Introduction to nonlinear time series analysis tools

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ICREA

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Outline

- Univariate time-series analysis
- Ordinal analysis
- Information theory measures
- Bivariate time-series analysis
- Applications

Nonlinear time-series analysis

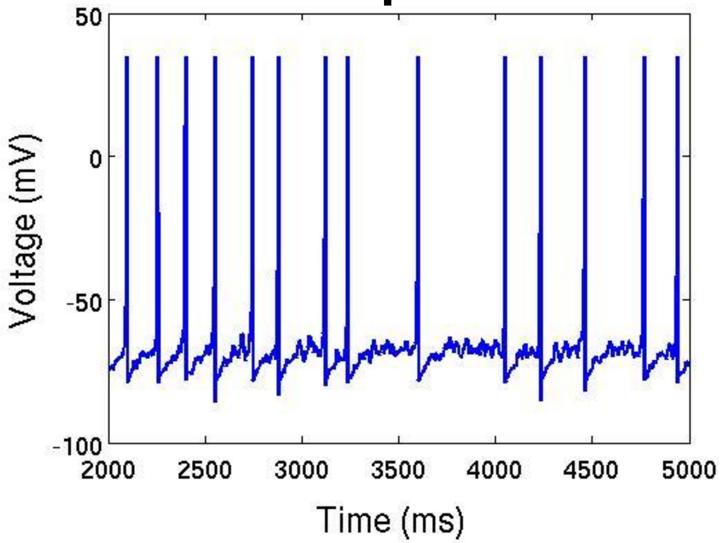
- Many methods have been developed to test for determinism, nonlinearity and correlations in data generated from complex systems (biomedical, geoscience, socio-economical, etc).
- The appropriateness of the method depends on the data
  - short or long;
  - stationary or not;
  - more or less noisy;
  - multi or single channel measurements,
  - discrete or continuous values,
  - etc.
- Different methods provide complementary information.

Visual inspection

- First step: Look at the data. Examine simple properties: auto correlation, Fourier spectrum, return map (x_i vs $x_{i+\tau}$), histogram, etc.

Optical spikes

Neuronal spikes



Similar type of processes generate these output signals?

Event-like description of a signal

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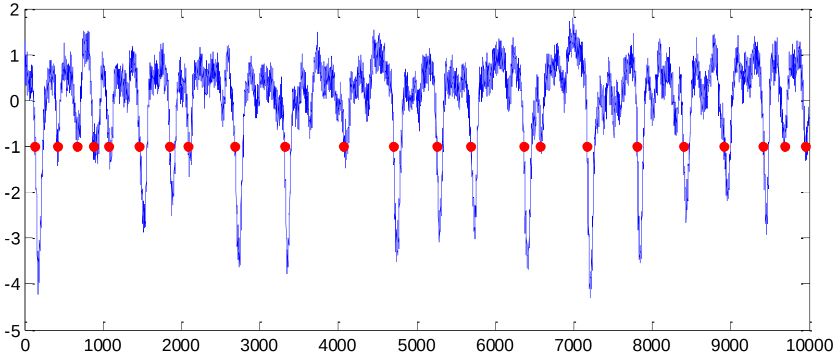
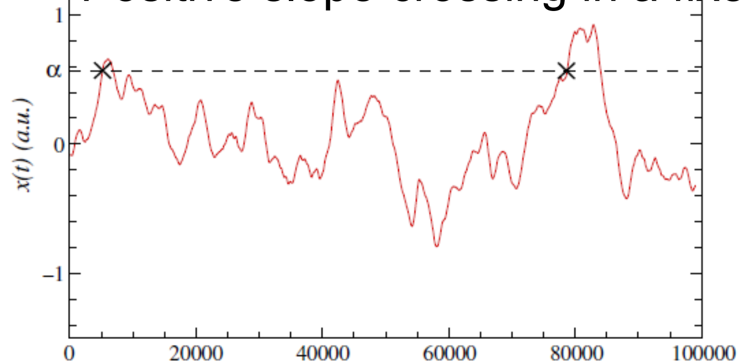
- Threshold crossings

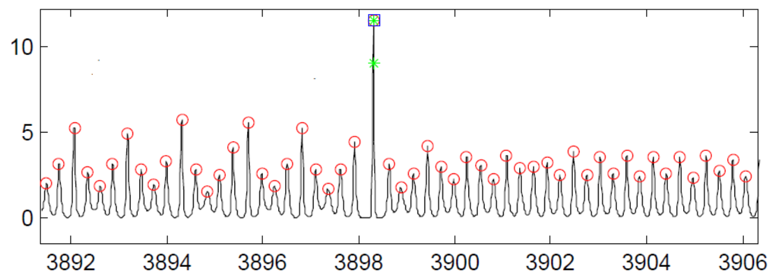


Positive slope crossing in a fixed α level

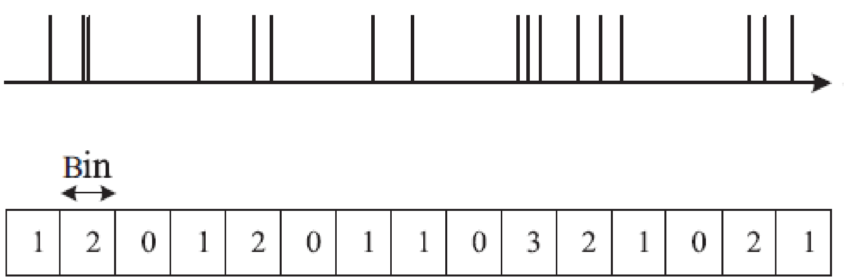


R. Friedrich et al. / Physics Reports 506 (2011) 87–162

- Extreme values



- Bin counting



… and many others.

Time intervals between events can be statistically independent (renewal) or not statistically independent (non-renewal process).

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Histogram analysis of response to periodic stimulation

Neuron inter-spike interval (ISI) distribution

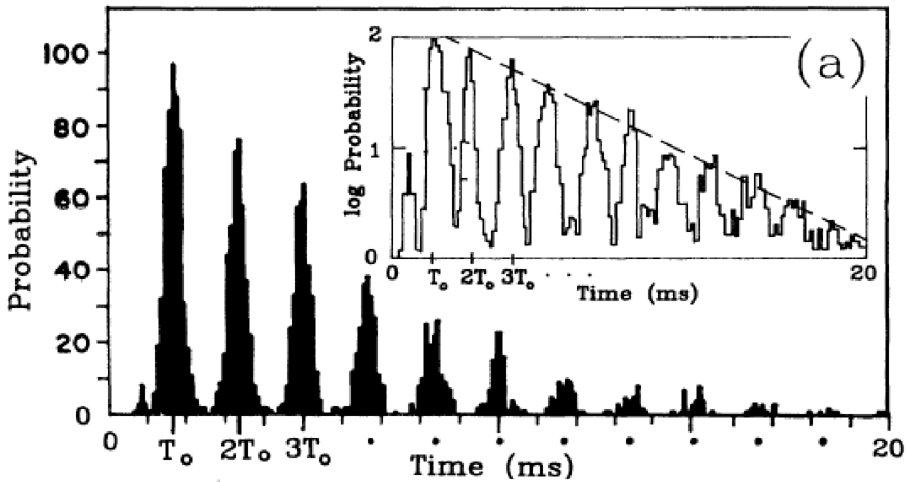
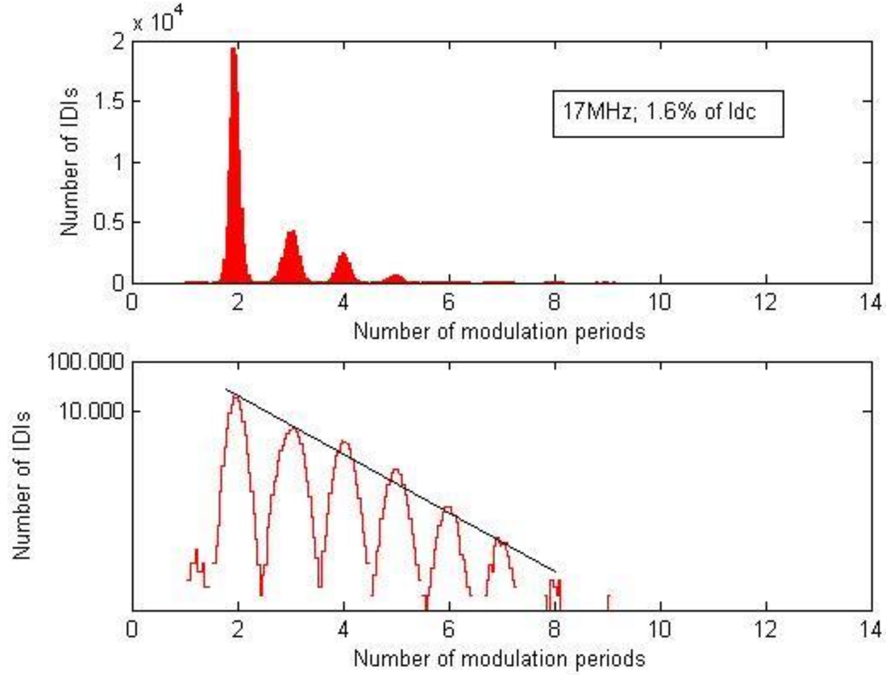


FIG. 1. (a) An experimental ISIH obtained from a single auditory nerve fiber of a squirrel monkey with a sinusoidal 80-dB sound-pressure-level stimulus of period $T_0 = 1.66$ ms applied at the ear. Note the modes at integer multiples of T_0. Inset:

A. Longtin et al, PRL 67 (1991) 656

Optical ISI distribution, data collected in our lab



when a sinusoidal signal is applied to the laser current

Return maps: ΔT_i vs. ΔT_{i+1}

Neuronal ISIs

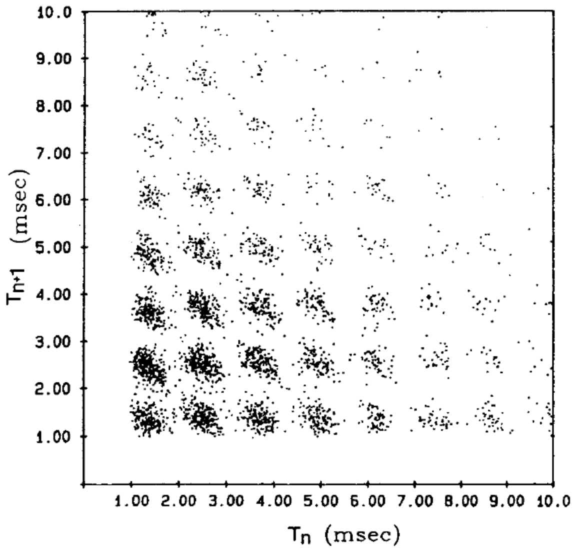
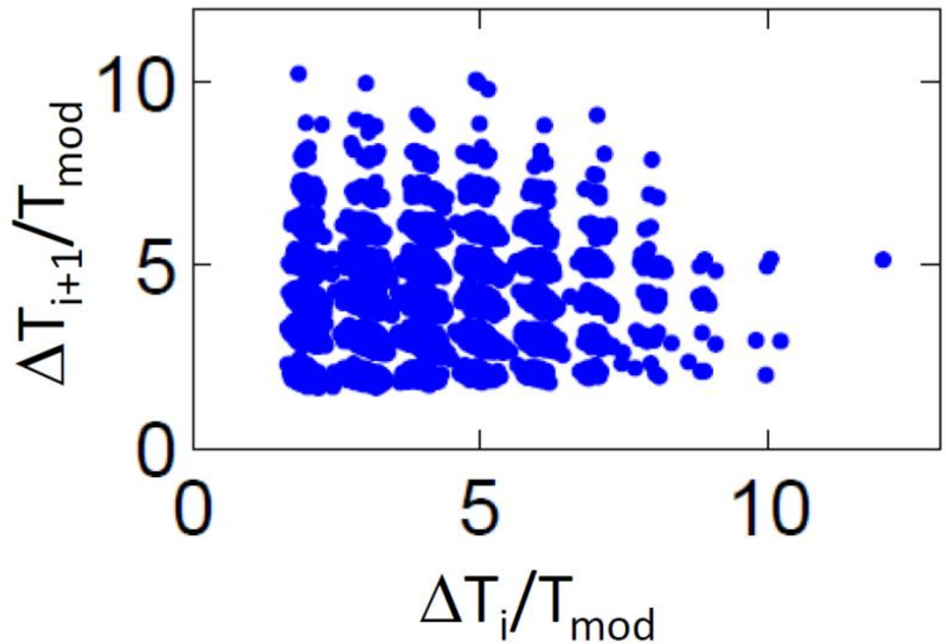


Fig. 4. Scatter plot of spike train data obtained from extracellular measurements of cat auditory fiber activity in response to an 800 Hz 60 dB sound pressure level pure tone presented to the outer ear. The stimulus is discontinuous (see

A. Longtin IJBC 3 (1993) 651

Optical ISIs



when a sinusoidal signal is applied to the laser current

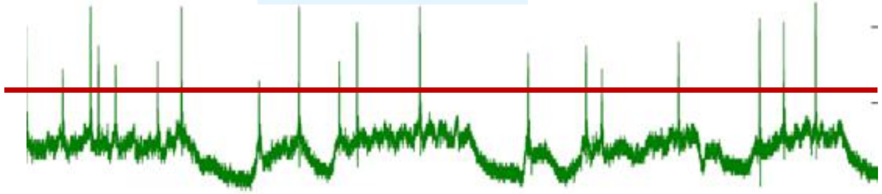
A. Aragoneses et al, Opt. Exp. (2014)

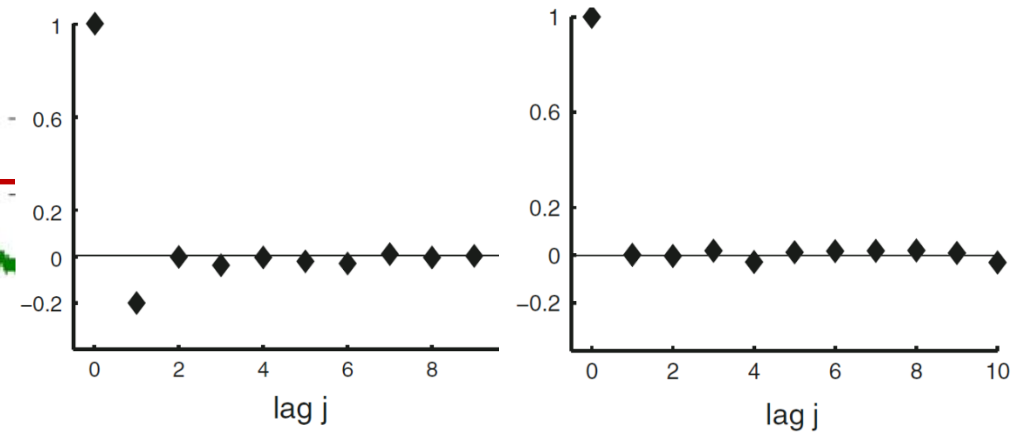
Inter-spike-intervals serial correlation coefficients

$$\{\ldots I_{i-1}, I_i, I_{i+1} \ldots\}$$

$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

$$I_i = t_{i+1} - t_i$$





Exp Brain Res (2011) 210:353–371

HOW TO INDENTIFY TEMPORAL STRUCTURES?
RECURRENT / INFREQUENT PATTERNS?

