Symbolic stochastic resonance in excitable systems

Cristina Masoller
Departament de Física
Universitat Politècnica de Catalunya
Campus Terrassa

www.fisica.edu.uy/~cris

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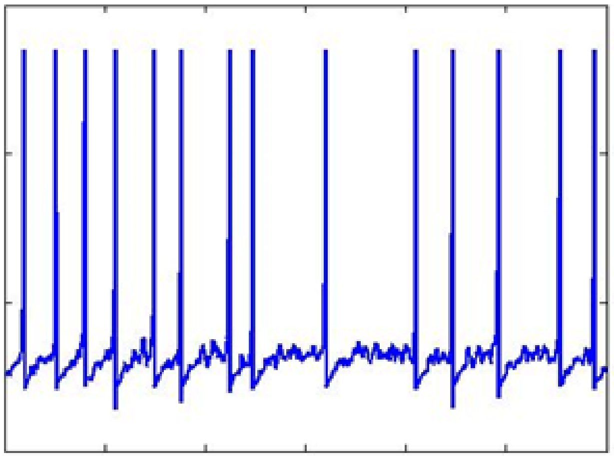
XXVI Sitges Conference, May 2019

Motivation: **How excitable systems respond to weak external forcing in noisy environments?**

Laser intensity

Time 10^{-6} s

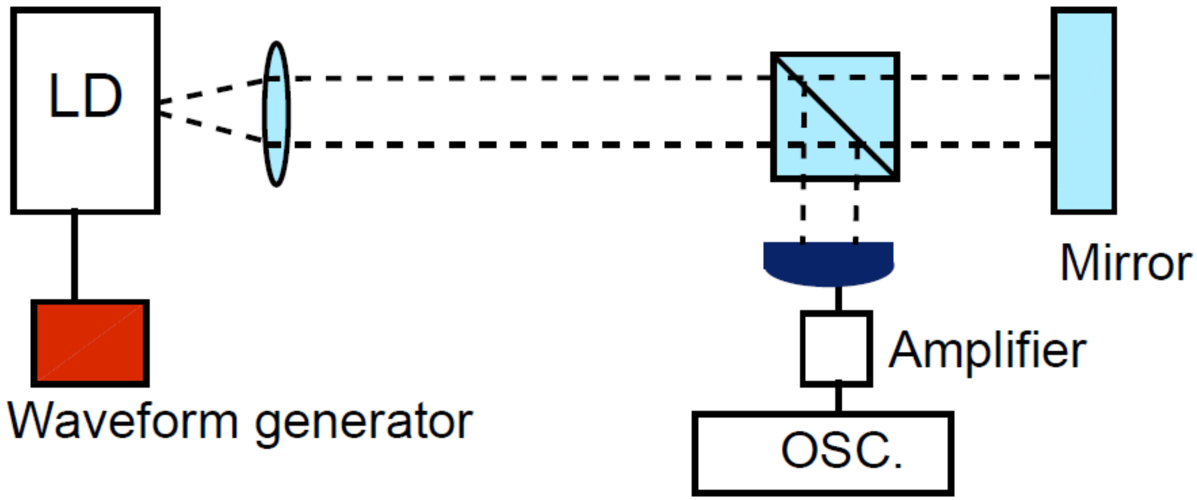
Neuron model

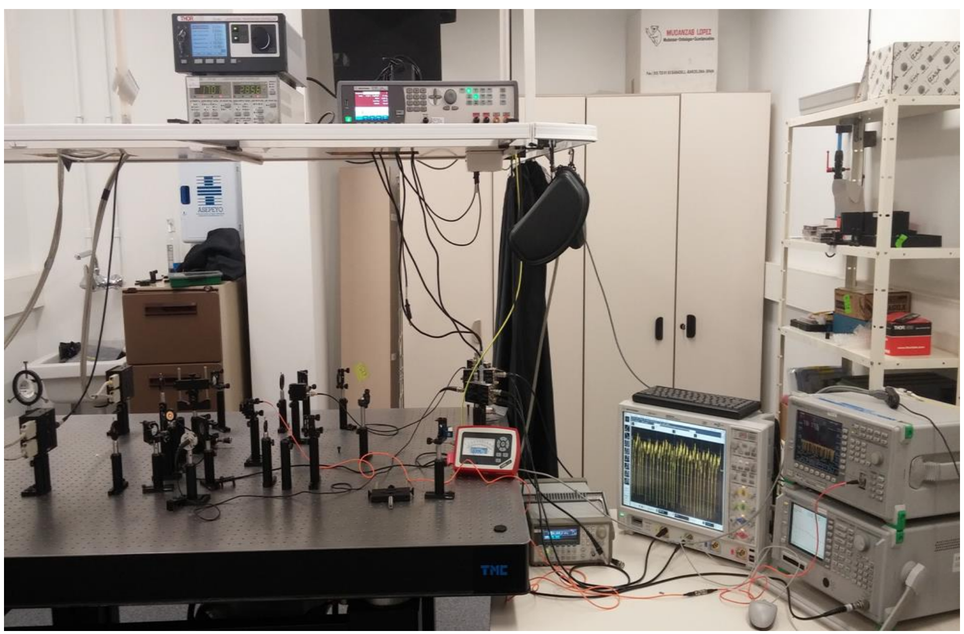


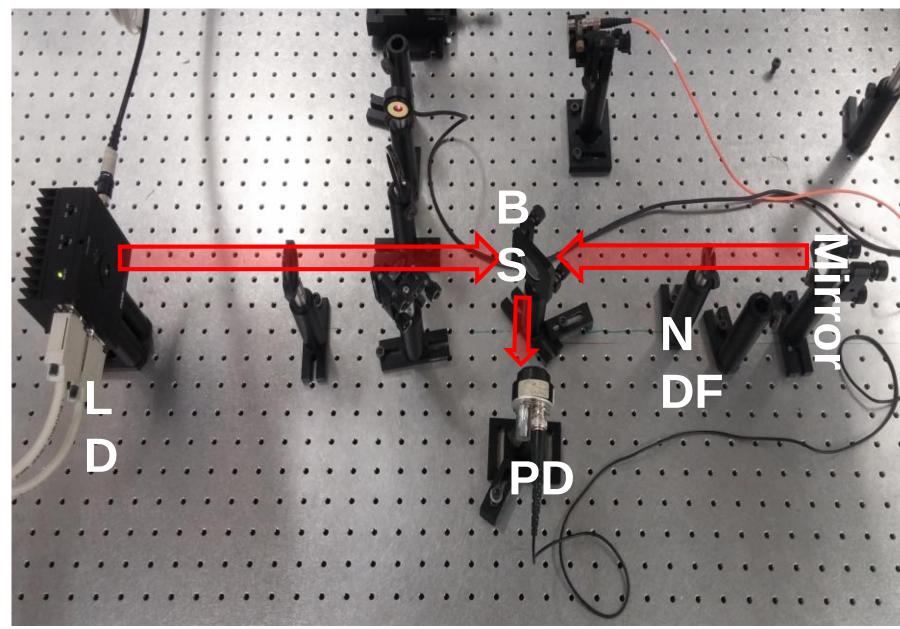
Time 10^{-3} s

Can lasers mimic real neurons?

How can a neuron (or a laser) “encode”, in a sequence of spikes, the information of a weak signal, in the presence of noise?


LD
Mirror
Amplifier
Waveform generator
OSC.




B
S
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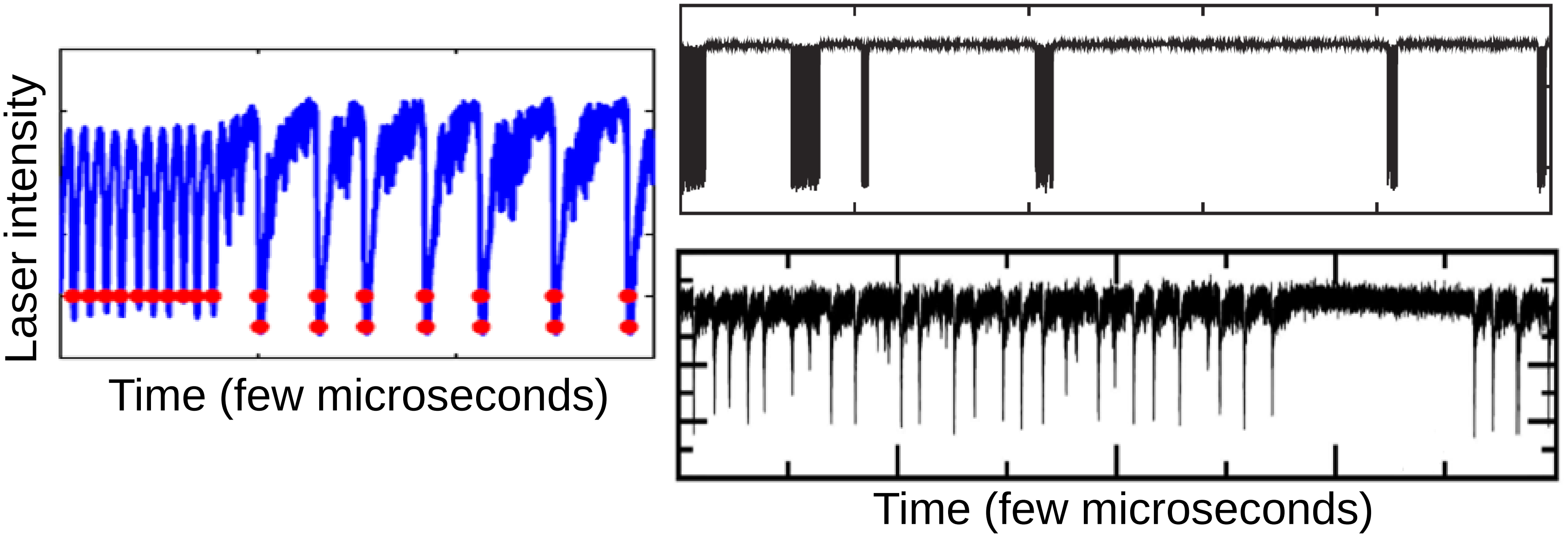
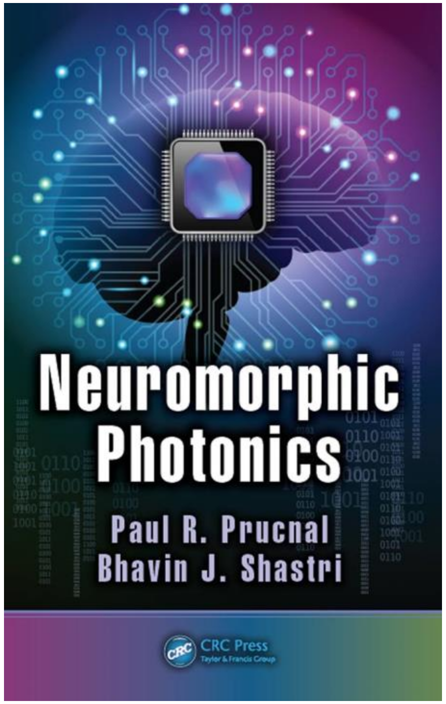
Mirror

