

Hilbert analysis unveils inter-decadal changes in large-scale patterns of surface air temperature variability

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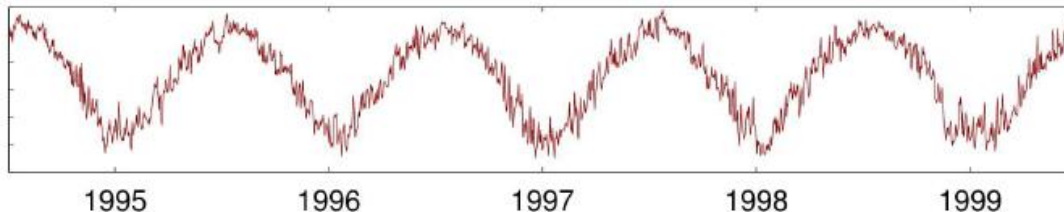
Workshop on Nonlinear Phenomena in the
Atmosphere and Ocean
Volga, July 2017



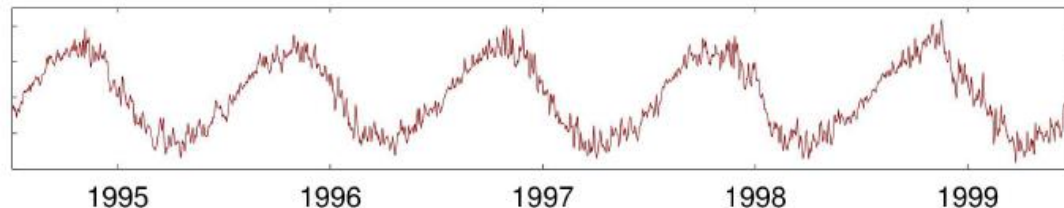
The Hilbert Transform of a real oscillatory signal

- Allows to calculate, **for each data point** in the time series, an instantaneous
 - Amplitude **$a(t)$**
 - Phase **$\varphi(t)$**
 - Frequency **$\omega(t)=d\varphi(t)/dt$**
- $HT[\sin(\omega t)]=\cos(\omega t)$

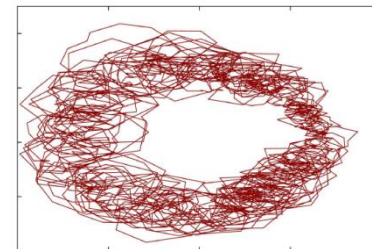
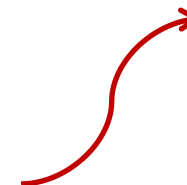
X



$HT[X]$

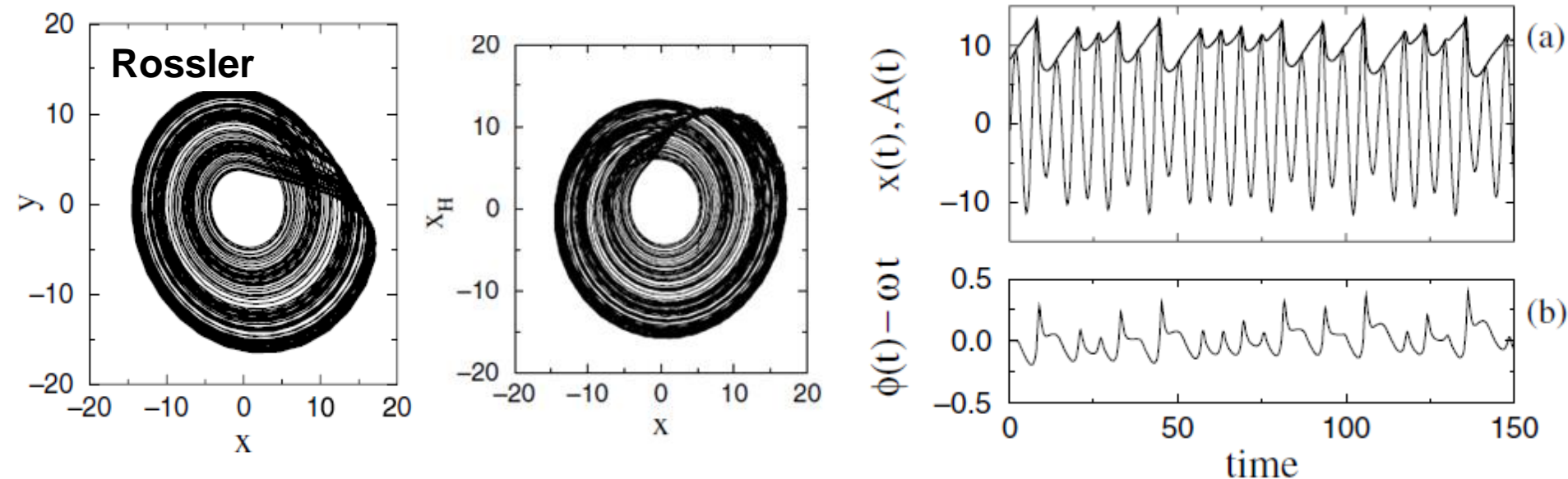


$Y=HT[X]$



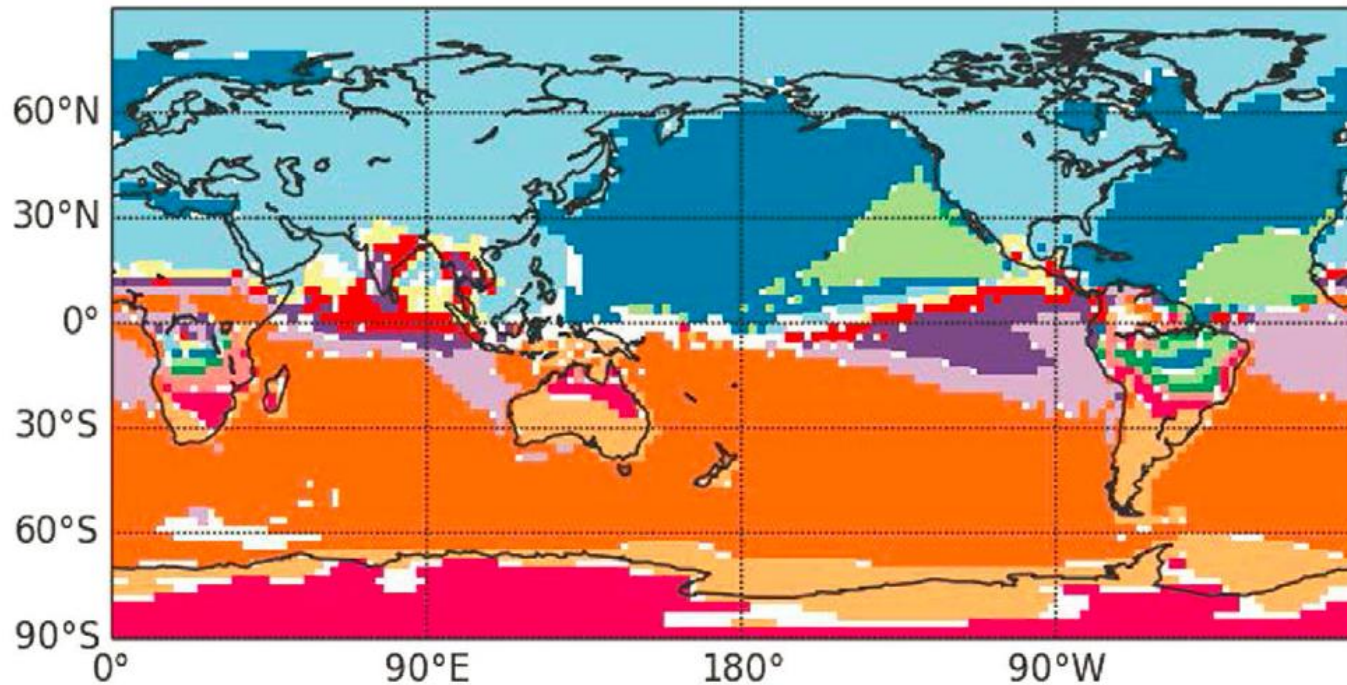
X

Extensively used to analyze output signals of complex systems
(physiological, neurological, etc.)

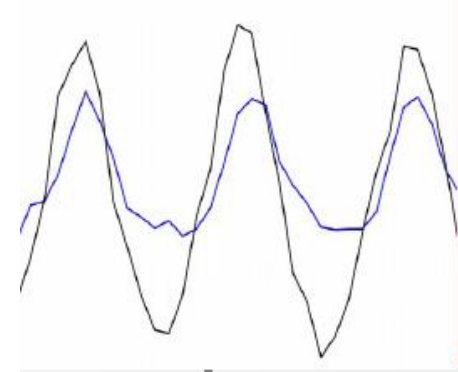


Can we use the amplitude, phase, frequency, to investigate
synchronization in climate data and quantify regional changes?

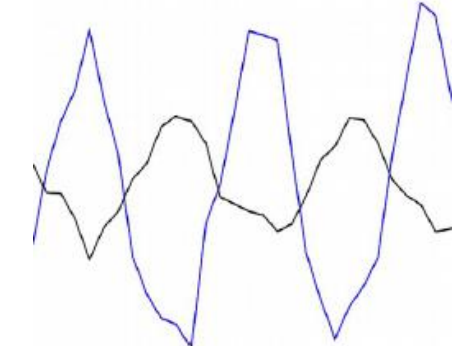
Well-defined regions with in-phase annual cycles of surface air temperature.

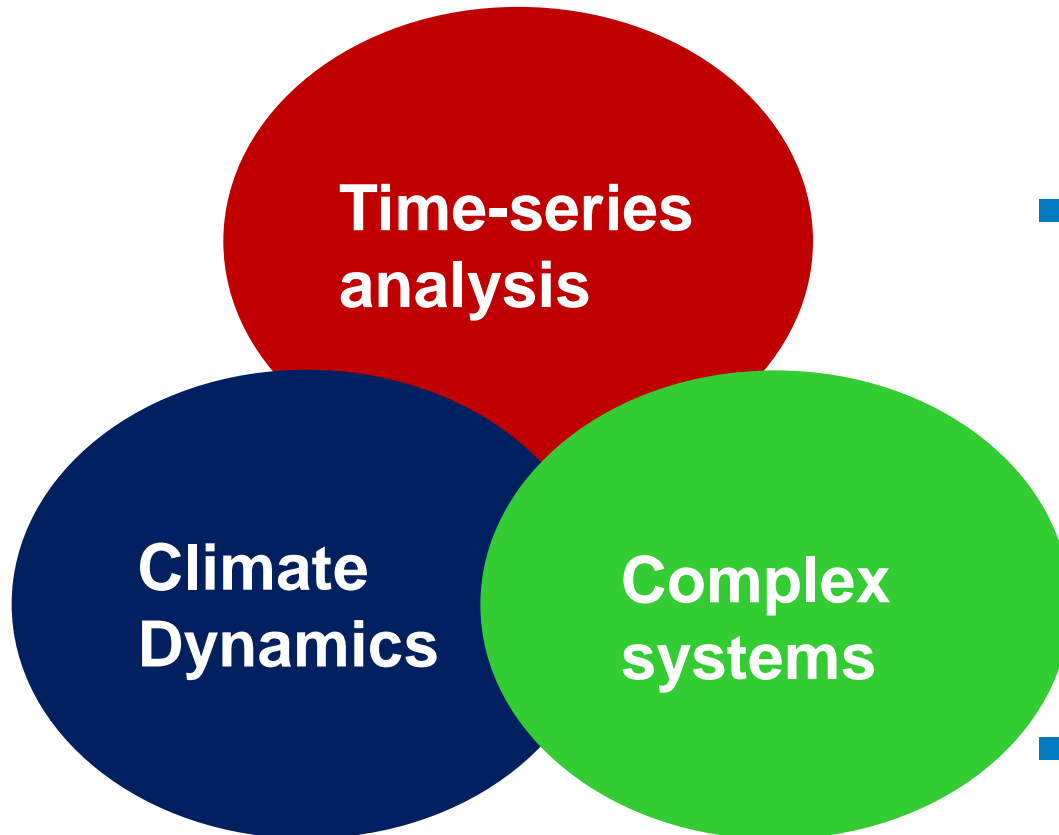


SAT in regions with the same color



SAT in regions with different color





- Data and Hilbert method
- Results
 - Univariate analysis
 - Bivariate analysis
 - Inter-decadal variations
- Summary