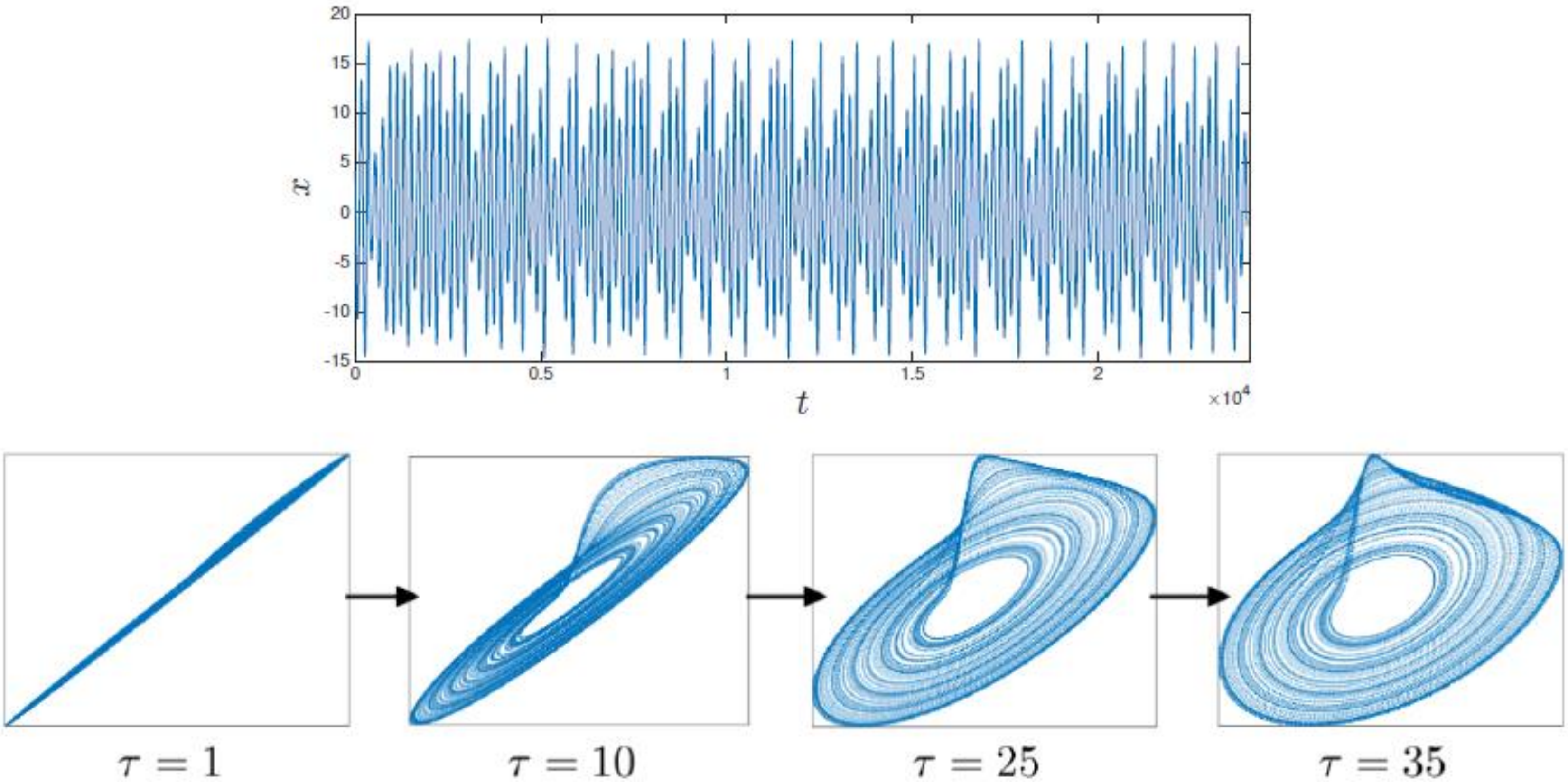
Next step: detect hidden structures

Several approaches to identify patterns and temporal ordering in the sequence

- Phase-space reconstruction methods
  - Time-delay coordinates
  - Derivative coordinates
- Symbolic methods
- Mapping the time series into a network

They allow for model verification, forecasting, classification of different types of behaviors, noise reduction, etc.

Reconstruction using delay coordinates



A problem: finding good embedding (lag τ, dimension d)

Bradley and Kantz, CHAOS 25, 097610 (2015)

Read more

Practical implementation of nonlinear time series methods: The TISEAN package

CHAOS VOLUME 9, 1999

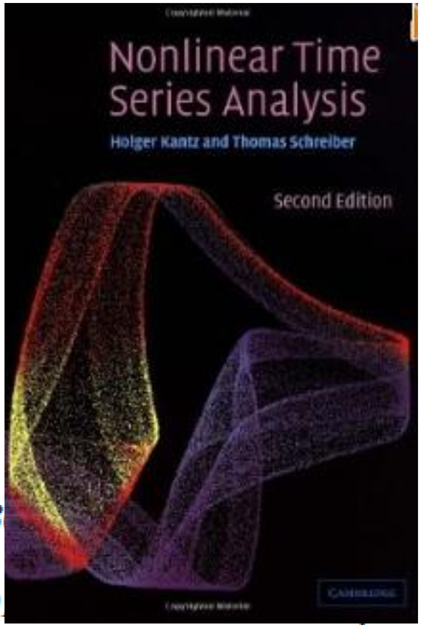
Rainer Hegger and Holger Kantz[a)]
Max Planck Institute for Physics of Complex Systems, Nöthnitzer Str. 38, D-01187 Dresden, Germany

Thomas Schreiber
Department of Physics, University of Wuppertal, D-42097 Wuppertal, Germany

CHAOS 25, 097610 (2015)

Nonlinear time-series analysis revisited

Elizabeth Bradley[1,a)] and Holger Kantz[2,b)]
[1]*Department of Computer Science, University of Colorado, Boulder, Colorado 803*
Santa Fe Institute, Santa Fe, New Mexico 87501, USA
[2]*Max Planck Institute for the Physics of Complex Systems, Noethnitzer Str. 38 D, 0*



Symbolic analysis

- The time series $\{x_1, x_2, x_3, \ldots\}$ is transformed (using an appropriated rule) into a sequence of symbols $\{s_1, s_2, \ldots\}$
- taken from an "alphabet" of possible symbols $\{a_1, a_2, \ldots\}$.
- Then consider "blocks" of D symbols ("patterns" or "words").
- All the possible words form the "dictionary".
- Then analyze the "language" of the sequence of words
  - the probabilities of the words,
  - missing/forbidden words,
  - transition probabilities,
  - information measures (entropy, mutual information, etc).

Threshold transformation (phase space partition)

- if $x_i > x_{th} \Rightarrow s_i = 0$; else $s_i = 1$

  transforms a time series into a sequence of 0s and 1s, e.g., {011100001011111…}
- Considering "blocks" of D letters gives the sequence of words. Example, with D=3:

  {011 100 001 011 111 …}
- The number of words (patterns) grows as **2^D**
- More thresholds allow for more letters in the "alphabet" (and more words in the dictionary). Example:

  if $x_i > x_{th1} \Rightarrow s_i = 0$;
  else if $x_i < x_{th2} \Rightarrow s_i = 2$;
  else $(x_{th2} < x_i < x_{th1}) \Rightarrow s_i = 1$.

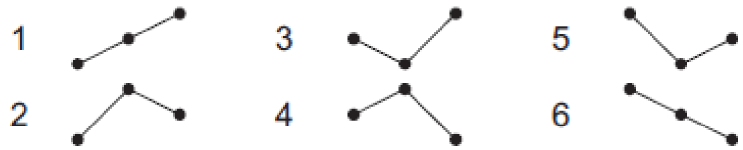
Alternative symbolic rule

- Ordinal transformation:

  if $x_i > x_{i-1} \Rightarrow s_i = 0$; else $s_i = 1$

  also transforms a time-series into a sequence of 0s and 1s <u>without</u> using a threshold

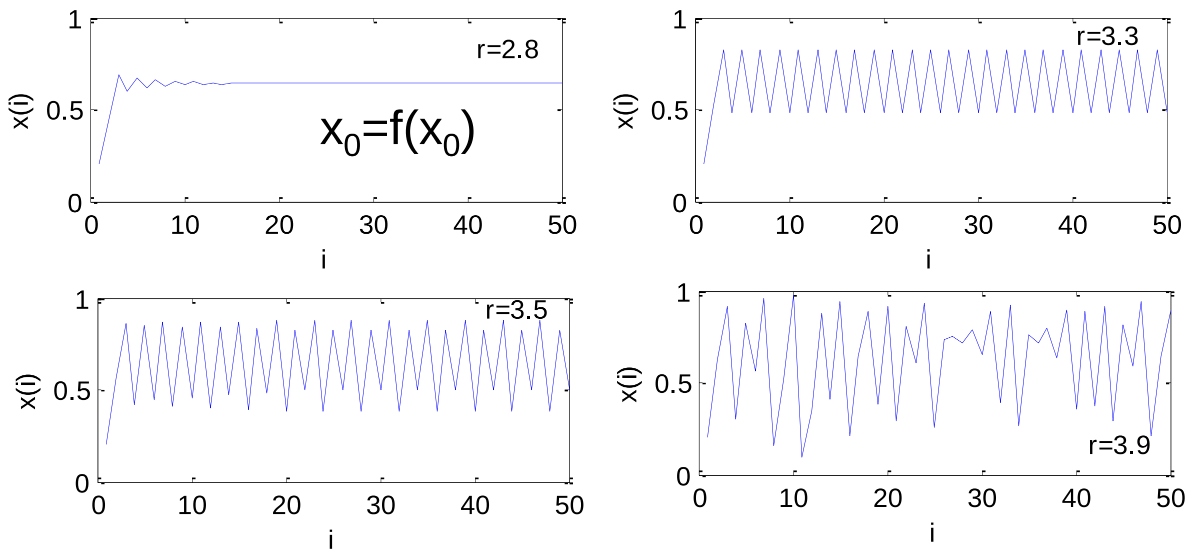
- "words" of D letters are formed by considering the order relation between sets of D values $\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$.

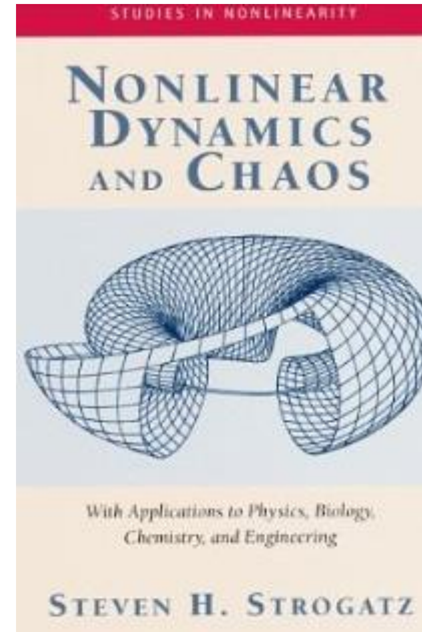


- The number of patterns grows as **D!**

Example: the logistic map

- $x(i+1)=\mathbf{r}\ x(i)[1-x(i)]$



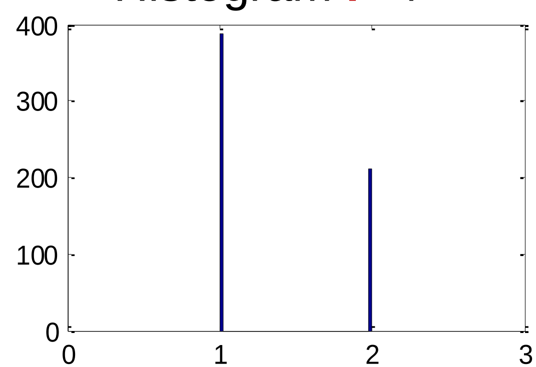


- Rule:

if $(x_i < x_{i+1}) \Rightarrow s_i = 1$;

else $s_i = 2$

Histogram **r**=4



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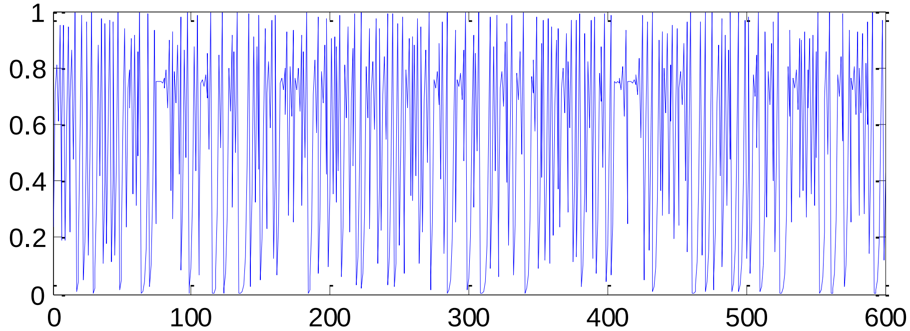
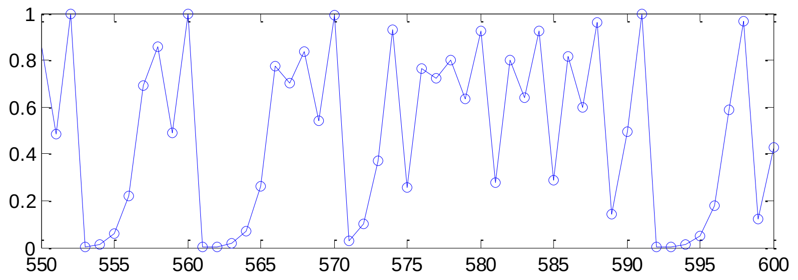
Logistic map: symbolic dynamics characterized with D=3 words

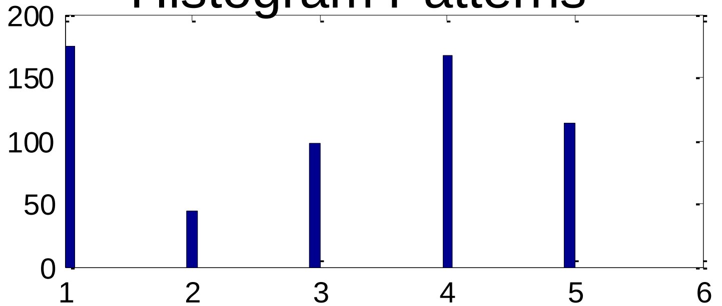
Time series



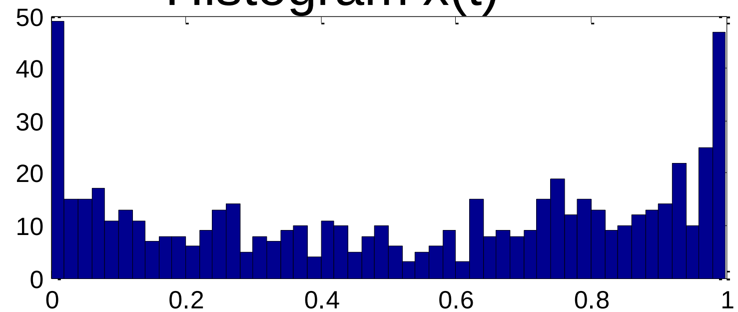
Detail

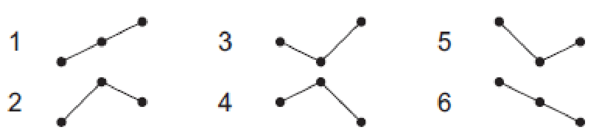


Histogram Patterns



Histogram x(t)





↑ **forbidden**

Take home message: ordinal analysis can yield information about more expressed (and/or missing) patterns in the data.

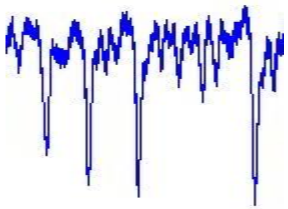
Ordinal analysis

- Proposed in 2002 (Bandt and Pompe PRL 88, 174102).

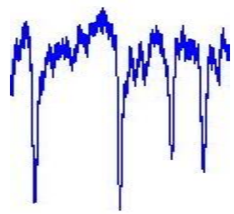
- It has been successfully applied to the analysis of signals
  - Financial
  - Biological, life sciences
  - Geosciences, climate
  - Physics, chemistry, etc

- Used to:
  - Distinguish stochasticity and determinism
  - Classify different types of dynamical behaviors (pathological, healthy)
  - Quantify complexity
  - Identify coupling and directionality, etc.

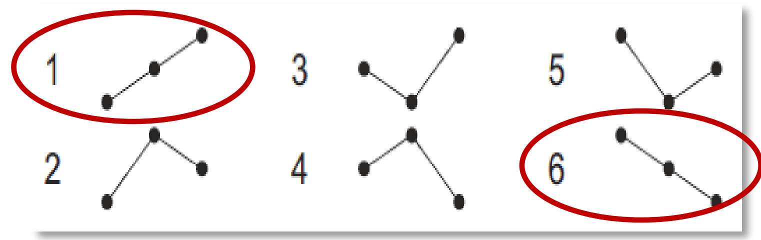
- **D=3**: correlations among 3 ***inter-spike-intervals*** (ISIs).



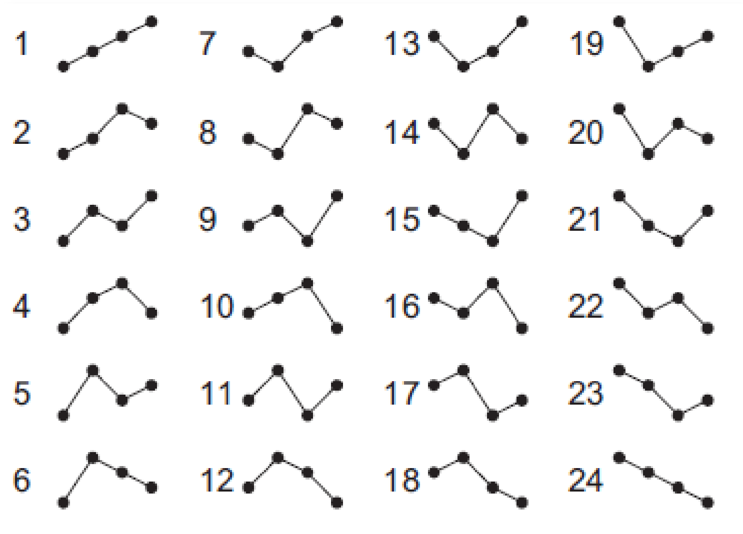
⇒012



⇒210



- The number of patterns grows as **D!**



- How to quantify the information?
  - Permutation entropy (more latter)

$$s_p = -\sum p_i \log p_i$$

- How to select optimal D? depends on:
  - The length of the data.
  - The length of the correlations

U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Read more

Entropy **2012**, *14*, 1553-1577; doi:10.3390/e14081553
Article

Permutation Entropy and Its Main Biomedical and Econophysics Applications: A Review

Massimiliano Zanin [1,2,3,] *, Luciano Zunino [4,5], Osvaldo A. Rosso [6,7] and David Papo [1]

Special Issue

- **Recent Progress in Symbolic Dynamics and Permutation Complexity Ten Years of Permutation Entropy**
- **The European Physical Journal Special Topics**
  Volume 222 / No 2 (June 2013)

comparison

Threshold transformation:

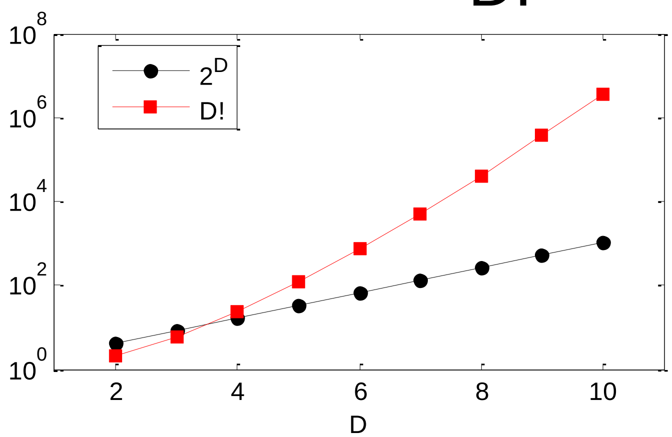
if $x_i > x_{th} \Rightarrow s_i = 0$; else $s_i = 1$

- Advantage: keeps information about the magnitude of the values.
- Drawback: how to select an adequate threshold ("partition" of the phase space).
- 2^D

Ordinal transformation:

if $x_i > x_{i-1} \Rightarrow s_i = 0$; else $s_i = 1$

- Advantage: no need of threshold; keeps information about the temporal order in the sequence of values
- Drawback: no information about the actual data values
- D!



