Identifying and characterizing regime transitions in complex dynamical systems

Cristina Masoller
Universitat Politecnica de Catalunya, Physics Department
Terrassa, Barcelona, Spain

Cristina.masoller@upc.edu

www.fisica.edu.uy/~cris

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Campus d'Excel·lència Internacional

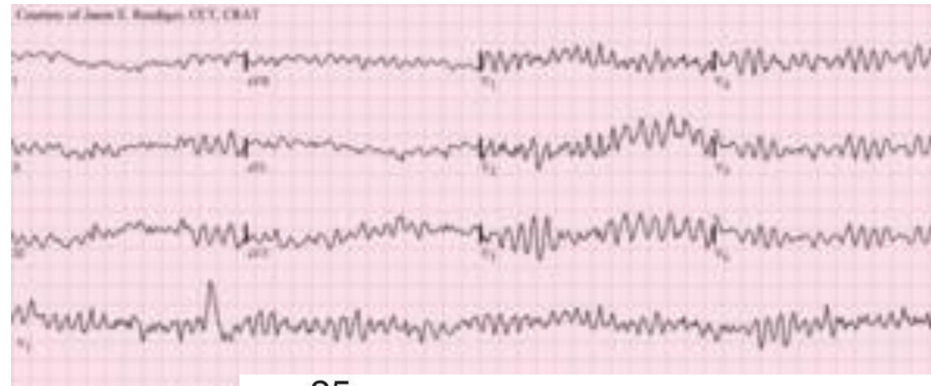
Medyfinol 2016
Valdivia, Chile

Dangerous regime transitions

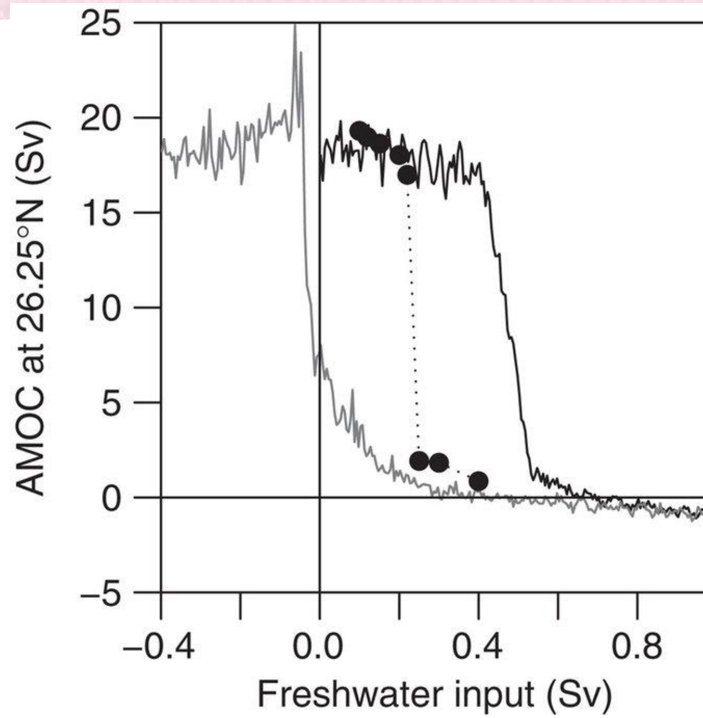
Electroencephalographs - EEGs

www.epilepsysociety.org.uk

Cardiac arrhythmia



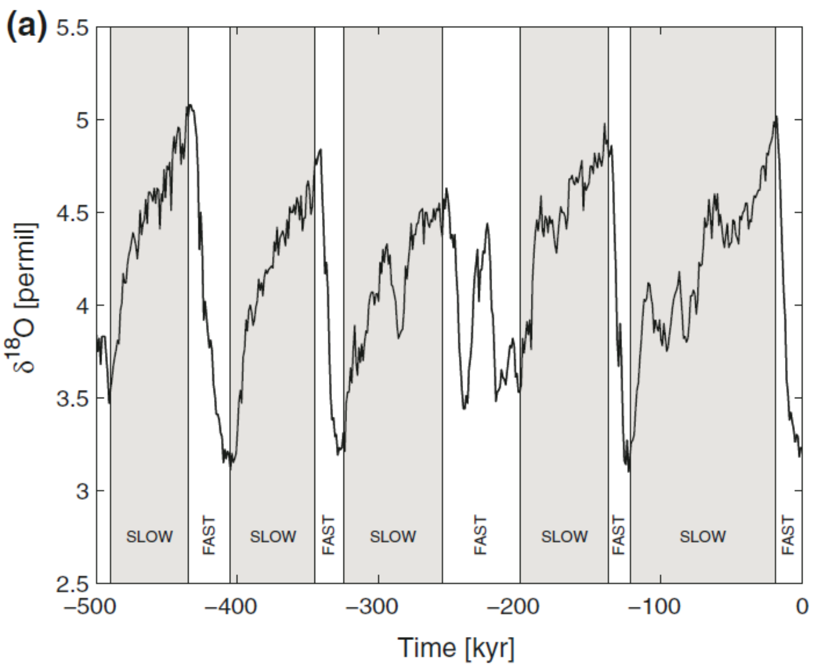
Wikipedia



Collapse of Atlantic Meridional Overturning Circulation
Nat. Comm. 2014

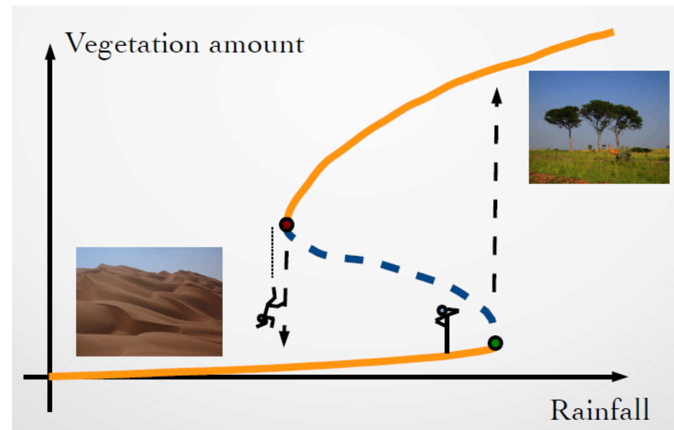
Motivation

A lot of work is being devoted to develop reliable diagnostic tools for identifying and characterizing regime transitions



long-term climatic signals reveal a slow-fast dynamics.

De Saedeleer, Crucifix and Wieczorek, Clim. Dyn. 2013

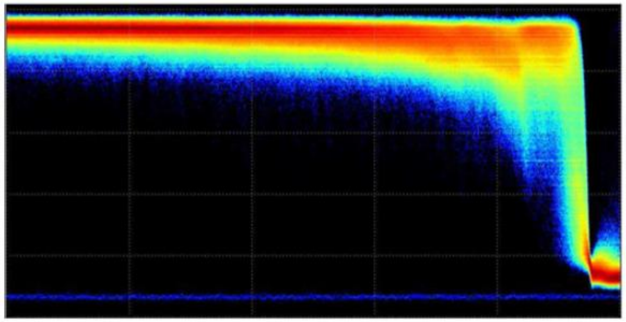


Bangladesh, Nature 2014

Dynamical regime transitions in optical systems

- **Polarization switching**

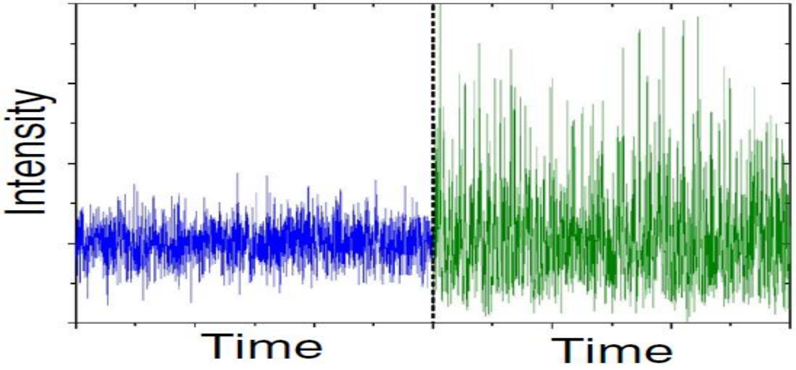
  Semiconductor laser output as the pump current increases



Time

- **Transition to turbulence**

  Raman fiber laser output as the pump power increases



- **How optical chaos emerges from noise: transition to coherence collapse**

  Semiconductor laser output as the pump current increases

Scope of this presentation

- Data analysis tools based on symbolic analysis and network representation of time-series provide new insight into these phenomena.

- Optical data can be useful for testing novel analysis tools.

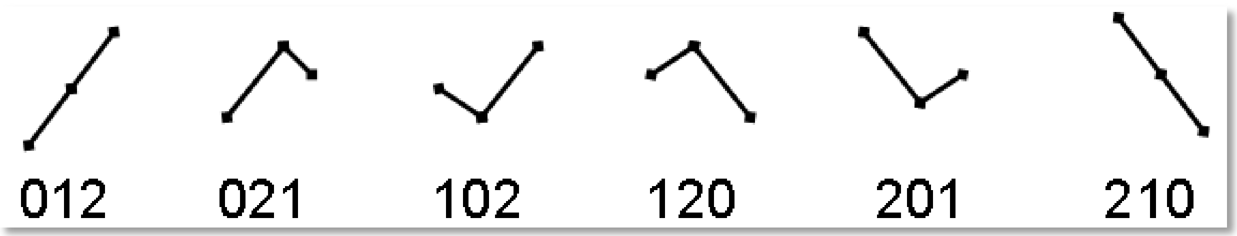
Outline

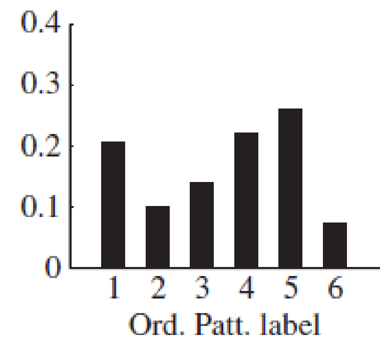
- Tools to analyze the data
- Early-warning signs of upcoming abrupt switching
- Laminar-turbulence transition
- Optical noise – chaos transition
- Summary

Method of nonlinear symbolic time-series analysis: ordinal patterns

- X= {…x_i, x_{i+1}, x_{i+2}, …}

Brandt & Pompe, PRL 88, 174102 (2002)





The OP probabilities allow identifying <u>more expressed and/or infrequent patterns</u> in the <u>*order*</u> of the sequence of data values.