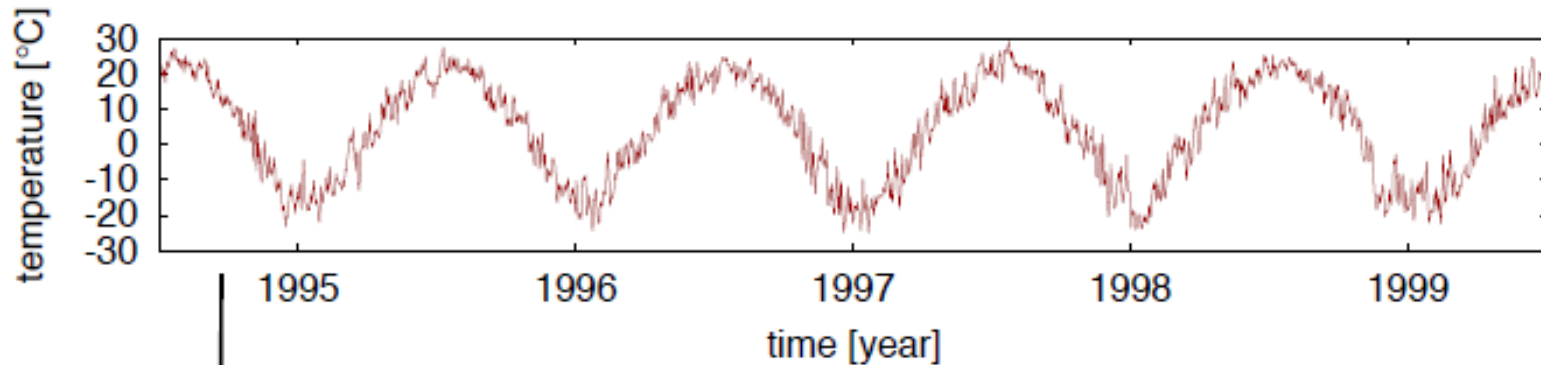
Surface Air Temperature Datasets

- NCEP/NCAR Reanalysis (from National Centers for Environmental Prediction and National Center for Atmospheric Research)
 - monthly-averaged surface air temperature (SAT)
 - spatial grid resolution of 2.5° (10512 points)
 - January 1958 – May 2012 (773 months)

- ERA-Interim (from European Centre for Medium-Range Weather Forecasts)
 - daily-averaged values of SAT
 - spatial resolution of 2° (16380 nodes)
 - January 1979 – April 2015 (13269 days)



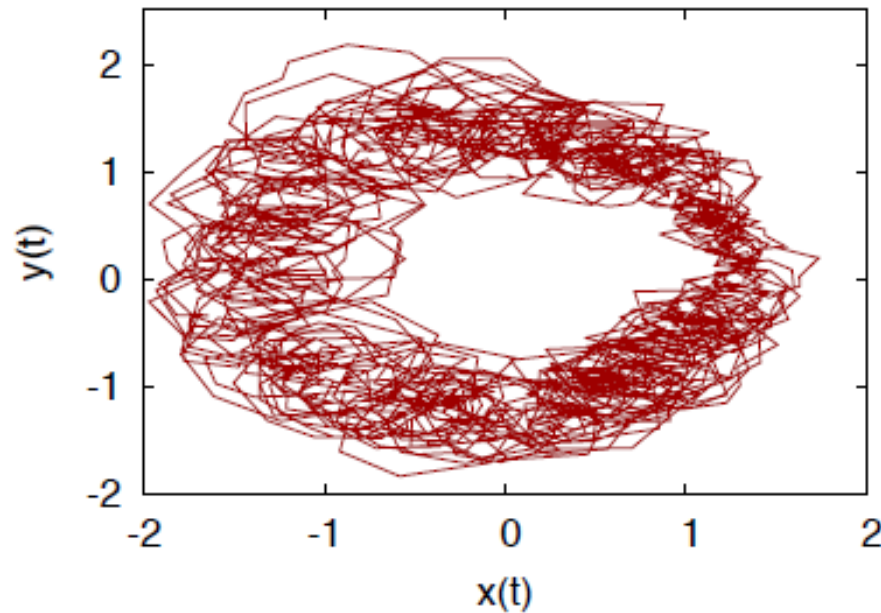
Hilbert analysis of surface air temperature



SAT series of
a site in
continental
east Asia

$x(t)$ detrended and normalised: $\langle x \rangle = 0$; $\sigma_x = 1$

$y(t) = H[x](t)$



→ amplitude: $A = \sqrt{x^2 + y^2}$

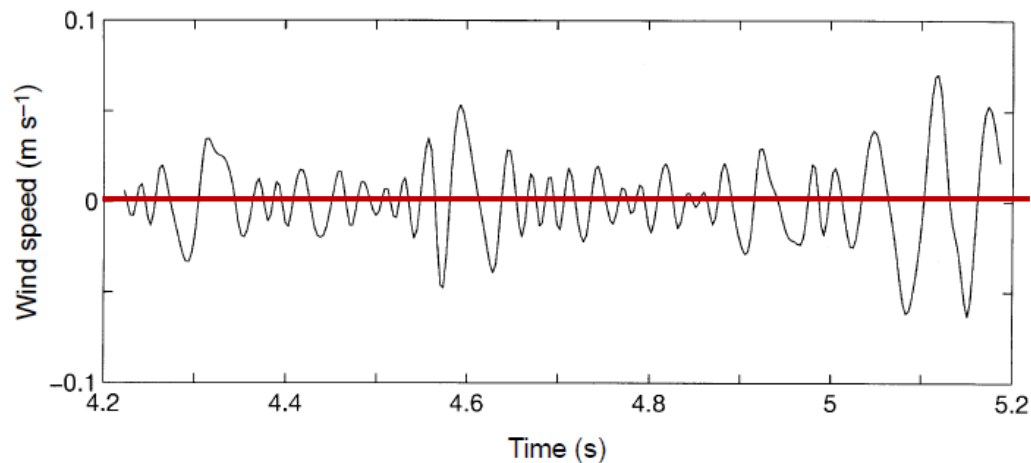
→ phase: $\varphi = \arctg\left(\frac{y}{x}\right)$ then unwrap

↓
frequency: $\omega = \frac{d\varphi}{dt}$

By construction:

$$x(t) = A(t) \cos \varphi(t)$$

- In order to obtain meaningful instantaneous frequency, the signal has to be ‘narrow band’
- For a narrow band signal the **number of extrema** and the **number of zero crossings** have to equal.

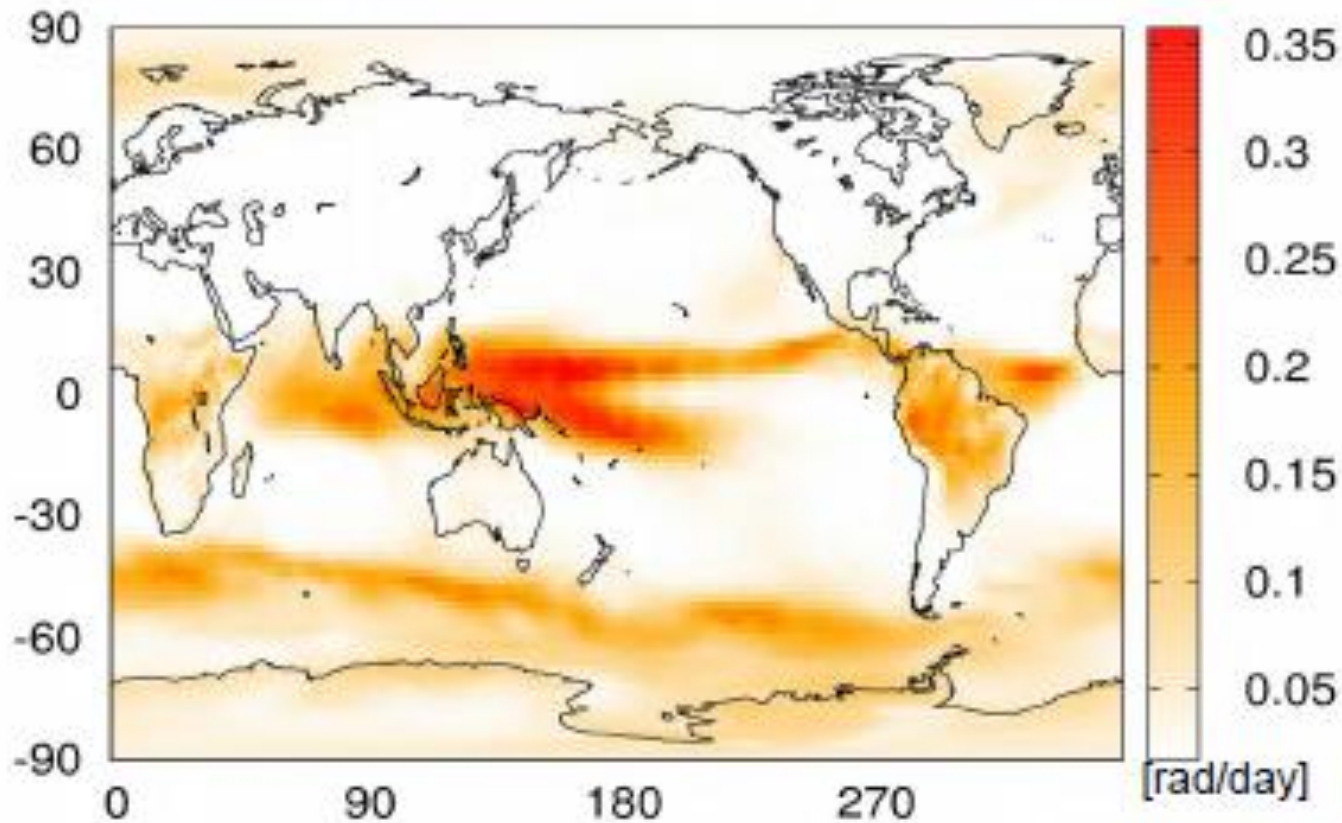


- This condition does not hold to daily SAT time series (“noisy” weather). However...

“Even in cases where the data do not meet the mathematical or algorithmic requirements ..., the results of nonlinear time-series analysis can be helpful in understanding, characterizing, and predicting dynamical systems.”

