

Influence of Periodic Modulation in Rare and Extreme Optical Pulses

Cristina Masoller

UPC, Terrassa, Barcelona, Spain

Cristina.masoller@upc.edu

www.fisica.edu.uy/~cris



UNIVERSITAT POLITÈCNICA
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MS33: Rare Events in Stochastic Systems





Sandro Perrone



Ramon Vilaseca



Jordi Zamora Munt



Dhananjay Bhiku



Jatin Ahuja

Indian Institute of Technology, Guwahati, Assam, India

What is a rogue wave?

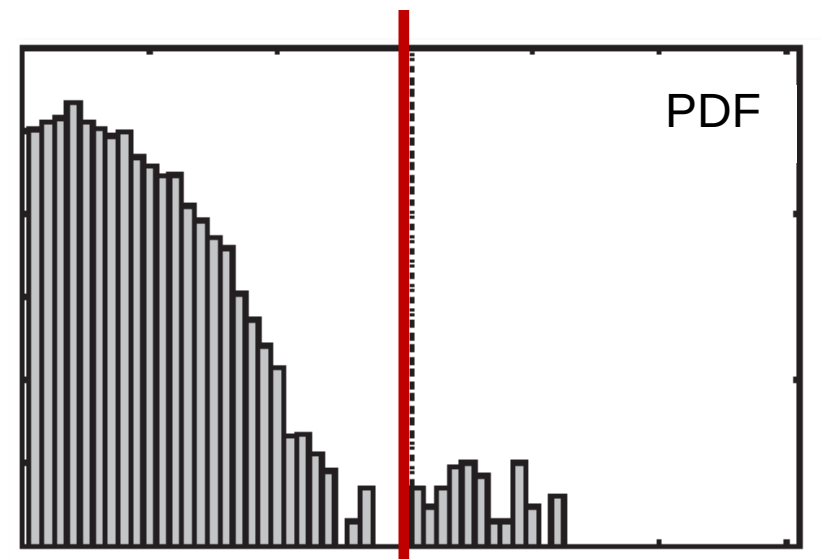
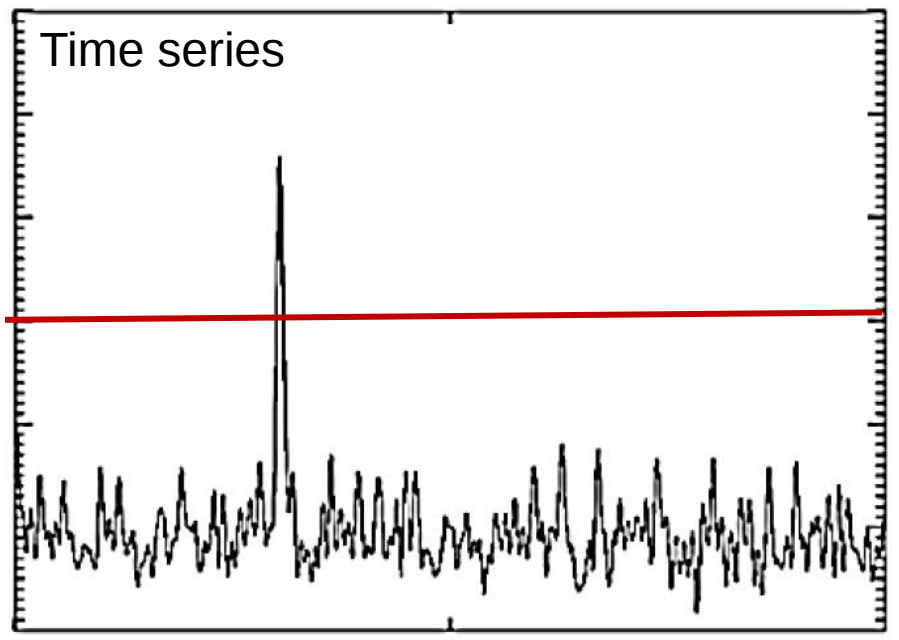
RWs are rare, ultra-high waves that fall outside (and far from) the main part of long-tailed probability distributions.



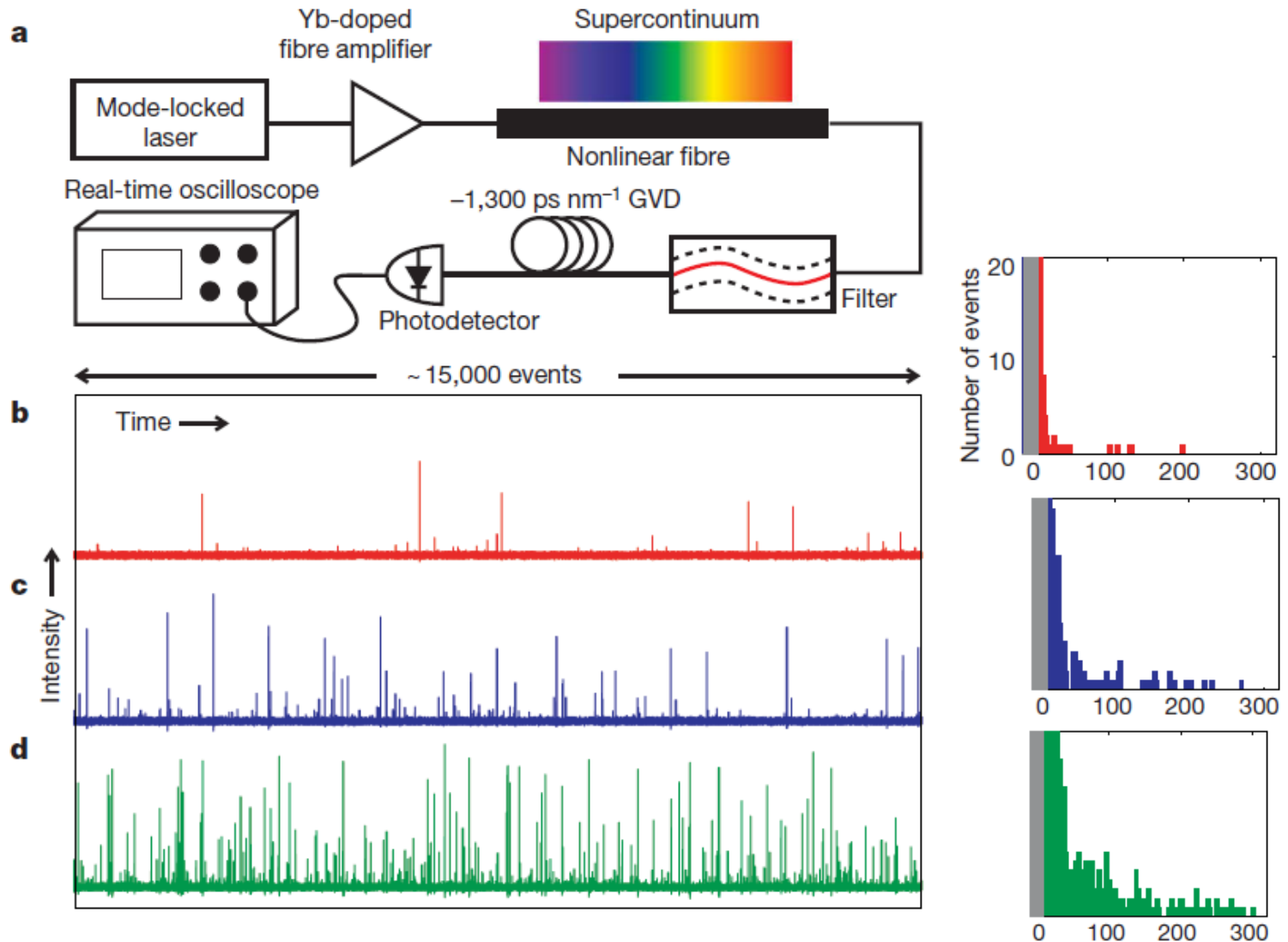
The Great Wave of Kanagawa, Katsushika Hokusai. Source: Wikipedia

Two more precise rogue wave definitions

- A RW: above **twice** the significant wave height (SWH).
SWH = $\langle \text{highest } 1/3 \text{ waves} \rangle$ that occur over a certain period of time.
- A RW: above a certain threshold ($\langle A \rangle + 4-8 \sigma$)



Optical rogue waves



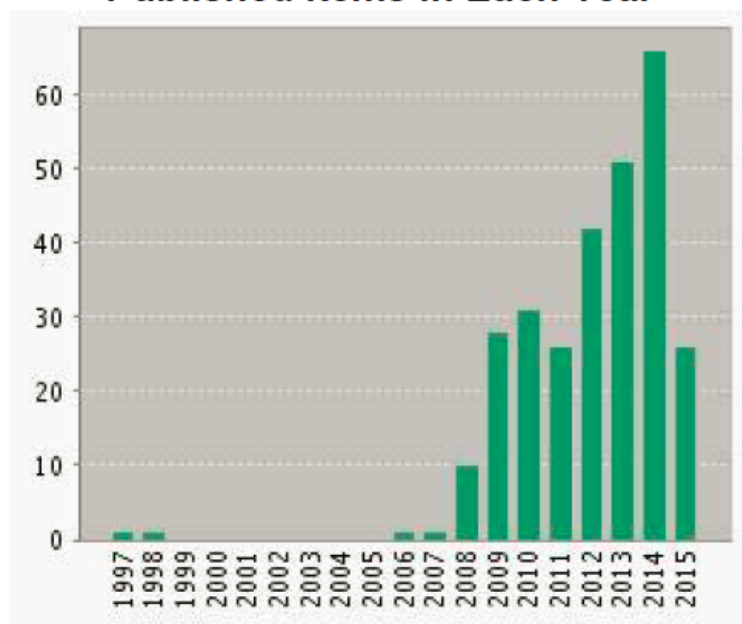
Rare and
extreme
intensity
pulses

Since then, optical rogue waves are attracting increasing attention

You searched for: **TOPIC:** (optical rogue waves) [...More](#)

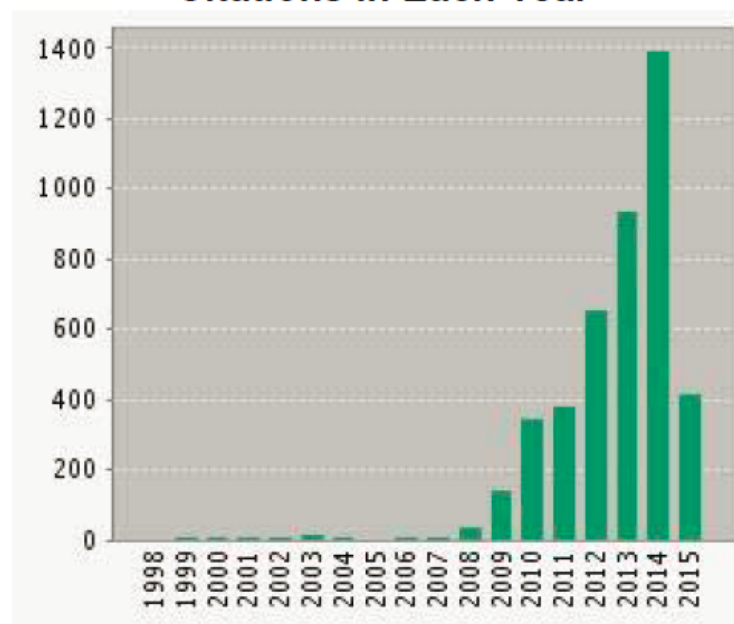
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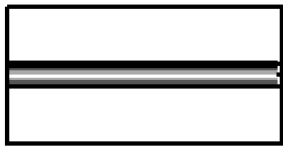


The latest 20 years are displayed.

