Subthreshold signal encoding in coupled FitzHugh-Nagumo neurons

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Workshop on Nonlinear Dynamics and Complexity
Volga, July 2017

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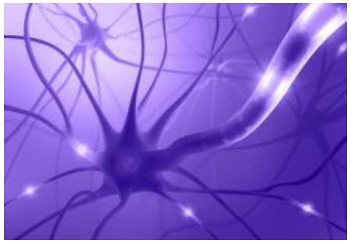
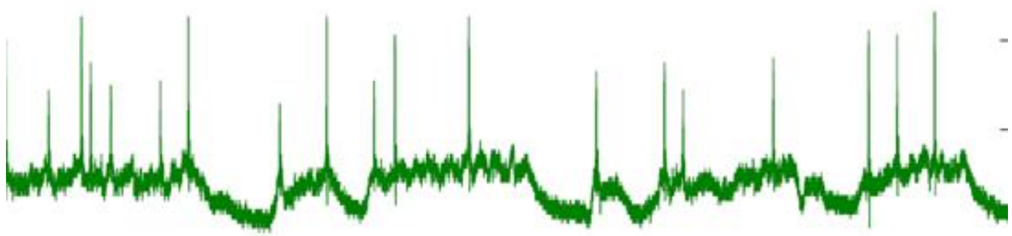
How neurons encode information?



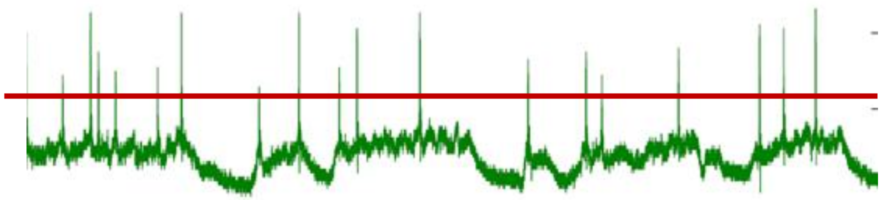


- In the spike **rate**?
- In the relative **timing** of the spikes?
- **Single** neuron encoding or **ensemble** encoding?
- If there are temporal correlations, how can they be detected and quantified?
- Our goal: try to understand how a neuron encodes a weak (subthreshold) periodic input in the output sequence of spikes.

Inter-spike-intervals (ISIs)

$$\{\ldots I_{i-1}, I_i, I_{i+1} \ldots\}$$

inter-spike-intervals



Empirical ISI distribution

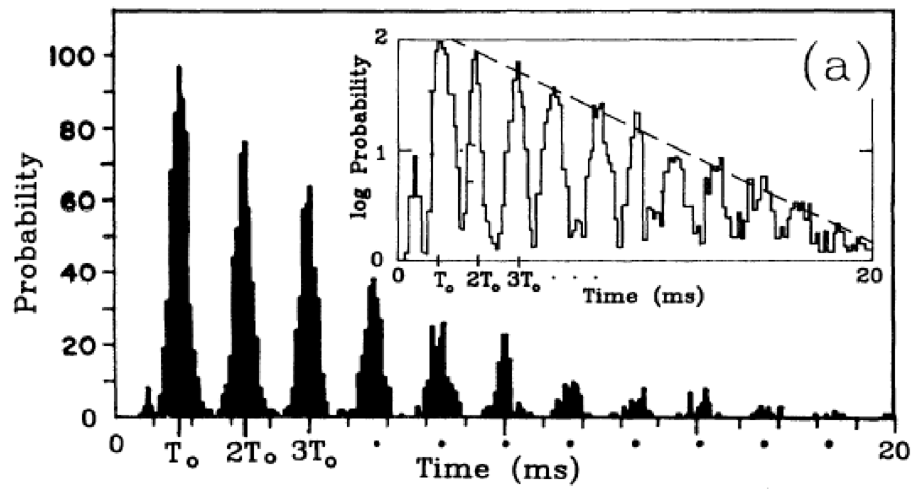


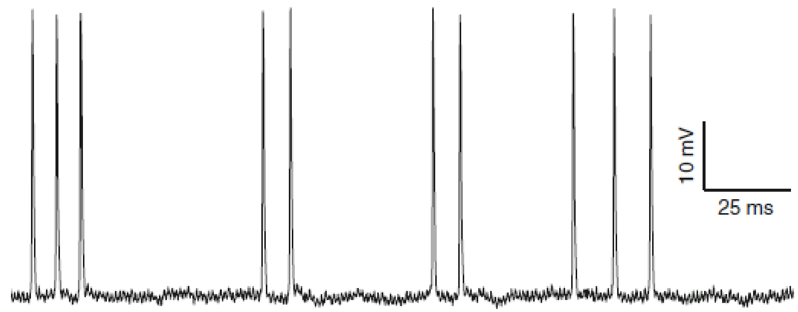
FIG. 1. (a) An experimental ISIH obtained from a single auditory nerve fiber of a squirrel monkey with a sinusoidal 80-dB sound-pressure-level stimulus of period $T_0 = 1.66$ ms applied at the ear. Note the modes at integer multiples of T_0. Inset:

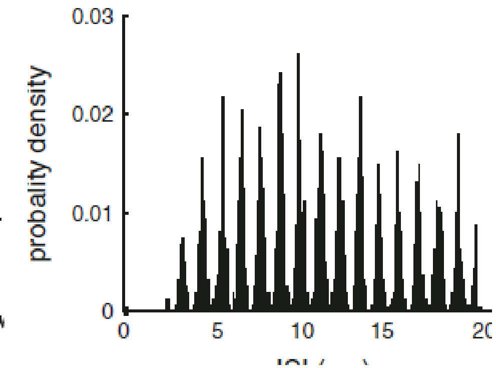
A. Longtin et al, PRL 67 (1991) 656

Serial correlations

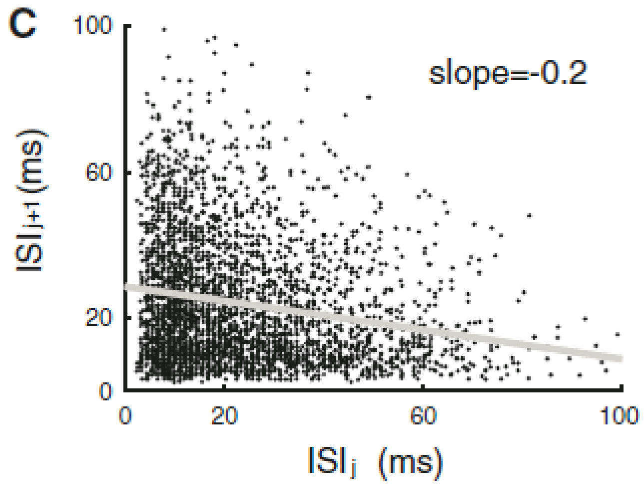
$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

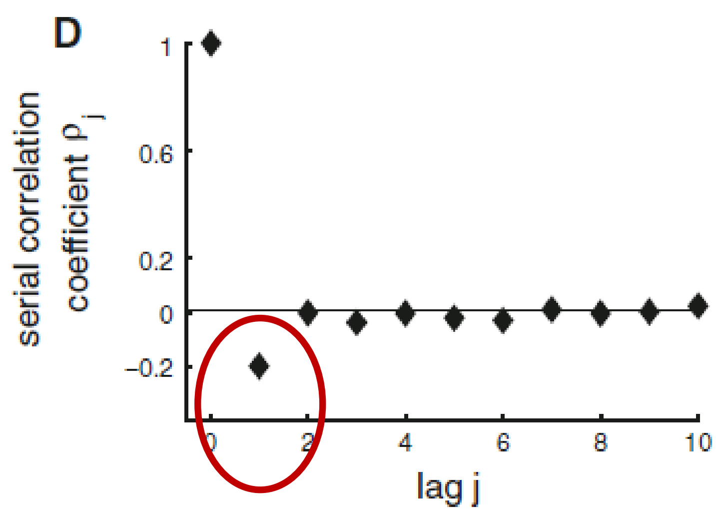
Membrane potential from a pyramidal cell of weakly electric fish





Nonrenewal spike train statistics: negative correlation at lag 1



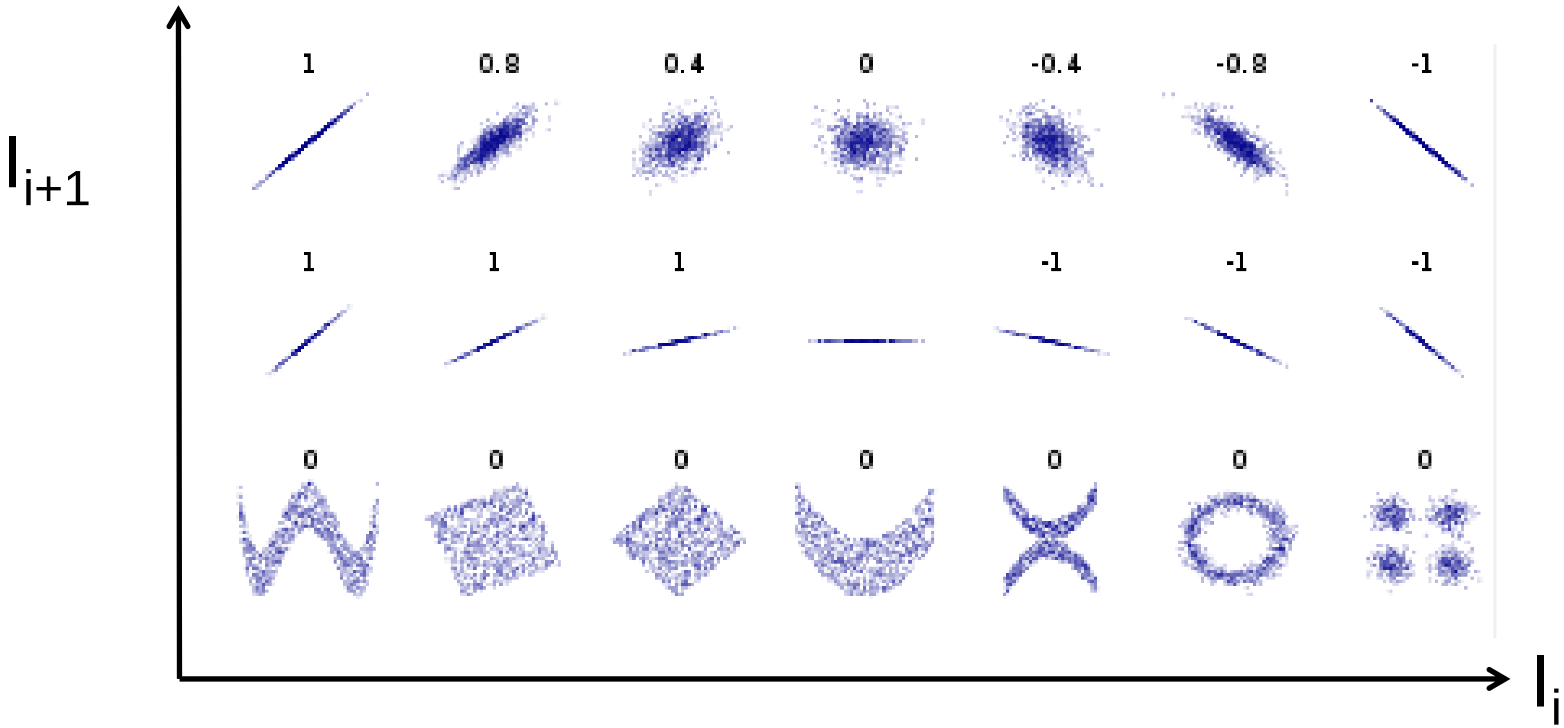


- Short ISIs tend to be followed by long ones and vice versa.
- Correlations at longer lags vanish.