The dynamics of a laser with feedback: excitability, tonic spikes and bursting. Similar to real neurons?



A. Aragoneses et al., Sci. Rep. ***4****, 4696 (2014).*
C. Quintero-Quiroz et al., Sci. Rep. ***6*** *37510 (2016).*

Uncovering similarities can be interesting. But useful? Maybe…



- Excitable lasers: building blocks of **photonic neurons**.
- Diode lasers: very very low cost, highly energy efficient.
- Very very fast.
- Main challenge: understand how lasers and neurons encode a **weak periodic** signal in the presence of **noise**.

Inter-spike-interval (ISI) distribution with sinusoidal forcing

Neuron empirical data

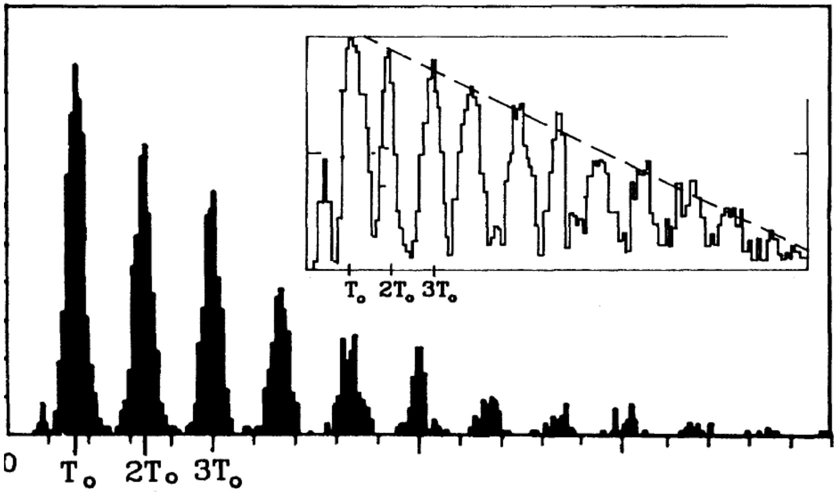
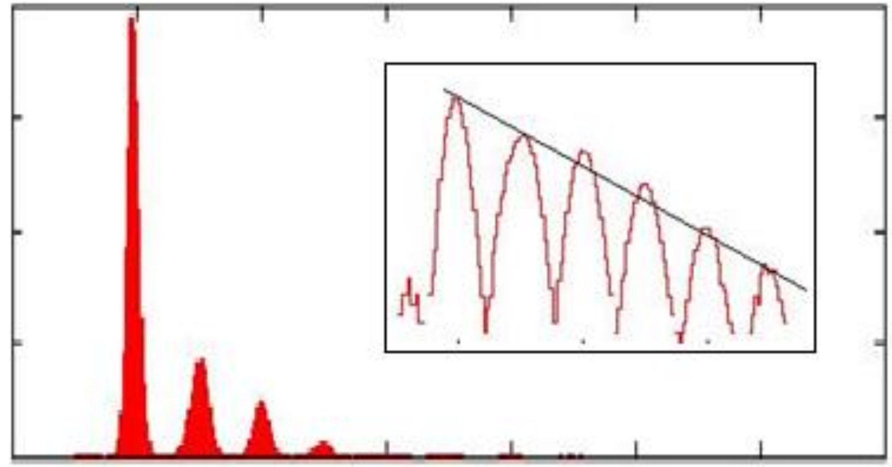


FIG. 1. (a) An experimental ISIH obtained from a single auditory nerve fiber of a squirrel monkey with a sinusoidal 80-dB sound-pressure-level stimulus of period $T_0 = 1.66$ ms applied at the ear. Note the modes at integer multiples of T_0. Inset:

A. Longtin et al. PRL (1991)

Laser empirical data



$2T_0$ $4T_0$

Experimental data when the laser current is modulated with a sinusoidal signal of period T_0.

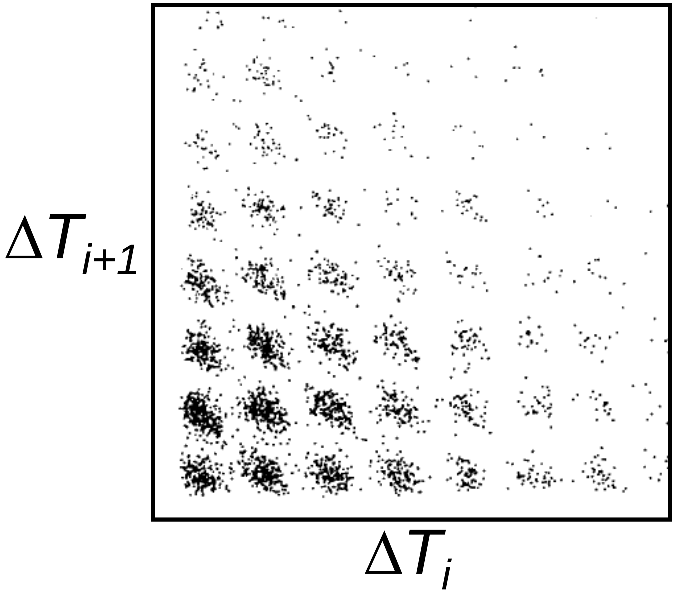
Aragoneses et al. Opt. Express (2014)

Stochastic resonance: noise-enhanced regularity.

Gammaitoni, Hänggi et al, Rev. Mod. Phys. 70, 223 (1998)

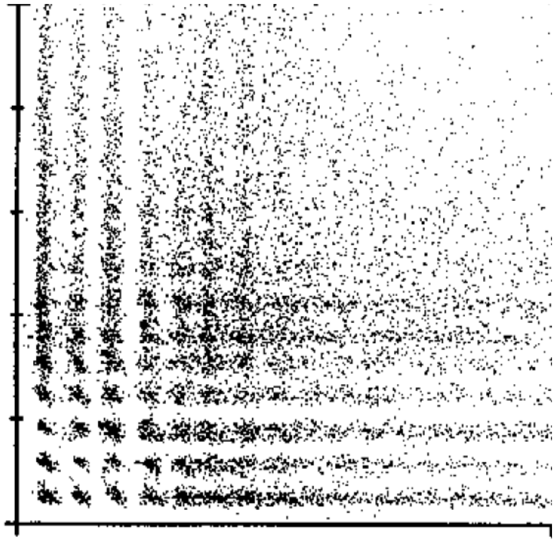
Similar spike timing? Return maps of inter-spike-intervals

Neuronal ISIs



A. Longtin, IJBC 1993

Laser ISIs



M. Giudici et al PRE 1997
A. Aragoneses et al., Opt. Exp. 2014

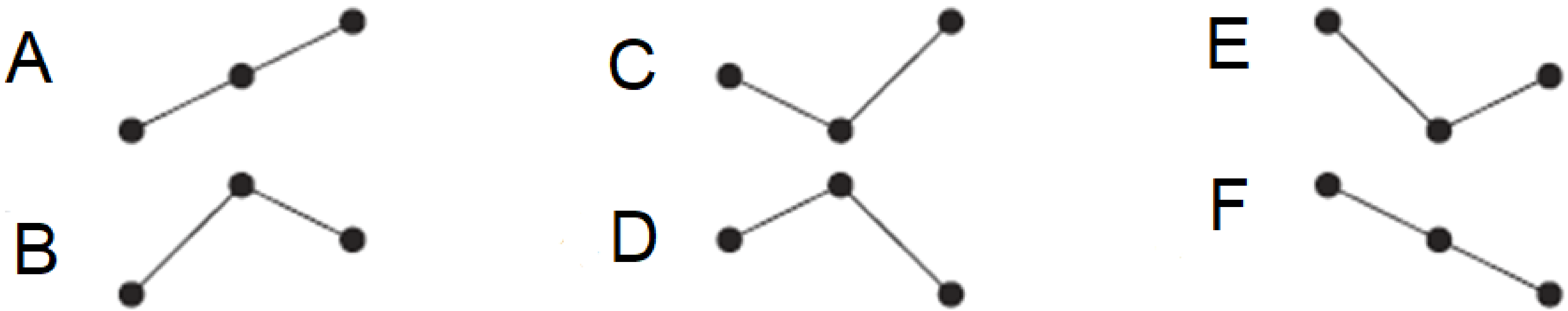
HOW TO INDENTIFY SIMILAR TEMPORAL PATTERNS?