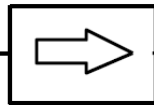
RWs in semiconductor lasers

Optically injected semiconductor lasers provide an inexpensive and controllable setup to study RWs

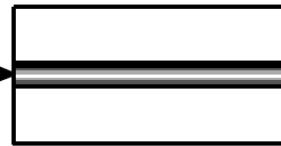
Master Laser



Isolator



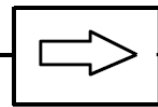
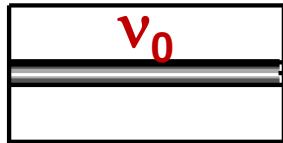
Slave Laser



Detection system
(photo detector,
oscilloscope,
spectrum
analyzer)

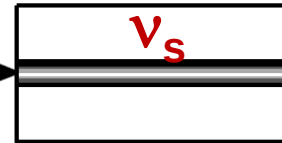
Key parameters and main dynamical regimes

Master Laser

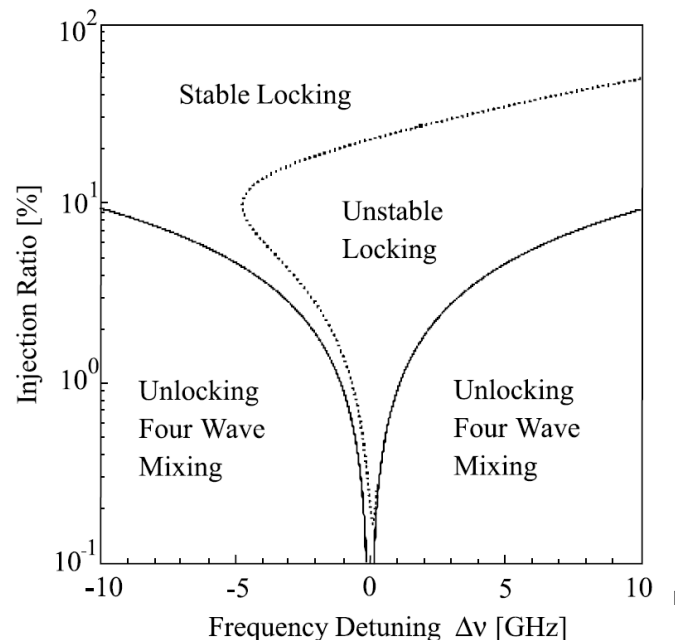


Isolator

Slave Laser



- Injection ratio
- Frequency detuning $\Delta\nu = \nu_s - \nu_0$

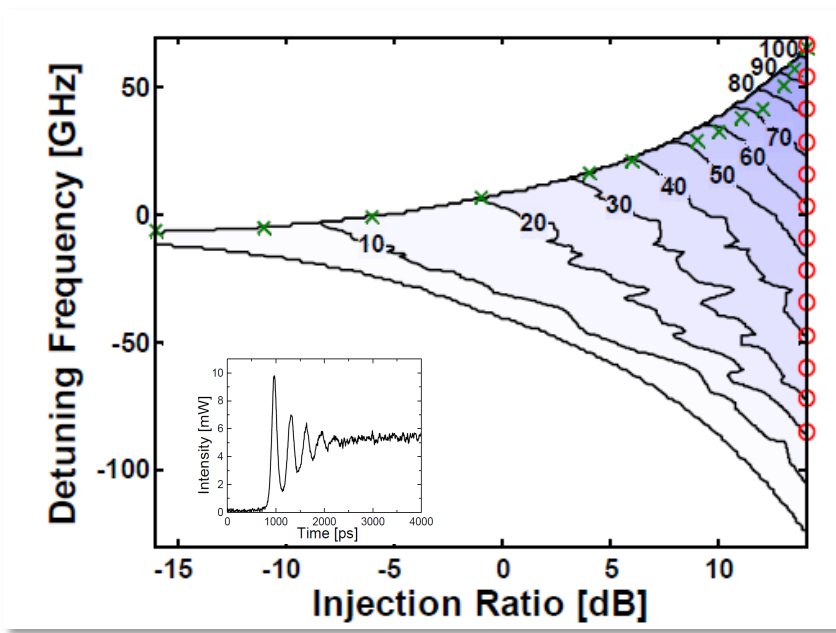


- Regimes:
 - Stable locking: stable output at the frequency of the master laser
 - Regular oscillations
 - Excitability and Chaos

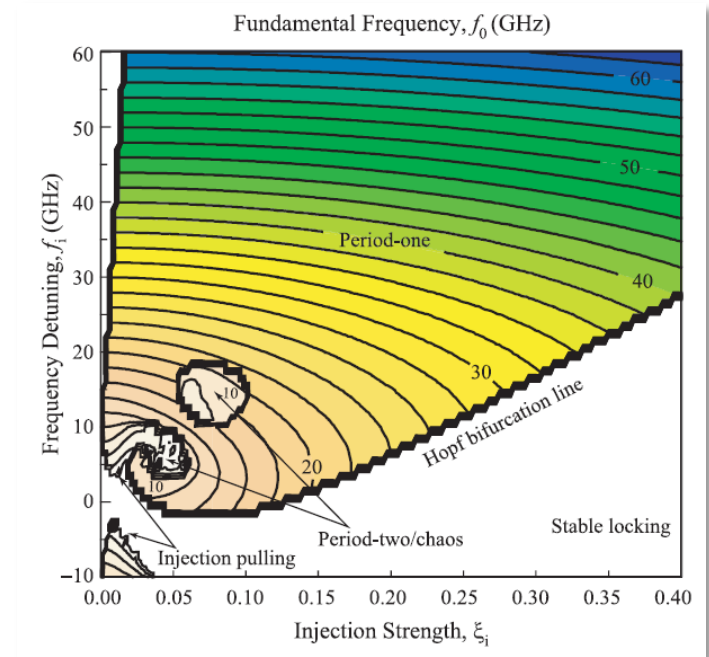
Adapted from J. Ohtsubo, *Semiconductor Lasers: Stability, Instability and Chaos*. Springer, 2006.

Practical use of optical injection

- Inside the locking region: increased resonance frequency and modulation bandwidth.
- Outside the locking region: regular oscillations, with tunable frequency (controlled by the injection parameters).



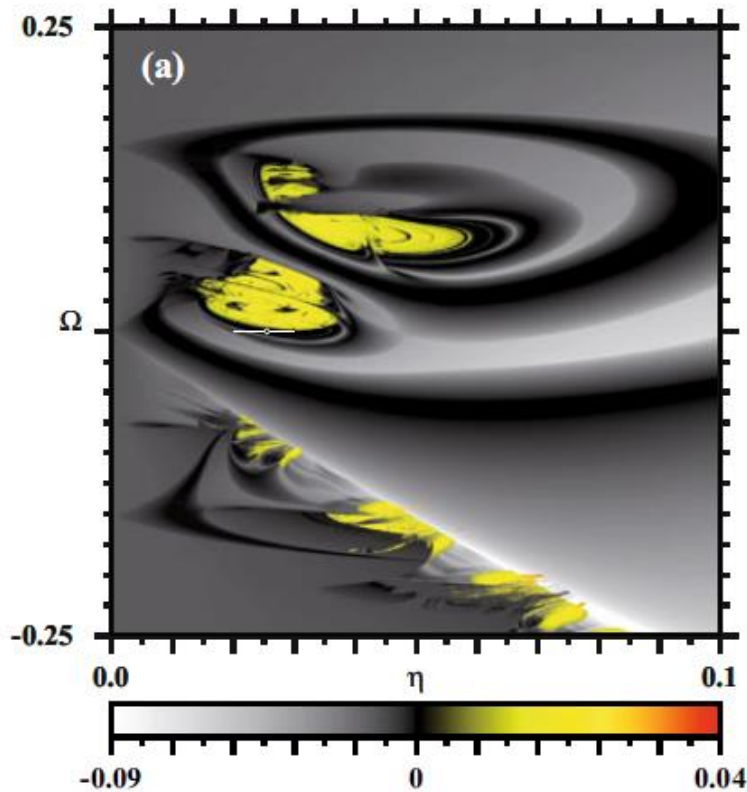
E. K. Lau et al, Opt. Express 16, 6609 (2008)



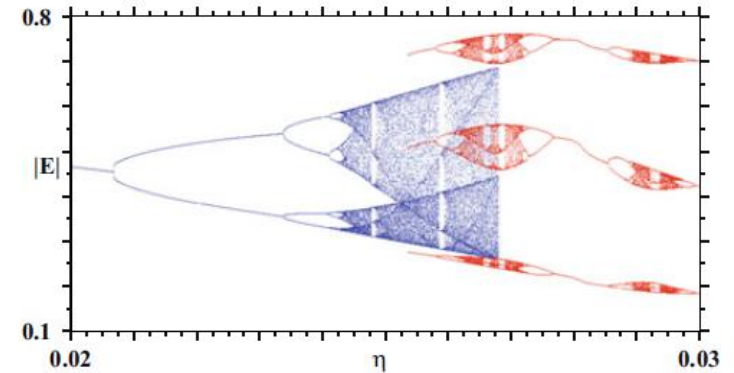
S-C Chan et al, Optics Express 15, 14921 (2007)

Chaotic oscillations, bistability, etc.