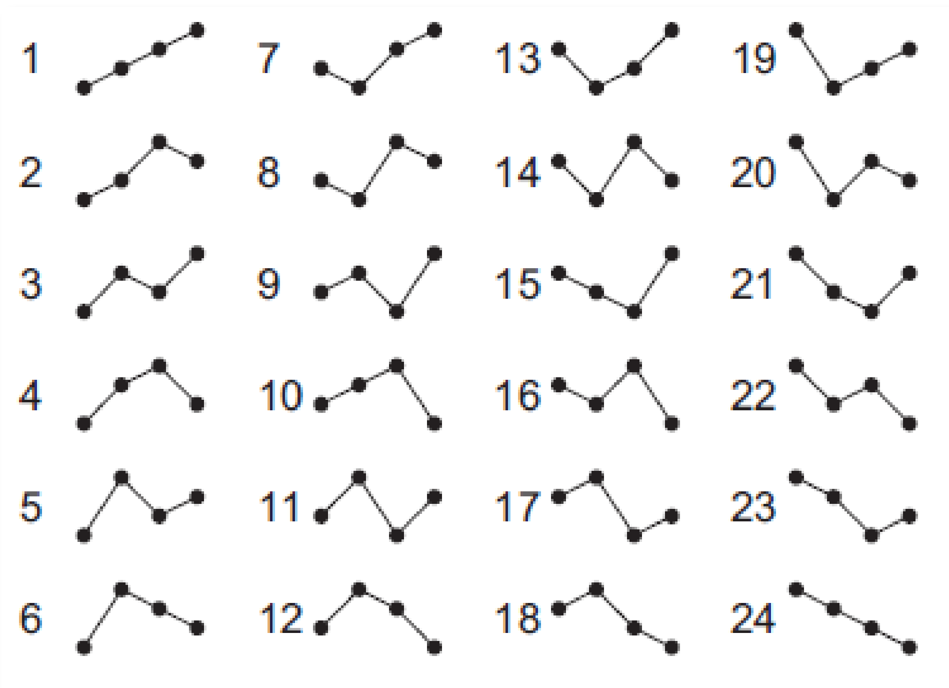
Random data?

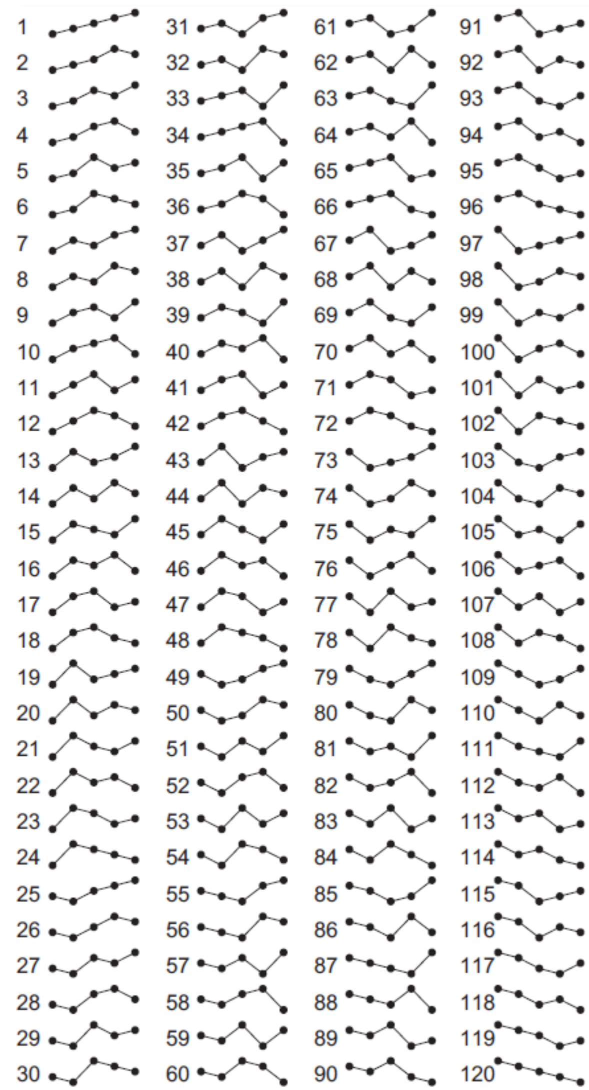
- Advantage: the probabilities uncover temporal correlations.
- Drawback: we lose information about the actual values.

⇒ Ordinal analysis provides complementary information to that gained with other analysis tools.

Read more: M. Zanin, L. Zunino, O. A. Rosso, and D. Papo, Entropy 14, 1553 (2012)

The number of patterns increases as D!

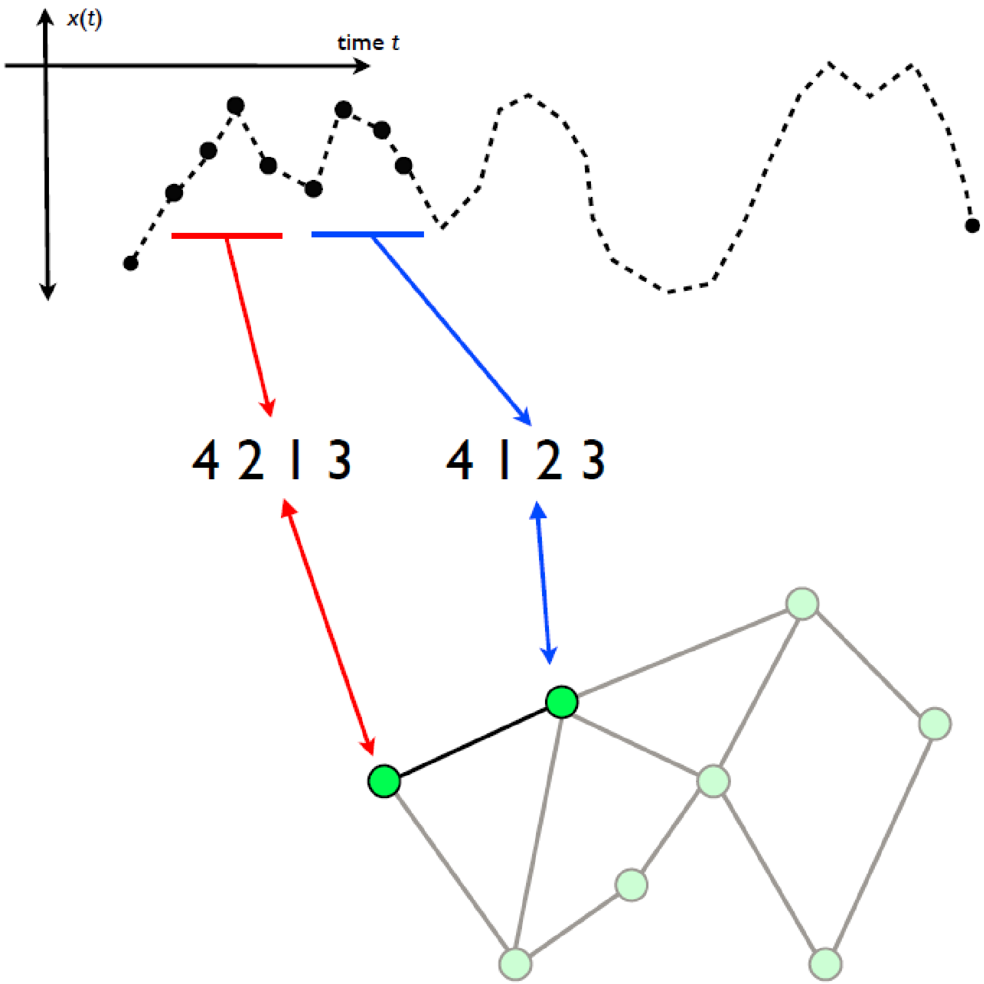




Opportunity: turn a time-series into a network by using the patterns as the "nodes" of the network.

U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

The network nodes are the "ordinal patterns", and the links?



- The links are defined in terms of the probability of pattern "β" occurring after pattern "α".
- Weighs of nodes: the probabilities of the patterns (**$\sum_i p_i=1$**).
- Weights of links: the probabilities of the transitions (**$\sum_j w_{ij}=1\ \forall i$**).

$\Rightarrow$ ***Weighted and directed network***

Adapted from M. Small (The University of Western Australia)

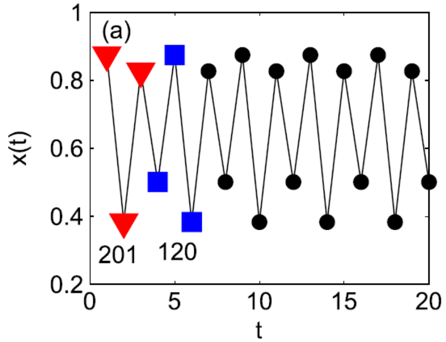
Three network-based diagnostic tools

- Entropy computed from the weights of the nodes (**permutation entropy**)

$$s_p = -\sum p_i \log p_i$$

- Entropy computed from weights of the links (**transition probabilities**, '01'→ '01', '01'→ '10', etc.)

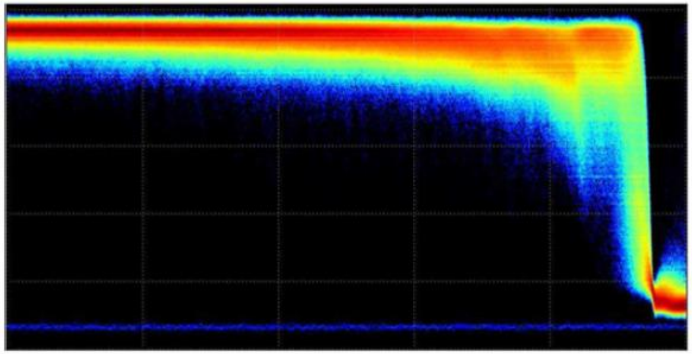
$$w_{ij} = \frac{\sum_{t=1}^{L-1} n\left[s(t) = i,\, s(t+1) = j\right]}{\sum_{t=1}^{L-1} n\left[s(t) = i\right]}$$



- Asymmetry coefficient: normalized difference of transition probabilities, P('01'→ '10') - P('10'→ '01'), etc.

$$a_c = \frac{\sum_i \sum_{j\neq i} \left| w_{ij} - w_{ji} \right|}{\sum_i \sum_{j\neq i} \left(w_{ij} + w_{ji} \right)}$$

(0 in a fully symmetric network; 1 in a fully directed network)