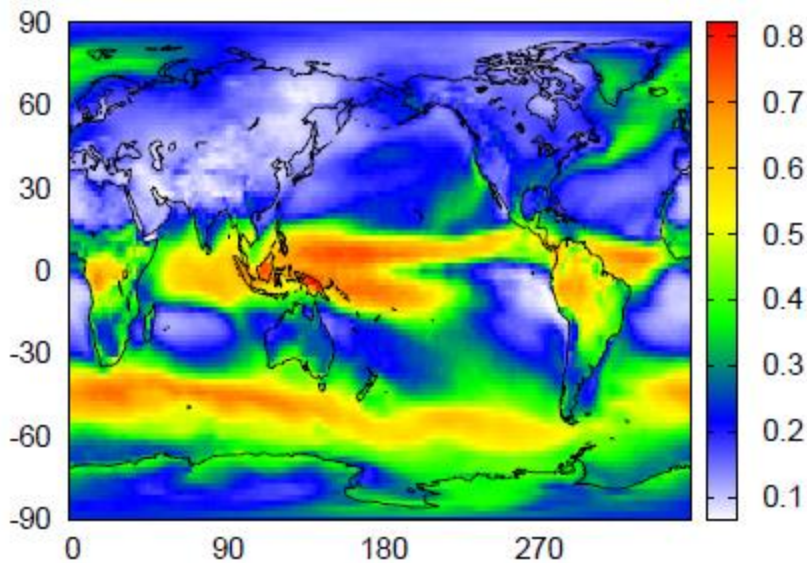
Elizabeth Bradley and Holger Kantz
Nonlinear time-series analysis revisited
Chaos 2015

Average frequency

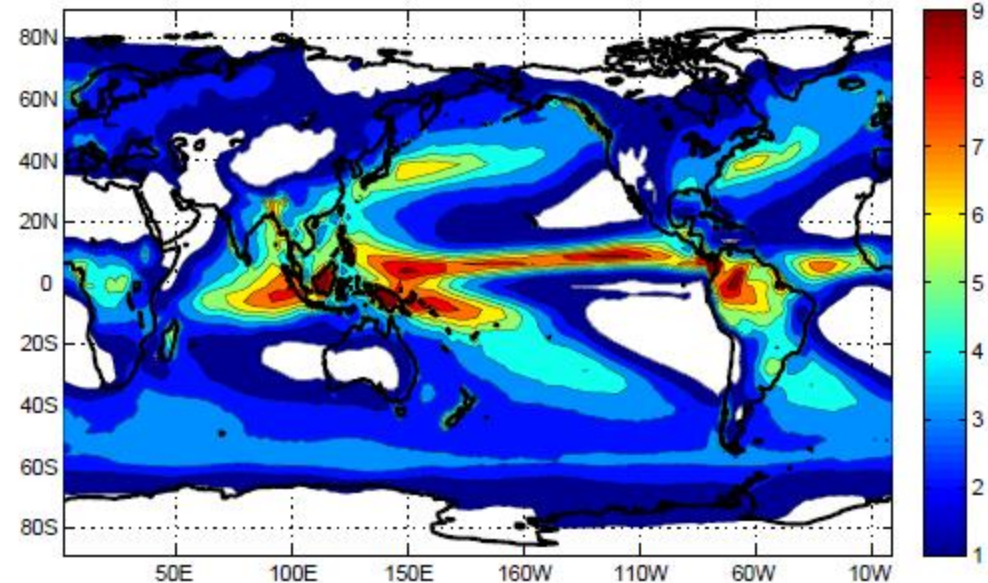


- The expected average frequency (one complete cycle in one year) is 0.017 rad/day

Standard deviation of frequency fluctuations



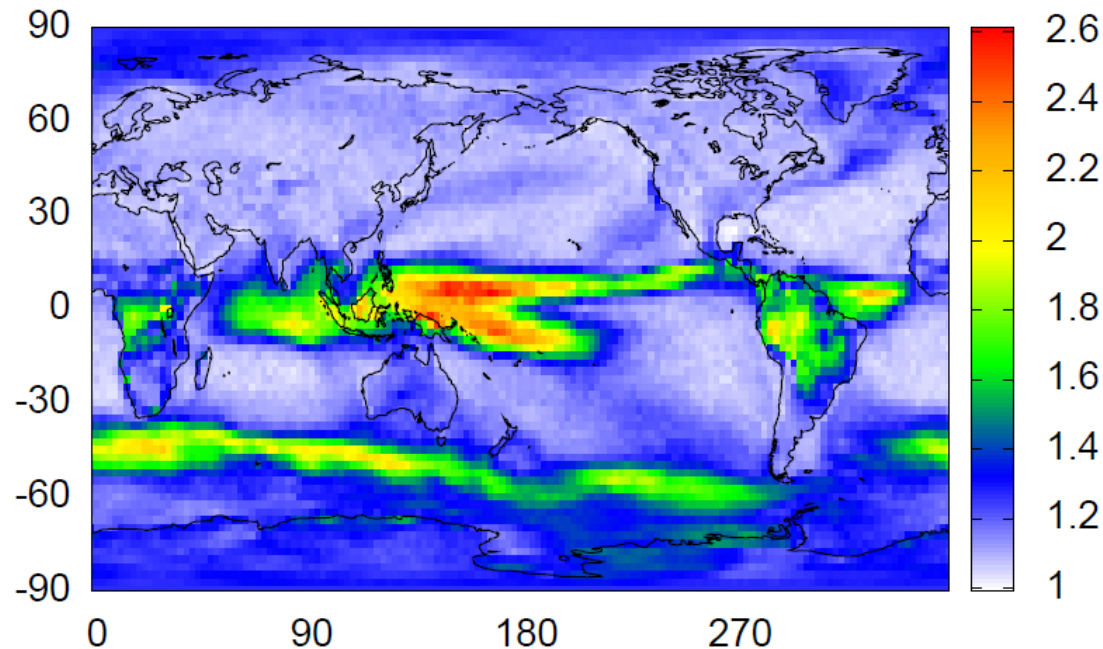
Annual mean precipitation (mm/day)



Consistency check

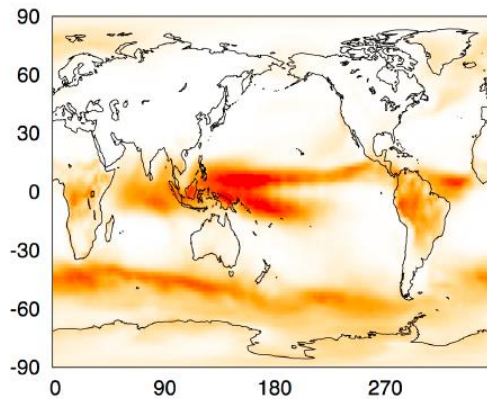
- A real signal has narrow band behavior if relative amplitude variations are very slow when compared with phase variations.

$$\left| \frac{d\phi}{dt} \right| \gg \left| \frac{1}{A} \frac{dA}{dt} \right| \Rightarrow \frac{|\omega|}{|(dA/dt)/A|} \gg 1$$