Exp Brain Res (2011) 210:353–371

$$SCC_1 = \frac{\langle (I_i - \langle I \rangle)(I_{i-1} - \langle I \rangle) \rangle}{\sigma^2}$$

However, SCCs detects *linear* relationships only

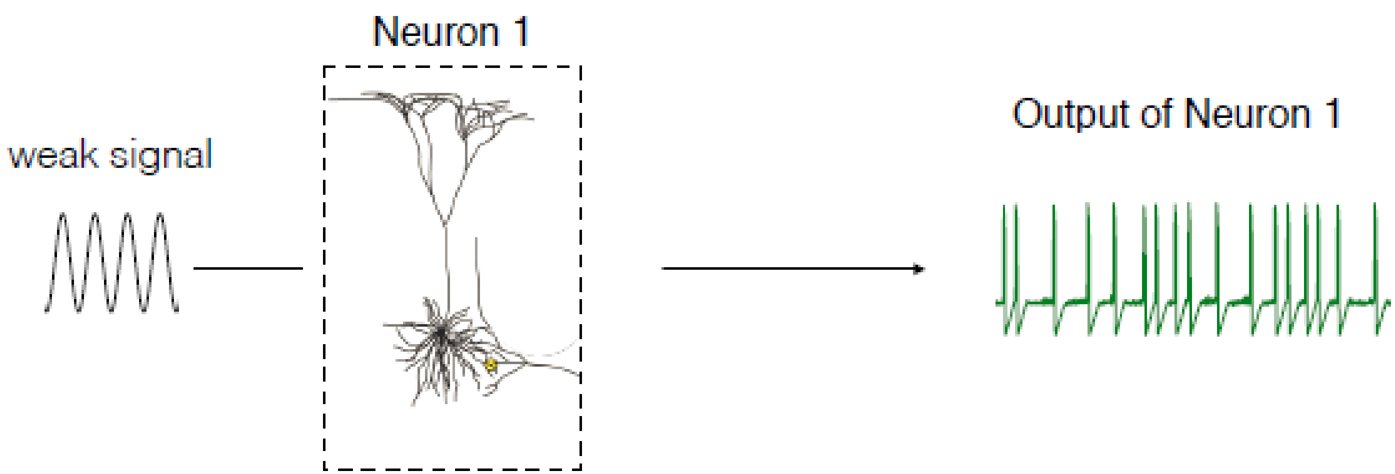


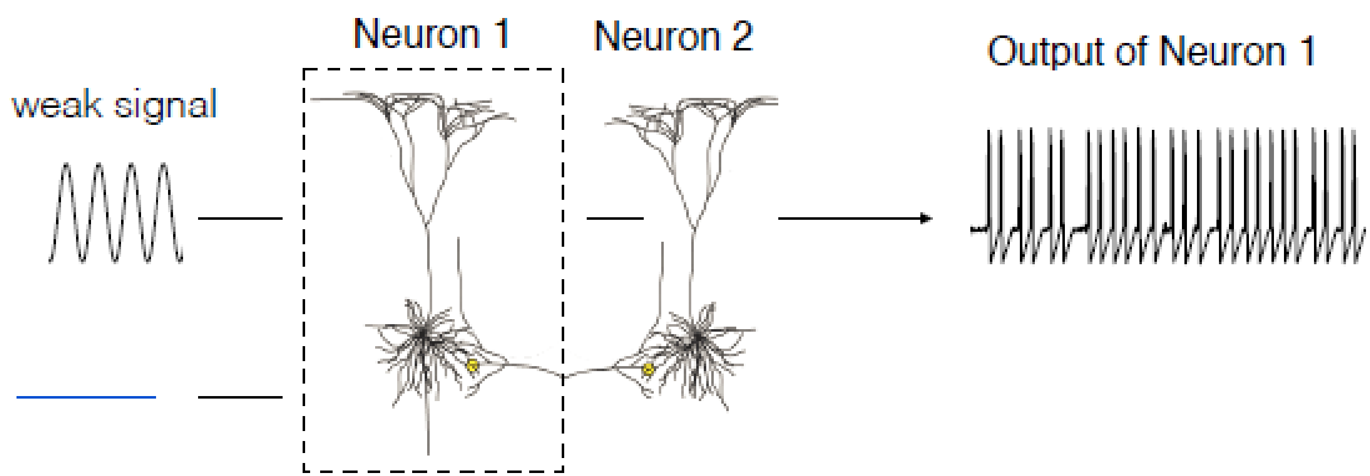
HOW TO INDENTIFY TEMPORAL STRUCTURES? RECURRENT / INFREQUENT PATTERNS?

Source: wikipedia

Outline

- Analysis method
- Single neuron
- Two coupled neurons





Symbolic method of time-series analysis

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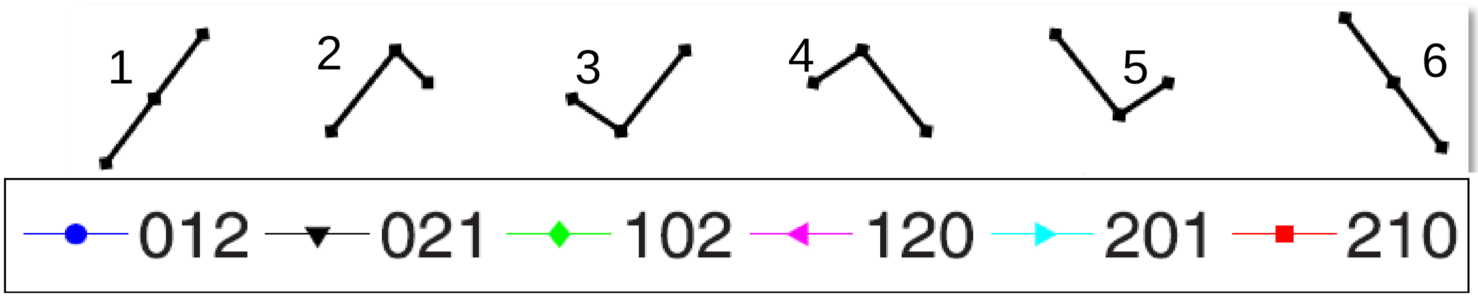
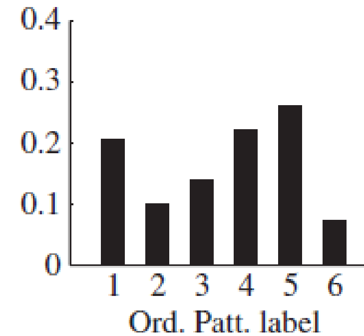
Symbolic method of time-series analysis

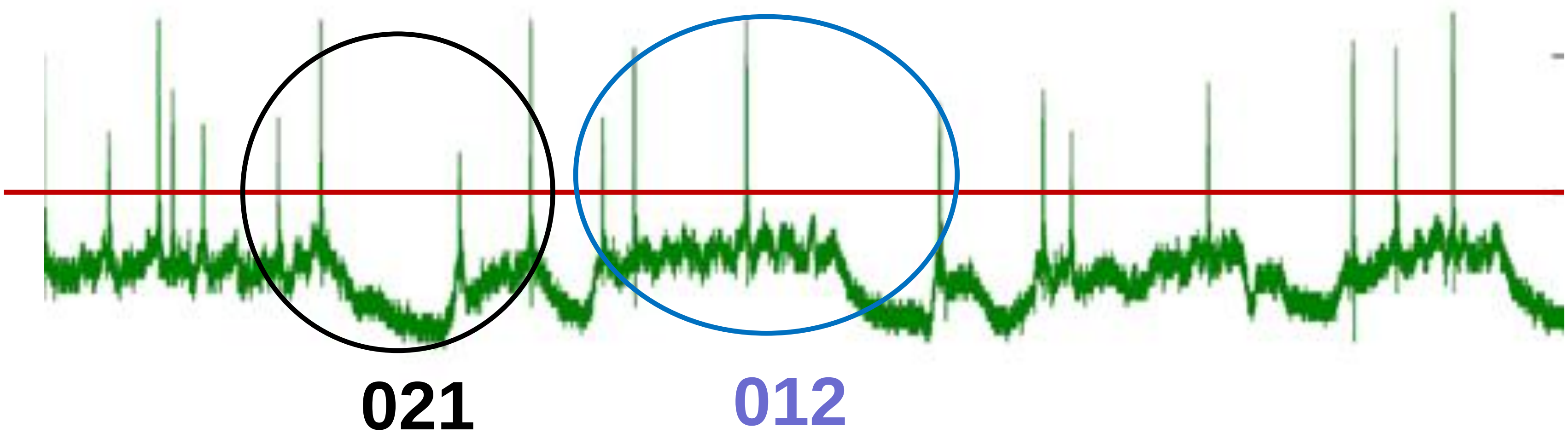
relative order of **three** intervals $\{\ldots \mathbf{I}_i, \mathbf{I}_{i+1}, \mathbf{I}_{i+2}, \ldots\}$



Ordinal probabilities



Example: (5, 1, 7) gives "102" because 1 < 5 < 7



Brandt & Pompe, PRL 88, 174102 (2002)

Analysis of single-neuron ISI sequences simulated with FHN model

- more/less frequent patterns encode information about subthreshold signal?

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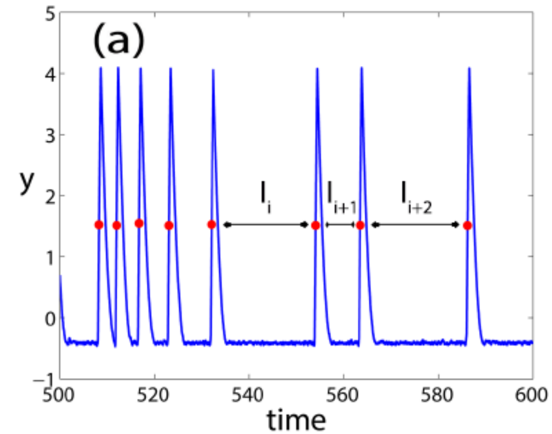
FHN model

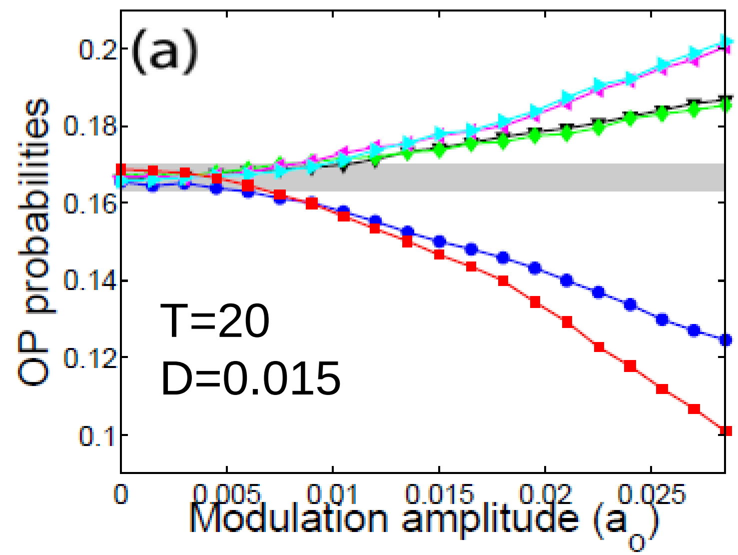
$$\epsilon\frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o\cos(2\pi t/T)} + D\xi(t),$$



a=1.05, ε=0.01

Gaussian white noise and <u>subthreshold</u> signal: **a_0** and **T** such that spikes are **noise-induced**.

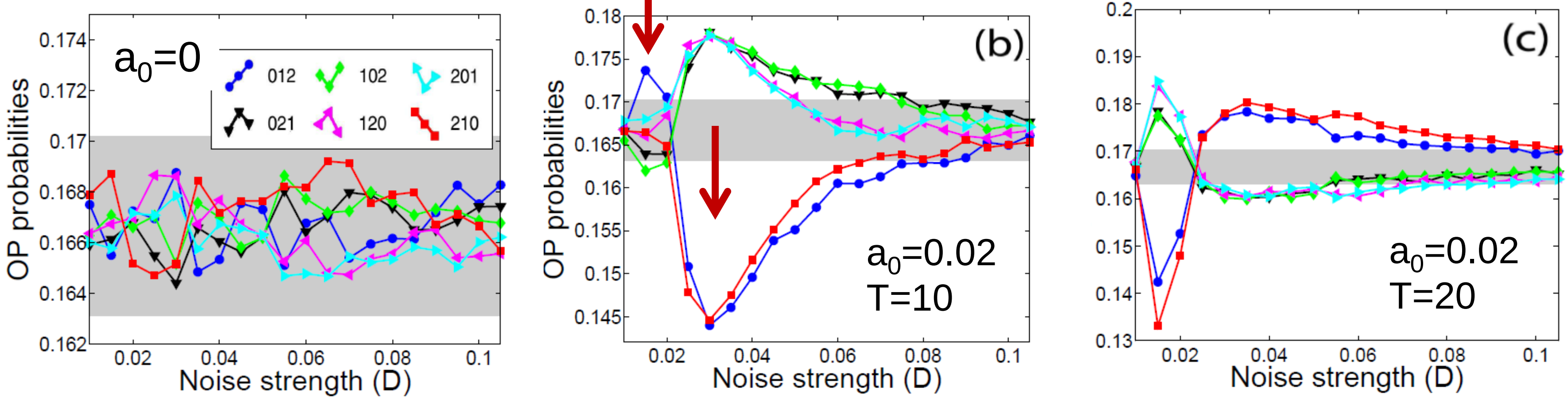
Time series with 100,000 ISIs simulated. Significance analysis with surrogates, 3σ confidence level.