Identifying early signs of upcoming transition

- "optical big data": useful for testing novel diagnostic tools

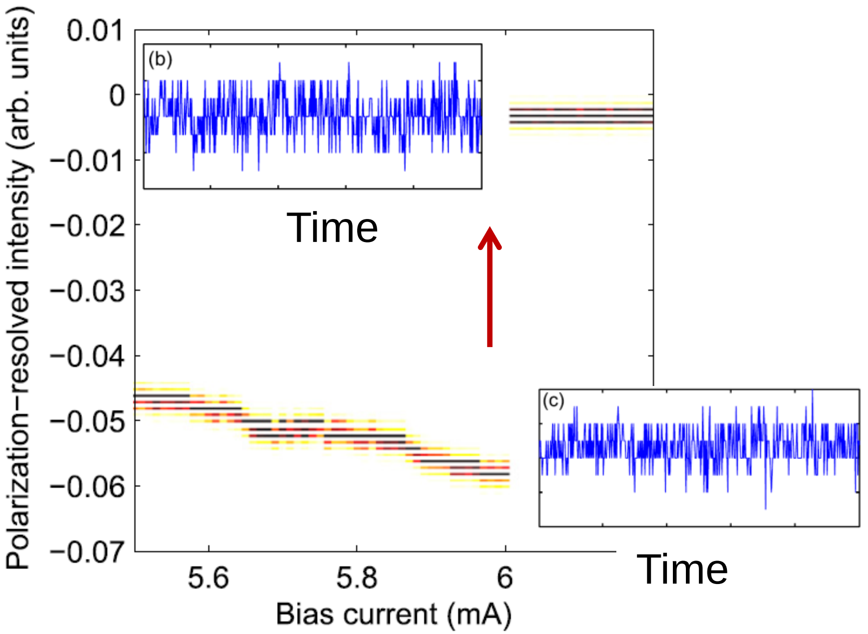
UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Campus d'Excel·lència Internacional

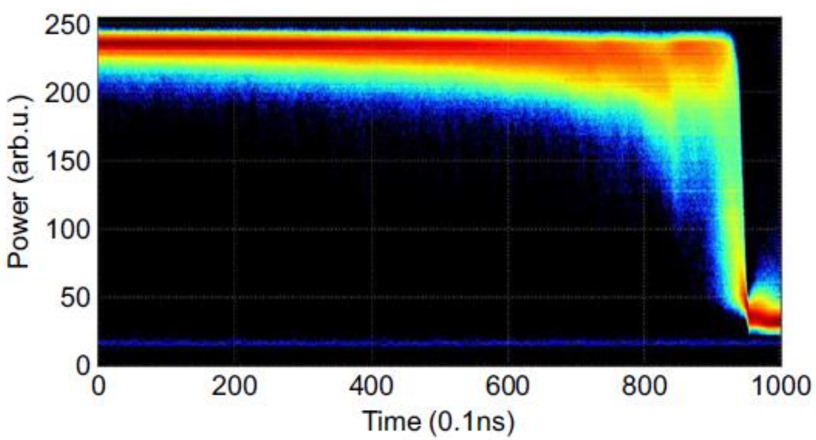
Experimental data from INLN & Bangor University (S. Barland & Y. Hong)

VCSEL polarization-resolved intensity: two sets of experiments

- Time series recorded with pump current constant in time.
- Record the turn-on of the orthogonal mode.



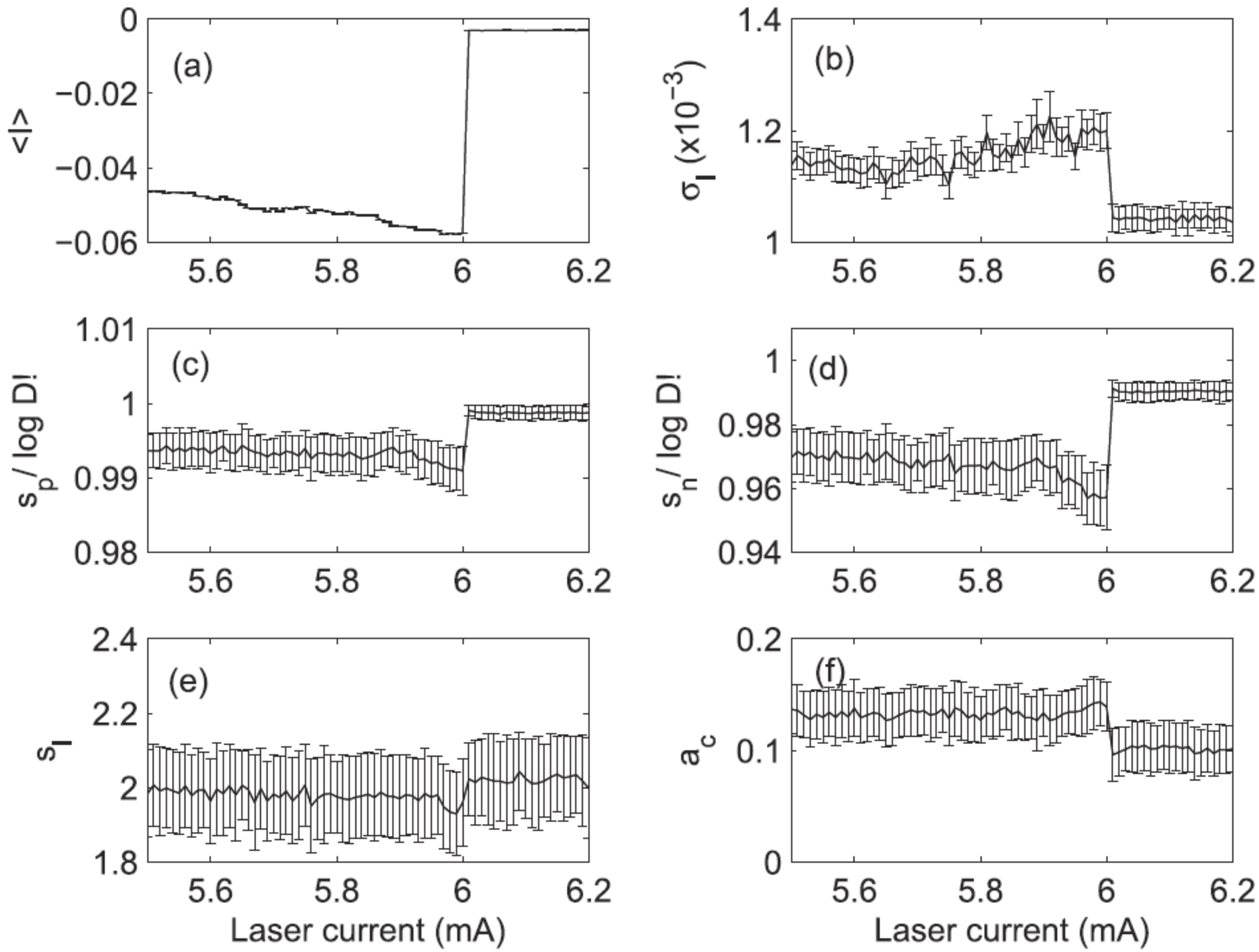
- Time series recorded with pump current varying in time.
- Record the turn-off of the fundamental mode.



Is it possible to anticipate the PS?

No if the mechanisms that trigger the PS are fully stochastic.

Results for constant pump current & turn-on of the orthogonal mode



⇒ Despite of the stochasticity of the time-series, the measures "anticipate" the PS.

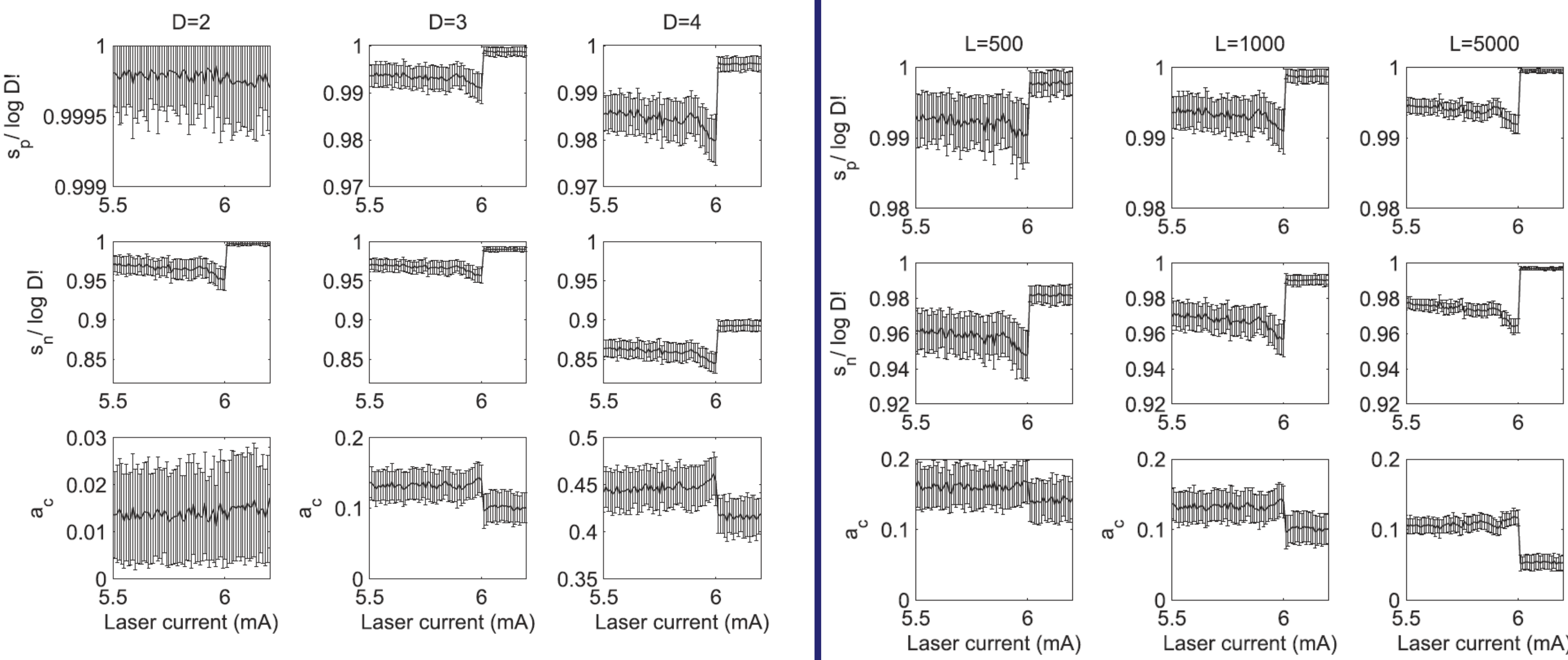
⇒ Deterministic mechanisms involved.

Error bars computed from 100 non-overlapping windows with L=1000 data points each. Length of the pattern D=3.

