

Optimal Entrainment of the Power Dropouts of a Semiconductor Laser with Optical Feedback to Current Modulation

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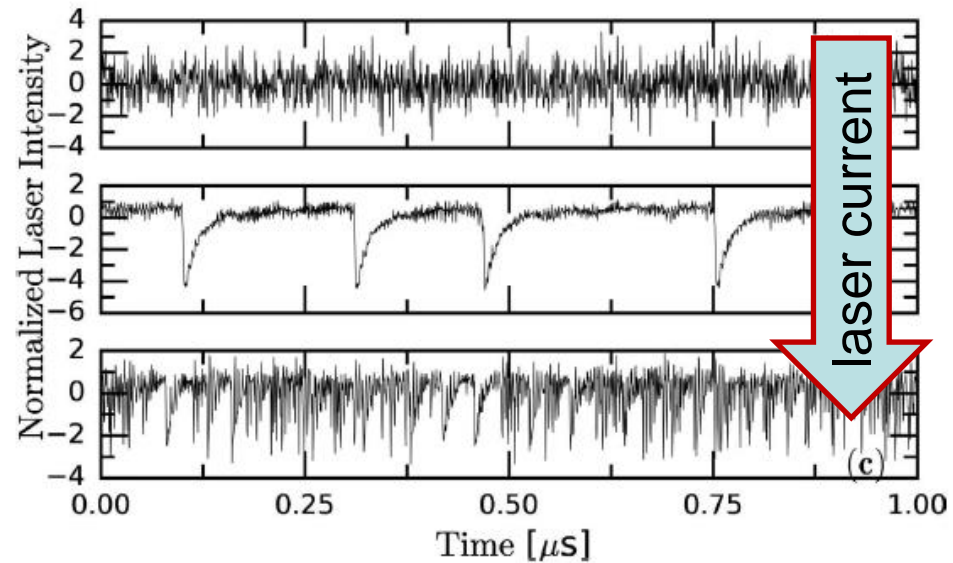
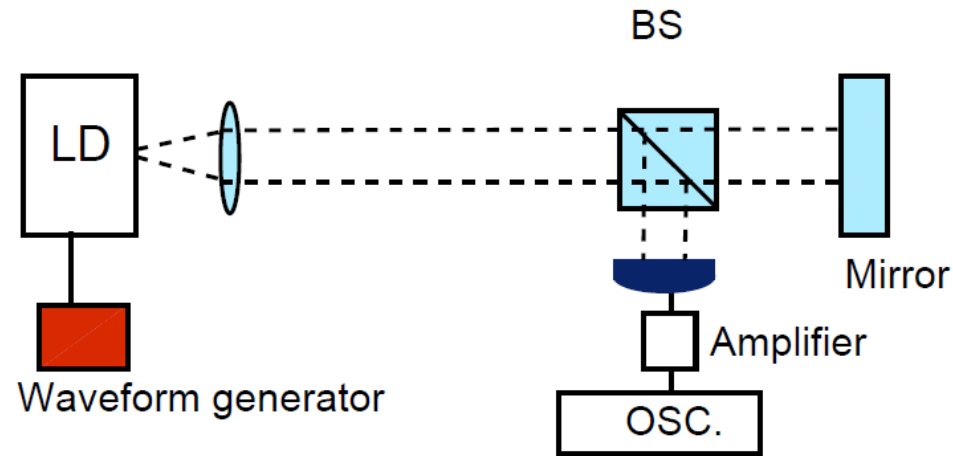


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- Characterization of optical spikes
- Control via small electric perturbations

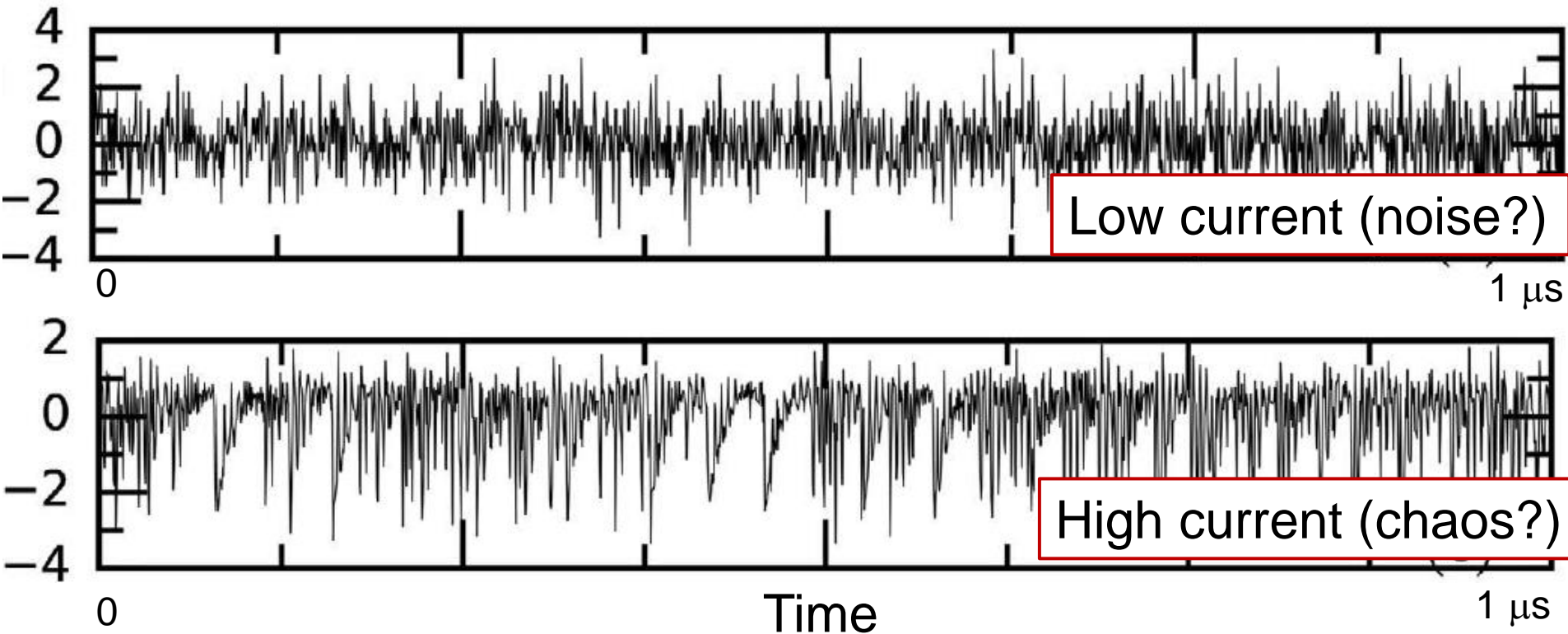
Dynamical transitions in a semiconductor laser with optical feedback



Video: [how complex signals emerge from optical noise](#)

Different dynamical regimes are difficult to distinguish.

