Temporal correlations in neuronal spikes induced by noise and periodic forcing

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Workshop "Advances in the collective behaviour of complex systems"
Potsdam, September 2016

ICREA

GOBIERNO DE ESPAÑA
MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD

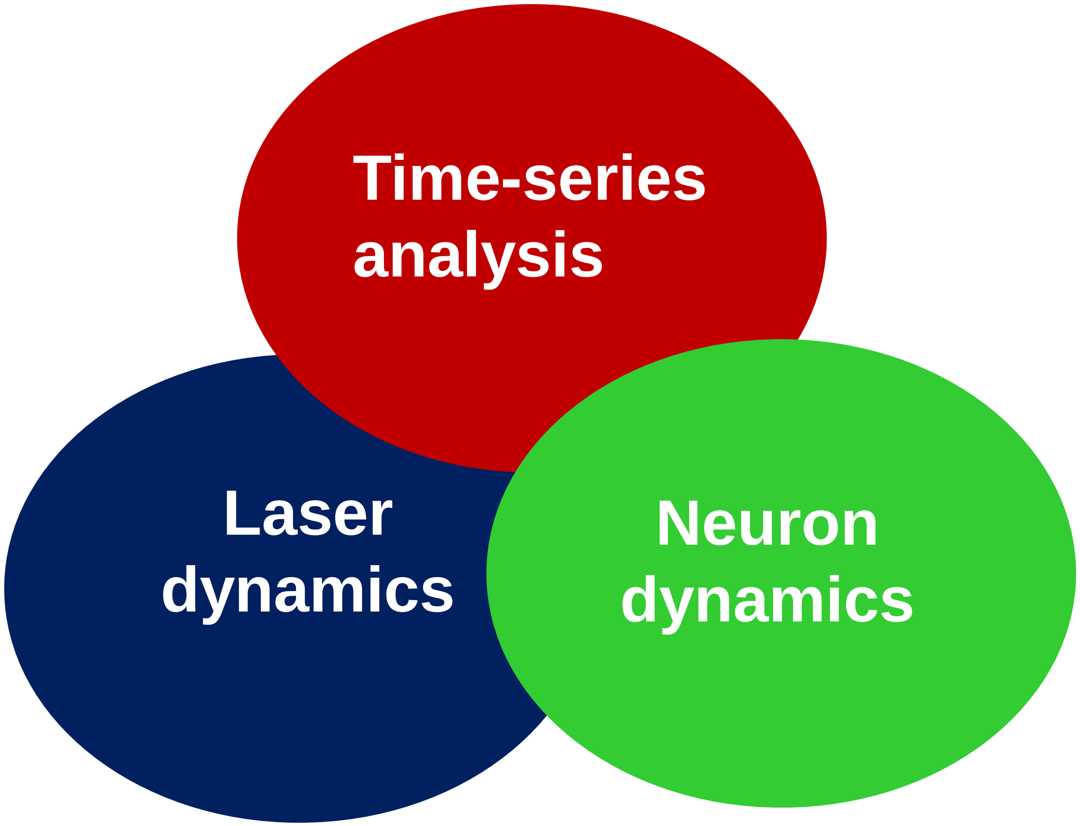
Over the years, Arkady' work has been a great source of inspiration

Wittenberg 2004

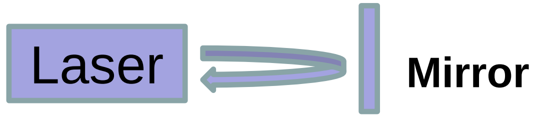
The University of Ohio, April 2016

HAPPY BIRTHDAY!

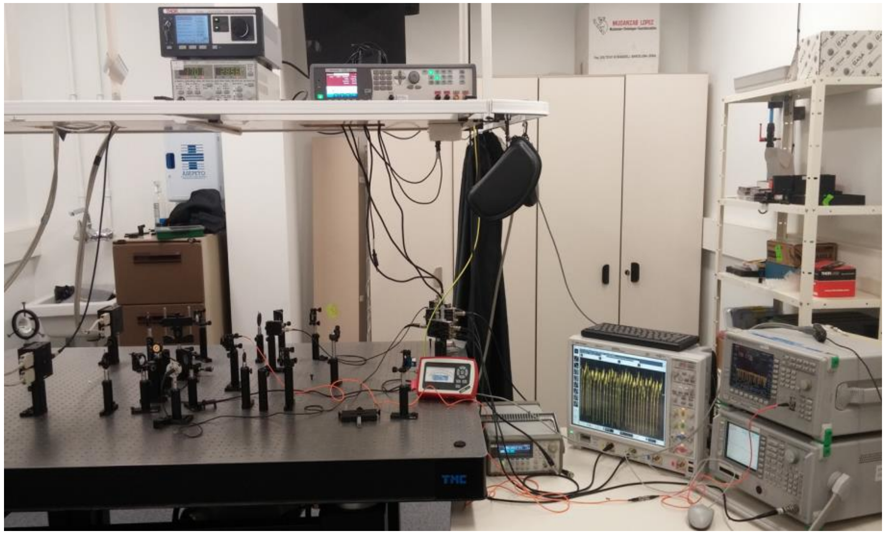
Outline

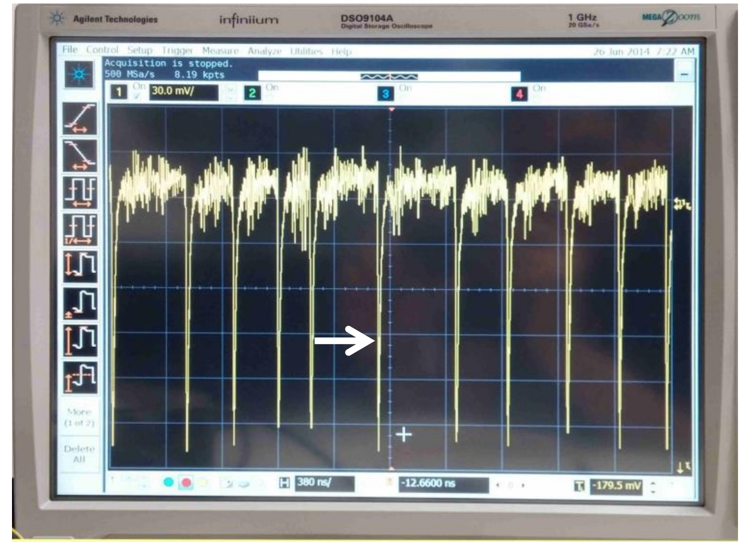


- Introduction
  - Motivation: spiking lasers that mimic neuronal behavior
  - Symbolic method of time-series analysis
- Results:
  - Response to a weak periodic input: comparison of optical and neuronal spikes
  - Analysis of ISI sequences generated by single-neuron models
- Summary



In our lab: experiments with semiconductor lasers





WHAT DO LASERS HAVE TO DO WITH NEURONS?

Similar statistics of inter-spike intervals?

MOTIVATION

Science 345, 668 (2014)

"a computer that is **inspired** by the brain."

Neuro-synaptic architecture allows to do things like image classification at a very low power consumption.

- Spiking lasers: photonic neurons?
- potential building blocks of brain-inspired computers.
- Ultra fast ! (micro-sec vs. mili-sec)

HOW SIMILAR NEURONAL AND OPTICAL SPIKES ARE?

VOLUME 78, NUMBER 5 PHYSICAL REVIEW LETTERS 3 FEBRUARY 1997

Coherence Resonance in a Noise-Driven Excitable System

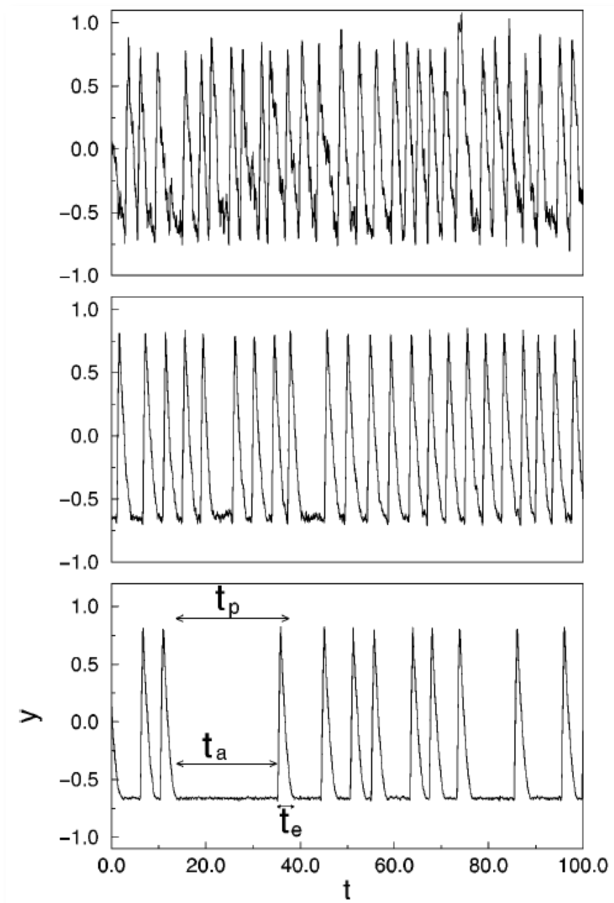
Arkady S. Pikovsky* and Jürgen Kurths*
Max–Planck–Arbeitsgruppe "Nichtlineare Dynamik" an der Universität Potsdam Am Neuen Palais 19, PF 601553, D-14415, Potsdam, Germany
(Received 9 August 1996)