C. Masoller et al, New J. Phys. 17, 023068 (2015)

Influence of the length of the pattern (D) and of length of the time-series (L)

L=1000 | **D=3**



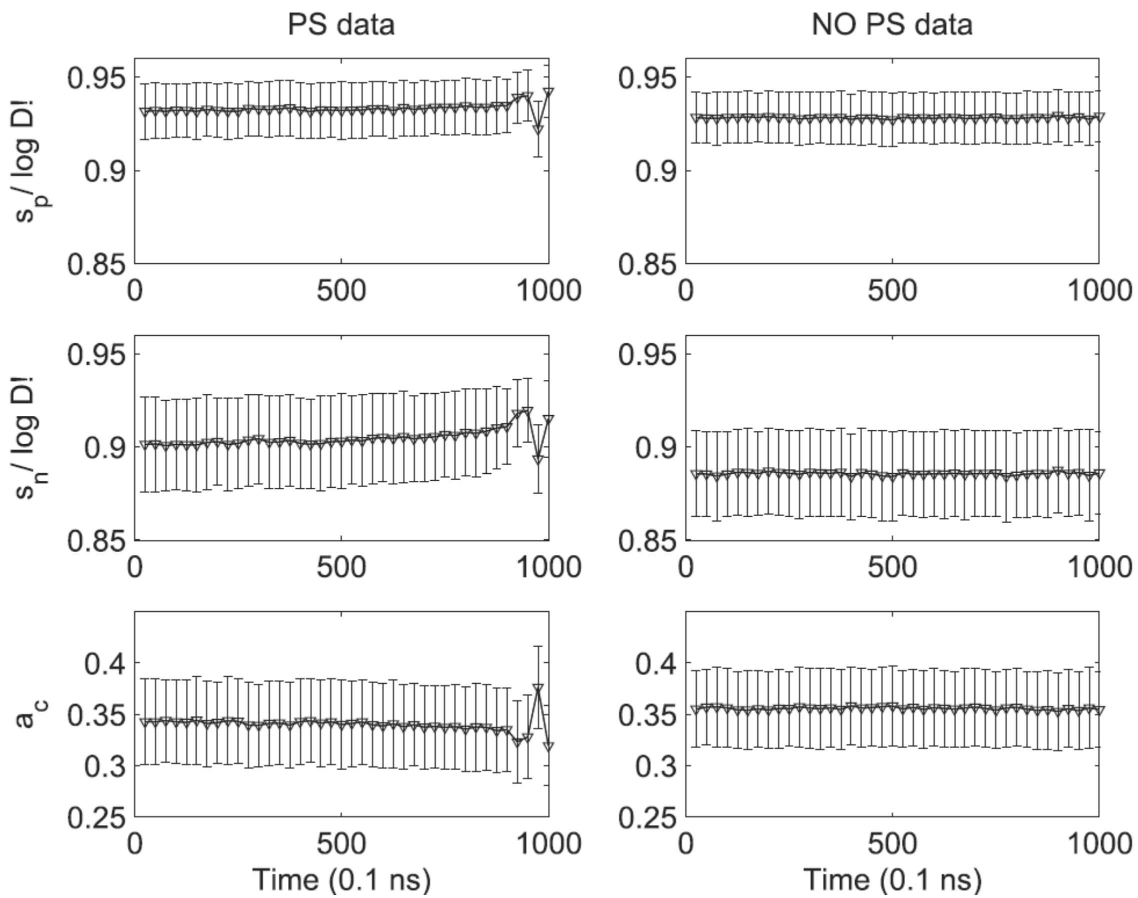
⇒ Transition detected even for short dataset (L=500 with D=3).
Open issues: quantify performance? optimal D depends on L?

C. Masoller et al, New J. Phys. 17 (2015) 023068

Results for time-varying pump current & turn-off of the fundamental mode

Slightly different optical feedback conditions result in PS or no PS.

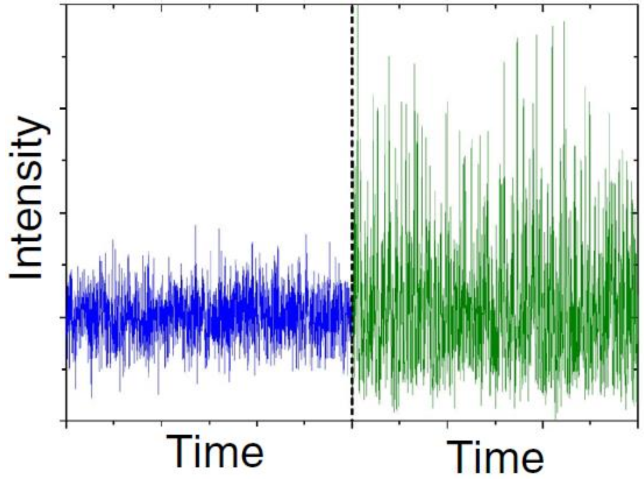
Analysis done with D=3, error bars computed with 1000 time series L=500.



Another open issue: comparison with other diagnostic tools

C. Masoller et al, New J. Phys. 17 (2015) 023068

Low --   High pump power



Characterizing the laminar-turbulence transition in a fiber laser

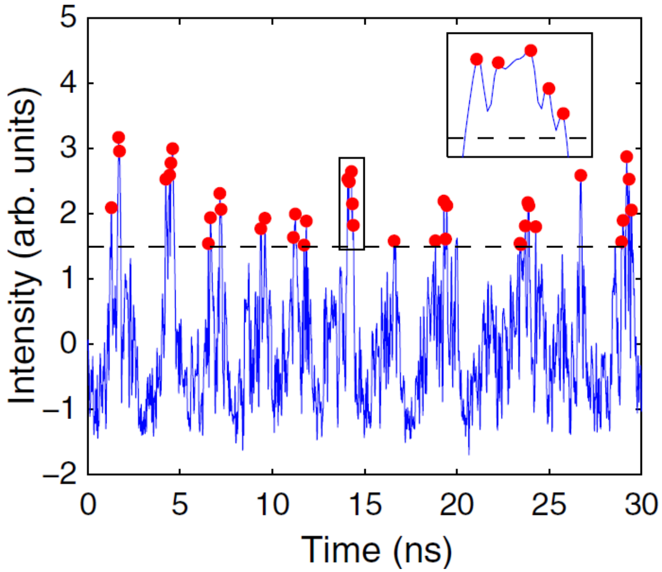
UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Campus d'Excel·lència Internacional

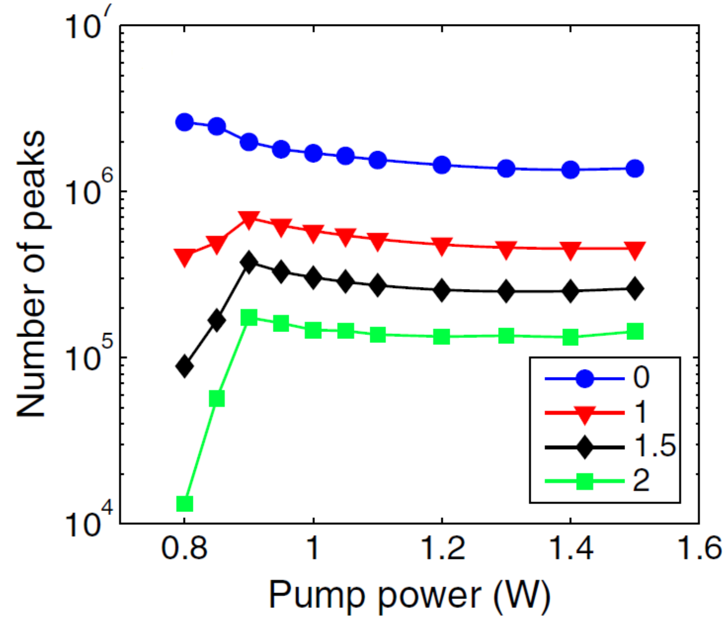
Experimental data from Aston University, UK
(Prof. Turitsyn' group)

Analysis of the intensity peaks higher than a threshold

Each time series is first normalized to $\langle I \rangle = 0$ and $\sigma = 1$



$$\{I_{\max,i}\}$$



In each time series 5×10^7 data points.
Sampling time dt = 12.5 ps

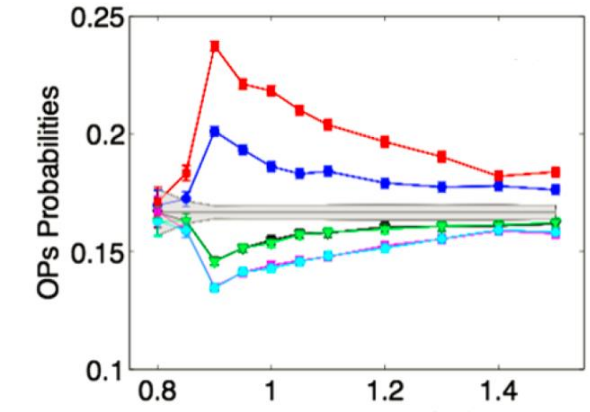
Th = 2: number of peaks $>10^4$ for all values of the pump power

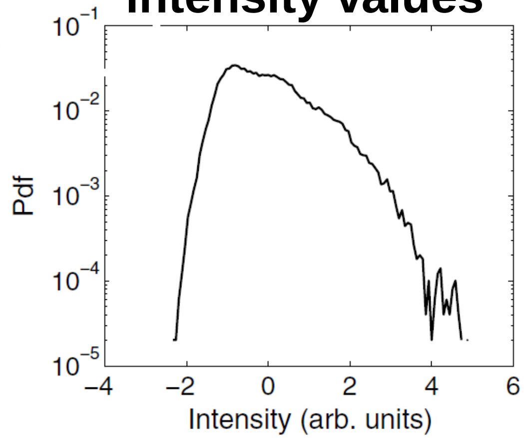
UNIVERSITAT POLITÈCNICA
DE CATALUNYA
BARCELONATECH

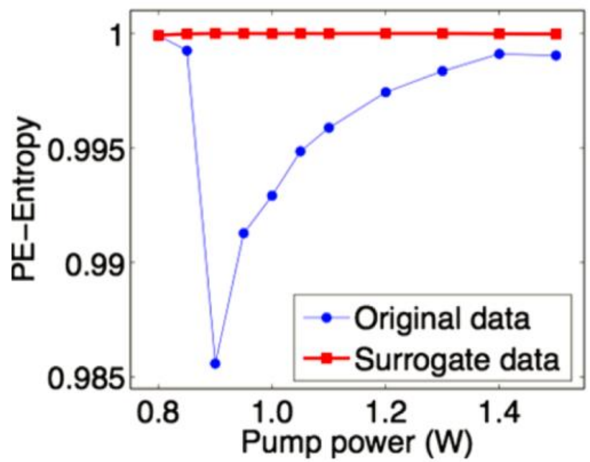
Campus d'Excel·lència Internacional

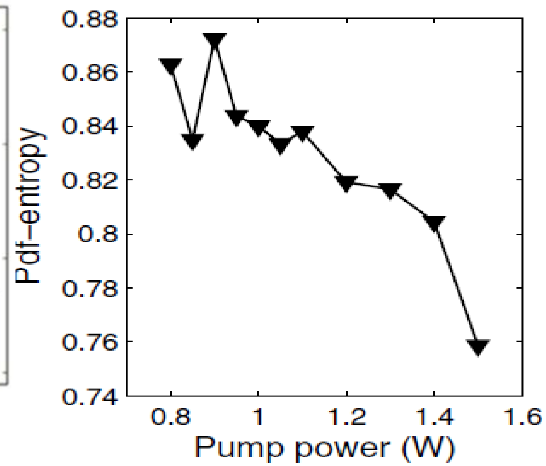
Ordinal analysis of the intensity peaks above Th = 2

Histogram of raw intensity values









- Sharp transition <u>not captured</u> by standard histogram analysis.
- Different entropy behavior.