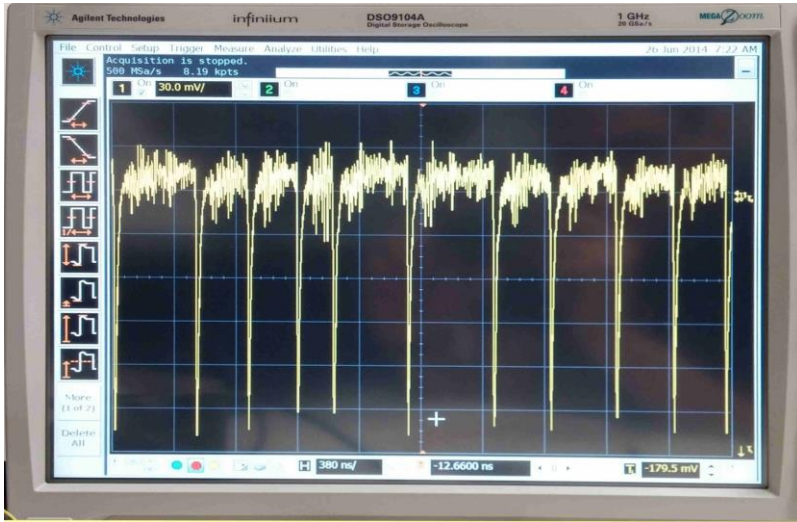
Laser output intensity



Can differences be quantified? With what reliability?

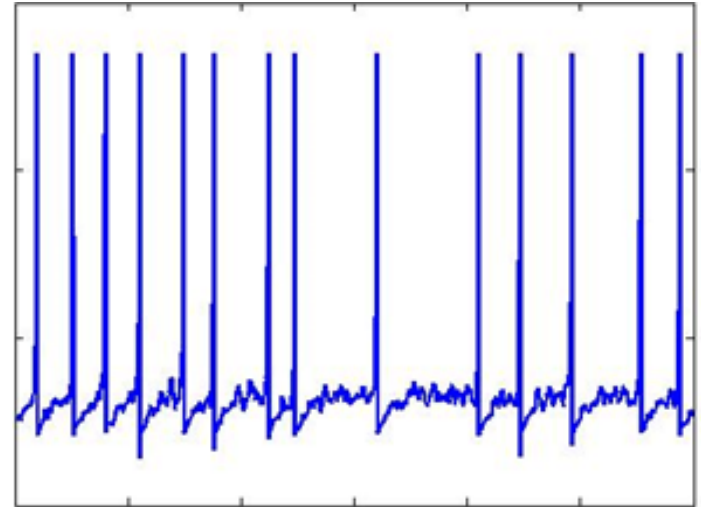
How similar these time series are?

Optical spikes



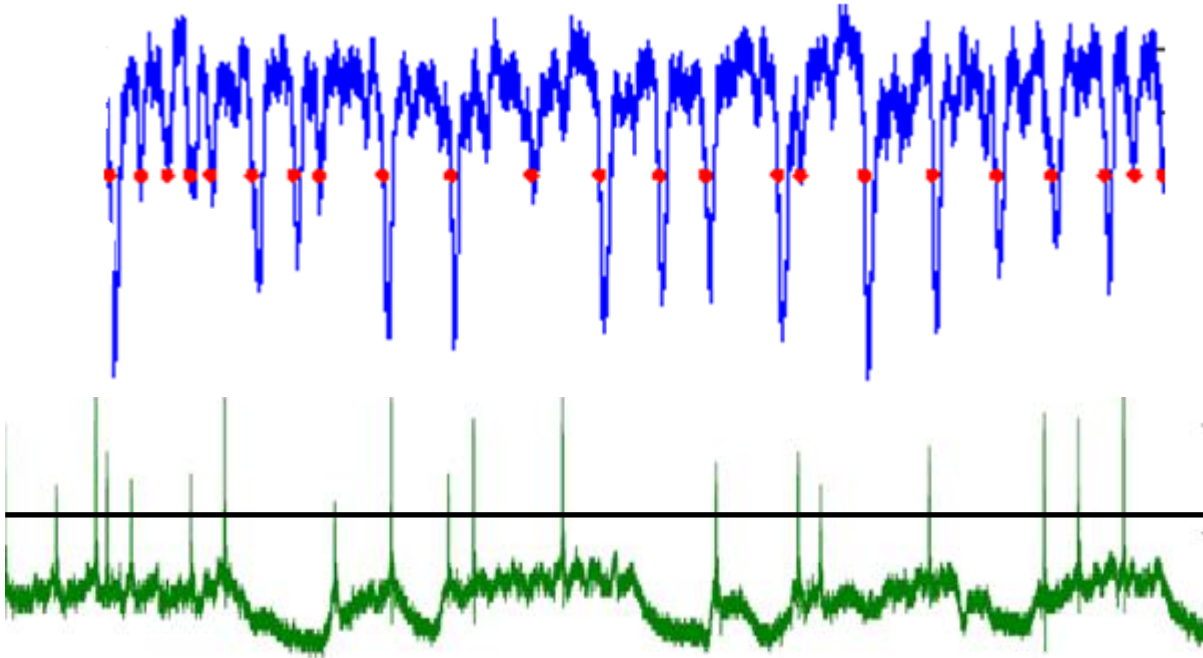
Time

Neuronal spikes



Time

Threshold crossings define ``events'' in a time series



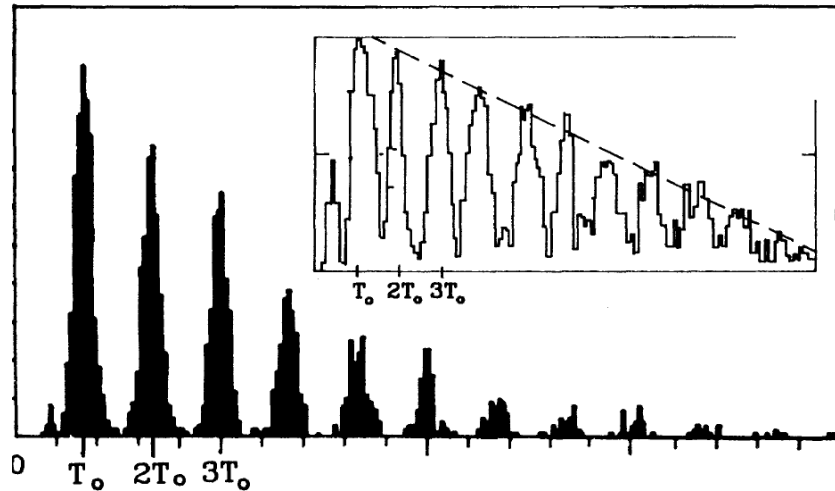
- Problems:
 - How to select the threshold?
 - Threshold dependent results?

inter-spike-intervals (ISIs):

$$\Delta T_i = t_{i+1} - t_i$$

ISI distribution indicates that neurons and lasers have a similar response to external periodic forcing

