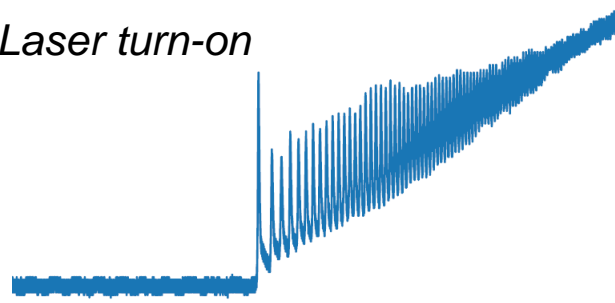
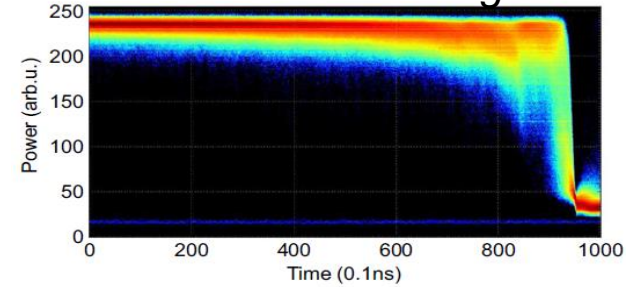


Laser turn-on



Polarization switching



In temporal networks correlation lags are  
informative of approaching bifurcations

Giulio Tirabassi and Cristina Masoller

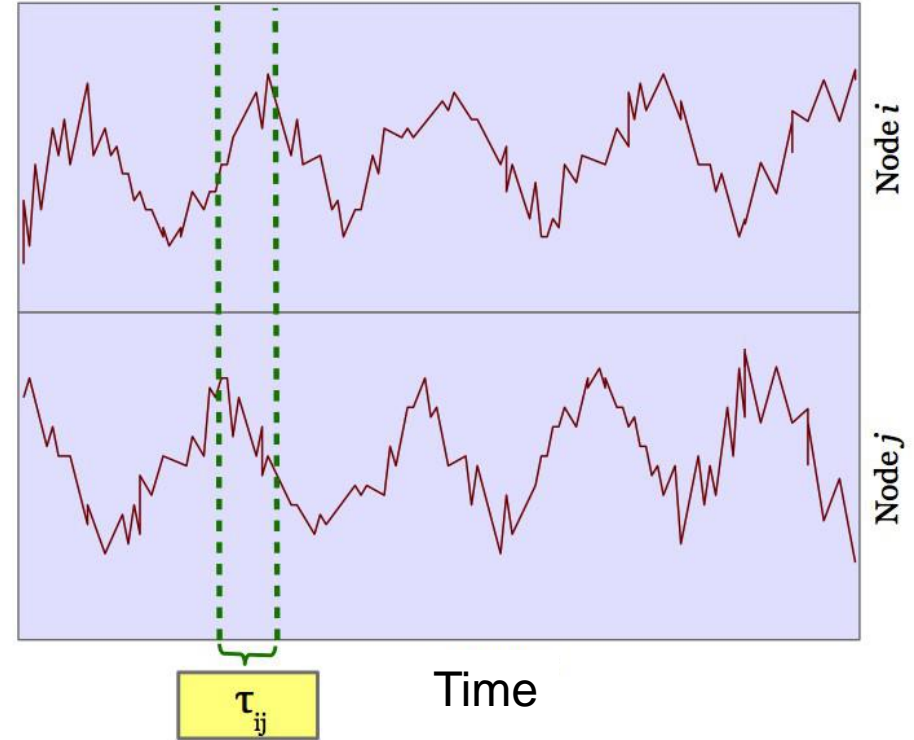
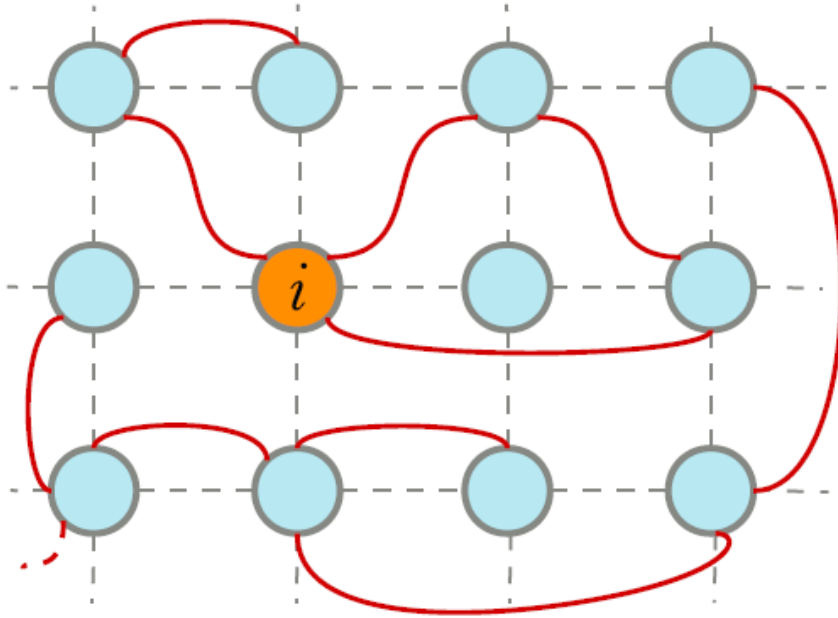
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COMPLEX NETWORKS 2022, Palermo  
November 9, 2022