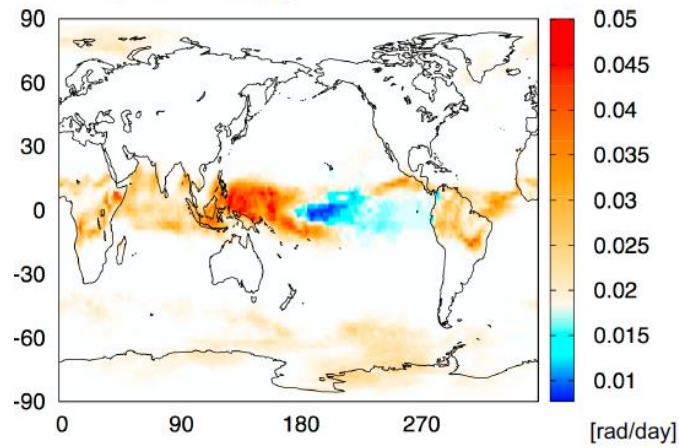


- SAT \rightarrow average in a window of D days \rightarrow Hilbert

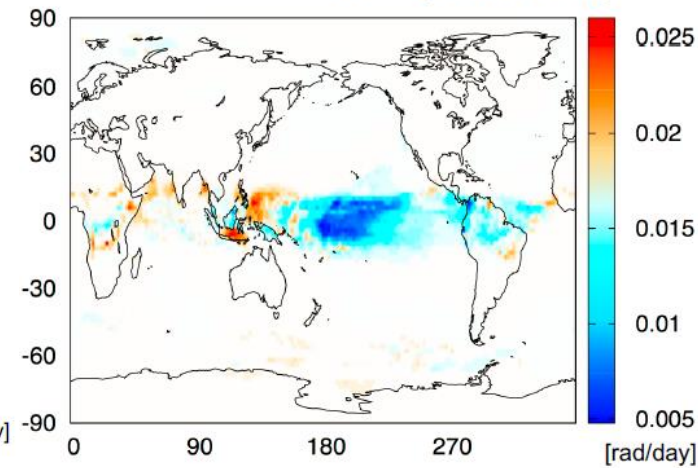
no smoothing



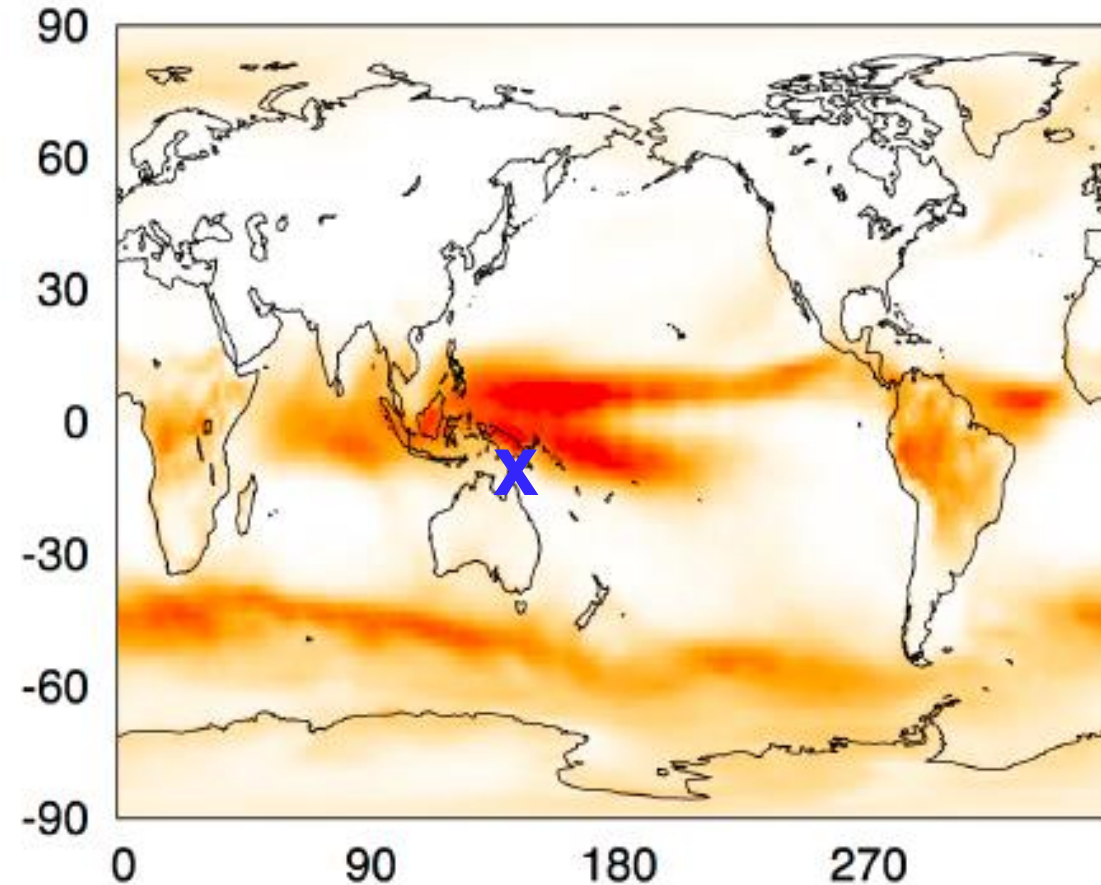
31-day smoothing



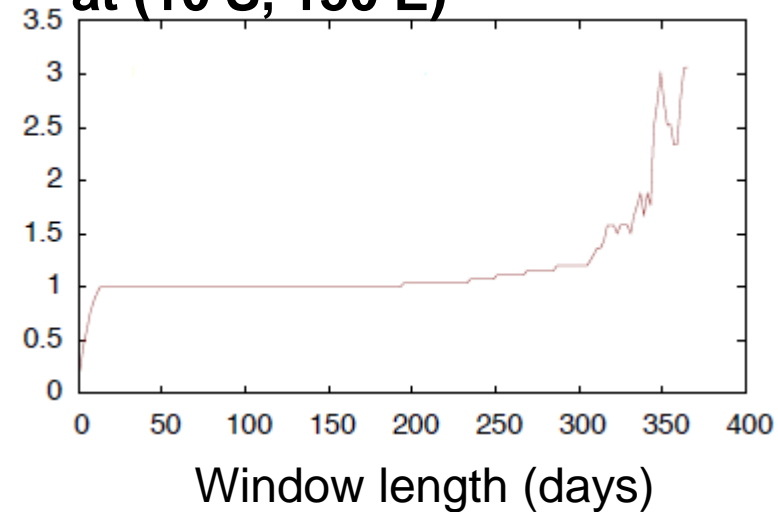
99-day smoothing

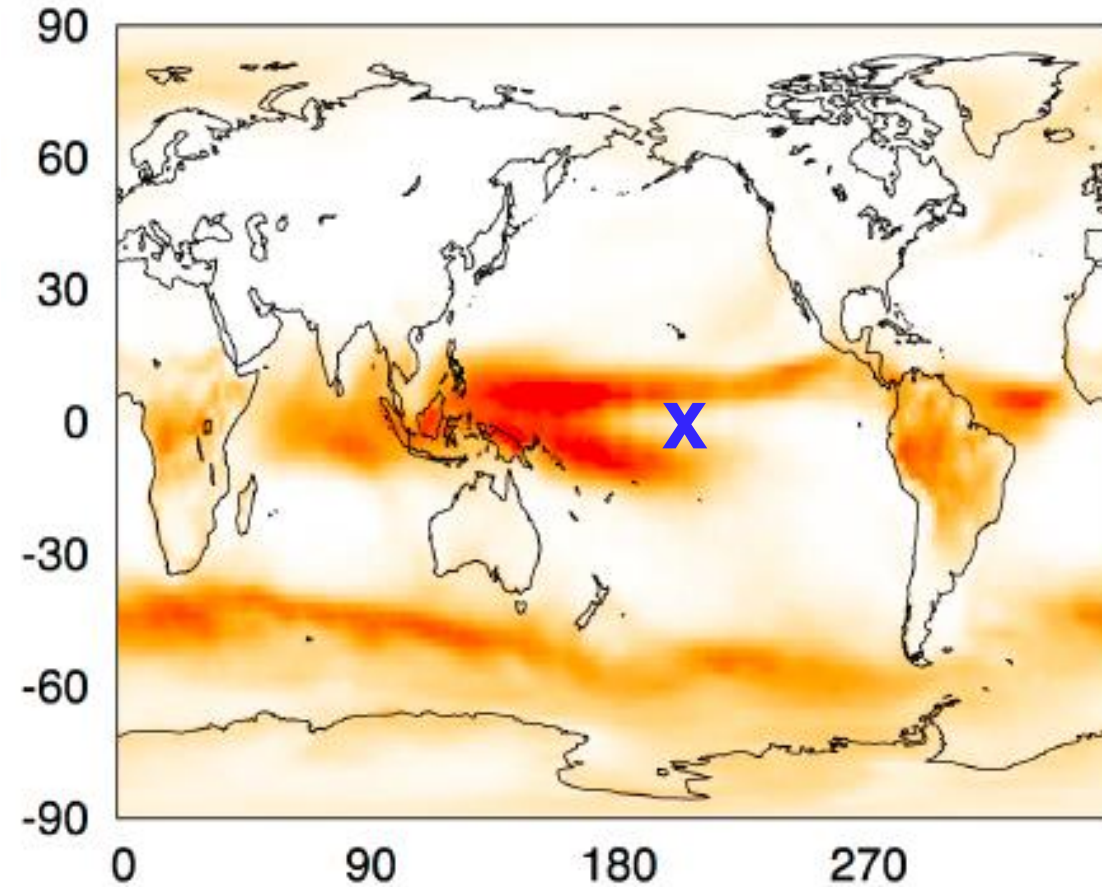


Influence of the length of SAT averaging window

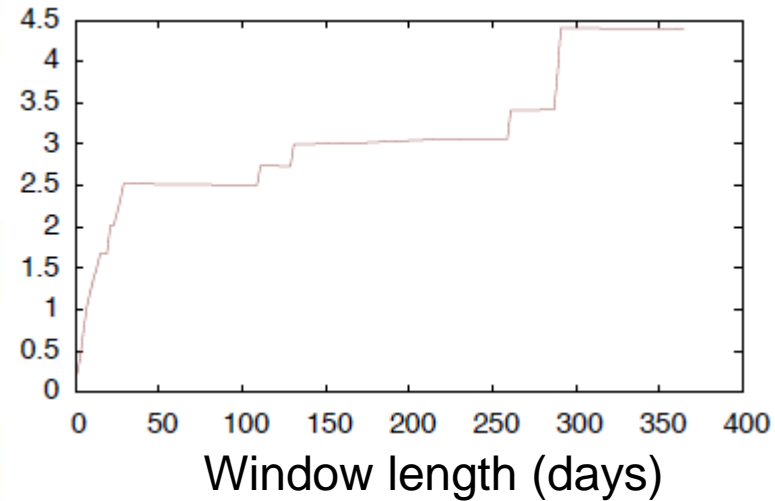


**Average Period (years)
at (10 S, 150 E)**





Average Period (years) at (0, 200 E)



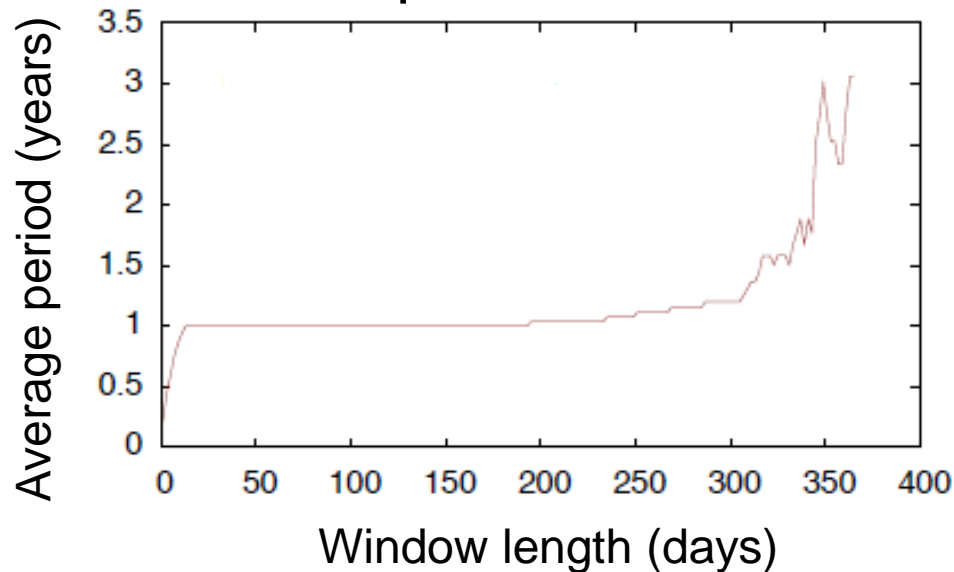
quasi-biennial oscillation
(QBO) ?

Comparison with synthetic data

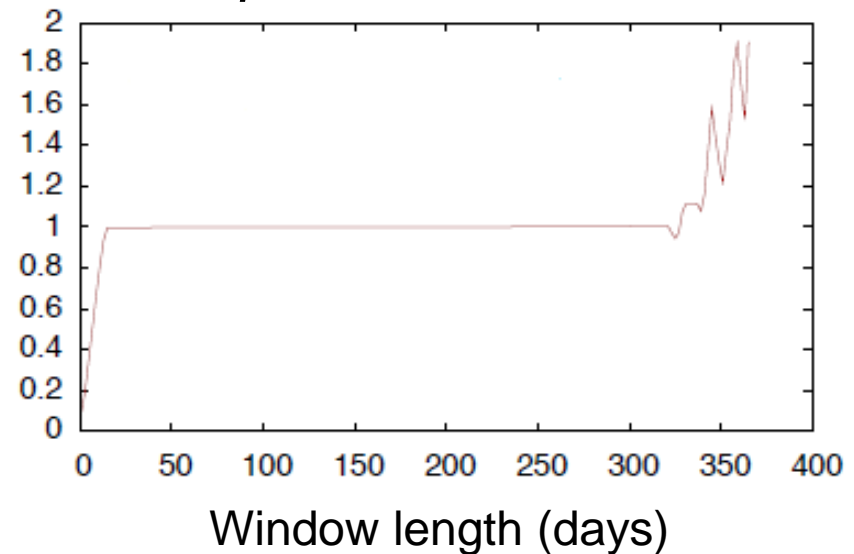
$$S(t) = \sqrt{1 - \alpha C} \sin(\omega_0 t) + \sqrt{\alpha} \xi_{\text{AR}(1)}$$