A. Aragoneses, L. Carpi, N. Tarasov, D. V. Churkin, M. C. Torrent, C. Masoller and S. K. Turitsyn, PRL 116, 033902 (2016)

Second diagnostic tool: horizontal visibility graph (HVG)

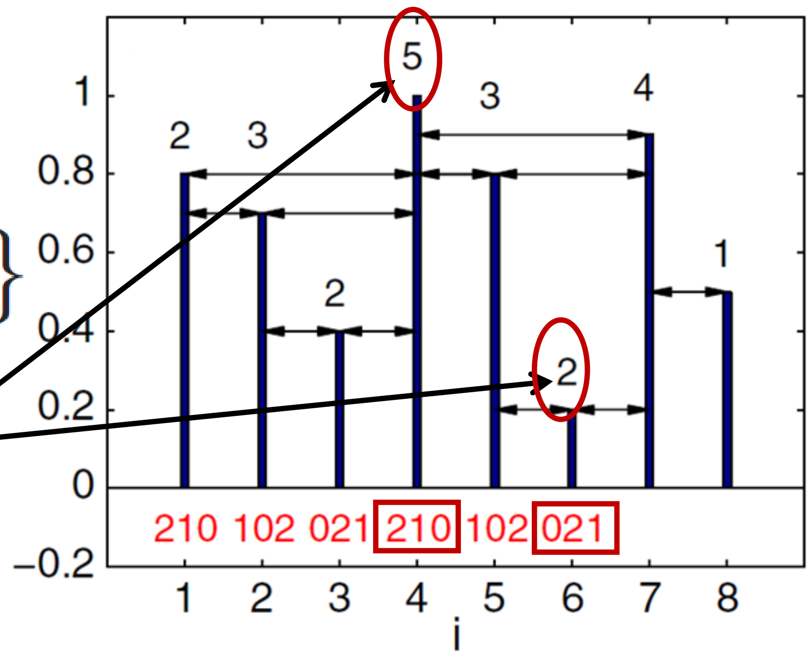
A time-series is represented as a graph, where each data point is a node

intensity peaks above Th = 2

$\{I_{\max,i}\}$

Number of links

Ordinal pattern D=3

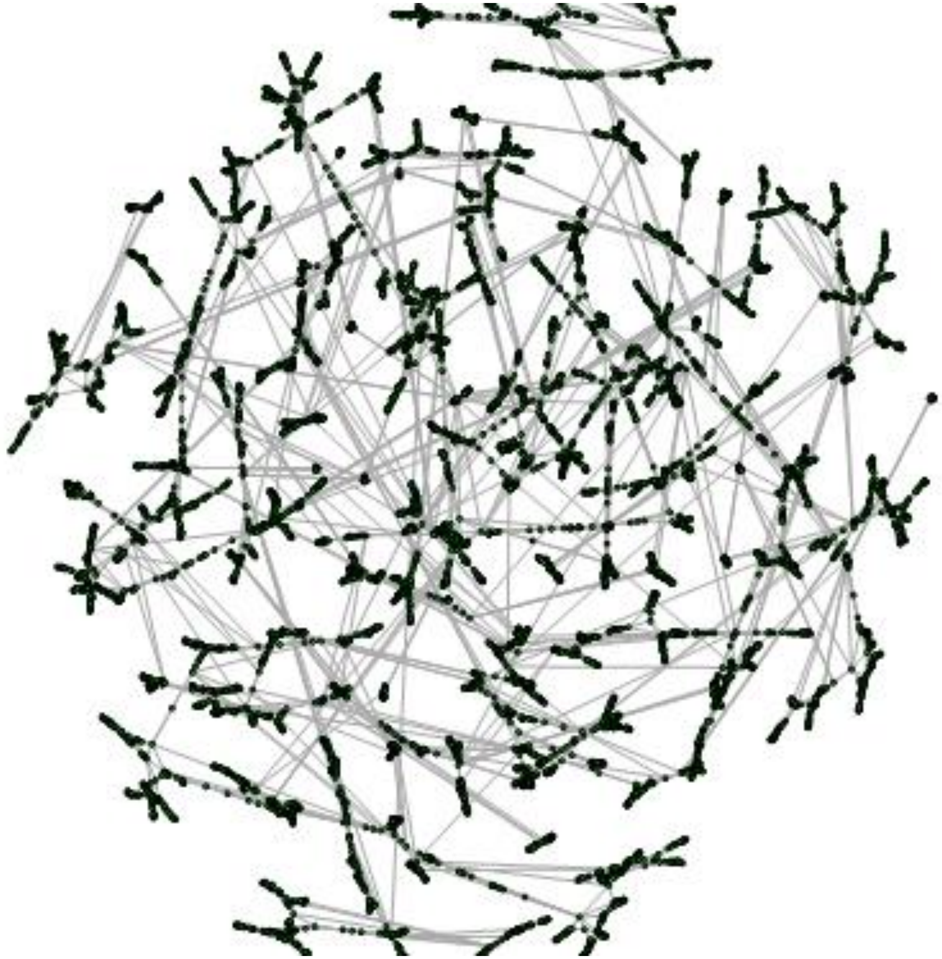


- Rule: data points i and j are connected if there is "visibility" between them: $I_{\max,i}$ and $I_{\max,j} > I_{max,n}$ for all n, $i<n<j$

$\Rightarrow$ ***Unweighted and undirected graph***

Read more HVG method: B. Luque et al, PRE 80, 046103 (2009)

The obtained graph

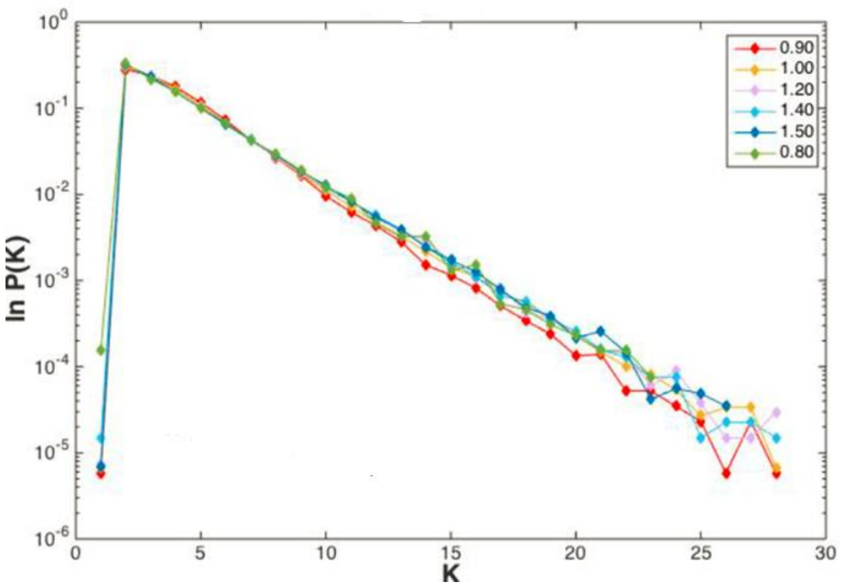


How to characterize this graph?

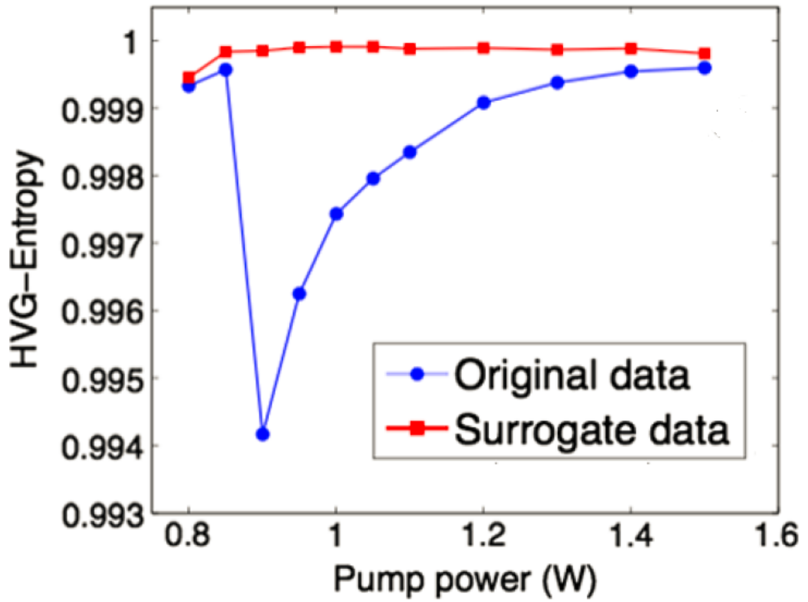
HVG analysis

⇒ Degree Distribution (distribution of the number of links)

- Degree distribution for various pump powers using Th=2.



- Entropy of the degree distribution (normalized to the entropy of Gaussian white noise)



⇒ sharp transition also seen with HVG technique.

Aragoneses et al, PRL 116, 033902 (2016)

UNIVERSITAT POLITÈCNICA
DE CATALUNYA
BARCELONATECH

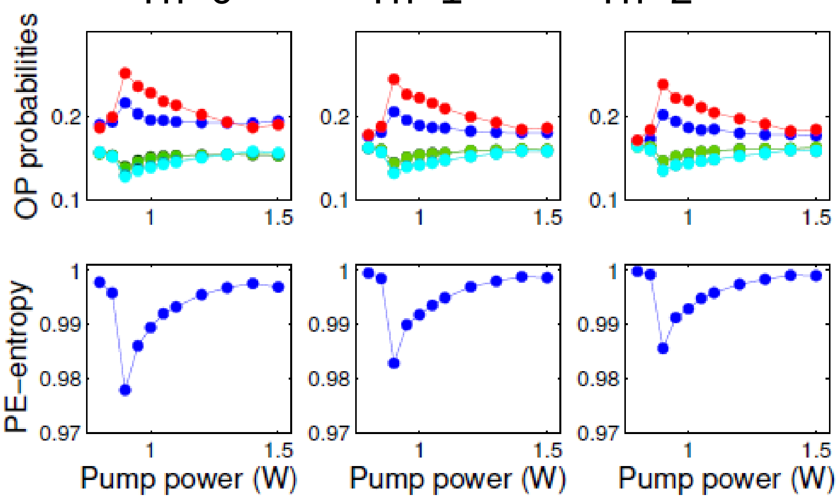
Campus d'Excel·lència Internacional

Influence of the threshold

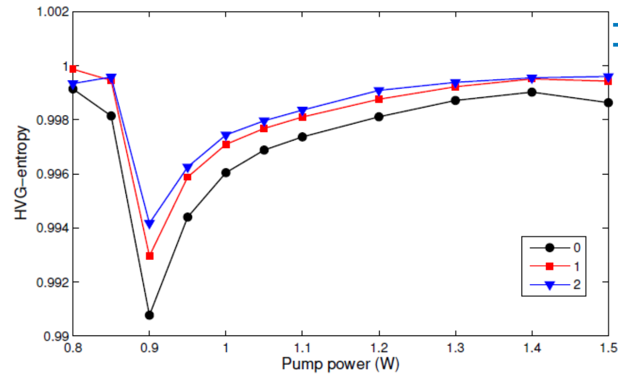
Raw data $\{\ldots I_i \ldots\} \Rightarrow \text{Th} \Rightarrow \{\ldots I_{max,i} \ldots\}$