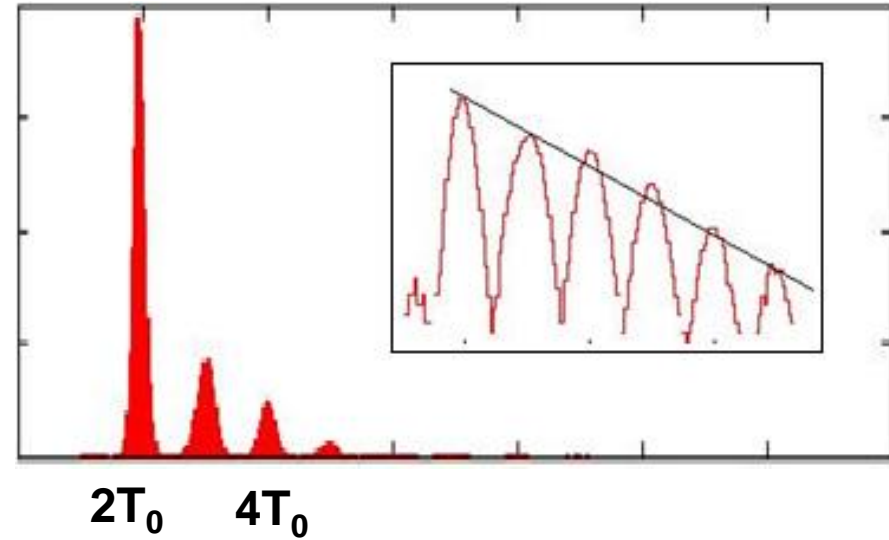
Neuron data



Single auditory nerve fiber of a squirrel monkey with a sinusoidal sound stimulus applied at the ear.

A. Longtin et al PRL (1991)

Laser data

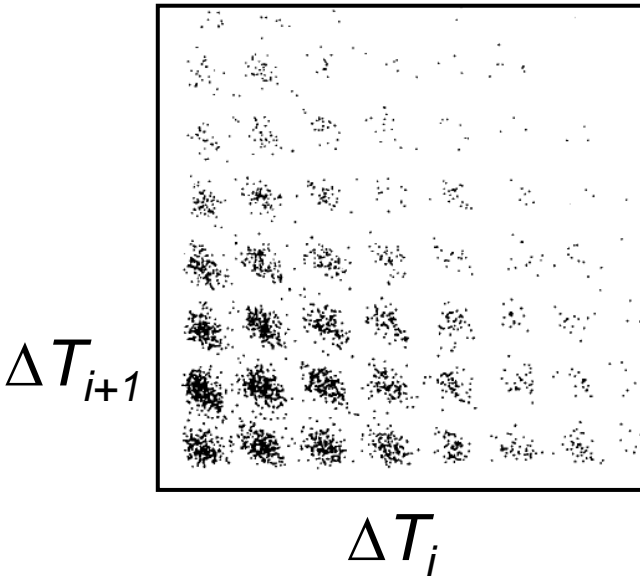


Data recorded in our lab when a sinusoidal signal is applied to the laser current.

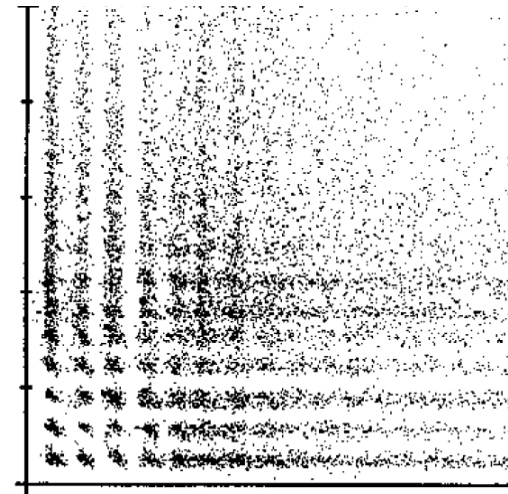
*A. Aragoneses et al
Optics Express (2014)*

Return maps also suggest that neurons and lasers have similar response to external periodic forcing

Neuronal ISIs



Laser ISIs



- How to identify temporal order?
- Are there more or less expressed “spike patterns” ?

A. Longtin
Int. J. Bif. Chaos (1993)

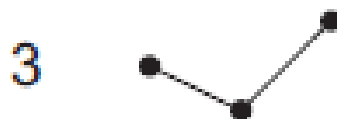
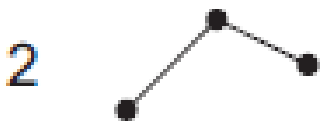
M. Giudici et al *PRE* (1997)
A. Aragonese et al
Optics Express (2014)

Different methods of time series analysis provide complementary information

- Many methods
 - Correlation analysis
 - Fourier analysis
 - Lyapunov & fractal analysis
 - Symbolic analysis
 - Wavelet analysis
 - Etc. etc.
- The method to be used depends on the data
 - Length
 - Noise
 - Resolution
 - Etc.

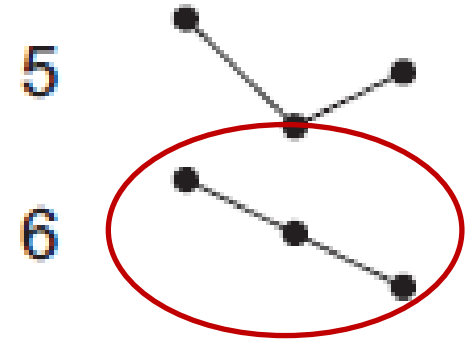
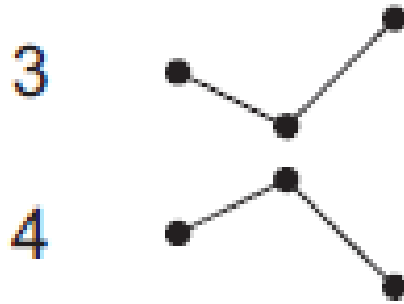
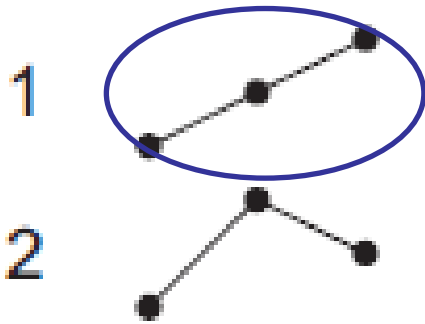
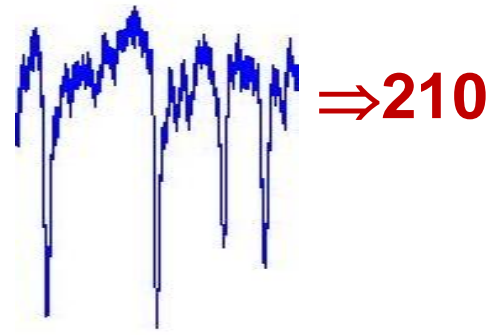
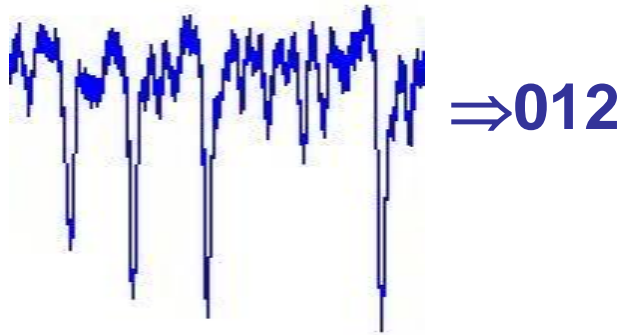
Ordinal analysis: a tool to look for patterns in data

- Consider a time series $X(t) = \{\dots, X_i, X_{i+1}, X_{i+2}, \dots\}$
- Which are the possible order relations among three data points?