$\omega_0 = 2\pi/365$
C = norm. factor
 $\beta = \text{AR}(1)$
persistence

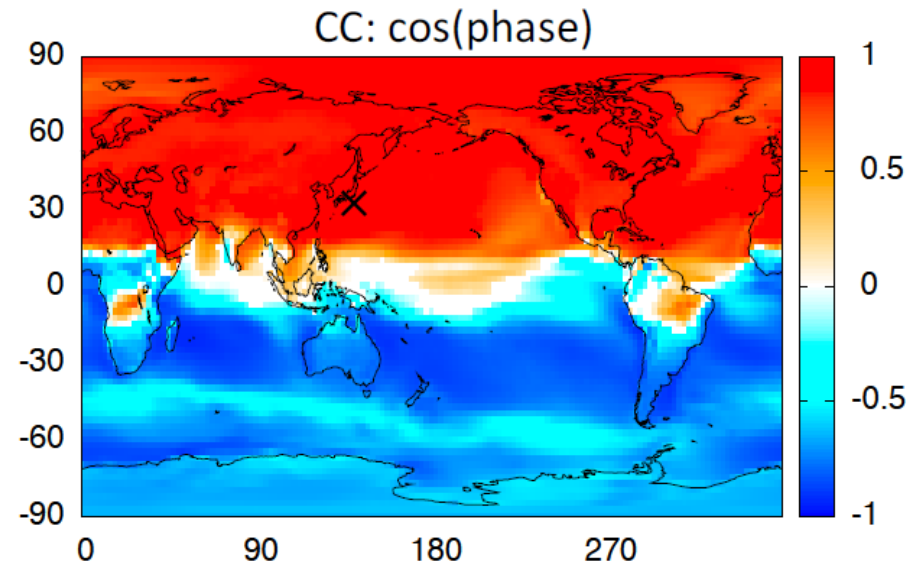
Empirical data



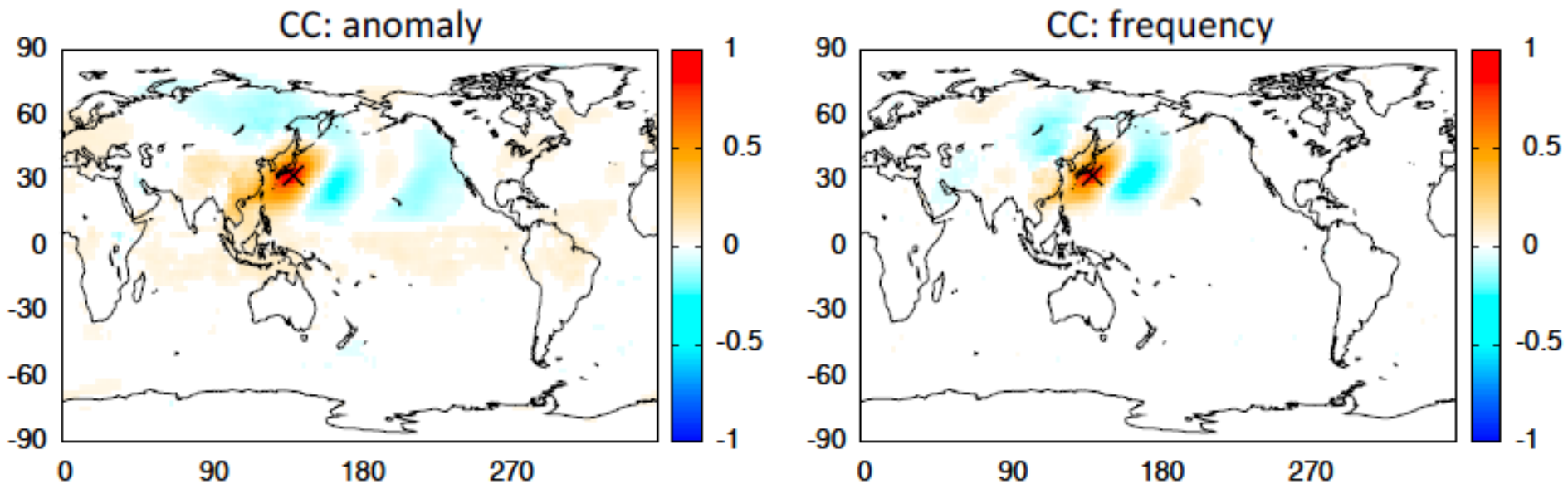
$\alpha = \beta = 0.5$



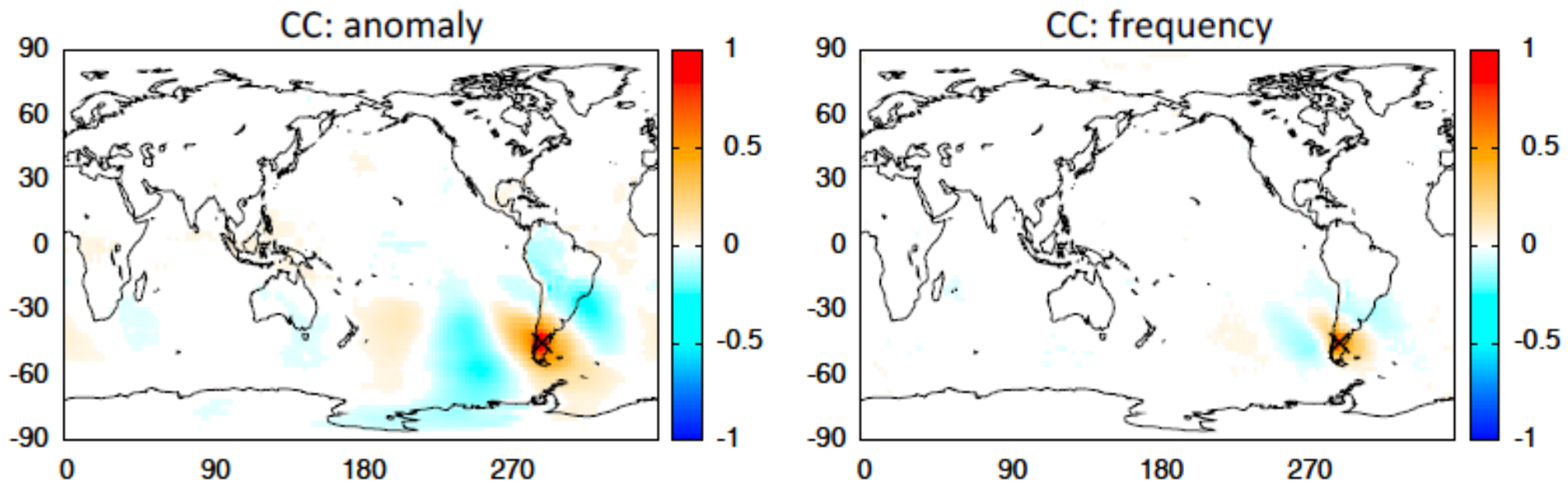
- Cos(phase) –typical year
- Cos(phase) –El Niño year
- Cos(phase) –La Niña year



Bivariate analysis: extra-tropics



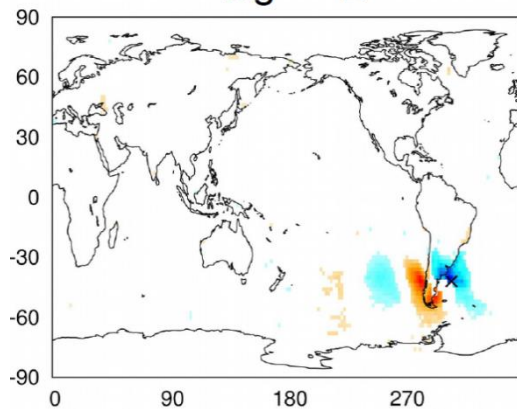
Significance: 100 surrogates (anomaly TS or Hilbert TS), then use 3σ confidence level



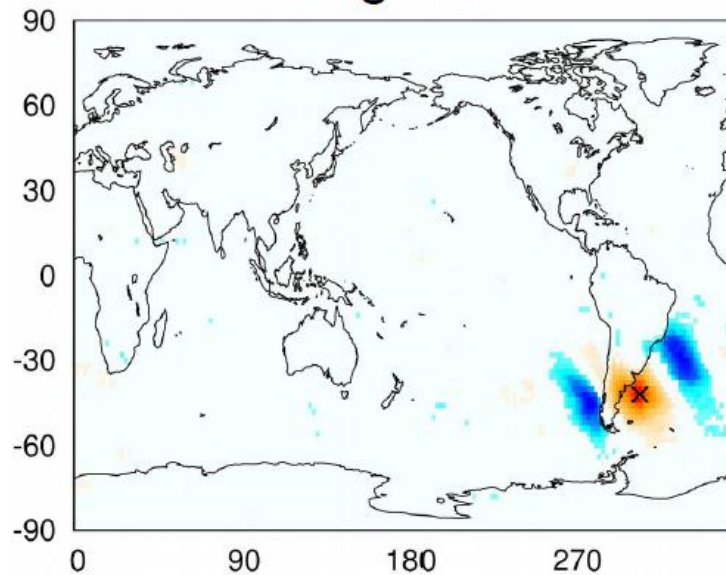
Lagged correlations?

- In the extra-tropics, lag (in days) cross-correlations of Hilbert frequencies.