Ordinal analysis

Th=0 Th=1 Th=2

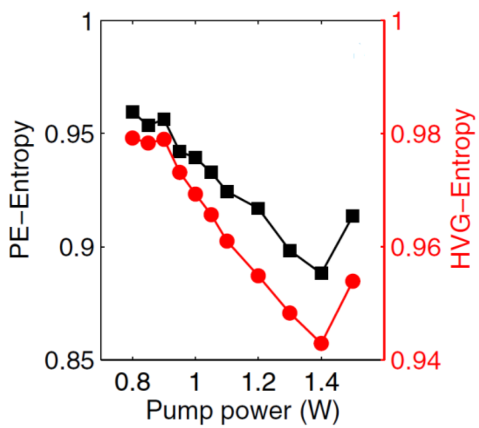


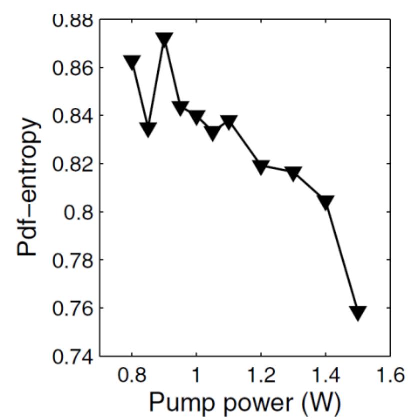
HVG analysis



⇒ sharp transition seen with different thresholds.

With <u>raw data</u>





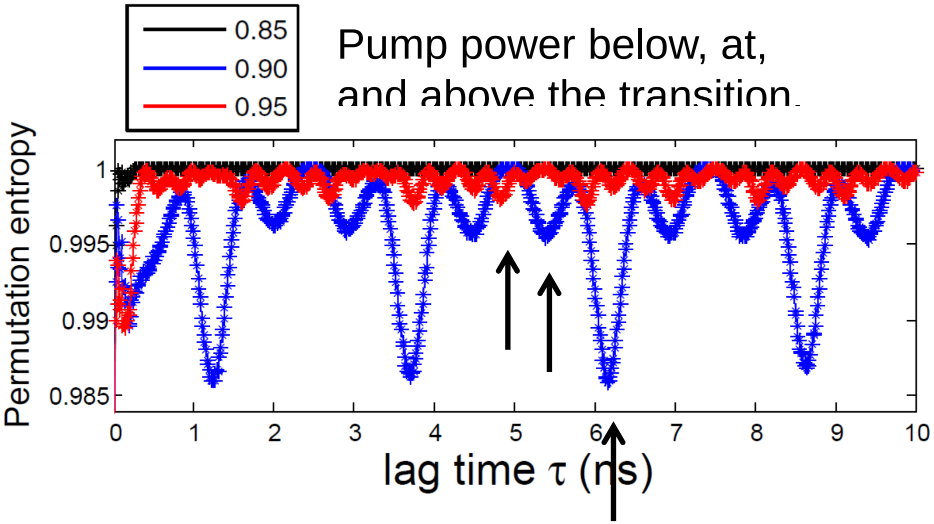
⇒ sharp transition not seen.
Can we obtain more info. from the raw data?

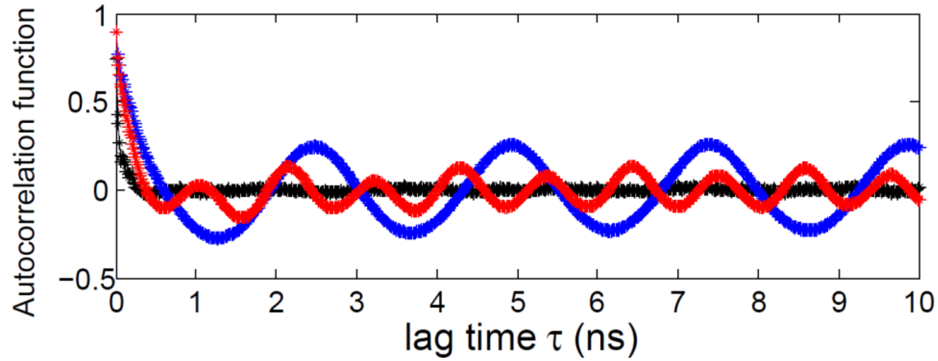
Aragoneses et al, PRL 116, 033902 (2016)

Ordinal analysis of lagged raw data

$$\{I_i\,, I_{i+\tau}, I_{i+2\tau}\,, \ldots\}$$

all data points, no threshold used

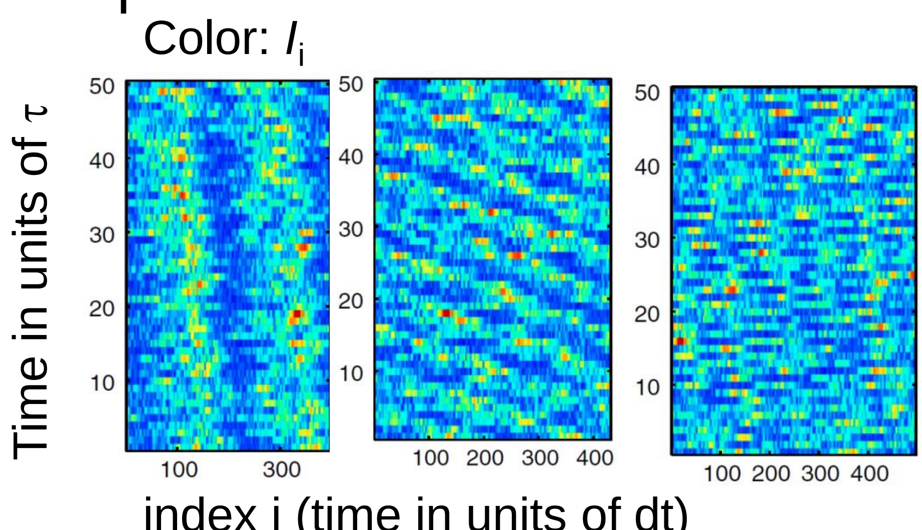




$\Rightarrow$ Sharp variations at the transition not captured.

Space-time representation

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ I_{2\tau+1} & I_{2\tau+2} & \cdots & I_{3\tau} \\ I_{\tau+1} & I_{\tau+2} & \cdots & I_{2\tau} \\ I_1 & I_2 & \cdots & I_\tau \end{bmatrix}$$



$\Rightarrow$ Different structures uncovered with different lags (sampling times).

How optical chaos emerges from noise?

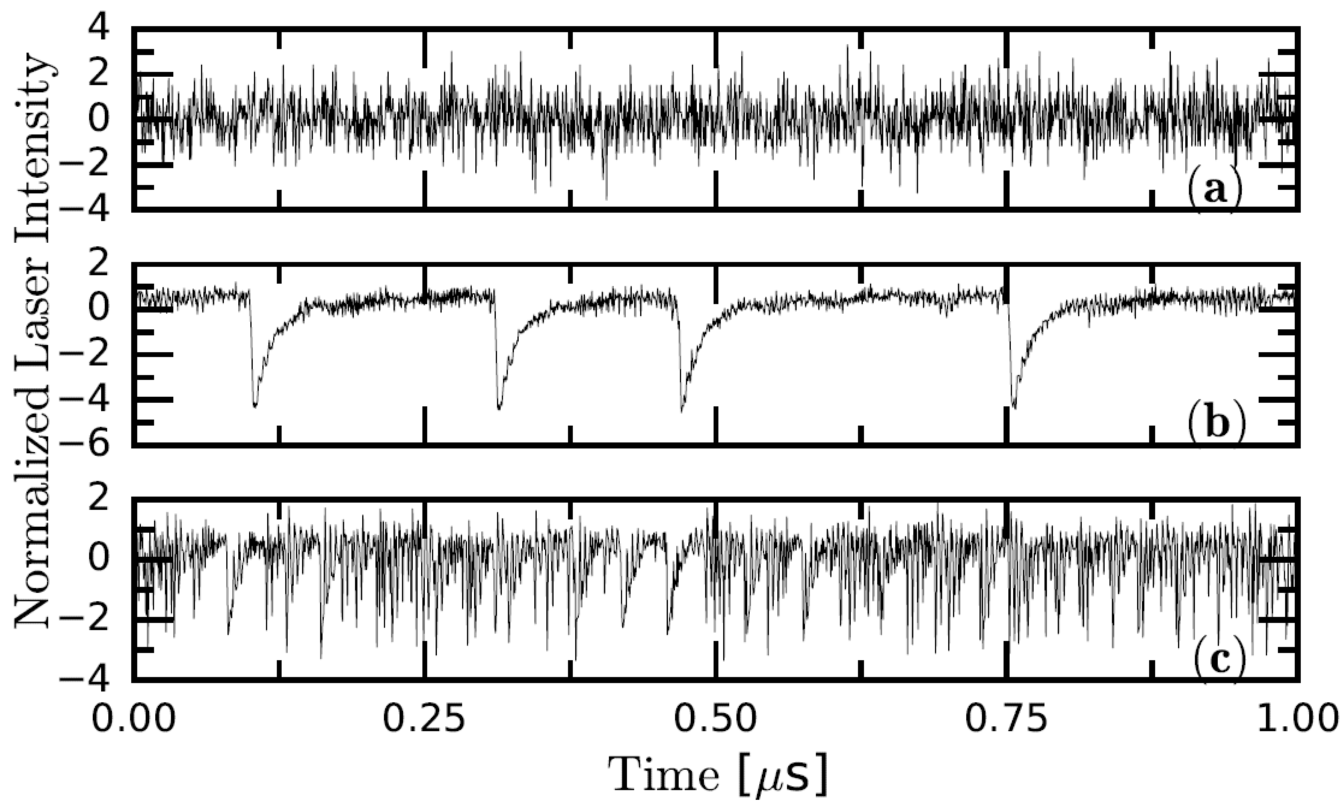
UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Campus d'Excel·lència Internacional

Noise-LFFs-CC transition

Experimental data recorded in our lab (zero mean, normalized to σ=1)

