Synchronization phenomena in coupled oscillators: from neurons to Kuramoto to the climate system

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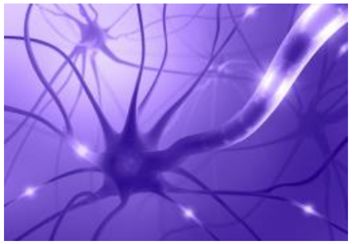
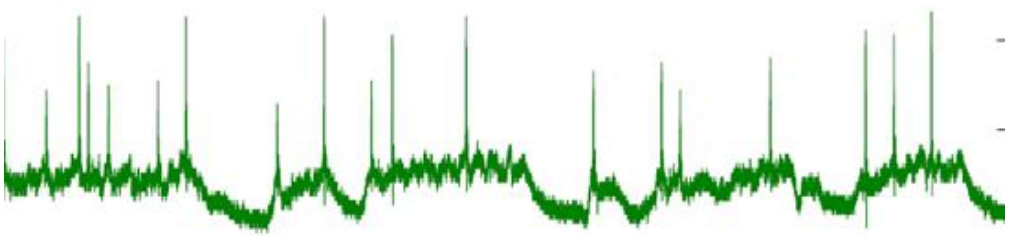
How neurons encode information?


- In the spike **rate**?
- In the relative **timing** of the spikes?
- **Single** neuron encoding or **ensemble** encoding?
- How can temporal correlations be detected and quantified?

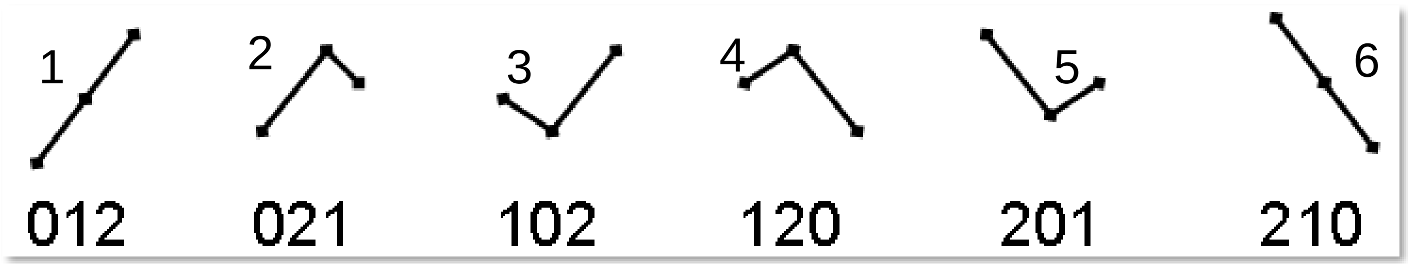
- Our goal: **crack the neural code**

- Begin by trying to understand how neurons encode a *weak (**subthreshold**) periodic signal*, in the presence of **noise**.

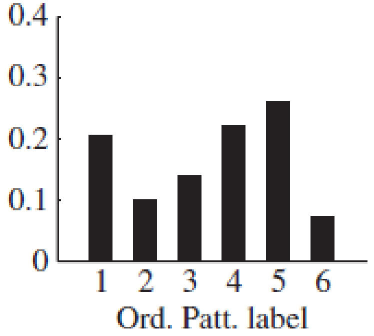
Symbolic method of analysis of inter-spike sequences: Relative order of **three** consecutive intervals

$I_i = t_{i+1} - t_i$

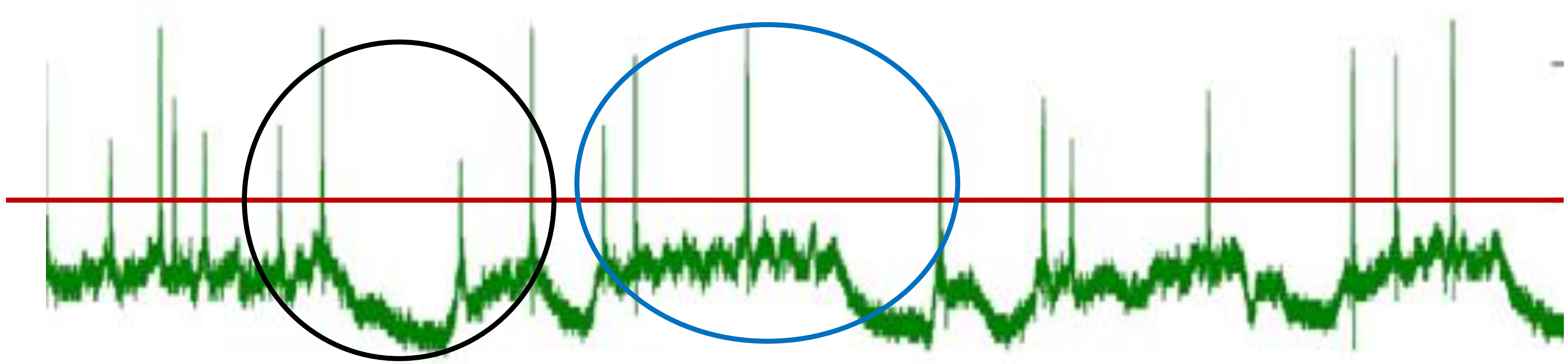
$\{\ldots \boldsymbol{I}_i, \boldsymbol{I}_{i+1}, \boldsymbol{I}_{i+2}, \ldots\}$



Ordinal probabilities



Example: (5, 1, 7) gives "102" because 1 < 5 < 7

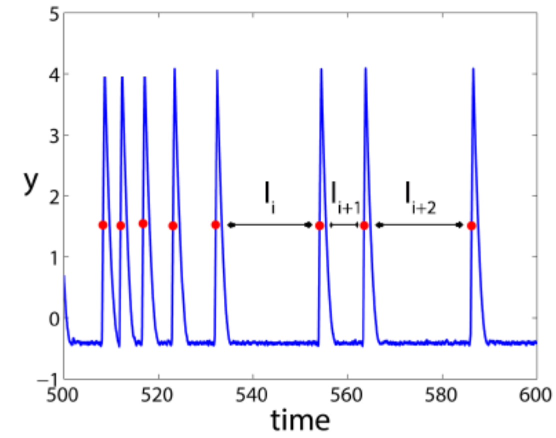


021 **012**

Brandt & Pompe, PRL 88, 174102 (2002)

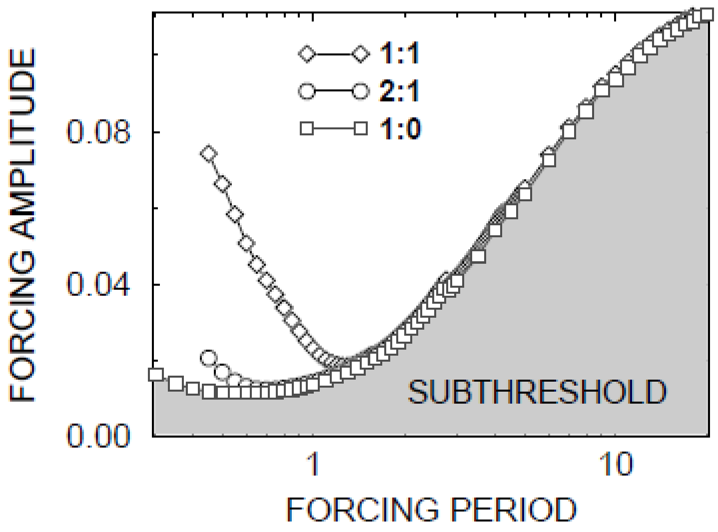
FitzHugh-Nagumo model

$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + a_o \cos(2\pi t/T) + D\xi(t),$$



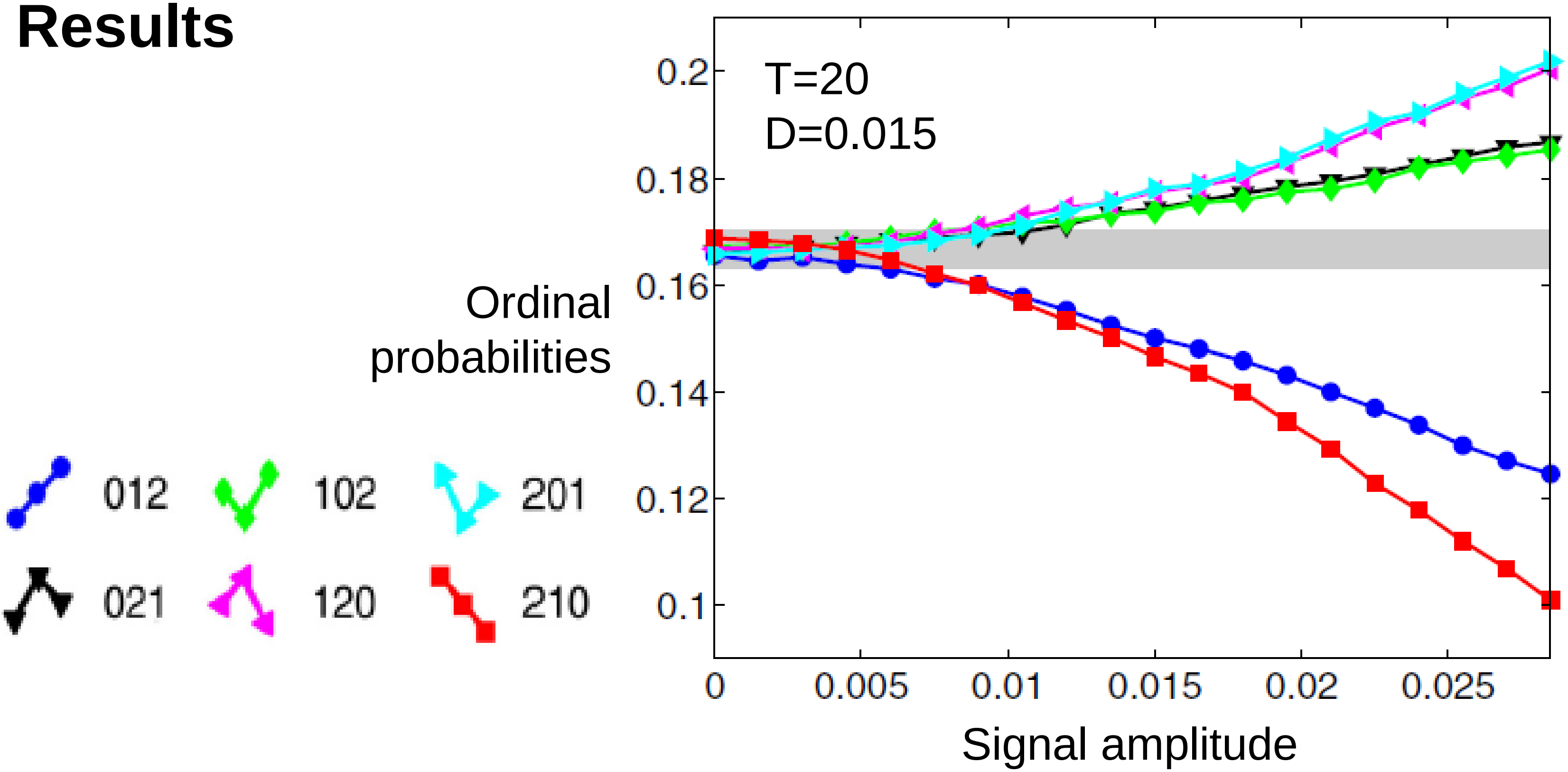
- Gaussian white noise and <u>subthreshold</u> signal: **a_0** and **T** such that spikes are **noise-induced**.