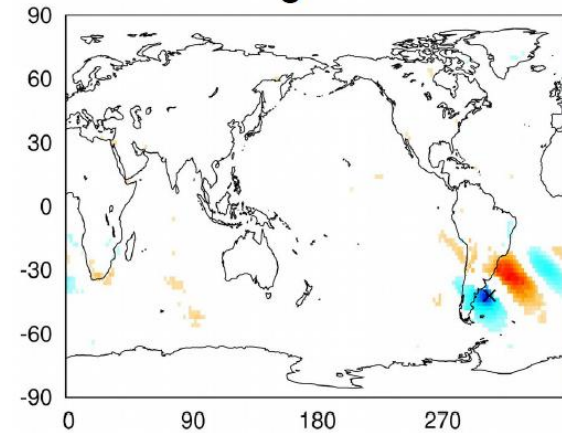
lag = -2



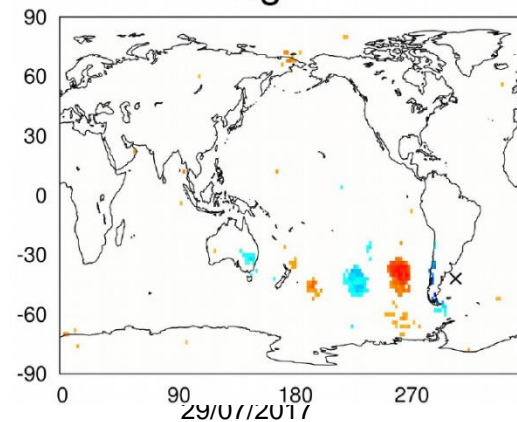
lag = 0



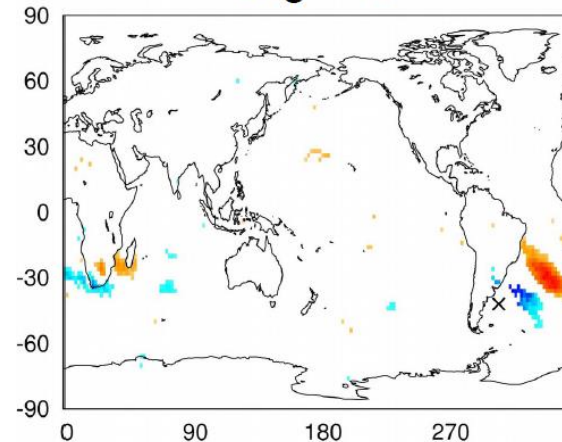
lag = 2



lag = -4

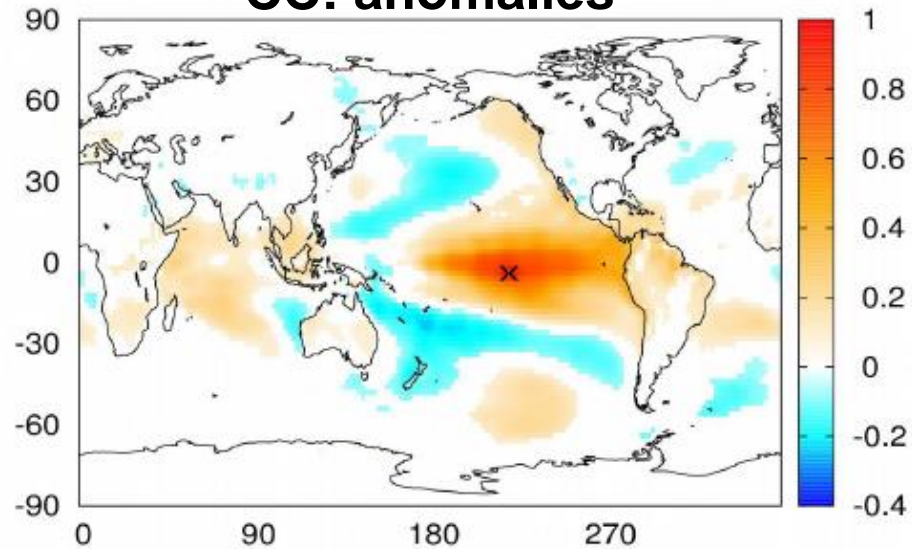


lag = 4

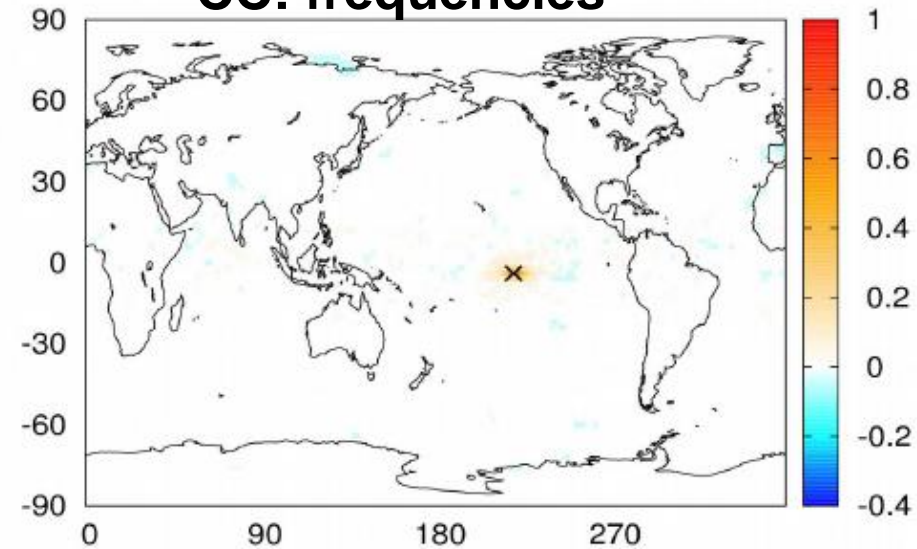


But in the El Niño region

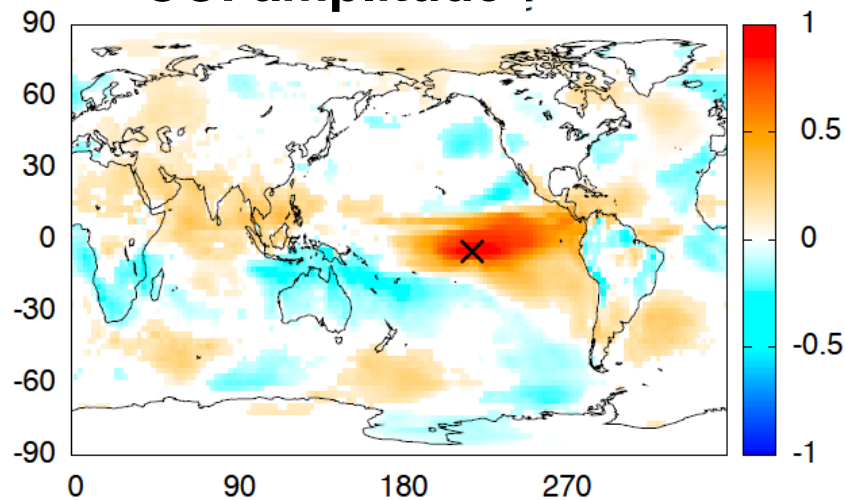
CC: anomalies



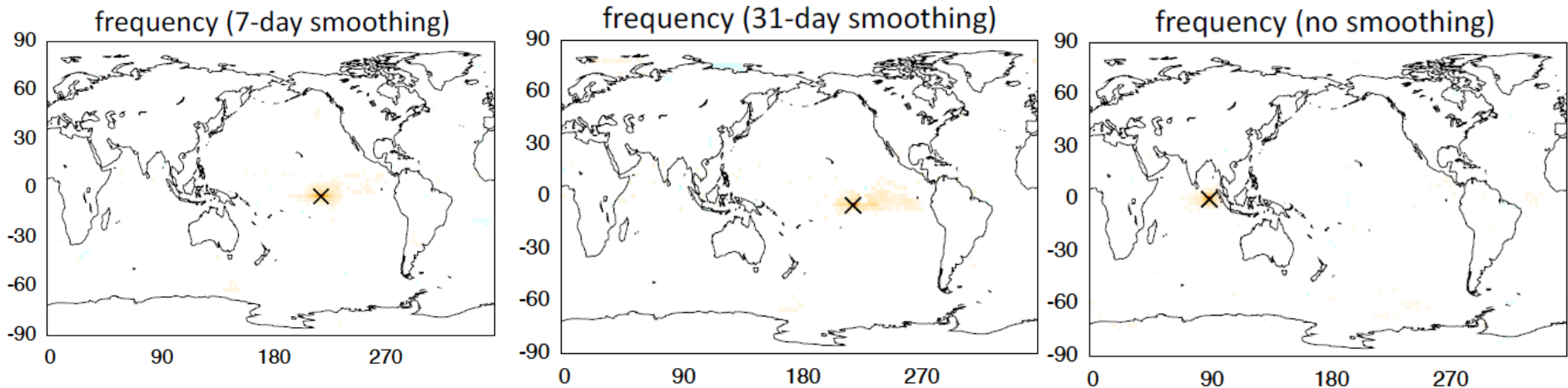
CC: frequencies



CC: amplitude



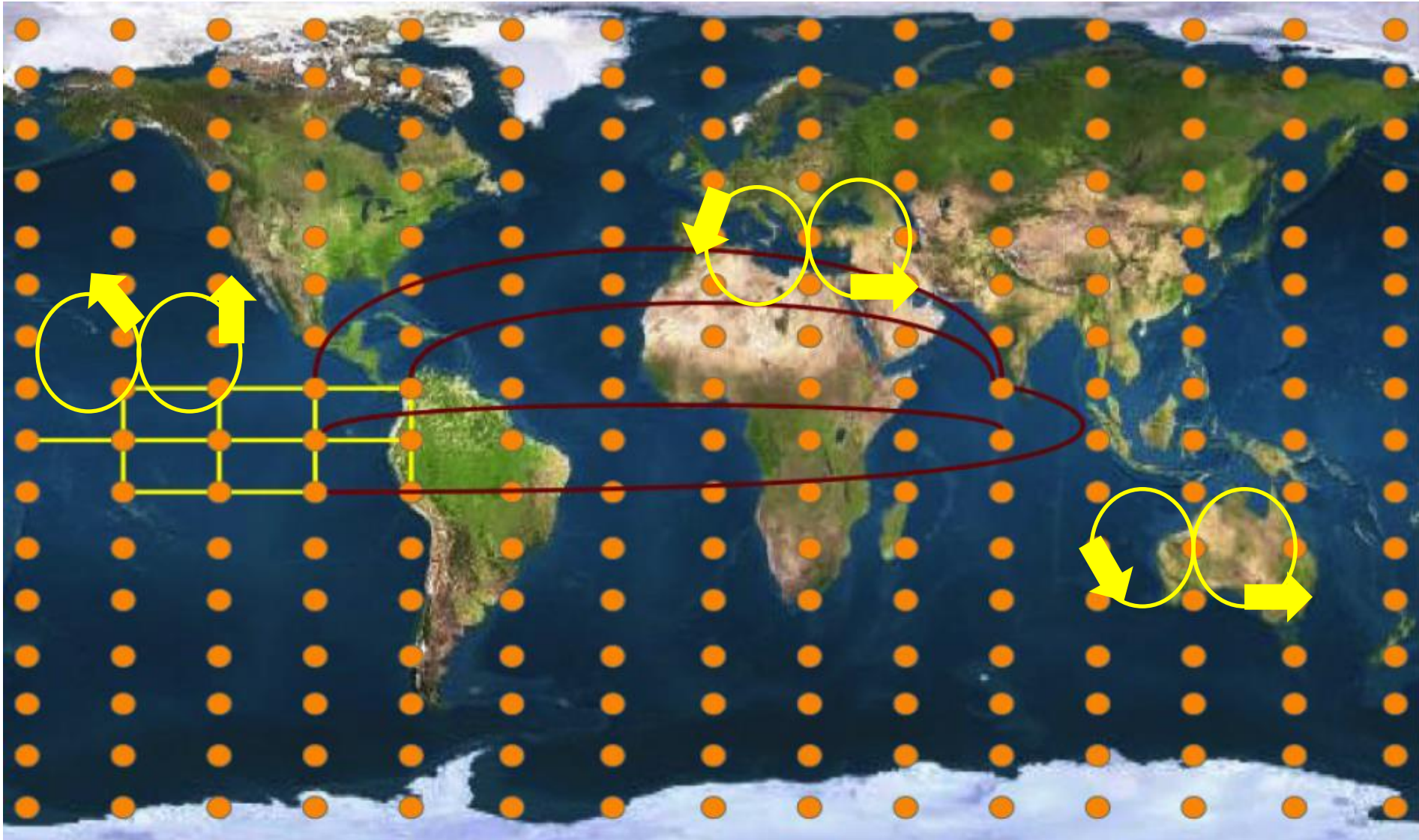
Pre-processing data: filtering fast SAT variability



Also, no significant correlations found in lagged time-series.

⇒ Atmospheric dynamics in the tropics has different time-scales from the extra-tropics