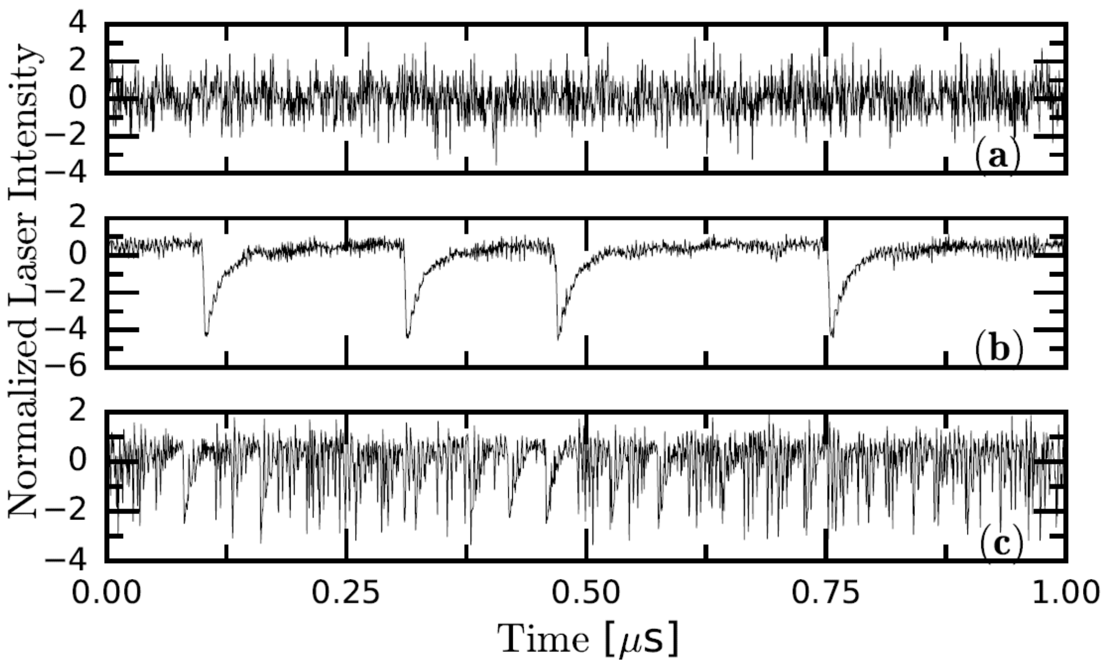
Increasing
pump current



D. Lenstra, B. Verbeek and A. Den Boef, IEEE J. Quantum Electron. 21, 674 (1985)

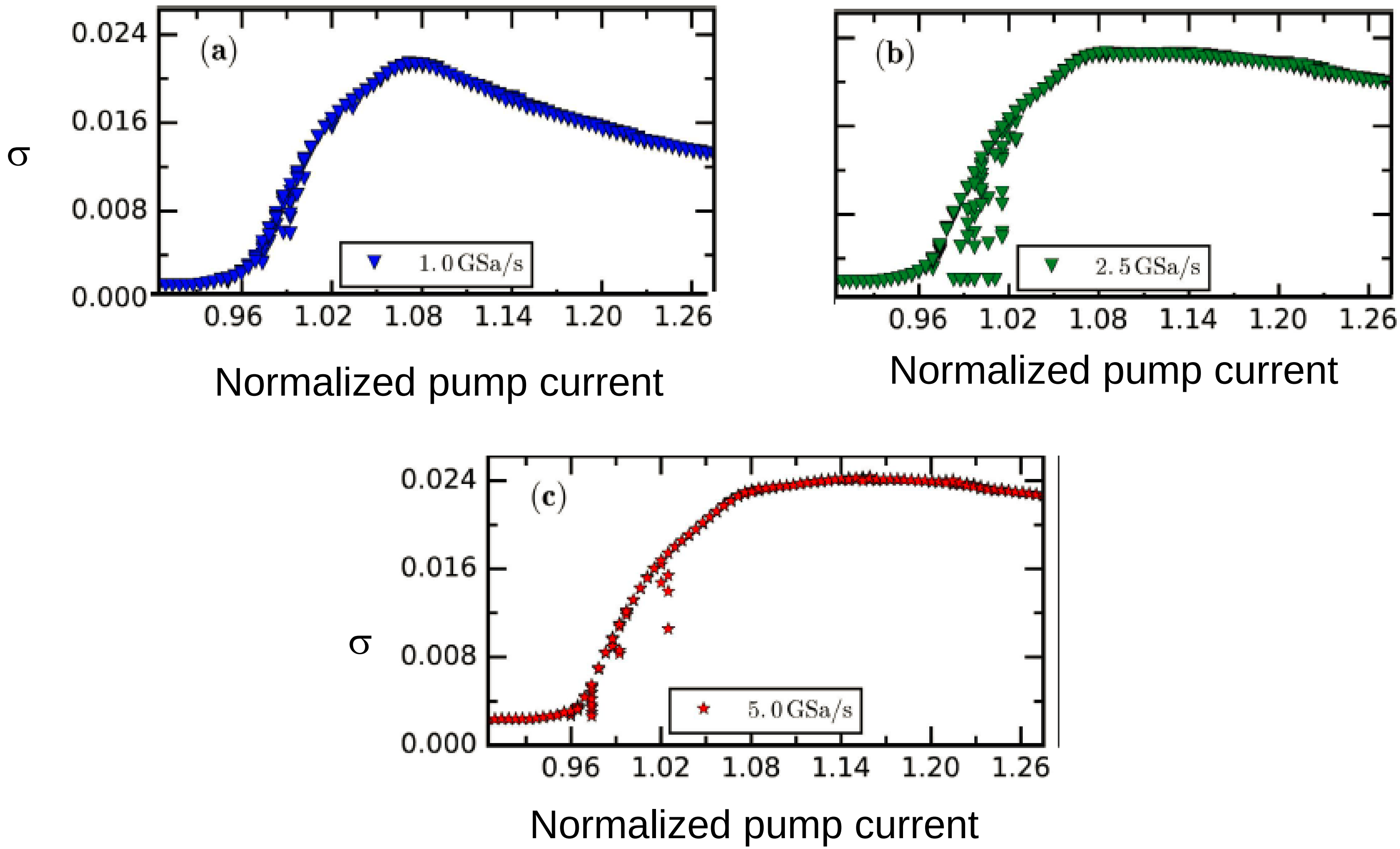
- Can we quantitatively identify the onset of LFFs and CC regimes?

- Can we quantify their different properties?