Constructing longer words

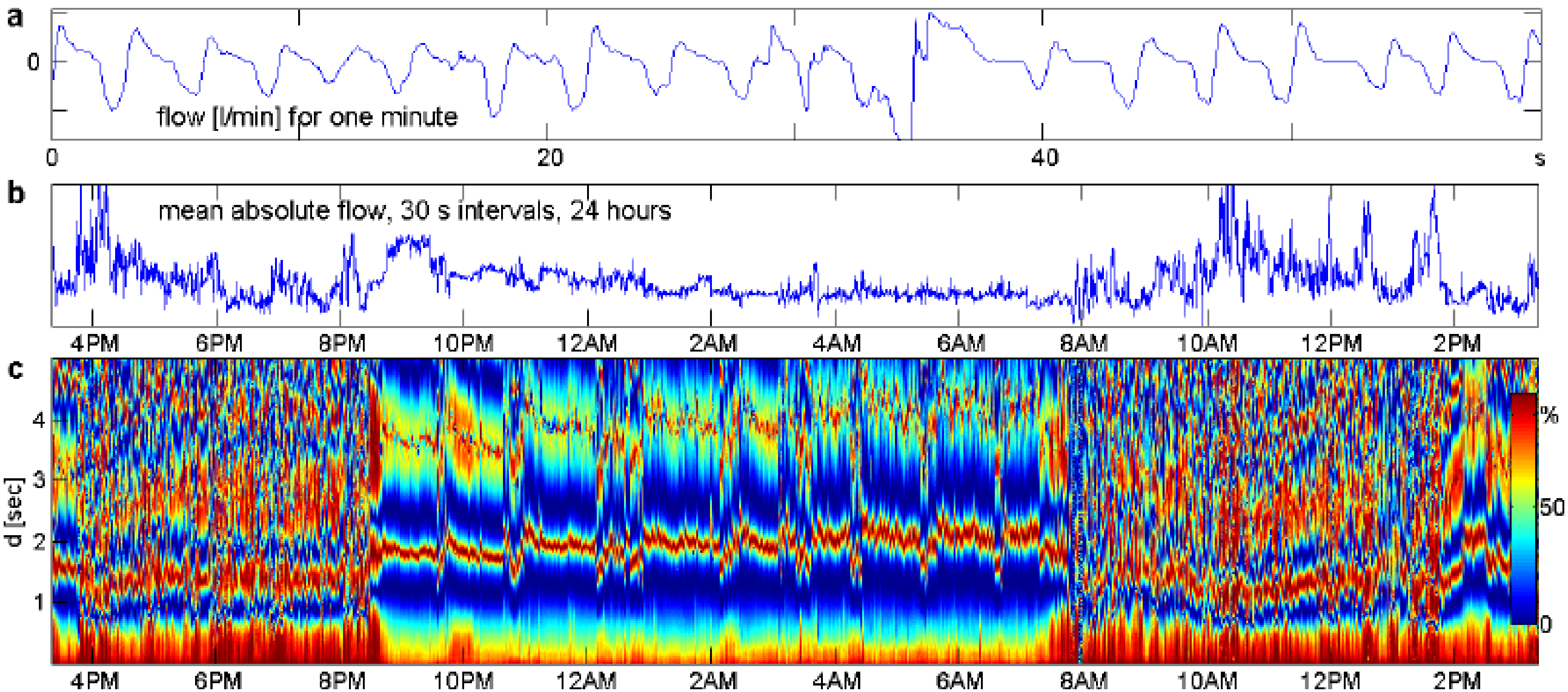
$$[\ldots\ x(t), x(t+1), x(t+2), x(t+3), x(t+4), x(t+5)\ldots]$$

- But long time series will be required to estimate the probabilities of the fast growing number of words in the dictionary (D!).
- Solution: a lag allows considering long time-scales without having to use words of many letters

$$[\ldots\ x(t), x(t+2), x(t+4), \ldots]$$

- Example: climatological data (assuming monthly sampled data)
  - Consecutive months: $[\ldots x_i(t), x_i(t+1), x_i(t+2)\ldots]$
  - One year: $[\ldots x_i(t), \ldots\ x_i(t+4), \ldots\ x_i(t+8)\ldots]$
  - Consecutive years: $[\ldots x_i(t), \ldots\ x_i(t+12), \ldots\ x_i(t+24)\ldots]$
  - etc

Nose breathing of a healthy volunteer in normal life



a: One minute of data without artefacts.

b: Mean absolute flow for each 30 seconds of 24 hours measurement.

c: persistence as a function of lag d, shows structure, needs no calibration.

$$\tau(d) = p_{123}(d) + p_{321}(d) - \frac{1}{3}$$

C. Bandt, arXiv:1411.3904v2

Next?

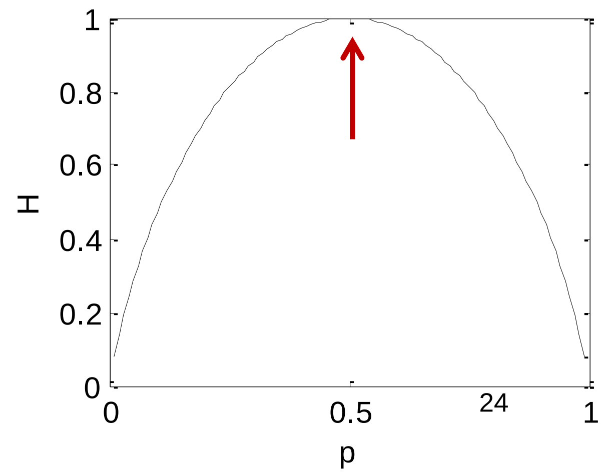
- Assuming that we have a suitable symbolic description of the time series.
- What information can we obtain from the sequence of "words"?
- How much information is in a time-series?
- Analogy with deciphering a foreign text.

Information theory measure: Shannon entropy

- The time-series is described by a set of probabilities $\sum_{i=1}^{N} p_i = 1$
- Shannon entropy: $H = -\sum_i p_i \log_2 p_i$
- Interpretation: "*quantity of* ***surprise*** *one should feel upon reading the result of a measurement*"

  K. Hlavackova-Schindler et al, Physics Reports 441 (2007)
- Simple example: a random variable takes values 0 or 1 with probabilities: p(0) = p, p(1) = 1 − p.
- $H = -p \log_2(p) - (1-p) \log_2(1-p)$.

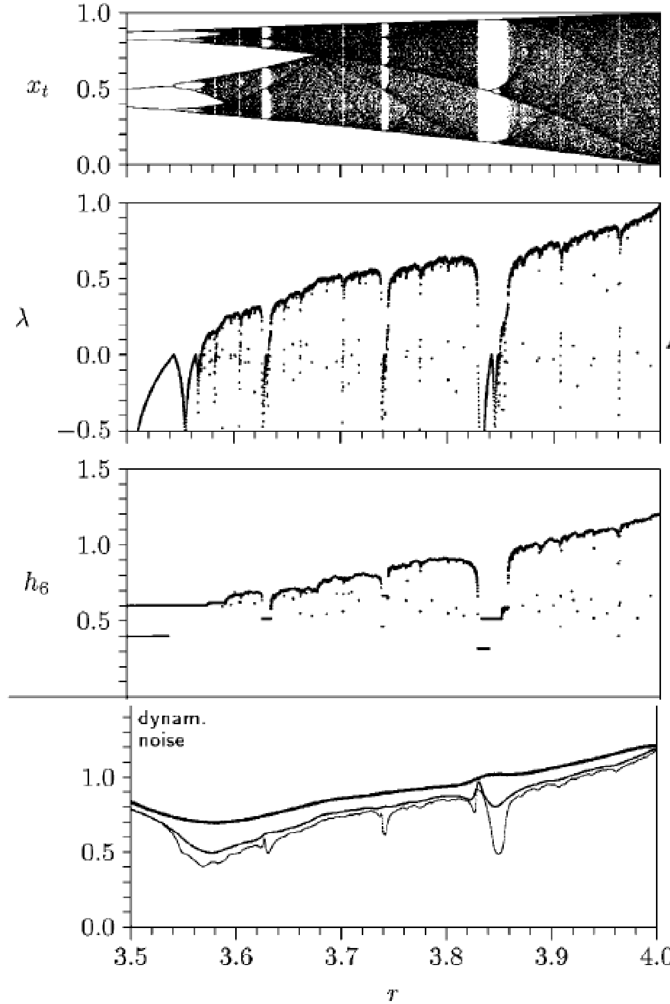
$\Rightarrow$ p=0.5: Maximum **unpredictability.**



Shannon entropy computed from ordinal probabilities: **Permutation Entropy**

Permutation entropy and Lyapunov exponent

- **x(i+1)=r x(i)[1-x(i)]**



$$|\delta_n| \approx |\delta_0| e^{n\lambda}$$

$$\lambda = \lim_{n\to\infty} \left\{ \frac{1}{n} \sum_{i=0}^{n-1} \ln |f'(x_i)| \right\}$$

Entropy per symbol:

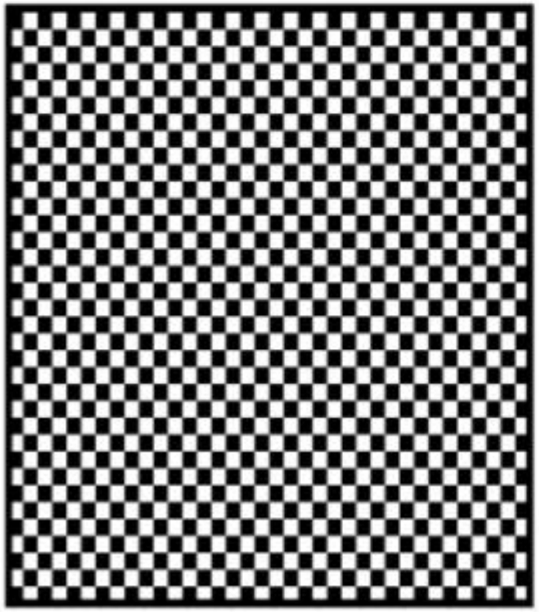
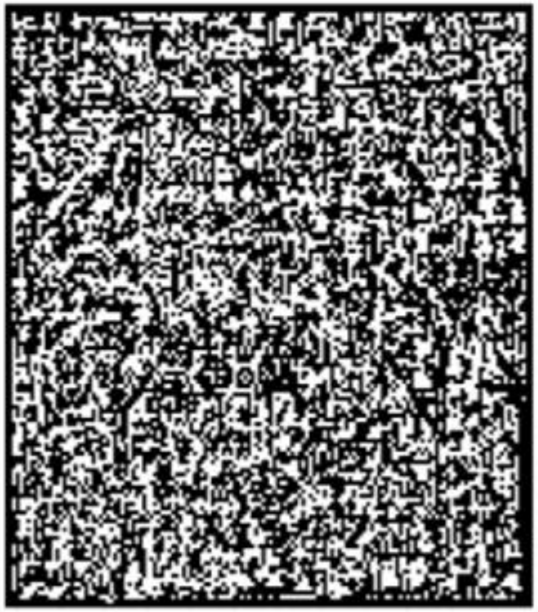
$$h_n = H(n)/(n-1)$$

Robust to noise

Bandt and Pompe
Phys. Rev. Lett. 2002

Entropy: measures unpredictability or disorder.
Complexity?

We would like to find a quantity "C" that measures **complexity**, as the entropy, "H", measures **unpredictability**, and, for low-dimensional systems, the Lyapunov exponent measures **chaos**.

Order	Chaos	Disorder
		
H = 0 C = 0	H ≠ 0 C ≠ 0	H = 1 C = 0

A useful complexity measure needs to do more than satisfy the boundary conditions of vanishing in the high- and low-entropy limits.

Maximum complexity occurs in the region between the system's perfectly ordered state and the perfectly disordered one.

Feldman, McTague and Crutchfield, Chaos 2008

Information measures

- Assuming that we know the probability distribution P=[pi, i=1,N] that characterizes a given system, we can use one of the following information measures

- Shannon entropy

$$I[P]=S_S[P]=-\sum_i p_i \ln p_i$$

- Tsallis entropy

$$I[P]=S_T^q[P]=\frac{1}{q-1}\left[1-\sum_i p_i^q\right]$$

- Renyi entropy

$$I\left[P\right]=S_R^q[P]=\frac{1}{1-q}\ln\left[\sum_i p_i^q\right]$$

etc

Normalized entropy

$$H[P] = \frac{I[P]}{I_{\max}} \qquad 0 \leq H[P] \leq 1$$

where $I_{\max} = I[P_e]$

P_e being the equilibrium probability distribution (that maximizes the information measure).

Example: if $I[P]$ = Shannon entropy

then $P_e = [p_i=1/N \text{ for } i=1,N]$

and $I_{max} = \ln(N)$

Disequilibrium Q

Measures the “distance“ from P to the equilibrium distribution, Pe

$$Q[P] = Q_0 D\left[P, P_e\right]$$

where Q_o is a normalization constant such that

$$0 \leq Q[P] \leq 1$$

Distance between P and Pe

- Euclidean

$$D_E[P,P_e] = \|P - P_e\|_E = \sum_i (p_i - 1/N)^2$$

- Wootters

$$D_W[P,P_e] = \cos^{-1}\left[\sum_i p_i^{1/2}(1/N)^{1/2}\right]$$

- Kullback relative entropy

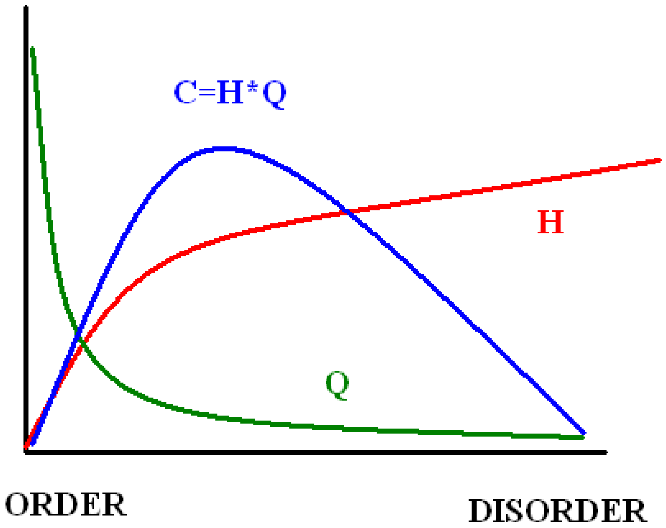
$$D_K[P,P_e] = K[P|P_e] = I[P_e] - I[P]$$

- Jensen divergence

$$D_J[P,P_e] = \frac{K[P|P_e] + K[P_e|P]}{2}$$

Read more: S-H Cha: *Comprehensive Survey on Distance/Similarity Measures between Probability Density Functions*, Int. J of. Math. Models and Meth. 1, 300 (2007)

Statistical complexity measure C



A family of complexity measures can be defined as:

$$C[P] = H_A[P] \cdot Q_B[P]$$

where

A = S, T, R (Shannon, Tsallis, Renyi)

B = E, W, K, J (Euclidean, Wootters, Kullback, Jensen)