J. M. Aparicio-Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016).

Role of the noise strength

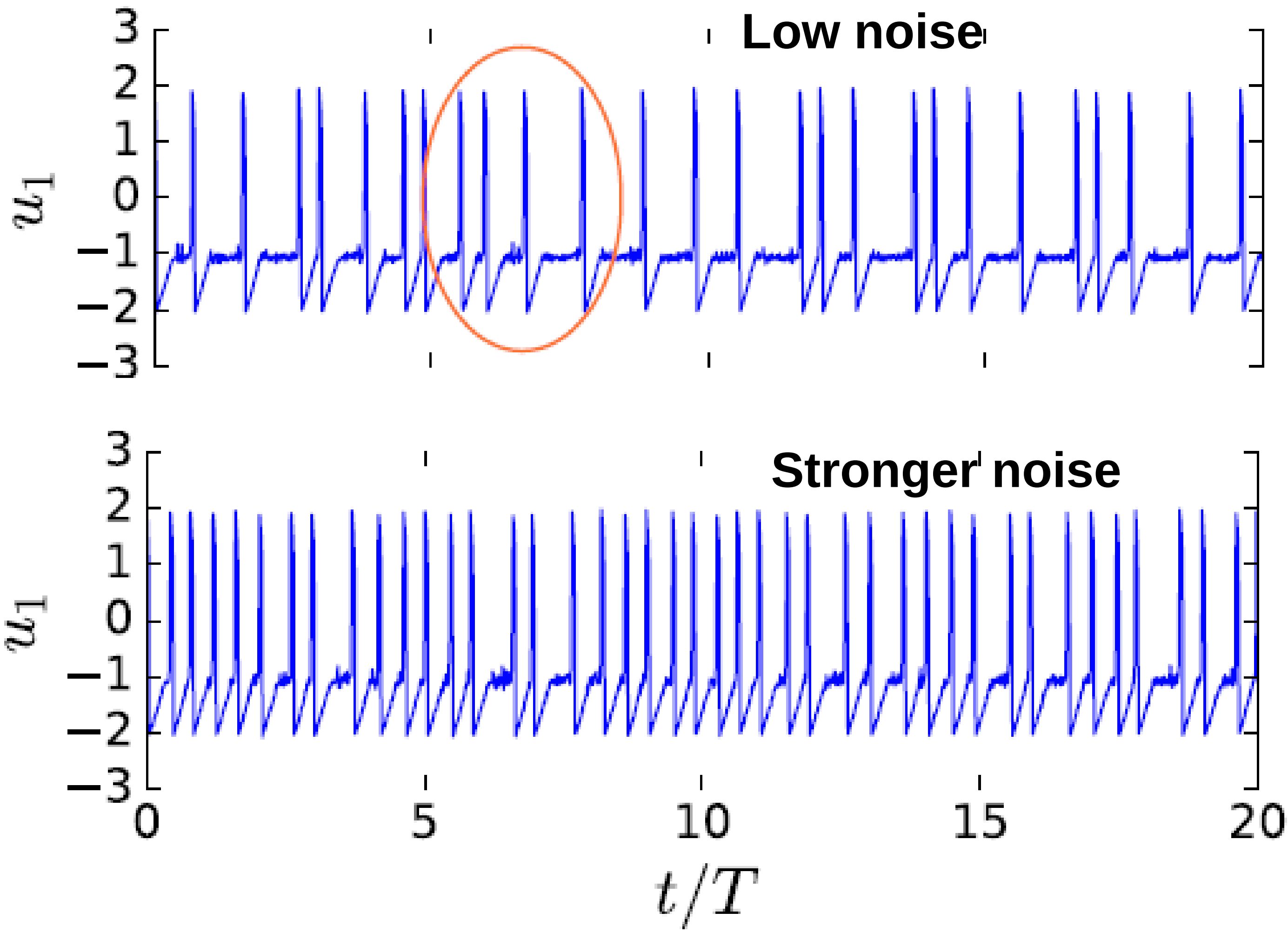
Without signal **With signal**



- No signal ⇒ no temporal ordering.
- Subthreshold periodic input induces preferred and infrequent patterns.
- They depend on the period and on the noise strength.
- Resonant-like behavior.

J. M. Aparicio-Reinoso M. C. Torrent and C. Masoller, PRE 94, 032218 (2016).

Time series with different P(012)

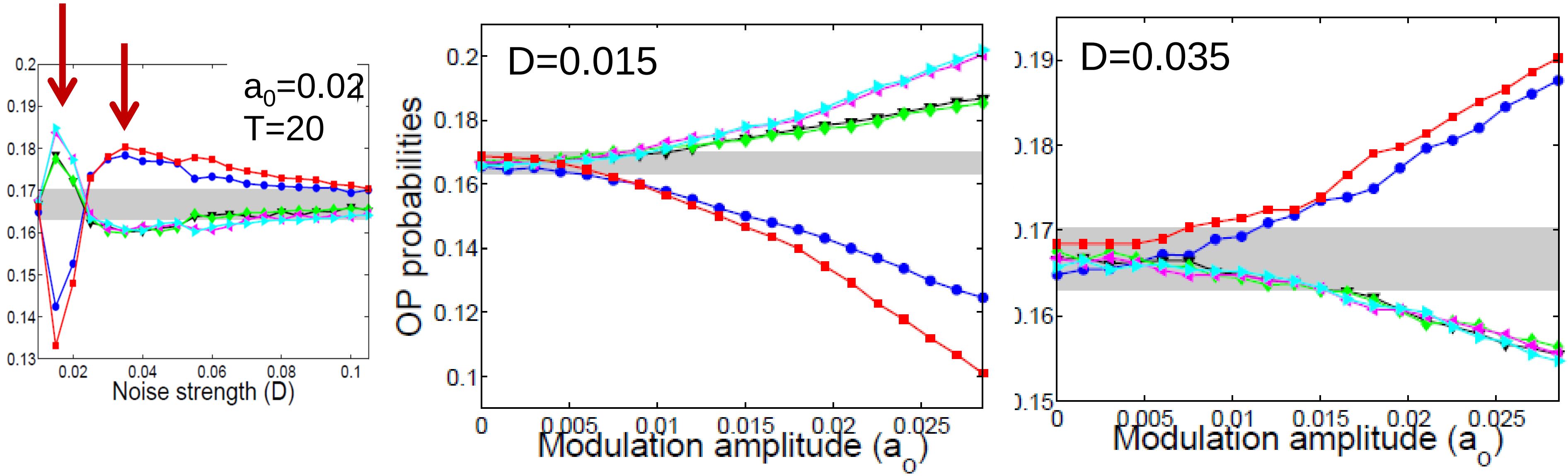


P(012)>1/6

P(012)<1/6

~ 40 spikes in 20 T

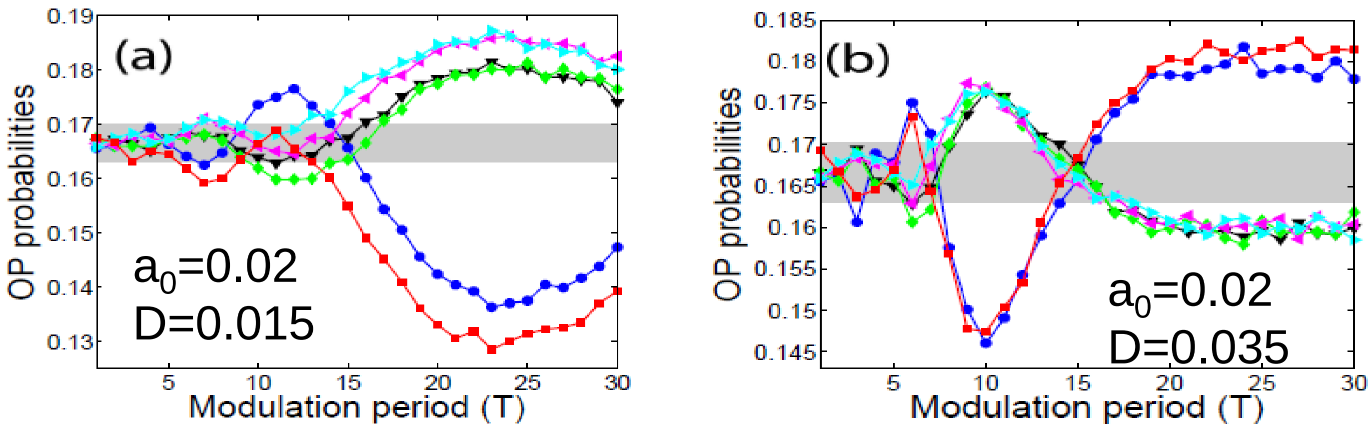
Role of the modulation amplitude



- The amplitude of the (weak) modulation does not modify the preferred or the infrequent patterns.
- Probabilities encode information about the amplitude of the signal.

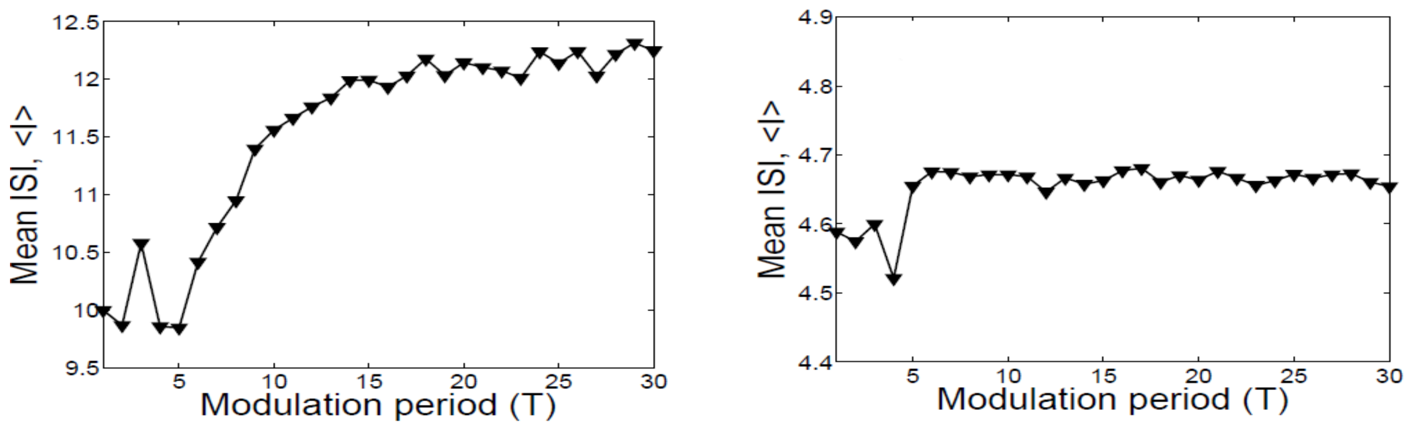
J. M. Aparicio-Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016).

Role of the modulation period



- More probable patterns depend on period and noise strength.

Which is the underlying mechanism? A change of the spike rate?



$\Rightarrow$ No direct relation.