Symbolic analysis of the sequence of time intervals: ordinal analysis

$$\{\ldots x_i,\ x_{i+1},\ x_{i+2},\ \ldots\}$$

Possible order relations among three numbers (e.g., 2, 5, 7)

{…2, 5, 7…}   {…5, 2, 7…}   {…7, 2, 5 …}

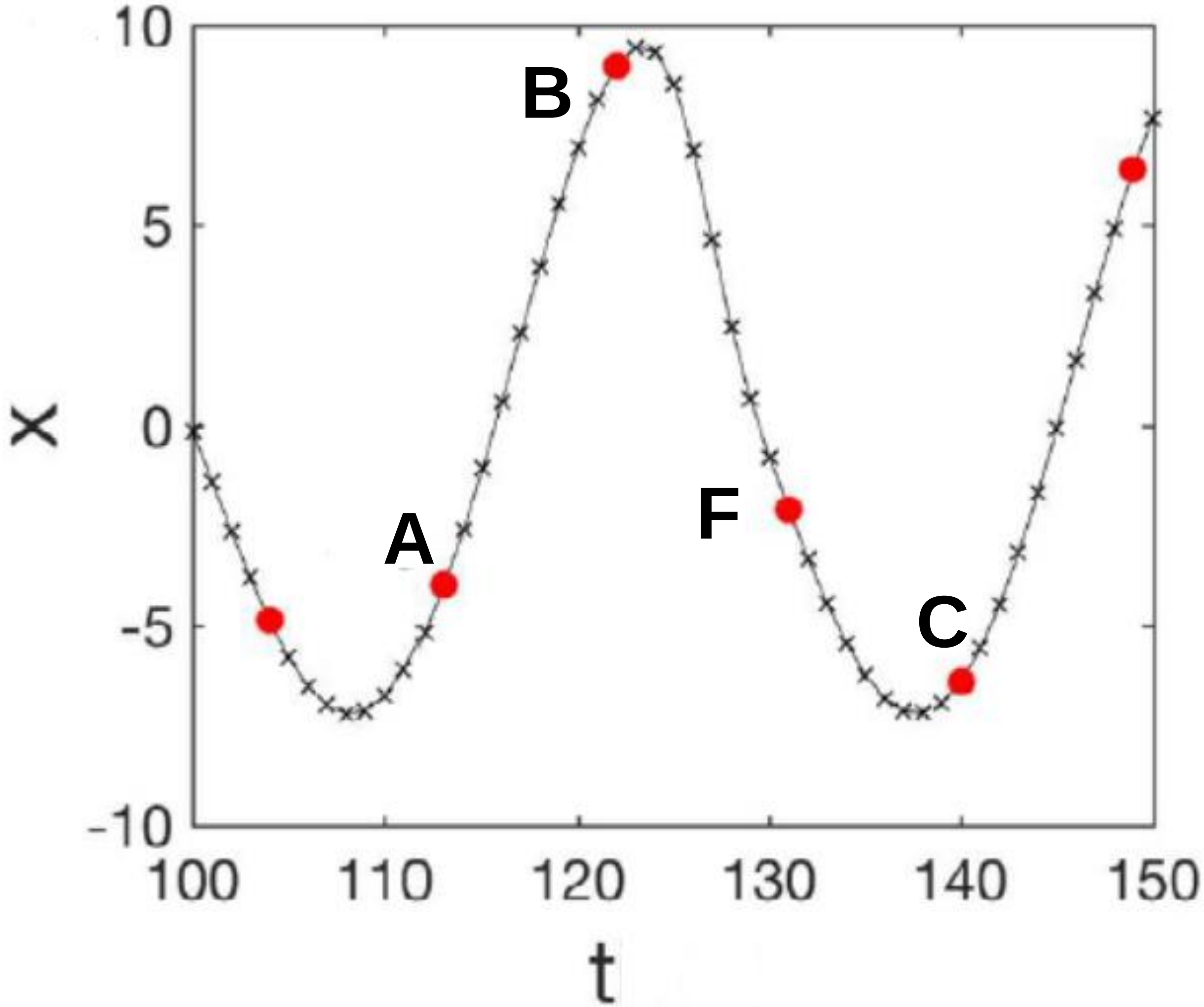


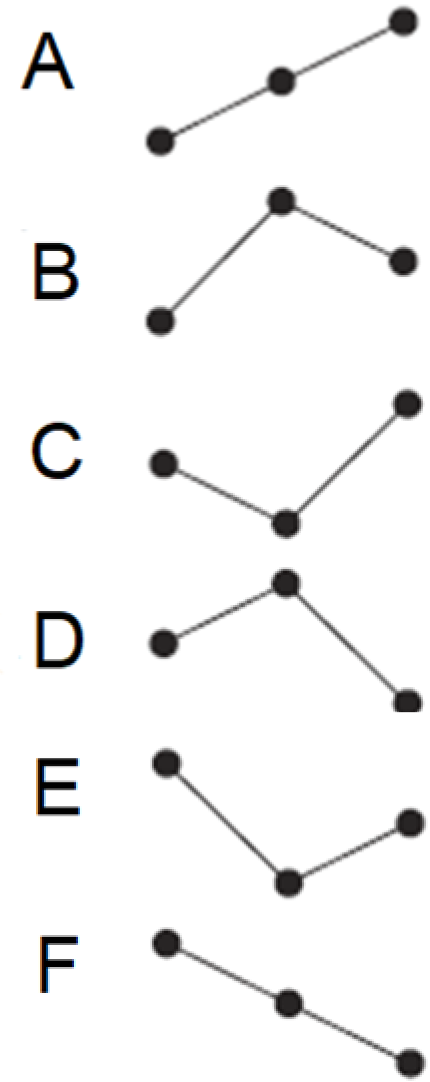
{…2, 7, 5…}   {…5, 7, 2…}   {…7, 5, 2…}

Drawback: information about the actual values is lost.

Bandt and Pompe, Phys. Rev. Lett. 2002

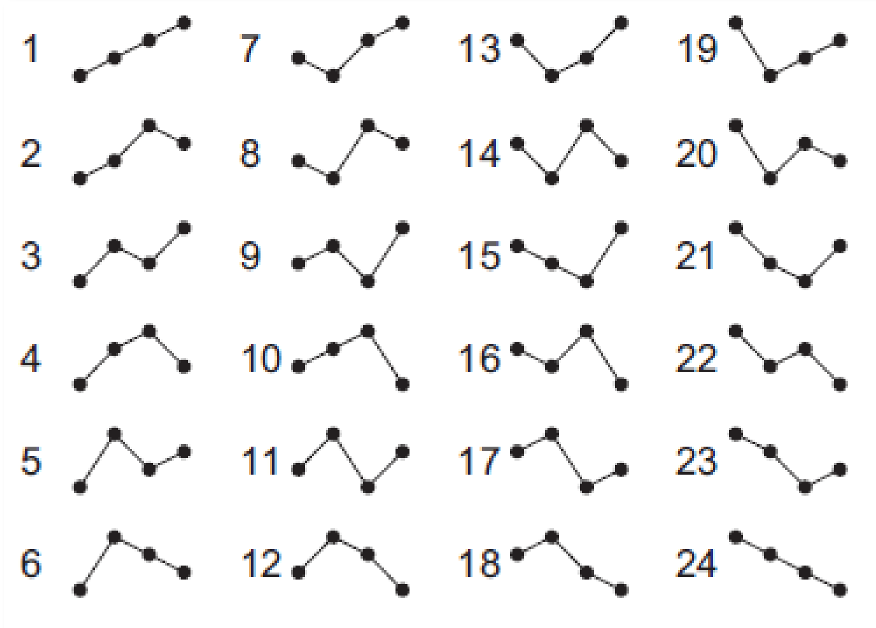
Which is the sequence of “letters” (patterns) defined by the red dots?