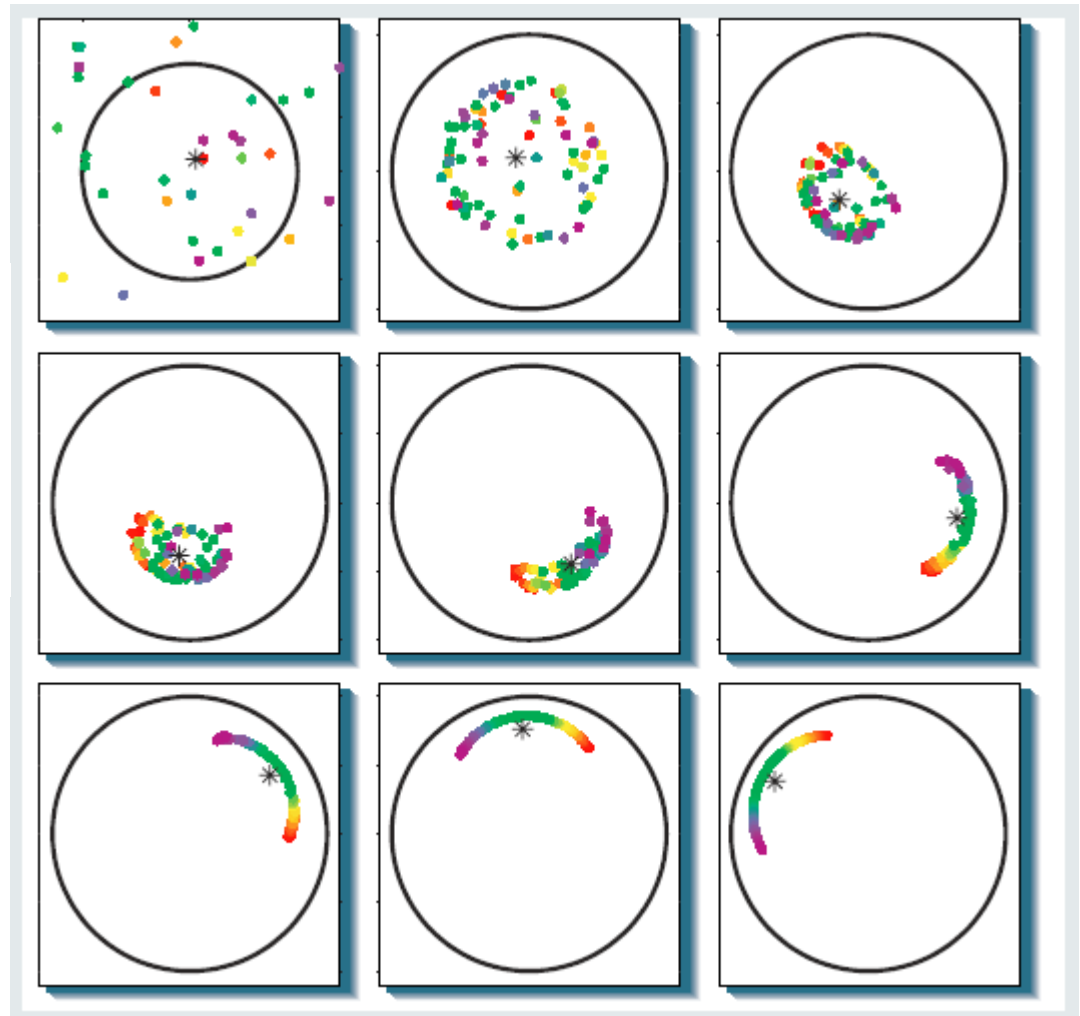
Network of individual oscillators

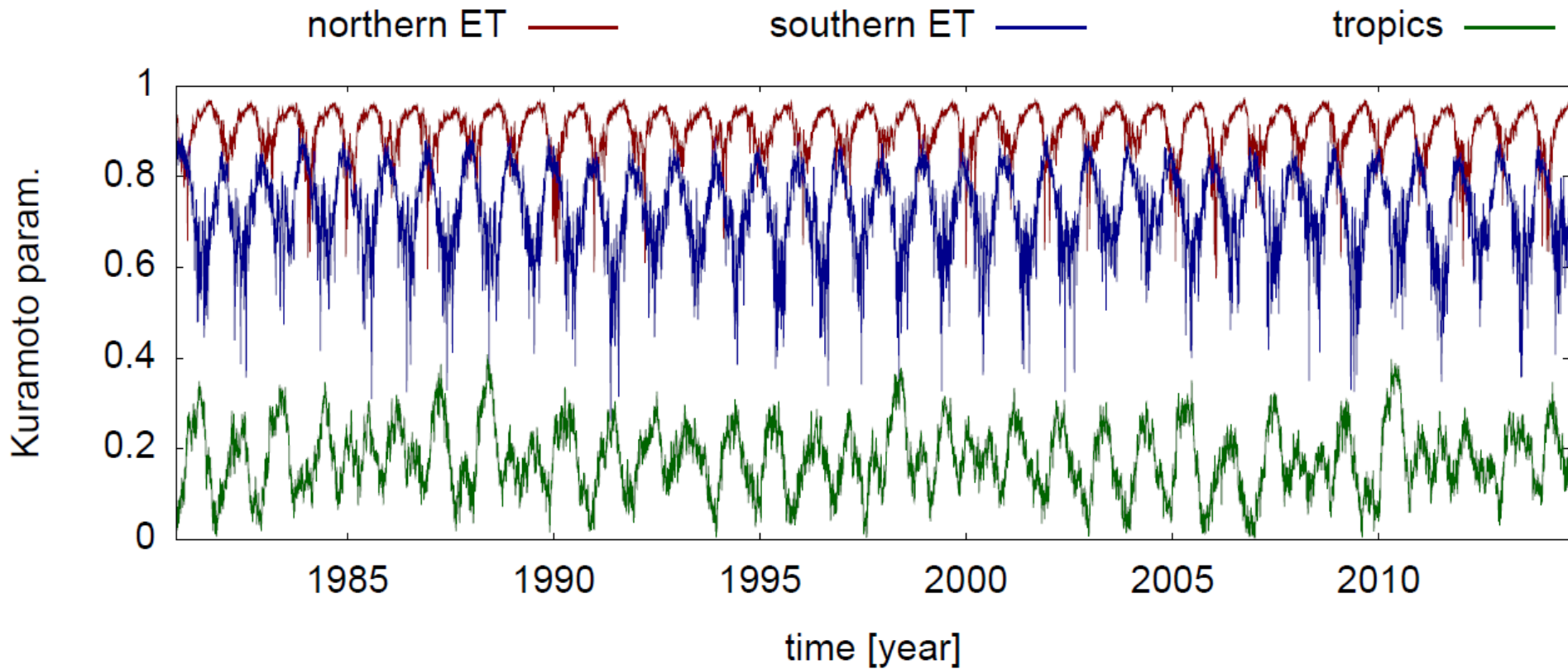


Quantifying phase synchronization

- Kuramoto order parameter

$$r(t) = \left| \frac{1}{N} \sum_{j=1}^N e^{i\theta_j(t)} \right|$$





Unveiling inter-decadal changes

-can amplitude and frequency variations be used as a quantitative measure of regional climate change?



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Relative inter-decadal variation (ERA-Interim, 37 years)

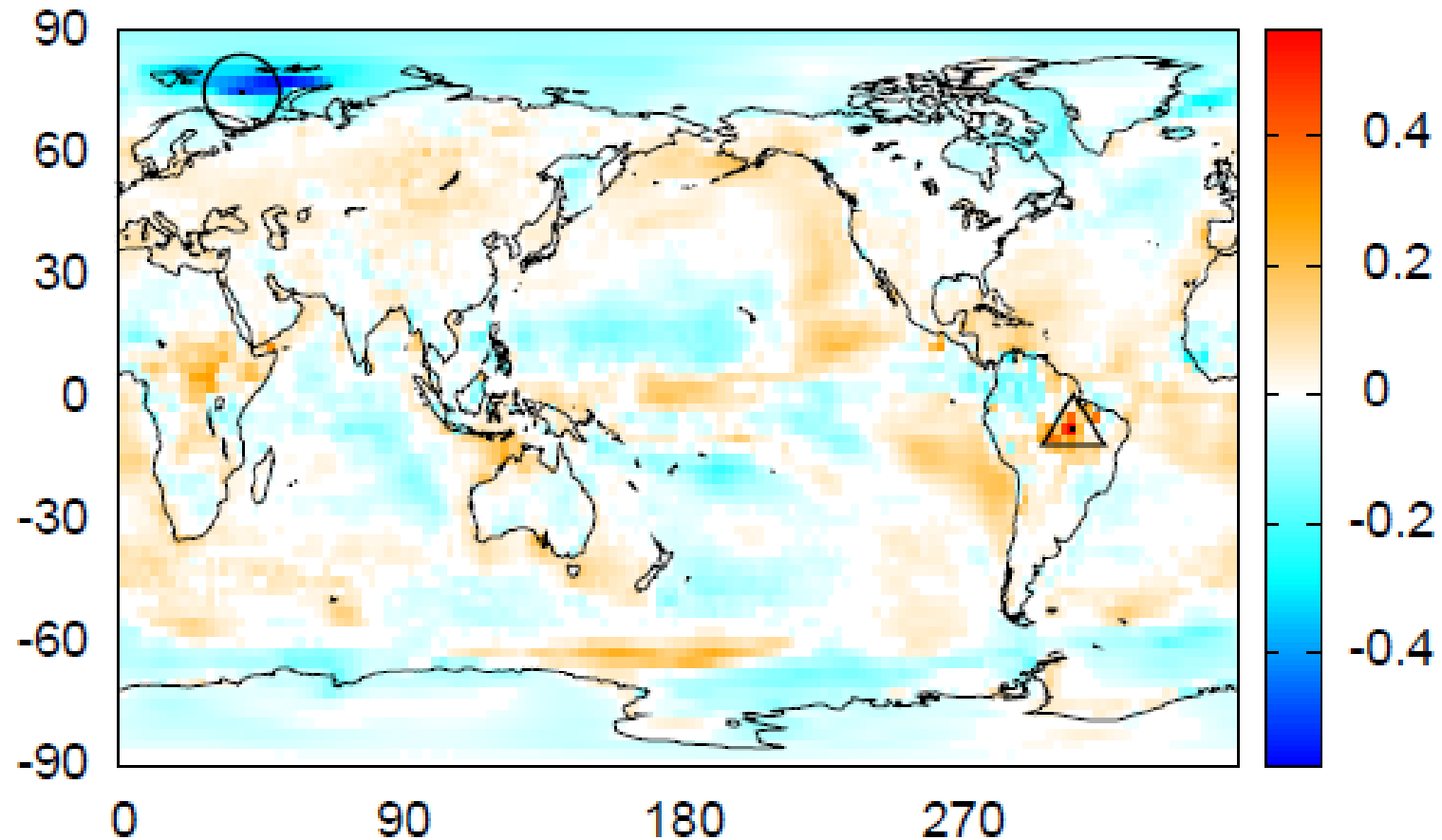
$$\Delta a = \langle a \rangle_{2016-2007} - \langle a \rangle_{1988-1979}$$

$$\frac{\Delta a}{\langle a \rangle_{2016-1979}}$$

Significance analysis: For each amplitude time series 100 shuffle surrogates were generated, and the relative change is significant if:

$$\frac{\Delta a}{\langle a \rangle} \geq \langle \cdot \rangle_s + 2\sigma_s \quad \text{or} \quad \frac{\Delta a}{\langle a \rangle} \leq \langle \cdot \rangle_s - 2\sigma_s$$

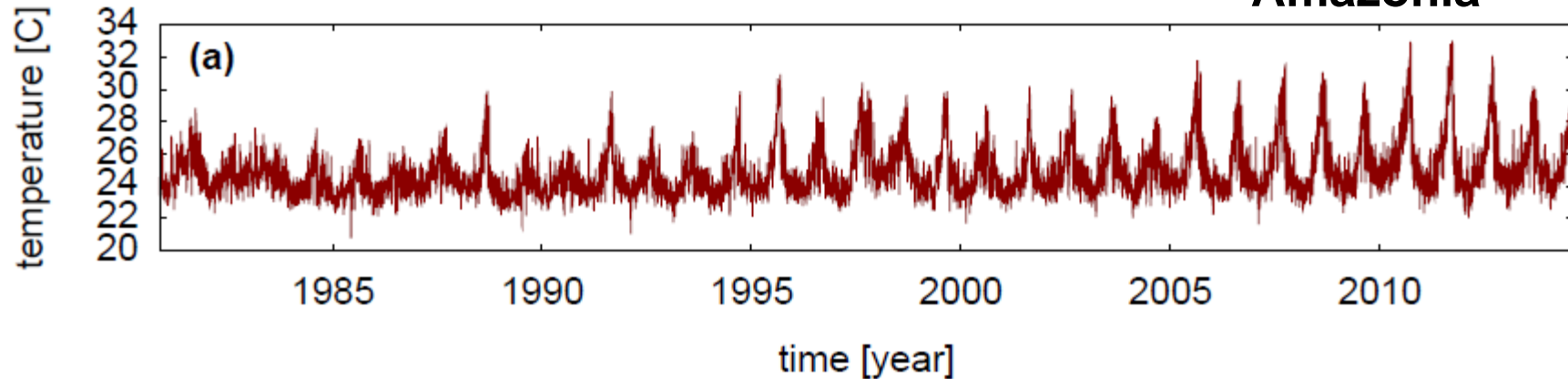
Relative change of time-averaged Hilbert amplitude



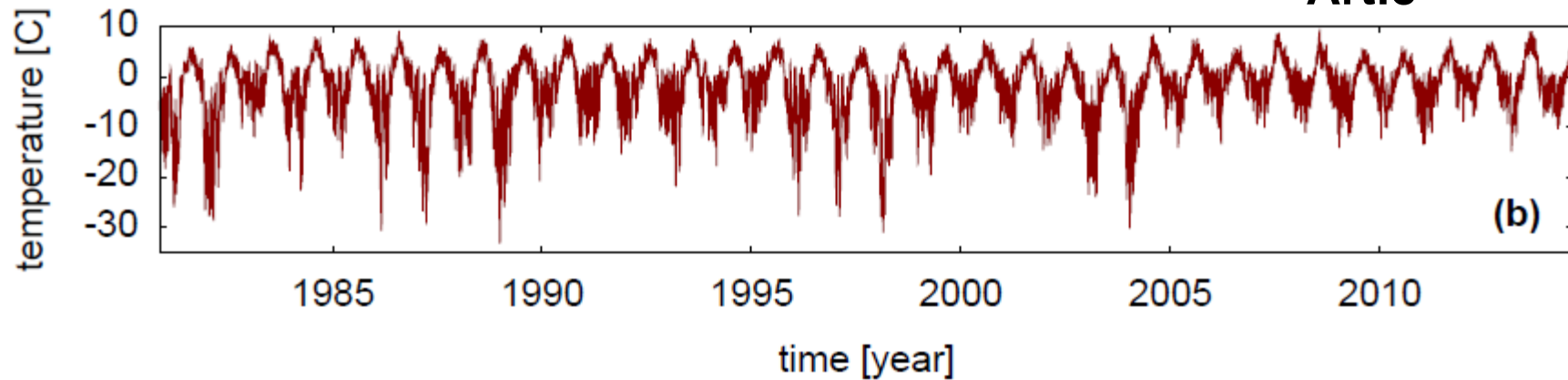
Melting of sea ice: during winter the air temperature is mitigated by the sea and tends to have more moderated values

Decrease of precipitation: the solar radiation that is not used for evaporation is used to heat the ground.