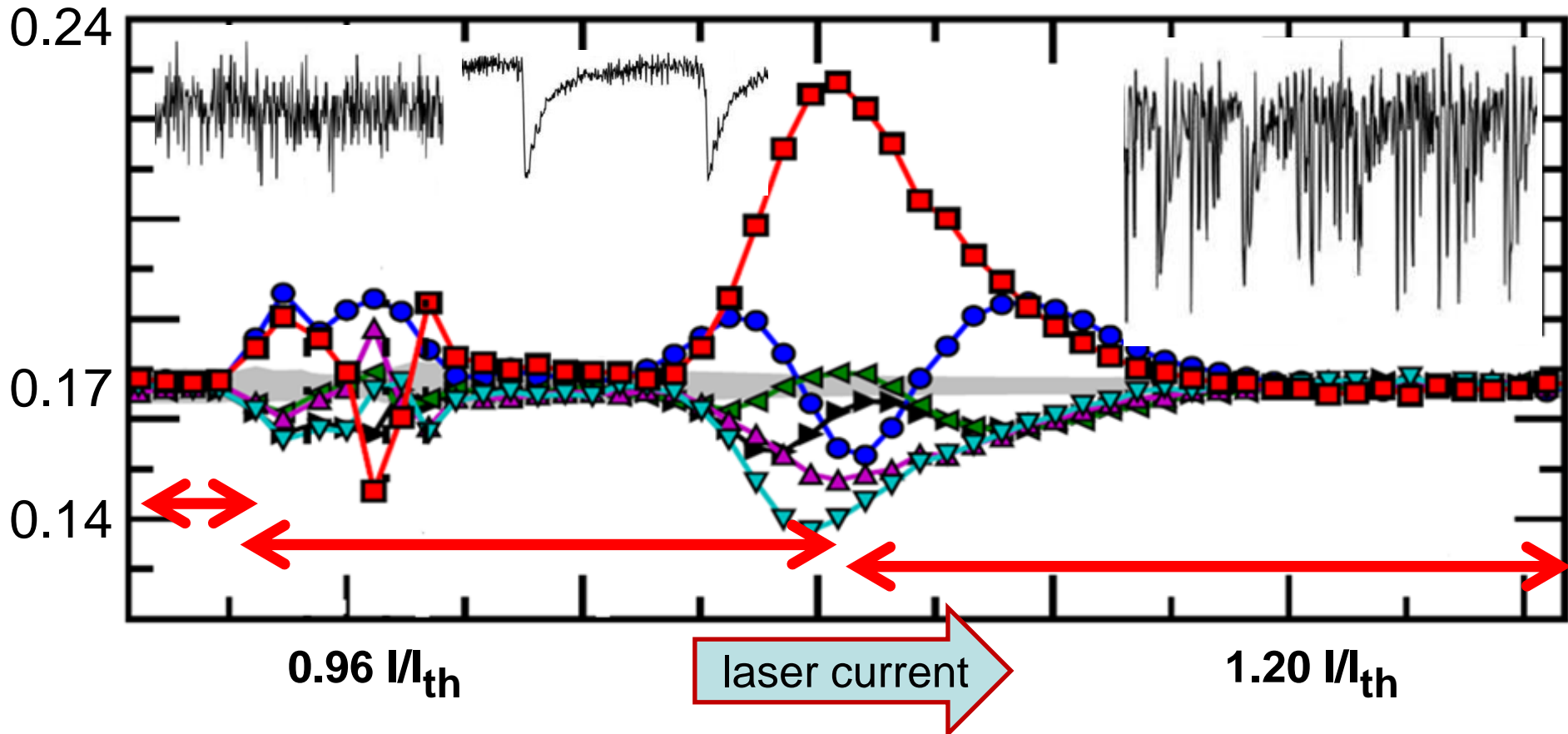


- Count how many times each “ordinal pattern” appears.
- Advantages: allows to identify temporal structures & is robust to noise.
- Drawback: information about actual data values is lost.

Ordinal analysis of inter-spike intervals

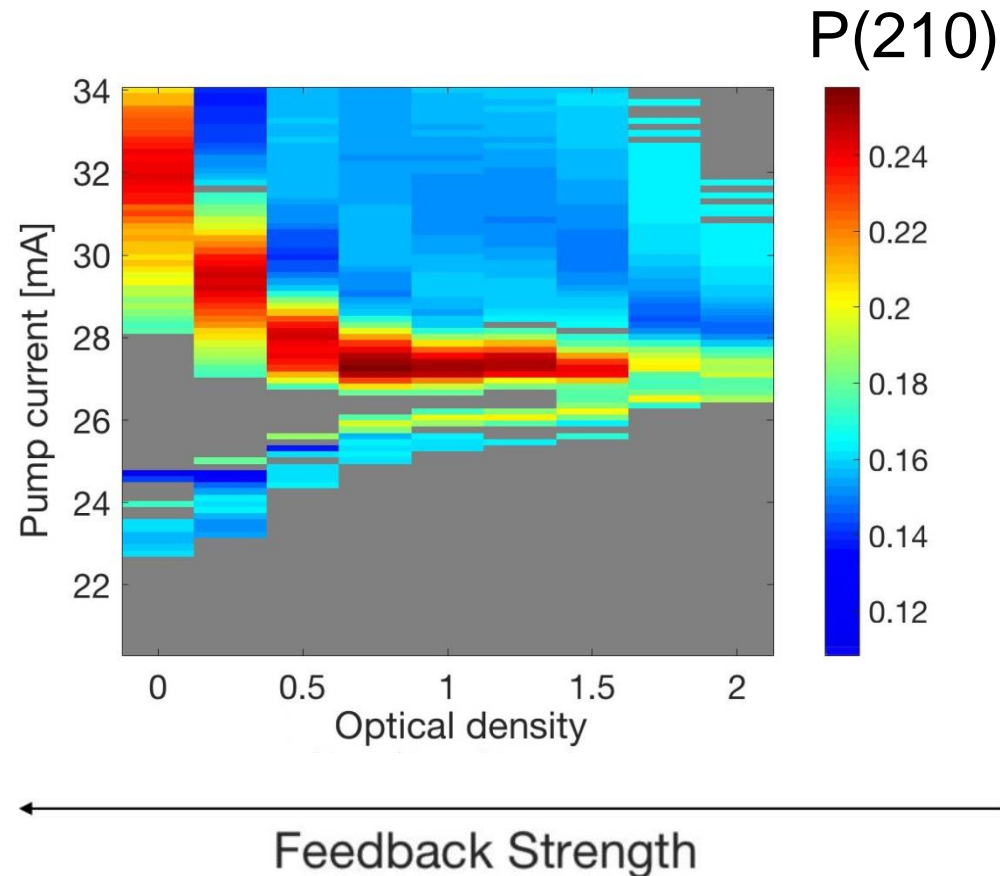


Ordinal analysis identifies the onset of different dynamical regimes, but does not distinguish “noise” and “chaos”