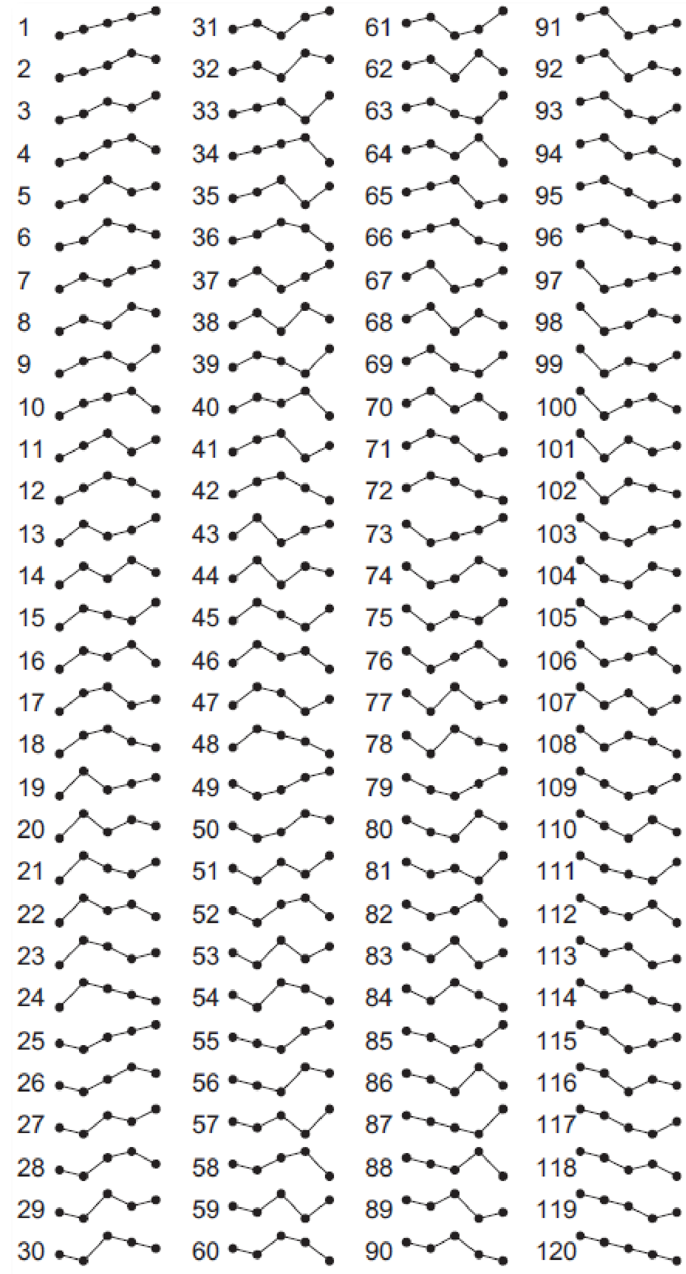


Number of patterns increases as D! (D=size of pattern)

$$\{\ldots x_i, x_{i+1}, x_{i+2}, x_{i+3} \ldots\}$$



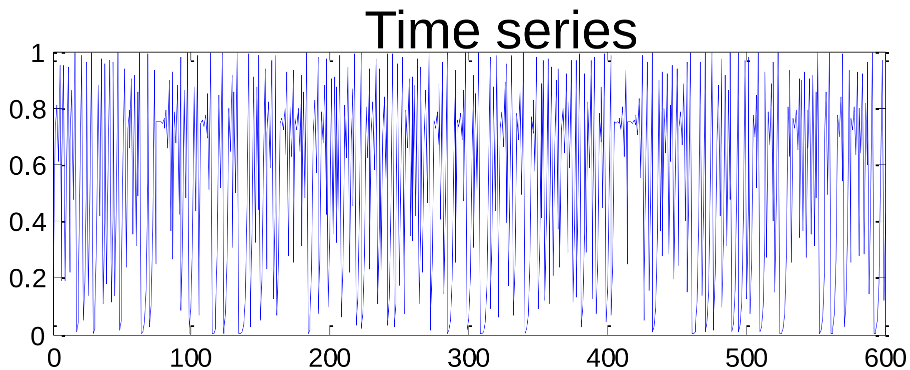
U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

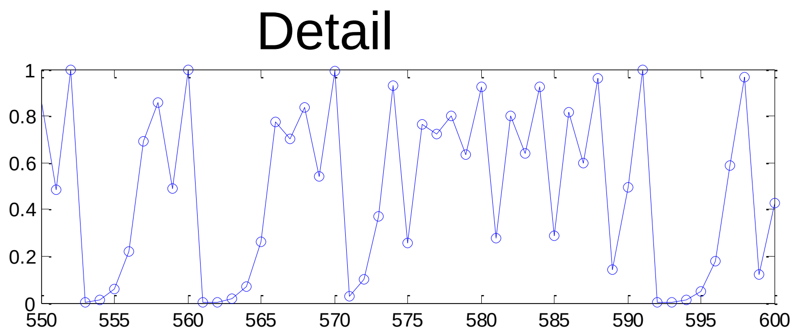


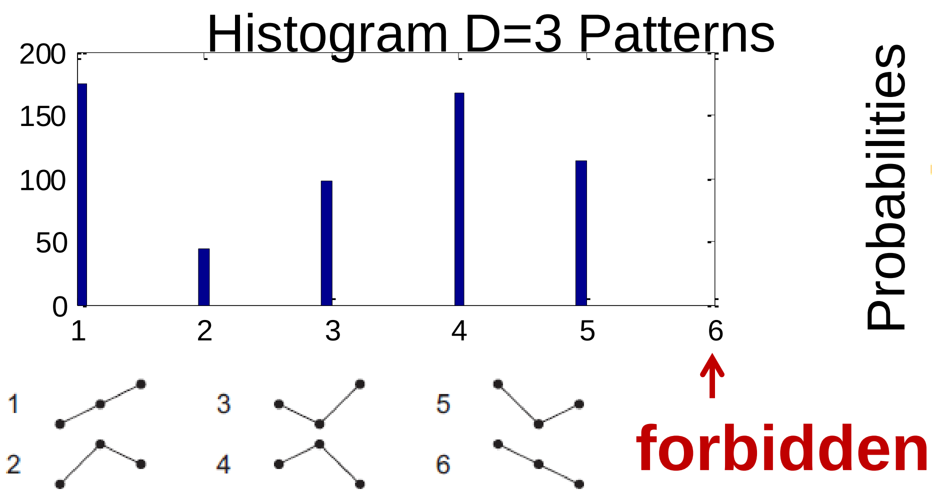
Permutation entropy: $s_p = -\sum p_i \log p_i$