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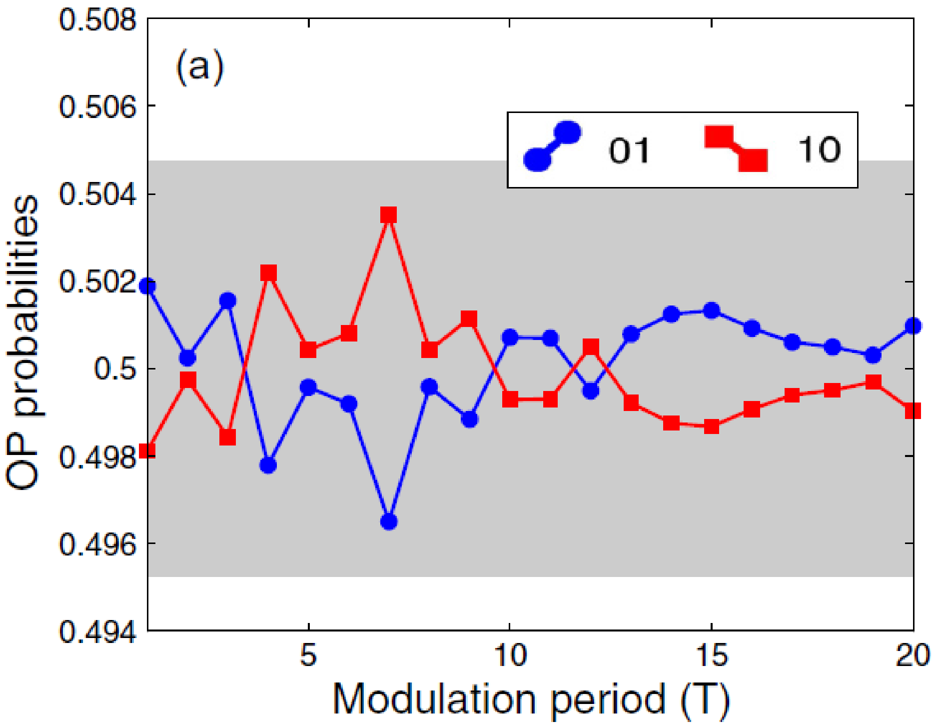
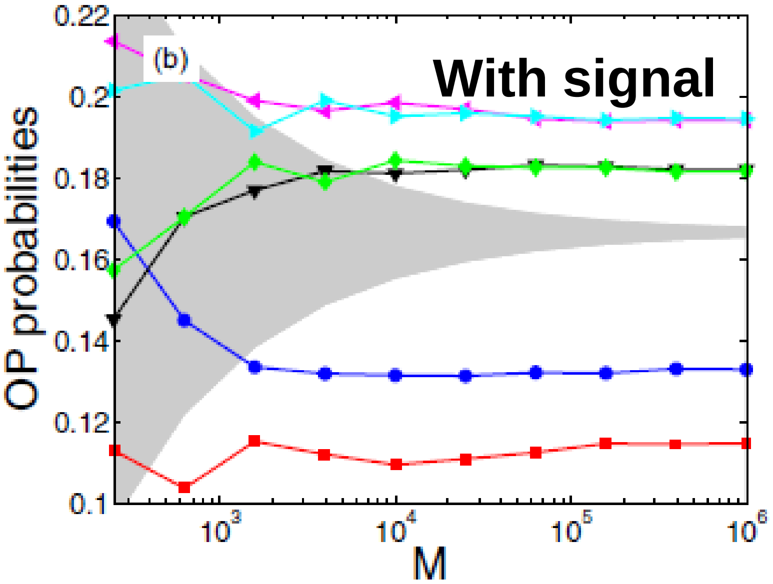
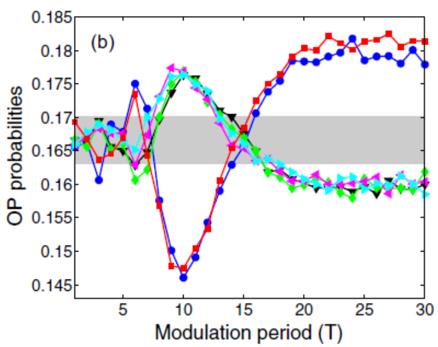
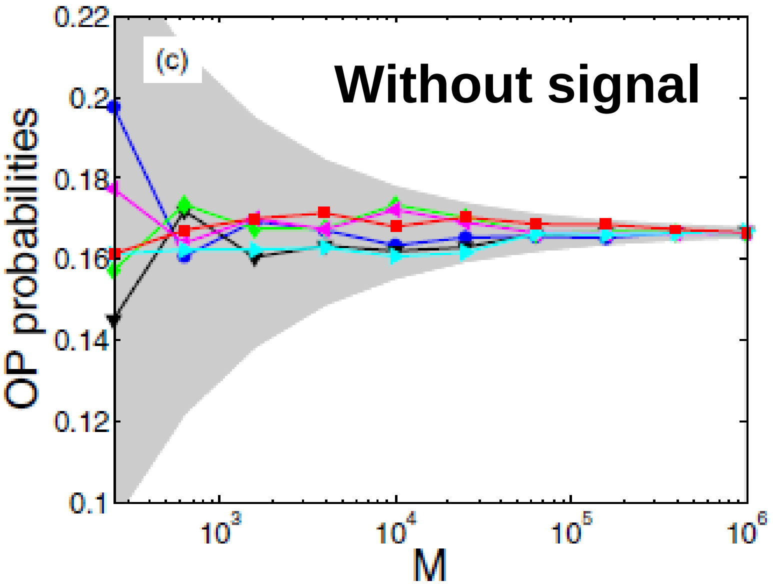
Role of the size of the pattern and of the length of dataset

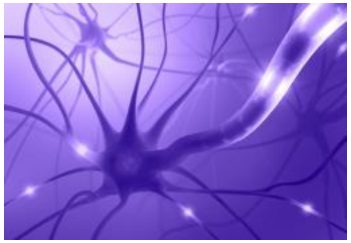


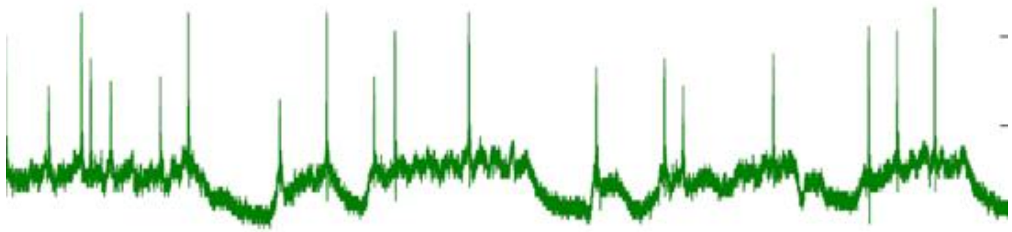




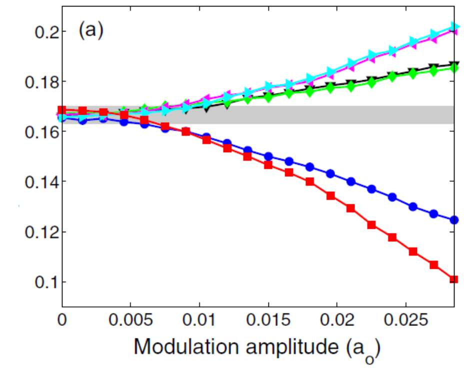


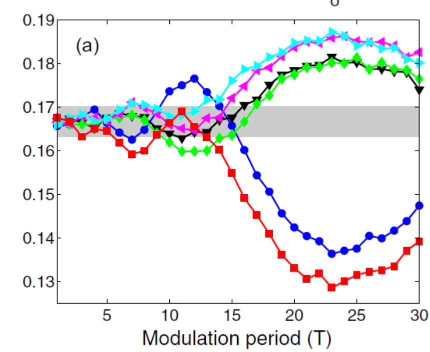
So… how neurons might encode characteristic features of an external stimulus?





- Periodic signal: amplitude and period might be encoded in more and less expressed patterns.
- <u>Single-neuron encoding</u>: very **slow** because long spike sequences are needed to estimate the probabilities.
- <u>Ensemble encoding</u>: very **fast** because few spikes are enough to compute the probabilities.

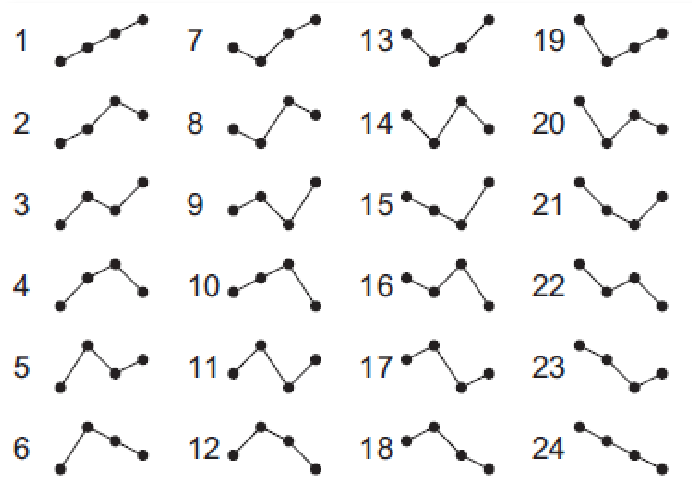




How many features can be encoded?

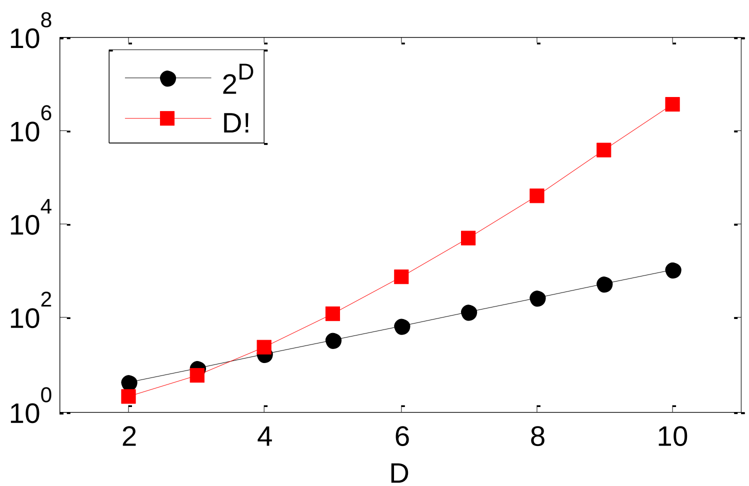
D=4

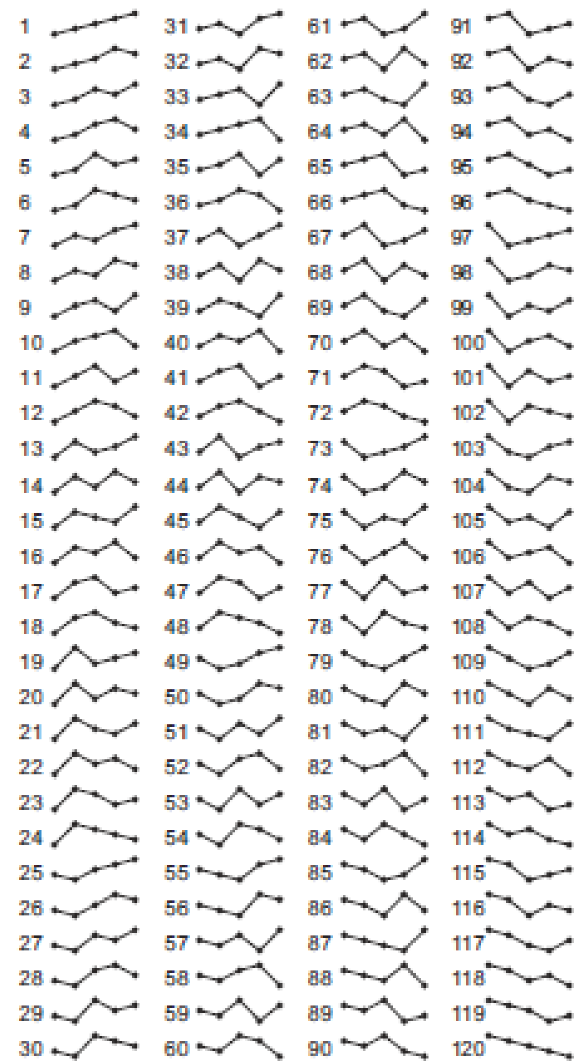
4!= 24



D=5:

120





U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Coupling to a second neuron

- can it improve the encoding of the subthreshold periodic signal?