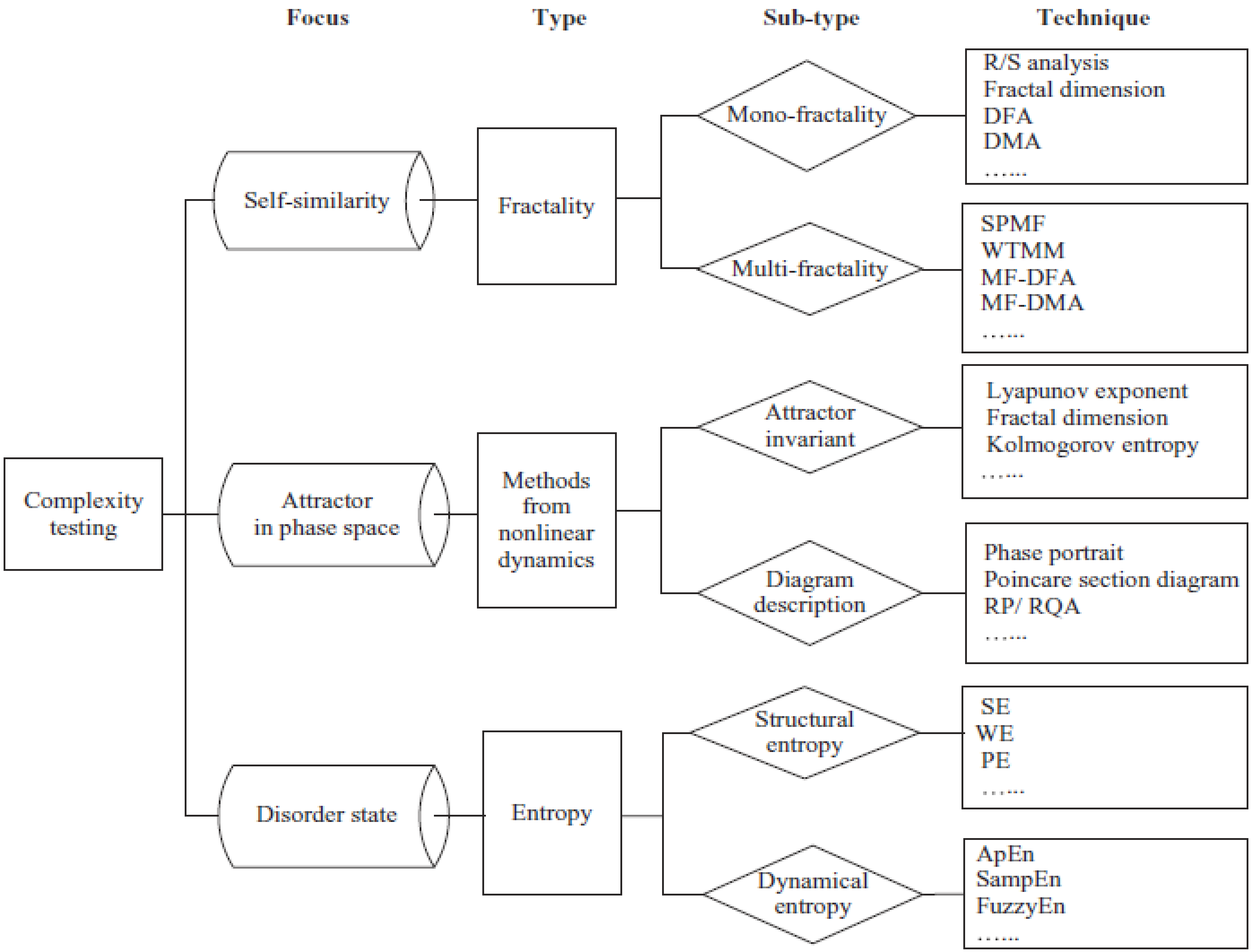
$$C_{LMC}[P] = H_S[P] \cdot Q_E[P]$$

Lopez-Ruiz, Mancini & Calbet, Phys. Lett. A (1995).
Celia Anteneodo & Plastino, Phys. Lett. A (1996).

$$C_{MPR}[P] = H_S[P] \cdot Q_J[P]$$

Martín, Plastino & Rosso, Phys. Lett. A (2003).

Many complexity measures have proposed in the literature



L. Tang et al. / Chaos, Solitons and Fractals 81 (2015) 117–135

Fractal objects: each part of the object is like the whole object but smaller

- Are characterized by a "fractal" dimension that measures roughness.

Romanesco broccoli
D=2.66

Human lung
D=2.97

Coastline of Ireland
D=1.22

Video: http://www.ted.com/talks/benoit_mandelbrot_fractals_the_art_of_roughness#t-149180

Kolmogorov Complexity

- The complexity of an object is a measure of the computability resources needed to specify the object.

Example: Let's consider 2 strings of 32 letters:

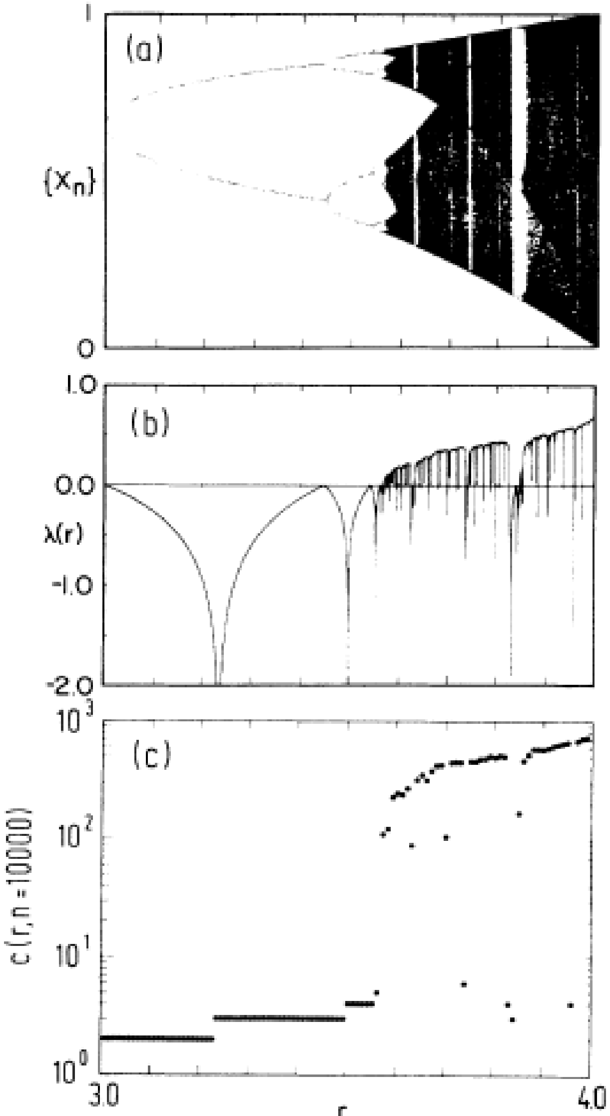
abababababababababababababababab

4c1j5b2p0cv4w1x8rx2y39umgw5q85s7

- The first string has a short description: "ab 16 times".
- The second one has no obvious simple description: complex or random?

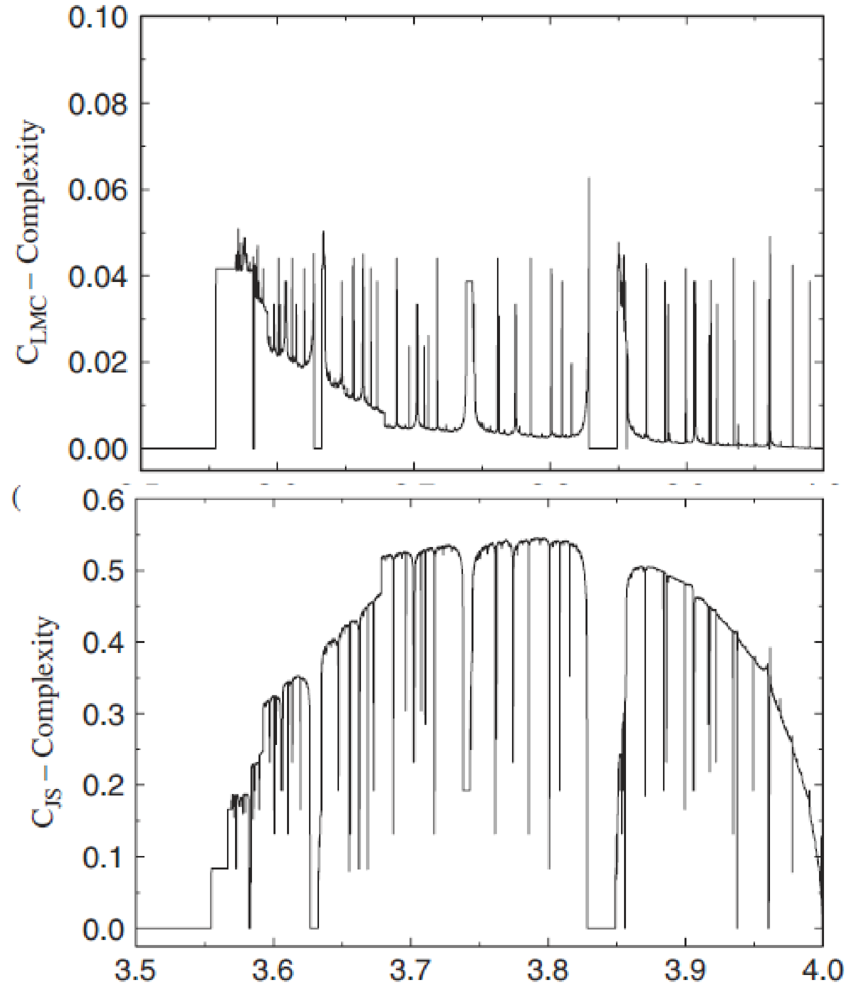
The complexity of the Logistic Map

- $x(i+1)=r\, x(i)[1-x(i)]$



Lempel & Zip complexity

Kaspar and Schuster, Phys Rev. A 1987



Euclidian distance

Jensen distance

Martín, Plastino, & Rosso, Physica A 2006

Read more

Review

Chaos, Solitons and Fractals 81 (2015) 117–135

Complexity testing techniques for time series data: A comprehensive literature review

Ling Tang [a], Huiling Lv [b], Fengmei Yang [b], Lean Yu [a,*]

[a] School of Economics and Management, Beijing University of Chemical Technology, Beijing 100029, China
[b] School of Science, Beijing University of Chemical Technology, Beijing 100029, China

nature physics

INSIGHT | REVIEW ARTICLES

PUBLISHED ONLINE: 22 DECEMBER 2011 | DOI: 10.1038/NPHYS2190

Between order and chaos

James P. Crutchfield

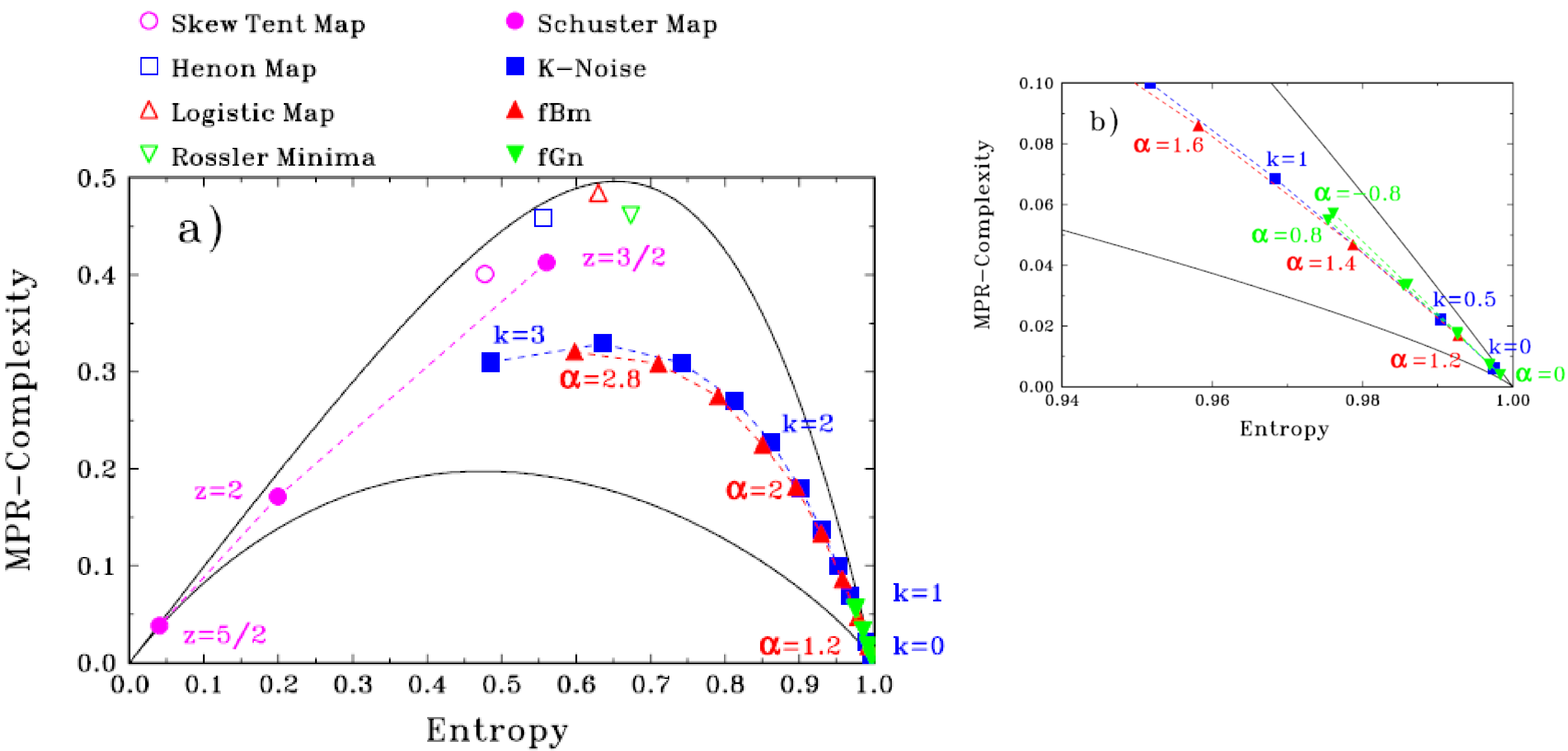
What is a pattern? How do we come to recognize patterns never seen before? Quantifying the notion of pattern and formalizing the process of pattern discovery go right to the heart of physical science. Over the past few decades physics' view of nature's lack of structure—its unpredictability—underwent a major renovation with the discovery of deterministic chaos, overthrowing two centuries of Laplace's strict determinism in classical physics. Behind the veil of apparent randomness, though, many processes are highly ordered, following simple rules. Tools adapted from the theories of information and computation have brought physical science to the brink of automatically discovering hidden patterns and quantifying their structural complexity.

Applications

PRL **99,** 154102 (2007) PHYSICAL REVIEW LETTERS week ending 12 OCTOBER 2007

Distinguishing Noise from Chaos

O. A. Rosso,[1,2] H. A. Larrondo,[3] M. T. Martin,[4] A. Plastino,[4] and M. A. Fuentes[5,6]



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Can be adapted to the analysis of 2D images

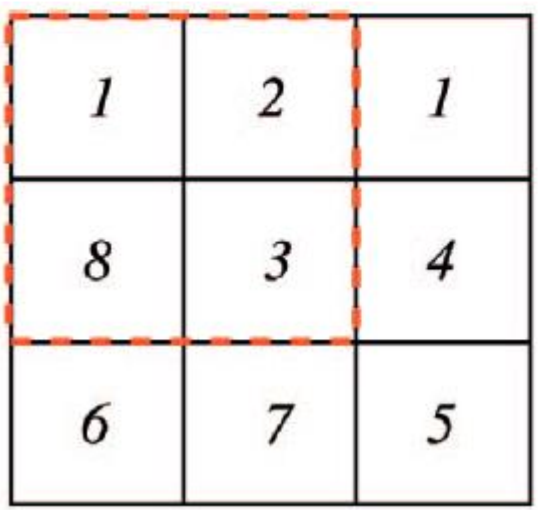
OPEN ACCESS Freely available online
PLoS one

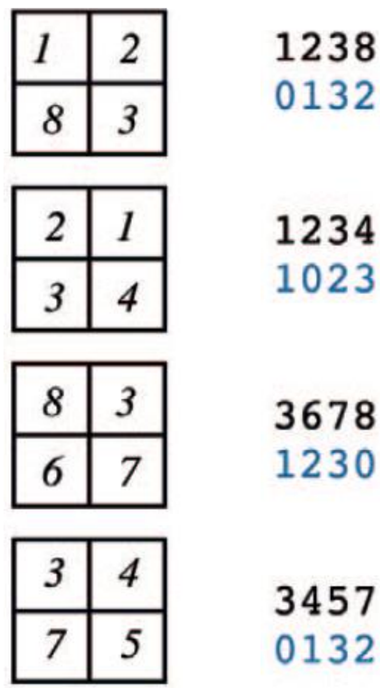
Complexity-Entropy Causality Plane as a Complexity Measure for Two-Dimensional Patterns

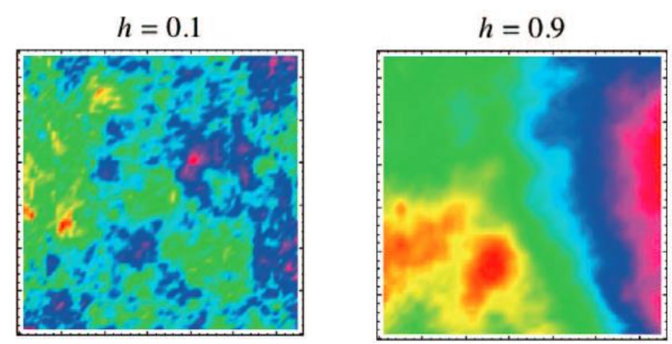
Haroldo V. Ribeiro[1]*, Luciano Zunino[2,3], Ervin K. Lenzi[1], Perseu A. Santoro[1], Renio S. Mendes[1]