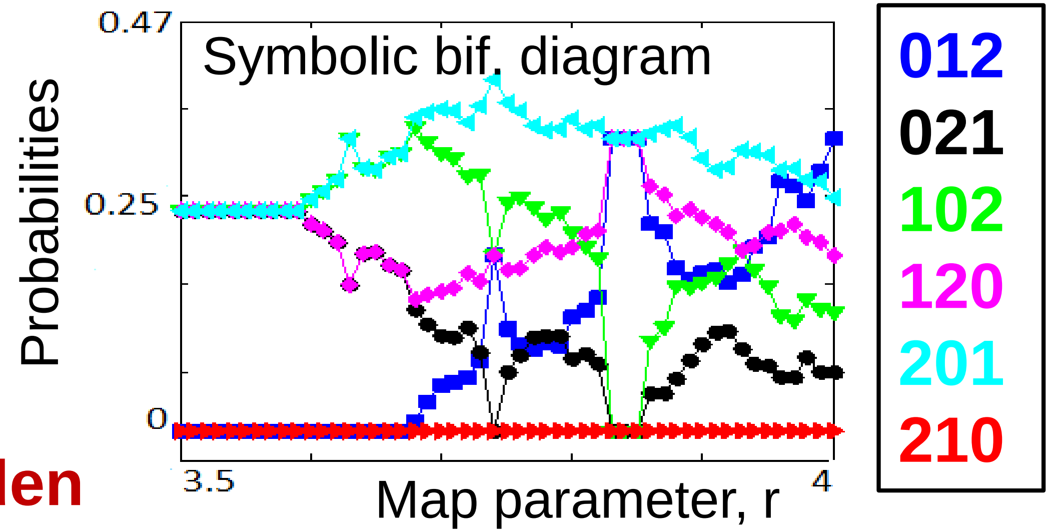
Example: Logistic map, D=3

$$x(i+1) = r\, x(i)[1 - x(i)]$$

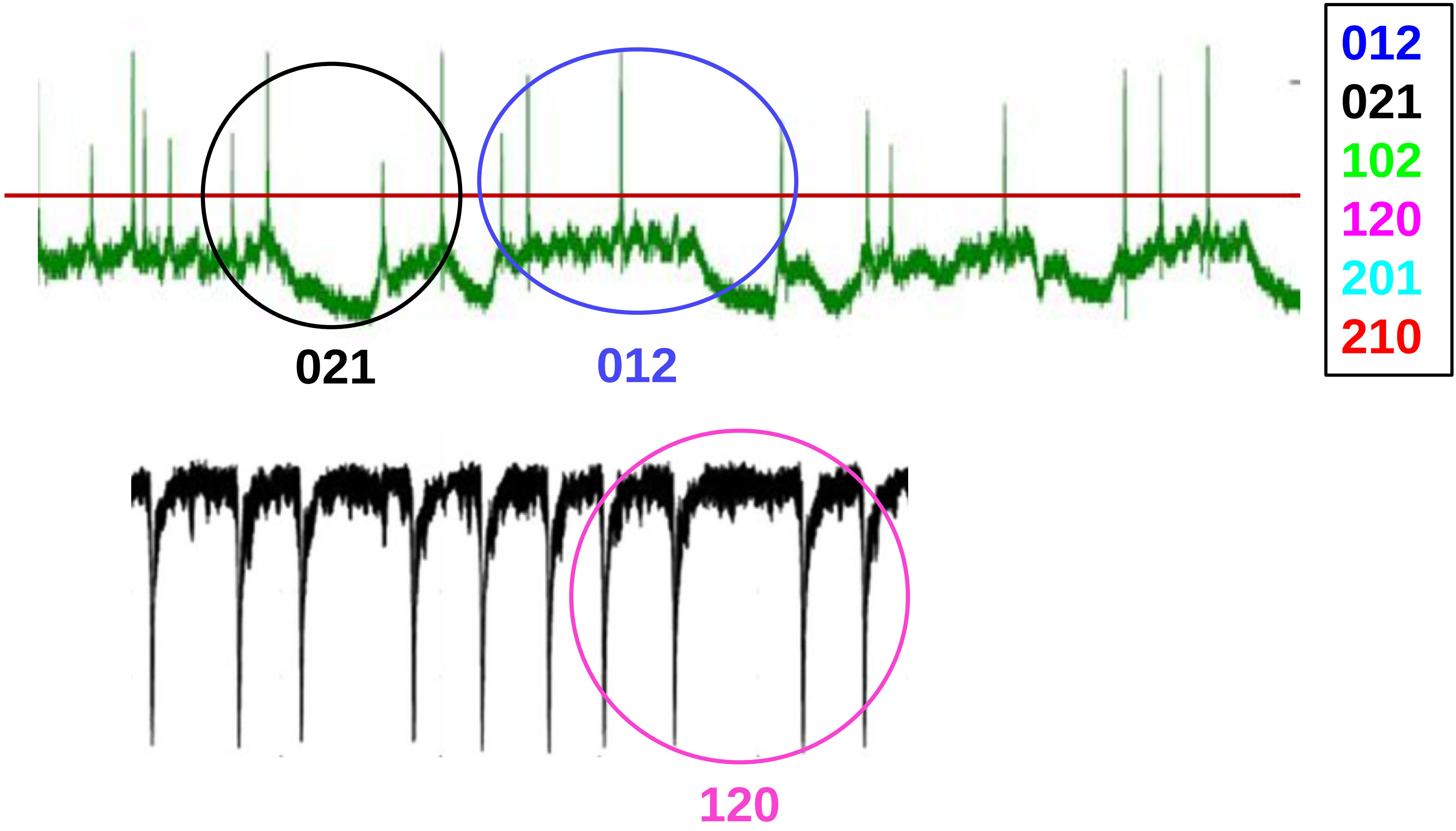






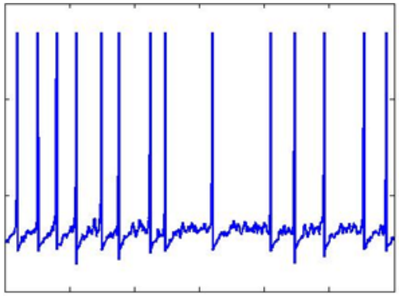


Analysis of D=3 patterns in spike sequences



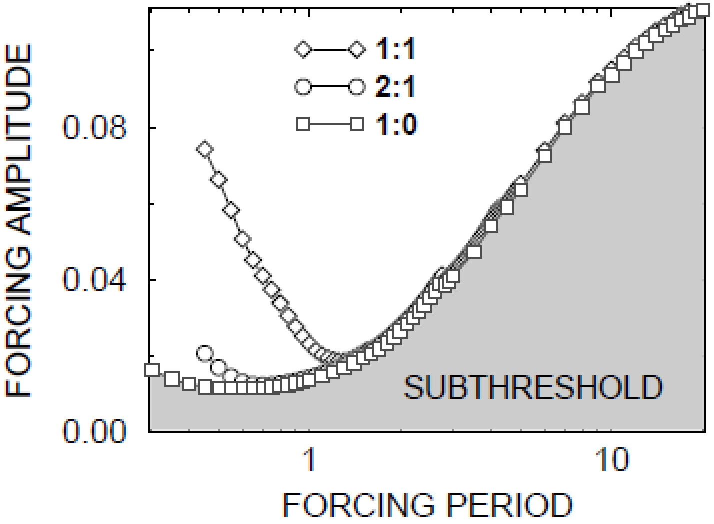
Modeling neuronal spikes with FitzHugh-Nagumo model

$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o \cos(2\pi t/T)} + D\xi(t),$$



- Gaussian white noise and <u>subthreshold</u> signal: **a_0** and **T** such that spikes are **noise-induced**.

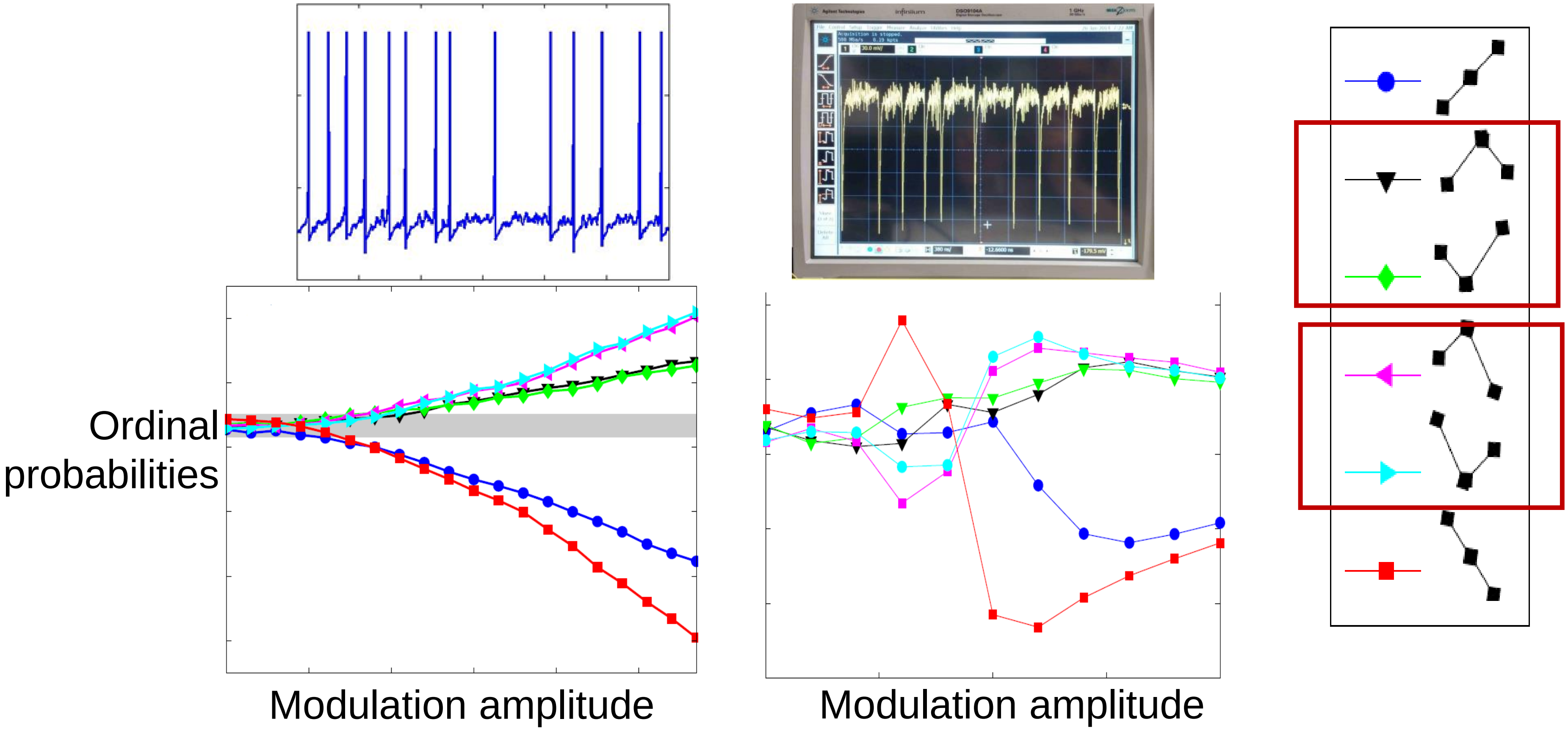
- Time series with M=100,000 spikes simulated (a=1.05, ε=0.01).



Longtin and Chialvo, PRL 1998

By analyzing the ordinal probabilities (from inter-spike intervals, ISIs) we uncover laser-neuron similarities