# Spatio-temporal system described by a “correlation network”

$C_{ij}$  = Pearson correlation coefficient

$$|C_{ij}(\tau_{ij})| > Th \Rightarrow A_{ij} = 1, \text{ else } A_{ij} = 0$$



Indicator of approaching bifurcation:  
variance of the distribution of lags

$$\sigma_{\tau} = (1/M) \sum_{i,j} (\tau_{ij} - \langle \tau \rangle)^2$$

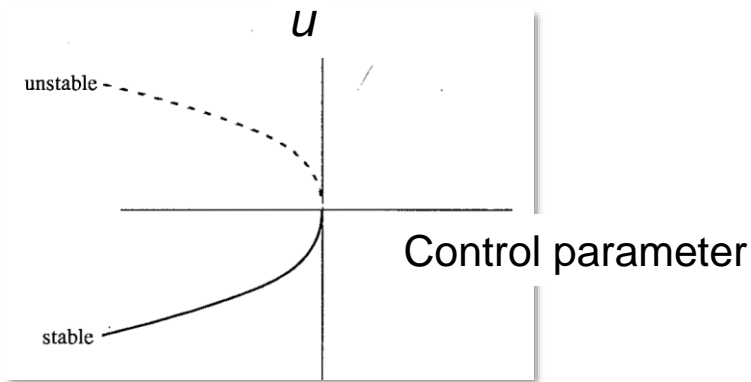
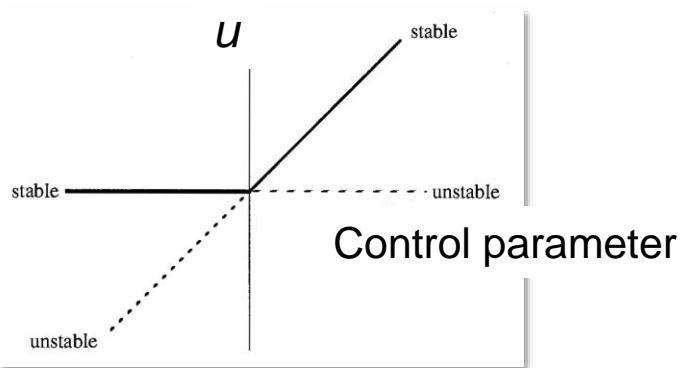
**Models:** Bistable reaction-diffusion, Fitzhugh–Nagumo, Vegetation extinction

$$\frac{\partial u}{\partial t} = -u^3 - \alpha u^2 + u(1 + \alpha) + D \frac{\partial^2 u}{\partial x^2} + \xi$$

$$\frac{\partial u}{\partial t} = \Lambda u - u^3 - \Sigma v - I + D_u \Delta u + \xi_u,$$

$$\frac{\partial v}{\partial t} = v(u - v) + D_v \Delta v + \xi_v.$$

**Bifurcations:** Trans-critical, saddle-node, subcritical and supercritical Hopf and a global bifurcation: travelling waves

