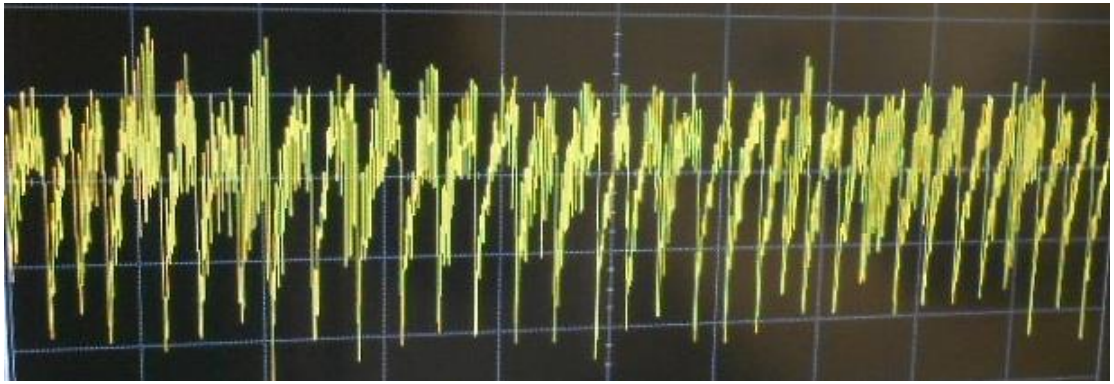

Optical spikes in the delayed Lang-Kobayashi equations: interplay of modulation and delay

Cristina Masoller

UPC, Barcelona, Spain

www.fisica.edu.uy/~cris

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NETT

GOBIERNO DE ESPAÑA MINISTERIO DE CIENCIA E INNOVACIÓN

Short Thematic Program on Delay Differential Equations

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES, TORONTO MAY 2015

Collaborators

- Taciano Sorrentino

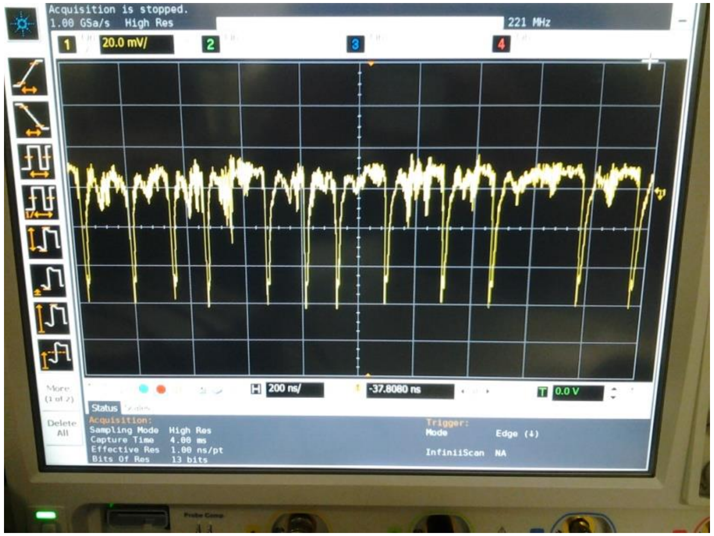
- Carlos Quintero

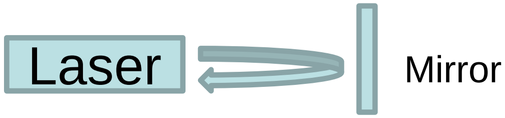
- Andres Aragoneses (now at Duke University, USA)

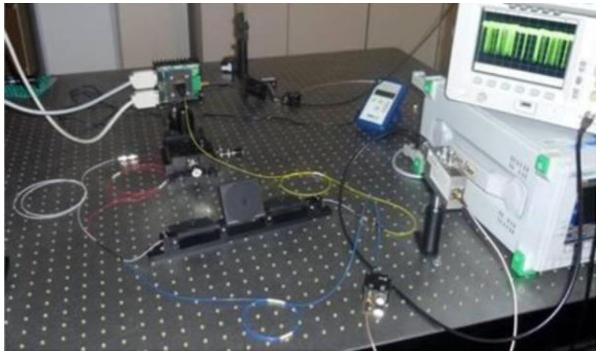
- Sandro Perrone (now at Leicester University, UK)

- Carme Torrent

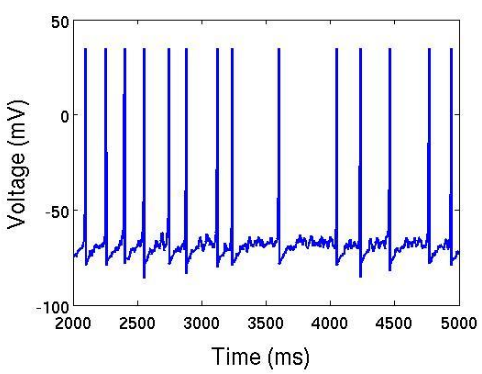
Research in our lab: nonlinear dynamics of semiconductor lasers

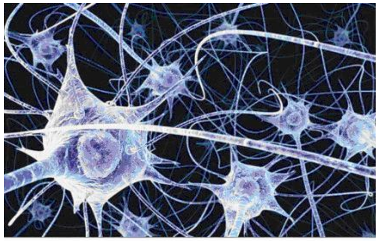






- Goal: are optical spikes similar to neuronal spikes?
- Potential for ultra-fast brain-inspired optical information processing? (**ms vs ns-µs**)
- Response to periodic forcing?





Outline

- Introduction

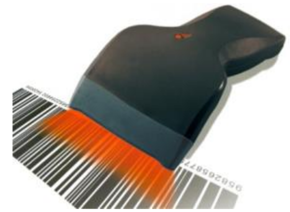
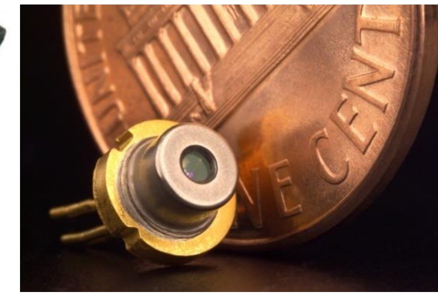
  The delayed Lang-Kobayashi equations: stochastic and high-dimensional dynamical system

- Method of analysis and experimental setup

- Results
  - Inferring signatures of determinism
  - Transitions & hierarchical clusters in the symbolic dynamics
  - Minimal model
  - Response to periodic forcing

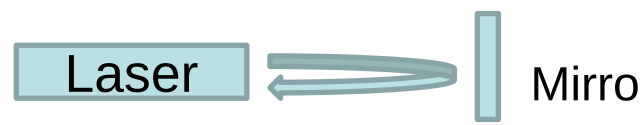
- Conclusions and take home message

Semiconductor lasers


- Widely used in:
  - Communications
  - Data storage (CDs, DVDs …)
  - Barcode scanners, laser printers, computer mice
  - Life sciences (imaging, sensing …)
  - Etc.

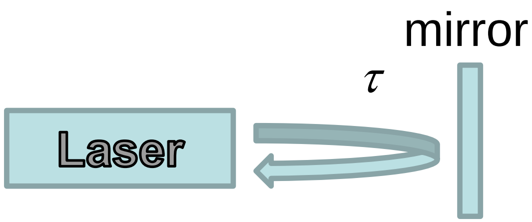
- Feedback induces nonlinear dynamics:
  - Multi-stability
  - Regular pulsing
  - Extreme pulses
  - Chaos, intermitency …



Kathy Ludge: "*Nonlinear Laser Dynamics: From Quantum Dots to Cryptography*", Wiley (2012). ISBN: 3527411003

Governing equations

R. Lang and K. Kobayashi, IEEE J. Quantum Electron. 16, 347 (1980)



$|E|^2$ ~ photon number (output intensity)

N ~ number of carriers (electron-holes)

$$\frac{dE}{dt} = \frac{1}{2\tau_p}(1+i\alpha)(G-1)E + \underbrace{\eta\, E(t-\tau)e^{-i\omega_0\tau}}_{\text{feedback}} + \underbrace{\sqrt{\beta_{sp}}\,\xi}_{\text{noise}}$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - G|E|^2\right)$$

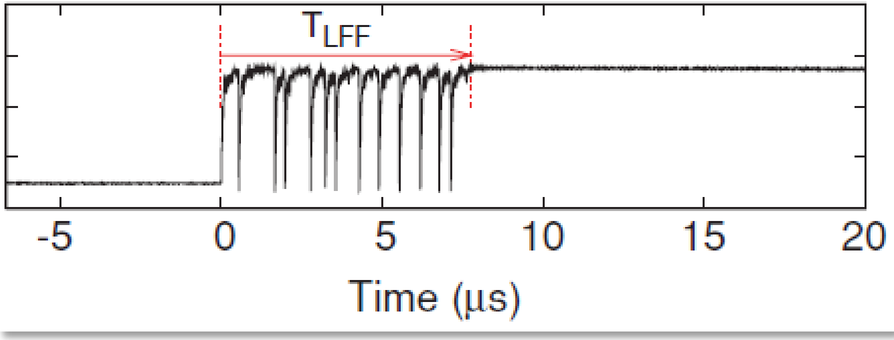
Gain: $G = N/(1+\varepsilon|E|^2)$

η = feedback strength
τ = feedback delay time
μ = pump current
(control parameter)

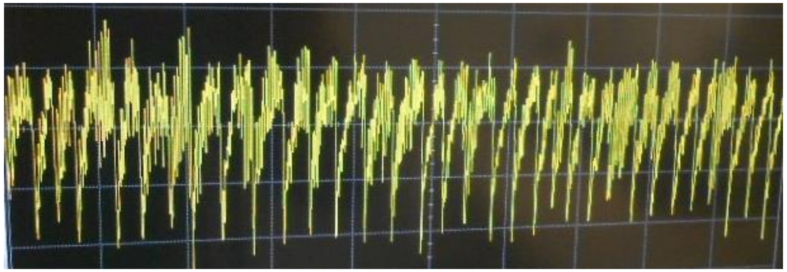
Model predictions

- In **deterministic** simulations: the spikes are **transient**.

Laser intensity



- But in **stochastic** model simulations: **bursts** of dropouts.
- In the experiments: which dropouts are triggered by **noise** and which ones are **deterministic**?



- Is there any **information** in the spike sequence?
- Can we infer signatures of underlying determinism?

A. Torcini et al, Phys. Rev. A 74, 063801 (2006)
J. Zamora-Munt et al, Phys Rev A 81, 033820 (2010)

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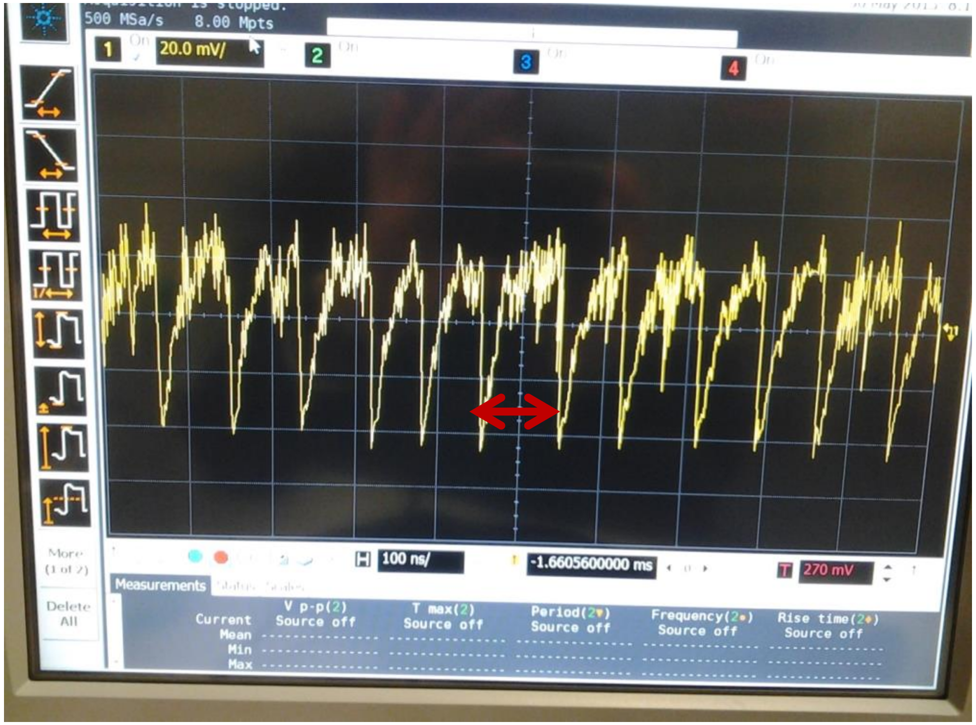
Problems and strategy

- Main problem: we can measure only one variable (the laser output intensity).
- Also a problem: the detection system (photodiode, oscilloscope) has a finite *bandwidth* that gives very limited temporal resolution.