Lyapunov diagram



Bifurcation diagram

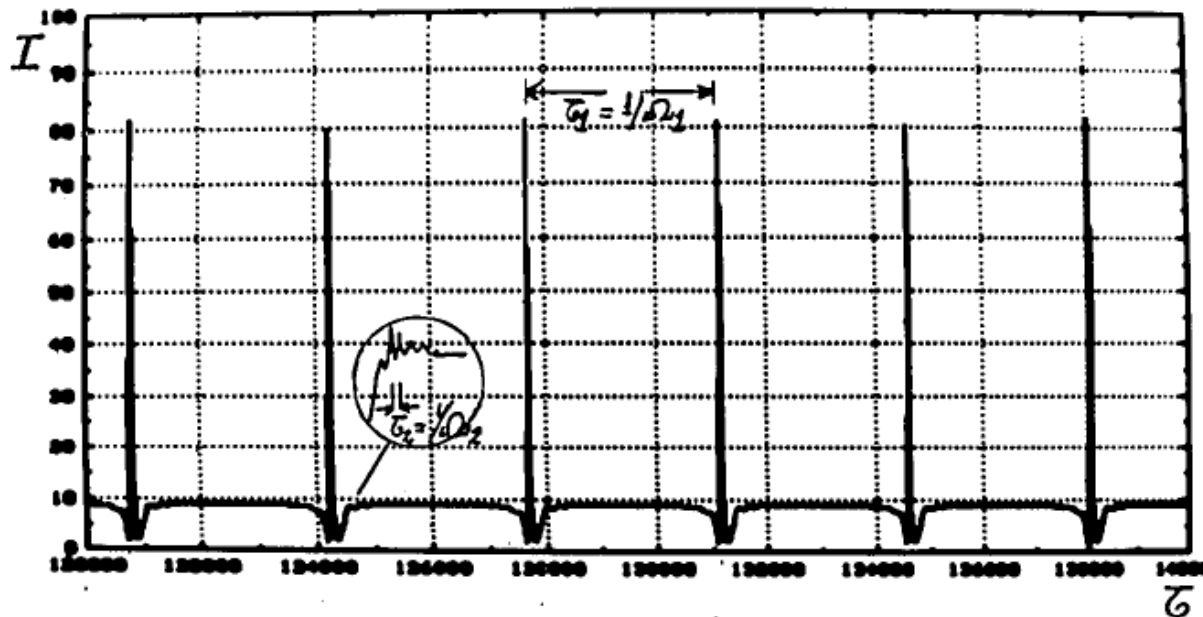


V. Kovanis et al, EPJD 58, 181 (2010)

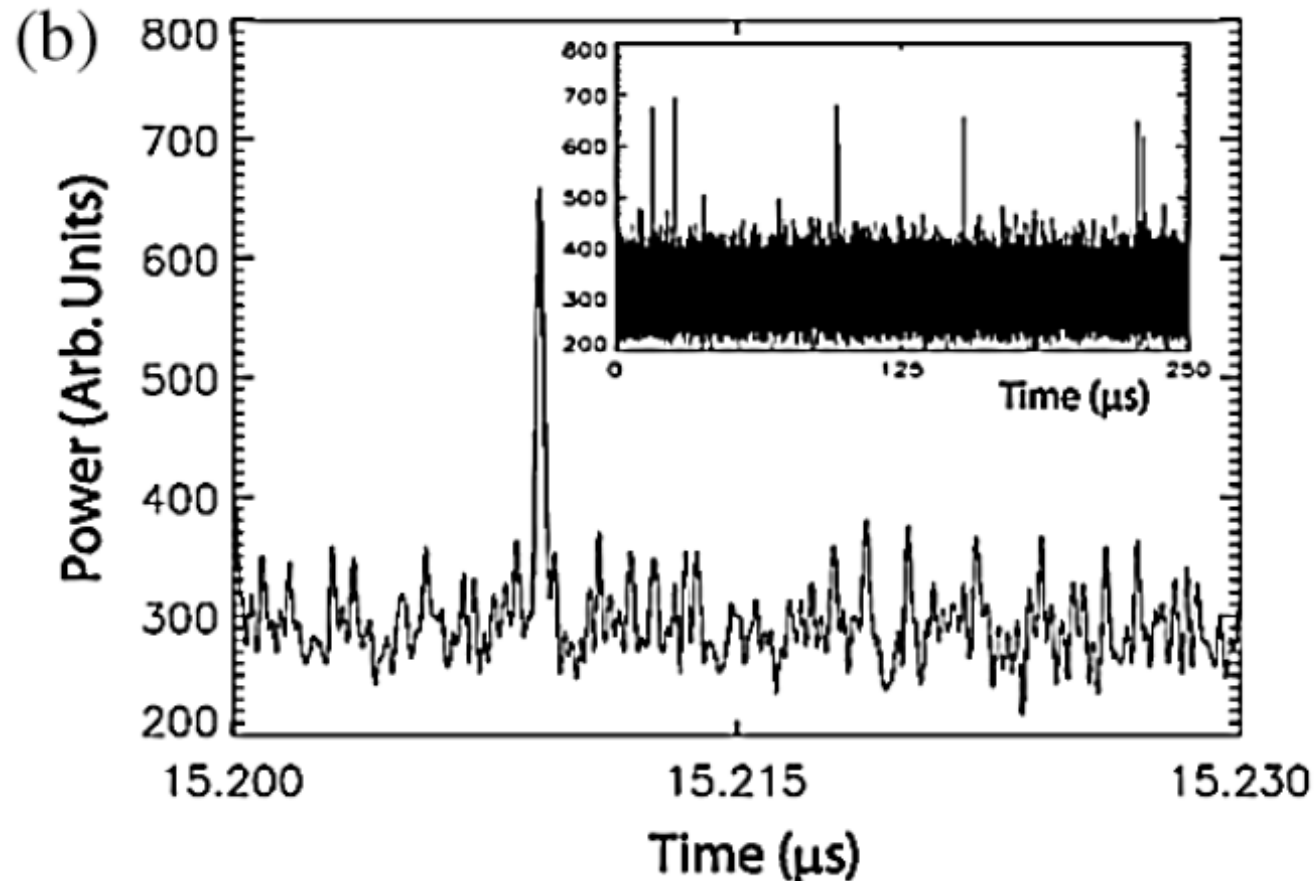
Instabilities in lasers with an injected signal

J. R. Tredicce, F. T. Arecchi, G. L. Lippi, and G. P. Puccioni

178 J. Opt. Soc. Am. B/Vol. 2, No. 1/January 1985



Experimental observation



C. Bonatto, M. Feyereisen, S. Barland, M. Giudici, C. Masoller, J. R. Rios Leite, and J. R. Tredicce, “*Deterministic optical rogue waves*”, PRL 107, 053901¹²(2011)

Governing equations

- Complex field, **E** (photon number $\propto |\mathbf{E}|^2$)
- Carrier density, **N**

$$\frac{dE}{dt} = \frac{1}{2\tau_p} (1 + i\alpha)(N - 1)E + \underbrace{i\Delta\omega + \sqrt{P_{\text{inj}}}}_{\text{optical injection}} + \underbrace{\sqrt{2\beta_{sp} / \tau_N} \xi(t)}_{\text{spontaneous emission}}$$

$$\underbrace{\frac{dN}{dt} = \frac{1}{\tau_N} (\mu - N - N|E|^2)}_{\text{Solitary laser}}$$

P_{inj}: injection strength

Δω: frequency detuning

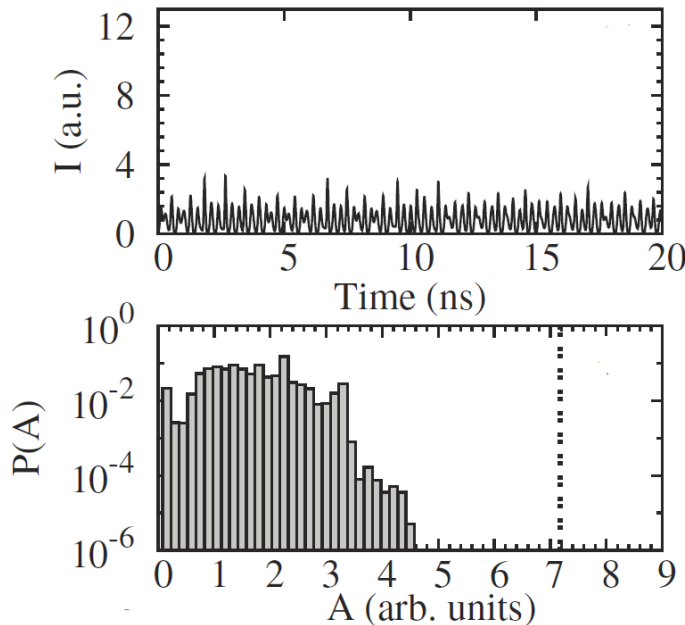
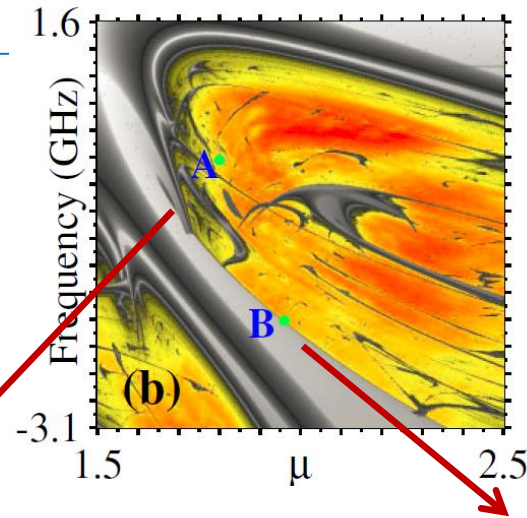
Solitary laser

μ: pump current parameter

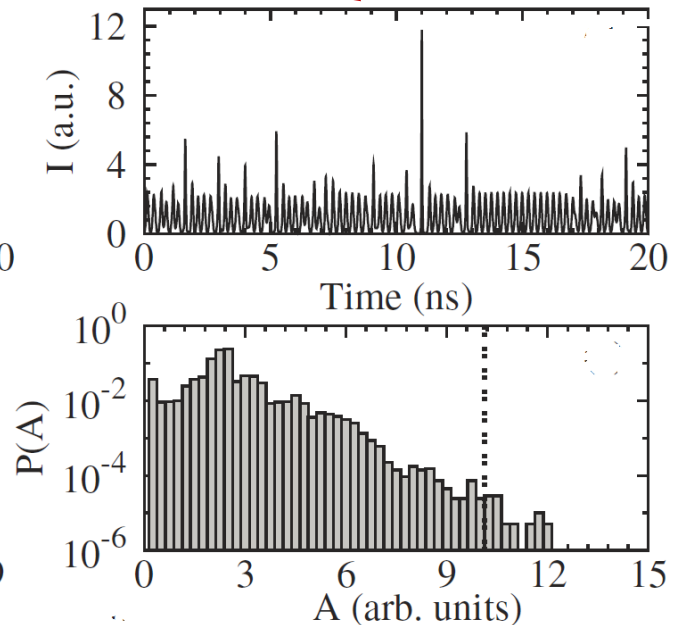
β_{sp}: noise strength

Deterministic simulations ($\beta_{sp}=0$)