1 Departamento de Física and National Institute of Science and Technology for Complex Systems, Universidade Estadual de Maringá, Maringá, Brazil, 2 Centro de Investigaciones Ópticas (CONICET La Plata - CIC), C.C. 3, Gonnet, Argentina, 3 Departamento de Ciencias Básicas, Facultad de Ingeniería, Universidad Nacional de La Plata, La Plata, Argentina

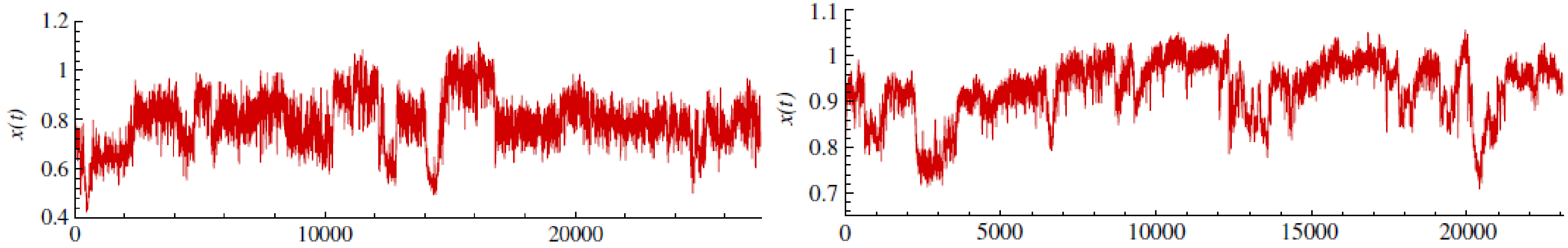
August 2012 | Volume 7 | Issue 8 | e40689





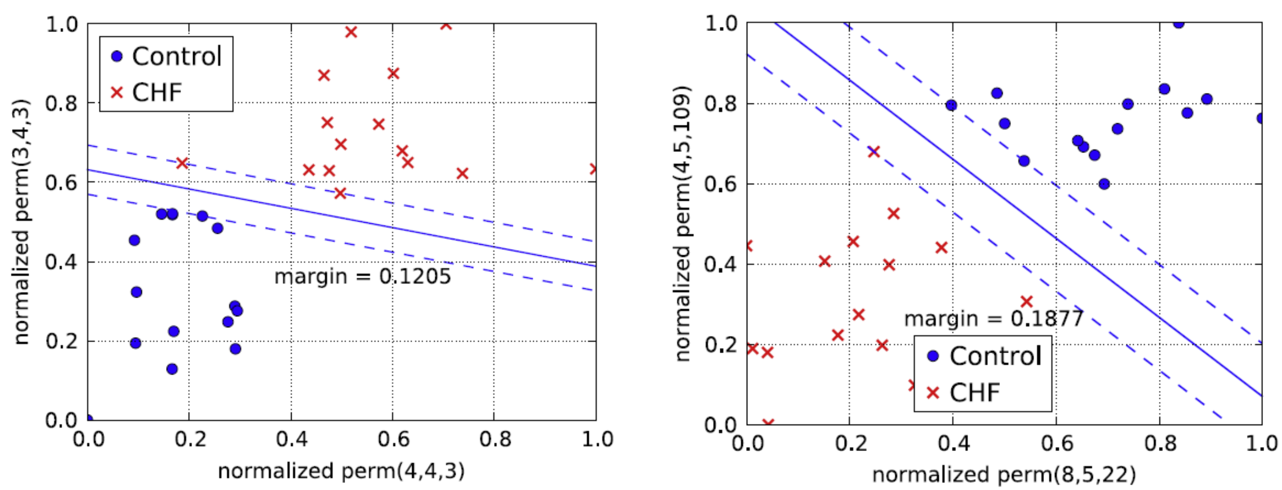


Classifying ECG-signals according to the frequency of words



Time series of inter-beat intervals x(t) versus interval number t for a typical person with congestive heart failure (right) and a healthy subject (left).

R. Friedrich et al. / Physics Reports 506 (2011) 87–162



Perm (i,D,lag) (the probabilities are normalized with respect to the smallest and the largest value occurring in the data set)

U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

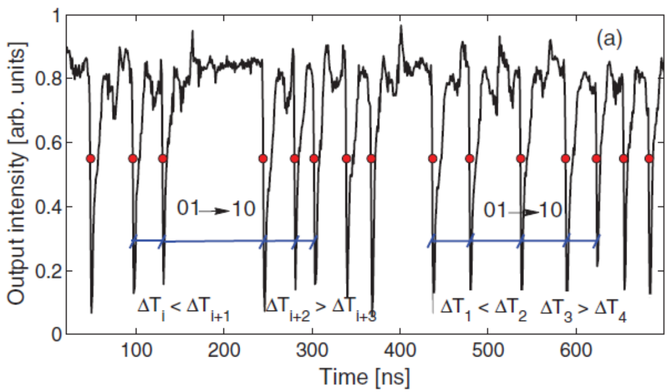
Transition to optical complexity

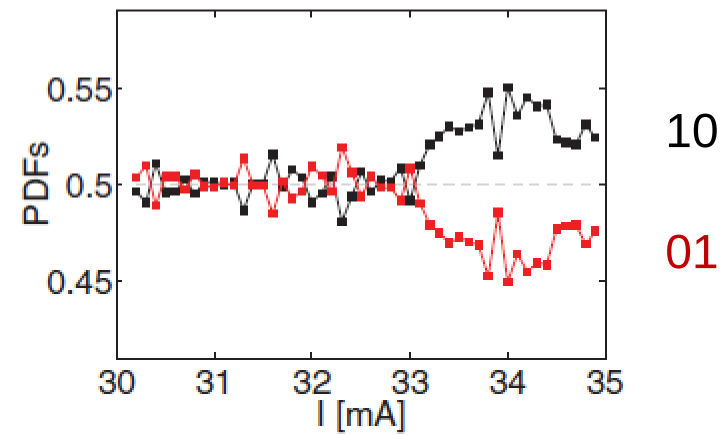
video

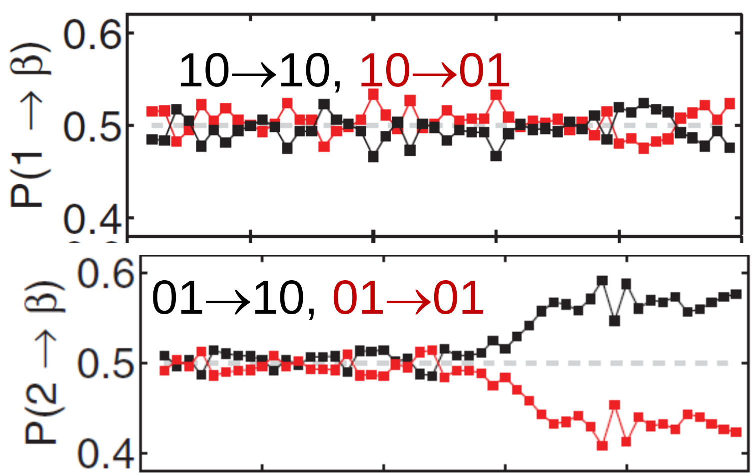
PHYSICAL REVIEW E **84**, 026202 (2011)

Language organization and temporal correlations in the spiking activity of an excitable laser: Experiments and model comparison

Nicolas Rubido,[1] Jordi Tiana-Alsina,[2] M. C. Torrent,[2] Jordi Garcia-Ojalvo,[2] and Cristina Masoller[2]







Consistent with stochastic dynamics at low pump current, signatures of determinism at higher pump currents.

Mapping a time series into a complex network

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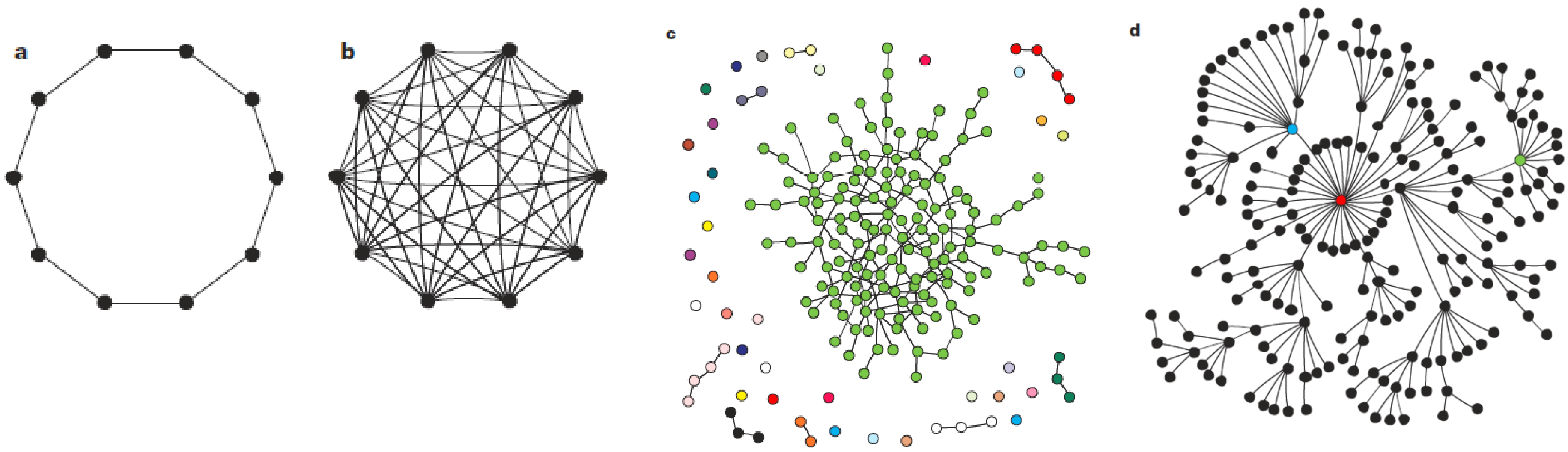
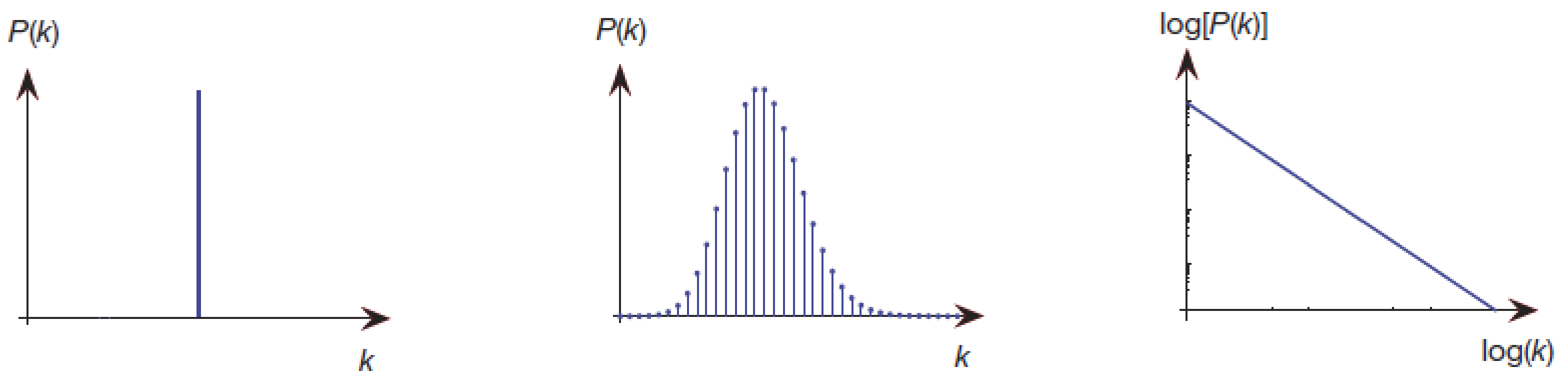
Networks = Graphs = vertices (nodes) + edges (links)

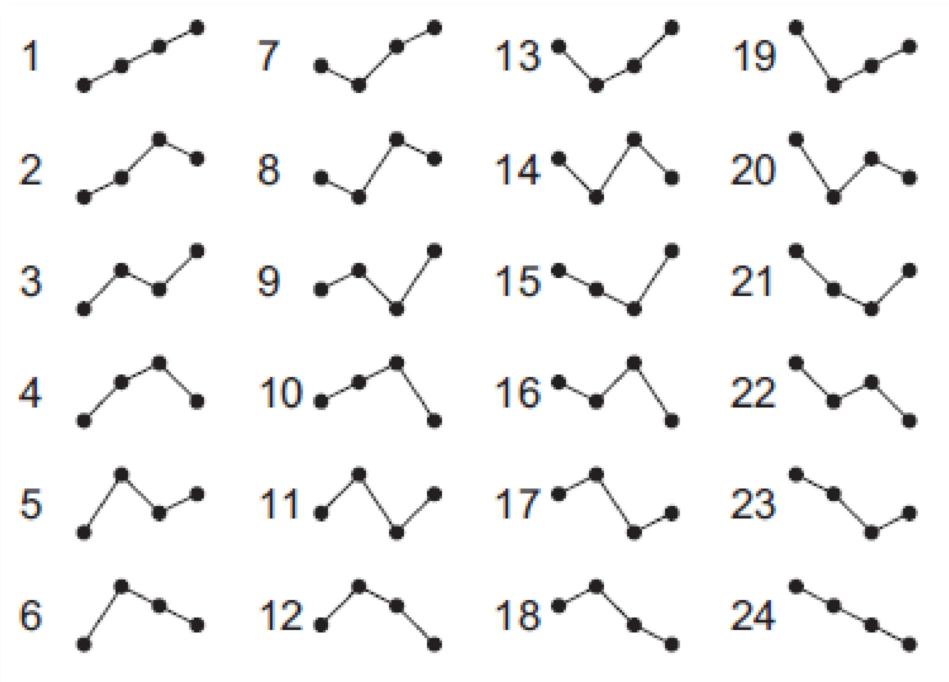


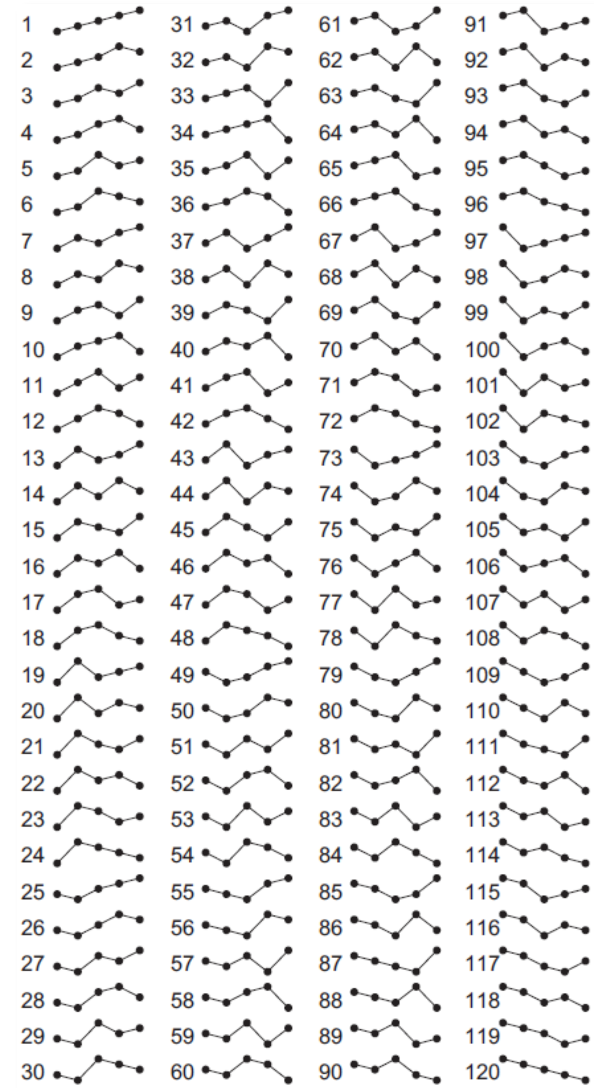
Degree (number of links of a node) distributions:



Strogatz, Nature 2001

The number of patterns increases as D!