FHN model

$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y\,,$$

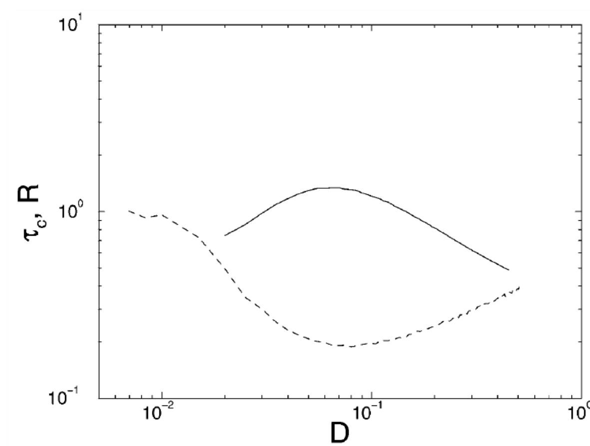
$$\frac{dy}{dt} = x + a + D\xi(t)$$

Increasing the noise strength



$$R_p = \frac{\sqrt{\mathrm{Var}\,(t_p)}}{\langle t_p \rangle}$$

$$\tau_c = \int_0^\infty C^2(t)\,dt$$



Experimental Evidence of Coherence Resonance in an Optical System

SCL with feedback

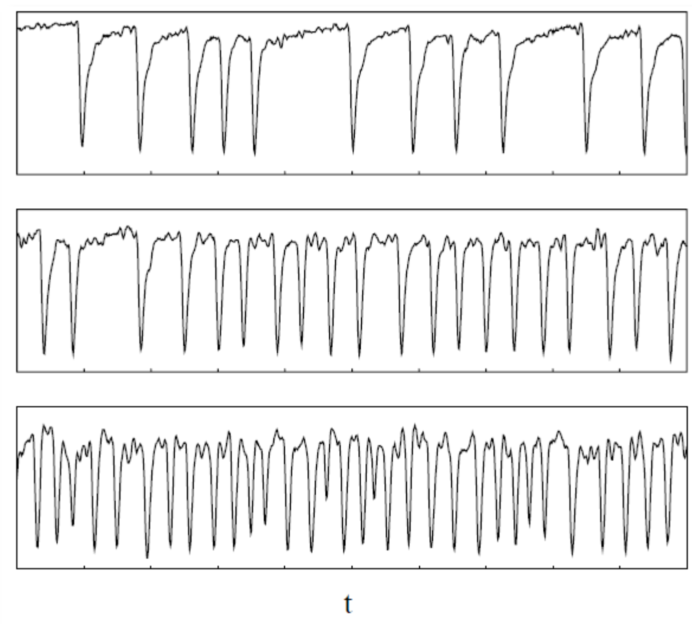
Giovanni Giacomelli
Istituto Nazionale di Ottica, Largo E. Fermi 6, 50125 Firenze, Italy

Massimo Giudici and Salvador Balle
Departamento de Física Interdisciplinar, Instituto Mediterraneo de Estudios Avanzados (CSIC-UIB), 07071 Palma de Mallorca, Spain

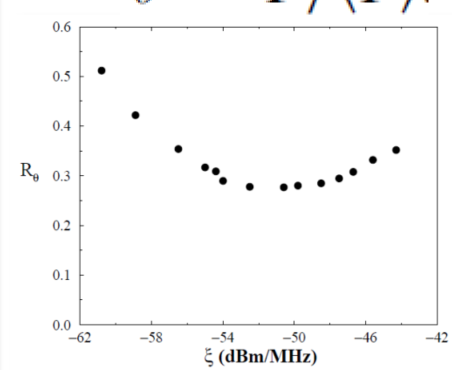
Jorge R. Tredicce
Institut Non-Linéaire de Nice, UMR 6618 Centre National de la Recherche Scientifique-Université de Nice Sophia-Antipolis, 06560 Valbonne, France

(Received 13 July 1999)

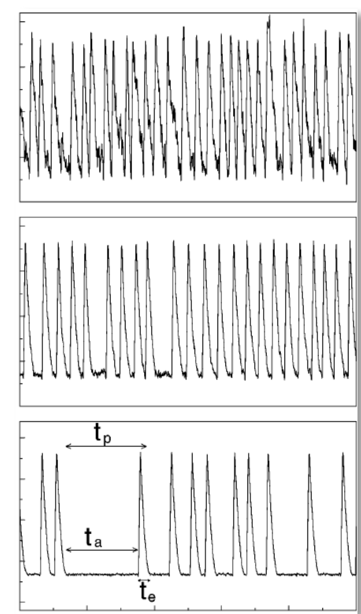
Increasing the noise strength



$$\theta = T/\langle T \rangle.$$



FHN model



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Similarity of neuronal & optical spikes?

Neuron inter-spike interval (ISI) distribution

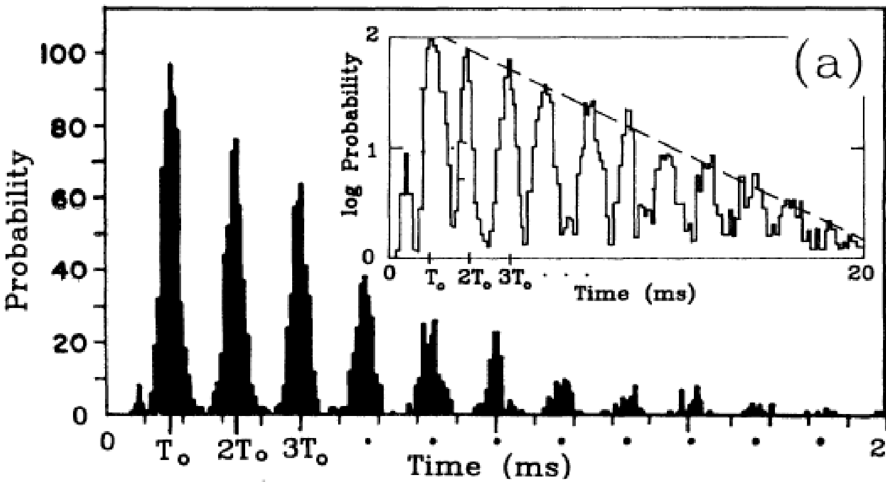
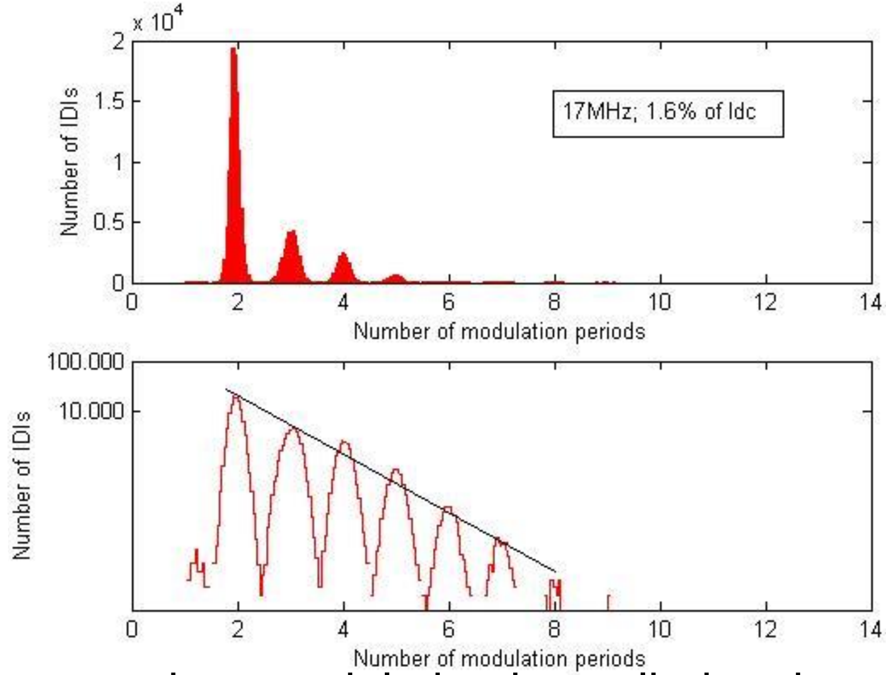


FIG. 1. (a) An experimental ISIH obtained from a single auditory nerve fiber of a squirrel monkey with a sinusoidal 80-dB sound-pressure-level stimulus of period $T_0 = 1.66$ ms applied at the ear. Note the modes at integer multiples of T_0. Inset:

A. Longtin et al, PRL 67 (1991) 656

Optical ISI distribution, data collected in our lab



when modulation is applied to the laser current

Neuronal ISIs

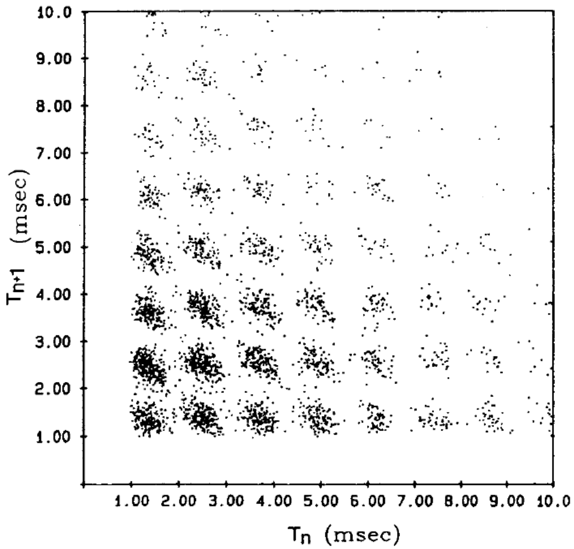
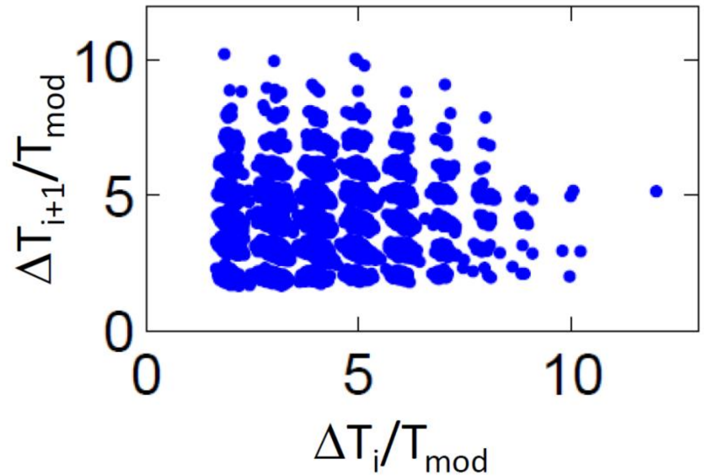
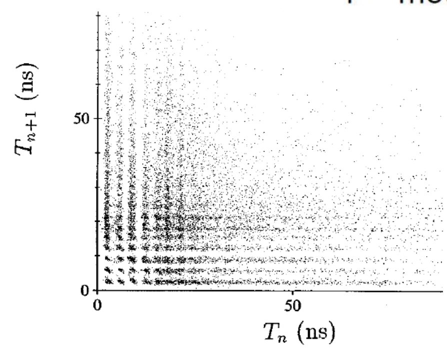