Lyapunov diagram
(detuning, pump current)



Chaos without RWs

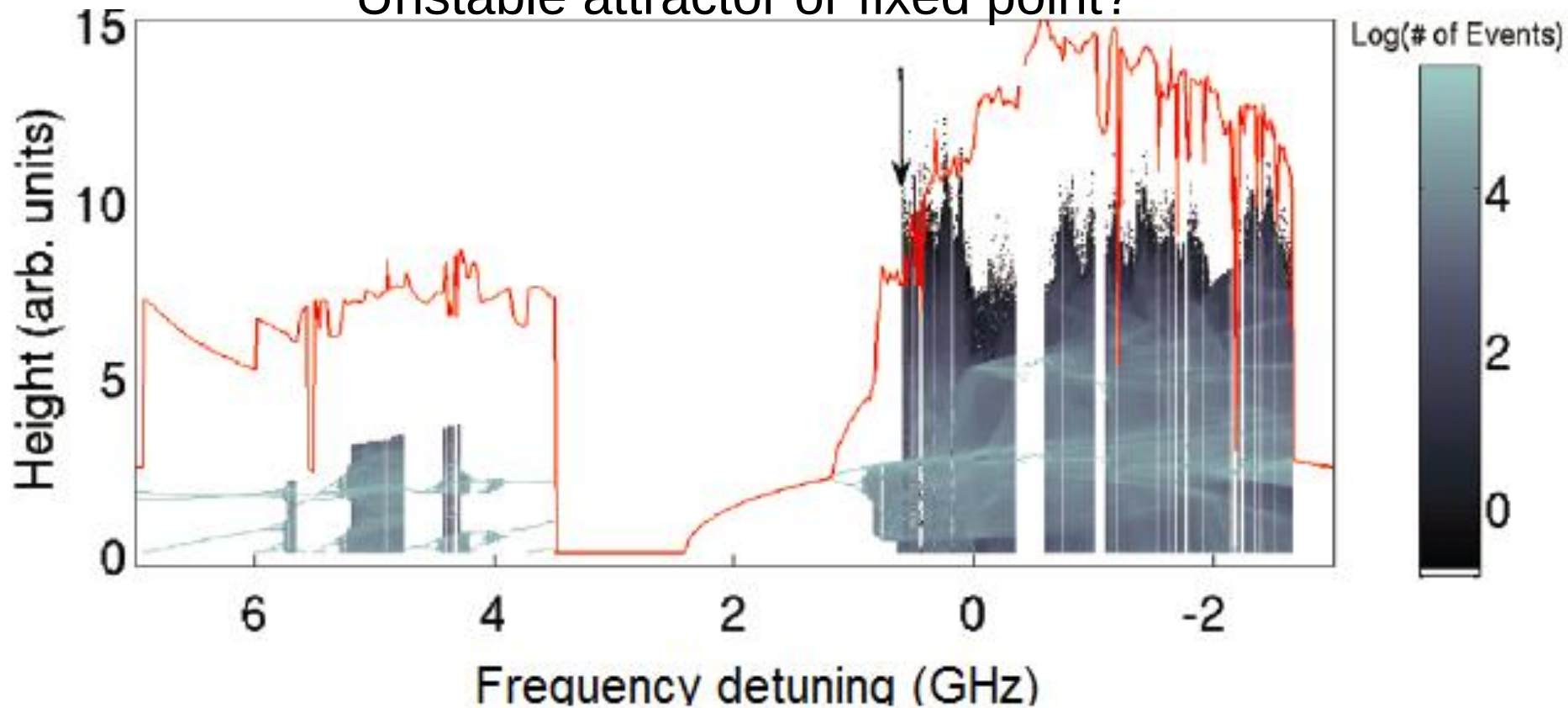


Chaos with RWs

- What triggers a rogue wave?
- Can rogue waves be predicted?
- Can rogue waves be controlled?

What triggers a RW?

Unstable attractor or fixed point?

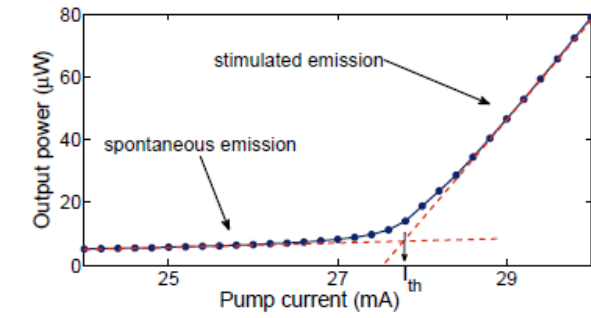
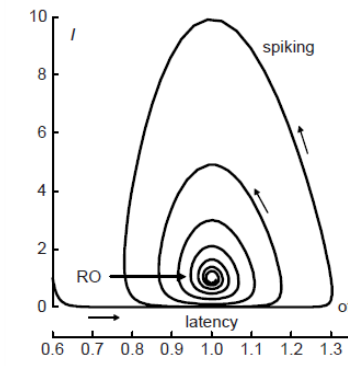
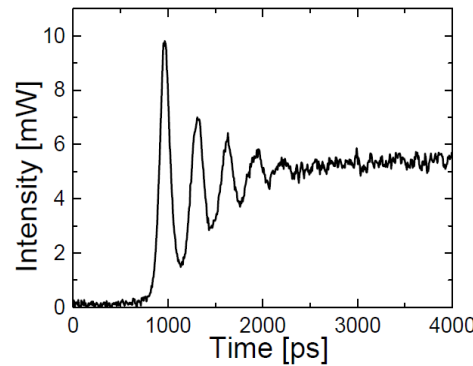


Threshold: $\langle H \rangle + 8\sigma$

Fixed point solutions

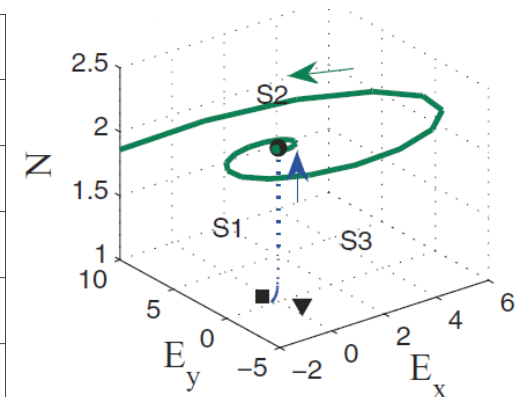
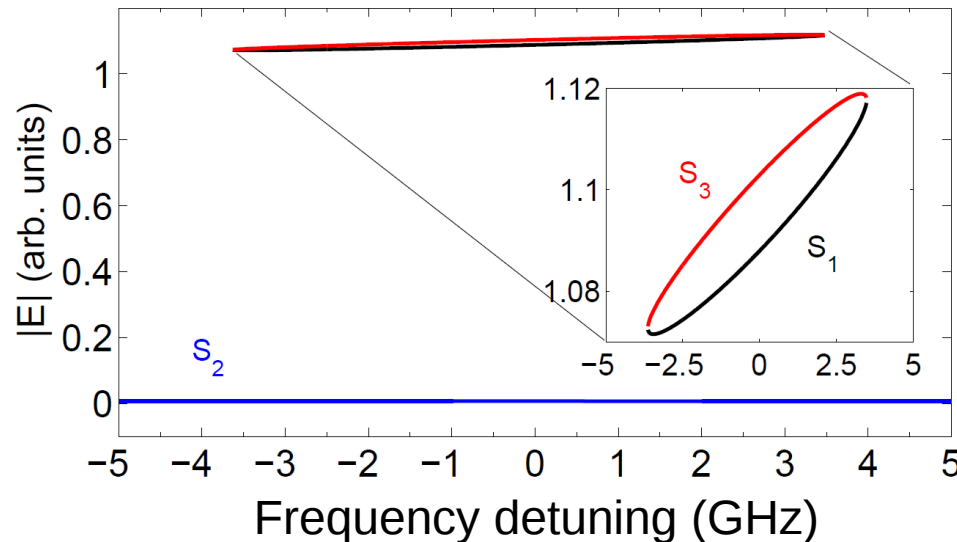
Solitary laser: two fixed points.

- transient spikes

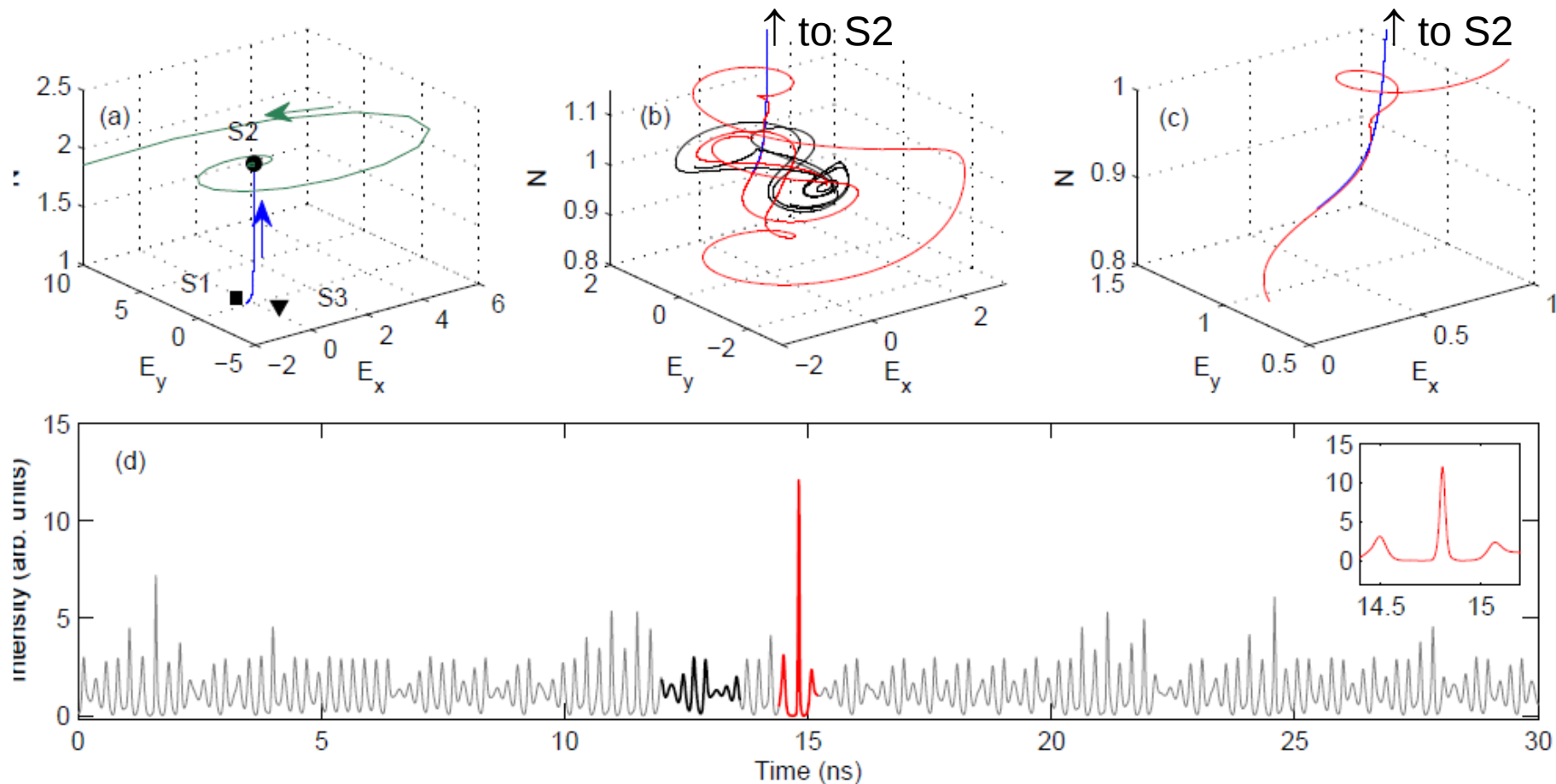


Injected laser: three fixed points.