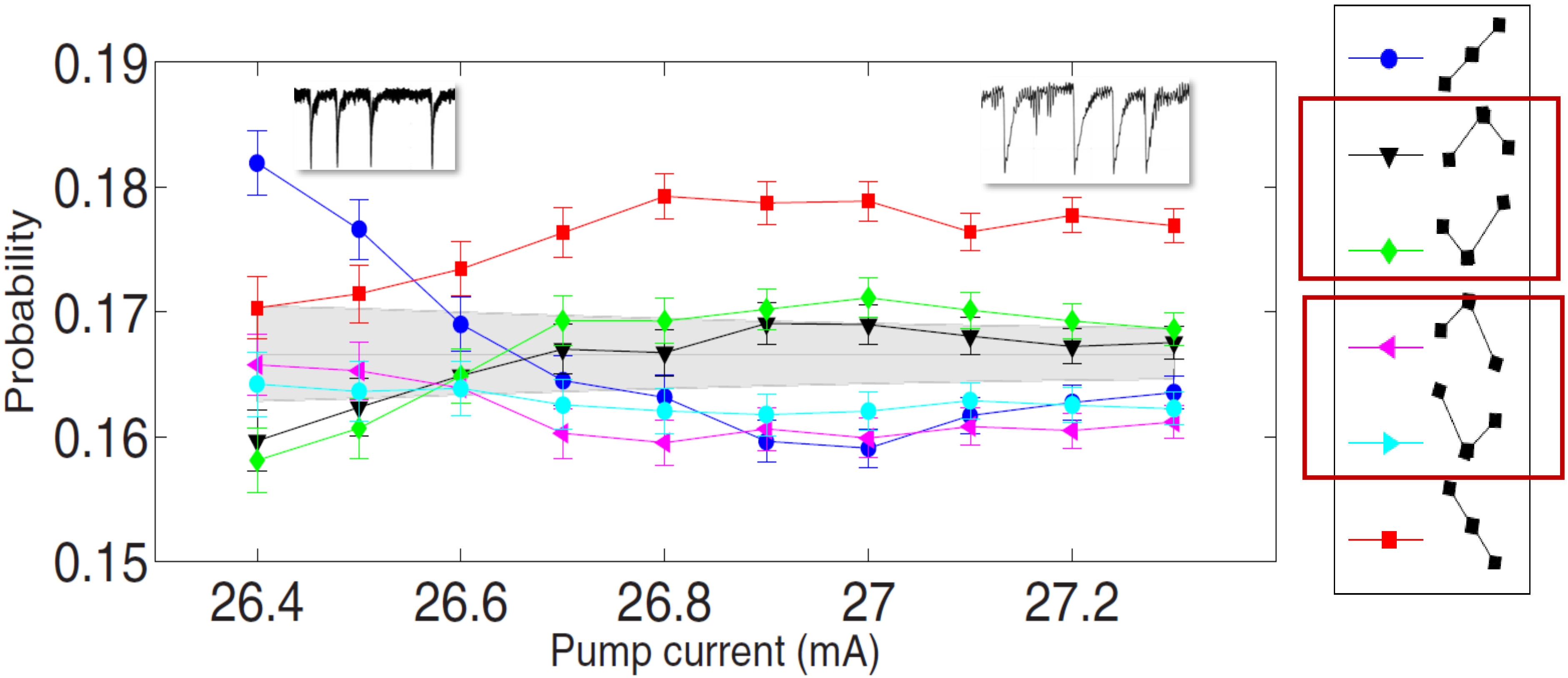
Neuron model | **Laser with feedback**



J. Aparicio-Reinoso et al, PRE 94, 032218 (2016)

A. Aragoneses et al, Sci. Rep. ***4****, 4696 (2014)*

Role of experimental control parameter: the laser current



75,000 – 880,000 spikes

Gray region: probabilities consistent with 1/6

A. Aragoneses et al, Sci. Rep. 4, 4696 (2014)

Minimal model? A modified circle map

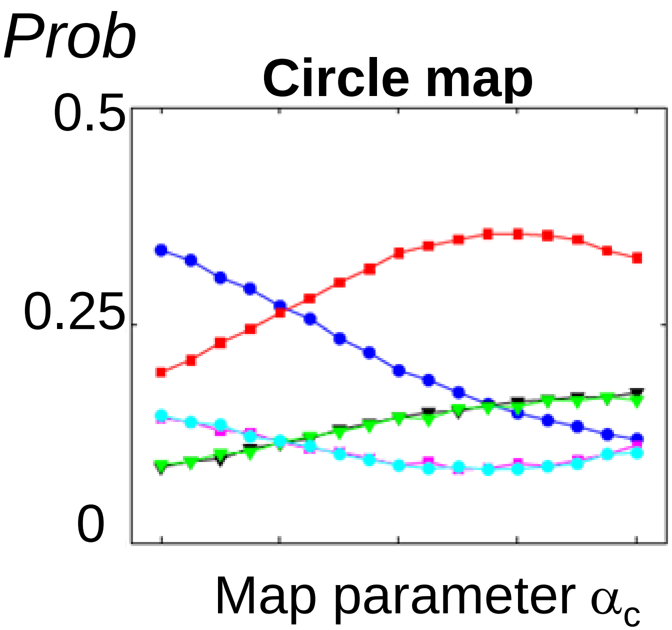
$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha_c \sin(4\pi\varphi_i)\right] + D\zeta$$

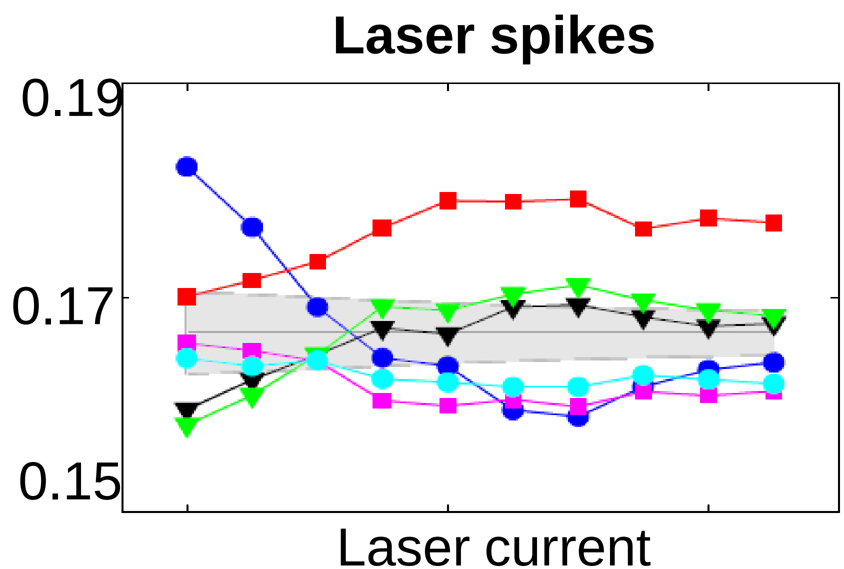
$$X_i = \varphi_{i+1} - \varphi_i$$

ρ = natural frequency / forcing frequency

K = forcing amplitude

D = noise strength

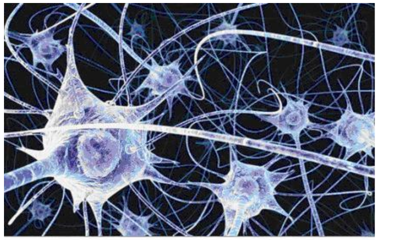




Same “clusters” & same hierarchical structure

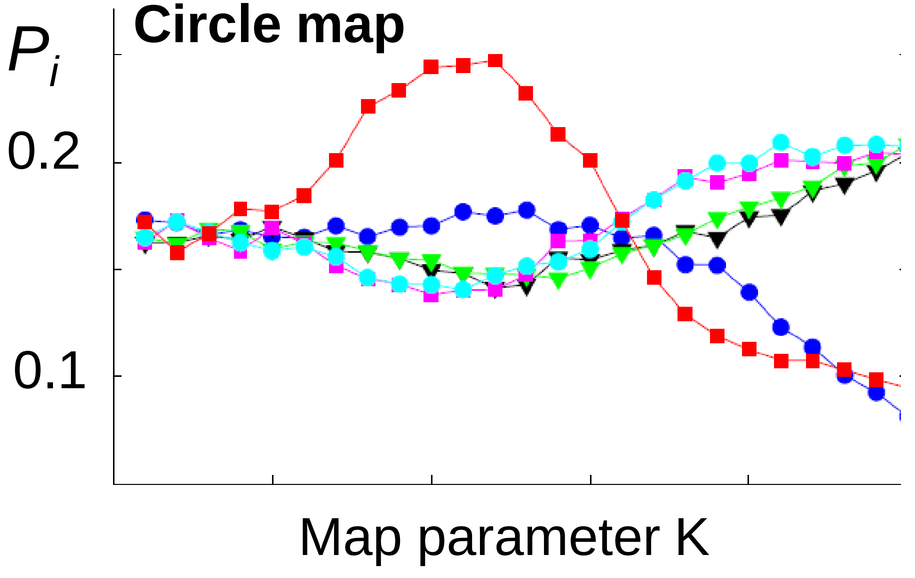
A. Aragoneses et al, Sci. Rep. 4, 4696 (2014)

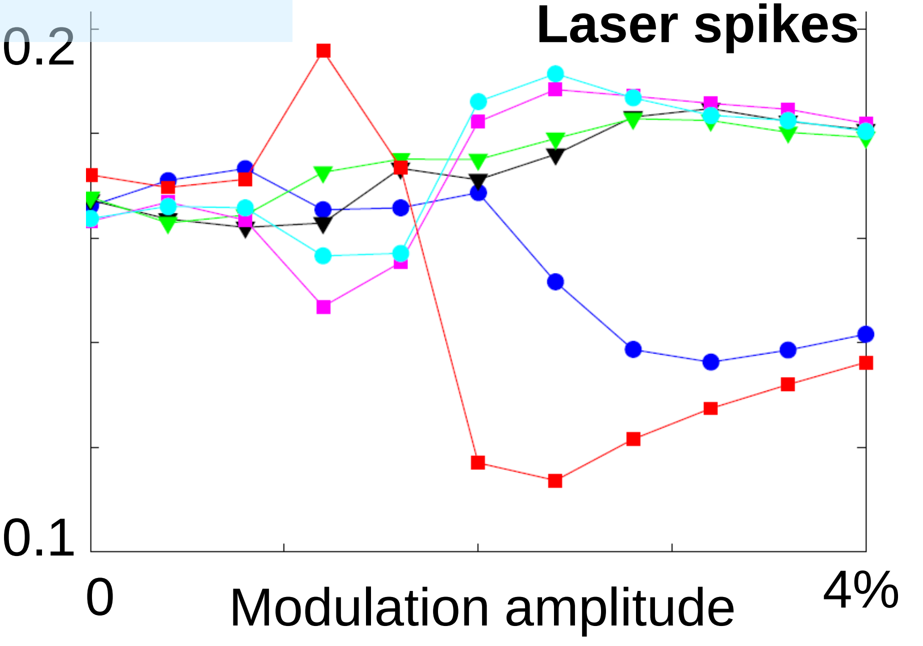
Connection with neurons: the circle map describes many excitable systems