- Time series with M=100,000 spikes simulated (a=1.05, ε=0.01).



Longtin and Chialvo, PRL 1998

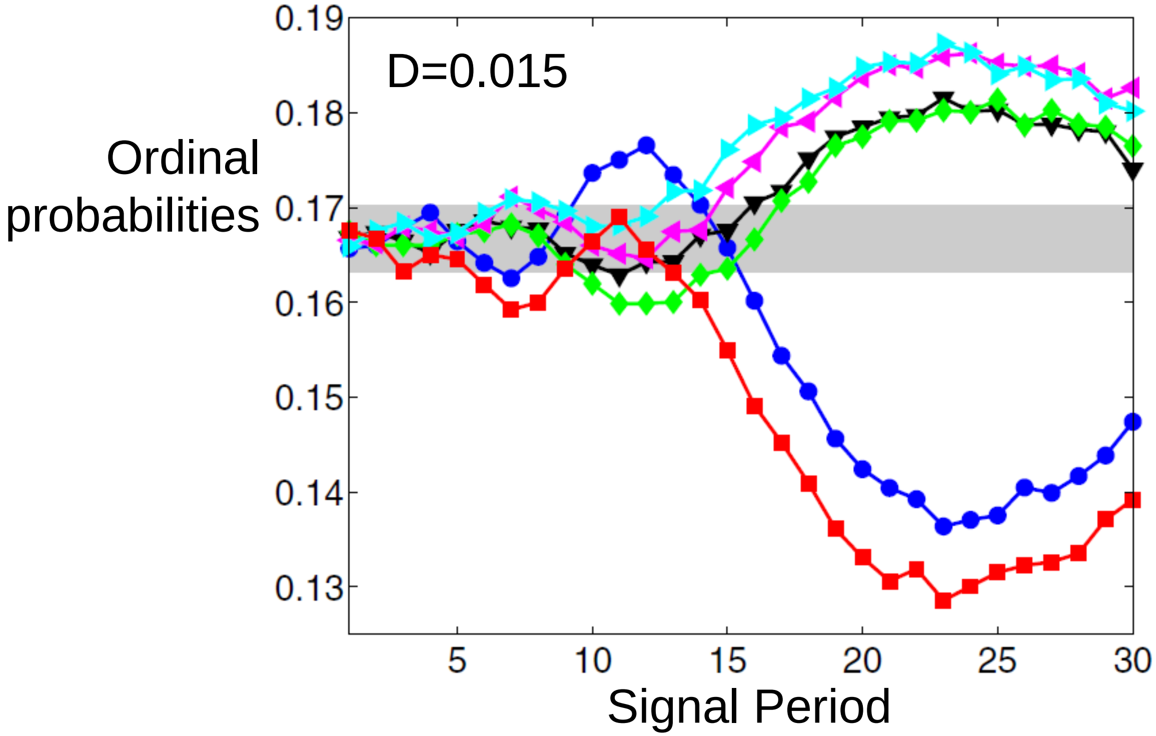
Results

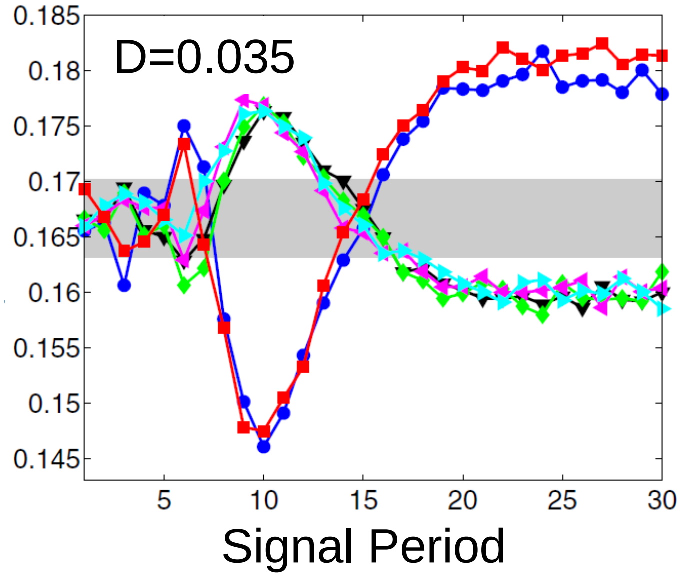


- Gray region: 3σ confidence level.

J. M. Aparicio-Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016)

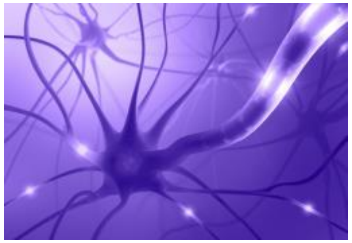
Role of the signal period and noise level

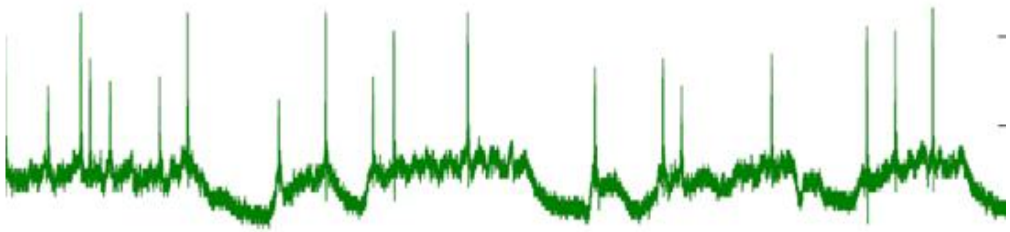




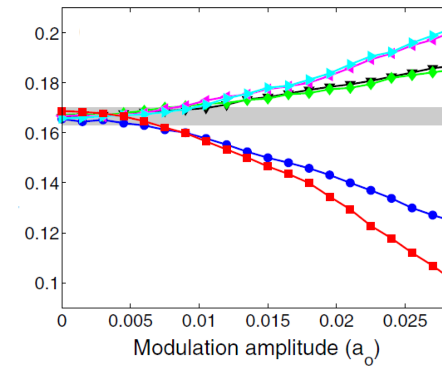
J. M. Aparicio-Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016)

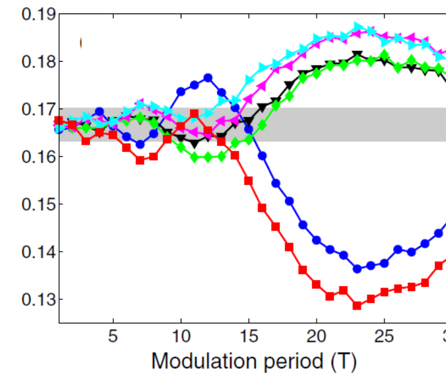
So… how neurons might encode a weak periodic signal?





- The amplitude and the period might be encoded in more and less expressed patterns.
- <u>Single-neuron encoding</u>: very **slow** because long spike sequences are needed to estimate the probabilities.
- <u>Ensemble encoding</u>: can be **fast** because few spikes per neuron are enough to estimate the probabilities.