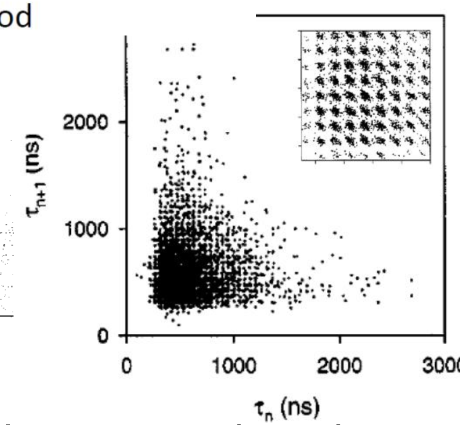
Fig. 4. Scatter plot of spike train data obtained from extracellular measurements of cat auditory fiber activity in response to an 800 Hz 60 dB sound pressure level pure tone presented to the outer ear. The stimulus is discontinuous (see

A. Longtin IJBC 3 (1993) 651

Optical ISIs

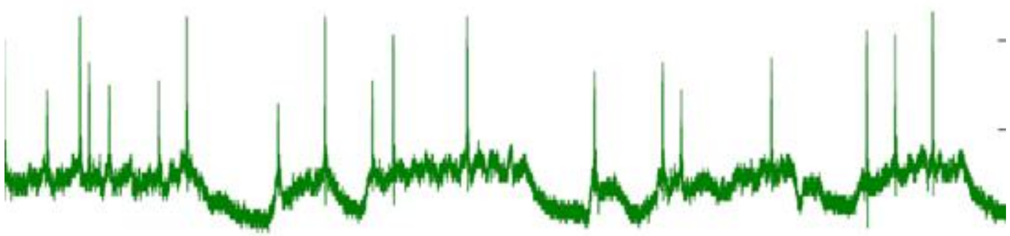






A. Aragoneses et al, Opt. Exp. (2014)
M. Giudici et al, PRE 55, 6414 (1997)
D. Sukow and D. Gautheir, JQE (2000)

How neurons encode information?



- In the spike rate?

- Is the **timing** of the spikes relevant?
  - Rate-based information encoding is slow.
  - Temporal codes transmit more information.

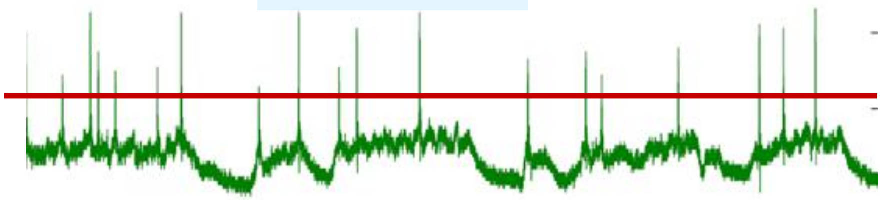
HOW TEMPORAL CORRELATIONS CAN BE IDENTIFIED AND QUANTIFIED?

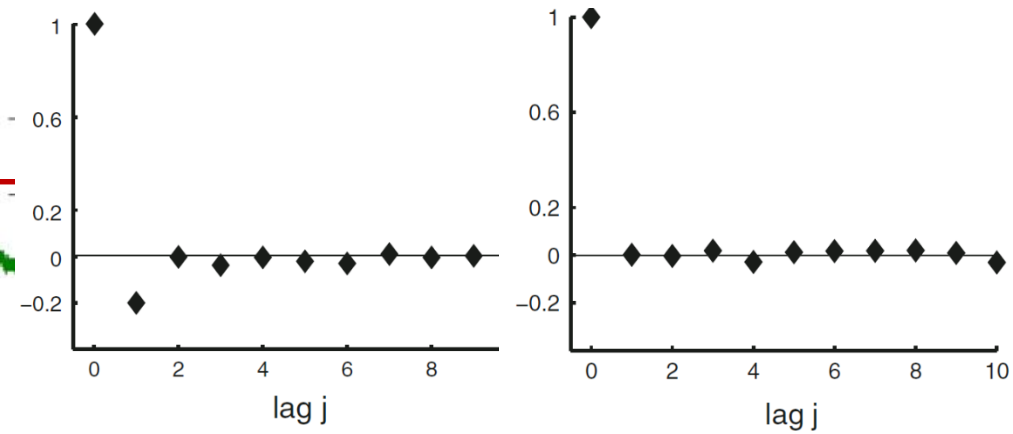
Inter-spike-intervals serial correlation coefficients

$$\{\ldots I_{i-1}, I_i, I_{i+1} \ldots\}$$

$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

$$I_i = t_{i+1} - t_i$$





Exp Brain Res (2011) 210:353–371

HOW TO INDENTIFY TEMPORAL STRUCTURES?
RECURRENT / INFREQUENT PATTERNS?

Symbolic method of analysis of ISI sequences

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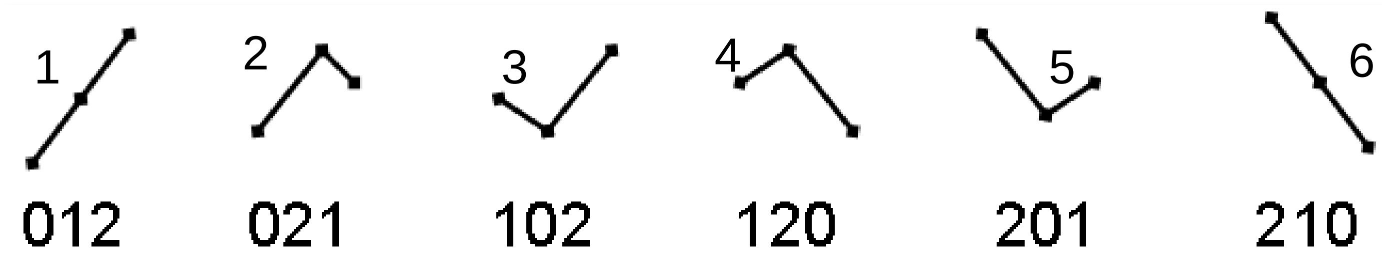
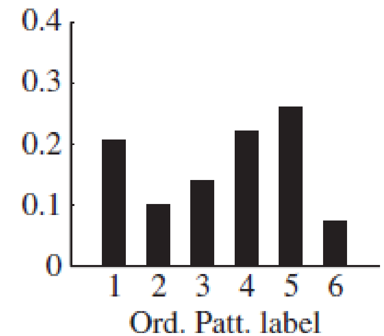
Ordinal analysis

Brandt & Pompe, PRL 88, 174102 (2002)

$$\{\ldots I_{i-1}, I_i, I_{i+1} \ldots\}$$





Random data $\Rightarrow$ OPs are equally probable

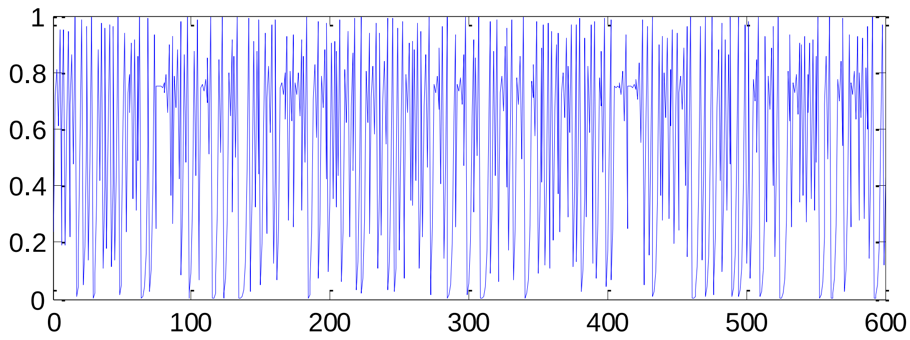
The OP probabilities allow to identify frequent patterns in the *ordering* of the data points

– Advantage: the probabilities uncover temporal correlations.

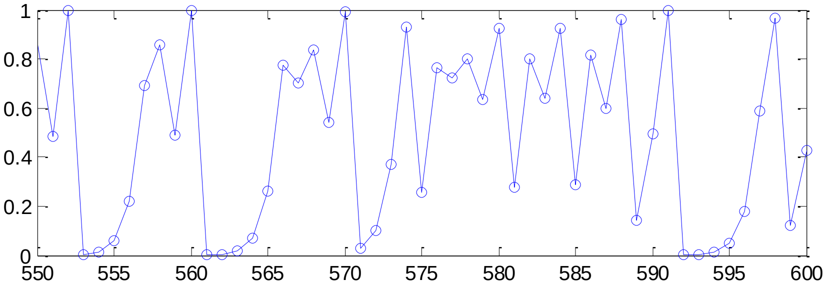
– Drawback: we lose information about the actual values.