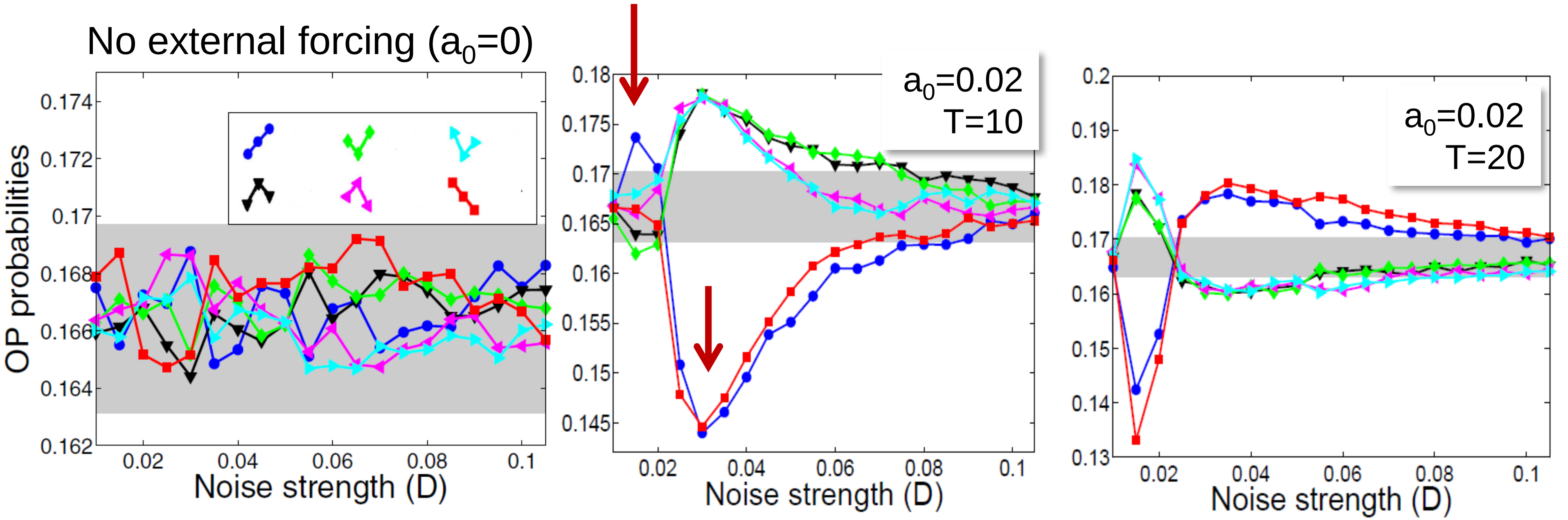
- Spike correlations in sensory neurons (Neiman and Russell, PRE 2005)
- Can we test its validity as a minimal model for the laser spikes?

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha_c \sin(4\pi\varphi_i)\right] + D\zeta$$





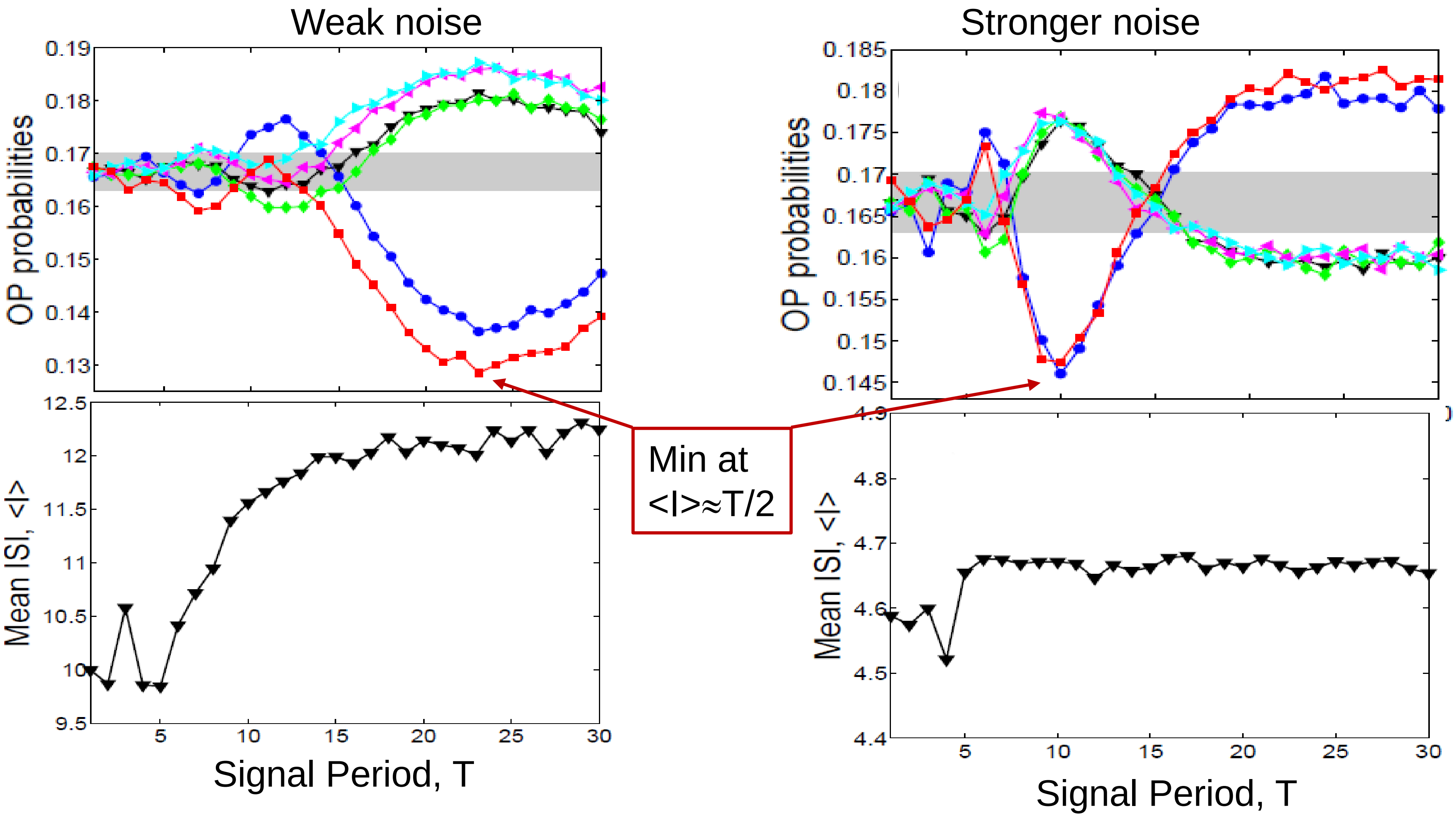
FHN neuron model: symbolic stochastic resonance



- The signal induces preferred and infrequent patterns.
- They depend on the period and on the noise strength.
- Resonant-like behavior.

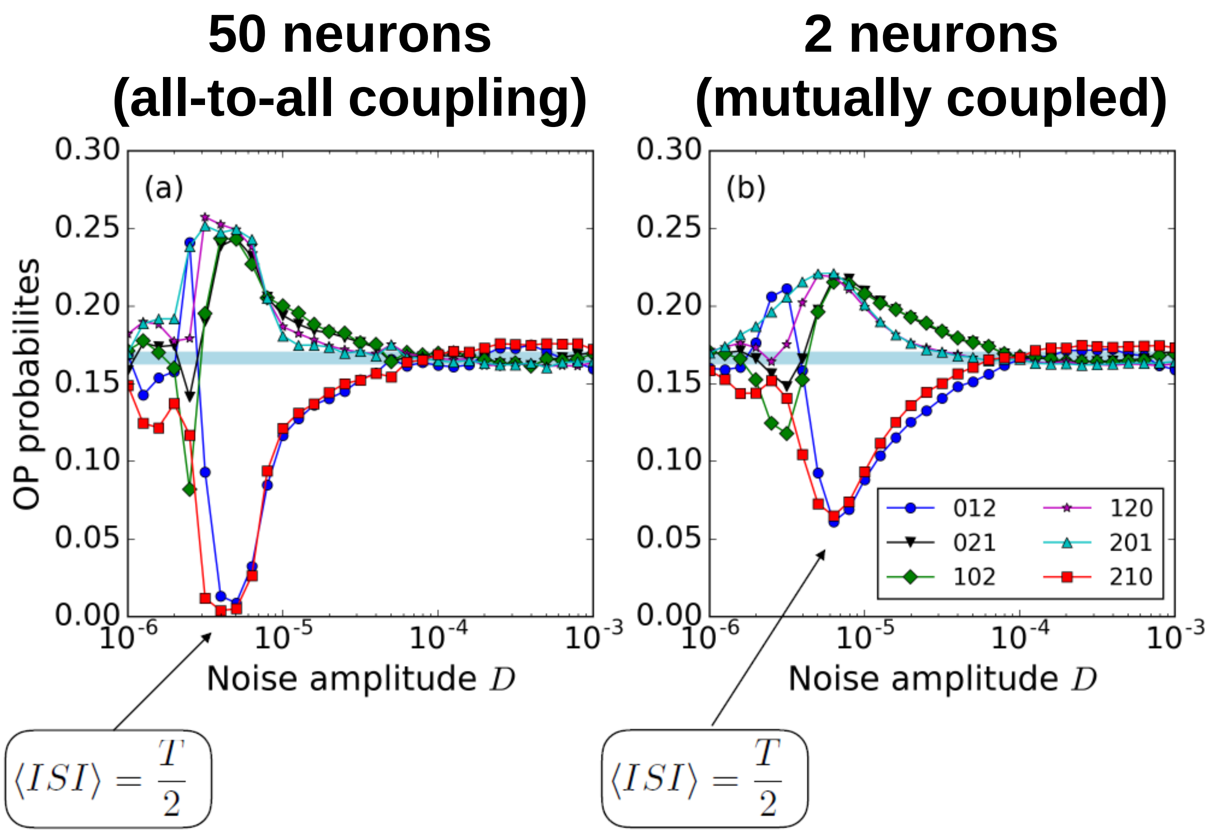
J. M. Aparicio-Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016)

