Optimal Entrainment of the Power Dropouts of a Semiconductor Laser with Optical Feedback to Current Modulation

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- Characterization of optical spikes
- Control via small electric perturbations
Dynamical transitions in a semiconductor laser with optical feedback

Video: how complex signals emerge from optical noise

Different dynamical regimes are difficult to distinguish.
Can differences be quantified? With what reliability?
How similar these time series are?

Optical spikes

Neuronal spikes

Time
Threshold crossings define ``events” in a time series

- Problems:
  - How to select the threshold?
  - Threshold dependent results?

inter-spike-intervals (ISIs):

\[ \Delta T_i = t_{i+1} - t_i \]
ISI distribution indicates that neurons and lasers have a similar response to external periodic forcing.

Neuron data

Single auditory nerve fiber of a squirrel monkey with a sinusoidal sound stimulus applied at the ear.

Laser data

Data recorded in our lab when a sinusoidal signal is applied to the laser current.

A. Longtin et al PRL (1991)

A. Aragoneses et al Optics Express (2014)
Return maps also suggest that neurons and lasers have similar response to external periodic forcing.

- How to identify temporal order?
- Are there more or less expressed “spike patterns”?

A. Longtin

M. Giudici et al *PRE* (1997)
A. Aragoneses *Optics Express* (2014)
Different methods of time series analysis provide complementary information

- Many methods
  - Correlation analysis
  - Fourier analysis
  - Lyapunov & fractal analysis
  - **Symbolic analysis**
  - Wavelet analysis
  - Etc. etc.

- The method to be used depends on the data
  - Length
  - Noise
  - Resolution
  - Etc.
Ordinal analysis: a tool to look for patterns in data

- Consider a time series \( x(t) = \{ \ldots, x_i, x_{i+1}, x_{i+2}, \ldots \} \)
- Which are the possible order relations among three data points?

1. Count how many times each “ordinal pattern” appears.
2. Advantages: allows to identify temporal structures & is robust to noise.
3. Drawback: information about actual data values is lost.

Bandt and Pompe PRL (2002)
Ordinal analysis of inter-spike intervals

\[ \Rightarrow 012 \quad \Rightarrow 210 \]
Ordinal analysis identifies the onset of different dynamical regimes, but does not distinguish "noise" and "chaos".

Grey region: probabilities are consistent with the uniform distribution ($P_i = 1/6 \approx 0.17 \ \forall \ i$) with 99.7% confidence level.

C. Quintero-Quiroz et al, Scientific Reports (2016)
P(210) identifies dynamical regimes in parameter space (pump current, feedback strength)

M. Panozzo et al, Chaos (2017)
Zooming into the region where spikes are well-defined, a transition is detected (not captured by correlation analysis)

\[ C_\tau = \frac{\langle (\Delta T_i - \langle \Delta T \rangle)(\Delta T_{i-\tau} - \langle \Delta T \rangle) \rangle}{\sigma^2} \]

A. Aragoneses et al., Scientific Reports (2014)
A modified circle map: simple minimal model

\[ \varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi} \left[ \sin(2\pi\varphi_i) + \alpha_c \sin(4\pi\varphi_i) \right] + D \zeta \]

\[ X_i = \varphi_{i+1} - \varphi_i \]

\[ \rho = \text{natural frequency} \]
\[ K = \text{forcing amplitude} \]
\[ D = \text{noise strength} \]

Circle map data

Empirical laser data

Same “clusters” & same hierarchical structure

A. Aragoneses et al Scientific Reports (2014)
Connection with neurons: the circle map describes many excitable systems

- The modified map describes spike correlations in sensory neurons (Neiman and Russell, PRE 2005)
- Can we test its validity as a minimal model for the laser spikes?

\[
\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi} \left[ \sin(2\pi \varphi_i) + \alpha_c \sin(4\pi \varphi_i) \right] + D\zeta
\]
Comparing with synthetic neuronal spikes: good agreement

FHN model with Gaussian white noise and weak sinusoidal input: spikes are noise-induced

\[
\begin{align*}
\epsilon \frac{dx}{dt} &= x - \frac{x^3}{3} - y, \\
\frac{dy}{dt} &= x + a + a_o \cos(2\pi t/T) + D\xi(t),
\end{align*}
\]

_Aparicio-Reinoso et al, PRE (2016)_
Ordinal probabilities uncover the regions of noisy locking

0.8 % 1.6 %
(a) (b)

P(10)

Laser current

Modulation frequency

\[ \langle T \rangle \approx 2 T_{mod} \]

2:1
3:1
4:1

\( P(3210) \)

How to control the laser spikes?  
How to quantify the degree of entrainment?
Inter-spike time interval distribution as a function of the frequency of the current modulation

\[ I_{th, sol} = 26.62 \text{ mA} \]
\[ I_{th} = 24.70 \text{ mA} \]

\[ I_{dc} = 27 \text{ mA} (f_0=15 \text{ MHz}), A_{mod} = 2.3\% \text{ of } I_{dc} \]

\[ \Rightarrow \text{“refractory time” clear} \]
\[ \Rightarrow \text{“locking” horizontal} \]
We test three modulation waveforms and quantify locking with the **success rate** and the **false positive rate**

\[
SR(\tau) = \frac{\text{# of spikes emitted in the interval } \tau}{\text{# of modulation cycles}}
\]

\[
FPR(\tau) = \frac{\text{# spikes that are not emitted in the time interval } \tau}{\text{Total # of spikes}}
\]
Quantification

Success rate

Modulation frequency (MHz)

Time/$T_{mod}$
Waveform comparison: in color code the success rate (red SR=1)


⇒ pulse-down waveform produces a wider locking region
And the false positives? (the natural, uncontrolled spikes)

Receiver operating characteristic (ROC) curves

Modulation amplitude %

Locked-unlocked transitions when the modulation frequency increases

\[ J. \text{Tiana et al., arXiv:1806.08950v1 (2018)} \]
Role of the laser current (controls the natural spike rate)

Success Rate vs. False positive rate for different laser currents (25 mA, 26 mA, 27 mA, 28 mA). The graphs show the Modulation and Amplitude percentage for each current level.
Influence of the modulation waveform

Pulsed-up

Sinusoidal

Pulsed-down
What did we learn?

- Transition to optical chaos: ordinal analysis distinguishes different regimes.
- Spike patterns that are more/less expressed are not always detected by correlation analysis.
- Minimal model identified (a modified circle map).
- Good agreement between optical & neuron (synthetic) spikes.
- ROC curves allow to quantify the entrainment quality.
- Regions of perfect 1:1 locking identified.

Ongoing work: potential for sensing applications?

To do in the future: The connection with the circle map needs to be explored.
Thank you for your attention

http://www.fisica.edu.uy/~cris

A. Aragoneses et al., Opt. Express 22, 4705 (2014)
T. Sorrentino et al., JSTQE 21, 1801107 (2015)
J.M. Aparicio-Reinoso et al., PRE 94, 032218 (2016)
M. Panozzo et al., Chaos 27, 114315 (2017)