Model

$$\epsilon \dot{u}_1 = u_1 - \frac{u_1^3}{3} - v_1 + \boxed{a_0 \cos(2\pi t/T)} + \boxed{\sigma_1 u_2} + \sqrt{2D}\xi_1(t)$$

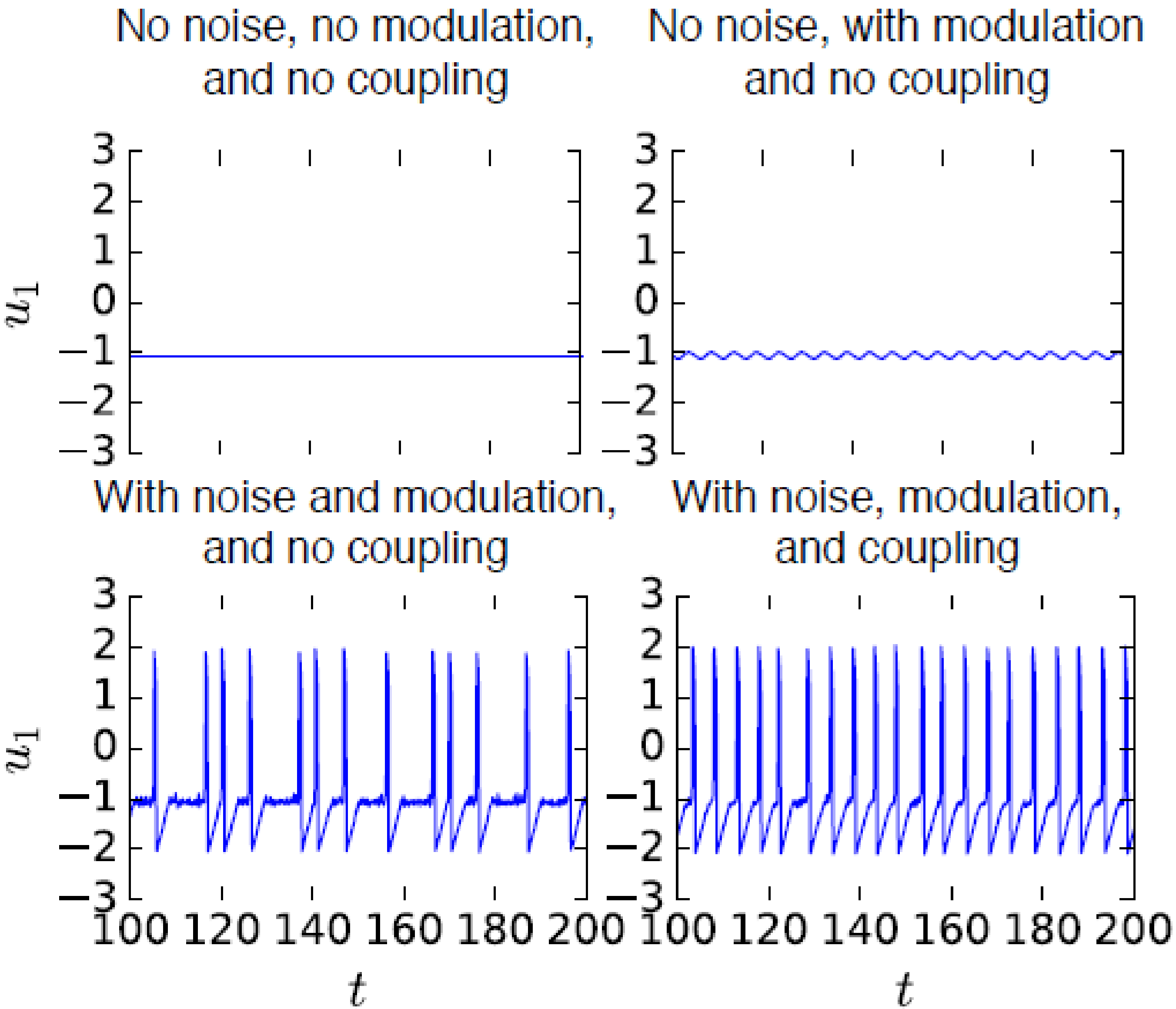
$$\dot{v}_1 = u_1 + a,$$

$$\epsilon \dot{u}_2 = u_2 - \frac{u_2^3}{3} - v_2 + \boxed{\sigma_2 u_1} + \sqrt{2D}\xi_2(t)$$

$$\dot{v}_2 = u_2 + a$$

- Identical neurons.
- Linear & instantaneous & asymmetric mutual coupling
- Coupling and noise both in the fast variable.
- a=1.05 and ε=0.01; parameters: a_0, T, D, σ_1, σ_2

We analyze the output of neuron 1



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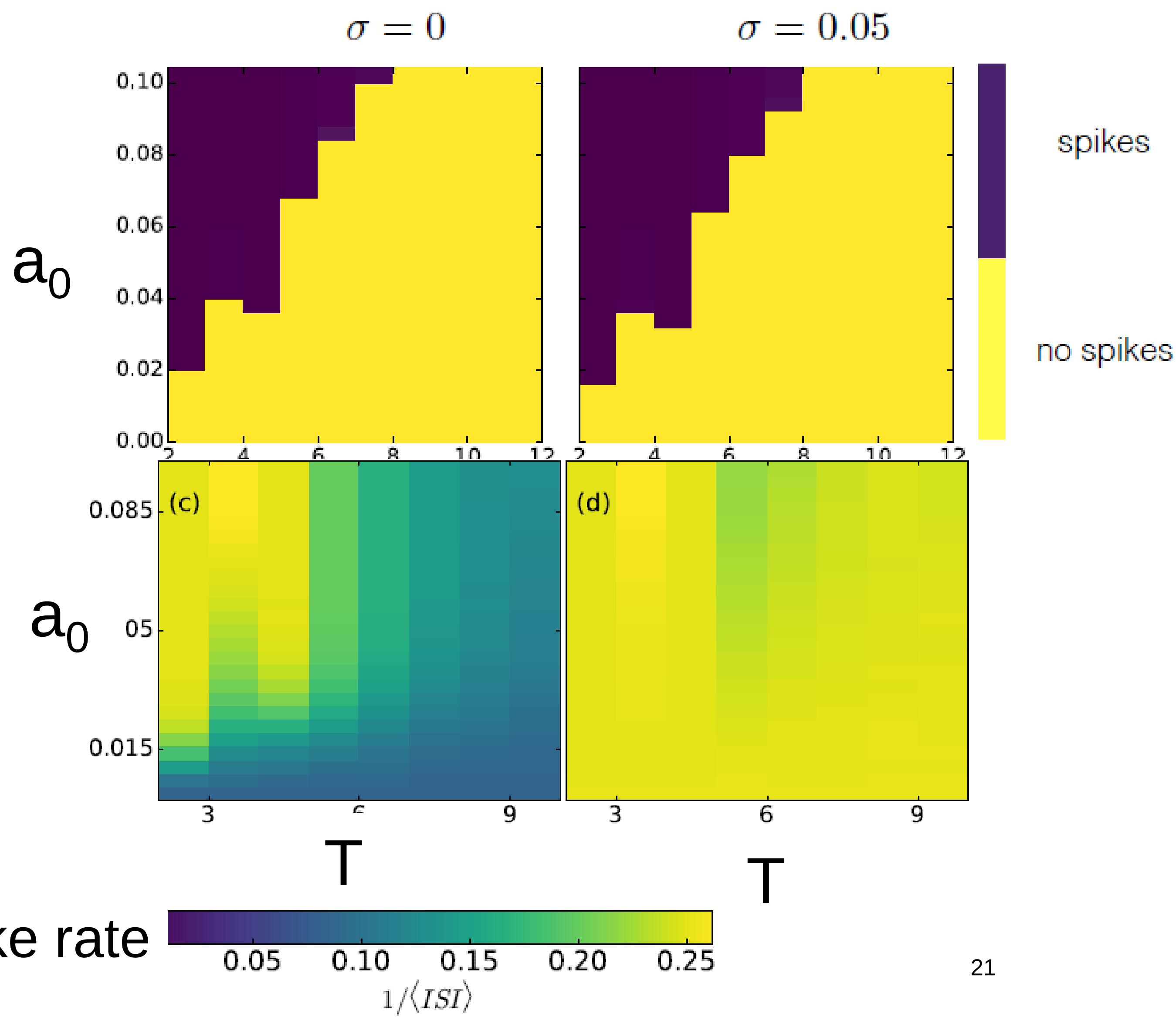
Identification of the subthreshold region

Without noise: signal is subthreshold if a_0 small and/or T long

With noise: coupling increases the spike rate



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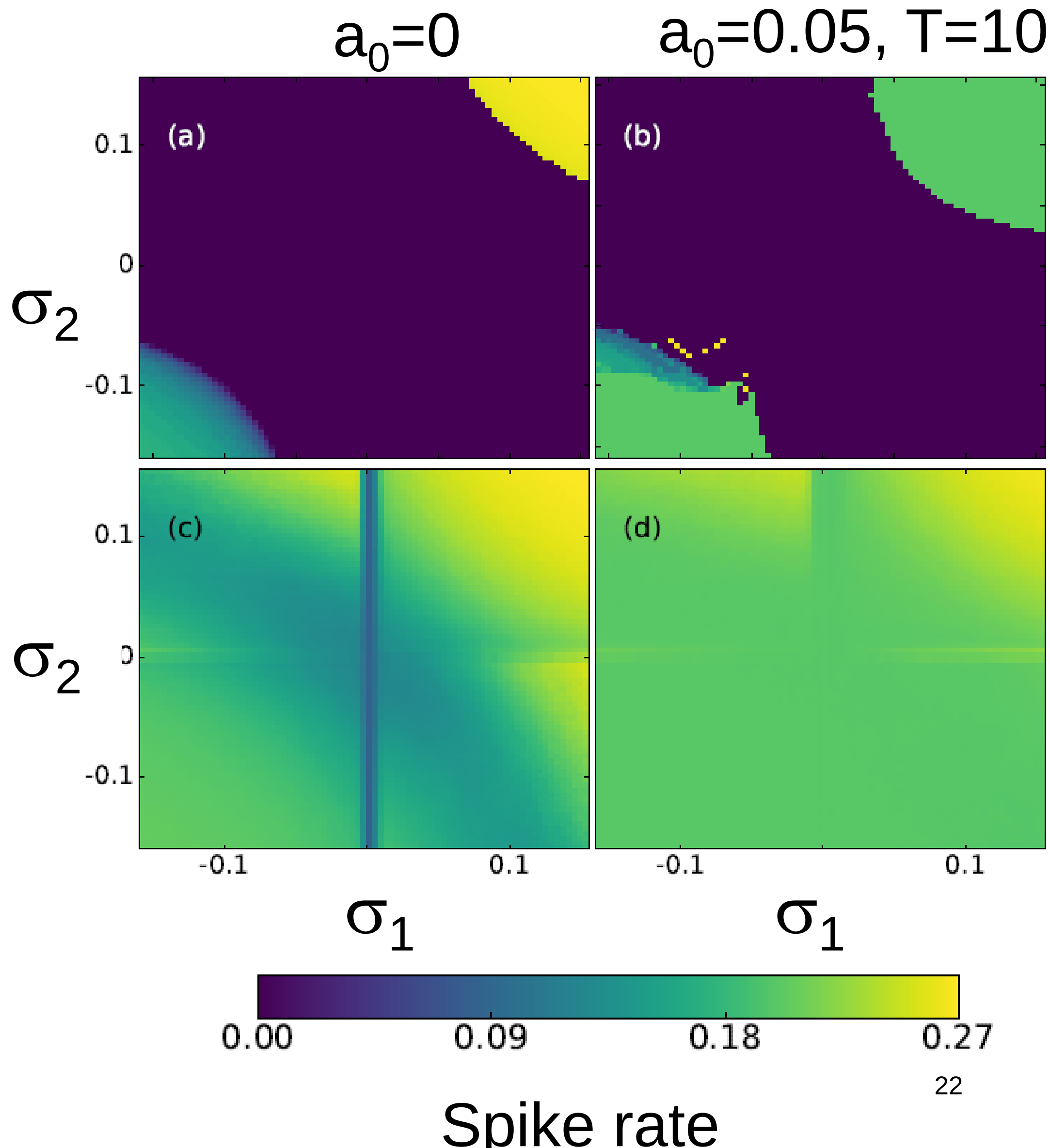
Role of coupling coefficients