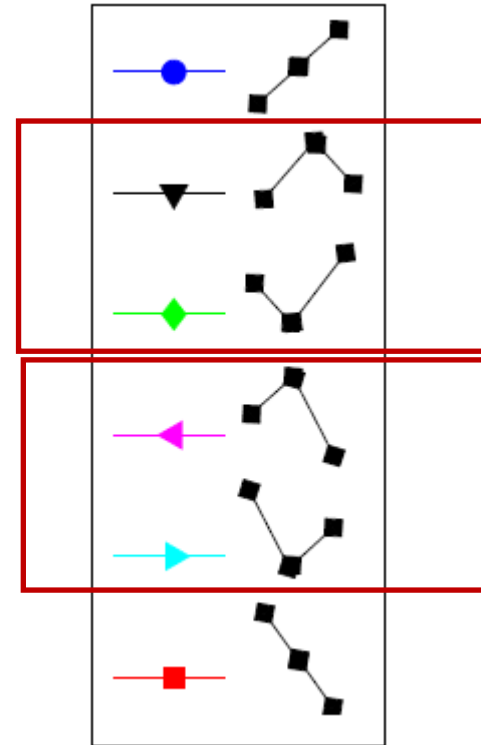
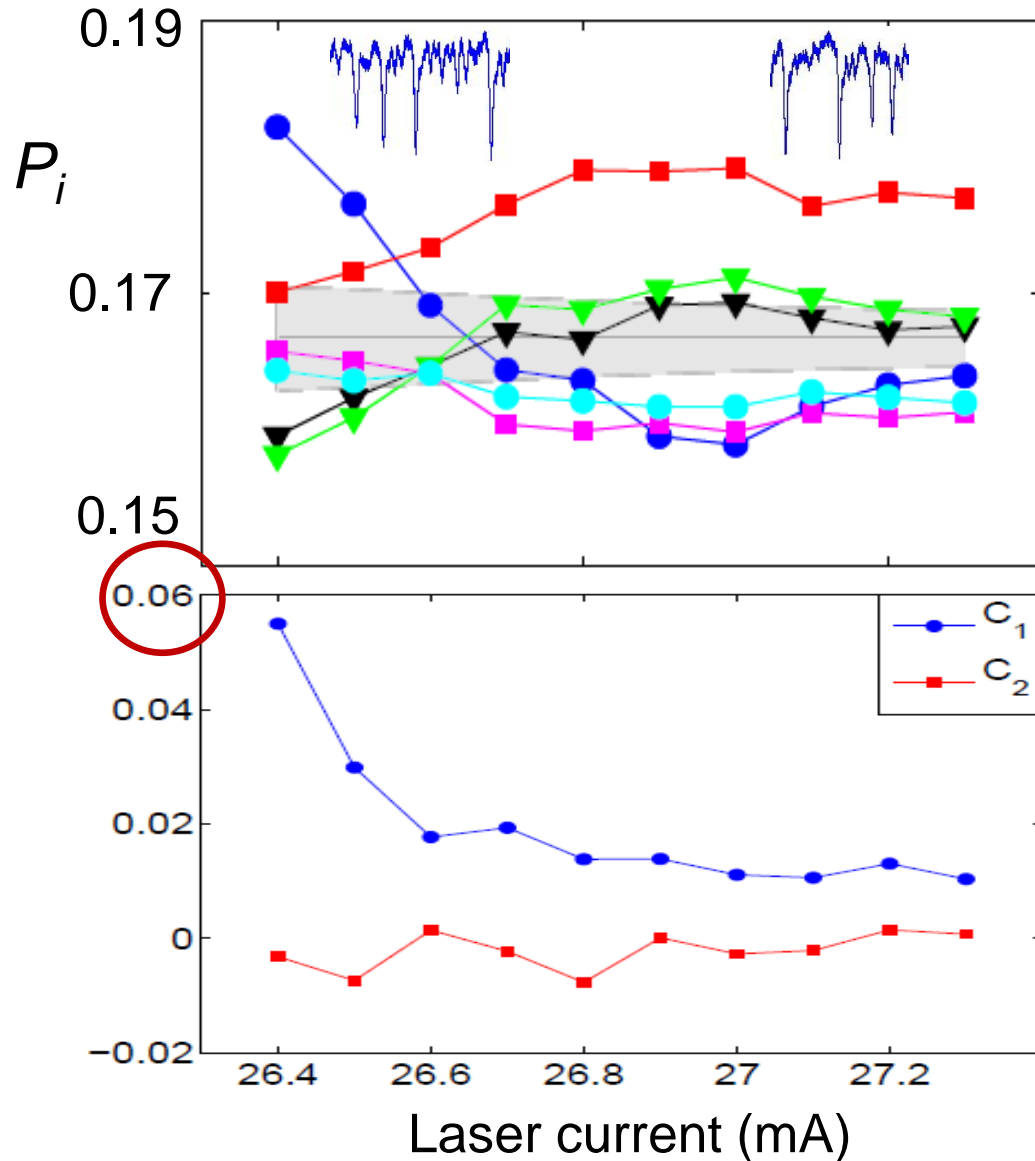
Grey region: probabilities are consistent with the uniform distribution ($P_i = 1/6 \cong 0.17 \forall i$) with 99.7% confidence level

P(210) identifies dynamical regimes in parameter space (pump current, feedback strength)



M. Panozzo et al, Chaos (2017)

Zooming into the region where spikes are well-defined, a transition is detected (not captured by correlation analysis)



$$C_\tau = \frac{\langle (\Delta T_i - \langle \Delta T \rangle) (\Delta T_{i-\tau} - \langle \Delta T \rangle) \rangle}{\sigma^2}$$

*A. Aragonese et al.,
Scientific Reports (2014)*

A modified circle map: simple minimal model

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi} [\sin(2\pi\varphi_i) + \alpha_c \sin(4\pi\varphi_i)] + D\zeta$$