To fix ideas: the logistic map $x(i+1)=4x(i)[1-x(i)]$

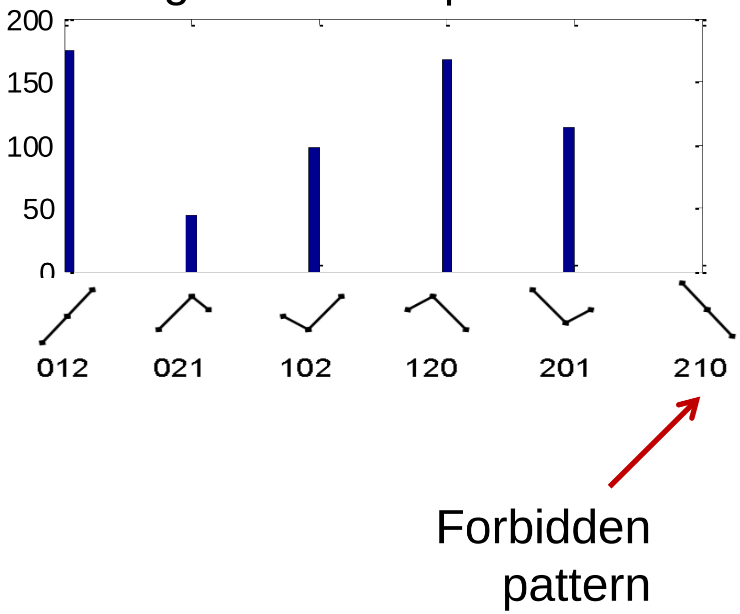
Time series



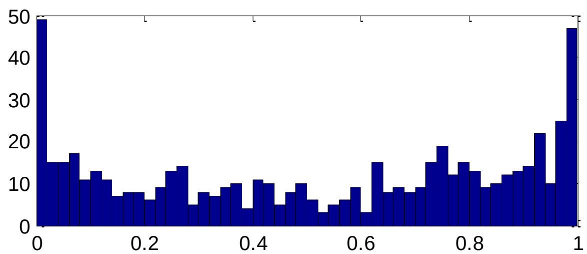
Detail



Histogram ordinal patterns D=3

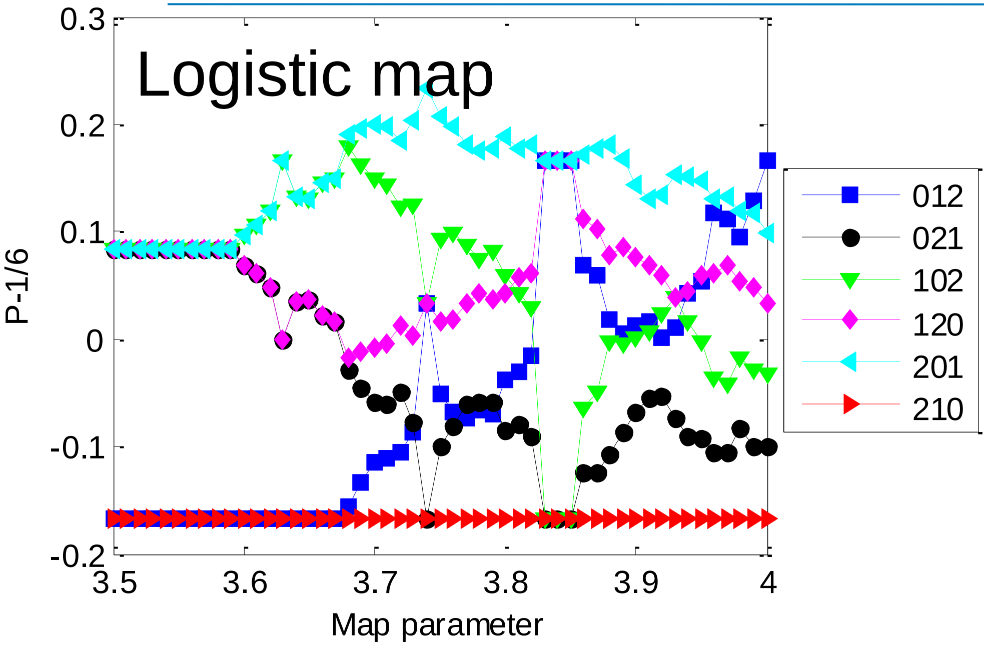


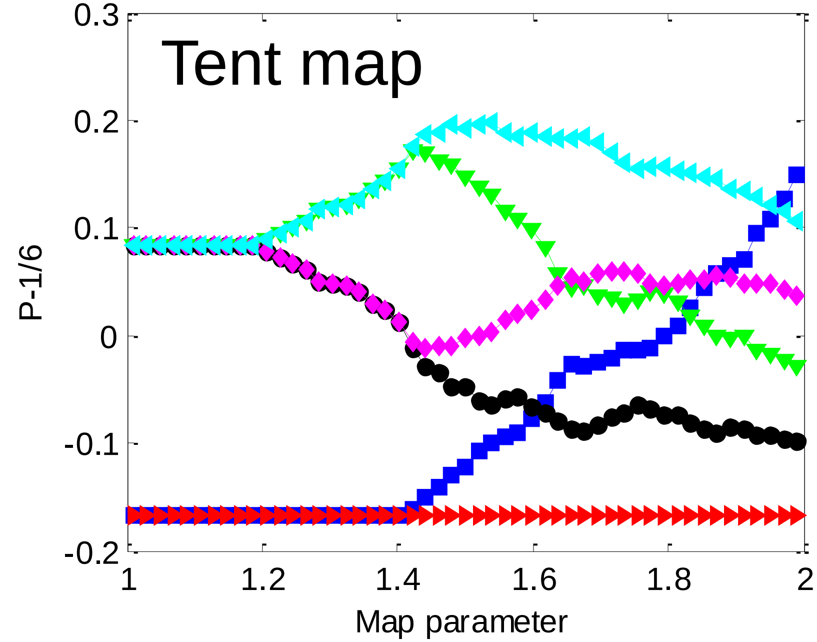
Histogram x(i)

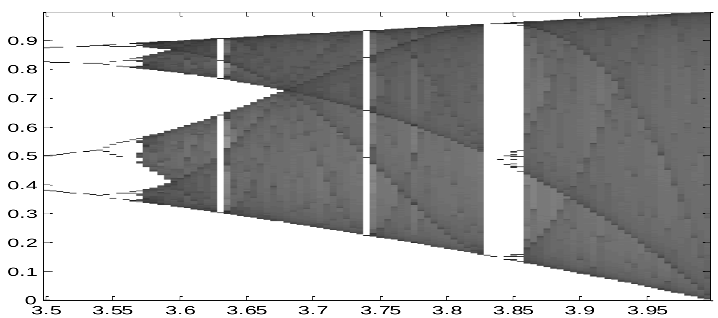


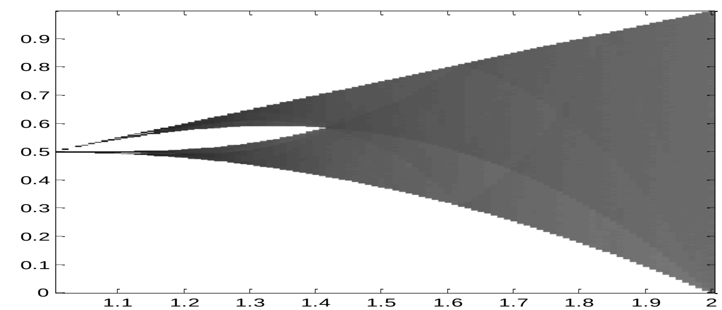
- Ordinal analysis provides complementary information.

Ordinal bifurcation diagrams

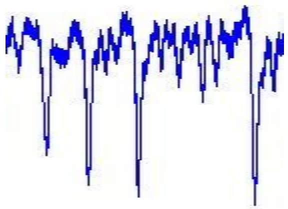




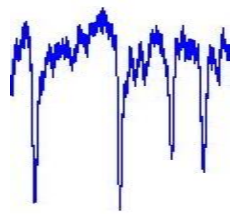




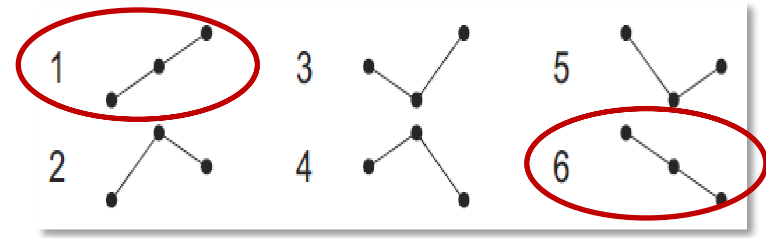
- With **D=3** we can study correlations among 4 spikes.