- Our strategy: **event-level description**. We analyze the sequence of **inter-spike-intervals (ISIs)**:

$$\boxed{\Delta T_i = t_{i+1} - t_i}$$

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Event level description of complex systems

- Examples:
  - **Intervals between threshold crossings and barrier crossings,**
  - **Neurons: inter-spike intervals (ISIs),**
  - **Human communication: inter-event user times (SMS, emails, Twitters).**
  - **Earth and climate: time-intervals between earthquakes, extreme events (tornados, rainfalls) etc.**

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- Introduction
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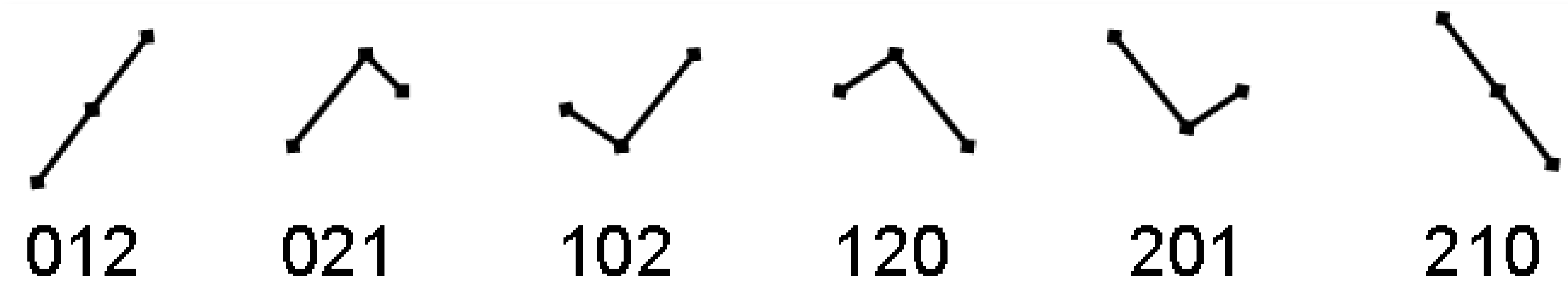
Ordinal Patterns

Brandt & Pompe, PRL 88, 174102 (2002)

- $X = \{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$. (in our case: sequence of **inter-spike-intervals**)



Example: (5, 1, 7) gives "102" because 1 < 5 < 7

– Advantage: the probabilities of the patterns unveil serial correlations.
– Drawback: the set (5,1,100) also gives "102".

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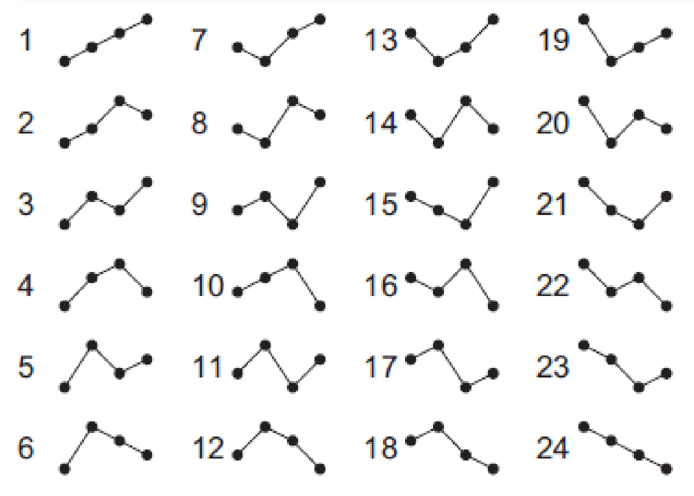
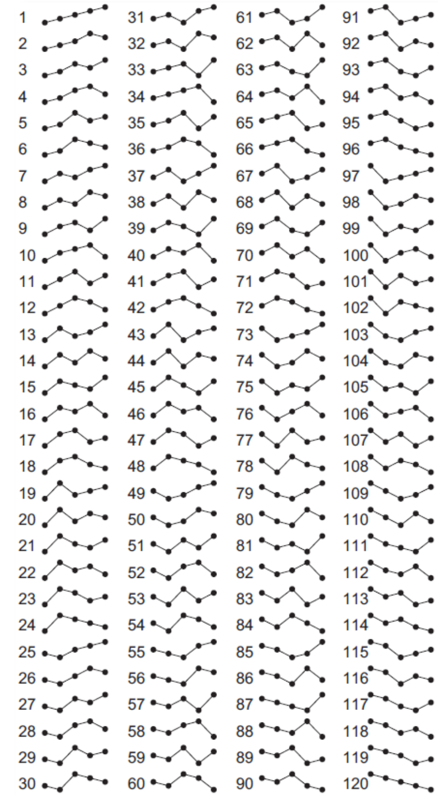
Number of possible ordinal patterns: D!

D=4

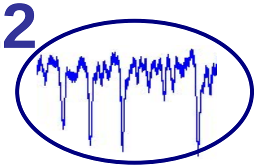


D=5

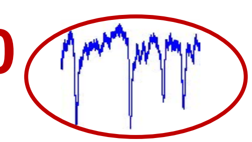


- How to select the size of the pattern? Optimal D depends on:
  - The length of the data.
  - The length of correlations in the data.
- For optical spikes: D=2 (D=3) unveil correlations among 3 (4) spikes

012



210



U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Ordinal Analysis

- Widely used to analyze the output signals of complex systems
  - **Financial, economical**
  - **Biological, life sciences**
  - **Geosciences, climate**
  - **Physics, chemistry, etc**

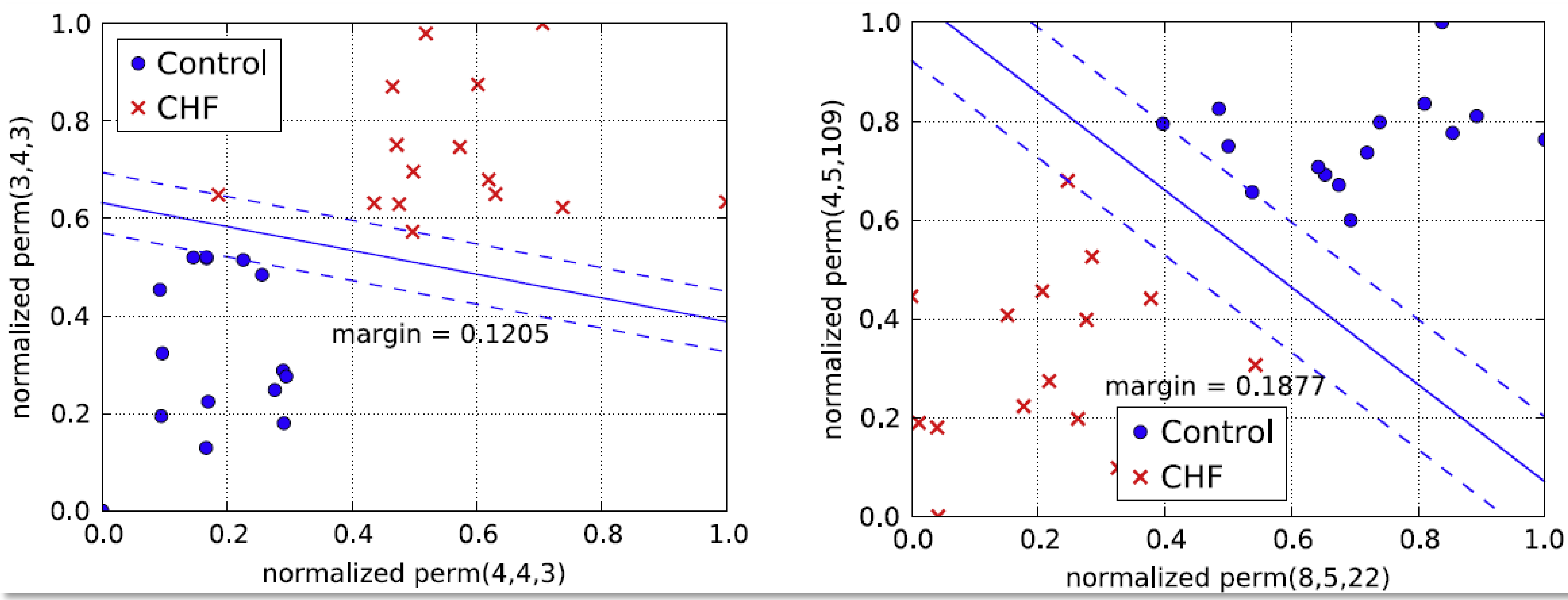
- The identification of patterns in the sequence of events allows for:
  - **Model validation, parameter estimation**
  - **Classification of dynamical behaviors (pathological, healthy)**
  - **Predictability - forecasting**

- Ordinal analysis has been able to:
  - **Distinguish stochasticity and determinism**
  - **Quantify complexity**
  - **Identify couplings and directionality.**

An illustrative example

Classifying cardiac biosignals using ordinal pattern statistics

congestive heart failure (CHF) vs healthy subjects.



U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Outline

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- Method of time-series analysis and experimental setup
- Results
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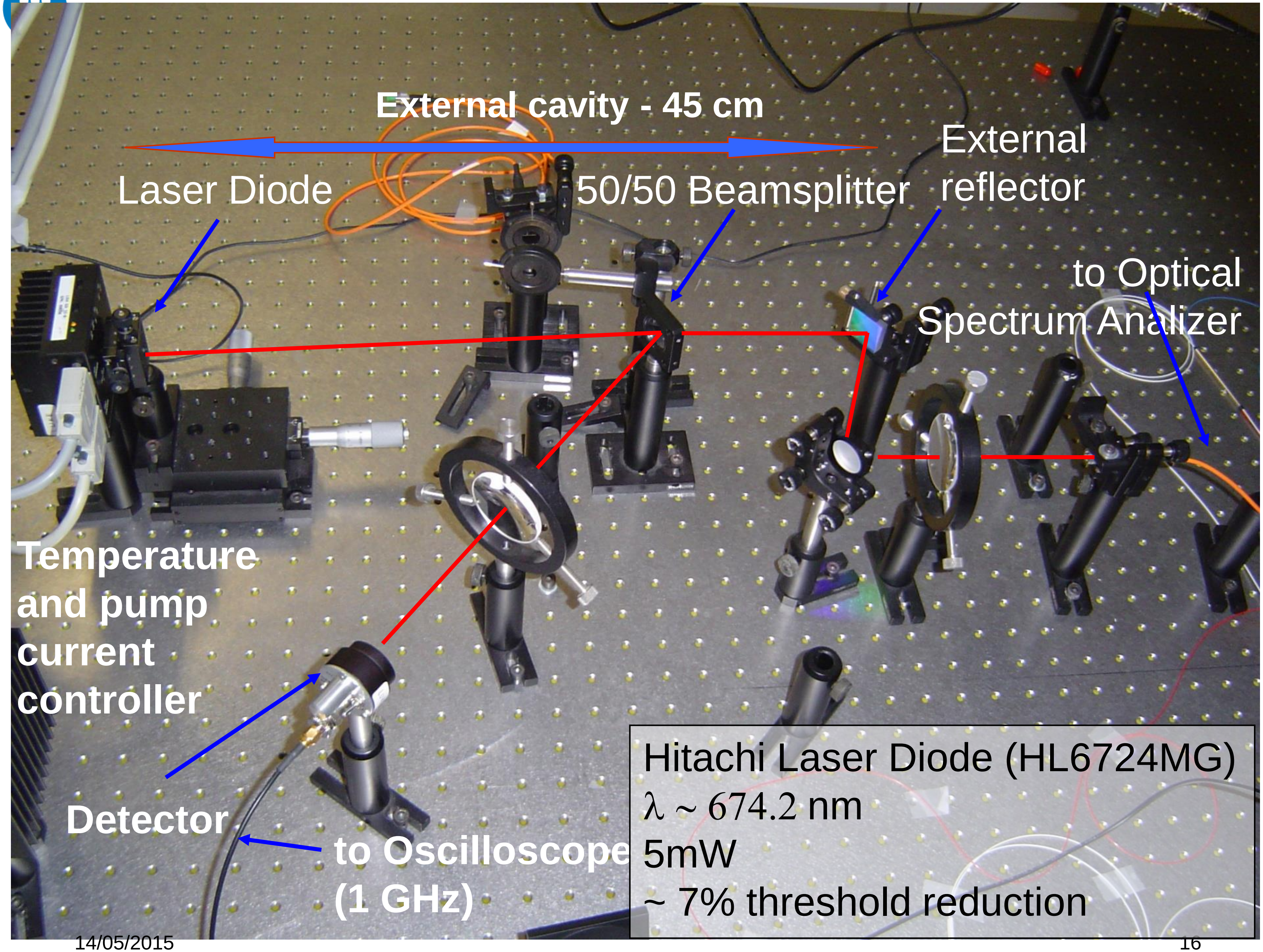

Hitachi Laser Diode (HL6724MG)
$\lambda \sim 674.2$ nm
5mW
~ 7% threshold reduction

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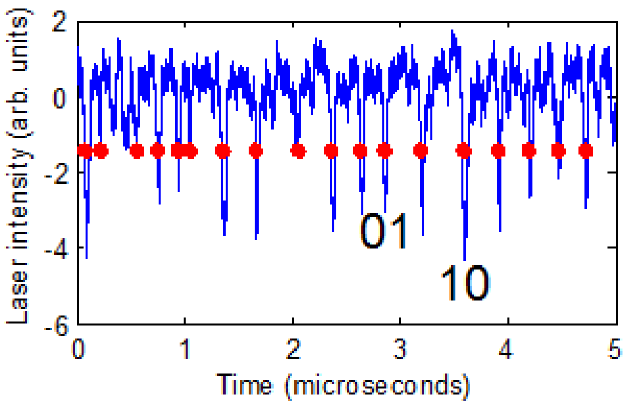
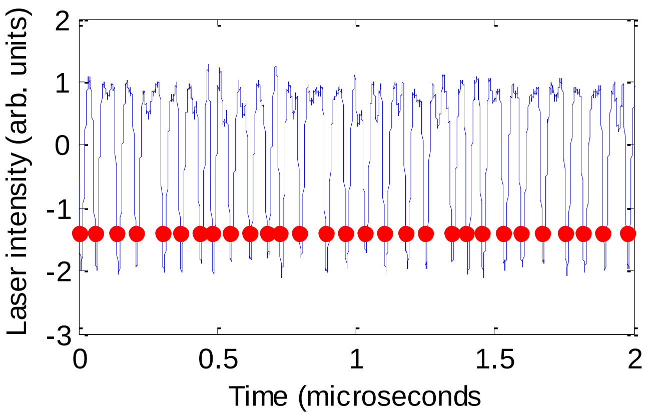
Spiking dynamics