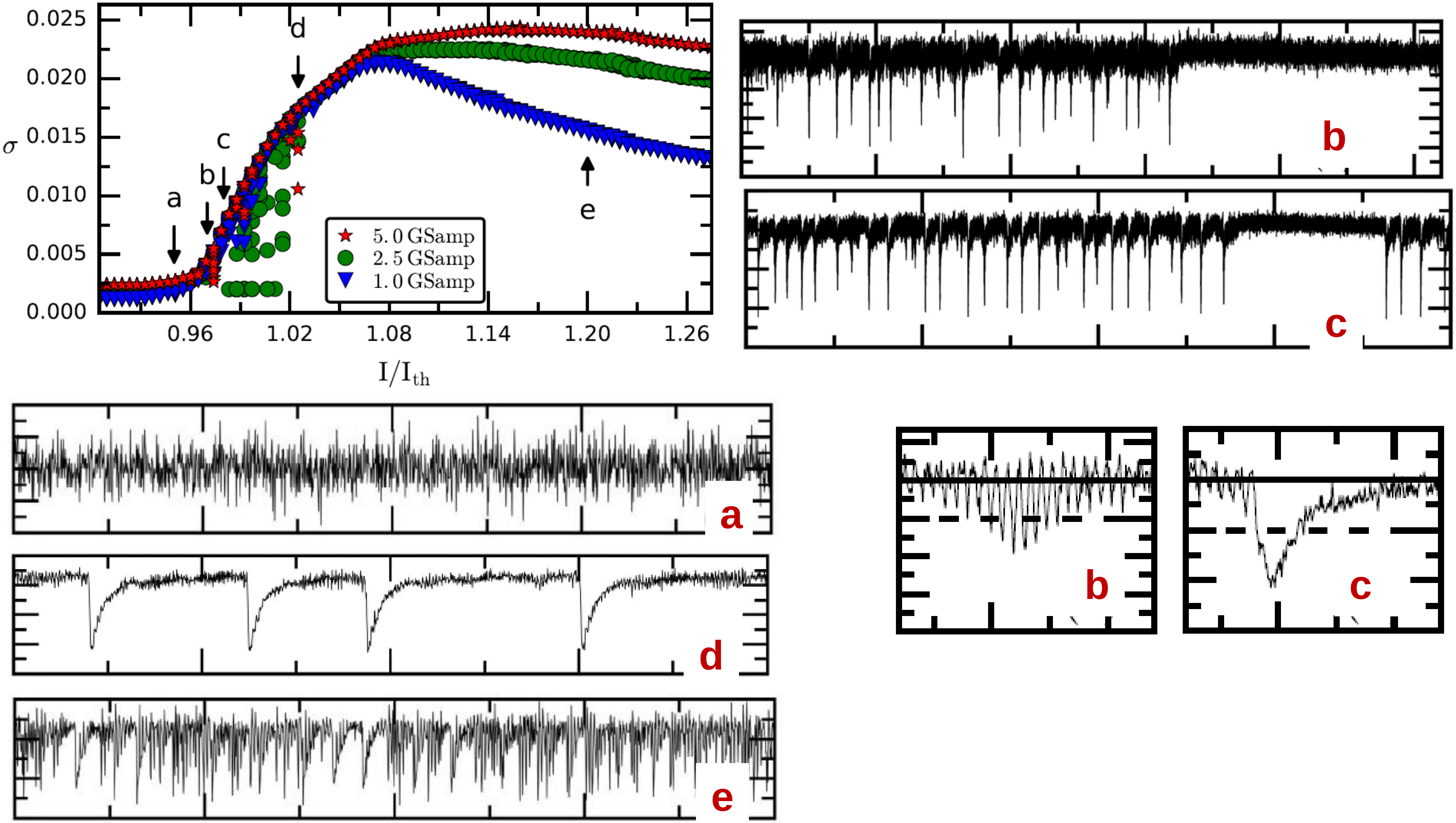


LFFs: slow-fast dynamics

Intensity PDF depends on the oscilloscope sampling time

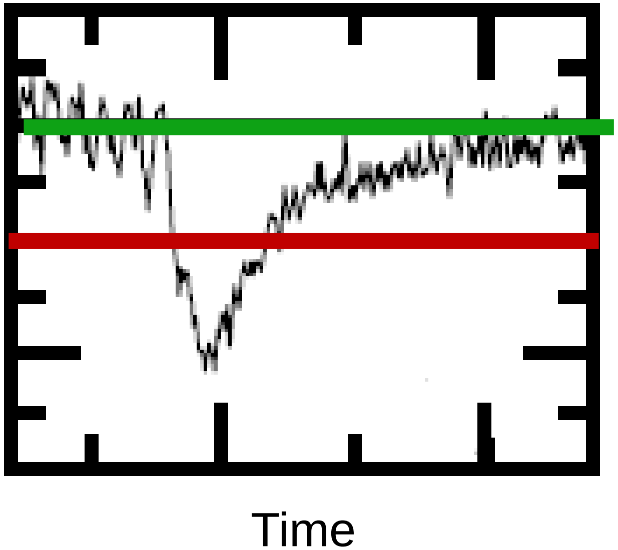


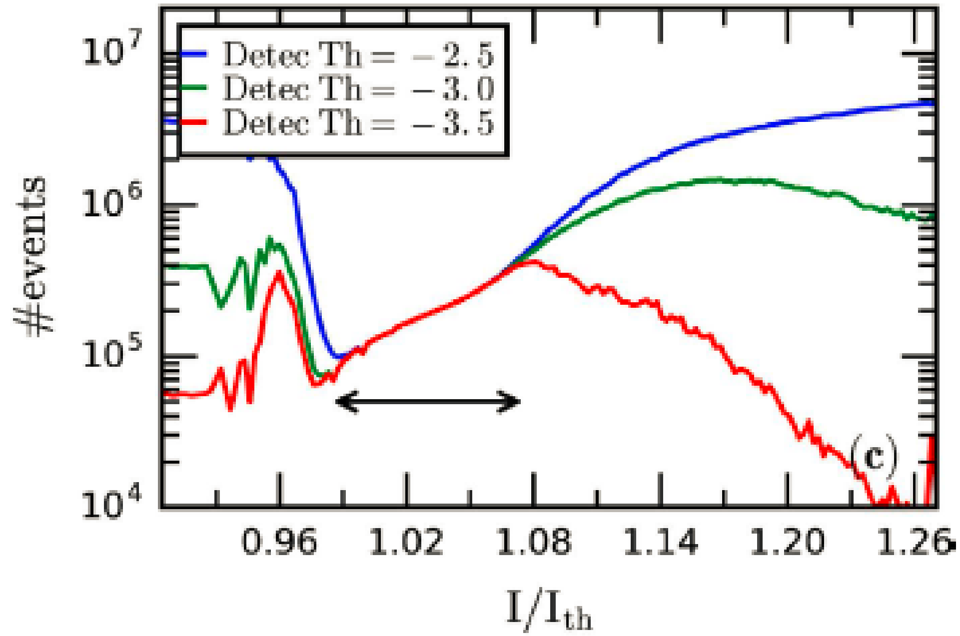
Identifying regime transition points

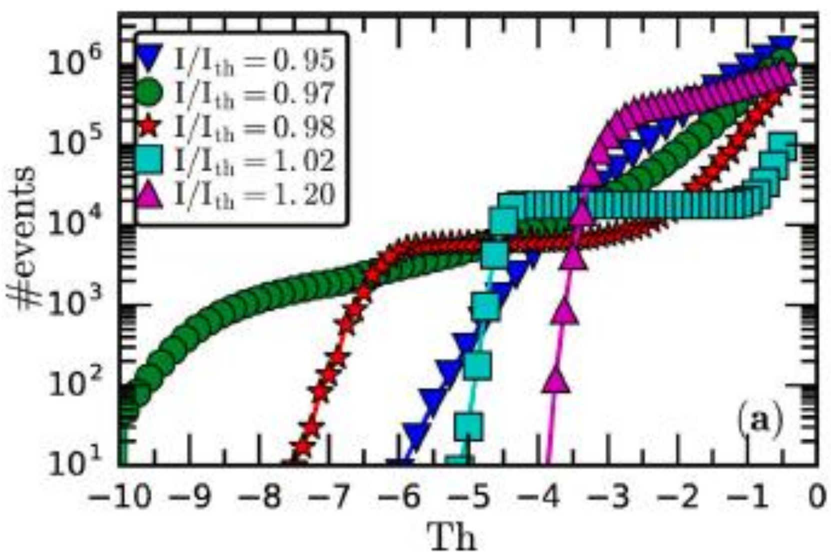


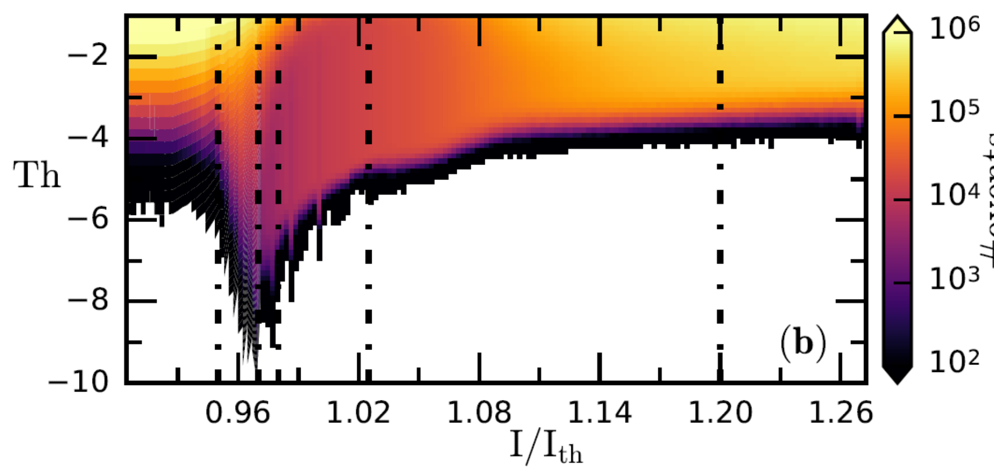
C. Quintero-Quiroz et al, Sci. Rep. 6 37510 (2016)

Number of threshold-crossing events

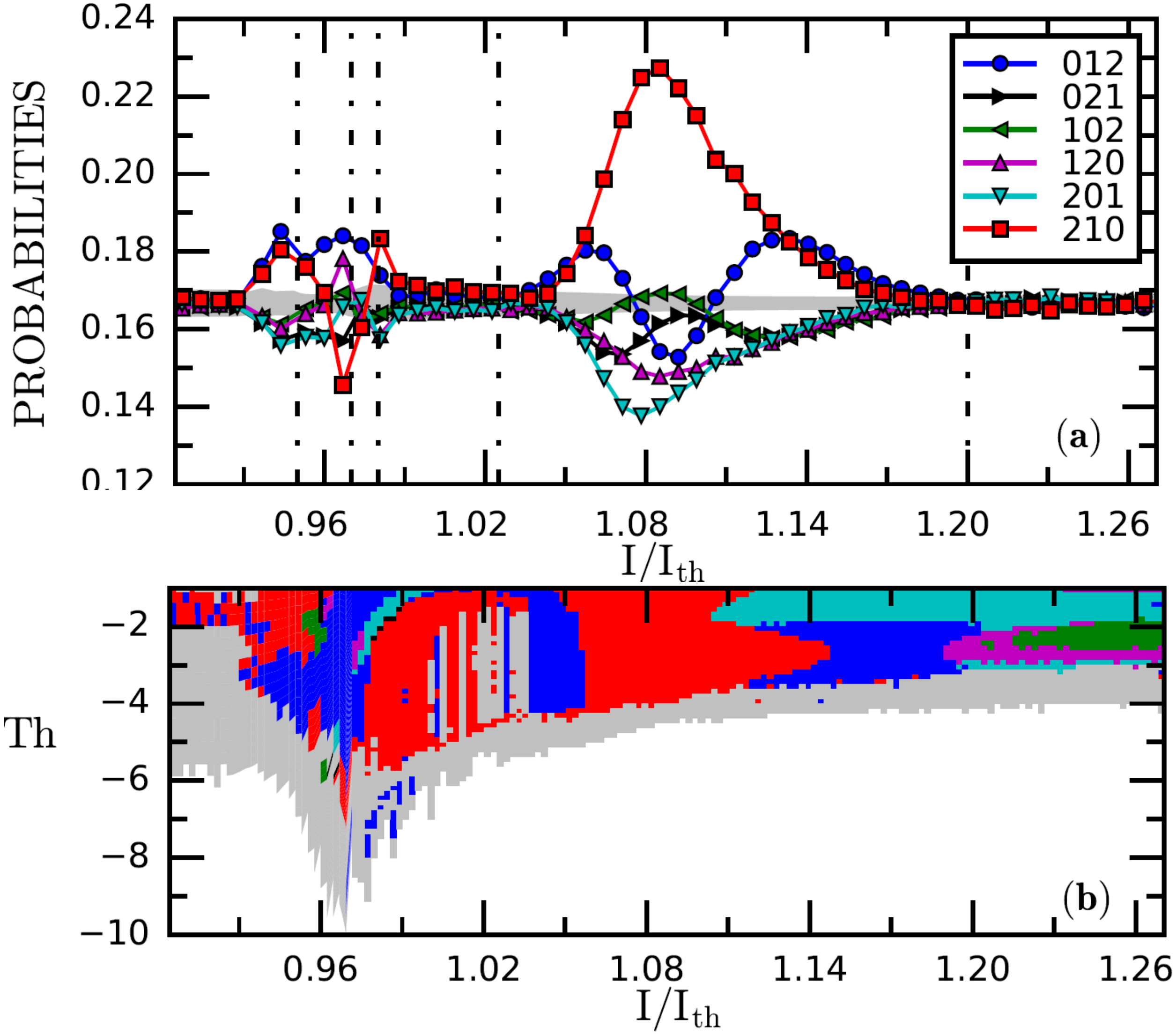








Ordinal probabilities



Conclusions

What did we learn?

- Take home message:
  - Ordinal time series analysis and the complex network approach are useful tools for characterizing transitions.

- Main conclusions
  - Early-warning signs of upcoming PS validated.
  - Laminar-turbulent transition: sharp transition seen in thresholded data but not in the raw data; particular time-scales identified at the transition.
  - Transition to optical chaos: low frequency fluctuations (LFFs) and the coherence collapse (CC) can be quantitatively distinguished.

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Campus d'Excel·lència Internacional

Collaborators


At UPC

- Carlos Quinteros
- Andres Aragoneses
- Laura Carpi
- Toni Pons
- Mari Carme Torrent



At URV (Tarragona)

- Alex Arenas, Sergio Gomez

Experimental data:

- PS data from INLN (S. Barland) and Bangor University (Y. Hong)
- Fiber laser data from Aston University (S. Turitsyn)

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Campus d'Excel·lència Internacional

THANK YOU FOR YOUR ATTENTION !

<cristina.masoller@upc.edu>

Papers at **http://www.fisica.edu.uy/~cris/**

- C. Masoller et al, "*Quantifying sudden changes in dynamical systems using symbolic networks*", NJP 17, 023068 (2015).
- A. Aragoneses et al, "*Unveiling temporal correlations characteristic of a phase transition in the output intensity of a fiber laser*", PRL 116, 033902 (2016).
- C. Quintero-Quiroz et al, "*Quantitative identification of dynamical transitions in a semiconductor laser with optical feedback*", Sci. Rep. 6, 37510 (2016).

ICREA

GOBIERNO DE ESPAÑA

MINISTERIO DE CIENCIA E INNOVACIÓN