- chaotic attractor develops from S3



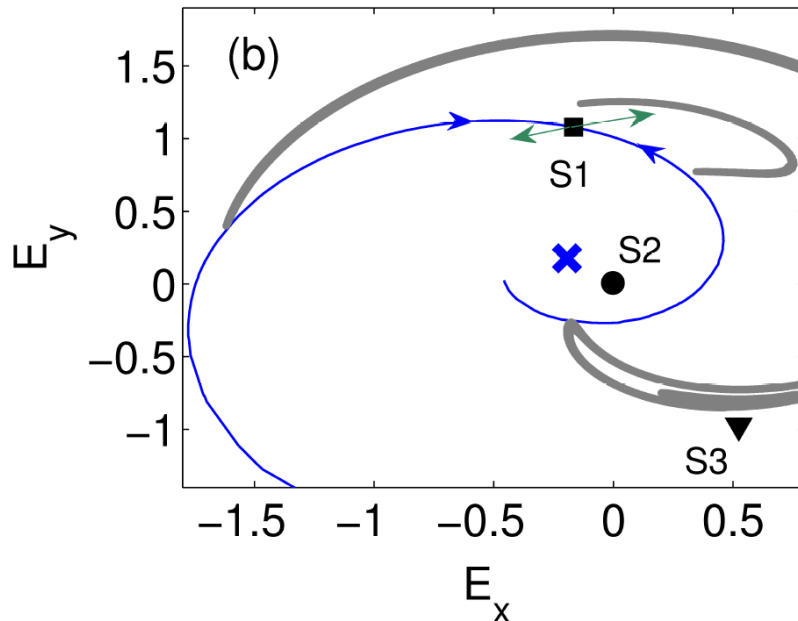
Stable manifold of S2: “RW door”



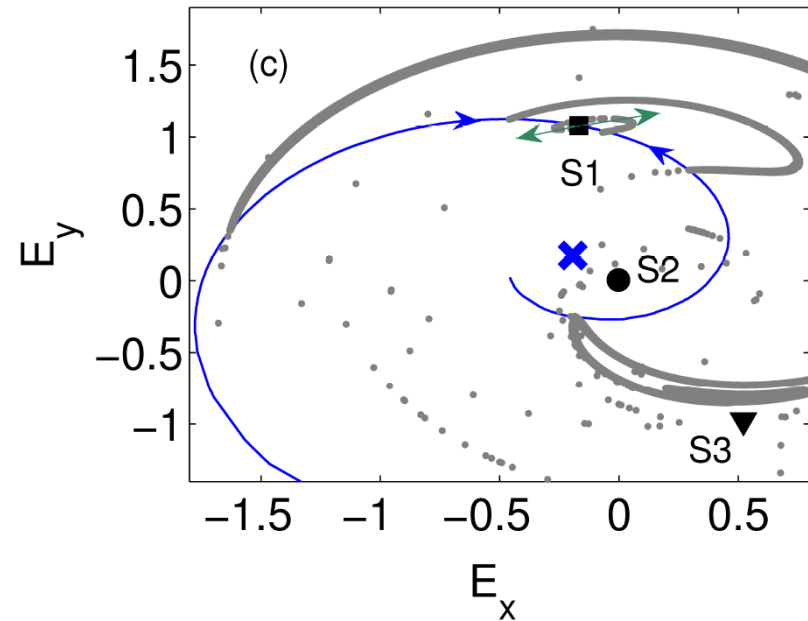
A **RW** is triggered when the trajectory closely approaches the stable manifold of S2 (RW door)

Why chaos with RWs and chaos without them?

Before attractor explosion



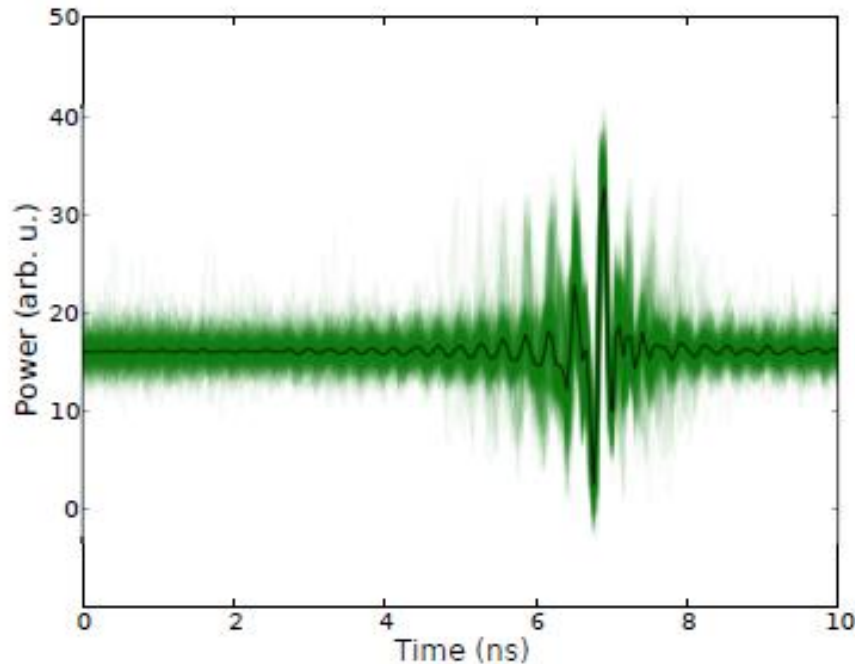
Just after attractor explosion



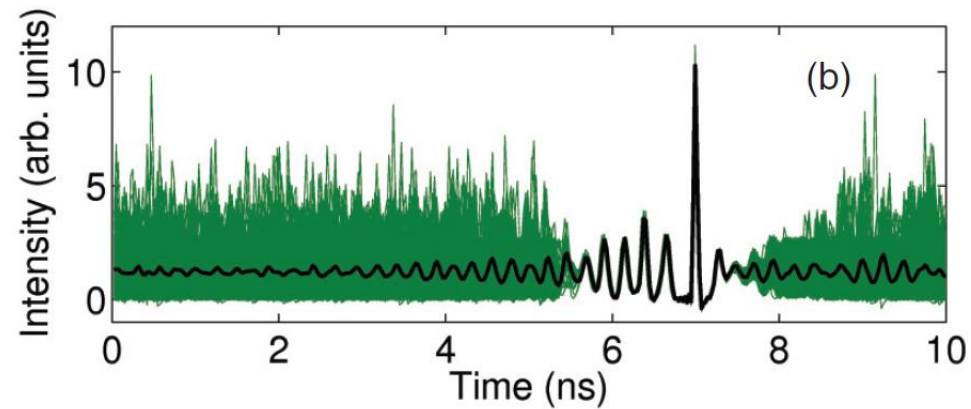
- In between the attractor originated from $S3$ and the “RW door” there is a “barrier”: the stable manifold of $S1$
- A **crises-like** process enables access to the region where the “RW door” is.

- What triggers a rogue wave?
- Can rogue waves be predicted?
- Can rogue waves be controlled?