Ensemble of FHN neurons

M. Masoliver

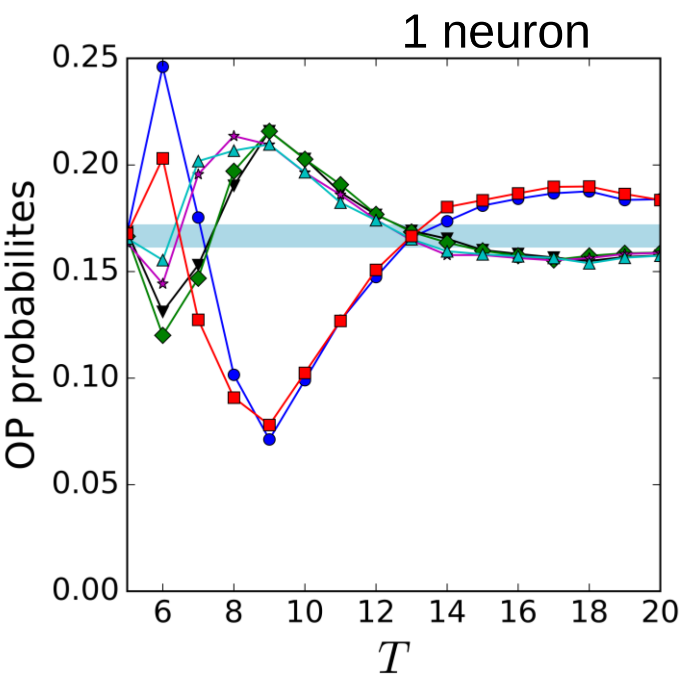
$$\epsilon \dot{u}_i = u_i - \frac{u_i^3}{3} - v_i + a_0 \cos(2\pi t/T) + \frac{\sigma_i}{\langle k \rangle} \sum_{j=0}^{N} A_{ij}(u_j - u_i) + \sqrt{2D}\xi_i \qquad j \neq i$$

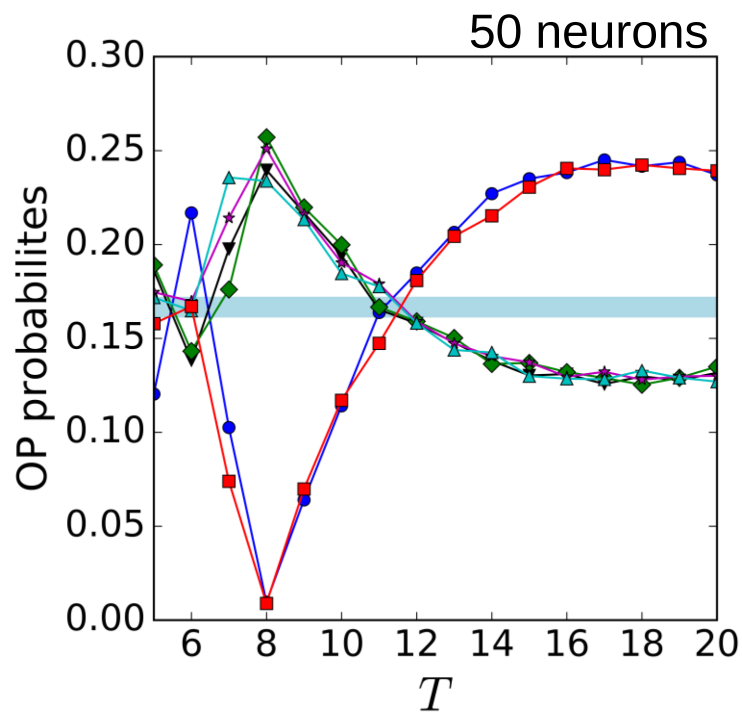
$$\dot{v}_i = u_i + a$$

The signal is applied to each neuron.

The signal is subthreshold for each individual neuron: with D = 0 and σ_i=0, no spikes.

Results





Visual perception: A subset of neurons in the human medial temporal lobe are *selectively* activated by different pictures of given individuals, landmarks or objects.

Quian Quiroga et al, Nature 435, 1102 (2005)

A single unit in the left posterior hippocampus is activated *exclusively* by different views of the actress Jennifer Aniston

Preliminary conclusion

- Encoding the “features” of the signal (amplitude and period) in the values of the ordinal probabilities is “plausible”.

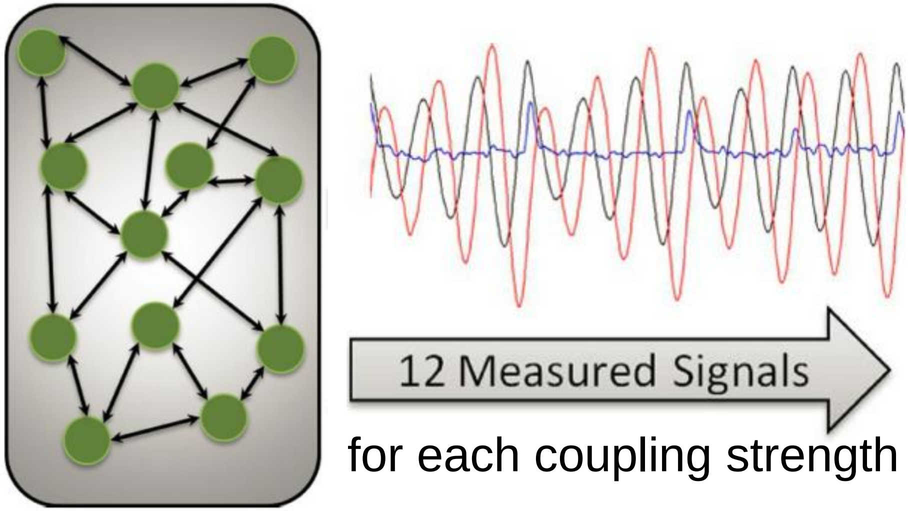
- Ongoing work (collaboration with Claudio Mirasso and Cristian Estarellas): test this encoding mechanism in another neuron model with non-linear synaptic coupling.

- Future work: consider signals with time-varying amplitude and/or period.

Dynamics of coupled oscillators

can lag-time information be used to
- anticipate the synchronization transition?
- infer the existing / nonexisting links?

(in collaboration with Immaculada Leyva)



Correlation analysis for anticipating sync. transition or inferring the existing undirected links

Observed time series in nodes i and j: $a_i(t)$, $a_j(t)$, $t=1, \ldots, T$

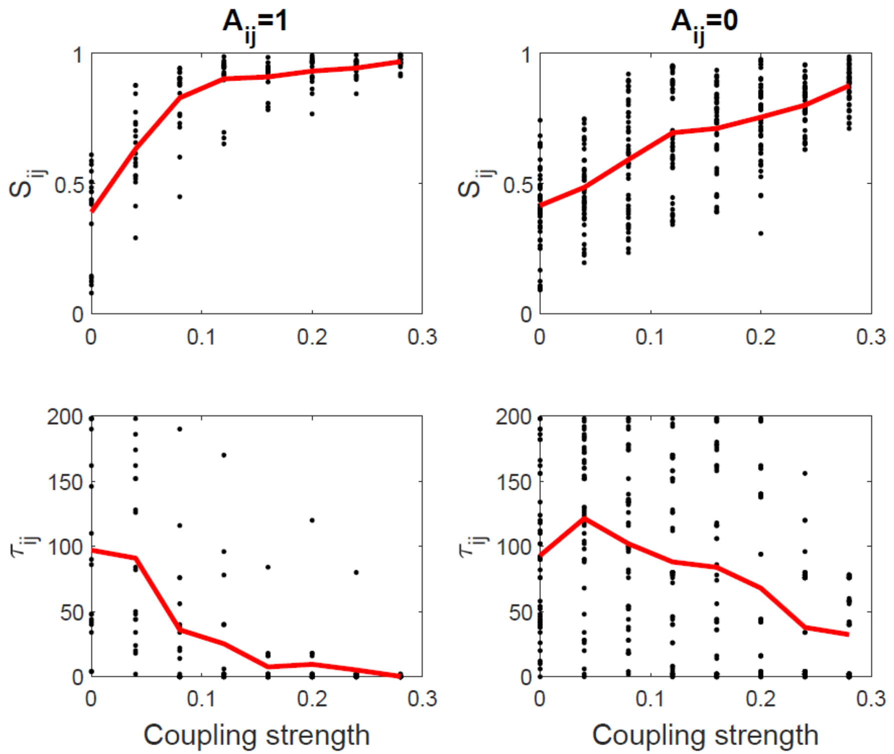
Lagged |cross correlation| : $CC_{ij}(\tau) = \frac{1}{T-\tau_{\max}} \left| \sum_{t=0}^{T-\tau_{\max}} a_i(t)\, a_j(t+\tau) \right|$

Statistical Similarity Measure:

$$S_{ij} = \max |CC_{ij}(\tau)|$$
$$= |CC_{ij}(\tau_{ij})| \qquad \tau_{ij} \text{ in } [0, \tau_{\max}]$$

How the distribution of CC_{ij} and τ_{ij} values change with the coupling strength?

- 12 experimental electronic chaotic circuits



G. Tirabassi, R. Sevilla-Escoboza, J. M. Buldú and C. Masoller, "*Inferring the connectivity of coupled oscillators from time-series statistical similarity analysis*", Sci. Rep. **5** 10829 (2015).