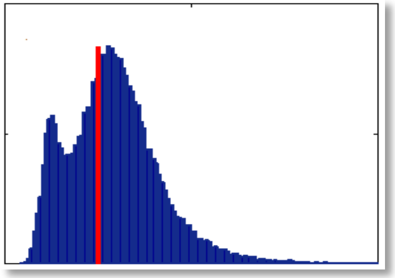
Low pump current

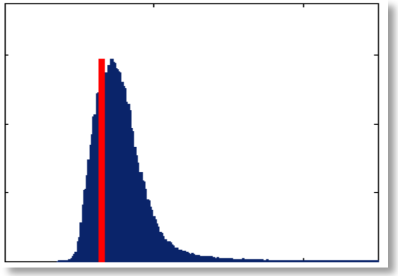


Higher pump current



ISI Histograms

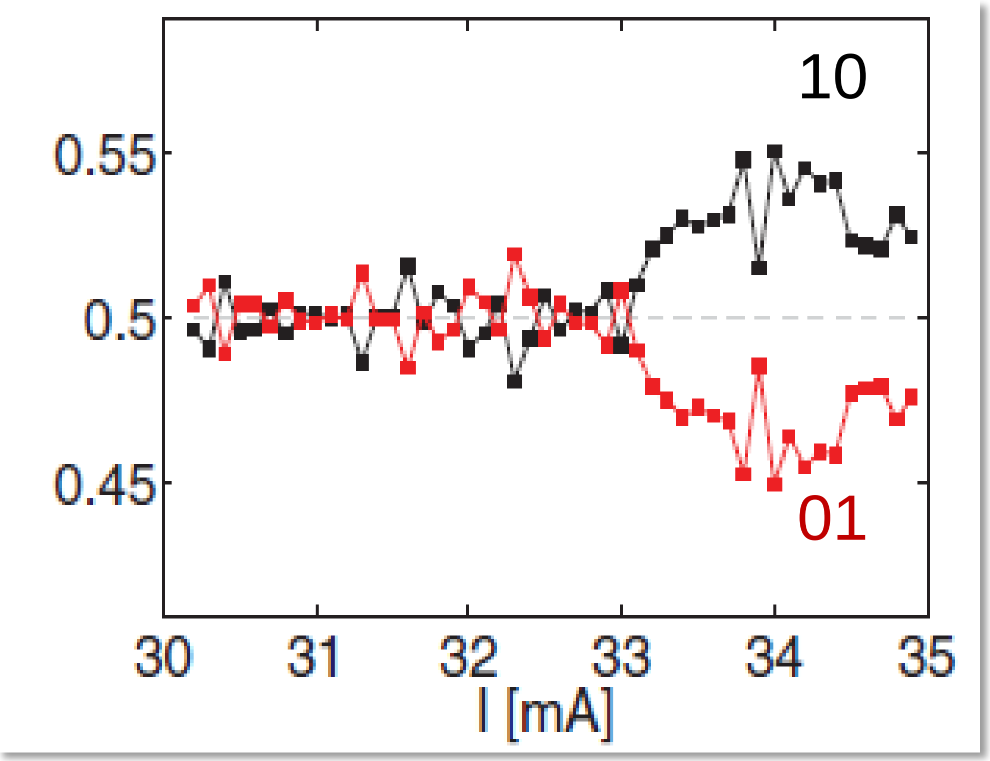




Probabilities of 01 and 10 reveal 3-spike correlations

Null Hypothesis: random spikes ⇒ P(01) = P(10)

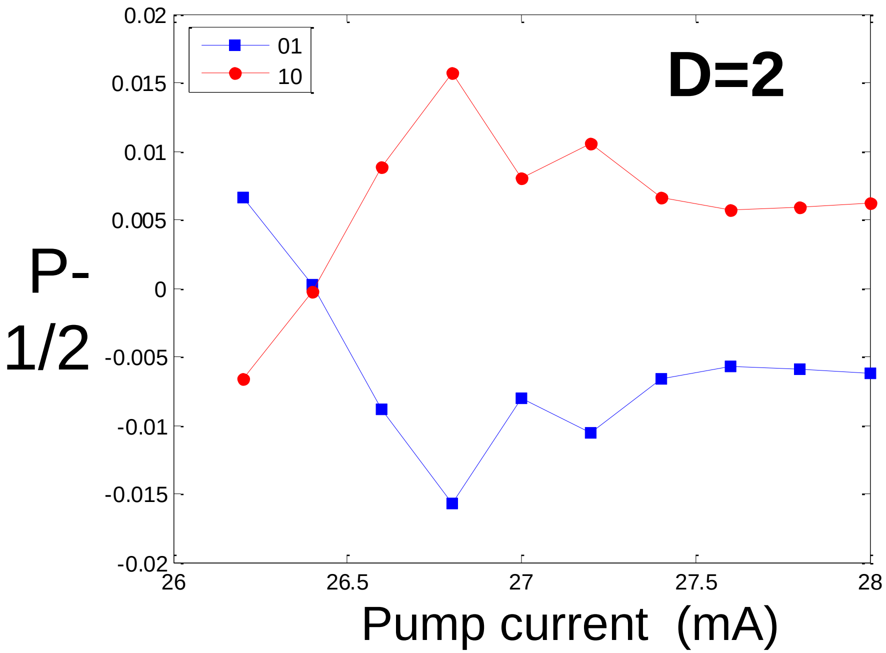
3 spike correlations

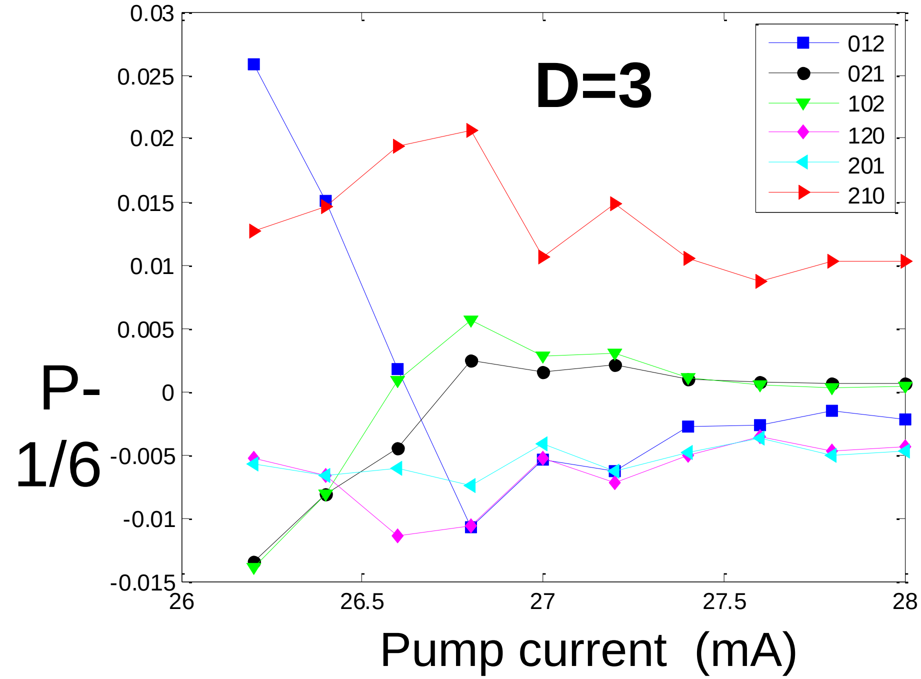


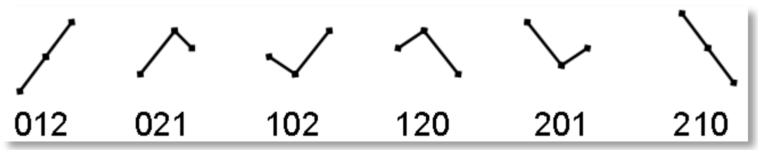
Consistent with stochasticity at low pump current, but signatures of determinism at high pump current.

N. Rubido et al, Phys. Rev. E 84, 026202 (2011)

At low pump current: are the spikes really random? New experiment







45000 - 220000 spikes

A. Aragoneses et al, Scientific Reports 3, 1778 (2013)

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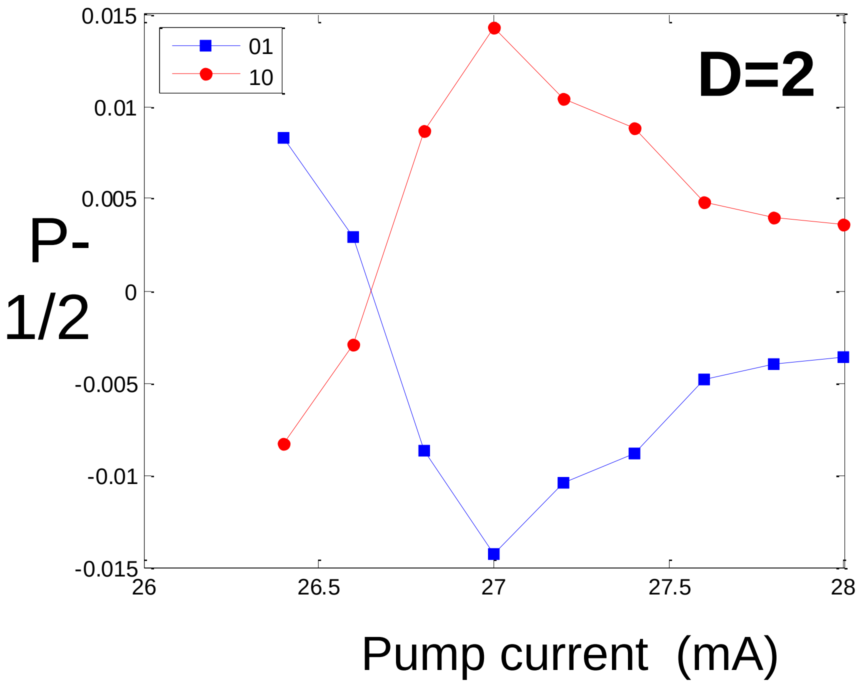
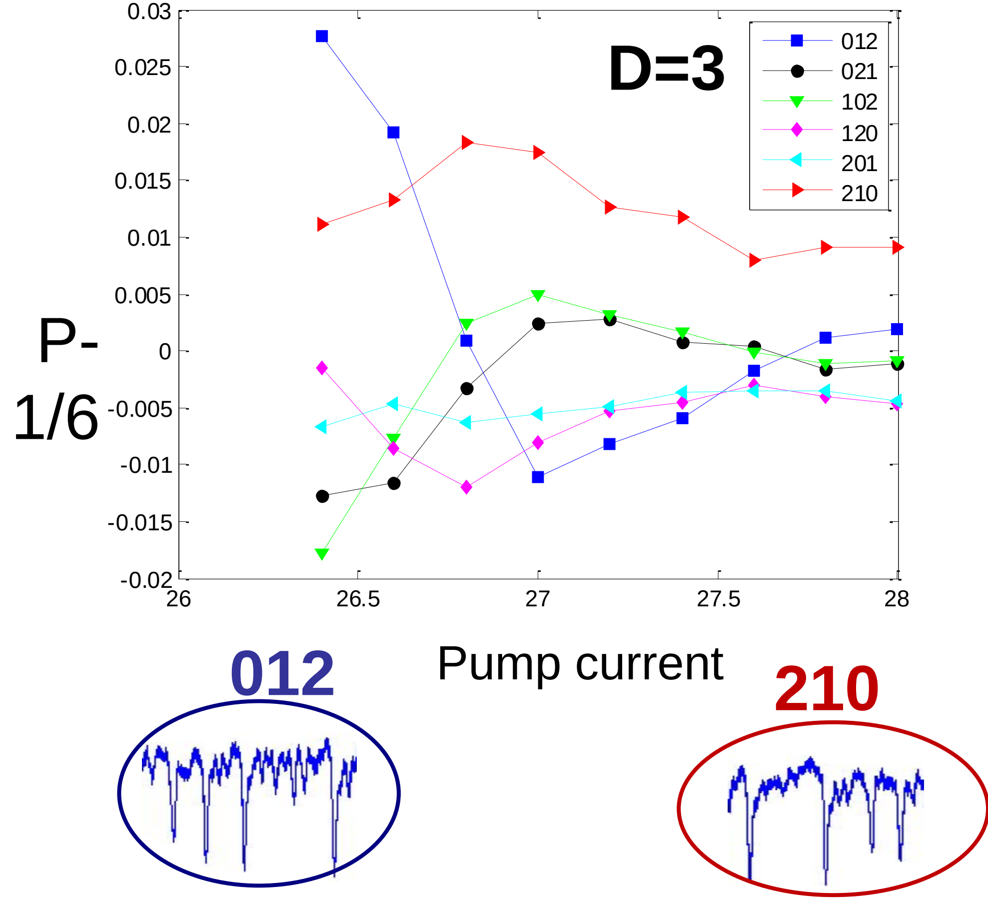
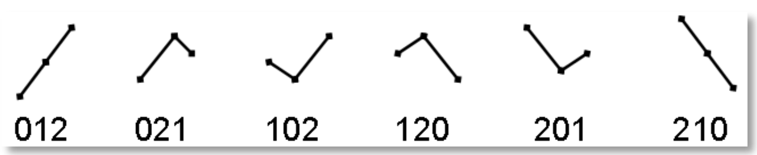
Also in another data set recorded at a different temperature (T=20 C)







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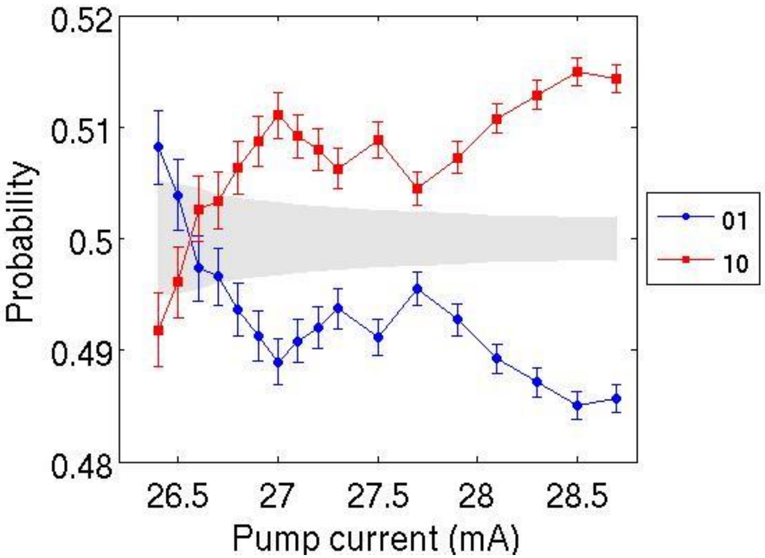
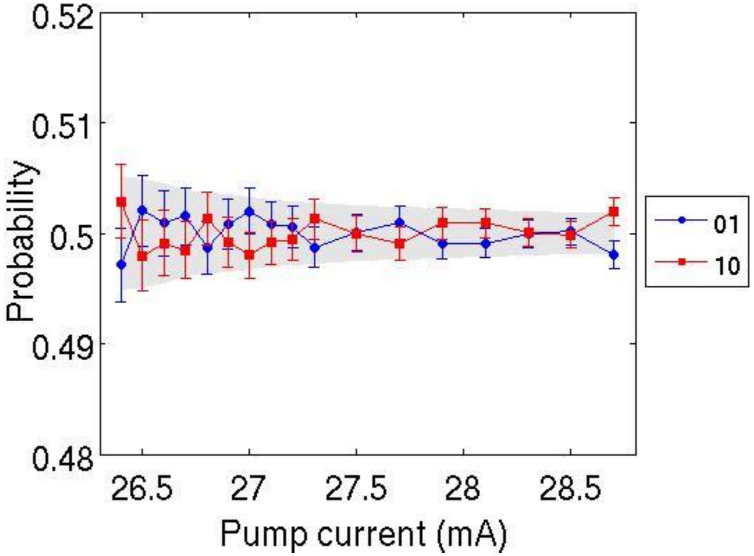
Are the results significant?