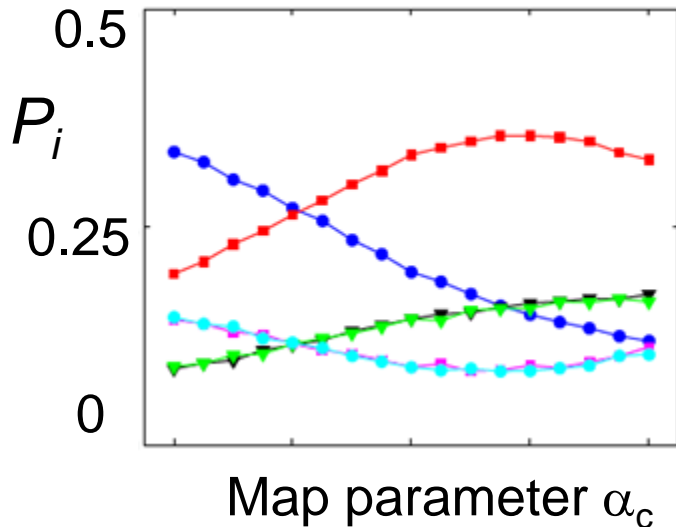
$$X_i = \varphi_{i+1} - \varphi_i$$

ρ = $\frac{\text{natural frequency}}{\text{forcing frequency}}$

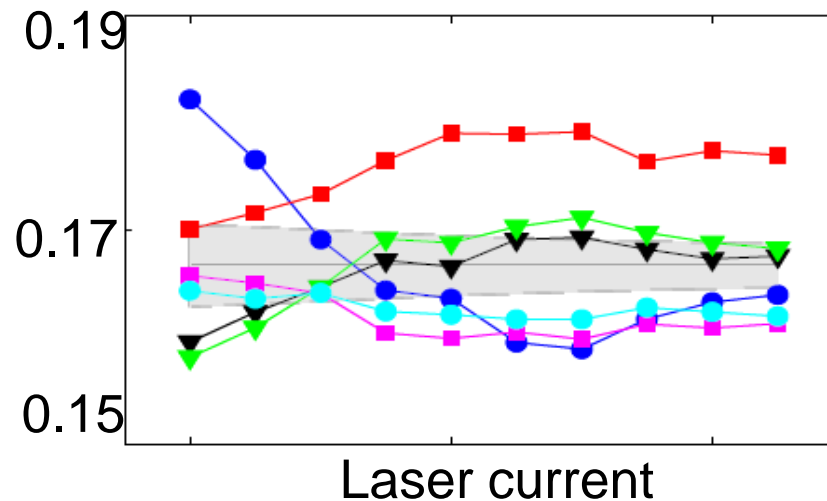
K = forcing amplitude

D = noise strength

Circle map data



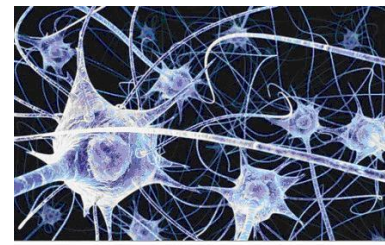
Empirical laser data



Same “clusters” & same hierarchical structure

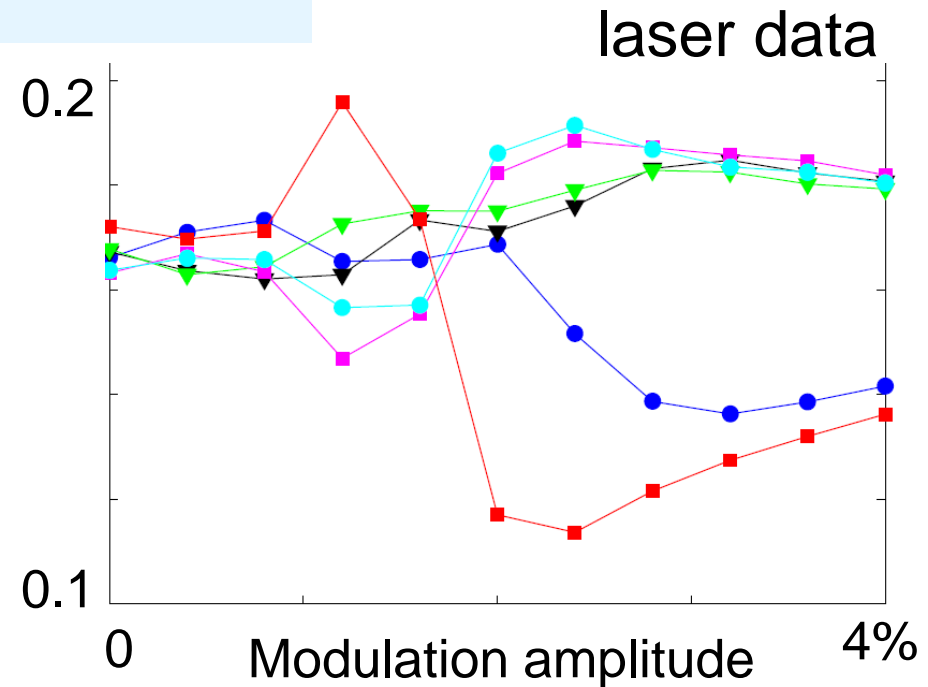
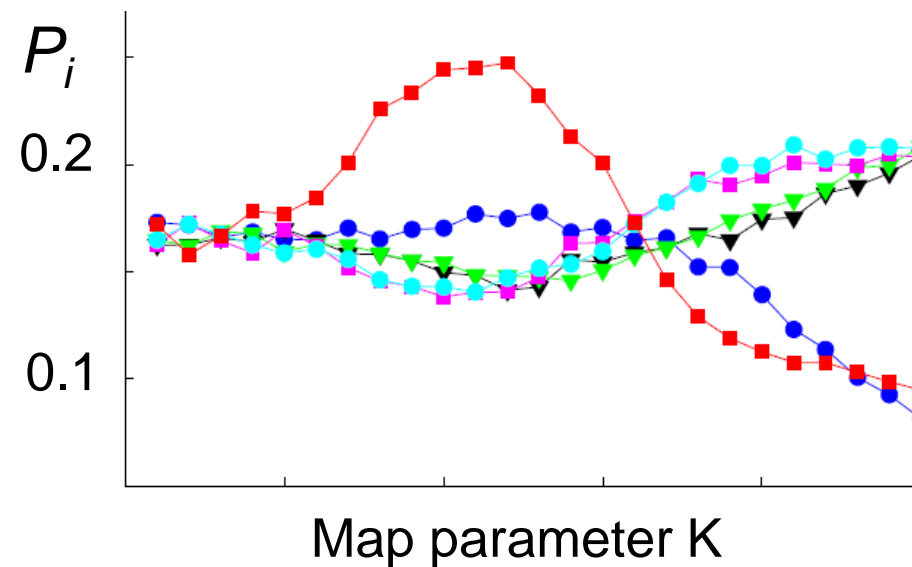
A. Aragonese et al Scientific Reports (2014)

Connection with neurons: the circle map describes many excitable systems



- The modified map describes spike correlations in sensory neurons (Neiman and Russell, PRE 2005)
- Can we test its validity as a minimal model for the laser spikes?