Using lag information to infer the links

If S_{ij} > TH the link i $\longleftarrow\!\!\longrightarrow$ j exists, otherwise, it does not exist

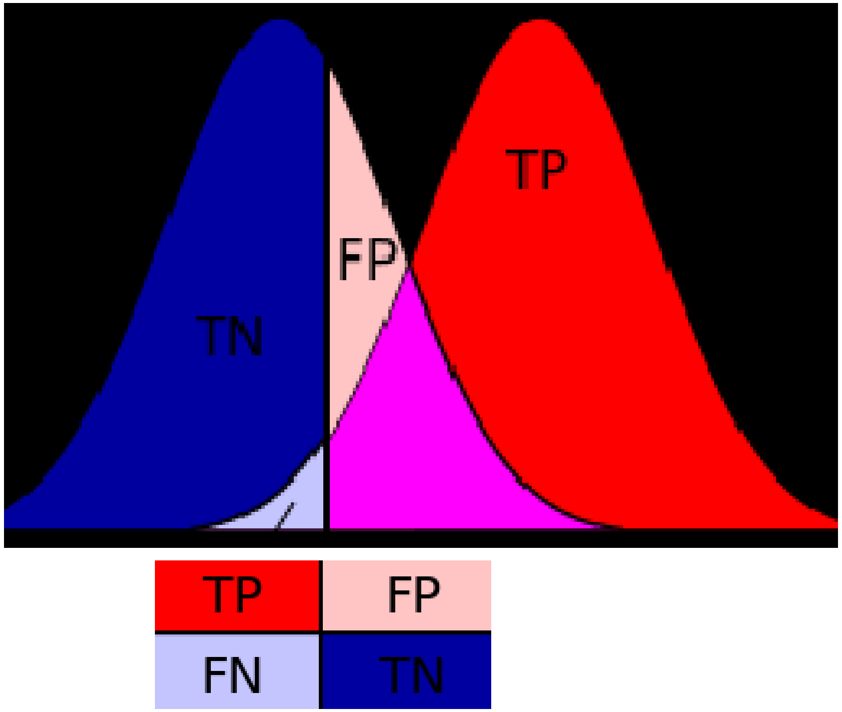
If $\tau_{ij} < \tau_{TH}$ the link i $\longleftarrow\!\!\longrightarrow$ j exists, otherwise, it does not exist

Three possible criteria:

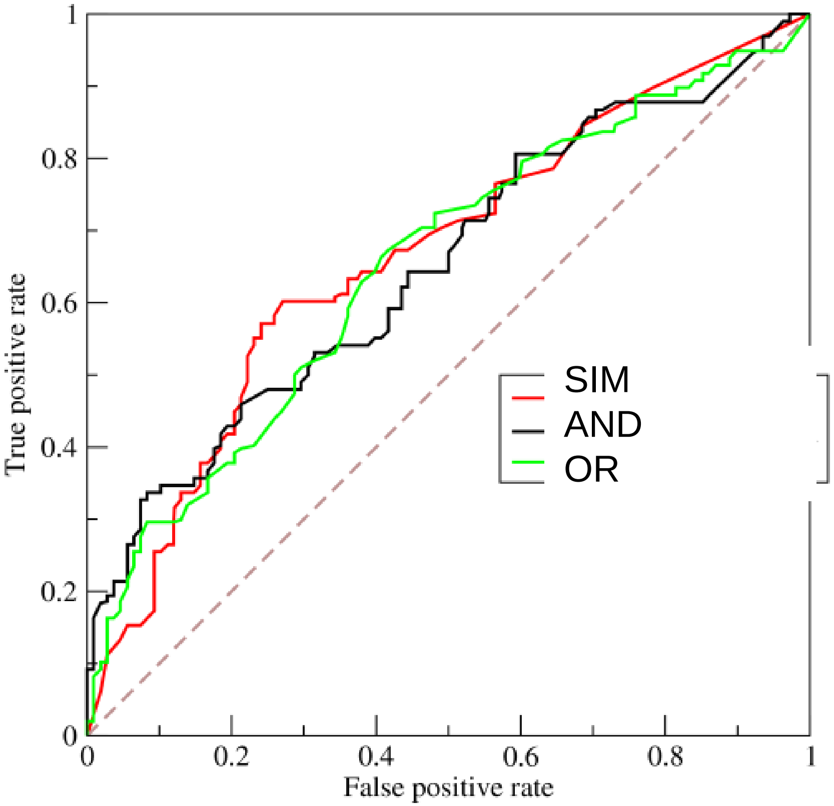
The link i $\longleftarrow\!\!\longrightarrow$ j exists if

- SIM : only SSM criteria holds (S_{ij} > TH)
- AND: both criteria hold (S_{ij} > TH and $\tau_{ij} < \tau_{TH}$)
- OR: at least one criteria holds (S_{ij} > TH or $\tau_{ij} < \tau_{TH}$)

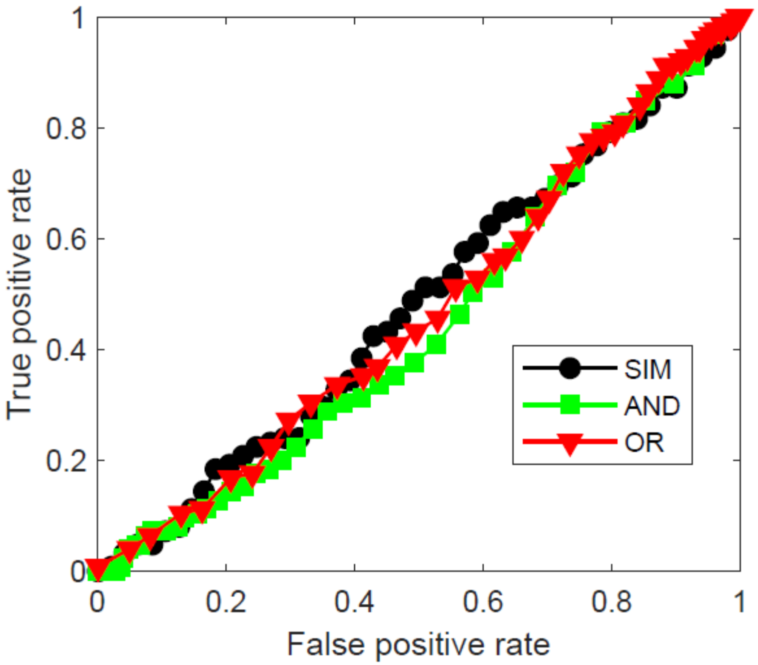
Quantifying the three criteria with receiver operating characteristic (ROC curve)