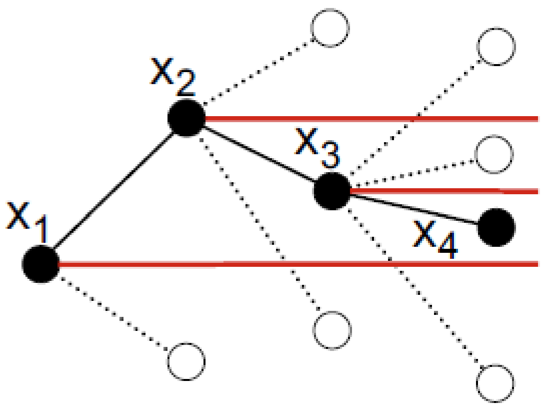
⇒**012**



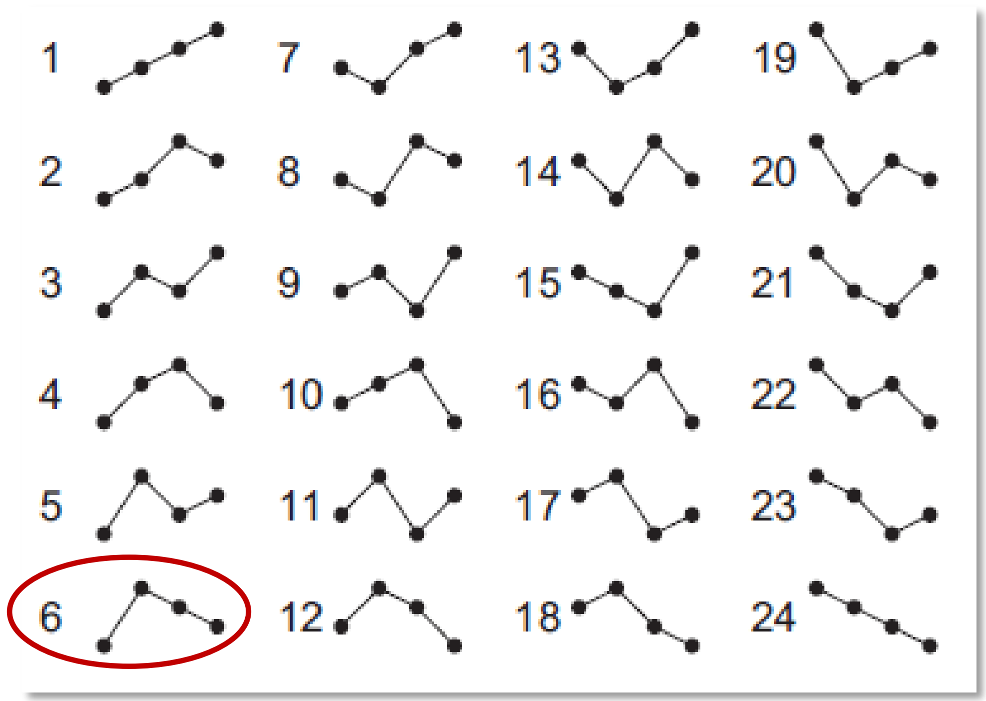
⇒**210**



- With **D=4**

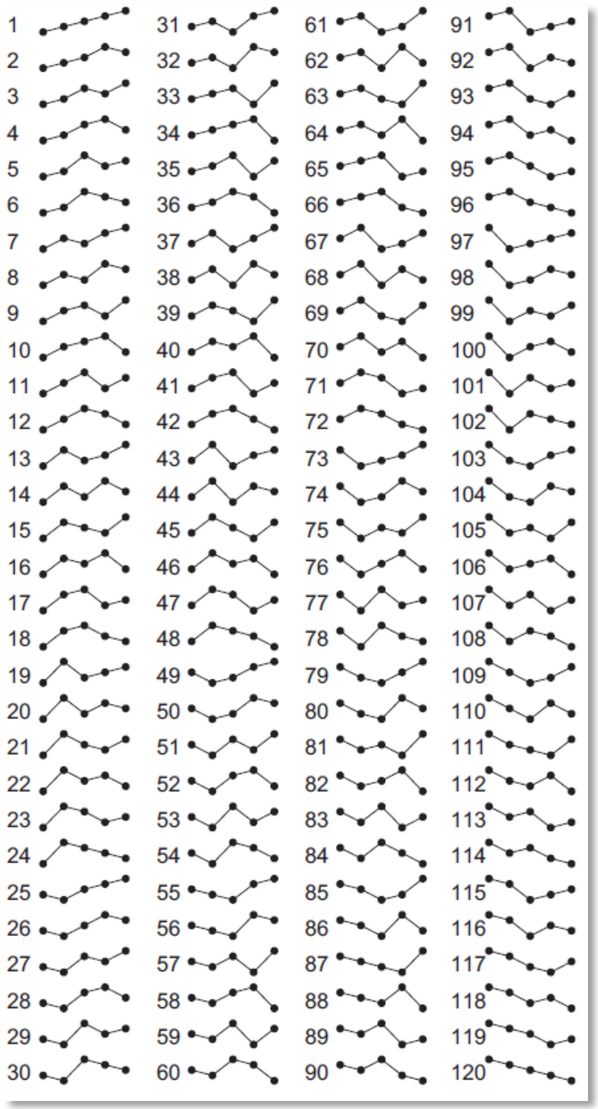


⇒ **0321**



U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

- The number of patterns grows with the length of the pattern as **D!**



D=5: 5!=120 patterns

- How to quantify the information?
  - Permutation entropy

$$s_p = -\sum p_i \log p_i$$

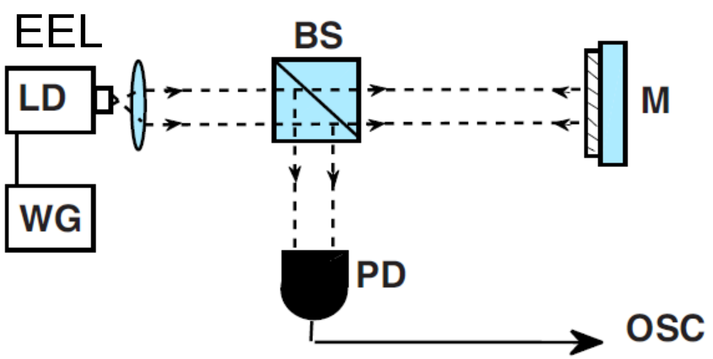
- How to select optimal D? depends on:
  - The length of the data.
  - The length of the correlations.

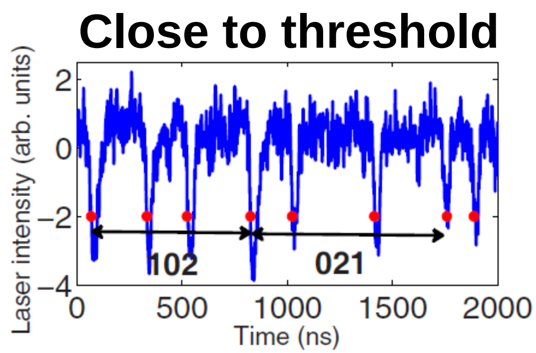
U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

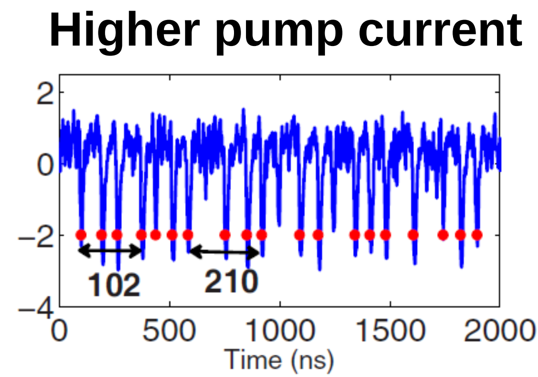
Contrasting empirical optical spikes with synthetic neuronal spikes

- do they have similar ordinal statistics?
- are there more/less frequent patterns?

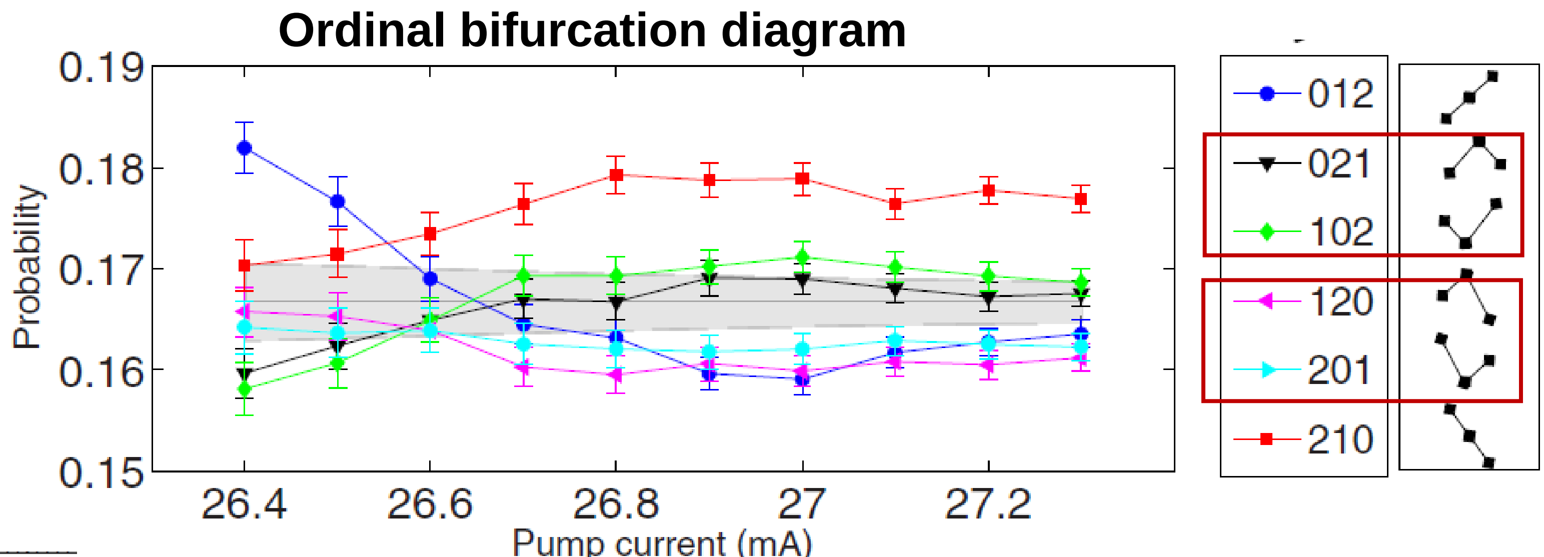
Ordinal analysis of ISI correlations







Grey region computed with binomial test: probabilities consistent with uniform distribution with 95% confidence level.



$$p \pm 3\sigma_p$$

$$\sigma_p = \sqrt{(p(1-p)/N)}$$

P =1 /6; ***N* > 10,000 ISIs**

A. Aragoneses, S. Perrone, T. Sorrentino, M. C. Torrent and C. Masoller, Sci. Rep. **4**, 4696 (2014)

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Minimal model of ISI correlations: modified circle map

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha_c \sin(4\pi\varphi_i)\right] + D\zeta$$