Recorded data

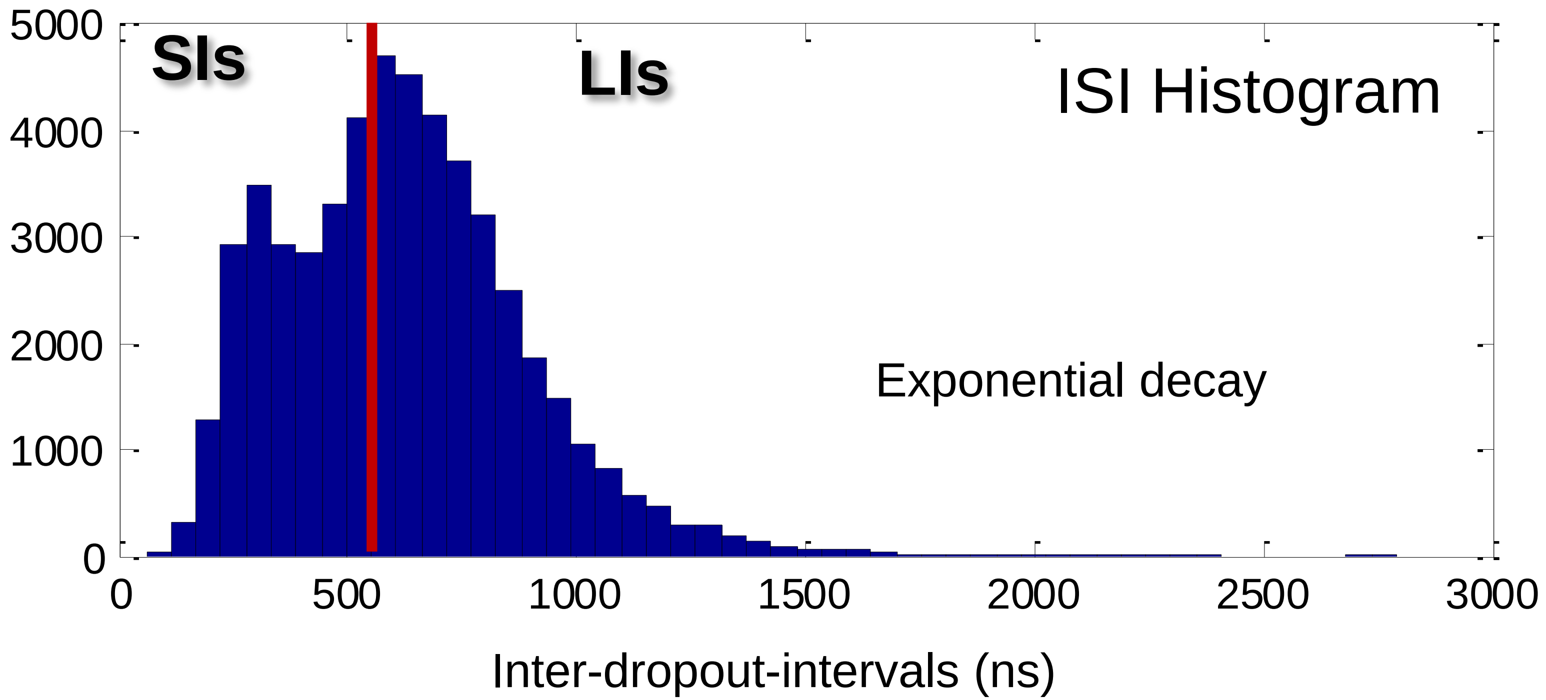


Surrogated data

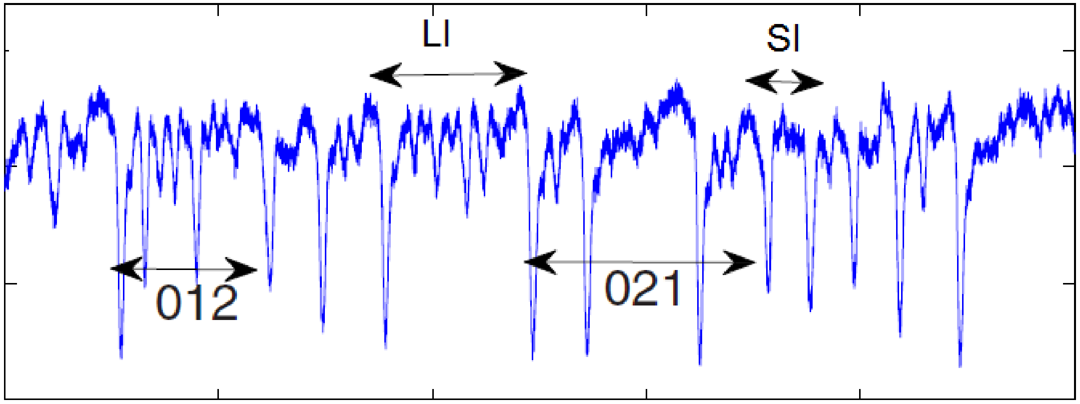


Error bars computed with a binomial test, gray region is consistent with N.H.

Which spikes are triggered by noise?

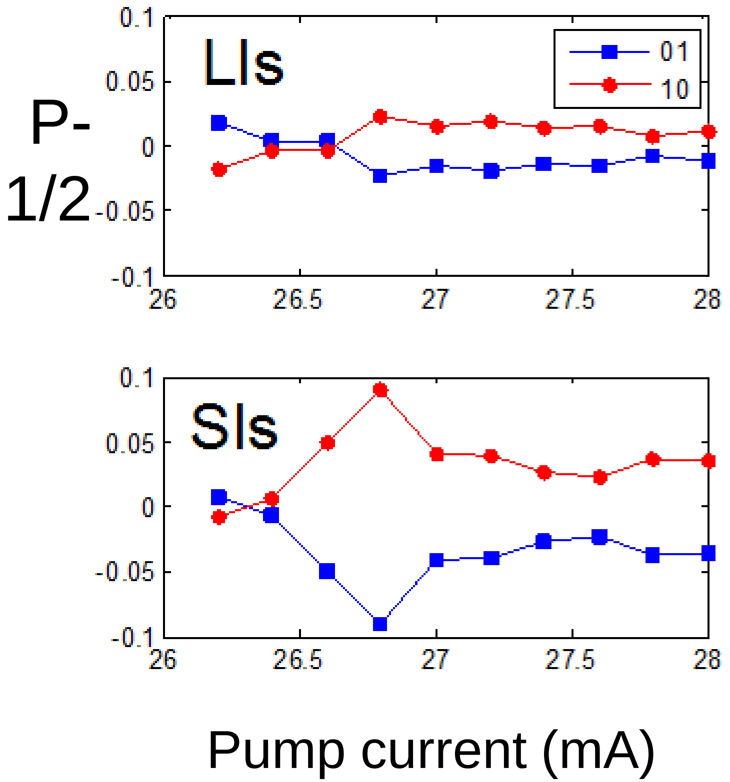


We use a **threshold** to classify the intervals as **short** or **long**



Constructing patterns with 2 consecutive SIs or LIs

- At high current: significant differences
  - LIs consistent with random events
  - SIs more deterministic.
- But at low current, the spikes can not be distinguished.



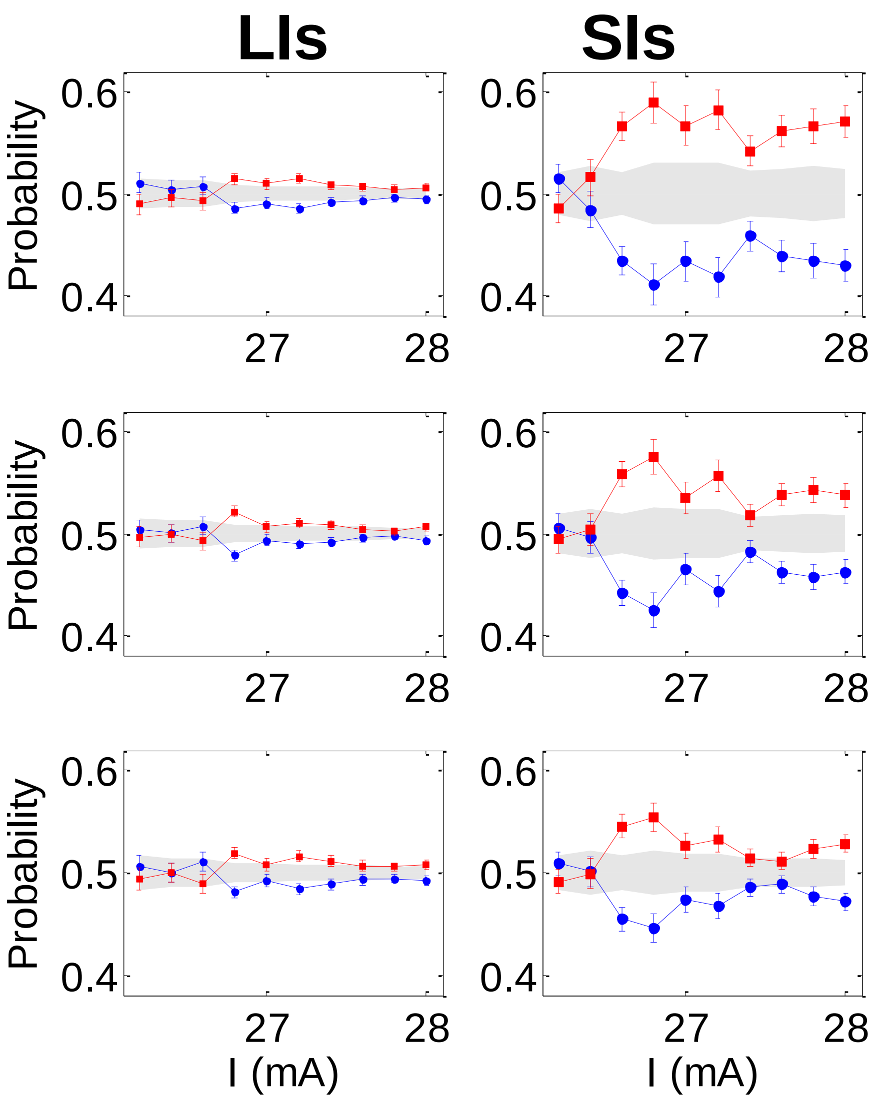
Similar results were obtained with D=3 OPs

Influence of the threshold used to classify as LIs and SIs

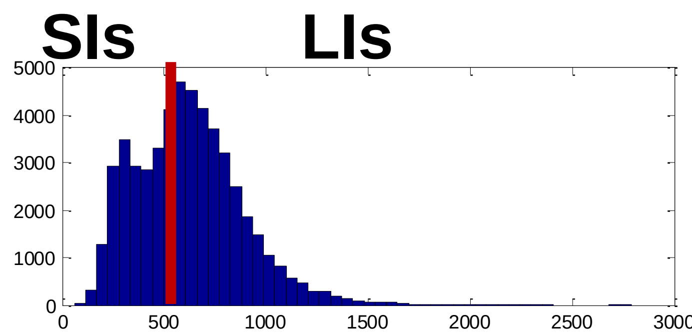
0.85 ΔT*

0.90 ΔT*

0.95 ΔT*



ΔT*= most probable value



Error bars computed with a binomial test, gray region consistent with NH

Tips to chose a good classification threshold

- LIs have statistical features as close as possible to random events:
  - Exponential distribution of values
  - Uniform distribution of pattern probabilities

- Good statistics: there are enough consecutive LIs and SIs
  - The NH region is sufficiently narrow
  - For the LIs, the error bars are in the NH region
  - For the SIs, the error bars are out of the NH region.