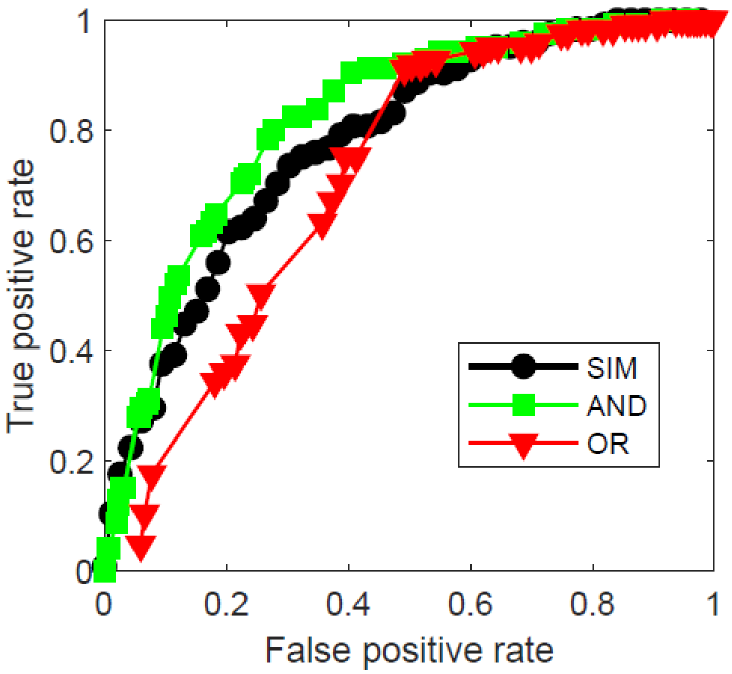
Source: wikipedia



Uncoupled oscillators

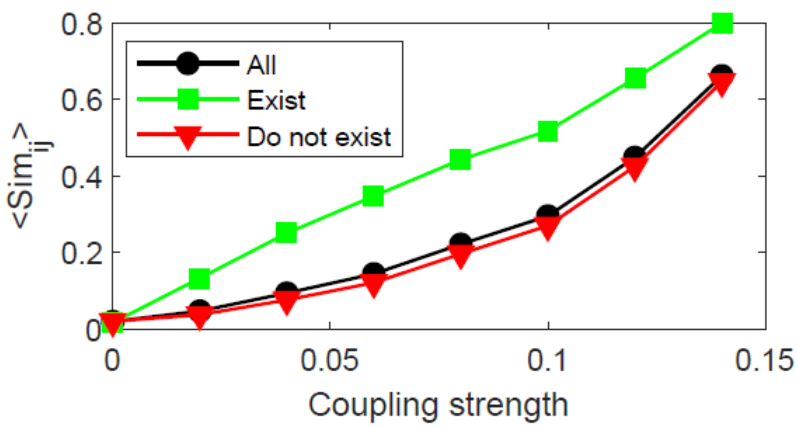


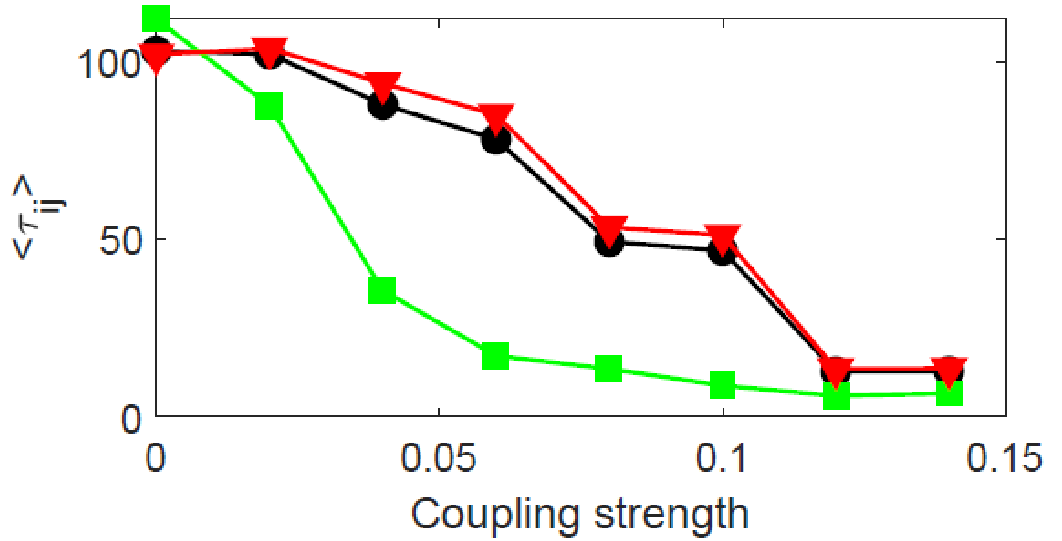
Coupled oscillators

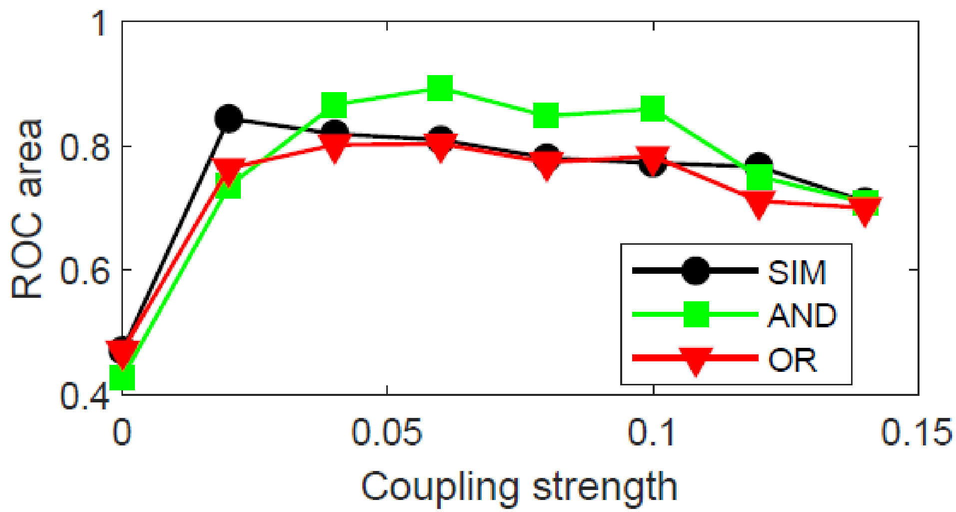


Very preliminary results

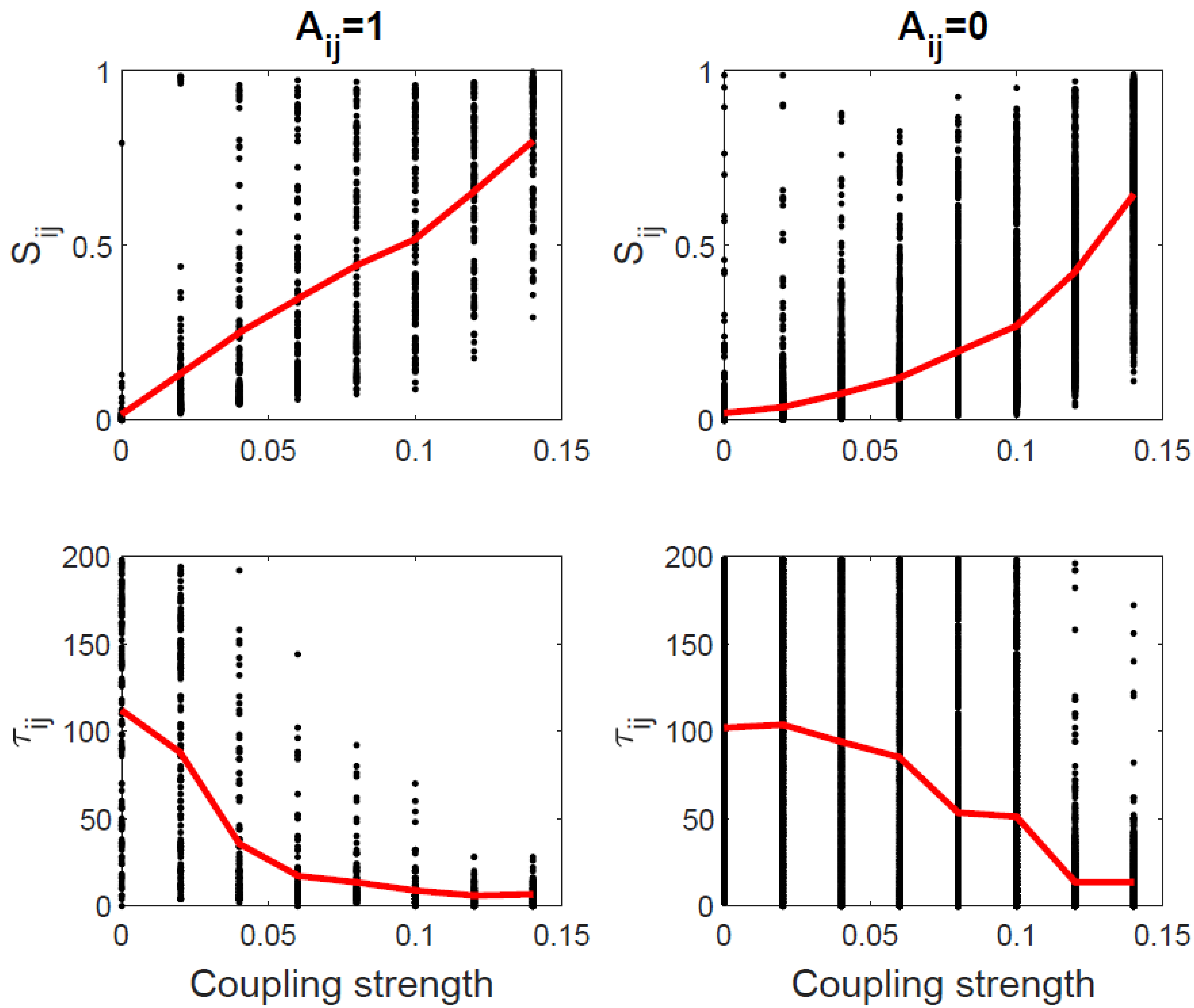
- 50 Kuramoto phase oscillators, 10% existing links, cross-correlation of $\cos(\varphi_i)$, $\cos(\varphi_j)$