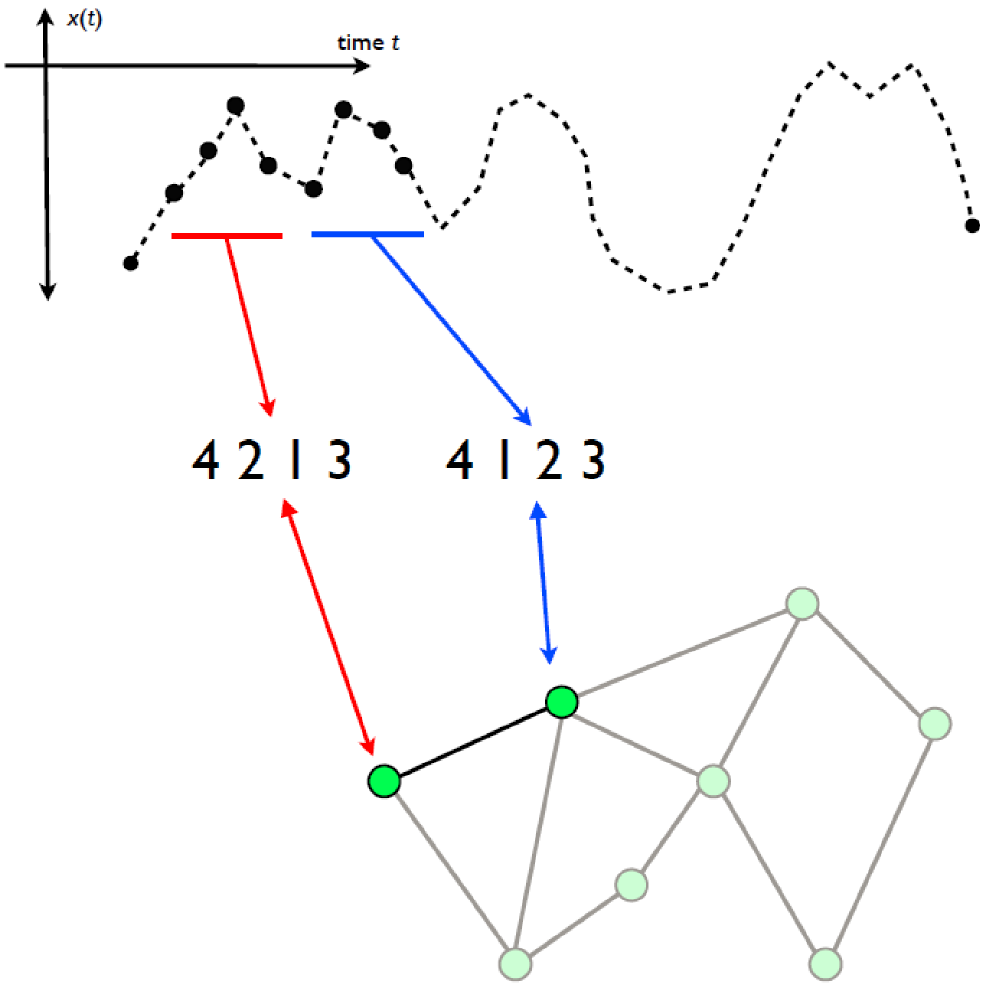




Opportunity: turn a time-series into a network by using the patterns as the "nodes" of the network.

U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

The network nodes are the "ordinal patterns", and the links?



- The links are defined in terms of the probability of pattern "β" occurring after pattern "α".
- Weighs of nodes: the probabilities of the patterns ($\sum_i p_i = 1$).
- Weights of links: the probabilities of the transitions ($\sum_j w_{ij} = 1 \; \forall i$).

$\Rightarrow$ ***Weighted and directed network***

Adapted from M. Small (The University of Western Australia)

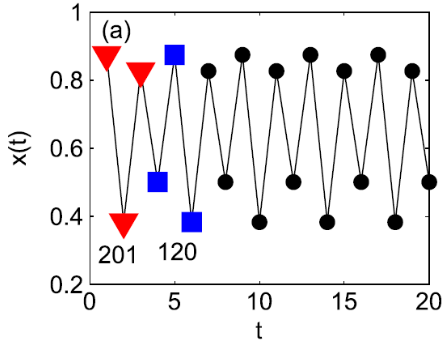
Three network-based diagnostic tools

- Entropy computed from the weights of the nodes (**permutation entropy**)

$$s_p = -\sum p_i \log p_i$$

- Entropy computed from weights of the links (**transition probabilities**, '01'→ '01', '01'→ '10', etc.)

$$w_{ij} = \frac{\sum_{t=1}^{L-1} n\left[s(t) = i, s(t+1) = j\right]}{\sum_{t=1}^{L-1} n\left[s(t) = i\right]}$$

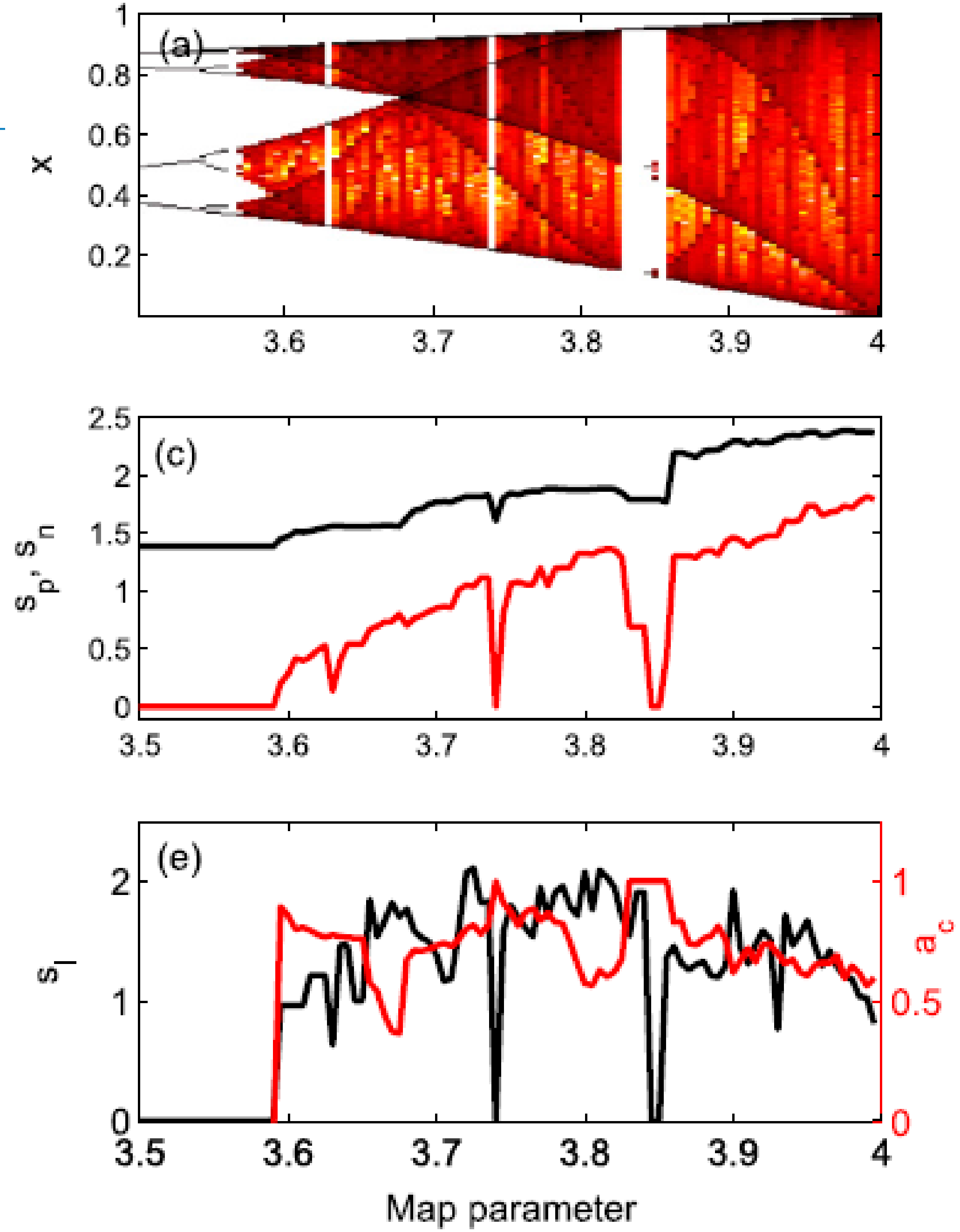


- Asymmetry coefficient: normalized difference of transition probabilities, P('01'→ '10') - P('10'→ '01'), etc.

$$a_c = \frac{\sum_i \sum_{j \neq i} \left| w_{ij} - w_{ji} \right|}{\sum_i \sum_{j \neq i} \left(w_{ij} + w_{ji} \right)}$$

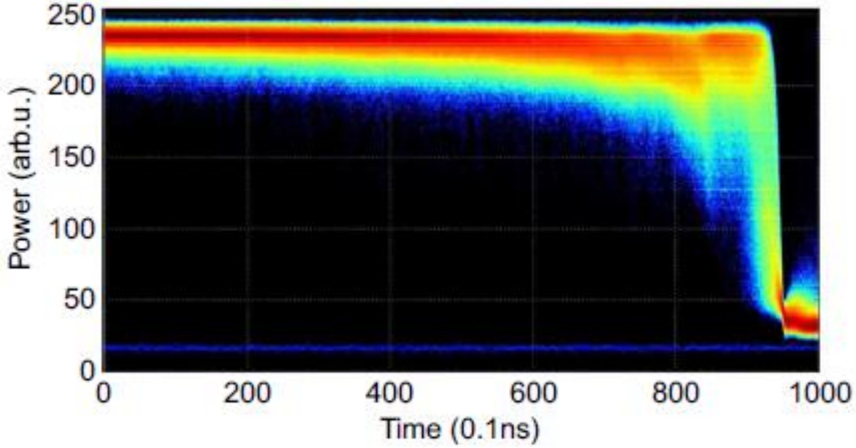
(0 in a fully symmetric network; 1 in a fully directed network)

- **x(i+1)=r x(i)[1-x(i)]**



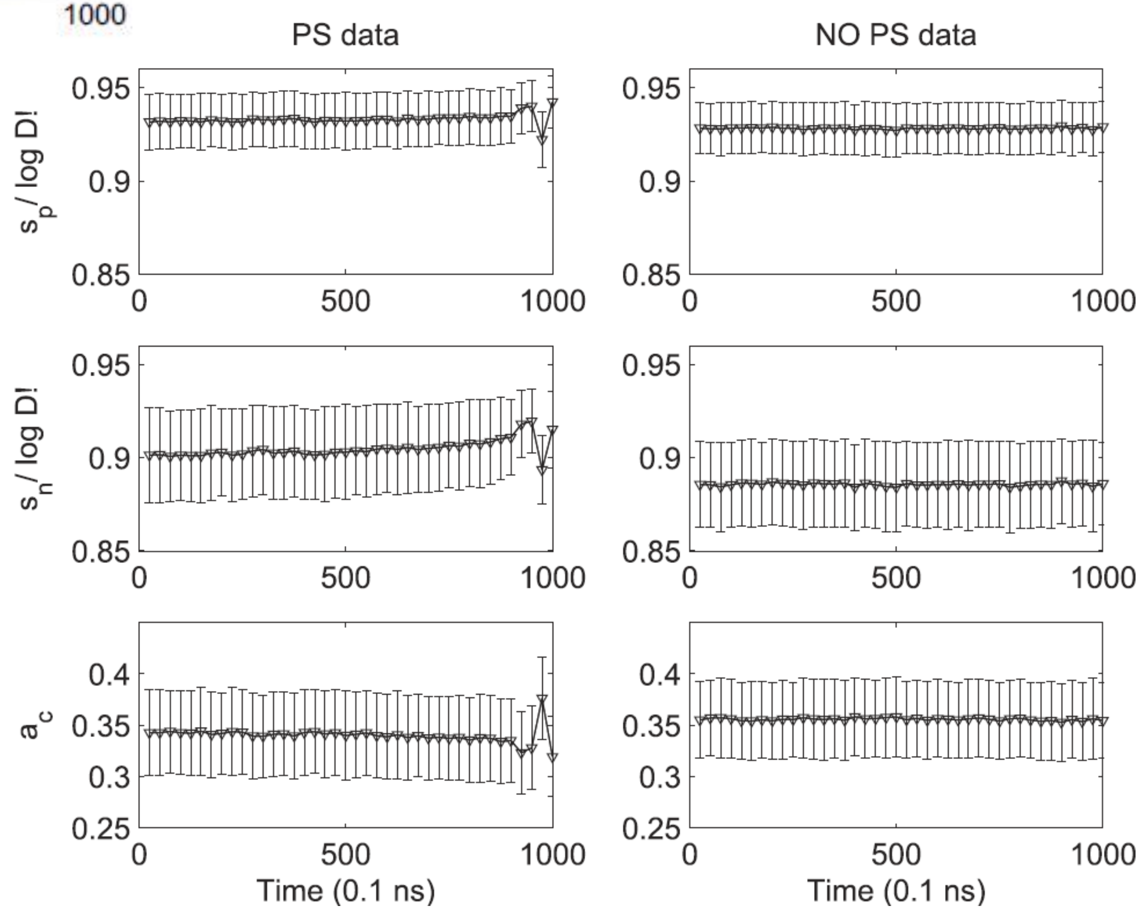
C. Masoller et al, NJP 2015

Empirical data



Slightly different optical feedback conditions result in PS or no PS.

Analysis done with D=3, error bars computed with 1000 time series L=500.



C. Masoller et al, New J. Phys. 17 (2015) 023068

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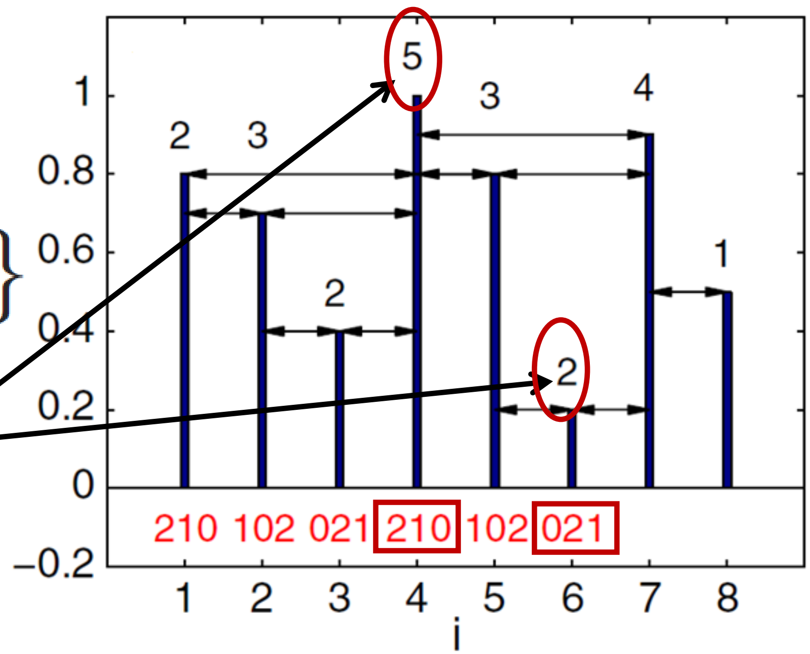
Another way to turn a time-series into a network: horizontal visibility graph (HVG)

A time-series is represented as a graph, where each data point is a node

intensity peaks above Th = 2

$\{I_{\max,i}\}$

Number of links



Ordinal pattern D=3

- Rule: data points i and j are connected if there is "visibility" between them: $I_{\max,i}$ and $I_{\max,j} > I_{max,n}$ for all n, $i<n<j$

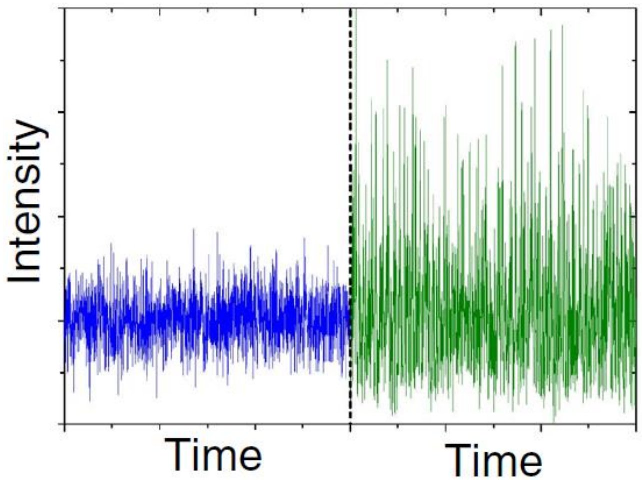
$\Rightarrow$ ***Unweighted and undirected graph***

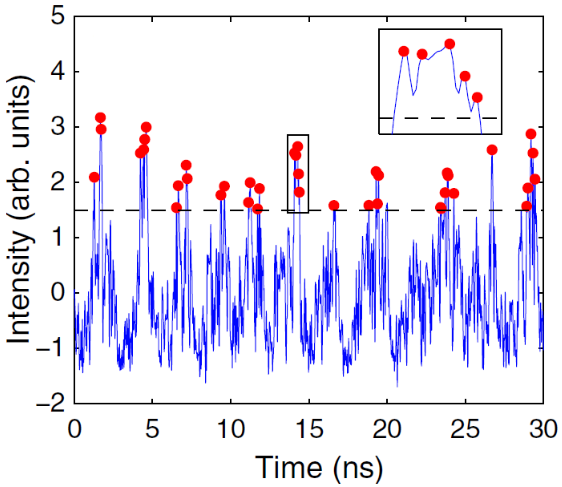
HVG method: B. Luque et al, PRE 80, 046103 (2009)