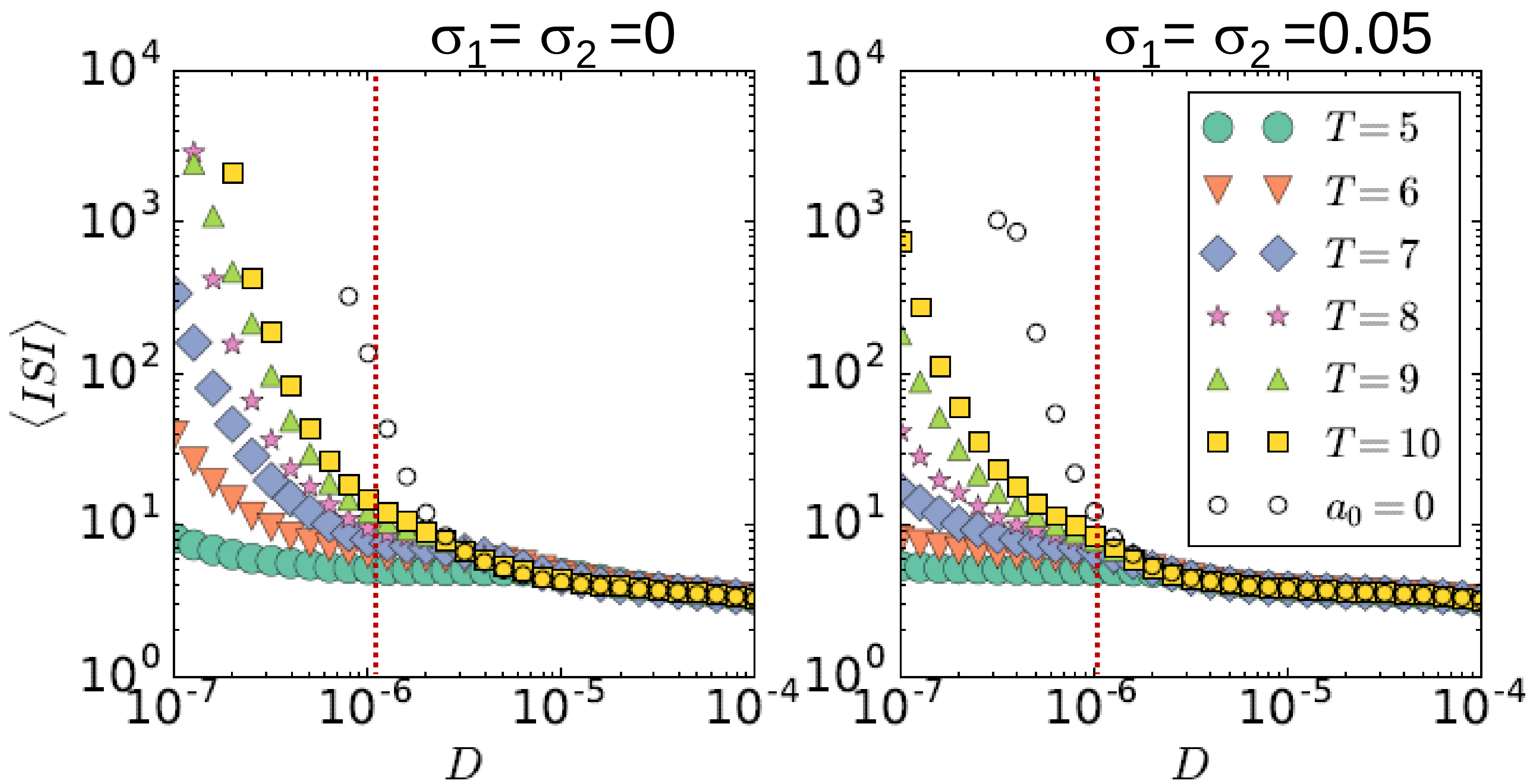
Without noise: large enough $|\sigma|$ induces spikes.

With noise: In the region of noise-induced spikes, The subthreshold signal increases the spike rate.

If σ_1=0, the spike rate of neuron 1 does not depend of neuron 2



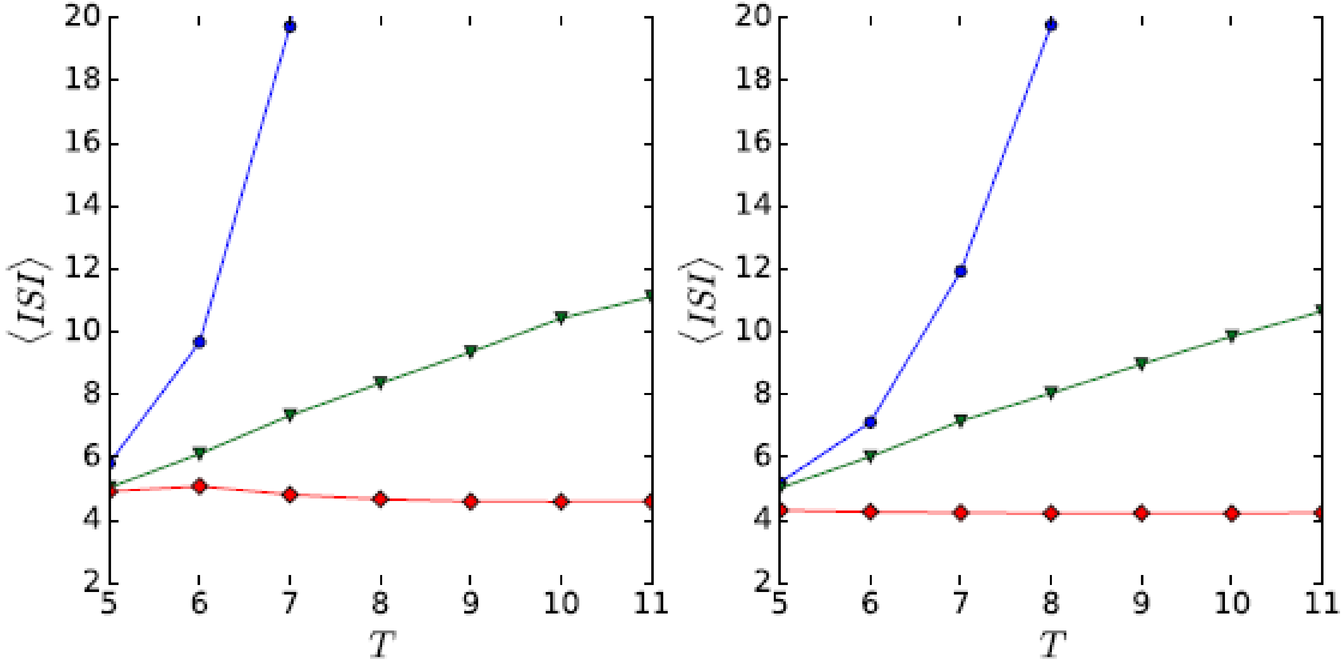
Noise dominated regime



The neuron fires at a lower noise level $\Rightarrow$ less noise is needed to encode the external signal.