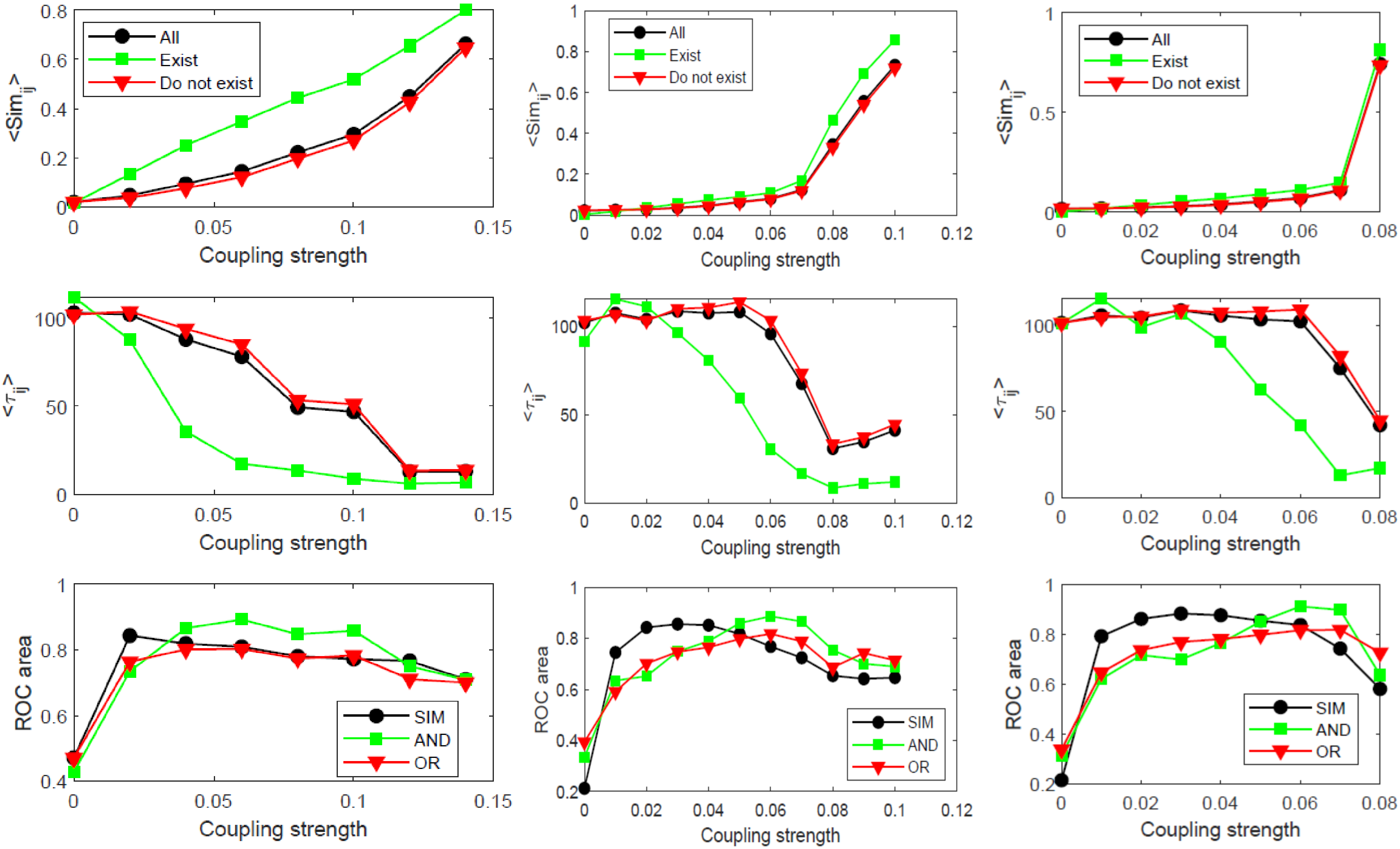




Variation of CC_{ij} and τ_{ij} values with the coupling strength

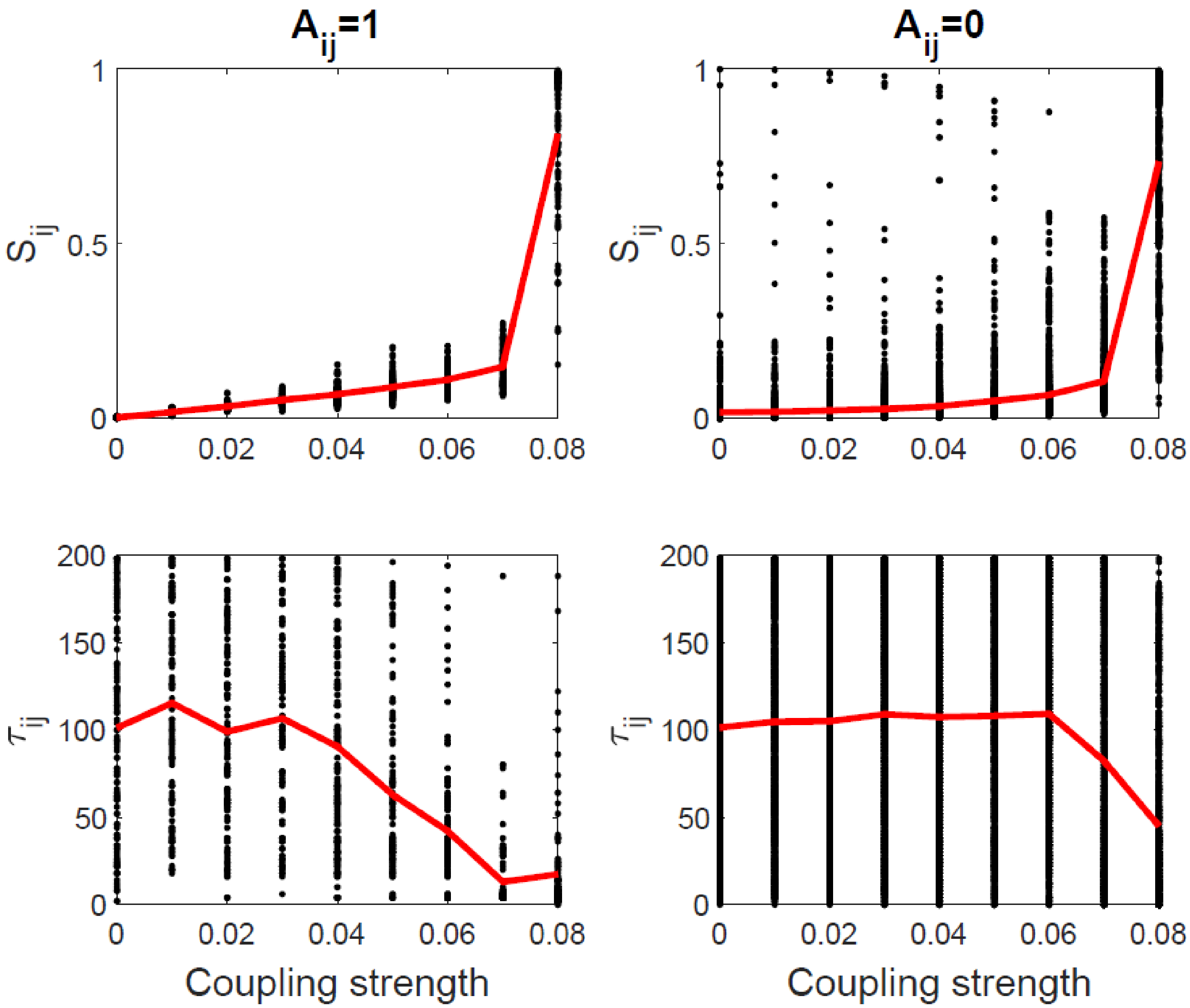


Changing the model parameters: different transition to synchronization



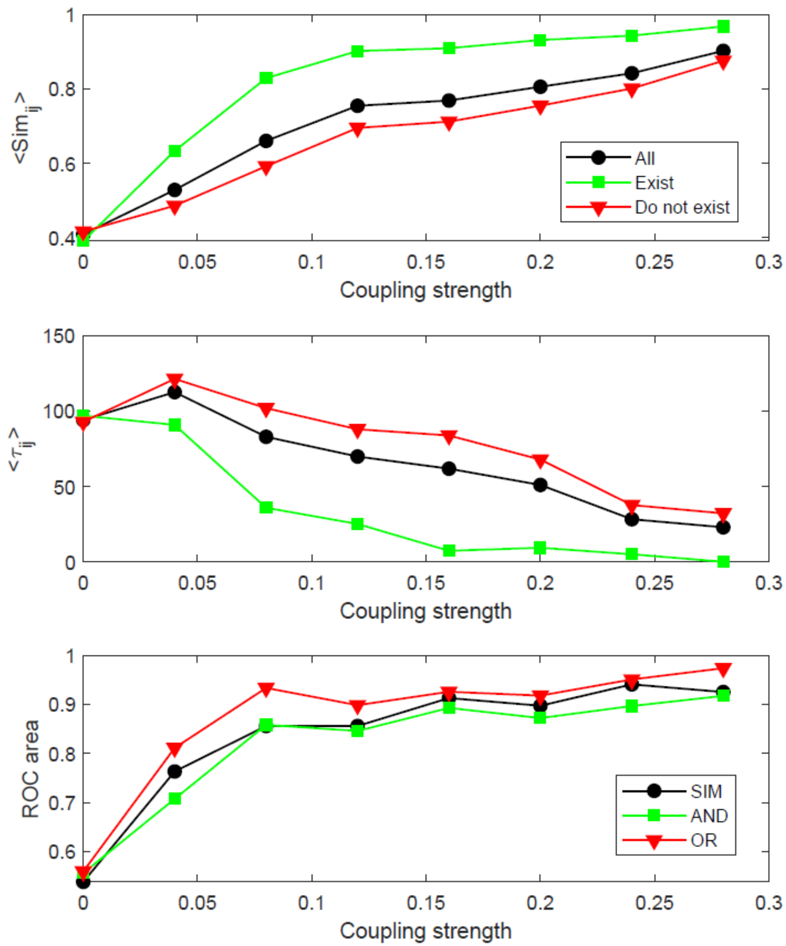
I. Leyva et al, PRL 108, 168702 (2012)