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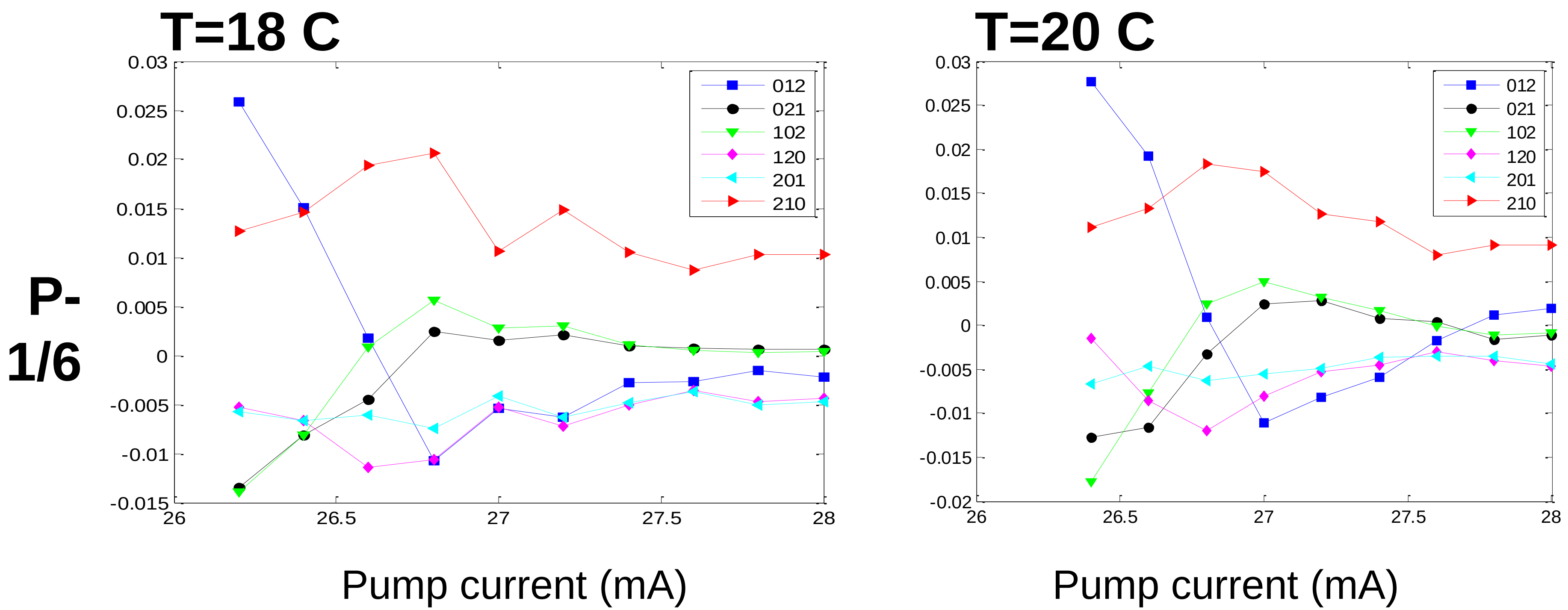
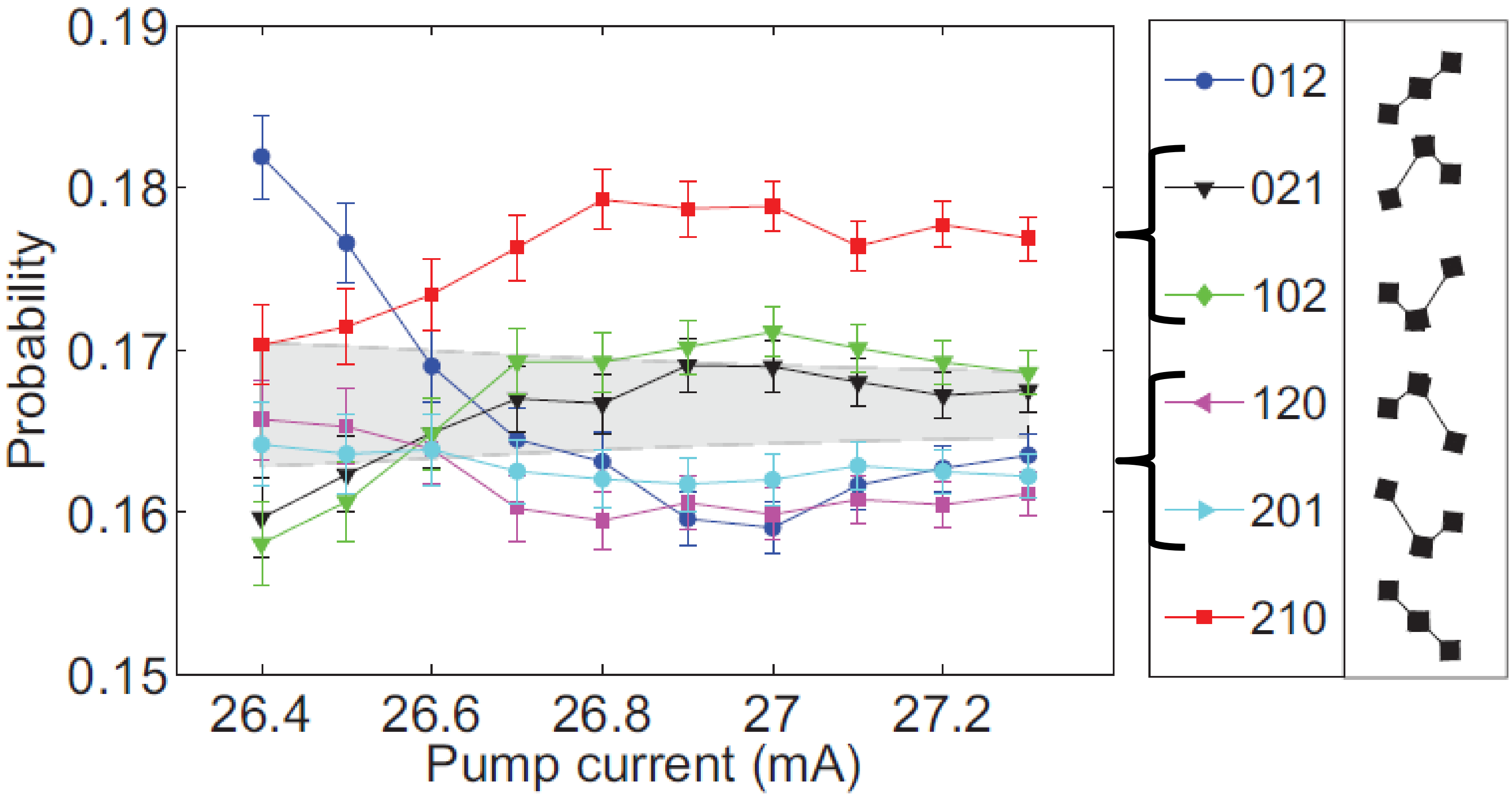
Ordinal analysis unveils new information



There is a hierarchical and clustered organization of the pattern probabilities

In another experiment: the same transition, hierarchy and clusters

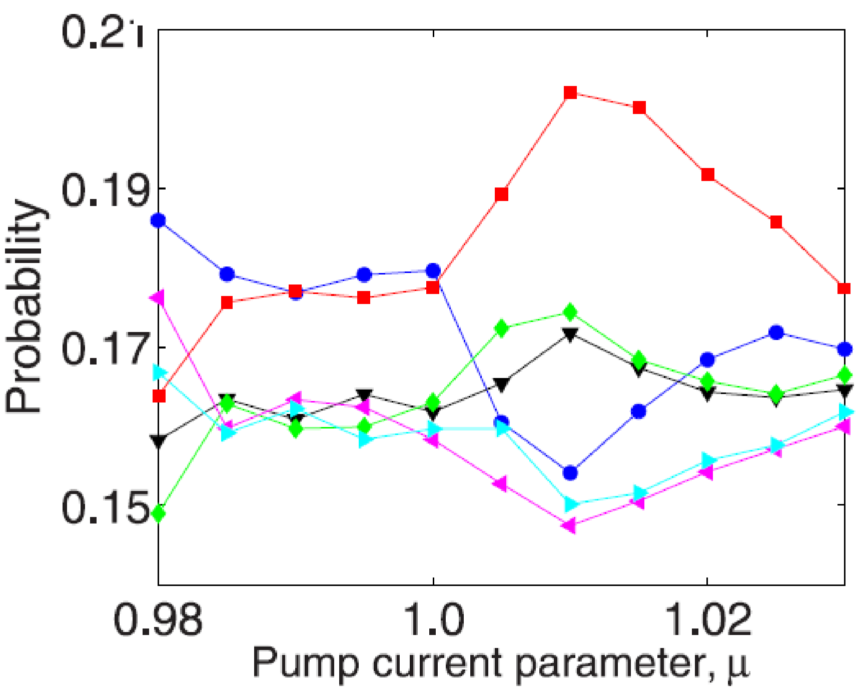


75,000 – 880,000 spikes
(different laser, new oscilloscope)

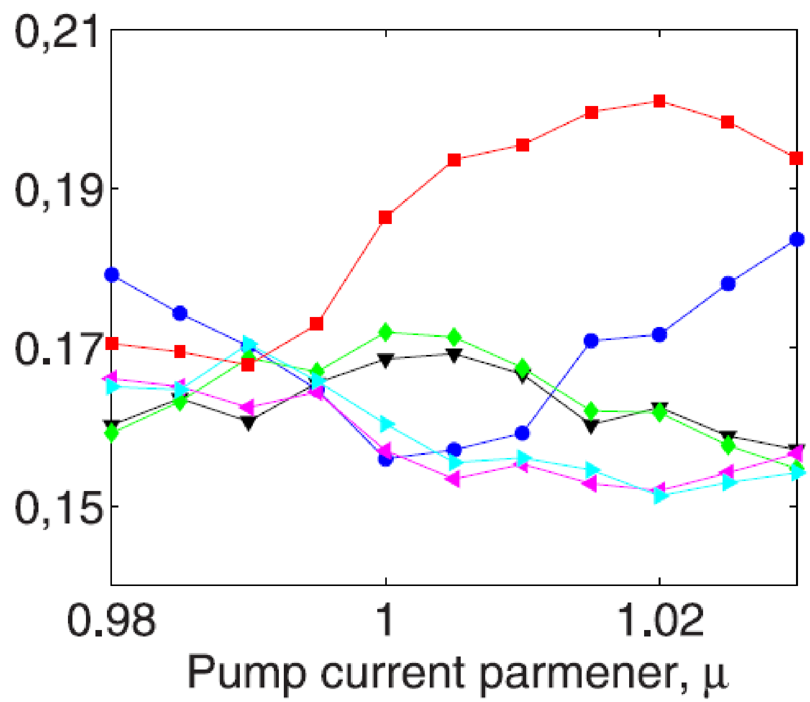
A. Aragoneses et al, Sci. Rep. 4, 4696 (2014)