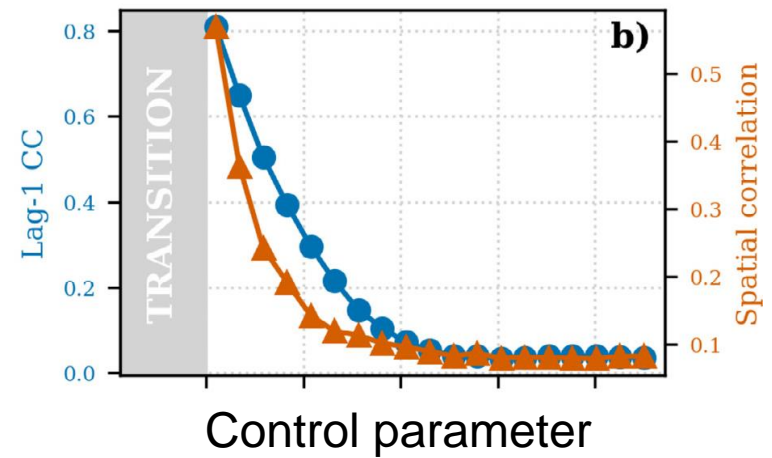
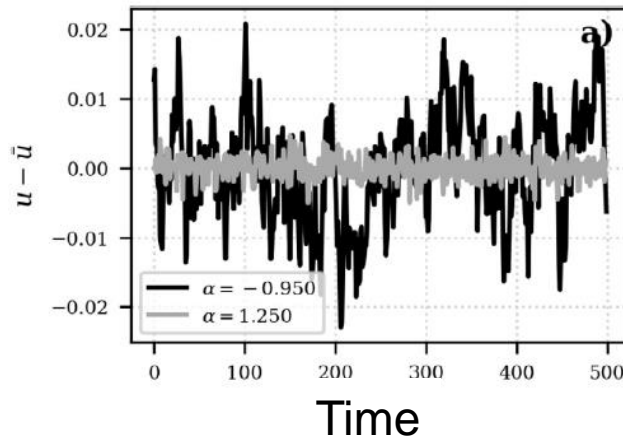


**Classical indicators:** lag-1 cross-correlation, spatial correlation

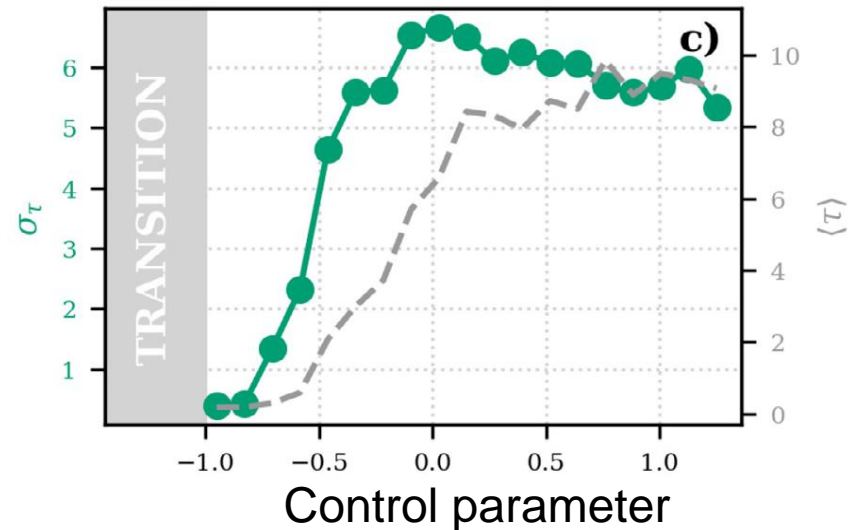
$$\text{lag-1 CC} = \langle |\langle u_i(t)u_i(t + 1) \rangle_t| \rangle_i$$

$$\text{Spatial CC} = \left\langle \left| \langle u_i(t)u_j(t) \rangle_{i,j} \right| \right\rangle_t$$

# Results



Indicator of approaching bifurcation:  
variance of the distribution of lags  
max or min before the bifurcation



## Reference:

G. Tirabassi, C. Masoller, “Correlation lags give early warning signals of approaching bifurcations”, Chaos, Solitons and Fractals 155, 111720 (2022)