Variation of CC_{ij} and τ_{ij} values with the coupling strength

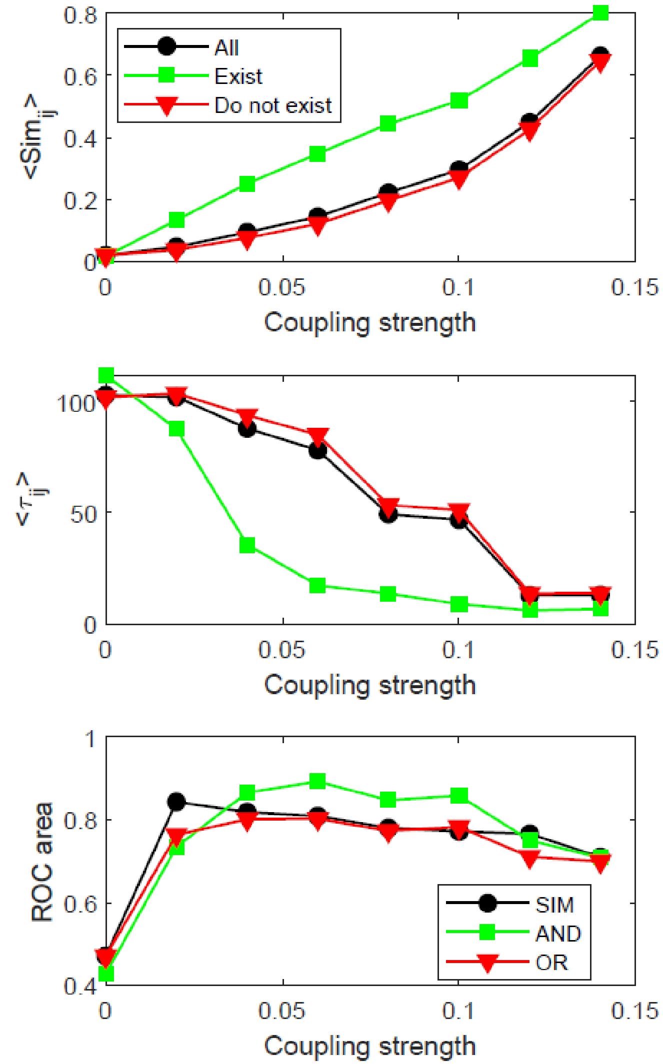


Comparison with experimental data

- 12 experimental electronic chaotic circuits



- 50 Kuramoto oscillators



G. Tirabassi et al, Sci. Rep. **5** 10829 (2015).

Preliminary conclusions…

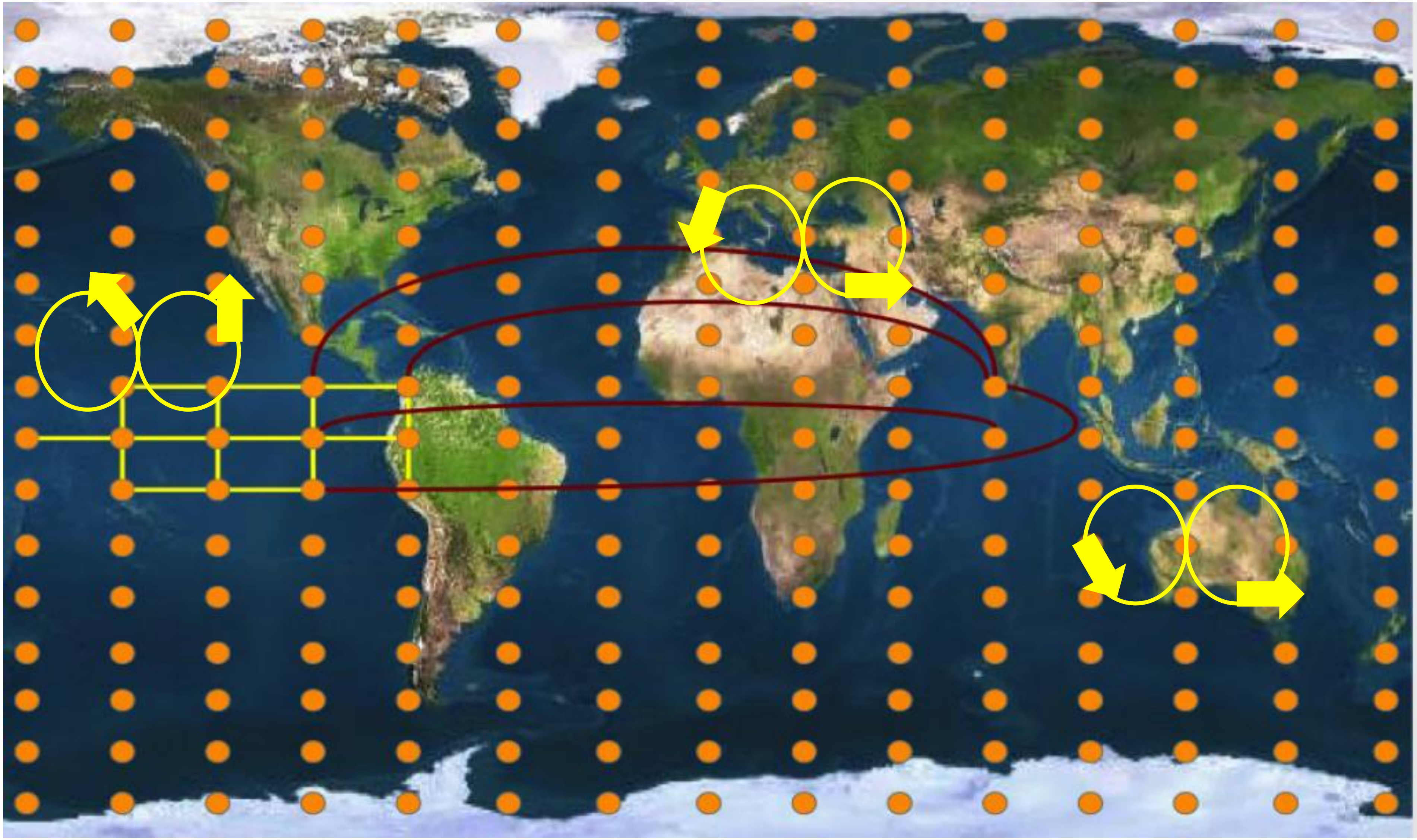
- If we know the system's connectivity, lag information seems to be useful for anticipating the transition to synchronization.

- If we don't know the system's connectivity, lag information does not seem to be useful for inferring the links (but it could be useful to reduce certain types of mistakes –the false positives or the false negatives).

The climate system

a network of mutually coupled oscillators forced by the solar cycle?

We analyze surface air temperature regularly sampled in space (>10.000 time series) and time (daily data from 1979 to 2016 gives > 13.000 data points in each TS).

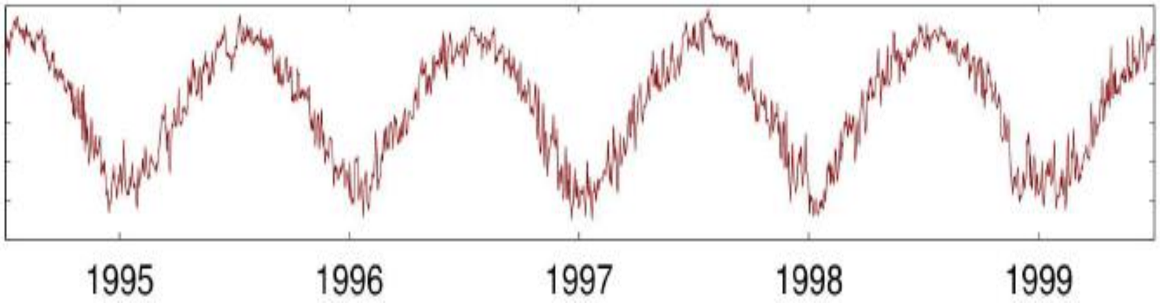


Hilbert phase analysis of raw SAT data

D. Zapalla

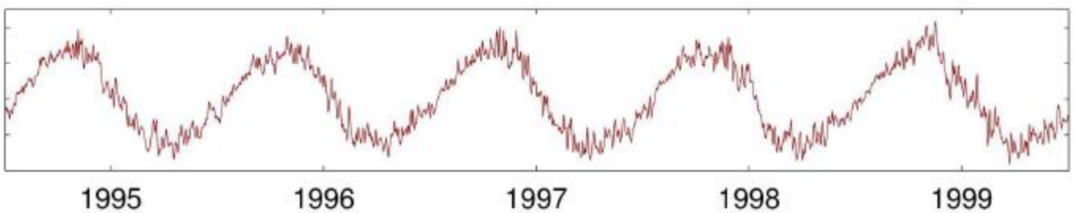
Each time series is first normalized to $\langle x \rangle = 0$ and $\sigma = 1$

x

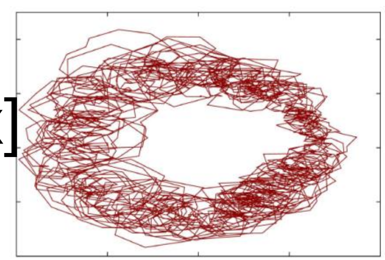


- HT[sin(ωt)]=cos(ωt)

HT[x]



y=HT[x]