Amazonia

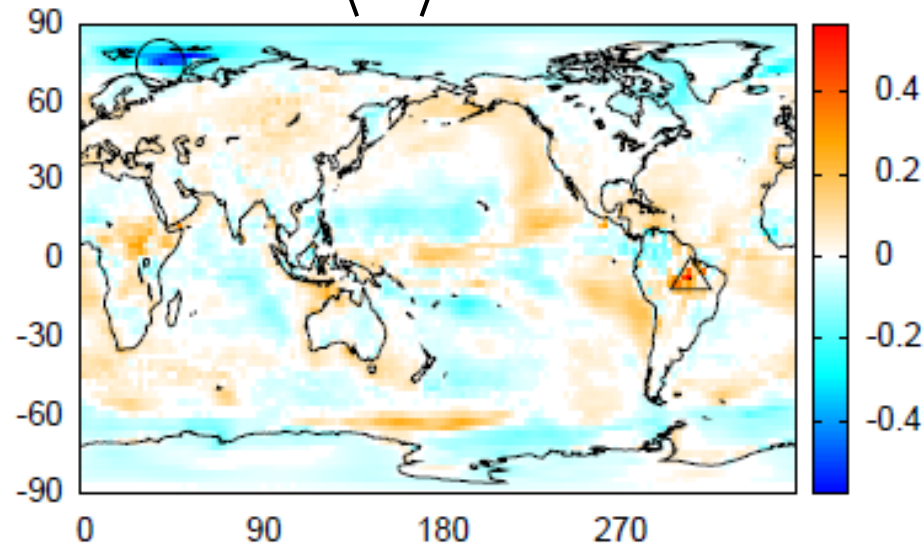


Artic

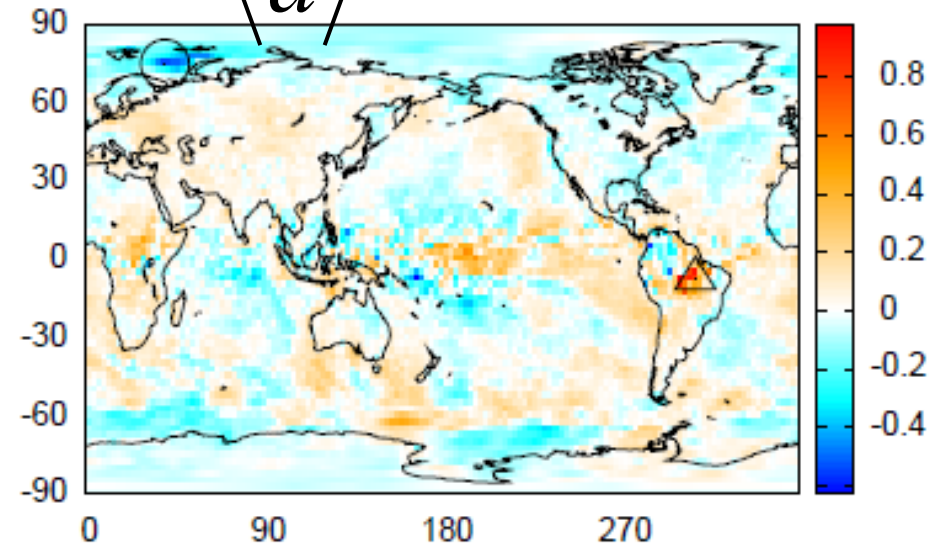


Hilbert amplitude variation consistent with climatology variation

$$\frac{\Delta a}{\langle a \rangle}$$



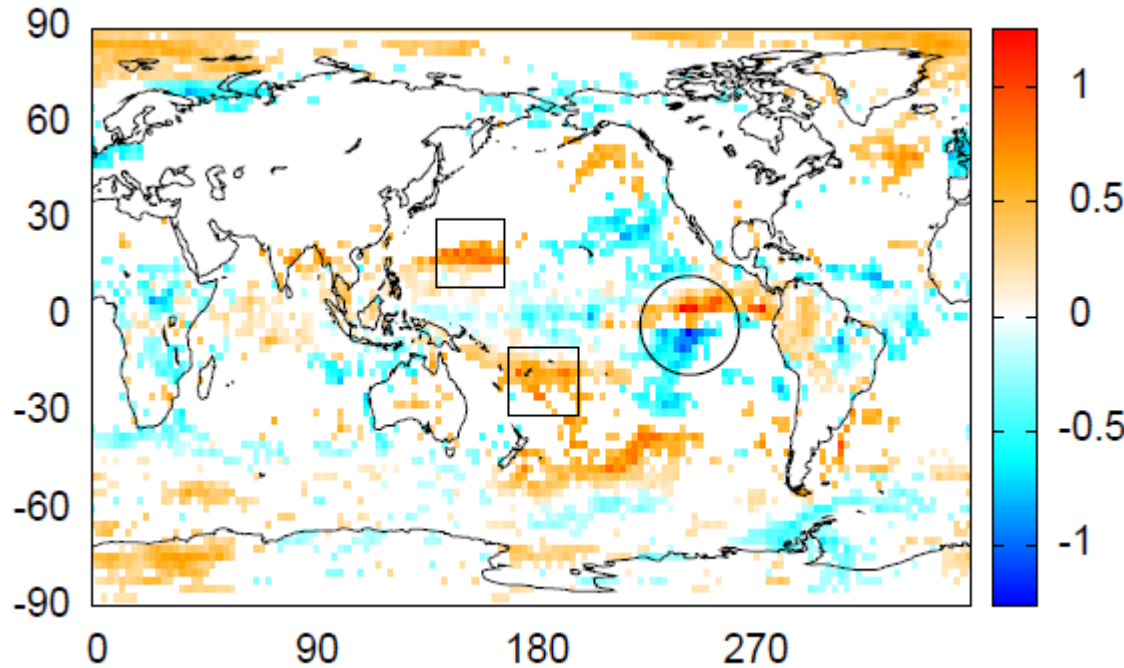
$$\frac{\Delta a^{c \text{lim}}}{\langle a \rangle^{c \text{lim}}}$$



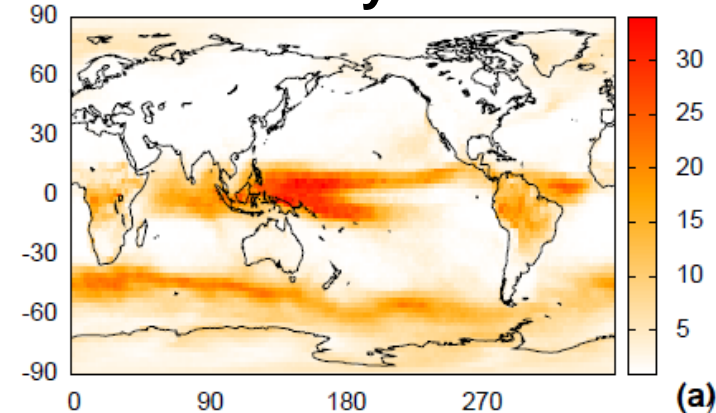
$$a_j^{(\text{clim})}(I) = \max[c_j^I(t)] - \min[c_j^I(t)]$$

I = time interval (2016—2007; 1988—1979; 2016—1979)

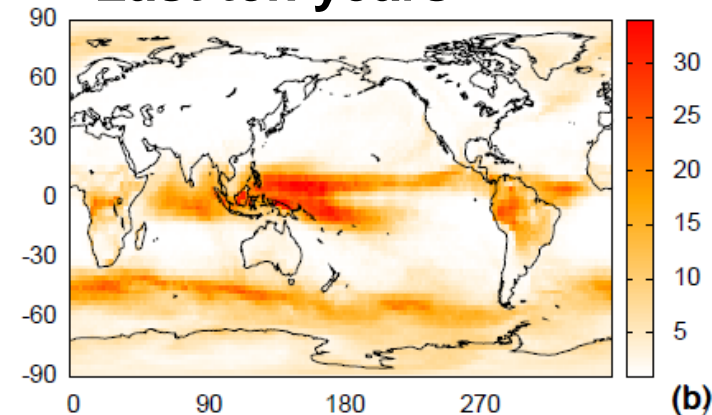
Relative change of time-averaged Hilbert frequency



First ten years



Last ten years

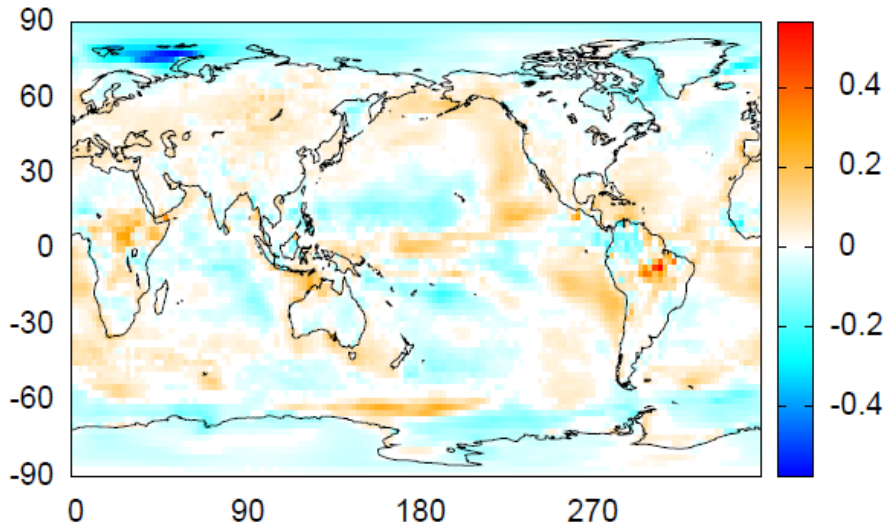


- ⇒ Consistent with a **shift towards north and a widening of the ITCZ**
- ⇒ Consistent with variation in the number of zero-crossings

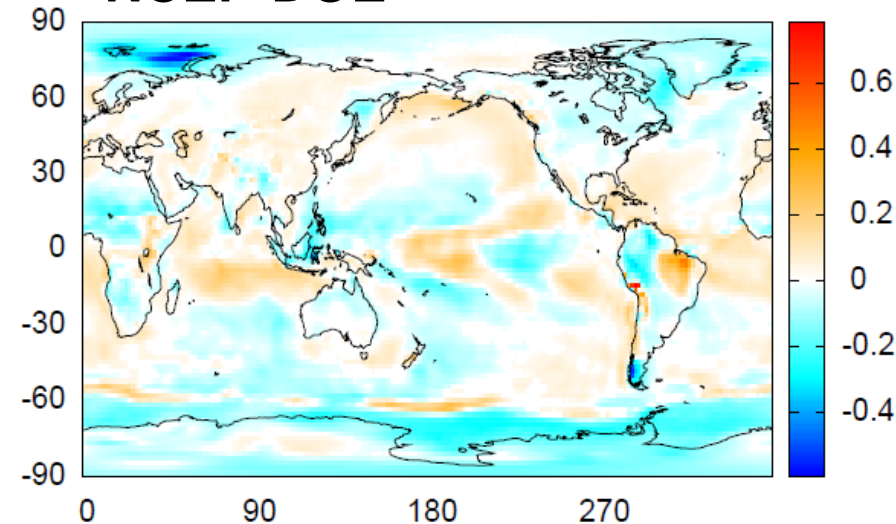
Model inter-comparison (same temporal period and resolution)

Average amplitude inter-decadal variation

ERA-Interim



NCEP-DOE



Good qualitative agreement (also in the maps of variations of average frequency the uncover ITCZ migration) but some differences are detected in certain regions.

Conclusions



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What did we learn?

- Hilbert analysis: allows to identify and quantify changes in climatic time series.
- Large variations of average amplitude (more than 50%):
 - ice melting in the Arctic
 - precipitation decrease in Amazonia
- Large variations of average frequency (more than 100%), consistent with an enlargement and northward migration of the inter-tropical convergence zone (ITCZ).
- Far reaching climatic consequences because ITCZ migration affects tropical atmospheric circulation.
- Ongoing work: predictive power? (attractor reconstruction, event synchronization applied to Hilbert time series, etc.)



THANK YOU FOR YOUR ATTENTION !

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<http://www.fisica.edu.uy/~cris/>

D. A. Zappala, M. Barreiro, and C. Masoller, “*Global atmospheric dynamics investigated by using Hilbert frequency analysis*”, Entropy **18**, 408 (2016).

D. A. Zappala, M. Barreiro, and C. Masoller, “*Quantifying interdecadal changes in large-scale patterns of surface air temperature variability*”, submitted (2017).