Experiments

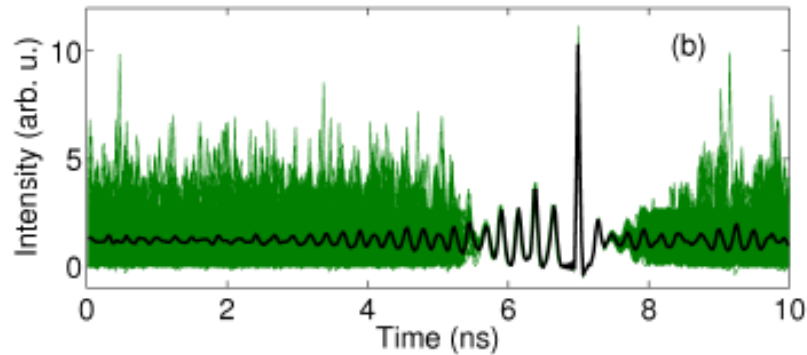


Simulations



Superposition of 500 time series at the RW peak

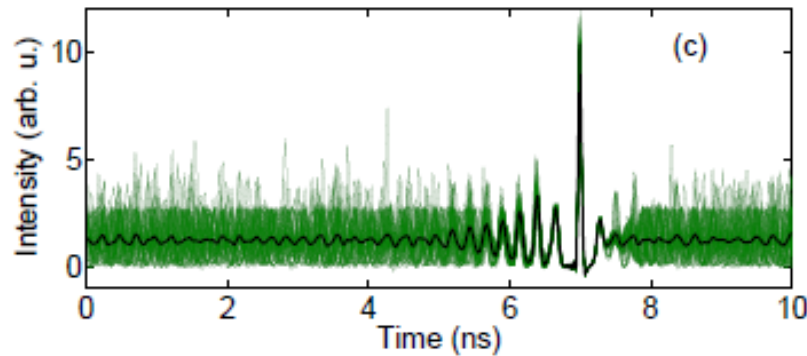
Influence of noise & RW threshold



$$\langle A \rangle + 8\sigma$$

$$\beta_{sp} = 0$$

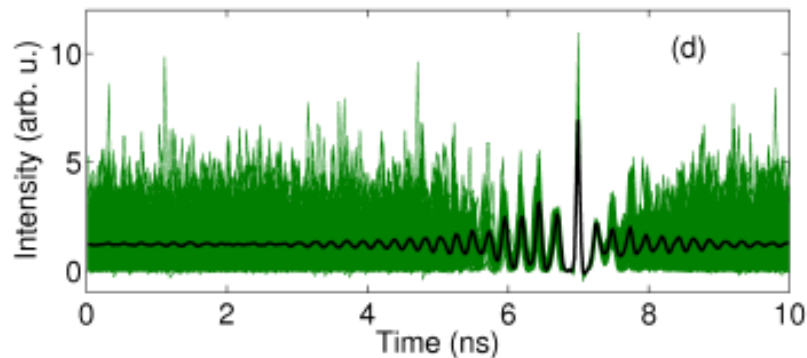
459 RWs



$$\langle A \rangle + 8\sigma$$

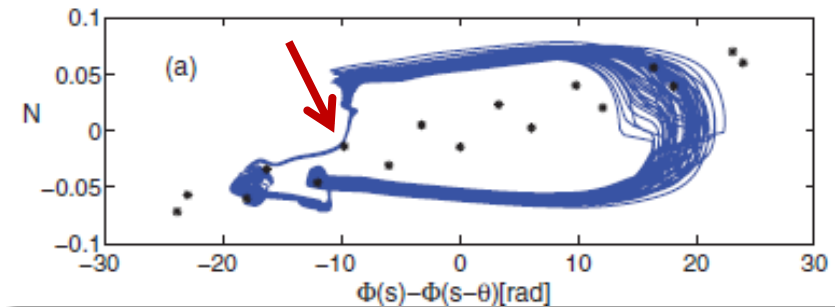
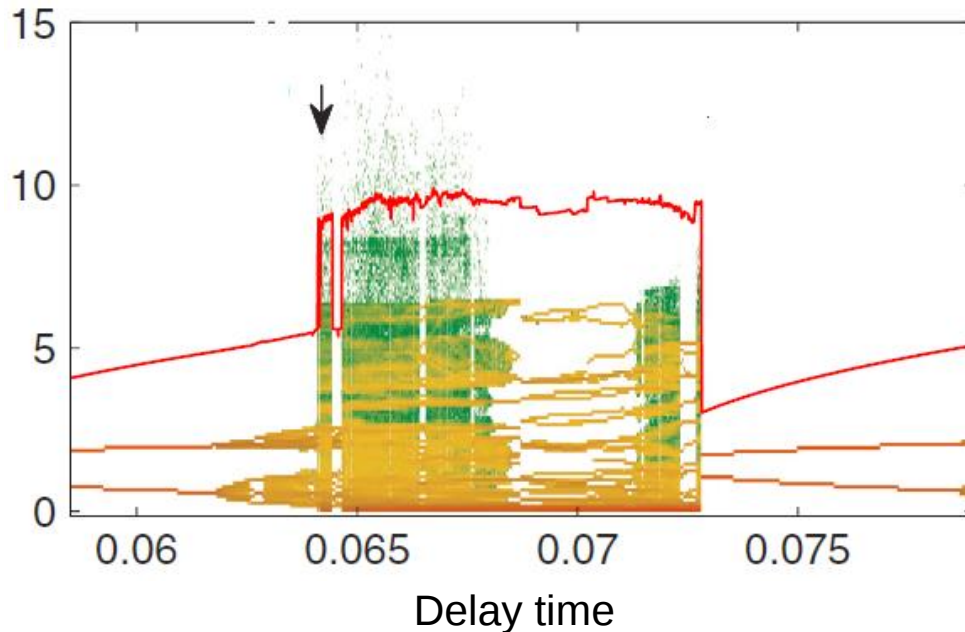
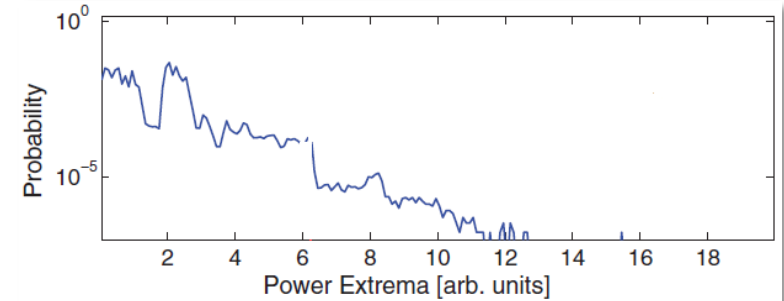
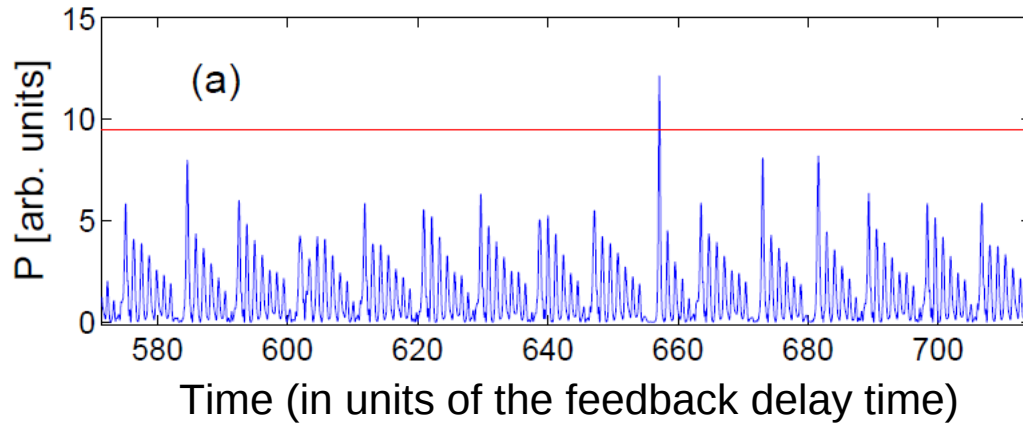
$$\beta_{sp} = 10^{-2}$$

53 RWs

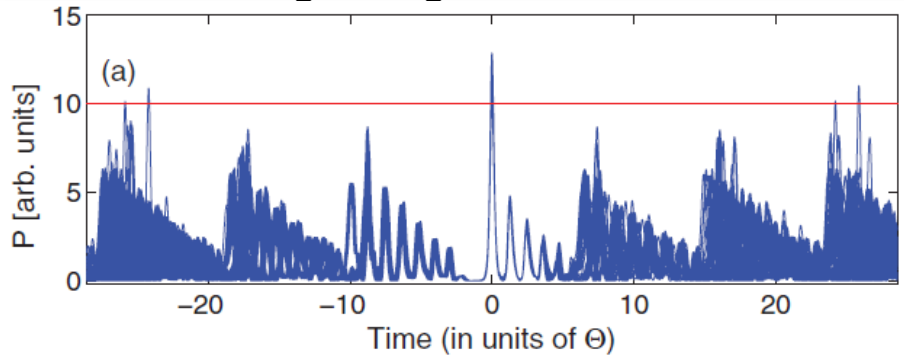


$$\langle A \rangle + 4\sigma$$

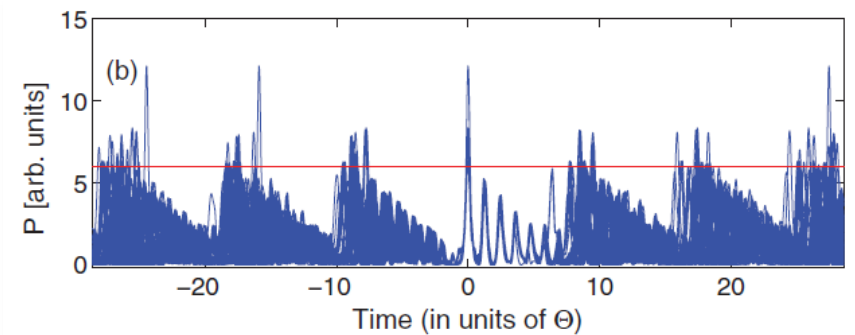
Similar behavior also in a laser with optical feedback



Using a high threshold



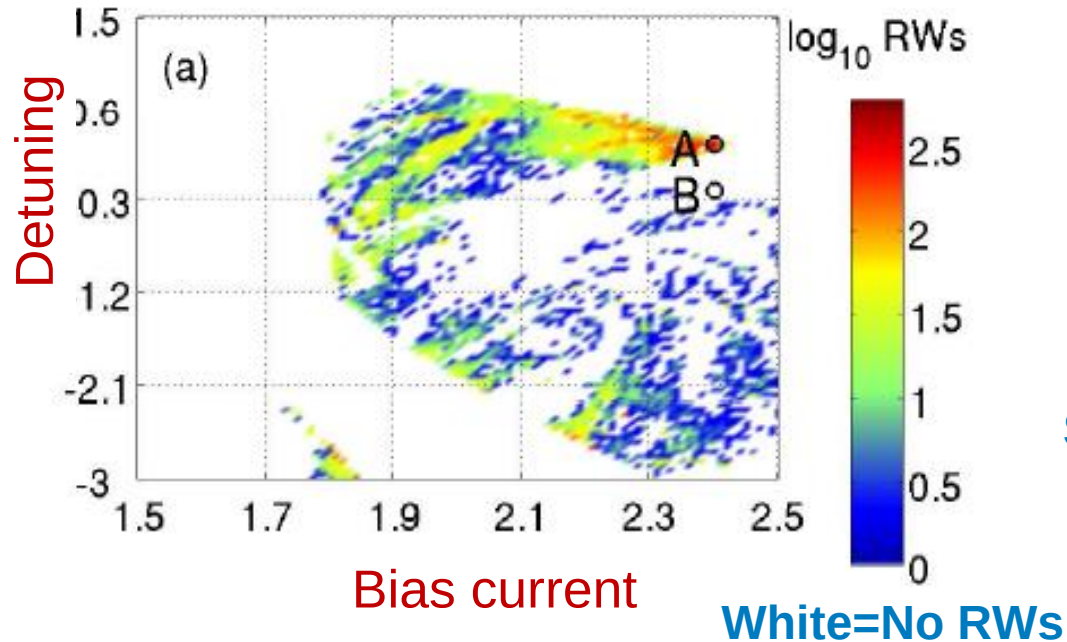
Using a lower threshold



- What triggers a rogue wave?
- Can rogue waves be predicted?
- Can rogue waves be controlled?