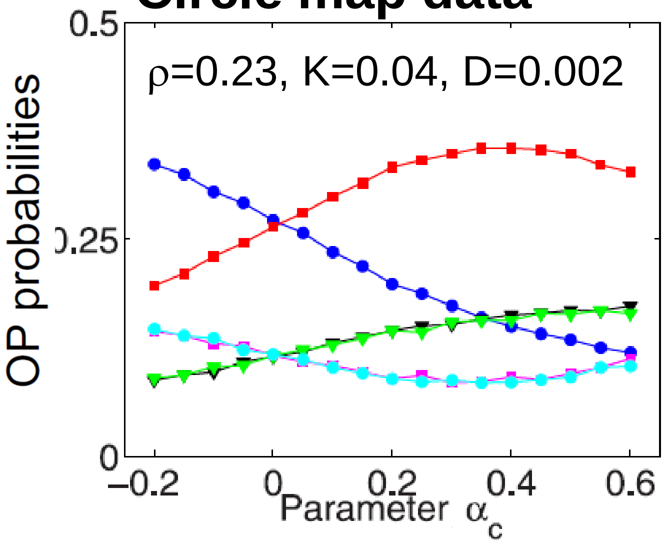
$$X_i = \varphi_{i+1} - \varphi_i$$

$\rho = \dfrac{\text{natural frequency}}{\text{forcing frequency}}$

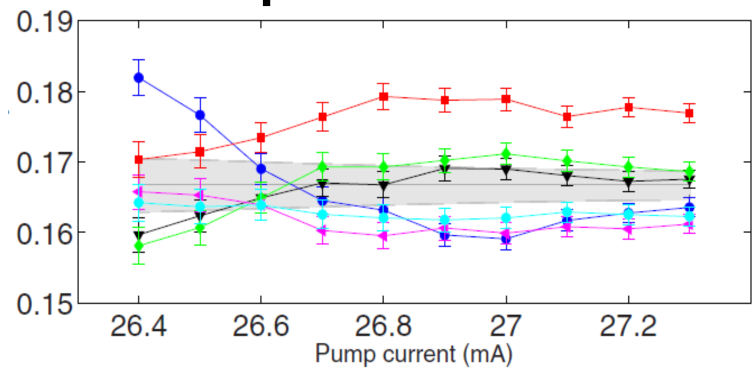
K = forcing amplitude

D = noise strength

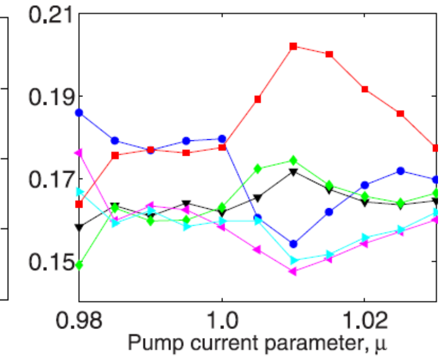
Circle map data



Empirical laser data



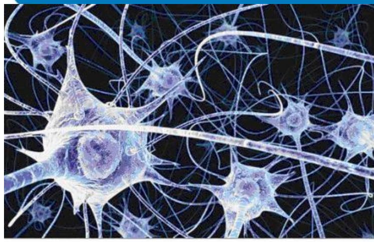
Lang-Kobayashi time-delay model



012 021 102 120 201 210

- Same "clusters" & same hierarchical structure.
- Modified circle map: minimal model for ordinal correlations.
- Same qualitative behavior found with other lasers & feedback conditions.

Model equations and parameters: A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)

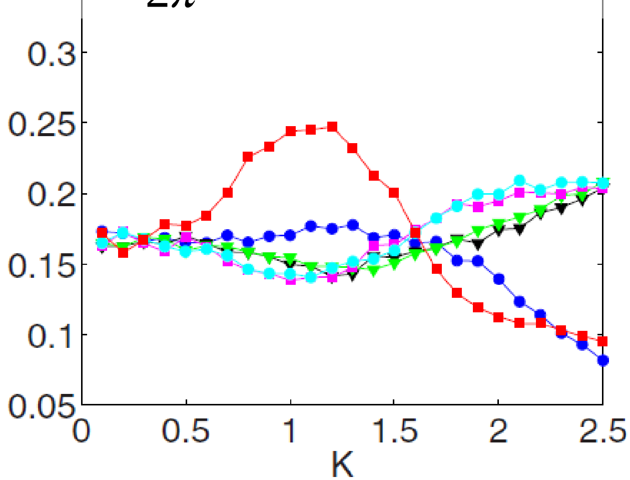


Connection with neurons

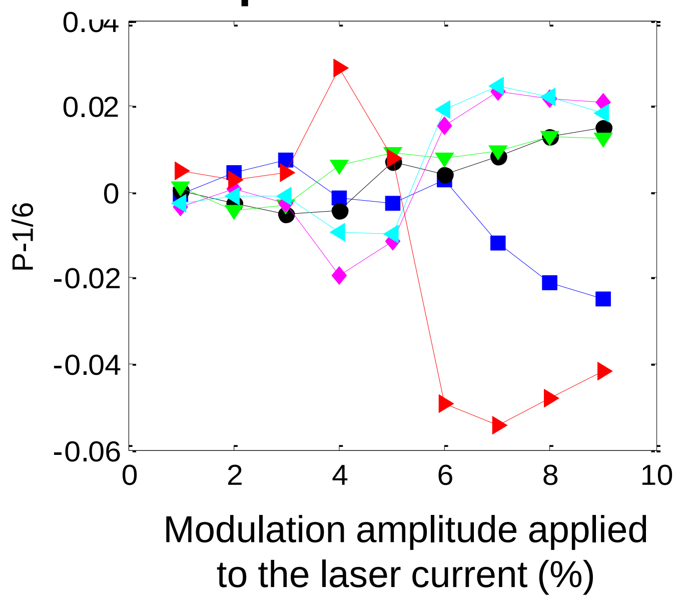
- The circle map describes many excitable systems.
- The modified circle map has been used to describe spike correlations in biological neurons.

  A. B. Neiman and D. F. Russell, *Models of stochastic biperiodic oscillations and extended serial correlations in electroreceptors of paddlefish*, PRE 71, 061915 (2005)

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha\sin(4\pi\varphi_i)\right]$$

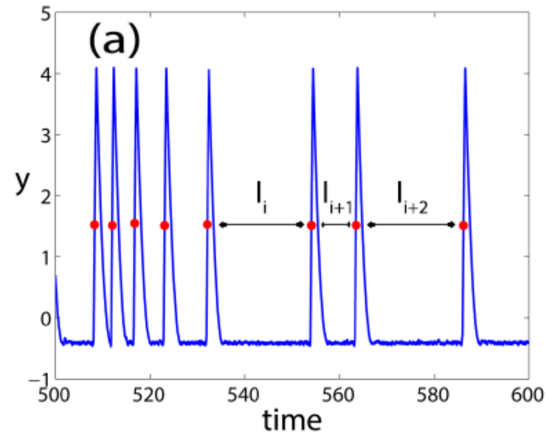


Empirical laser data



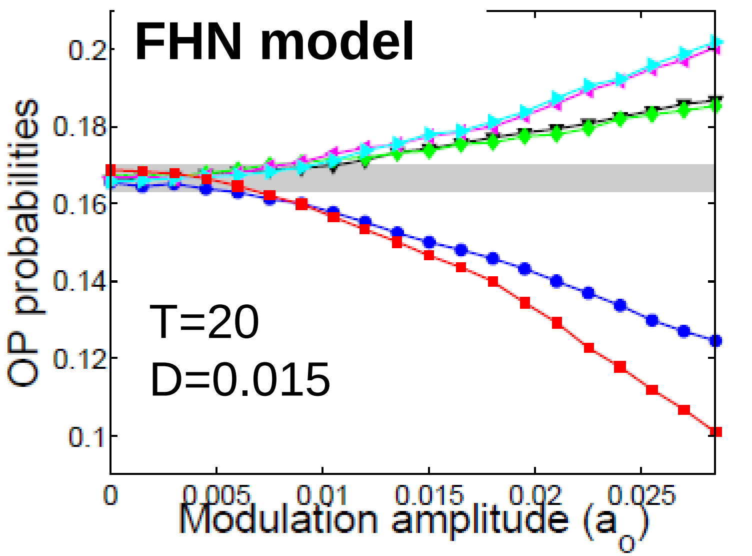
FHN model

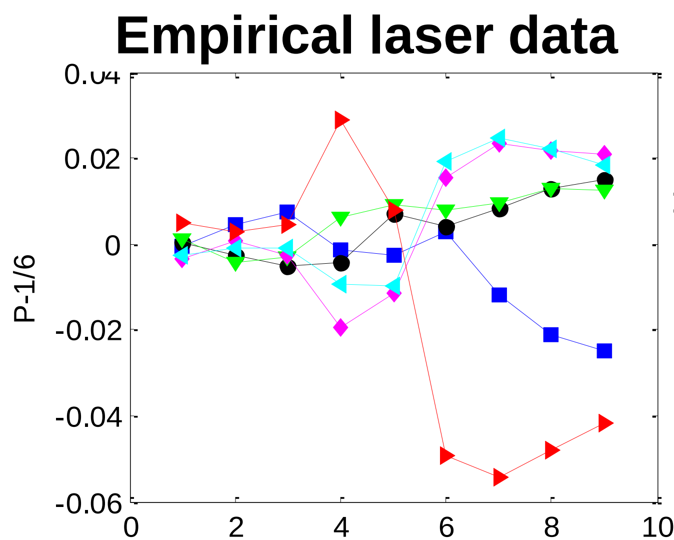
$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o \cos(2\pi t/T)} + D\xi(t),$$



Gaussian white noise and <u>subthreshold</u> (weak) modulation: **a_0** and **T** such that spikes are only noise-induced.

Time series with 100,000 ISIs simulated.





Modulation amplitude applied to the laser current (%)

$\Rightarrow$ Good qualitative agreement

Analysis of synthetic ISI sequences generated by single-neuron models

- more/less frequent patterns encode information?