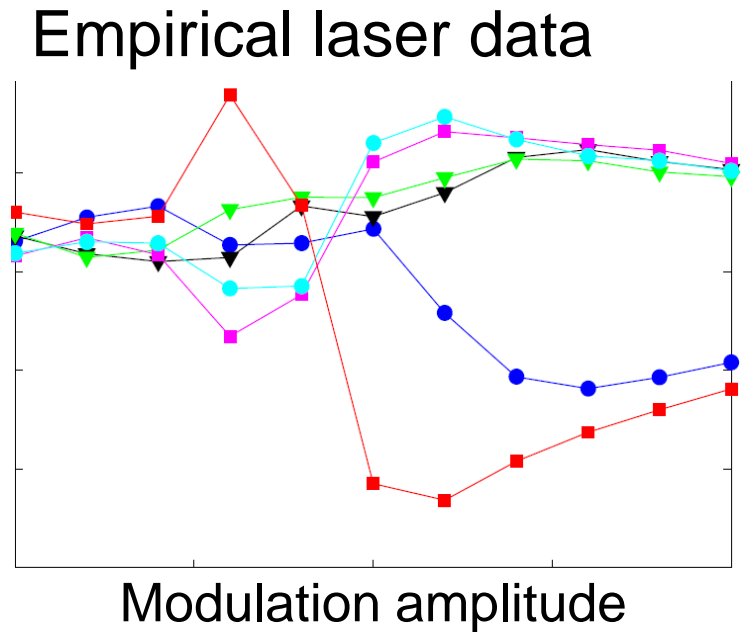
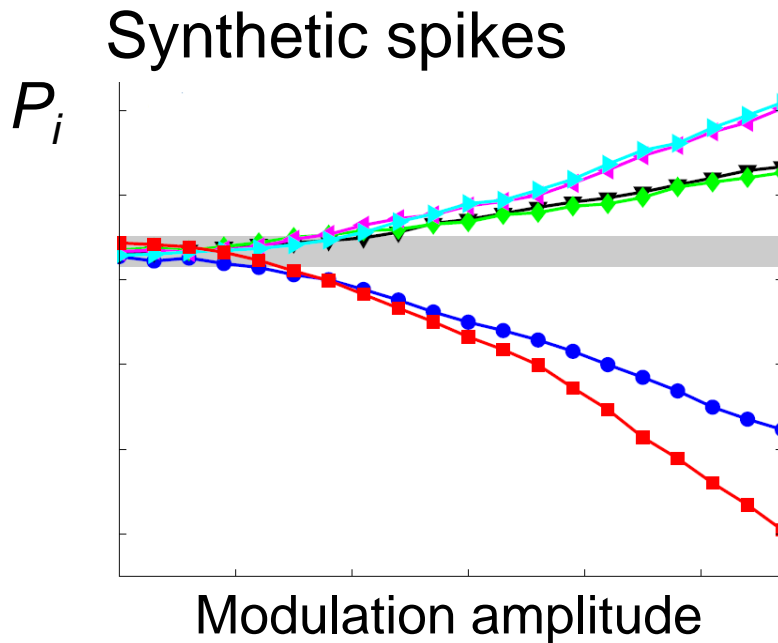
$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi} [\sin(2\pi\varphi_i) + \alpha_c \sin(4\pi\varphi_i)] + D\zeta$$



Comparing with synthetic neuronal spikes: good agreement

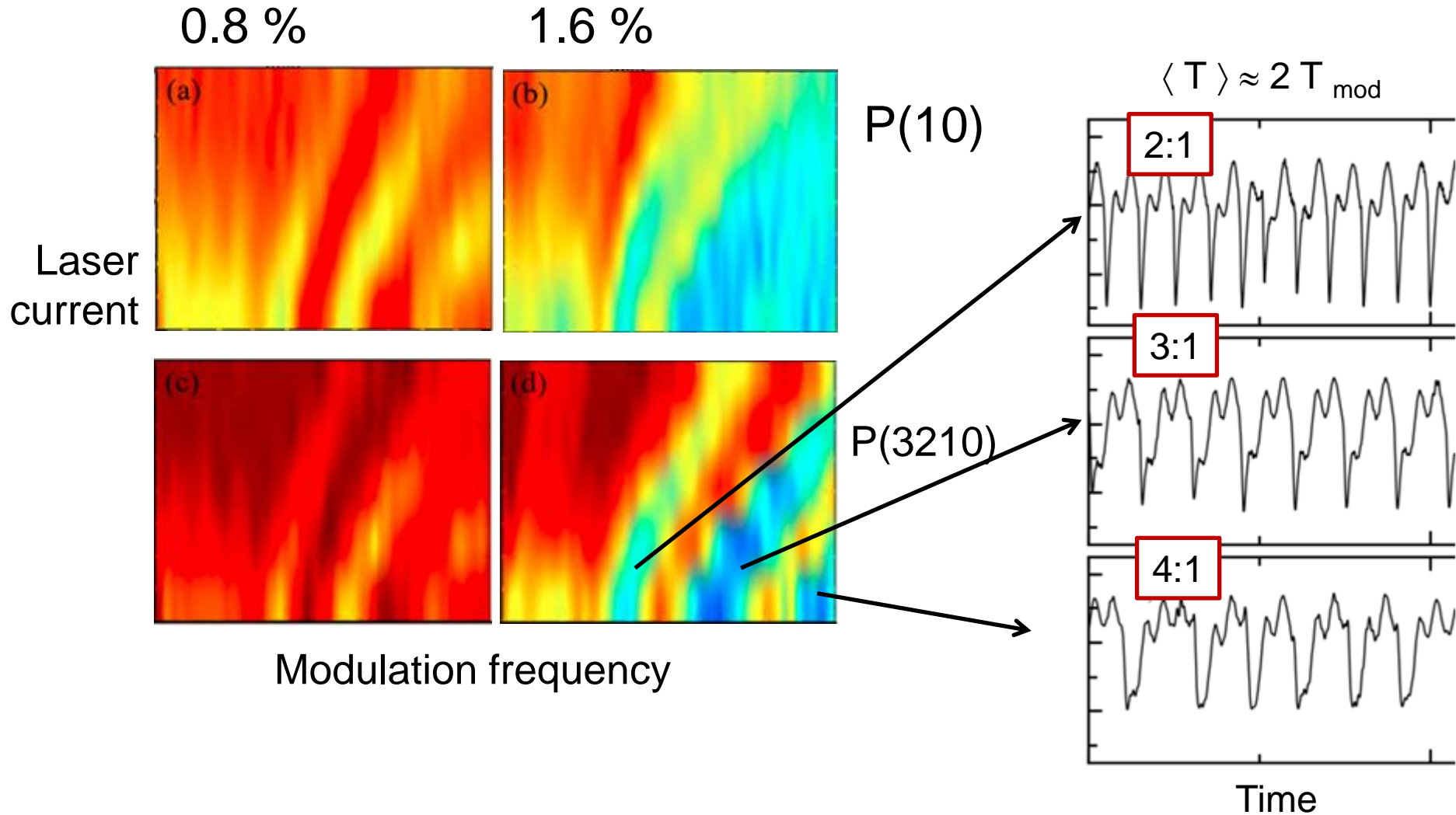
FHN model with Gaussian white noise and **weak** sinusoidal input: spikes are noise-induced

$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o \cos(2\pi t/T)} + D\xi(t),$$



Aparicio-Reinoso et al, PRE (2016)

Ordinal probabilities uncover the regions of noisy locking



How to *control* the laser spikes? How to *quantify* the degree of entrainment?