Influence of noise

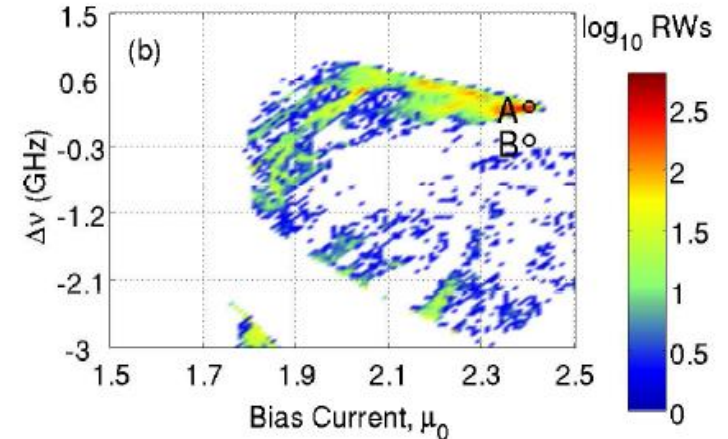
Number of RWs in noiseless simulations



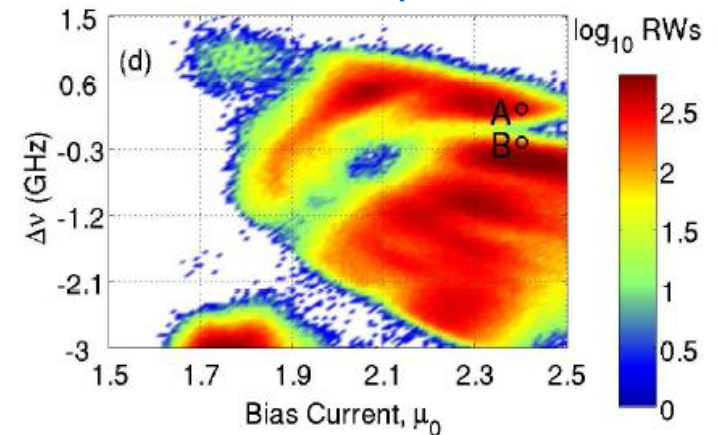
In point A: Deterministic RWs

In point B: no RWs

Weak noise ($\beta_{sn}=0.0001$)



Stronger noise ($\beta_{sp}=0.01$)



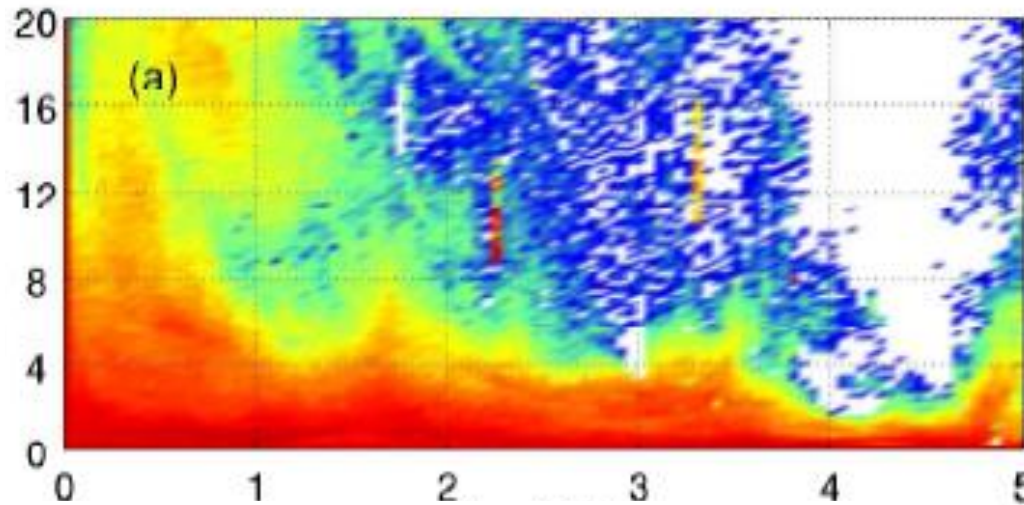
Weak noise reduces the number of RWs,
strong noise increases it

Influence of current modulation when RWs are deterministic (point A)

$$\mu = \mu_0 + \mu_{\text{mod}} \sin(2\pi f_{\text{mod}} t)$$

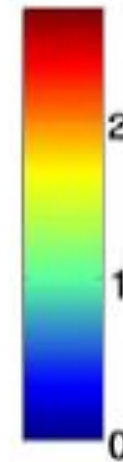
Modulation amplitude (%)

$\beta_{\text{sp}}=0$



Modulation frequency (GHz)

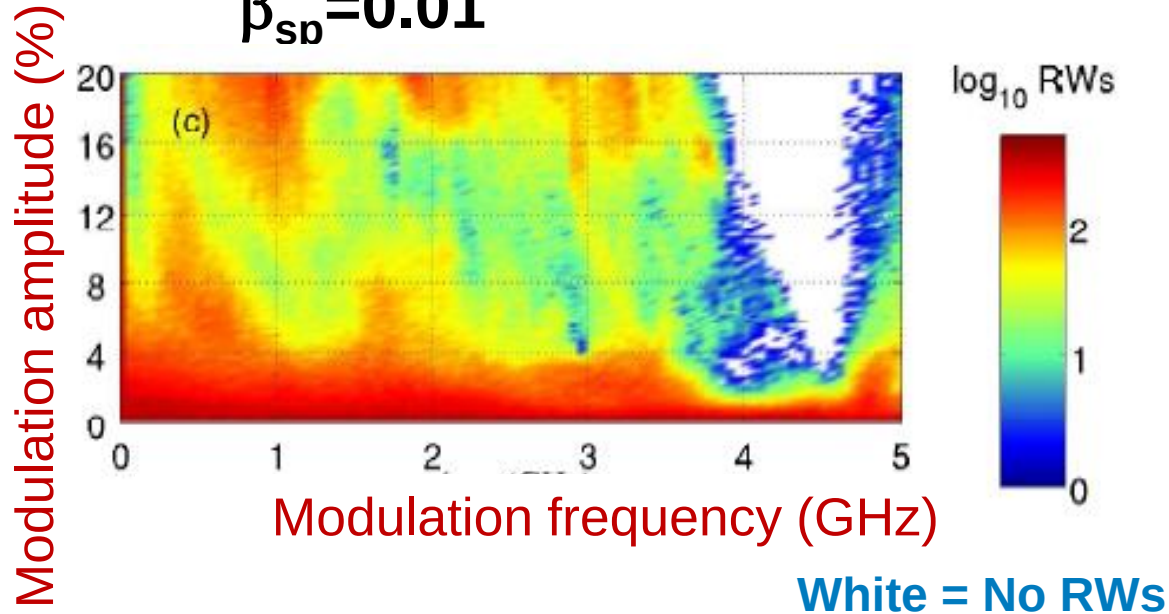
$\log_{10}$ RWs



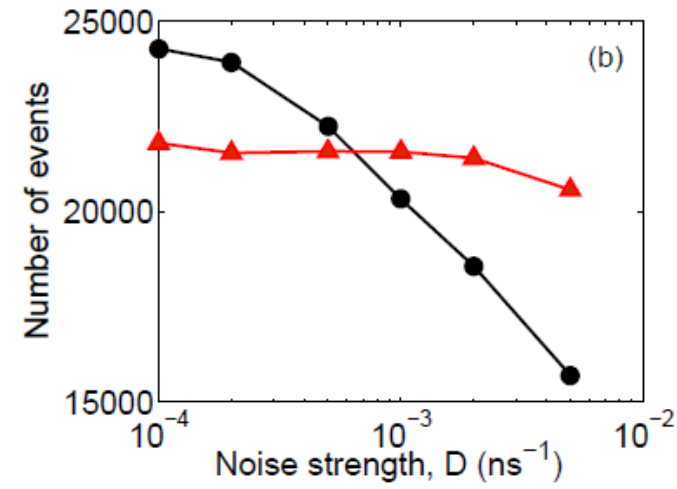
White = No RWs

Current modulation with frequency close to the **natural resonance frequency** fully suppresses RWs: there is a “**safe parameter region**” where no RWs occur.

$$\beta_{sp}=0.01$$



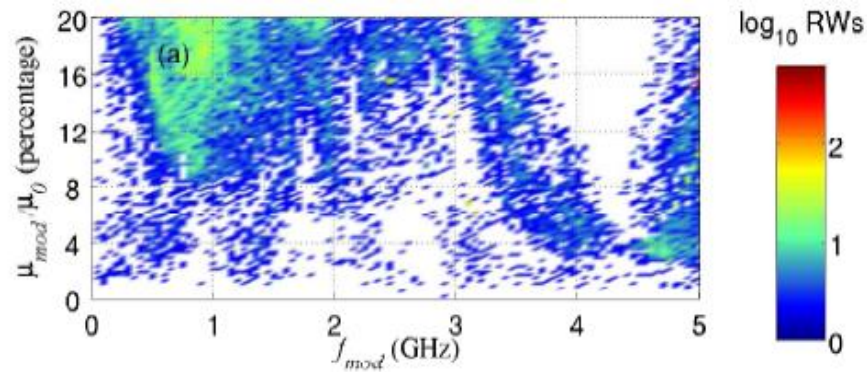
The “safe region” is robust to noise.