LK model in good agreement with observations

- Low feedback

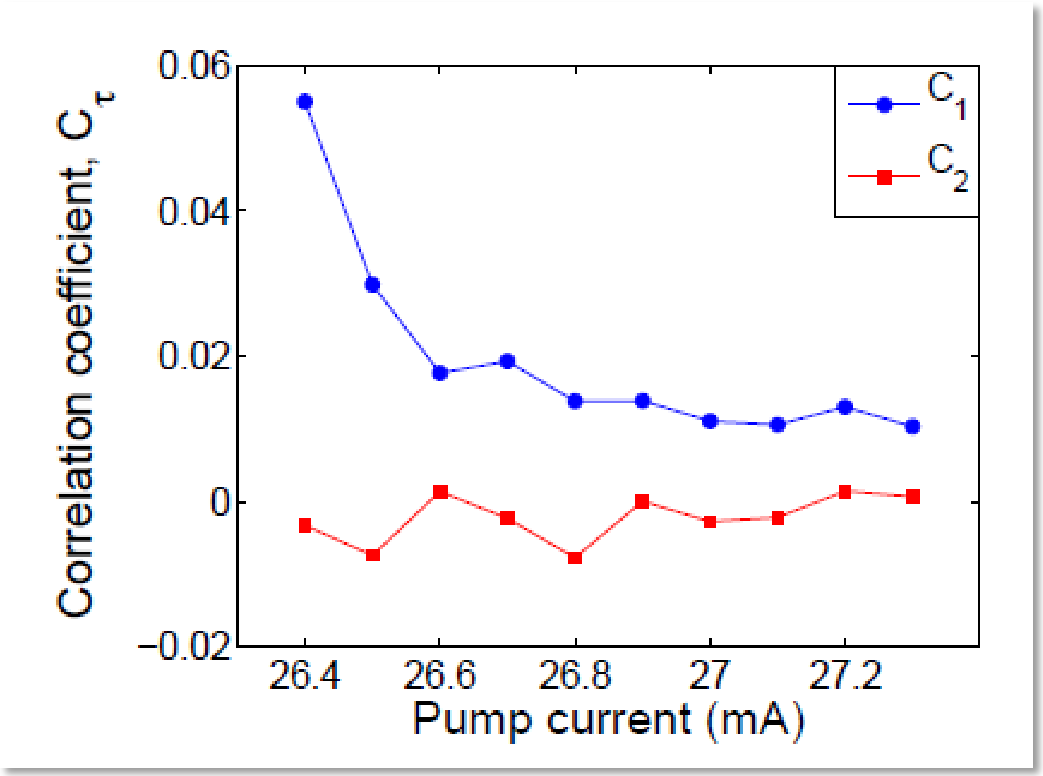


- Stronger feedback



A. Aragoneses et al, Sci. Rep. 4, 4696 (2014)

The transition is not detected by traditional autocorrelation analysis



First and second order correlation coefficient of the empirical ISI sequence

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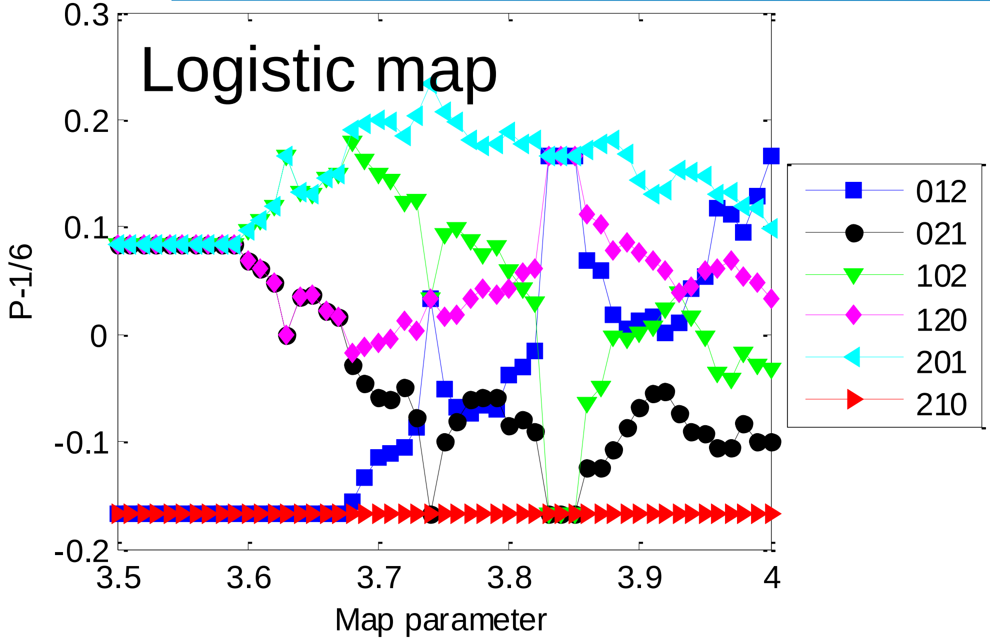
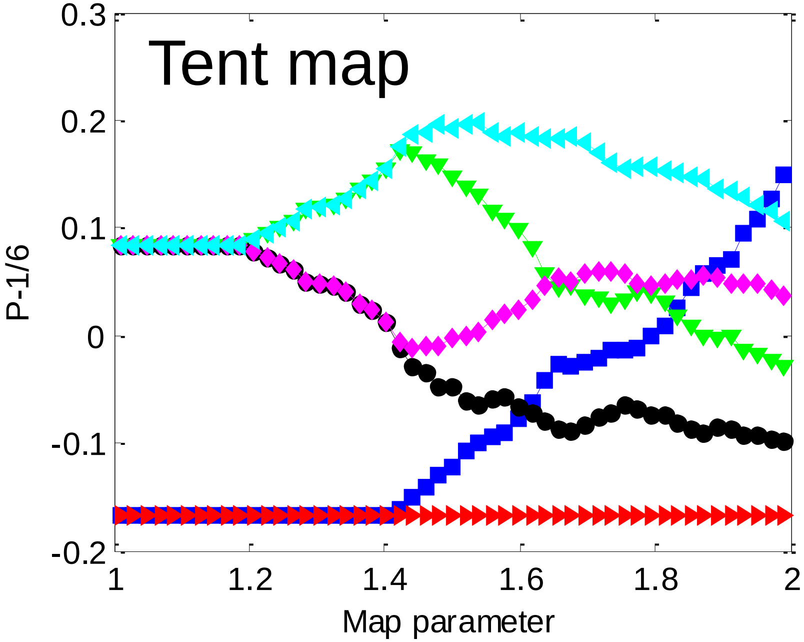
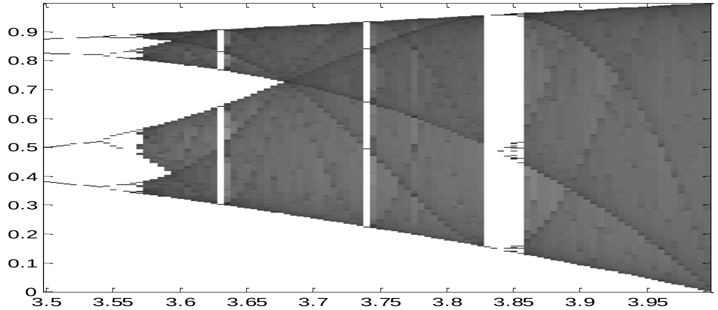
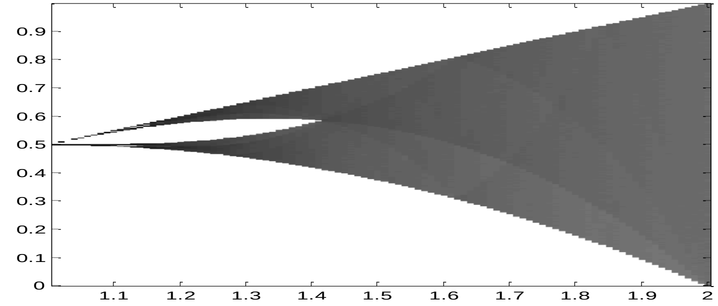
Can we find a minimal model that displays these features?









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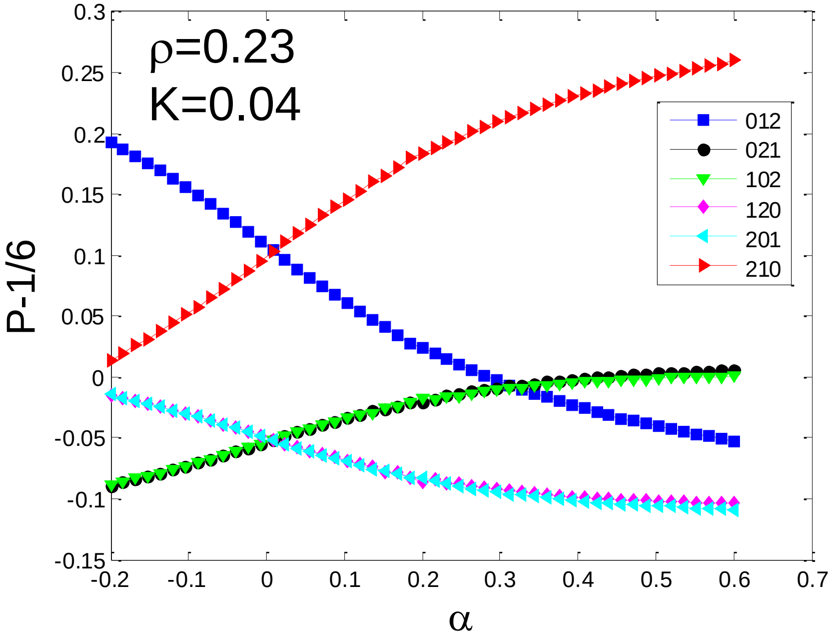
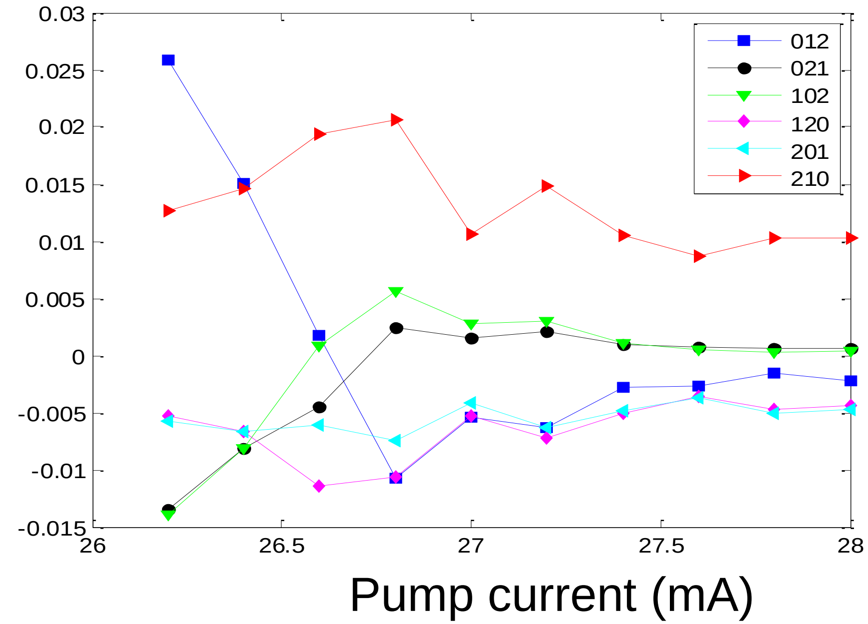
A modified circle map: minimal phenomenological model

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha \sin(4\pi\varphi_i)\right]$$

$$X_i = \varphi_{i+1} - \varphi_i$$





Neiman and Russell, *Models of stochastic biperiodic oscillations and extended serial correlations in electroreceptors of paddlefish*, PRE 71, 061915 (2005)

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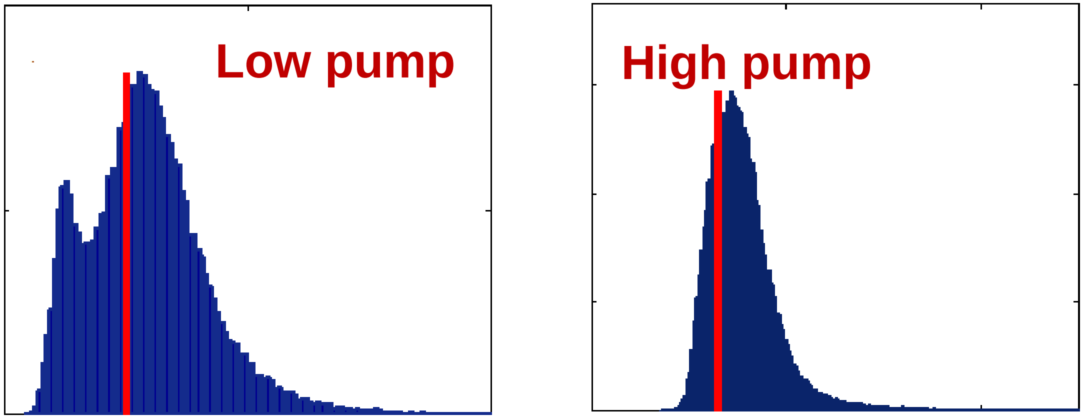
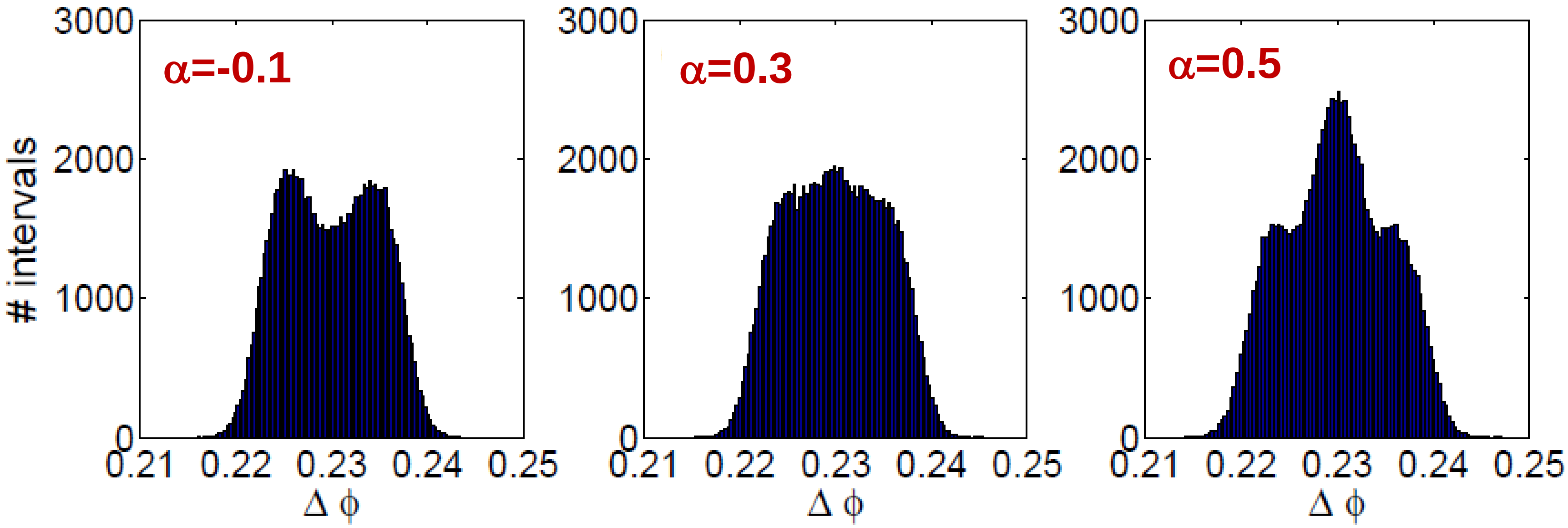
Good minimal model only for ISI correlations; not for ISI distribution





Outline

- Introduction
- Method of time-series analysis and experimental setup
- Results
  - Inferring signatures of determinism
  - Transitions & hierarchical clusters in the symbolic dynamics
  - Minimal model
  - Response to periodic forcing
- Conclusions and take home message

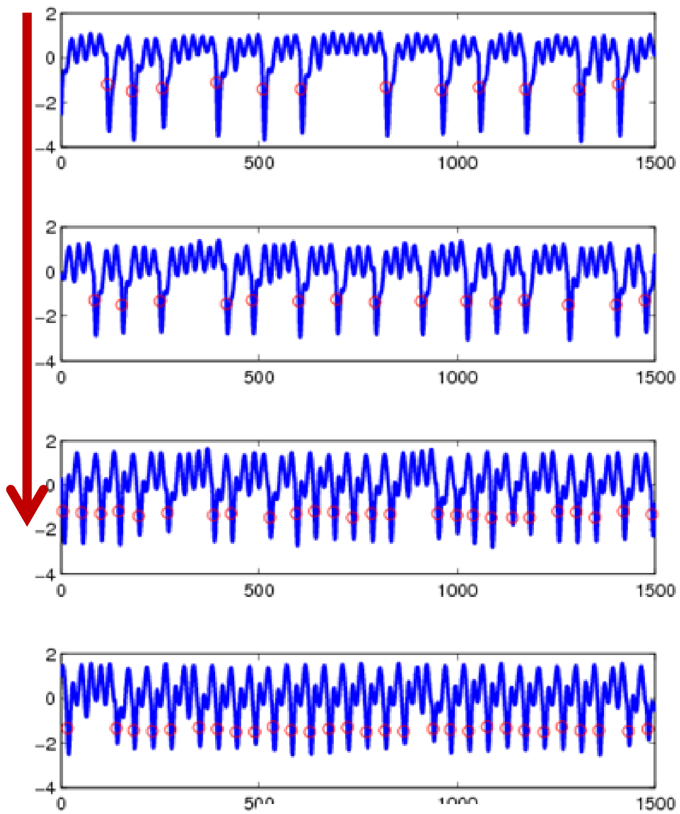
Response to periodic modulation

Relevant for understanding neuronal encoding of external stimuli

Laser intensity:

