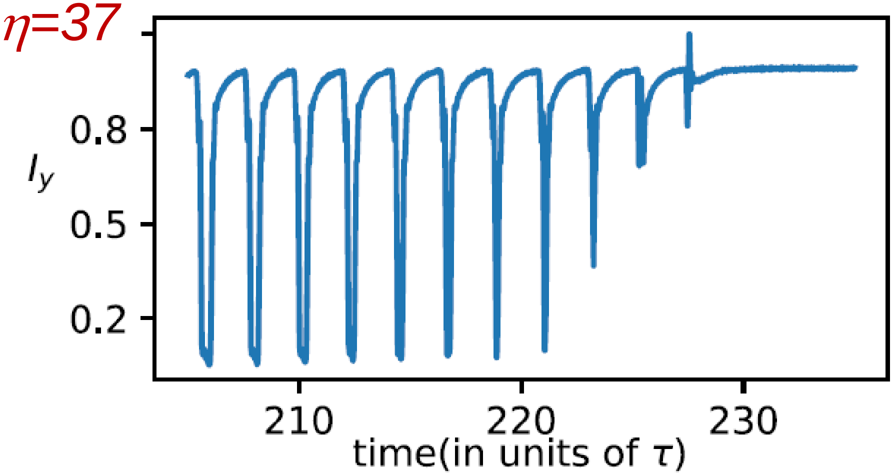
$$\frac{dE_{y,i}}{dt} = j\Delta E_{y,i} + k(i + j\alpha_l)(g_{y,i} - 1 - \beta)E_{y,i} + \eta E_{x,3-i}(t - \tau)e^{-j\omega_0\tau} + \sqrt{\beta_{sp}}\xi_{y,i},$$

$$\frac{dN_i}{dt} = \varepsilon_N(\mu - N_i - g_{x,i}I_{x,i} - g_{y,i}I_{y,i}),$$

η=36



η=37



Attractor reconstruction

$$z = \mathcal{F}(x, y) = F(x) + \gamma y$$

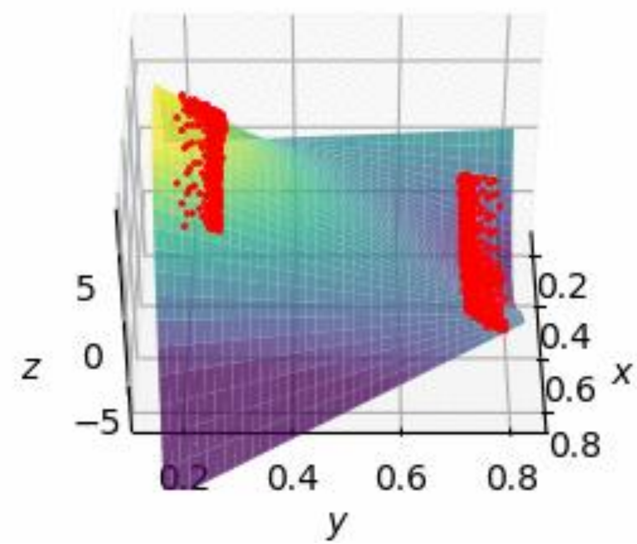
?

Try a polynomial function

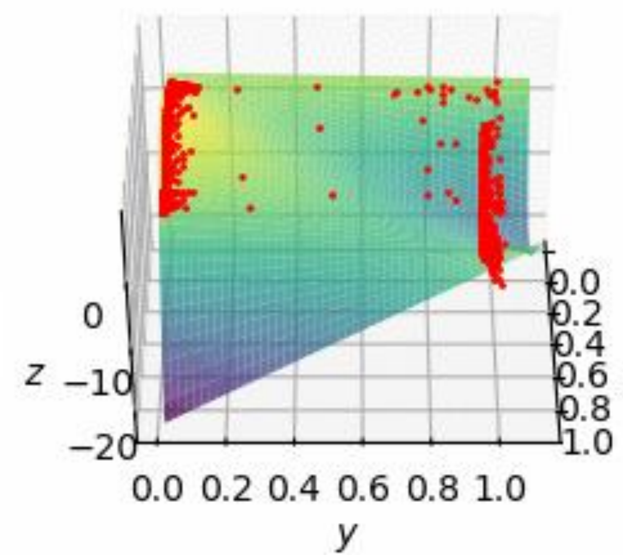
$$\mathcal{F}(x, y) = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 y + A_5 xy + A_6 x^2 y$$

Can we fit numerically the unknown parameters?