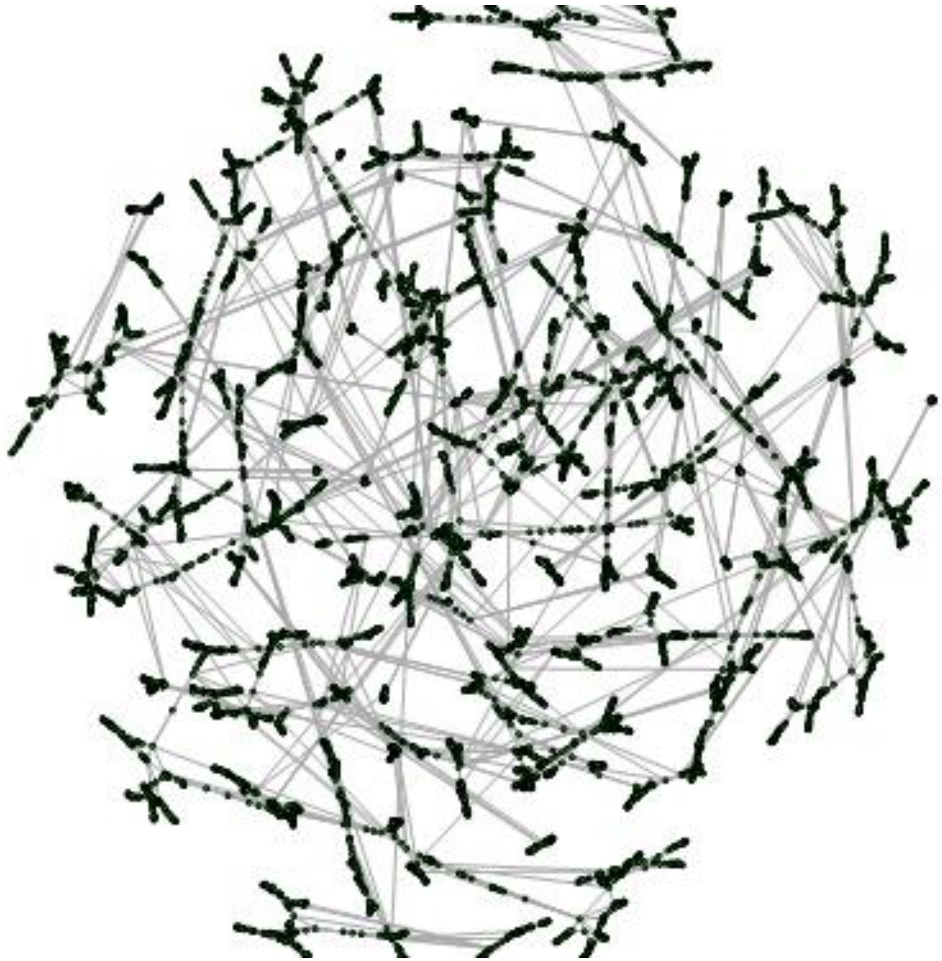
The obtained graph

Intensity of a fiber laser

Low -- High pump power





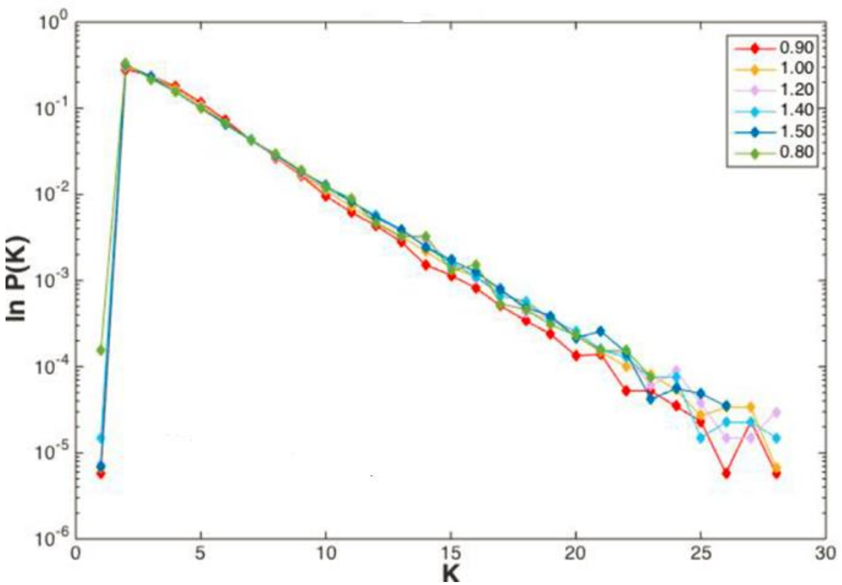


How to characterize this graph?

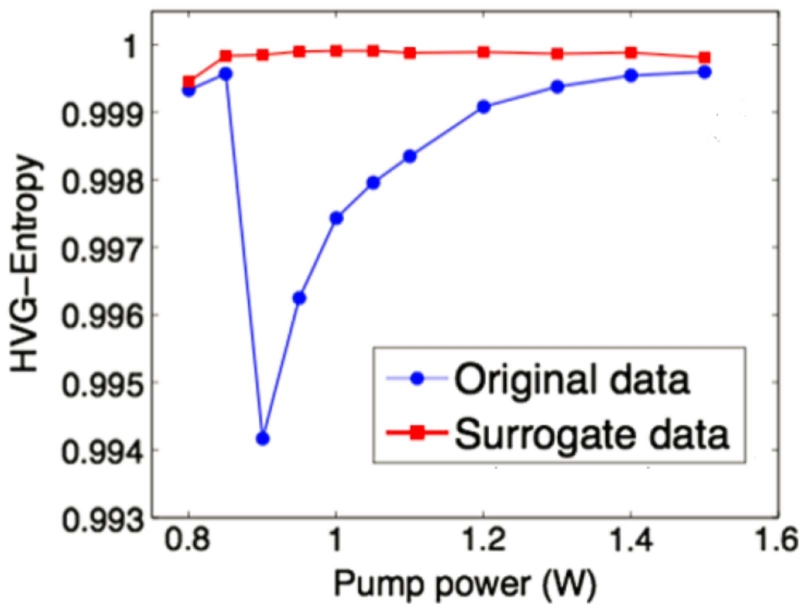
HVG analysis

⇒ Degree Distribution (distribution of the number of links)

- Degree distribution for various pump powers using Th=2.



- Entropy of the degree distribution (normalized to the entropy of Gaussian white noise)

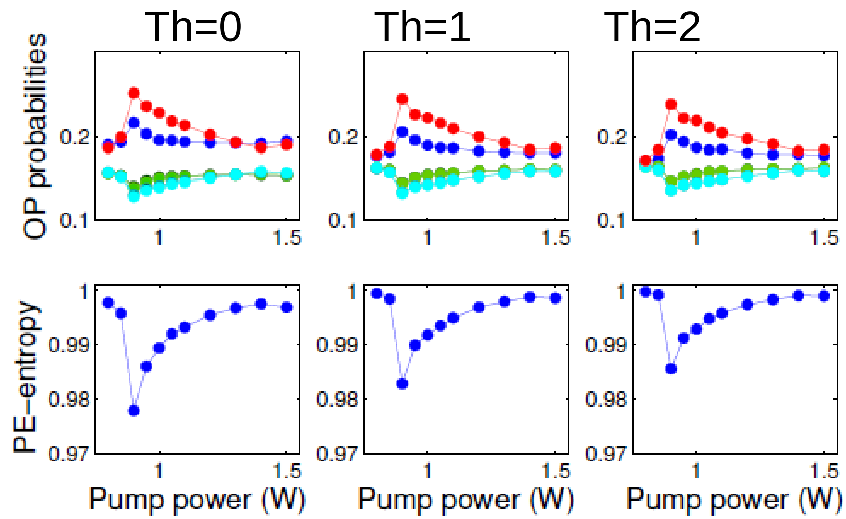


⇒ sharp transition detected.

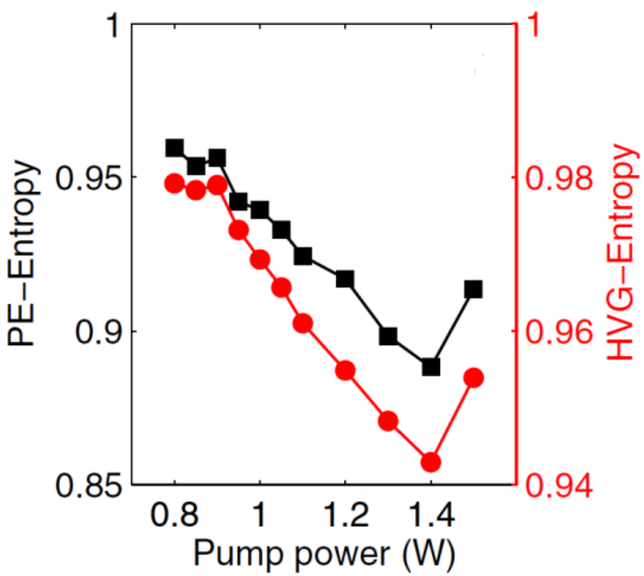
Aragoneses et al, PRL 116, 033902 (2016)

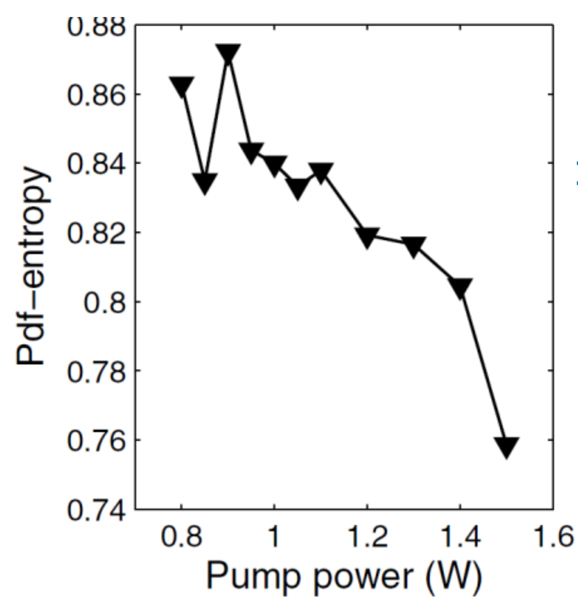
Threshold vs not threshold

- Sharp transition also seen with ordinal analysis



- But with raw data





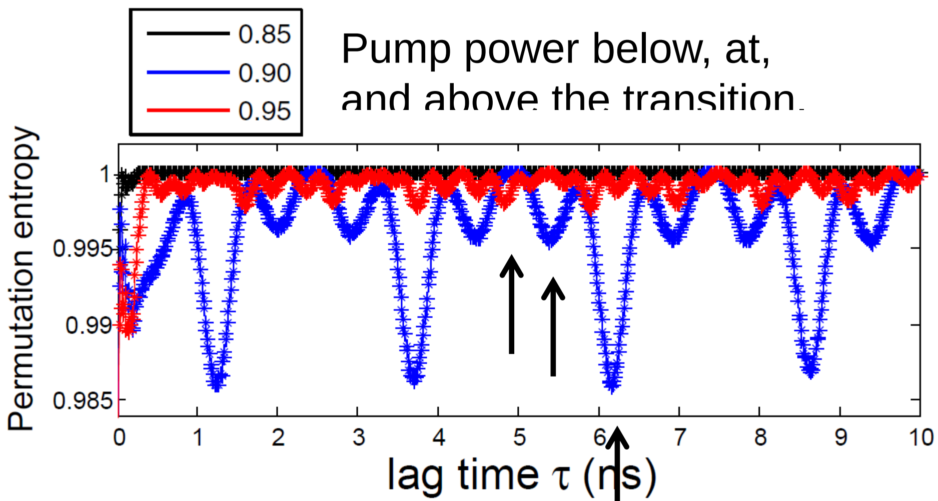
⇒ sharp transition not seen.

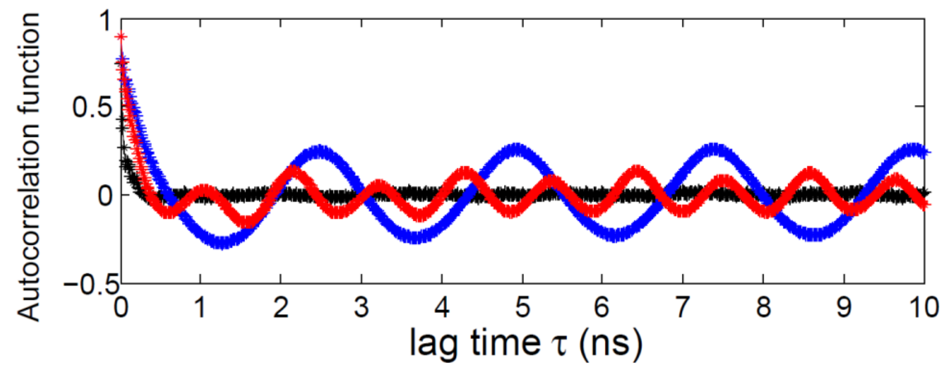
Space-time representation of a time-series

$$\{I_i, I_{i+\tau}, I_{i+2\tau}, \dots\}$$

all data points, no threshold used

Pump power below, at, and above the transition.

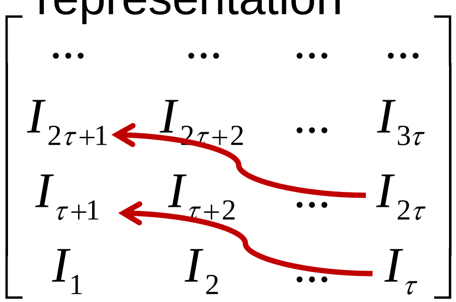


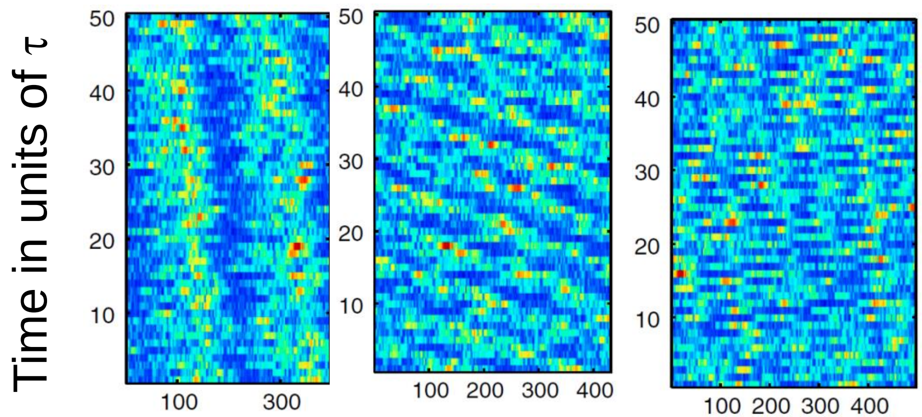


⇒ Sharp variations at the transition not captured.

Color: I_i

Space-time representation





index i (time in units of dt)

⇒ Different "spatial " structures uncovered with different lags (sampling times).

Bi-variate time-series analysis

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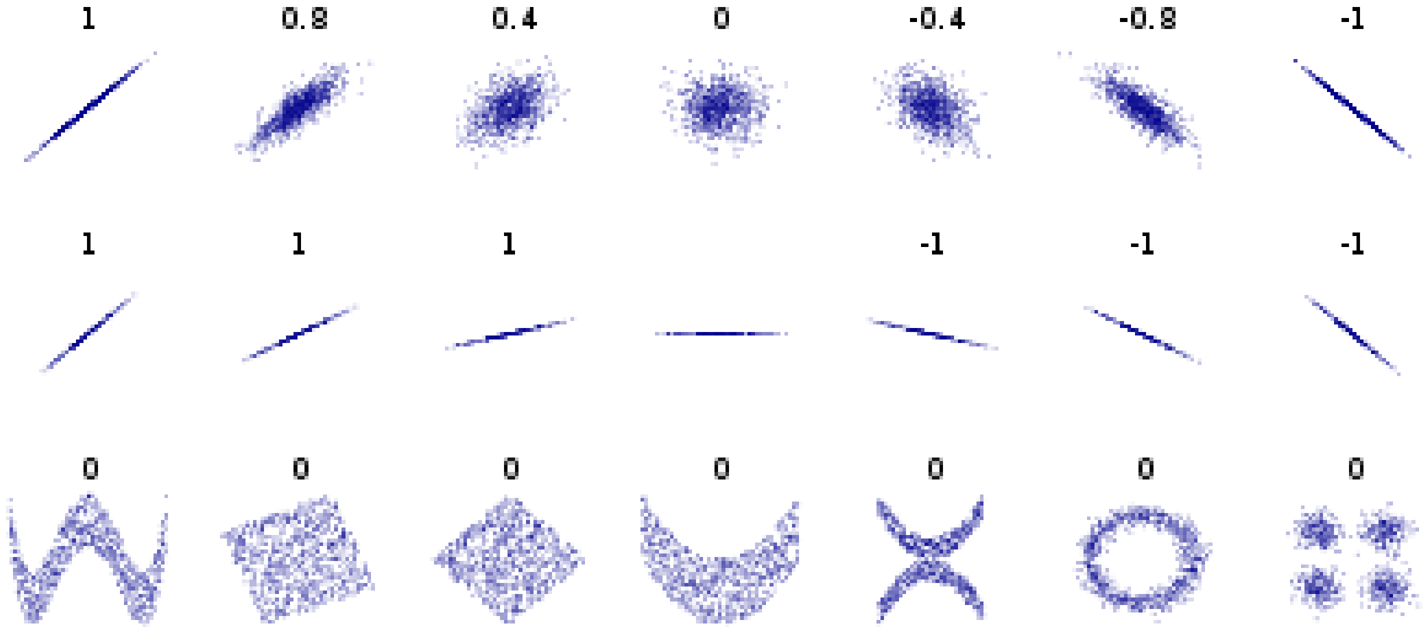
Two time series X, Y: Cross-correlation analysis

$$\rho_{X,Y}(\tau) = \frac{\mathrm{cov}(X(t), Y(t+\tau))}{\sqrt{\sigma_X^2 \sigma_Y^2}} = \frac{E[(X(t)-\mu_X)(Y(t+\tau)-\mu_Y)]}{\sqrt{\sigma_X^2 \sigma_Y^2}}$$