Relation between the period and the mean ISI

$\sigma_1 = \sigma_2 = 0$ $\sigma_1 = \sigma_2 = 0.05$



Weak noise

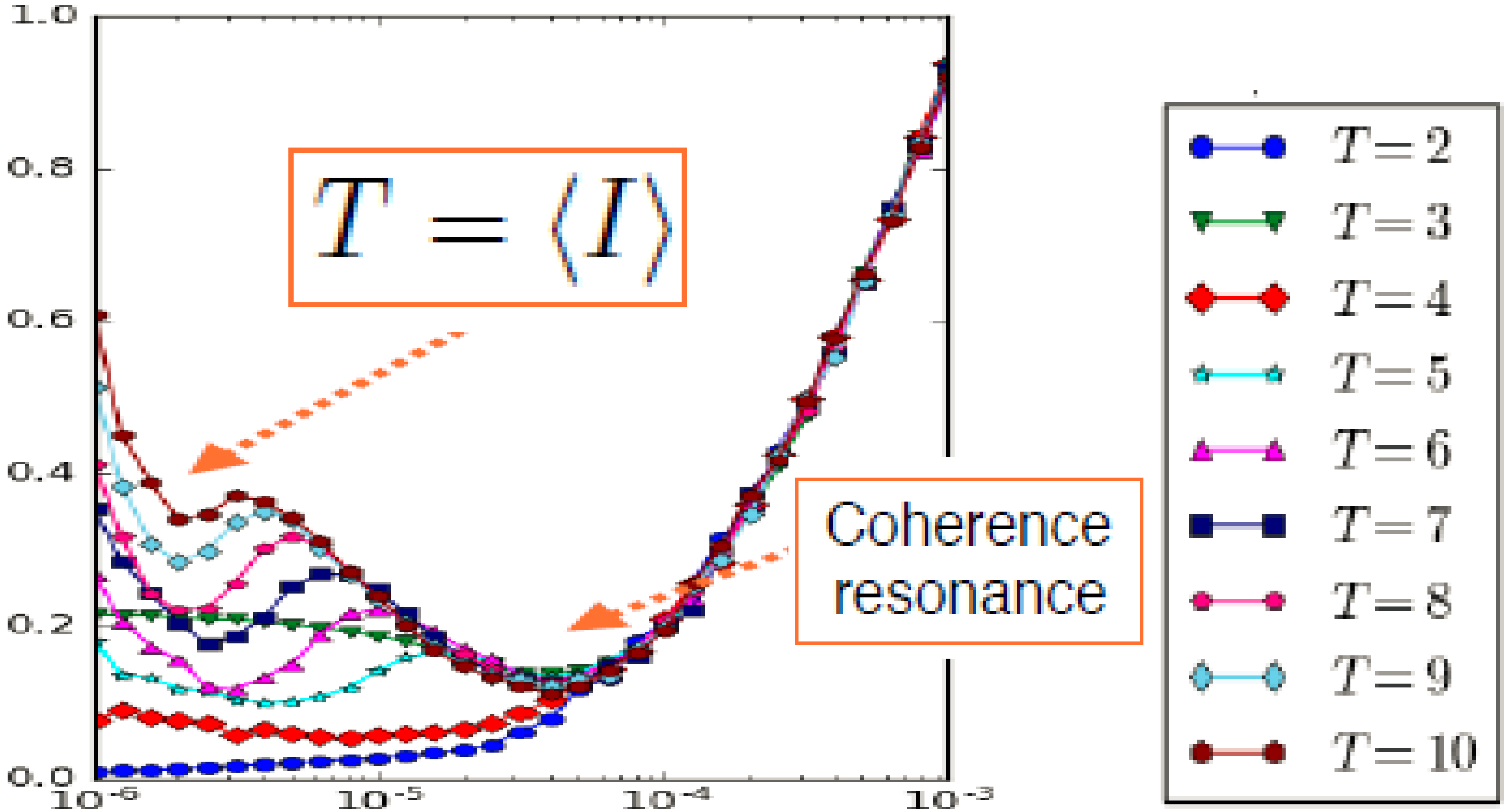
Intermediate noise

Strong noise

$\Rightarrow$ Three regimes, robust to coupling

Double resonance

$$R = \frac{\sigma}{\langle ISI \rangle}$$



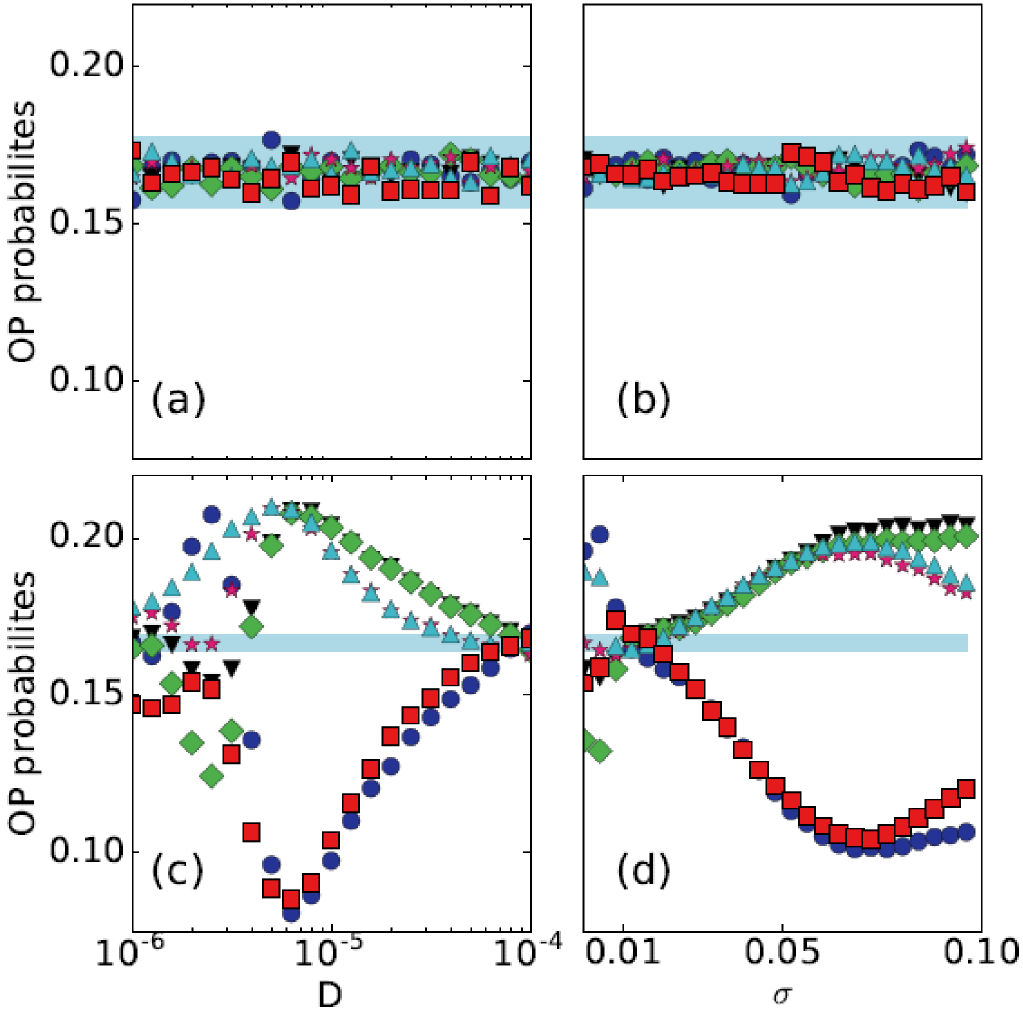
Noise strength

Subthreshold signal induces temporal correlations

$\sigma_1 = \sigma_2 = 0$ $\sigma_1 = \sigma_2 = 0.05$

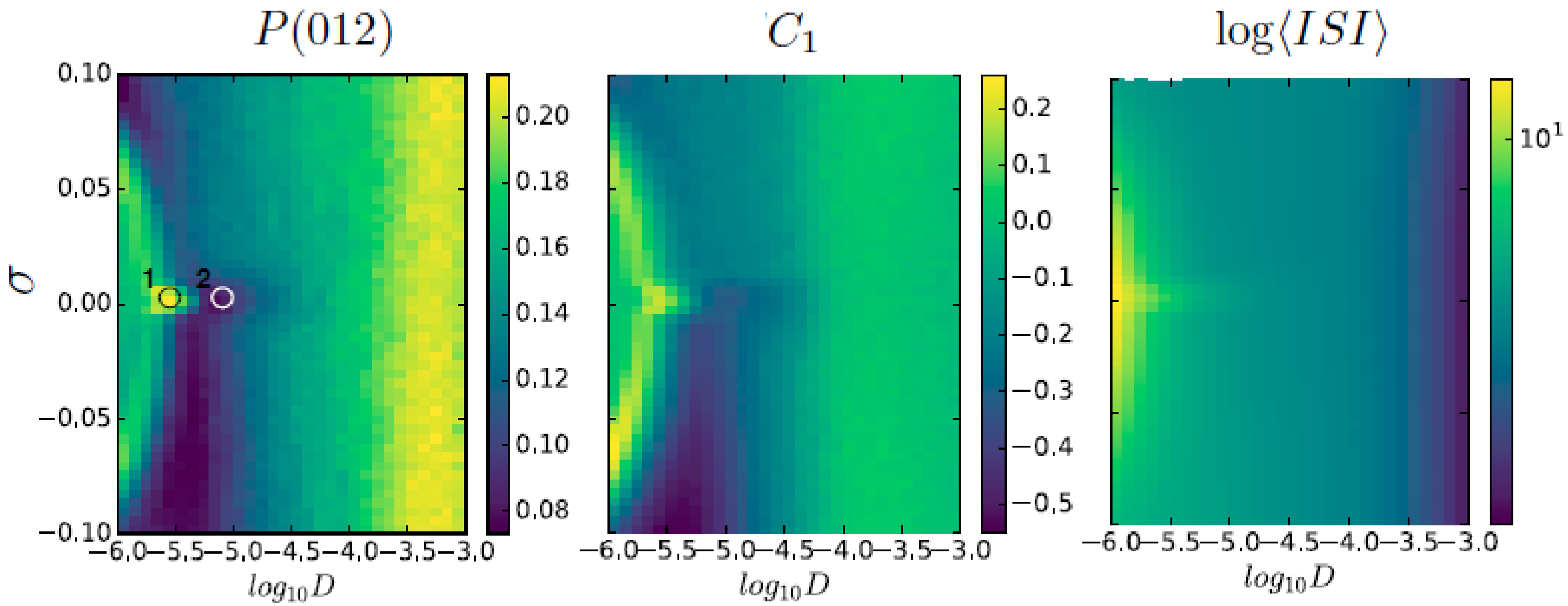
$a_0=0$

$a_0=0.05$ T=10



$\Rightarrow$ Similar effect of increasing noise or coupling

Coupling vs noise



Parameters: $a_0 = 0.05$ $T = 10$

$\Rightarrow$ For weak noise, coupling plays a role similar to noise

$\Rightarrow$ For strong noise, coupling plays no role

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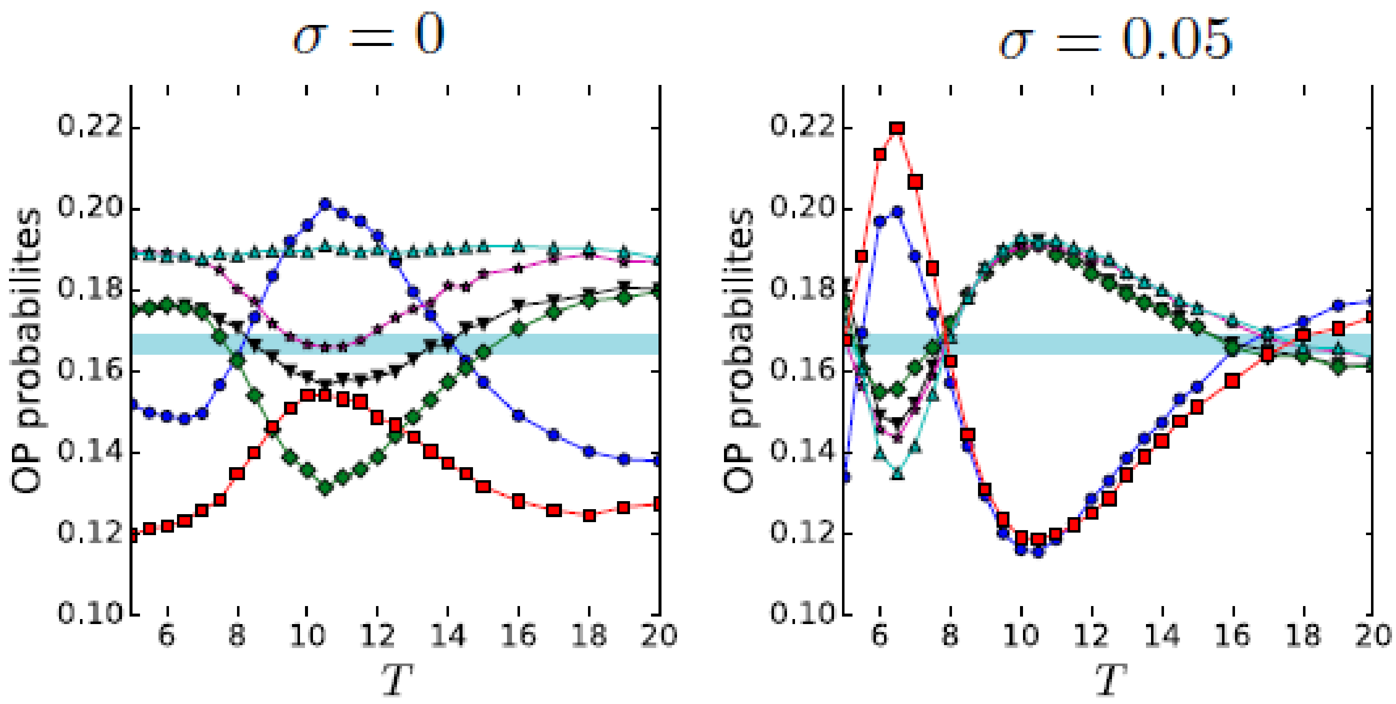
Influence of coupling in the ordinal probabilities



Parameters: $D = 2 \cdot 10^{-6}$ and $a_0 = 0.05$

$\Rightarrow$ With coupling the period of the subthreshold signal is still encoded in the ordinal probabilities

$\Rightarrow$ Coupling changes the preferred & infrequent patterns

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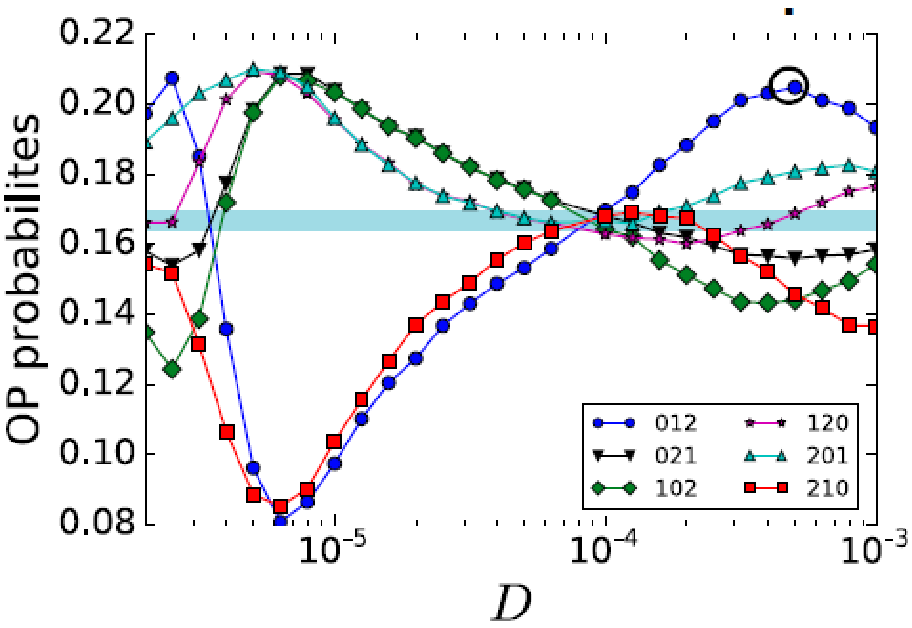
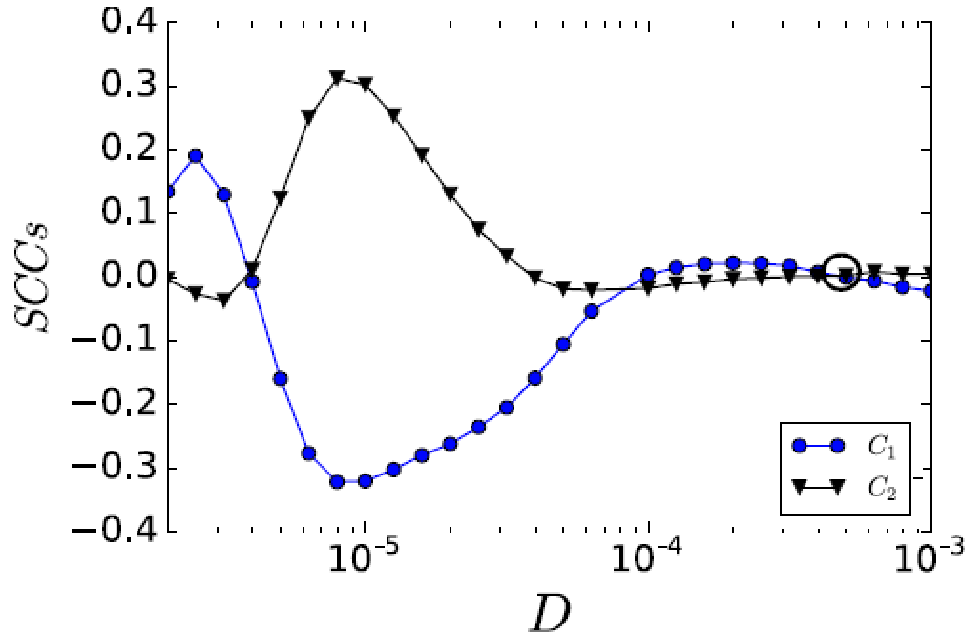
Comparing ordinal probabilities and serial correlation coefficients

$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

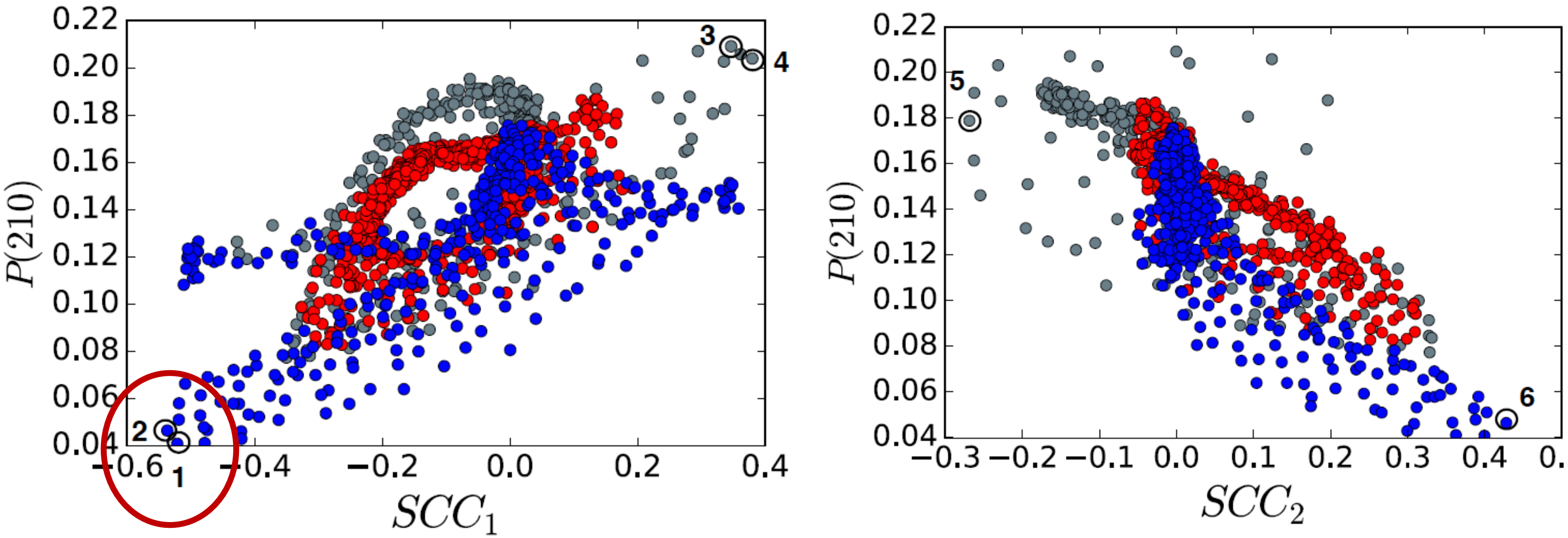
$a_0 = \sigma = 0.05$, T=8





⇒ For strong noise, correlation coefficients at lag 1 and 2 vanish but ordinal analysis detects more / less expressed patterns.