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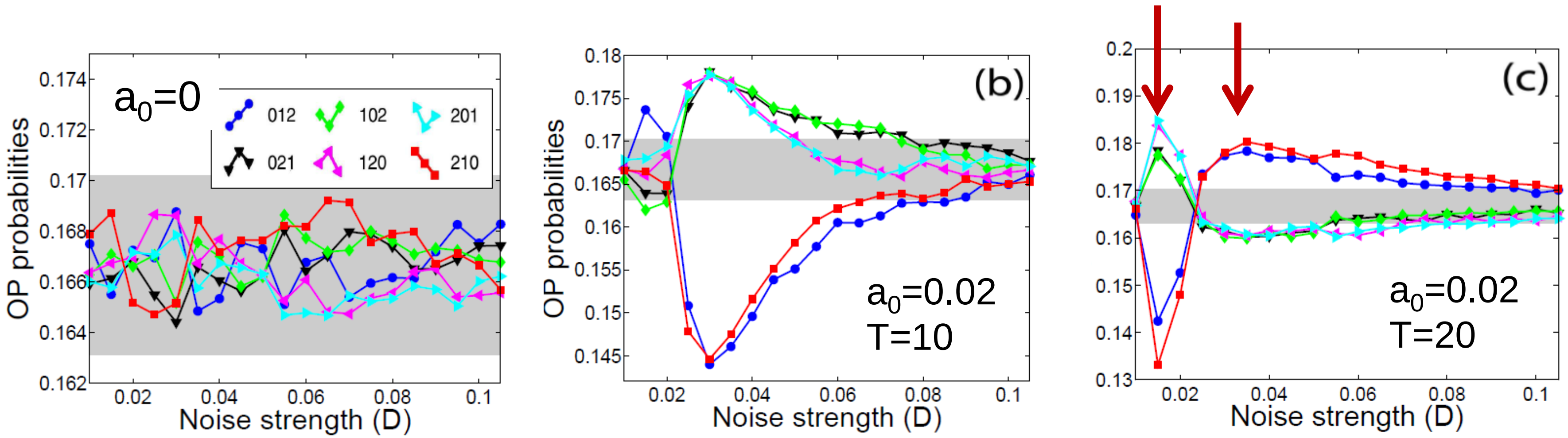
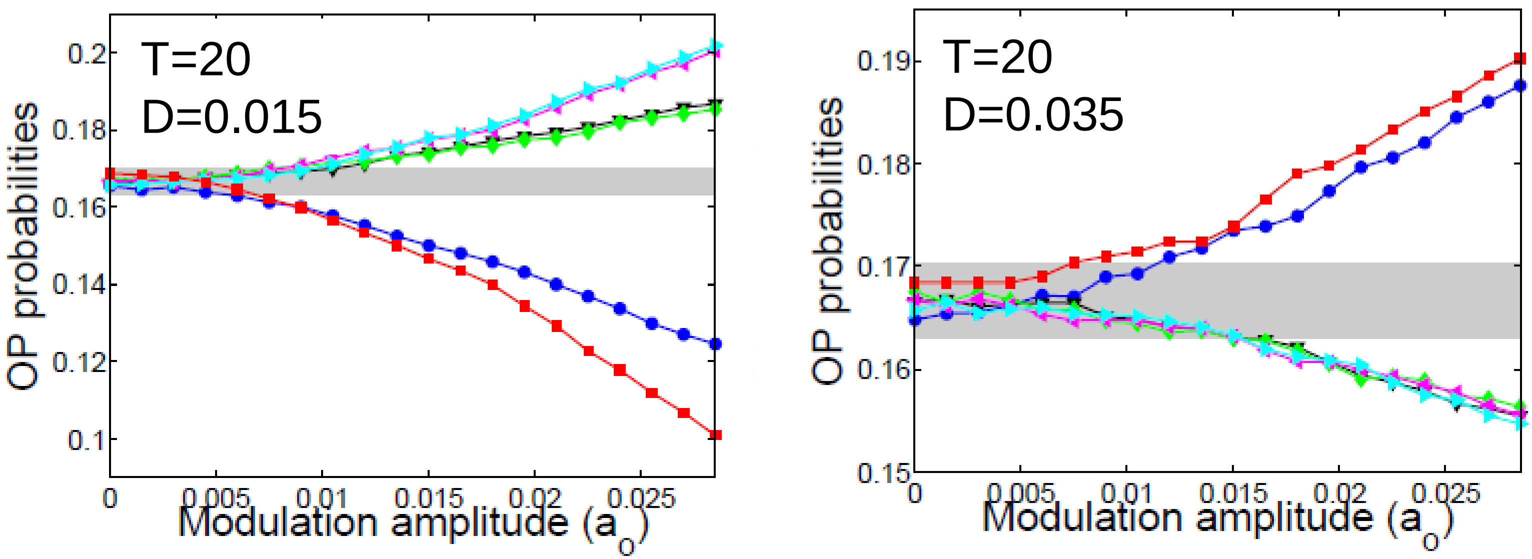
FHN model: role of the noise strength

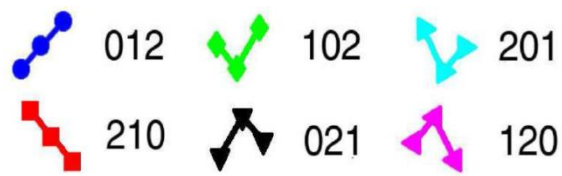


- No noise-induced temporal ordering.
- External periodic input induces temporal ordering.
- Preferred ordinal patterns depend on the noise strength.
- Resonant-like behavior.

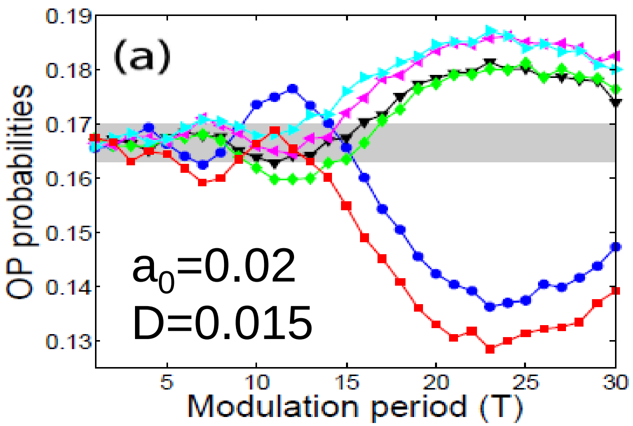
Role of the modulation amplitude

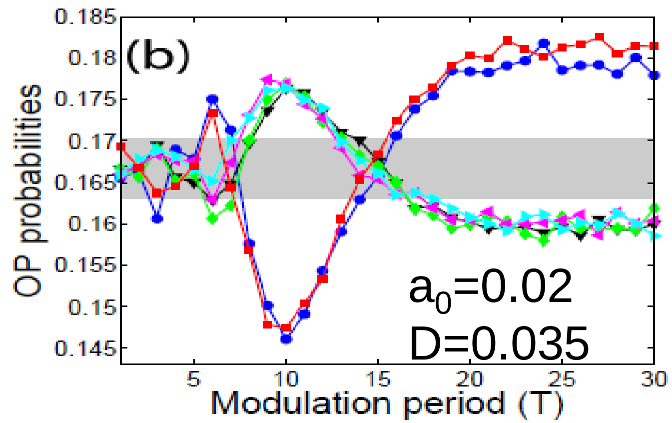


- The amplitude of the (weak) modulation does not modify the preferred and the infrequent patterns.



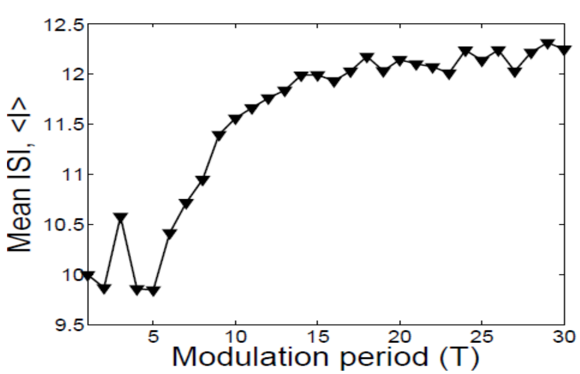
Role of the modulation period

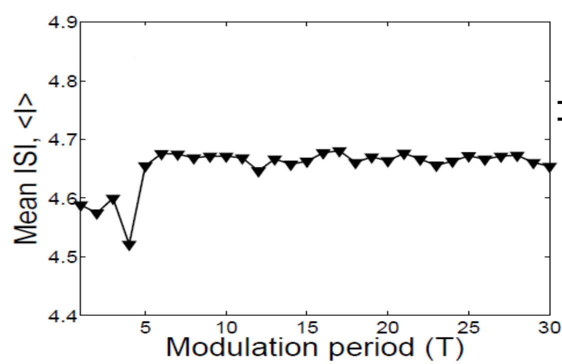




- More probable patterns depend on the period of the external input and on the noise strength.

Which is the underlying mechanism? A change of the spike rate?