η=36



η=37



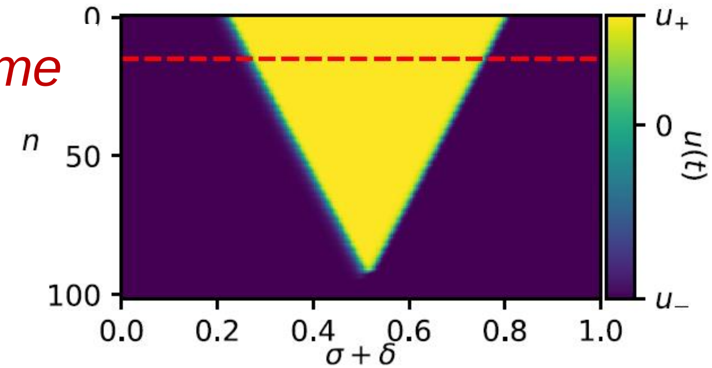
C. Quintero-Quiroz, M. C. Torrent and C. Masoller, "*State space reconstruction of spatially extended systems and of time delayed systems from the time series of a scalar variable*", Chaos 28, 075504 (2018).

Space-time analogy

Can we fit the time-delay system as a spatially extended system using the space-time representation?

$$\{I_1, I_2, \ldots I_\tau, I_{\tau+1}, \ldots\}$$

Time



Space "ϕ"

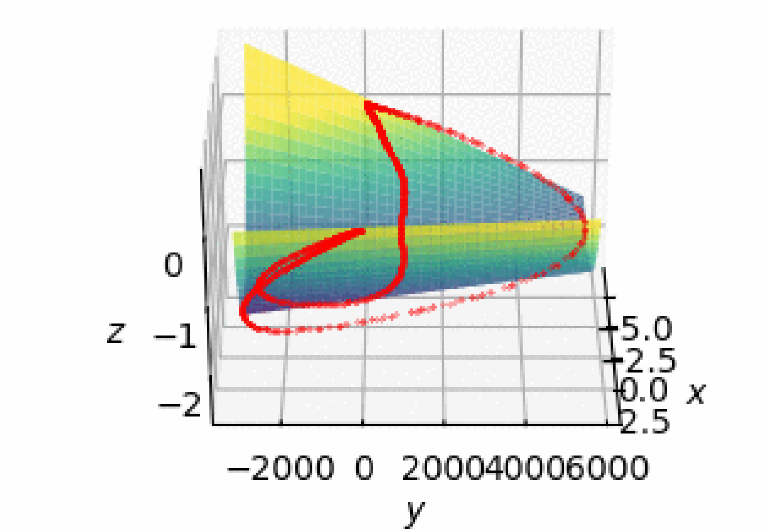
Time delay system

$$x = u, \quad y = u_\tau, \quad z = du/dt$$

Spatially extended system

$$x = u, \quad y = \partial_\phi^2 u, \quad z = \partial_t u$$

These partial derivatives have to be estimated numerically



$\Rightarrow$ Yes, we can obtain a good fit

However, the relation between the fitted parameters and the model parameters is unclear and needs to be further investigated.

Summary

- The evolution of a bi-stable time-delay system and a bi-stable spatially-extended system can be described, in a 3D phase space (x,y,z), by the same polynomial potential.
- More complicated bistable time-delay systems can be described, approximately, in the same way.

Future work: can the 3D representation be useful to investigate regime transitions, coupling and causal relations?

Thank you for your attention

http://www.fisica.edu.uy/~cris

C. Quintero-Quiroz et al., Chaos 28, 075504 (2018)
arXiv: 1801.08340

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