Relation of P(210) and serial correlation coefficients



Varying a_0 and T while keeping σ and D constant

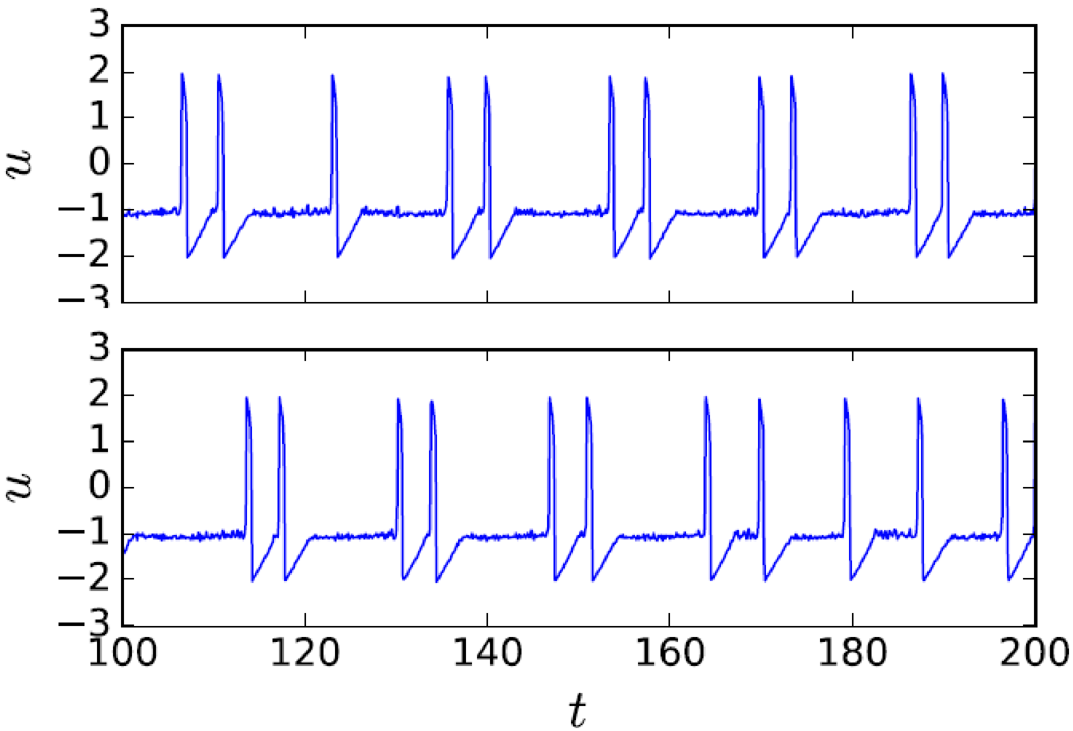
Varying s and D while keeping a_0 and T constant

Varying T and D while keeping a_0 and σ constant

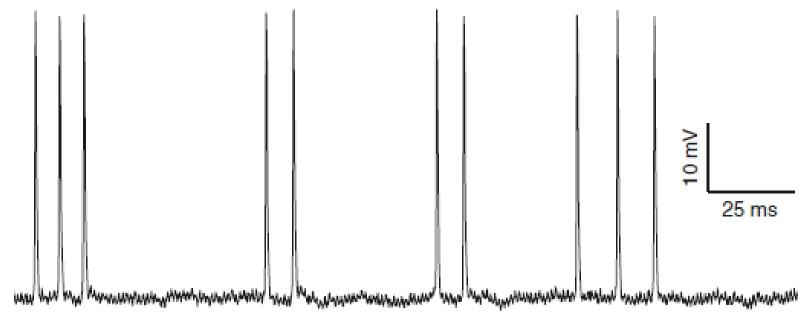
Ordinal analysis and linear SCCs provide complementary information: spike sequences with similar probabilities can be distinguished by different SCCs and vice-versa

Time series with negative SCC1 and low probability of 210

Synthetic



Empirical (weak electric fish)



- Short ISIs tend to be followed by long ones and vice versa.

M Exp Brain Res (2011) 210:353–371

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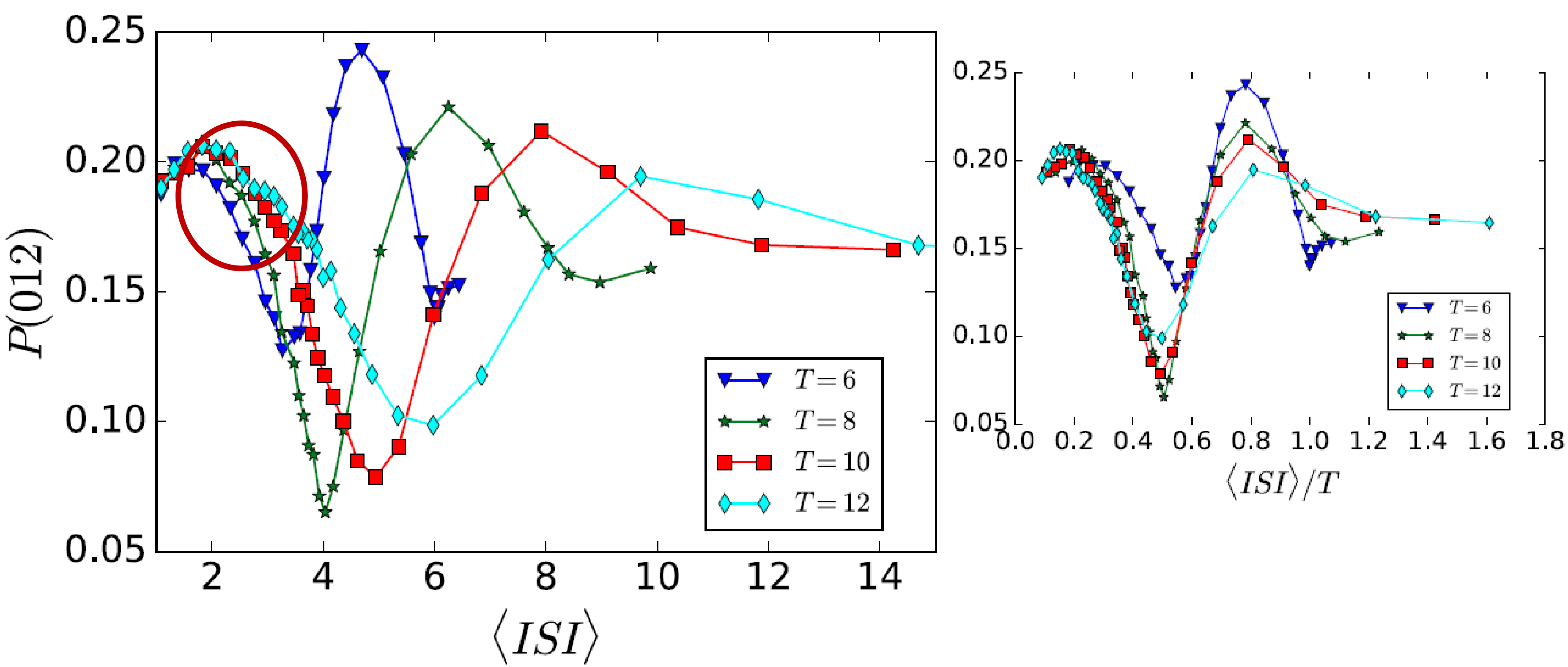
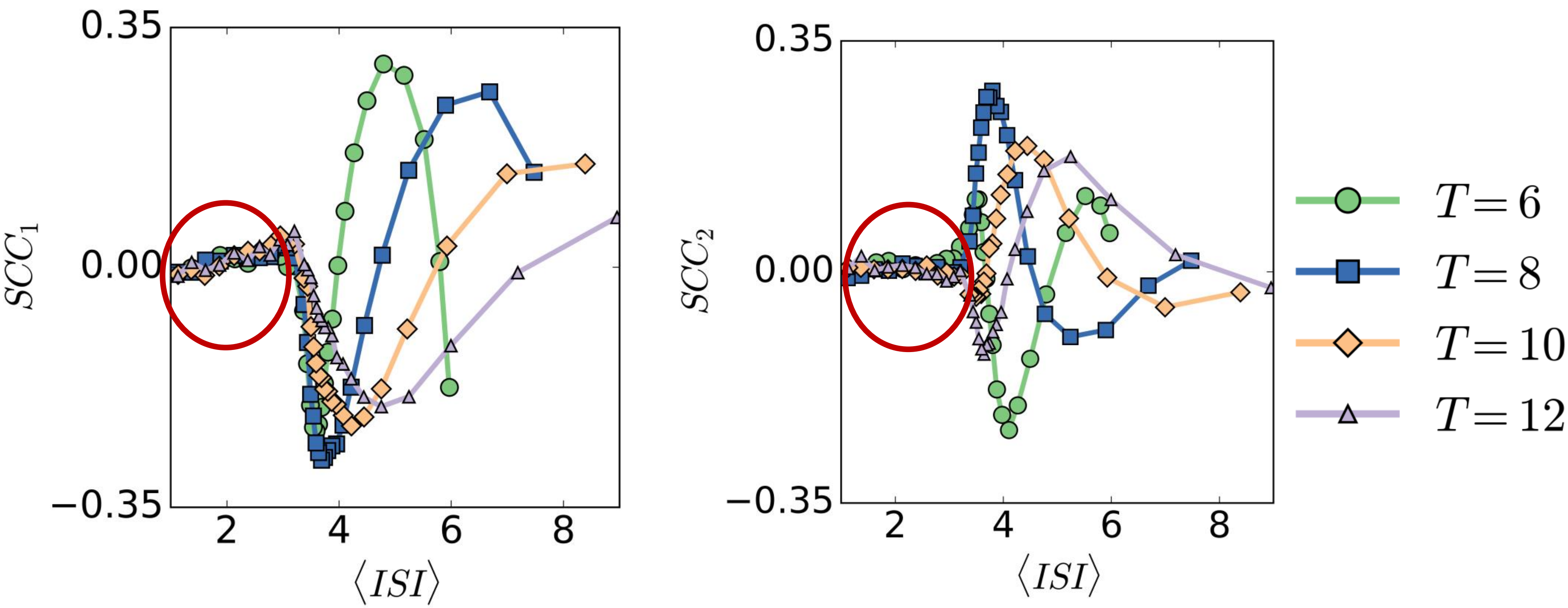
Relation between mean ISI and P(012)

Noise increases



- P(012) minimum when ⟨ISI⟩≈T/2 (two spikes each period)



- SCC1=SCC2=0, and the mean ISI is the same for the different periods, but the ordinal probabilities are different.

Conclusions

What did we learn?

- Take home message:
  - Ordinal analysis is useful for uncovering patterns in data
  - It detects nonlinear temporal correlations which might not be captured by linear analysis.

- Main conclusions:
  - Neuron fires at lower noise level when coupled.
  - For weak noise, coupling and noise have similar effects.
  - Coupling changes the preferred/infrequent patterns.

- Future work:
  - Neuronal ensemble.
  - Is it possible to optimize signal transmission to neuron 2?
  - Compare with empirical data.

THANK YOU FOR YOUR ATTENTION !

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http://www.fisica.edu.uy/~cris/

Emergence of spike correlations in periodically forced excitable systems

J. A. Reinoso, M. C. Torrent and C. Masoller, PRE 94, 032218 (2016).

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M. Masoliver and C. Masoller, in preparation (2017).

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