Increasing modulation amplitude

Tendency to lock

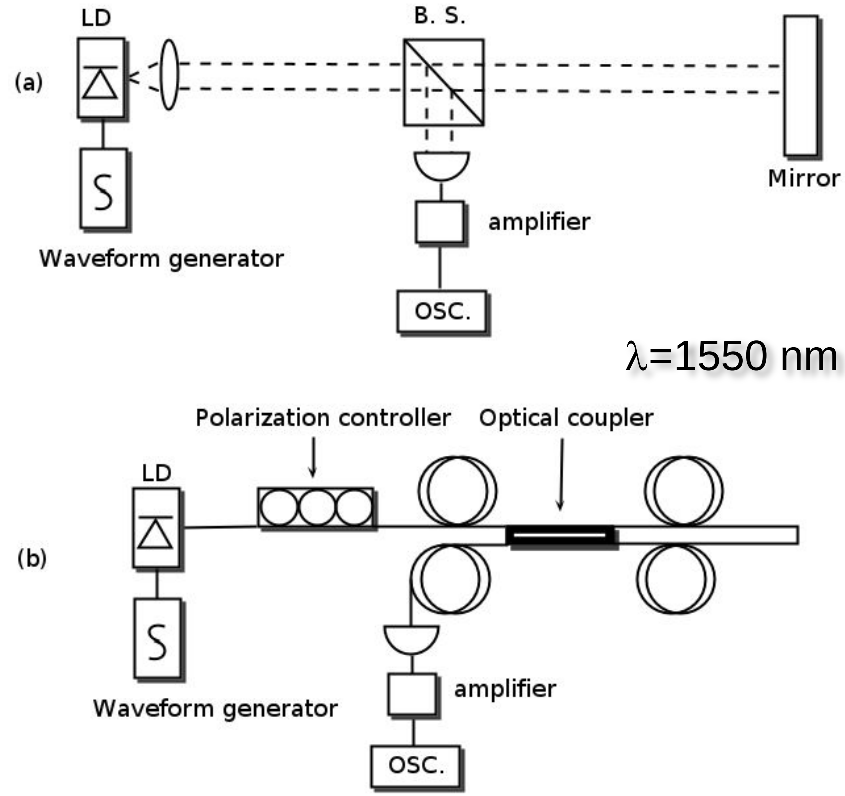


Time (ns)

Two experiments:

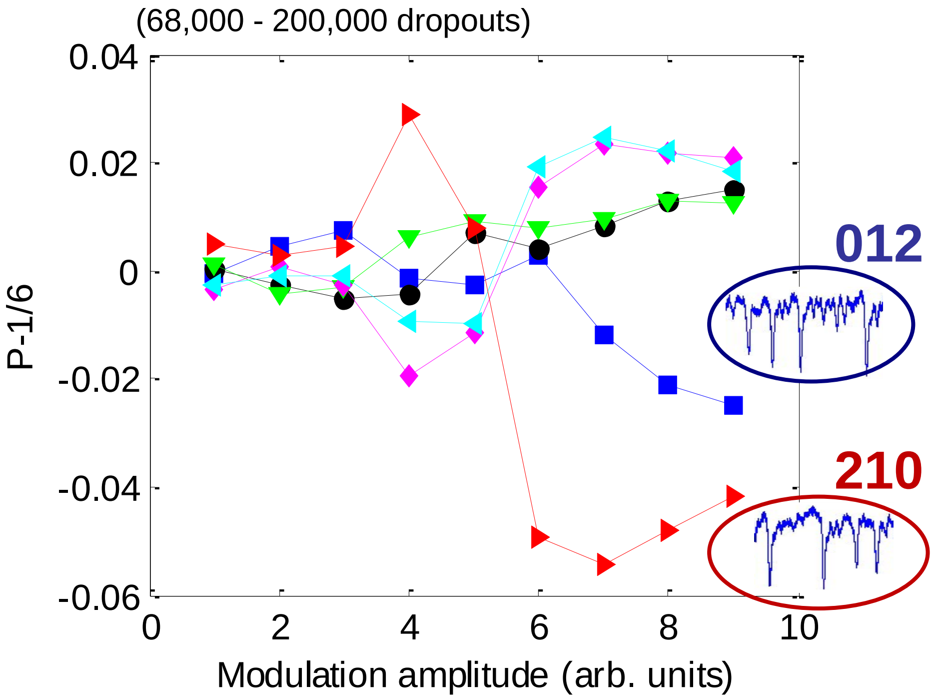
λ=660 nm

λ=1550 nm

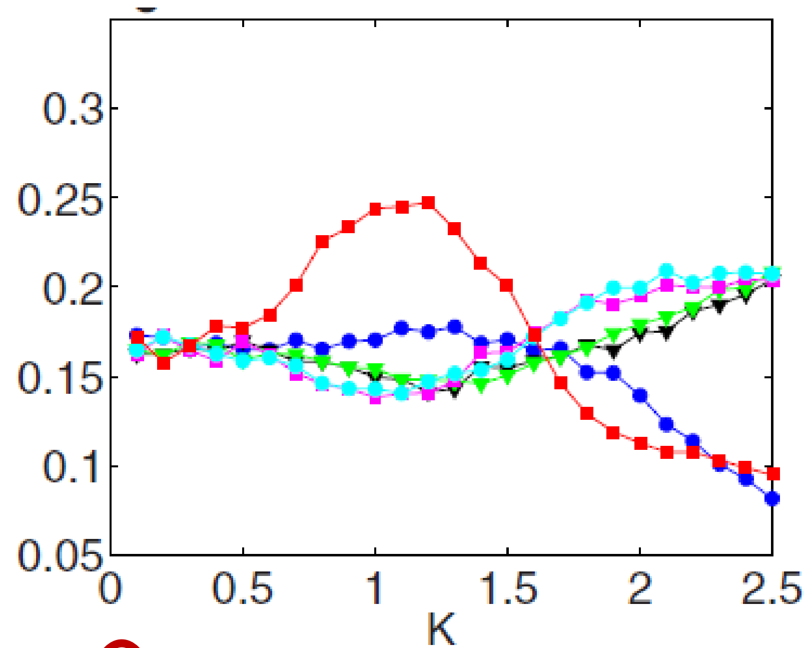


Experiments - minimal model comparison

Experiments @ 660 nm



Circle map

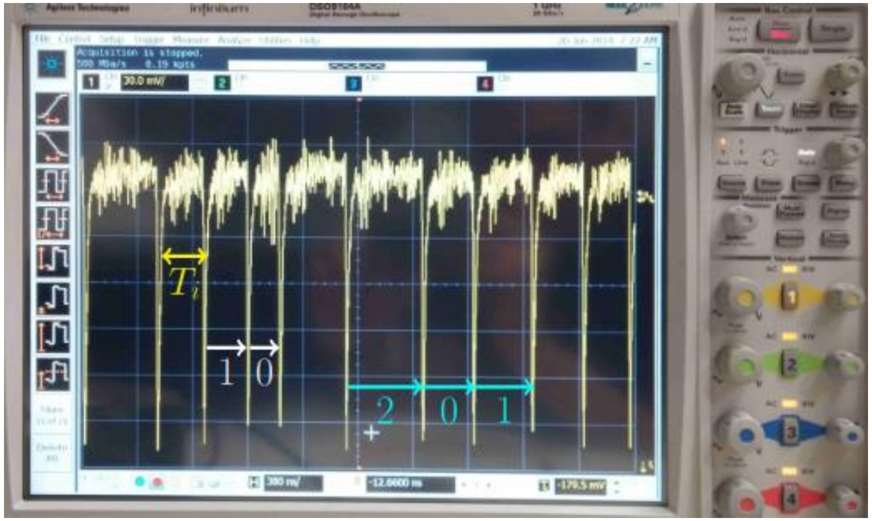


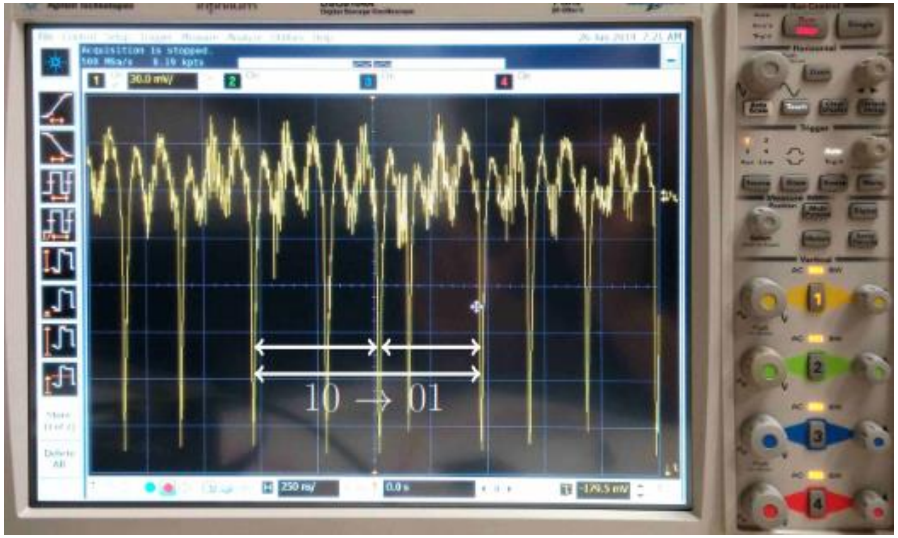
$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha\sin(4\pi\varphi_i)\right] + D\zeta$$