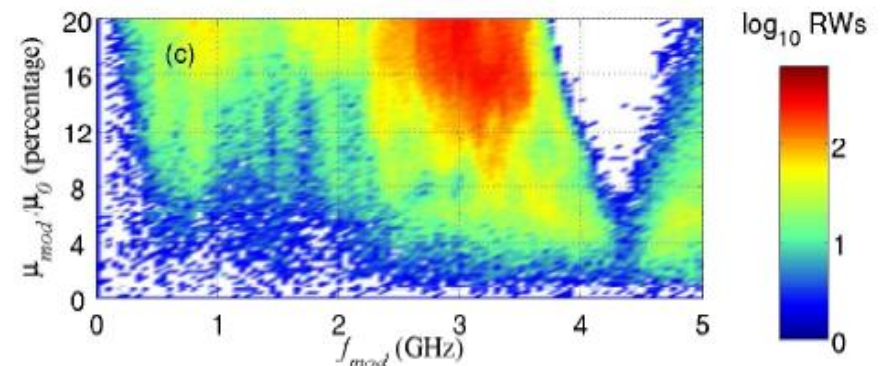
- Without modulation: noise reduces the number of RWs
- With modulation: noise does not affect the RW number

Influence of current modulation when there are no deterministic RWs (Point B)

$$\beta_{sp}=0$$



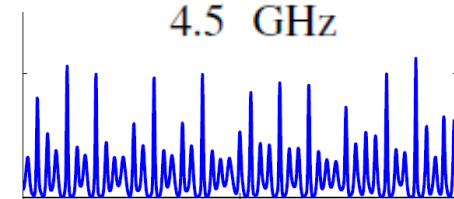
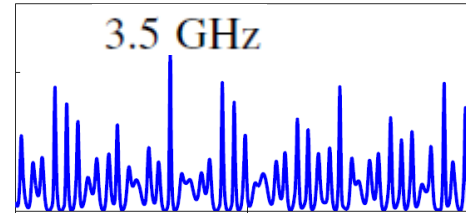
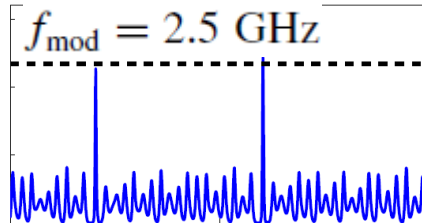
$$\beta_{sp}=0.01$$



White = No RWs

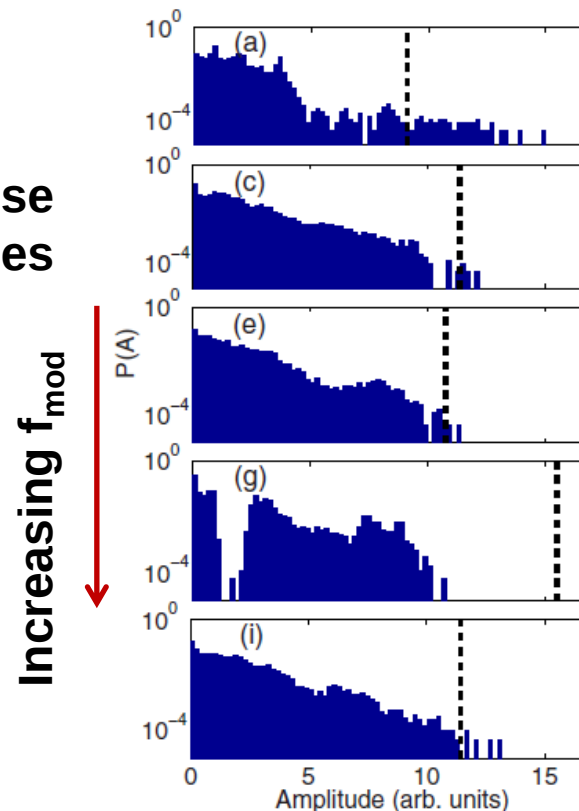
Current modulation can induce RWs but the “safe region” remains and is robust to noise.

Why RWs are suppressed?



High pulses become regular, therefore, they are not RWs.

Distribution of pulse
amplitudes



Threshold = $\langle A \rangle + 6 \sigma$

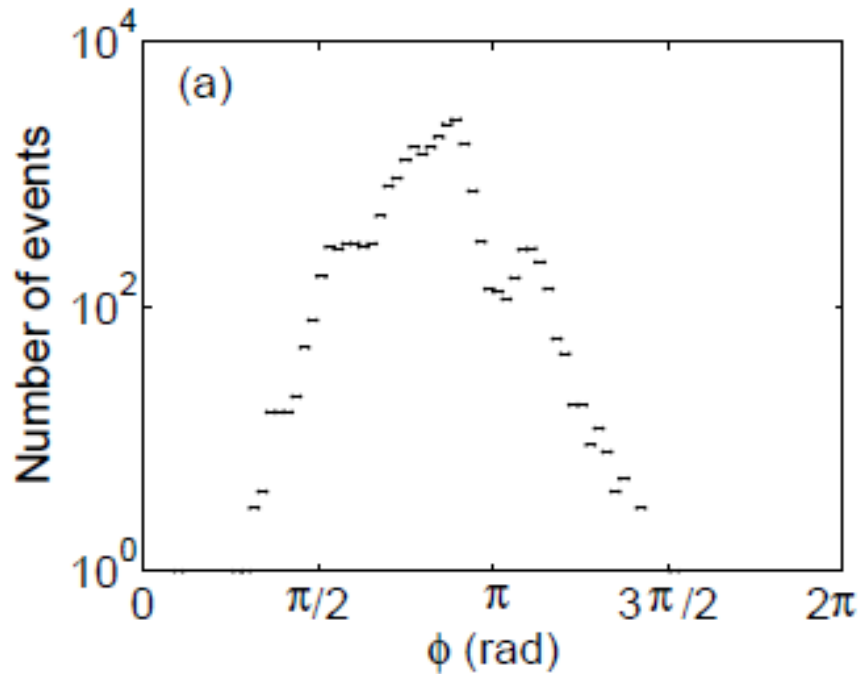
← RW threshold
increases.

Analogy: avalanche risk

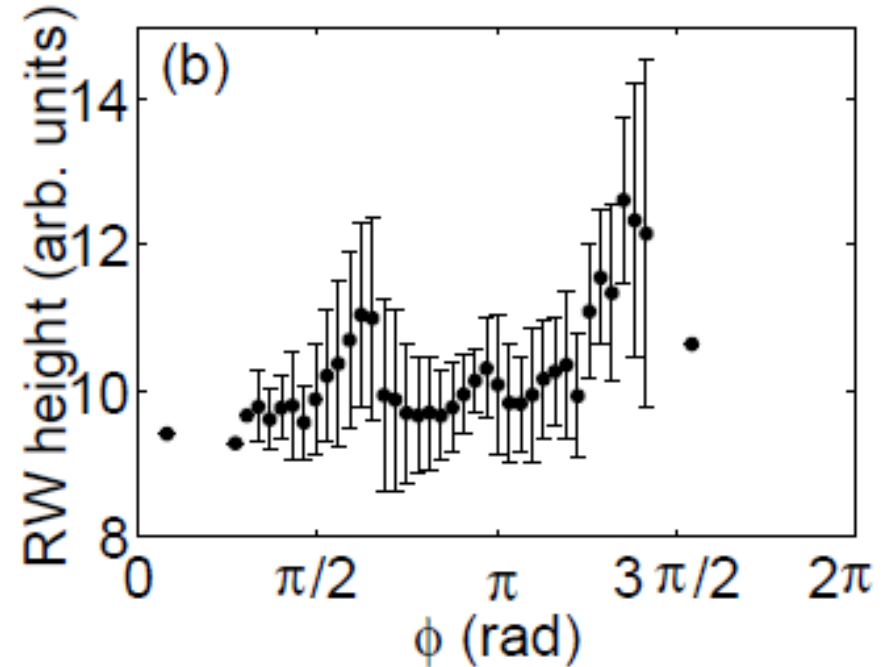
- Modulation suppressed extreme pulses:
We can think of controlled avalanches in a snow-covered mountain that reduce the accumulated snow that could feed a large and dangerous avalanche.



Outside the “safe parameter region”: role of the **modulation phase**



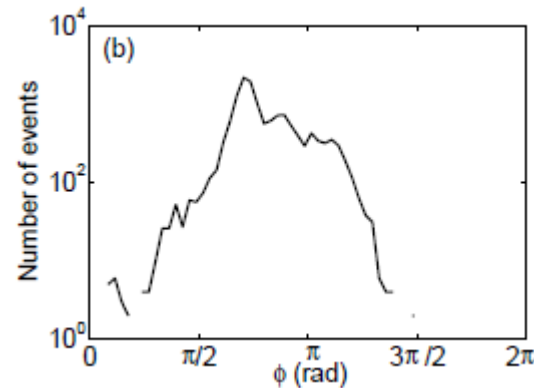
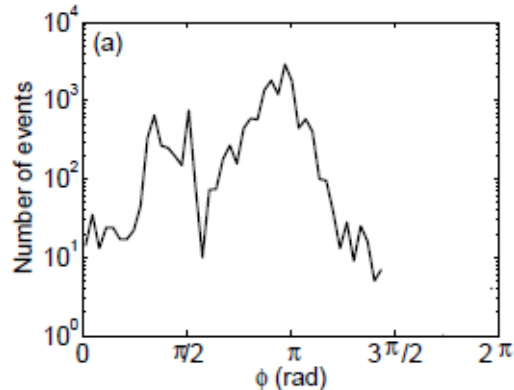
“**safe phase window**”
where no RWs occur.



highest (most extreme) RWs
occur before “safe phase
window”.

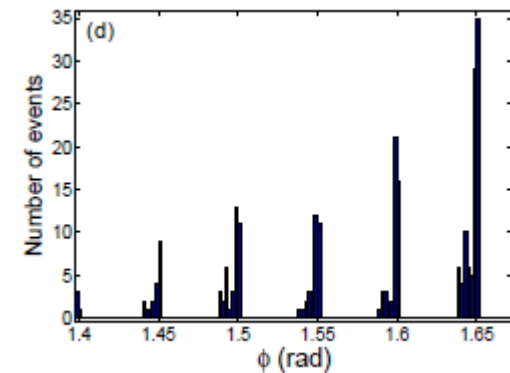
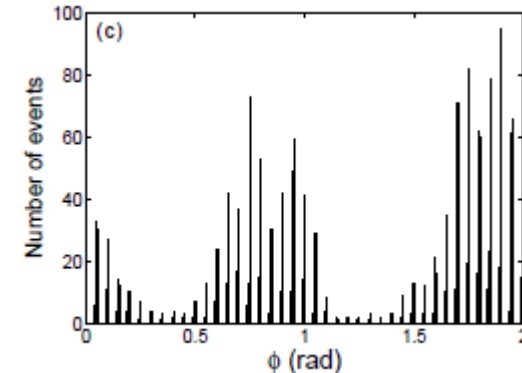
Influence of the modulation frequency

Slow modulation



“safe” phase window remains

Faster modulation



No “safe” phase window but RWs likely occur at specific values of the modulation phase

What we have learned

- In our system RWs can be **deterministic**, generated by a crisis-like process.
- RWs can be **predicted** with a certain anticipation time.
- RWs can be **controlled** by noise and/or modulation.
 - Within a “safe parameter region”: modulation fully suppresses RWs.
 - Outside this safe region: RWs occur in an specific window (or for specific values) of the modulation phase.

THANK YOU FOR YOUR ATTENTION !

<crisrina.masoller@upc.edu>

papers available at:

<http://www.fisica.edu.uy/~cris/>

- C. Bonatto et al, *Deterministic optical rogue waves*, PRL 107, 053901 (2011).
- J. Zamora-Munt et al, *Rogue waves in optically injected lasers: origin, predictability and suppression*, PRA 87, 035802 (2013).
- S. Perrone et al, *Controlling the likelihood of RWs in an optically injected semiconductor laser via direct current modulation*, PRA 89, 033804 (2014).
- J. Ahuja et al, *Rogue waves in injected semiconductor lasers with current modulation: role of the modulation phase*, Optics Express 22, 28377 (2014).

C. Masoller