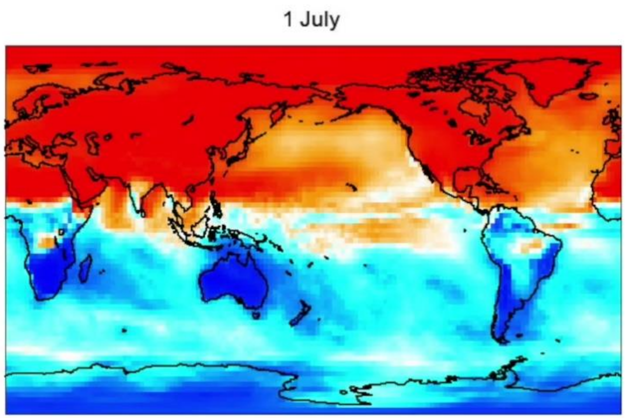
x

$$a(t) = \sqrt{[x(t)]^2 + [y(t)]^2}$$

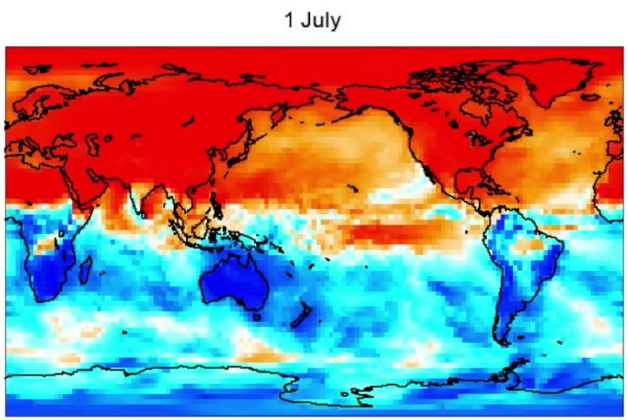
$$\varphi(t) = \arctan[y(t)/x(t)]$$

Hilbert phase dynamics: temporal evolution of the cosine of the phase

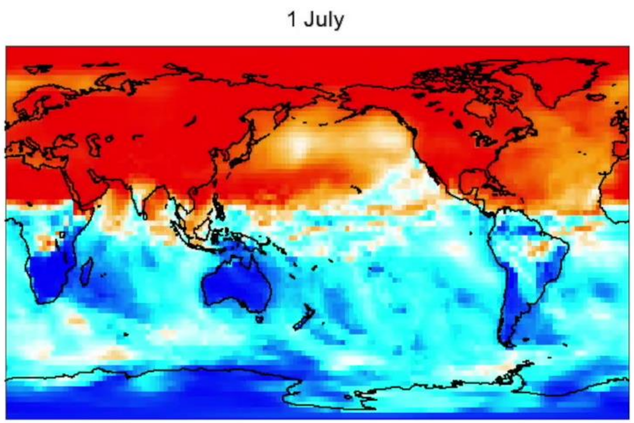
Typical year



El Niño year

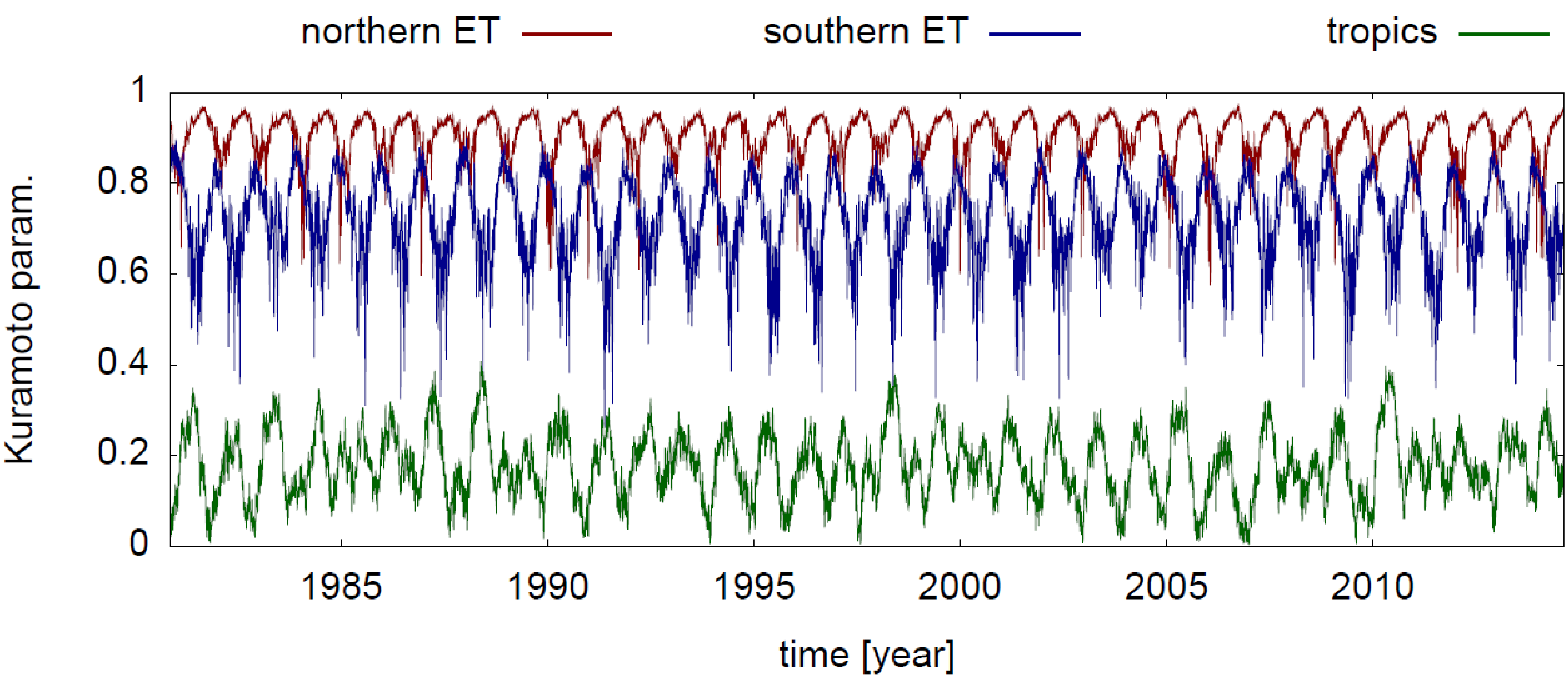


La Niña year



After using the Hilbert transform to obtain phase time series, we calculate the Kuramoto order parameter

$$r(t) = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j(t)} \right|$$



Relative decadal variations

$$\Delta a = \langle a \rangle_{2016-2007} - \langle a \rangle_{1988-1979}$$

$$\frac{\Delta a}{\langle a \rangle_{2016-1979}}$$

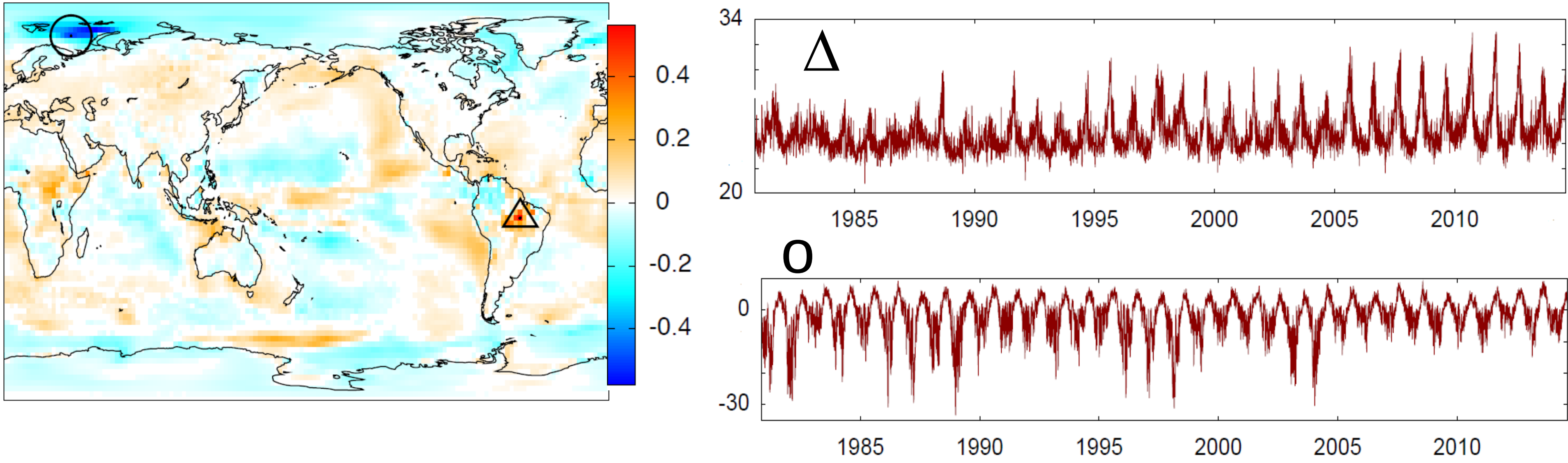
Relative variation is considered significant if:

$$\frac{\Delta a}{\langle a \rangle} \geq \langle . \rangle_s + 2\sigma_s \quad \text{or} \quad \frac{\Delta a}{\langle a \rangle} \leq \langle . \rangle_s - 2\sigma_s$$

100 “block” surrogates

D. A. Zappala, M. Barreiro and C. Masoller, “*Quantifying changes in spatial patterns of surface air temperature dynamics over several decades*”, Earth Syst. Dynam. **9**, 383 (2018)

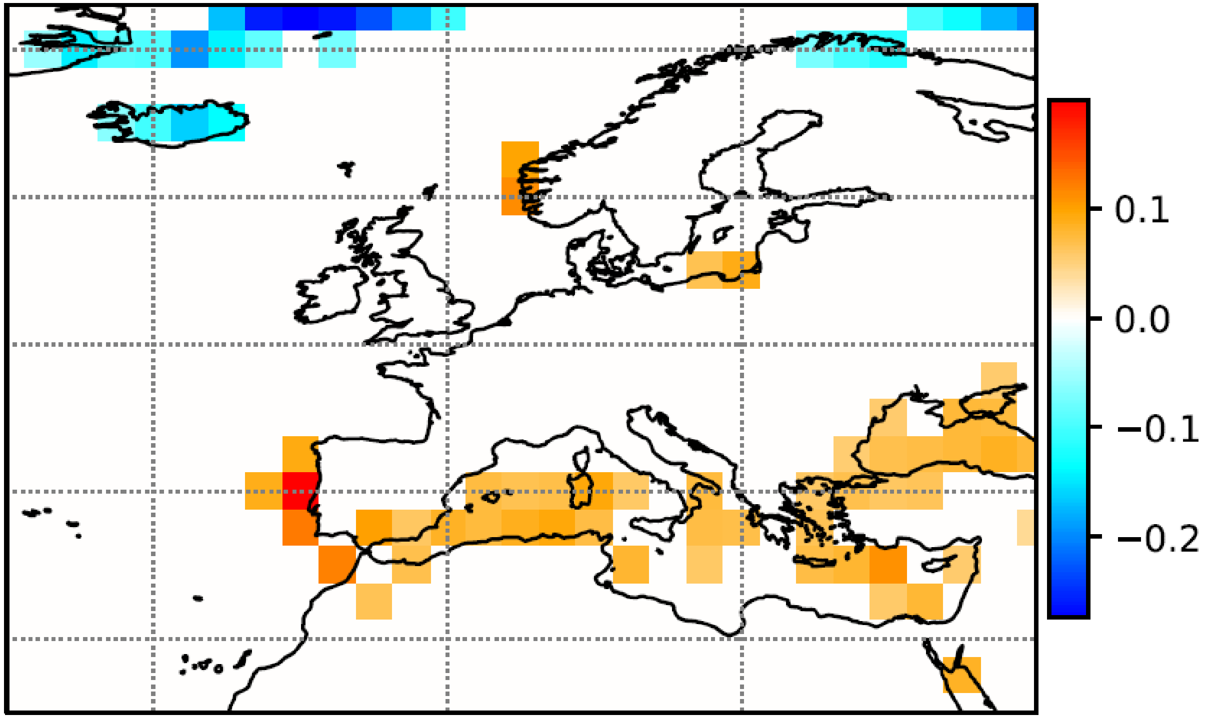
Relative variation of average Hilbert amplitude uncovers regions where the seasonal cycle increased/decreased



- **Decrease of precipitation**: the solar radiation that is not used for evaporation is used to heat the ground.

- **Melting of sea ice**: during winter the air temperature is mitigated by the sea and tends to be more moderated.

D. A. Zappala et al., Earth Syst. Dynam. **9**, 383 (2018)



[D. A. Zappala et al., Earth Syst. Dynam. **9**, 383 (2018)]

So…

- Hilbert analysis identifies inter-decadal variations in atmospheric data.

- Quantifies phase synchronization (mainly due to common solar forcing).

- Ongoing work: synchronization “events” occur before (or follow) El Niño / La Niña events?

THANK YOU FOR YOUR ATTENTION !

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Subthreshold signal encoding in coupled FitzHugh-Nagumo neurons

M. Masoliver and C. Masoller, Scientific Reports 8, 8276 (2018).

Anticipating the transition to synchrony in coupled phase oscillators through lag-time analysis

I. Leyva and C. Masoller, in preparation.

Quantifying changes in spatial patterns of surface air temperature dynamics over several decades

D. Zappala, M. Barreiro, and C. Masoller, Earth Syst. Dynam. 9, 383, (2018).