Detects linear relationships



Source: wikipedia

Brain functional network

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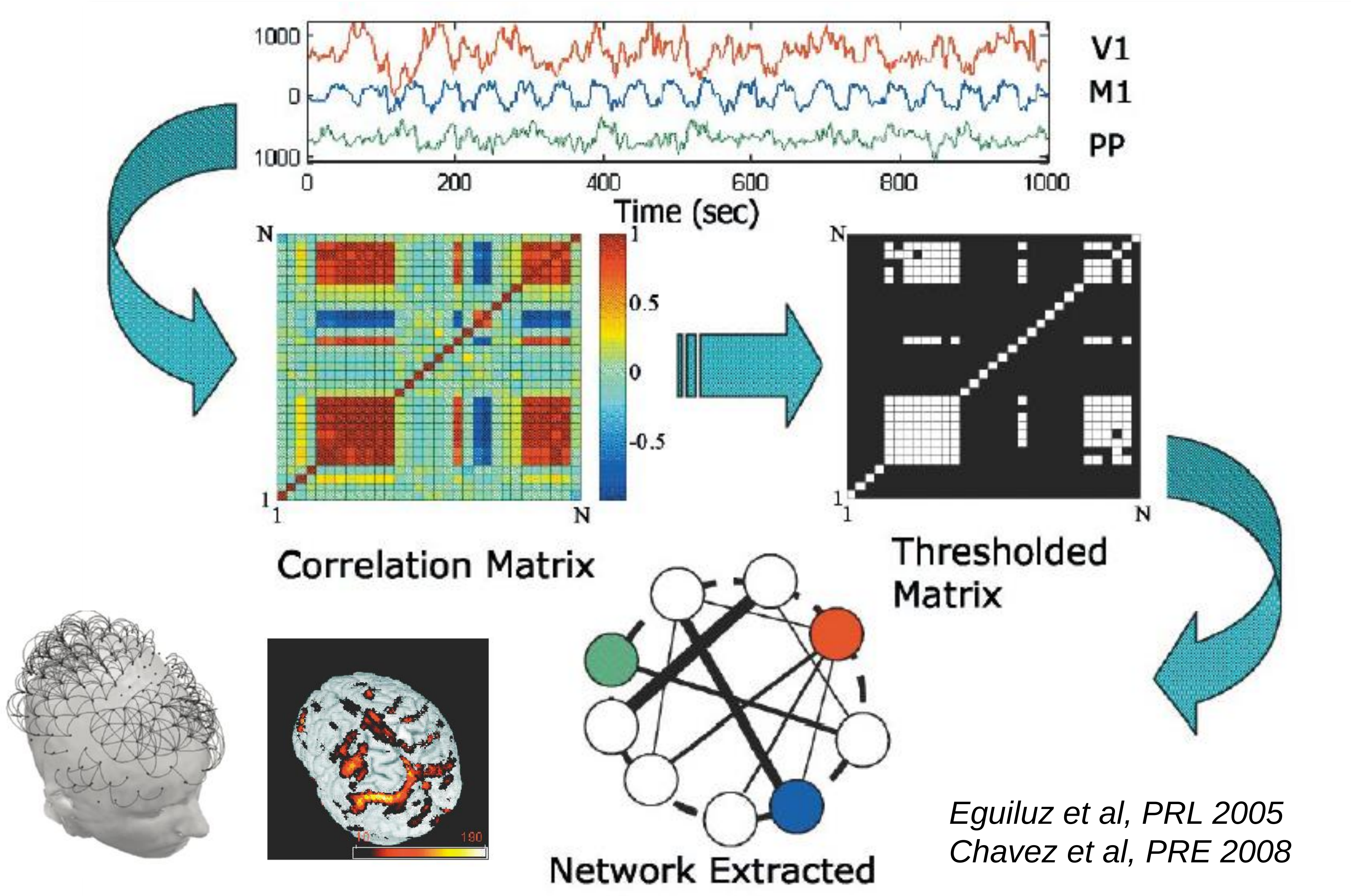

Eguiluz et al, PRL 2005
Chavez et al, PRE 2008

Statistical similarity measure: mutual information

- Joint entropy: $H(X,Y) = -\sum_{i=1}^{m_X}\sum_{j=1}^{m_Y} p(x_i, y_j)\log p(x_i, y_j)$

- If X and Y are independent: H(X,Y) = H(X) + H(Y)

- Mutual Information: MI(X,Y) = H(X) + H(Y) – H(X,Y)

$$MI(X,Y) = \sum_{i=1}^{m_X}\sum_{j=1}^{m_Y} p(x_i, y_j)\log\frac{p(x_i, y_j)}{p(x_i)p(y_j)}$$

- It reflects the reduction in uncertainty of one variable by knowing the other one.
- X and Y are independent $\Leftrightarrow$ MI(X,Y) = 0.

- However, computing probabilities from histograms give MI values that fluctuate or are systematically overestimated.

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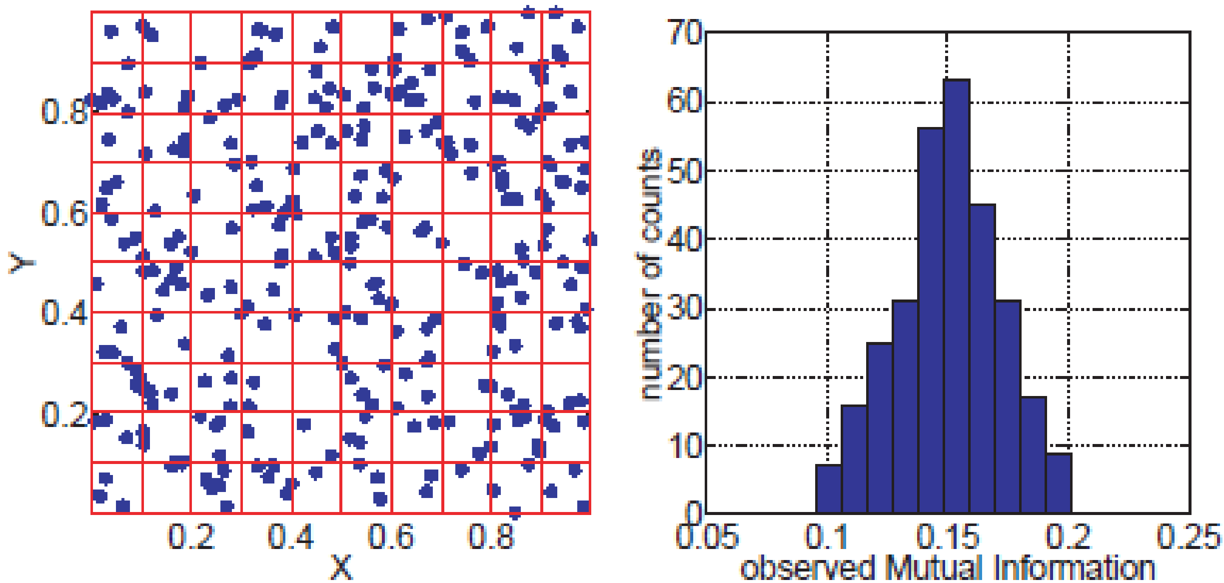
Problem with mutual information



$\langle I(X,Y)^{estimated}\rangle \approx 0.15 \pm 0.02$

The statistical significance of CC and MI values needs to carefully analyzed.

Fig. 1. Naive estimation of the mutual information for finite data. Left: The dataset consists of $N = 300$ artificially generated independent and equidistributed random numbers. The probabilities are estimated using a histogram which divides each axis into $M_X = M_Y = 10$ bins. Right: The histogram of the estimated mutual information $I(X,Y)$ obtained from 300 independent realizations.

R. Steuer et al, Bioinformatics 18, suppl 2, S231 (2002).

Read more

BIOINFORMATICS
Vol. 18 Suppl. 2 2002
Pages S231–S240

The mutual information: Detecting and evaluating dependencies between variables

R. Steuer[1], J. Kurths[1], C. O. Daub[2], J. Weise[2] and J. Selbig[2]

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ScienceDirect
PHYSICS REPORTS
Physics Reports 441 (2007) 1–46
www.elsevier.com/locate/physrep

Causality detection based on information-theoretic approaches in time series analysis

Katerina Hlaváčková-Schindler[a,*], Milan Paluš[b], Martin Vejmelka[b], Joydeep Bhattacharya[a,c]

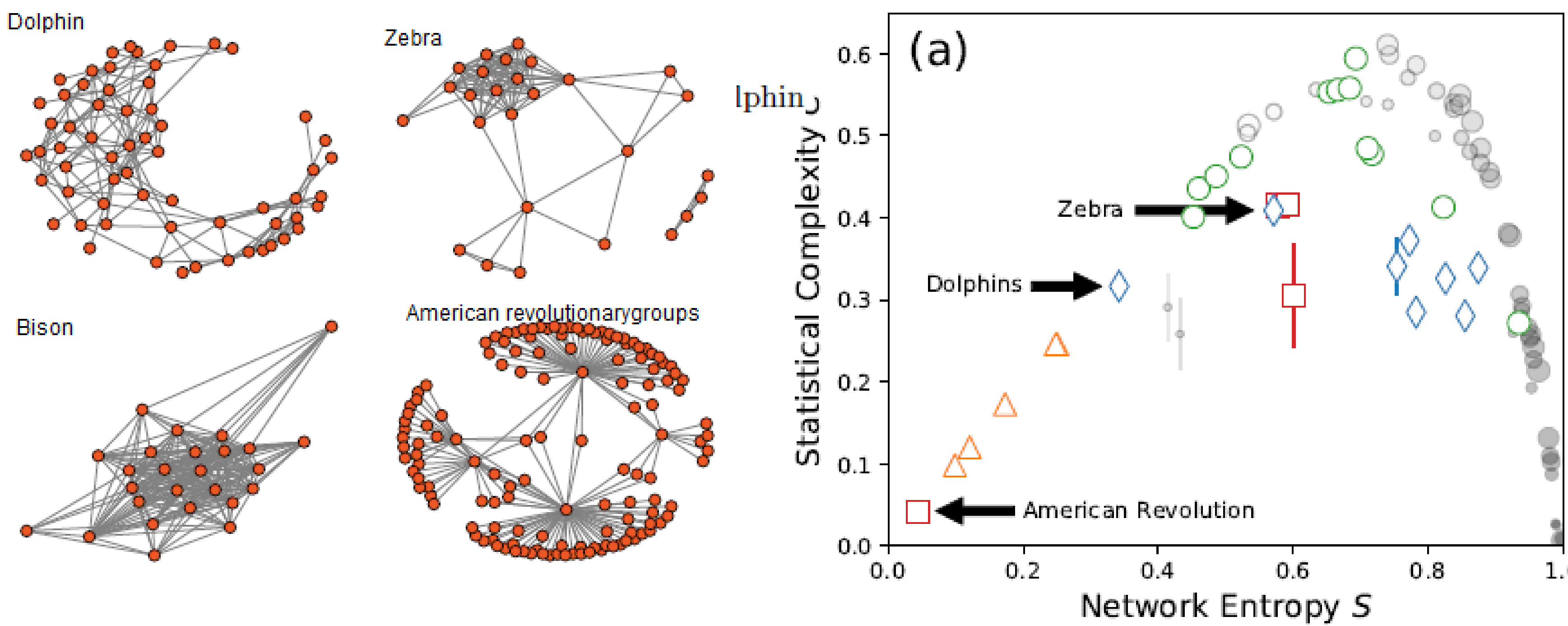
Beyond bi-variate analysis

Mapping and discrimination of networks in the complexity-entropy plane

Marc Wiedermann,[1,2,*] Jonathan F. Donges,[1,3] Jürgen Kurths,[1,2] and Reik V. Donner[1]

[1] *Potsdam Institute for Climate Impact Research — Telegraphenberg A31, 14473 Potsdam, Germany, EU*
[2] *Department of Physics, Humboldt University — Newtonstr. 15, 12489 Berlin, Germany, EU*
[3] *Stockholm Resilience Centre, Stockholm University — Kräftriket 2B, 114 19 Stockholm, Sweden, EU*

(Dated: April 26, 2017)

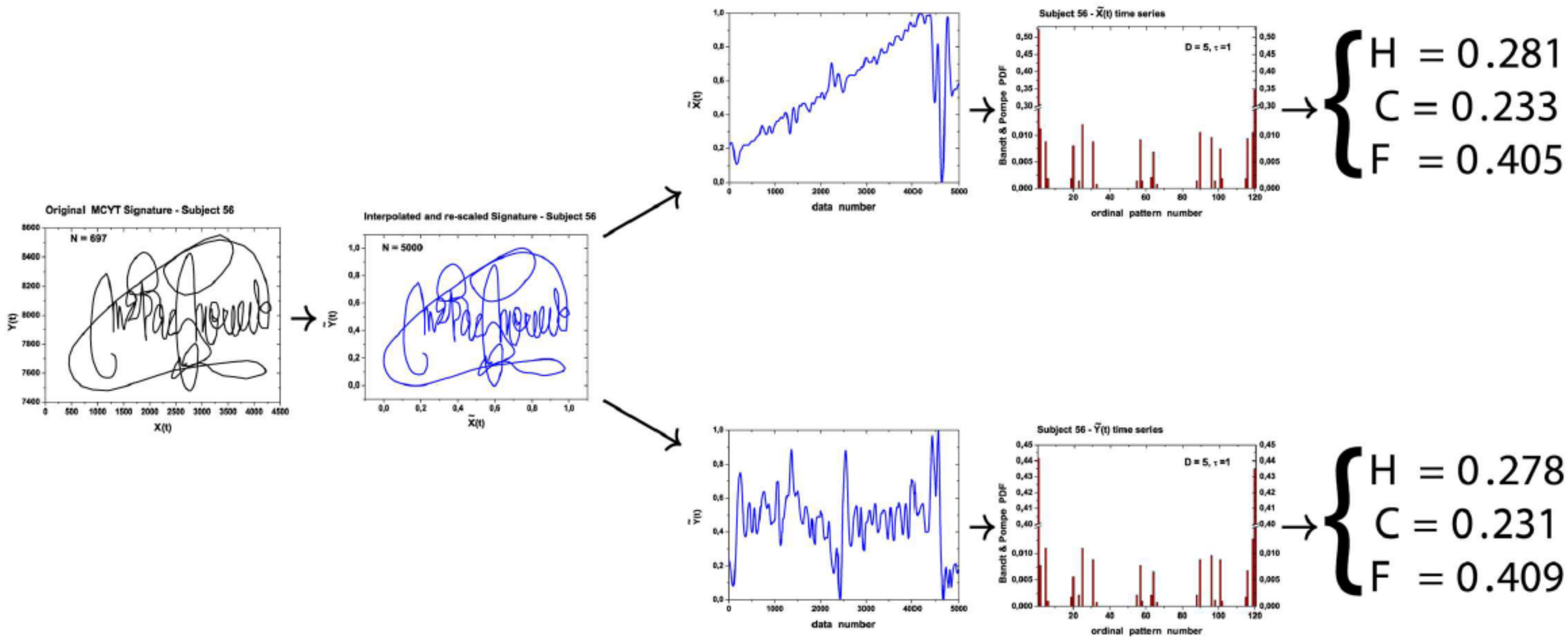


arXiv: 1704.07599v1

RESEARCH ARTICLE

Classification and Verification of Handwritten Signatures with Time Causal Information Theory Quantifiers

Osvaldo A. Rosso[1,2,3]*, Raydonal Ospina[4], Alejandro C. Frery[5]



PLOS ONE | DOI:10.1371/journal.pone.0166868 December 1, 2016

Perturbational complexity index (PCI)

A Theoretically Based Index of Consciousness Independent of Sensory Processing and Behavior
Adenauer G. Casali *et al.*
Sci Transl Med **5**, 198ra105 (2013);

- Electroencephalographic **index of human consciousness**.
- PCI is calculated by
  - perturbing the cortex with transcranial magnetic stimulation (TMS) to engage distributed interactions in the brain (integration) and
  - compressing the spatiotemporal pattern of these electrocortical responses to measure their algorithmic complexity (information).

Conclusions

What did we learn?

- Take home messages:
  - Symbolic analysis, network representation, spatiotemporal representation, etc., are useful tools for investigating complex signals.
  - Different techniques provide *complementary* information.

"…nonlinear time-series analysis has been used to great advantage on thousands of real and synthetic data sets from a wide variety of systems ranging from roulette wheels to lasers to the human heart. Even in cases where the data do not meet the mathematical or algorithmic requirements, the results of nonlinear time-series analysis can be helpful in understanding, characterizing, and predicting dynamical systems…"

Bradley and Kantz, CHAOS 25, 097610 (2015)

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<cristina.masoller@upc.edu>

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