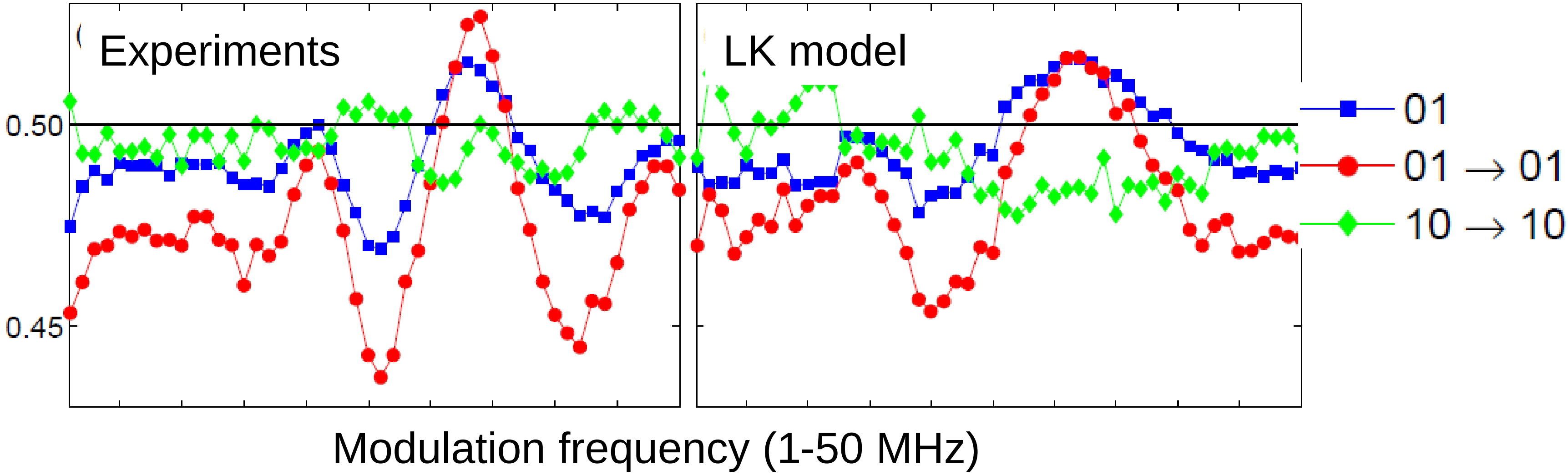
Similar observations @ 1550 nm

Interpretation: locking to external forcing

Experiments – LK model comparison







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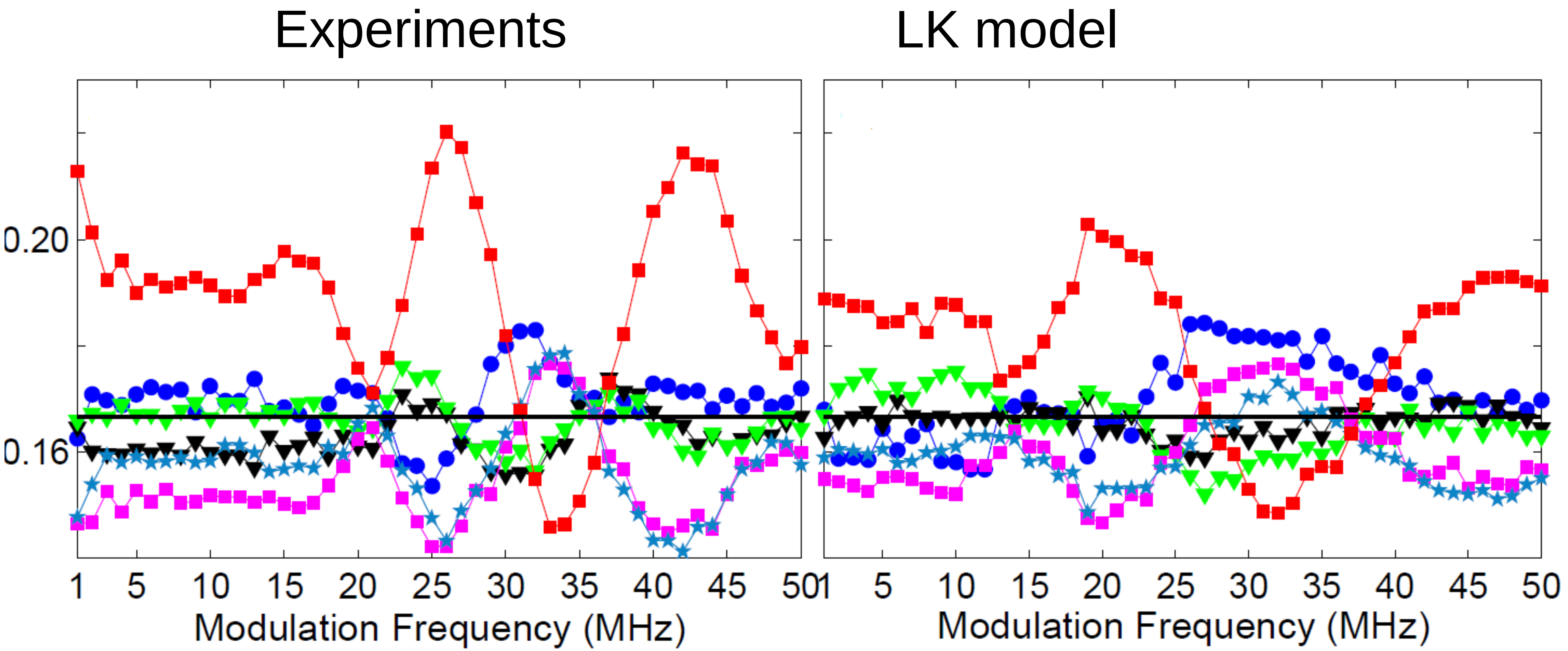
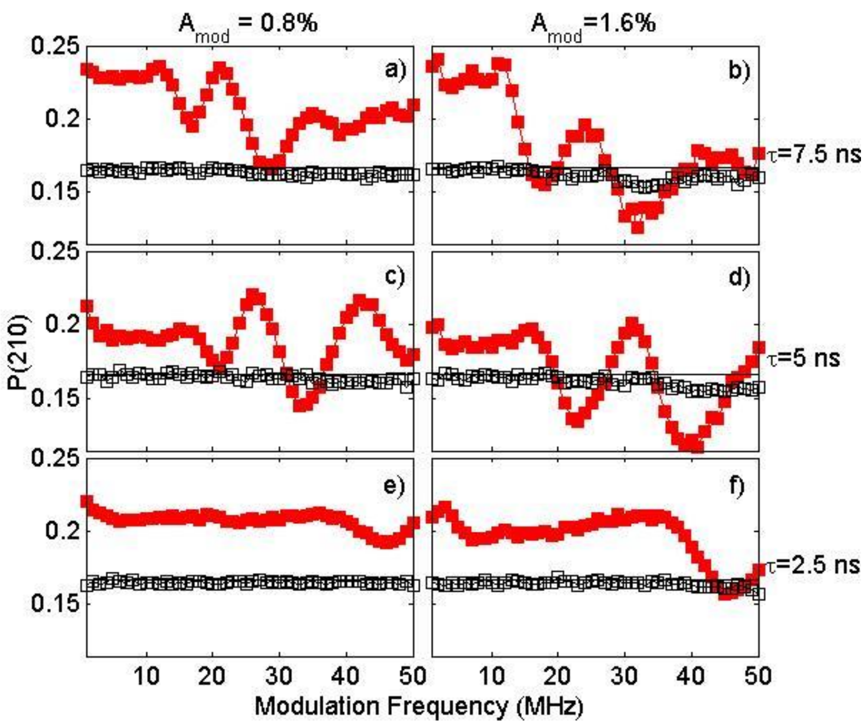
Weak modulation: influence of the modulation frequency



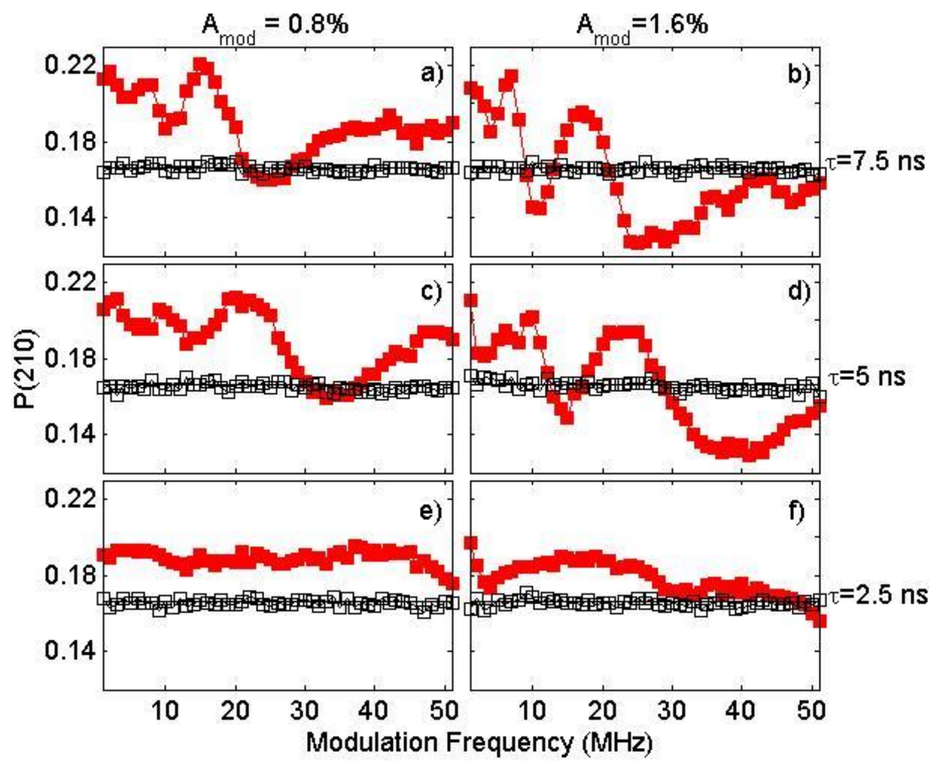
T. Sorrentino et al, Opt. Express 23, 5571 (2015)

Influence of the delay time (controls the natural spike rate)

Experiments



LK model

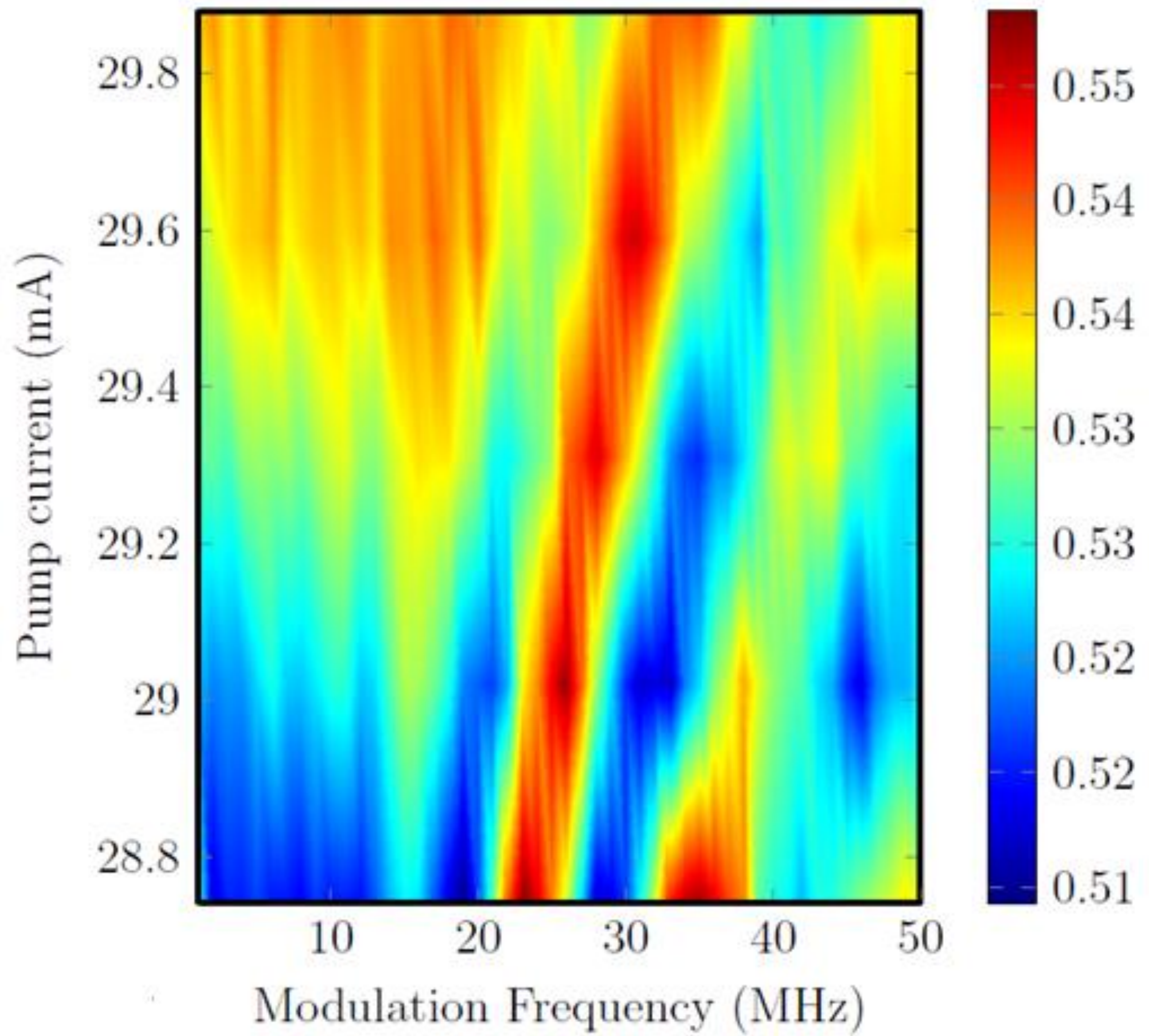


T. Sorrentino et al to appear in IEEE JSTQE (2015)

A valuable tool for identifying noisy n:m locking

Probability of "10" from empirical data.

Pump current: modifies the natural (unmodulated) spike rate



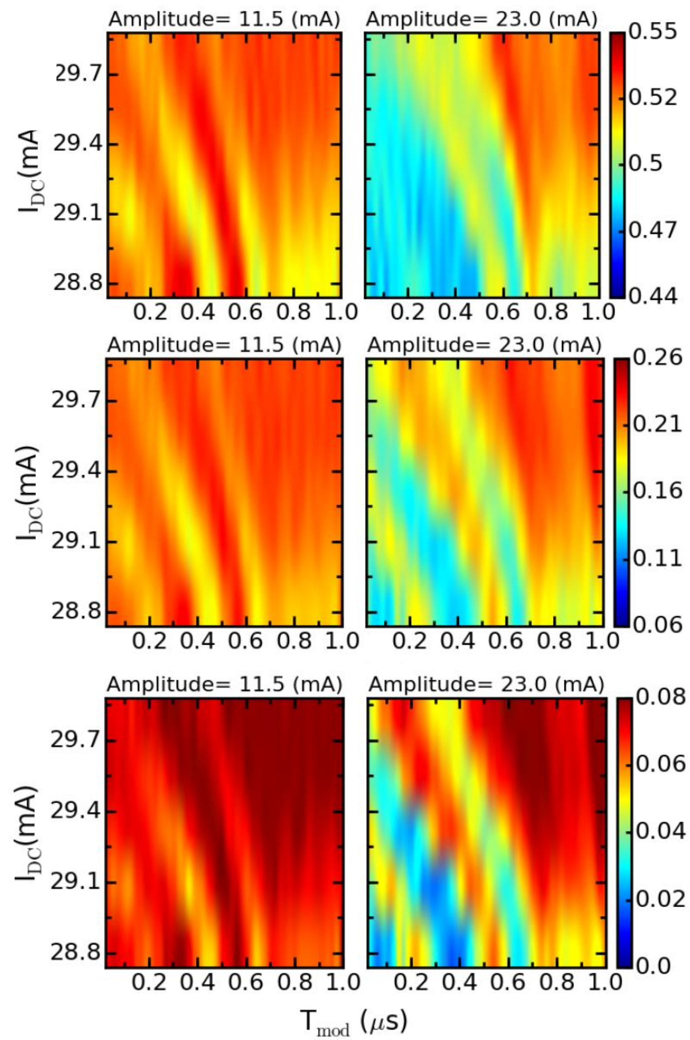
Uncovering longer correlations

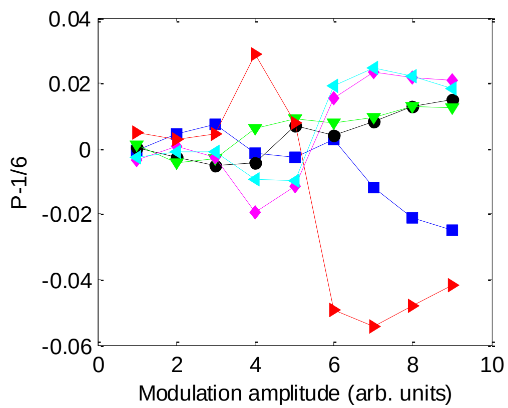
Weak modulation   Stronger modulation

Probability of "10"

Probability of "210"

Probability of "3210"





Outline

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Conclusions

- New method proposed to identify signatures of determinism in the apparently random sequence of spikes of a laser with time-delayed feedback.
- The method allows to classify the optical spikes in two categories.
- New symbolic states found, with a clear hierarchical and clustered organization.
- Very good agreement with time-delayed LK model.
- Minimal model identified. Robust under external forcing.
- Present work: towards trying to understand why the modified circle map is a good minimal model.

NETT

GOBIERNO DE ESPAÑA

MINISTERIO DE CIENCIA E INNOVACIÓN

Take home message

- Stochastic time-delayed systems are complex and high-dimensional.
- Event-level description + ordinal analysis: powerful method to analyze their output signals.

– useful for understanding data, uncovering patterns,

– for model comparison, parameter estimation,

– for classifying events,

– for forecasting events.

Thank you for your attention!

Papers @ **www.fisica.edu.uy/~cris**

– J. Zamora-Munt et al, Phys. Rev A 81, 033820 (2010).
– N. Rubido et al, Phys. Rev. E 84, 026202 (2011).
– A. Aragoneses et al, Scientific Reports 3, 1778 (2013).
– A. Aragoneses et al, Scientific Reports 4, 4696 (2014).
– T. Sorrentino et al, Opt. Express 23, 5571 (2015).