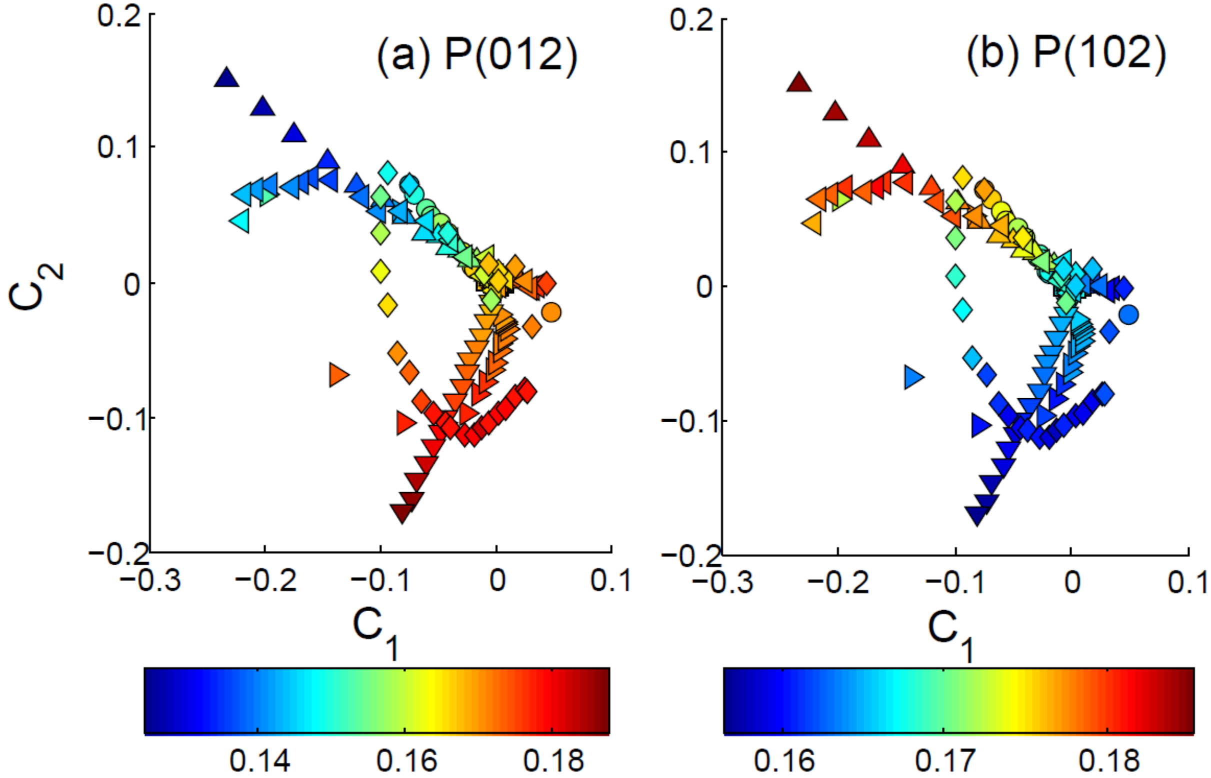




$\Rightarrow$ No direct relation.

$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

Relation between ordinal probabilities and C_1, C_2

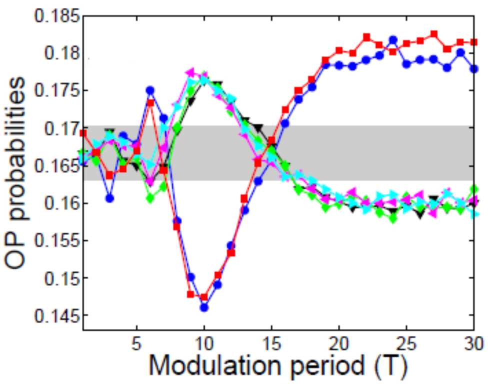


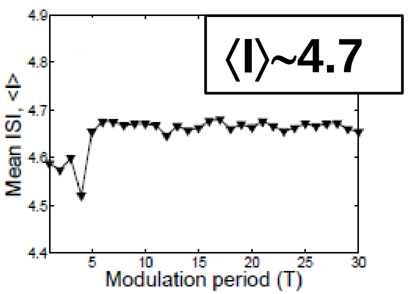
Varying noise strength, modulation amplitude and period: all datasets collapsed
$\Rightarrow$ clear trend with C_2, no trend with C_1

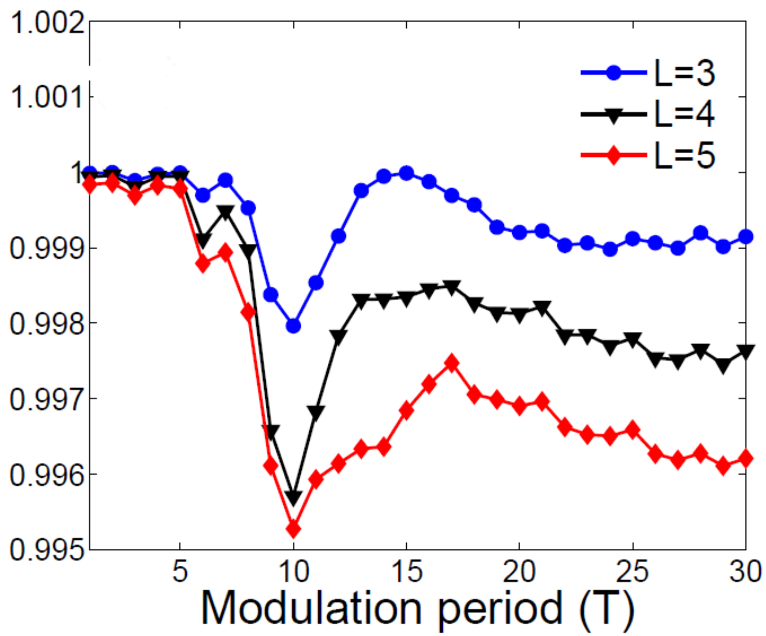
Are there longer temporal correlations?

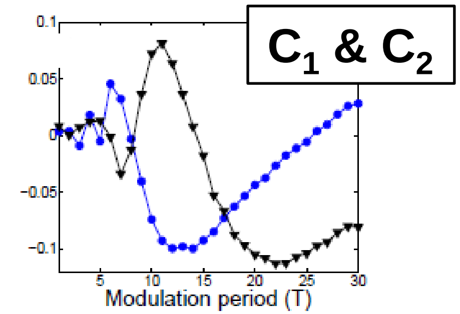
- **Permutation entropy**: computed from ordinal probabilities & normalized to maximum value.

$$S_{PE} = \frac{-\sum p_i \log p_i}{\log L!}$$



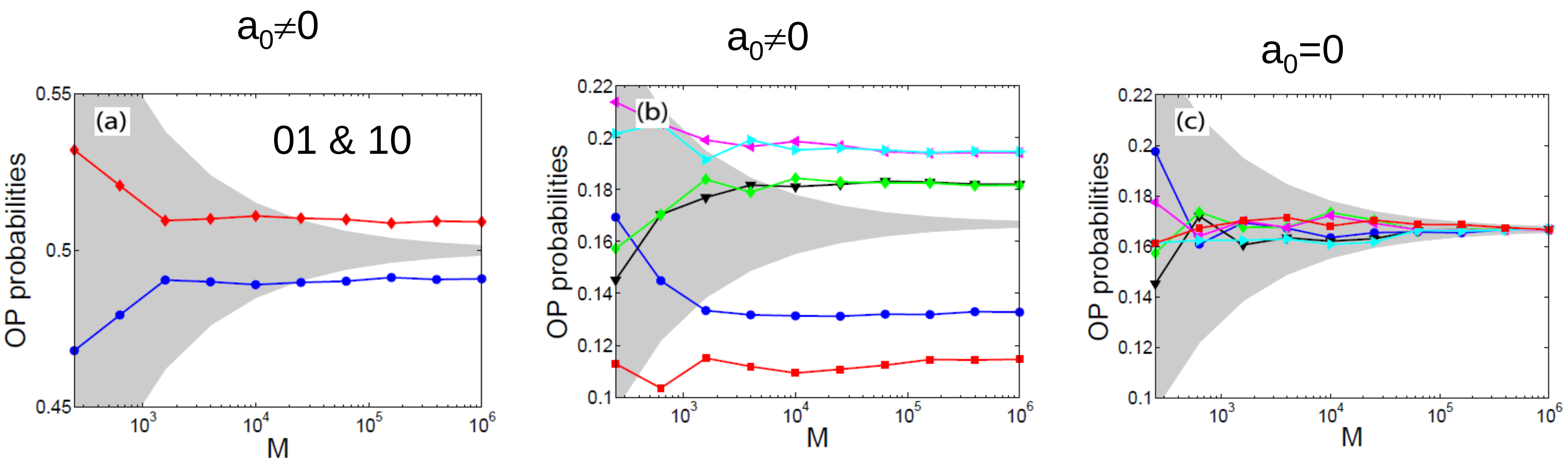






- Modulation period T > ⟨I⟩ induces long temporal correlations
- Sharp transition seen in S_{pe} at T=10 not detected by C_1 or C_2

Statistically significant results? Influence of the length of the data



- Long datasets are need for a robust estimation of the ordinal probabilities.

Conclusions

What did we learn?

- Take home message:
  - ordinal analysis is useful for understanding data, uncovering patterns,
  - for model comparison, parameter estimation, classifying events, etc.
  - robust to noise and artifacts in the data.
- Main conclusions
  - Correlations in optical & neuronal spike sequences compared: good qualitative agreement.
  - Minimal model for optical spikes identified: a modified circle map.
  - FHN model with subthreshold modulation and Gaussian white noise
    - There are preferred ordinal patterns which depend on the noise strength and on the period of the input signal, but not on (weak) amplitude of the signal.
    - resonance-like behavior: certain periods and noise levels maximize the probabilities of the preferred patterns, enhancing temporal order.
- Open issues (ongoing and future work):
  - Hierarchical & clustered structure: universal feature of excitable systems?
  - Mathematical insight: can we calculate the probabilities analytically?
  - Role of coupling? induce preferred/infrequent patterns?
  - Compare with empirical data (single-neuron ISI sequences)

THANK YOU FOR YOUR ATTENTION !

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Papers at: http://www.fisica.edu.uy/~cris/

- ***Unveiling the complex organization of recurrent patterns in spiking dynamical systems***

  A. Aragoneses, S. Perrone, T. Sorrentino, M. C. Torrent and C. Masoller,

  Sci. Rep. 4, 4696 (2014).
- ***Emergence of spike correlations in periodically forced excitable systems***

  J. A. Reinoso, M. C. Torrent, C. Masoller

  PRE in press (2016) http://arxiv.org/abs/1510.09035
- ***Analysis of noise-induced temporal correlations in neuronal spike sequences***

  J. A. Reinoso, M. C. Torrent, C. Masoller

  EPJST in press (2016).