Role of the signal period



J. M. Aparicio-Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016)

The resonance is enhanced by neuronal coupling



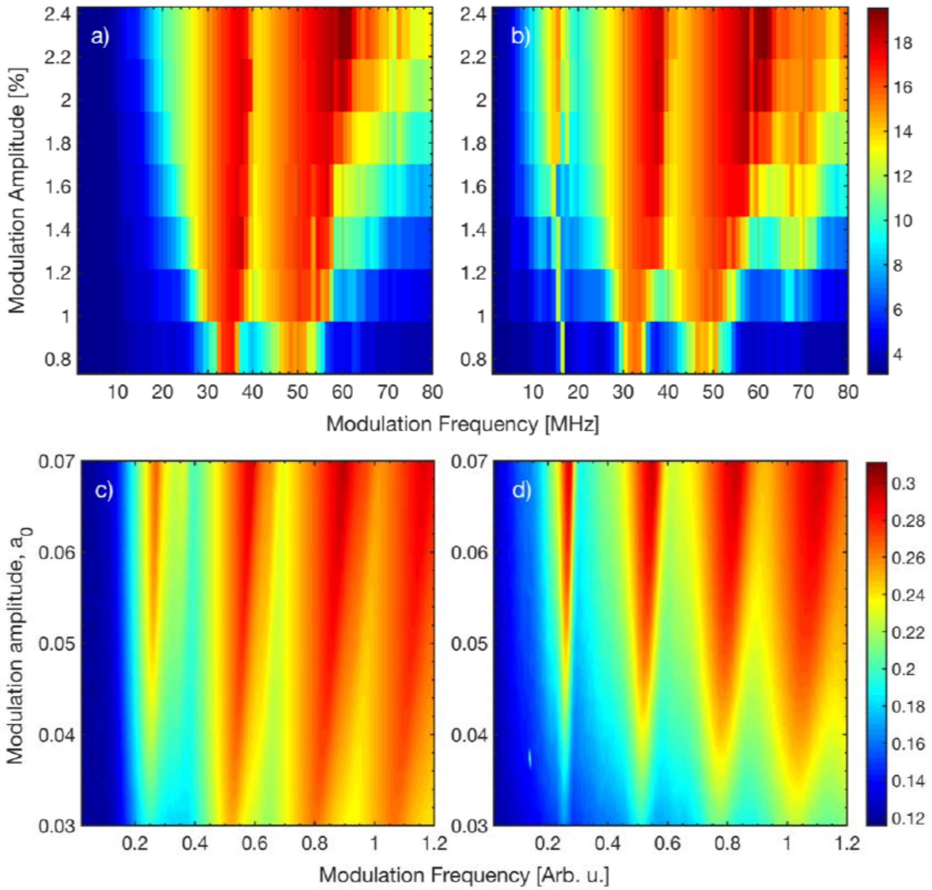
M. Masoliver and C. Masoller, "Neuronal coupling benefits the encoding of weak periodic signals in symbolic spike patterns", arXiv:1905.01933 (2019)

Comparison: how a diode laser and a neuron encode a weak periodic signal? spike rate code?

Sinusoidal   Pulsed signal

Laser with optical feedback (experiments modulating the laser current)

Neuron model (with the same input signal)



Spike rate in color code

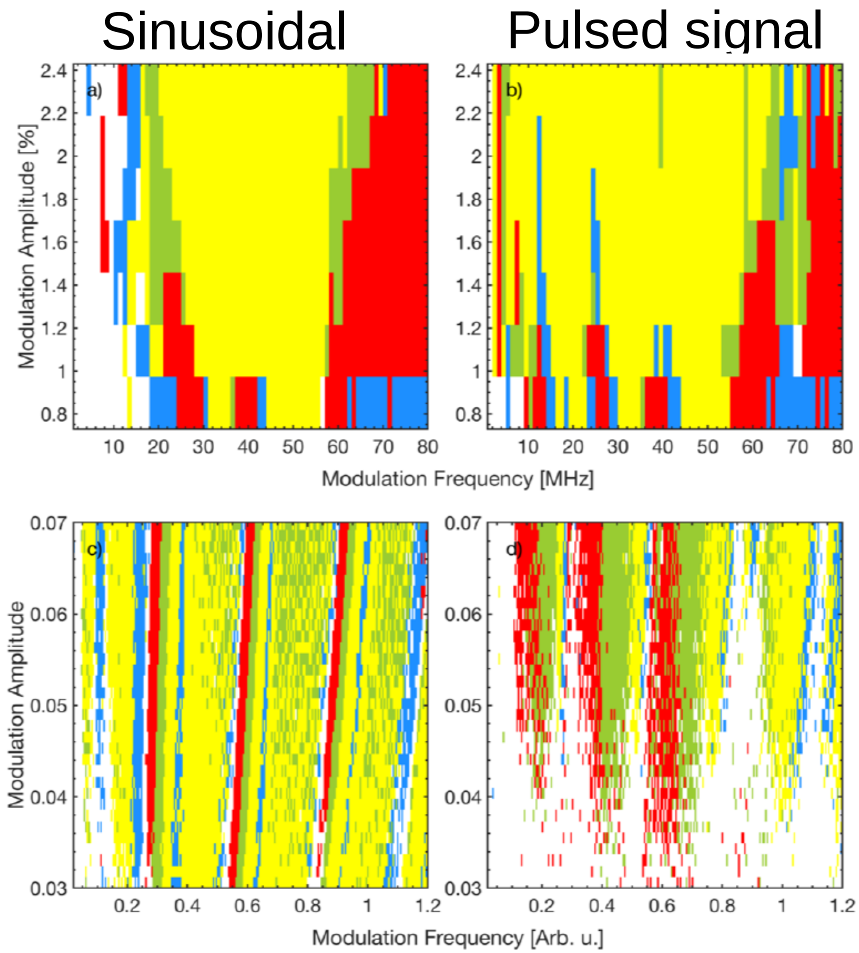
J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, "Comparing the dynamics of periodically forced lasers and neurons", submitted (2019)

Temporal code?

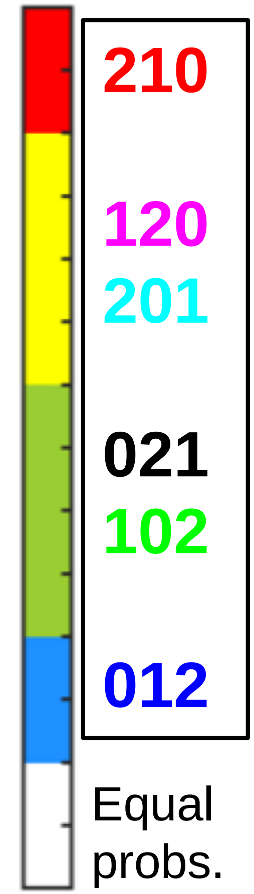
Ordinal analysis unveils differences in spike timing

Laser with optical feedback

Neuron model



Most probable pattern in color code



J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, "Comparing the dynamics of periodically forced lasers and neurons", submitted (2019)

Take home message

- We have studied how neurons and lasers encode a weak periodic signal in their output spike sequences.
- Using symbolic analysis applied to the inter-spike-intervals we uncovered preferred and infrequent spike patterns.
- They depend on the amplitude and the period of the signal, and the noise strength.
- Symbolic resonance: the probabilities of the “trend” patterns take minimum values when $\langle ISI \rangle \approx T/2$.
- We found similarities between neurons and lasers, but also some differences.
- We found a minimal model (a modified circle map) that describes the variation of the ordinal probabilities with experimental parameters.



M. Masoliver C. Quintero J. Tiana

A. Aragoneses et al,
Sci. Rep. 4, 4696 (2014).

J. A. Reinoso et al,
PRE 94, 032218 (2016).

J. Tiana et al,
PRE 99, 022207 (2019).

M. Masoliver & C. Masoller,
arXiv:1905.01933 (2019).

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