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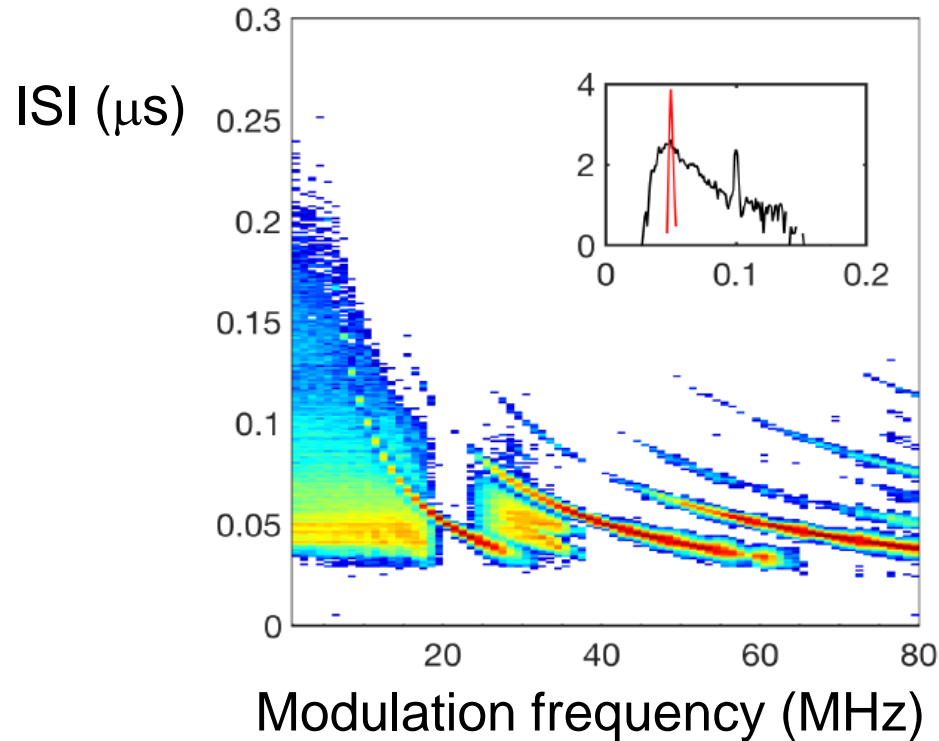
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Inter-spike time interval distribution as a function of the frequency of the current modulation

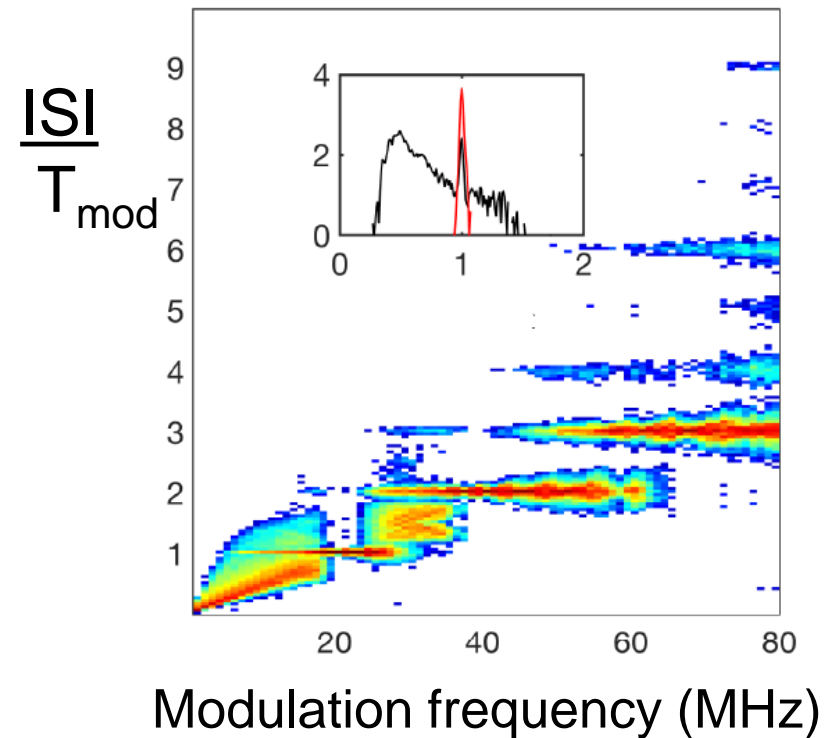
$$I_{th, sol} = 26.62 \text{ mA}$$

$$I_{dc} = 27 \text{ mA} (f_0 = 15 \text{ MHz}), A_{mod} = 2.3\% \text{ of } I_{dc}$$

$$I_{th} = 24.70 \text{ mA}$$

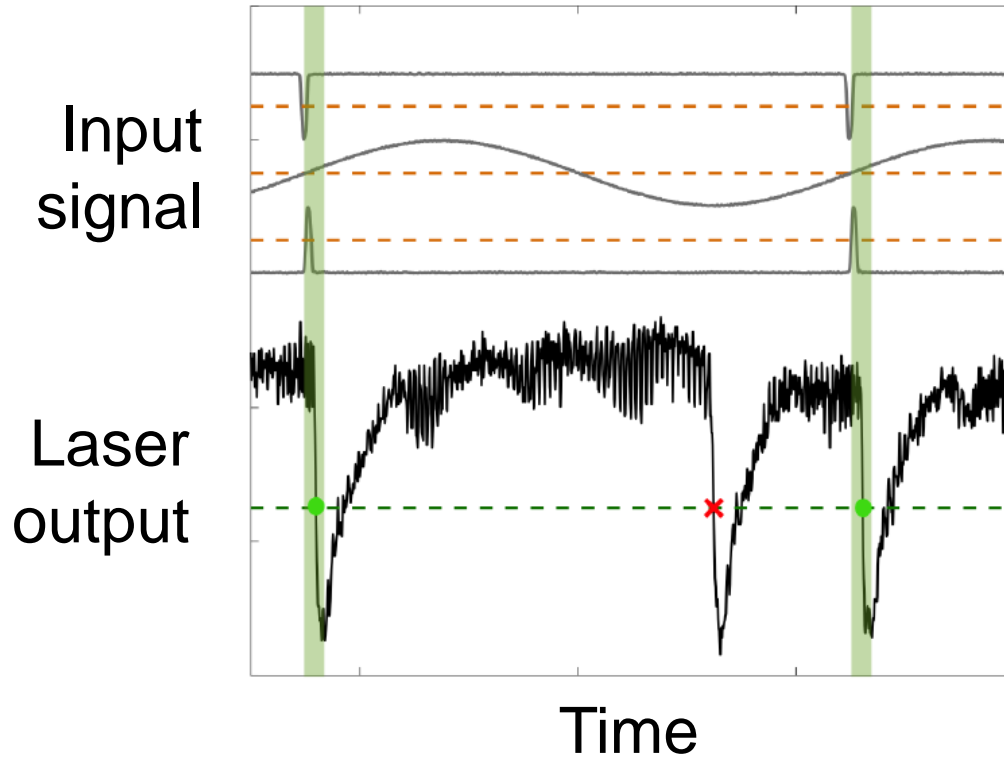


\Rightarrow “refractory time” clear



\Rightarrow “locking” horizontal

We test three modulation waveforms and quantify locking with the success rate and the false positive rate

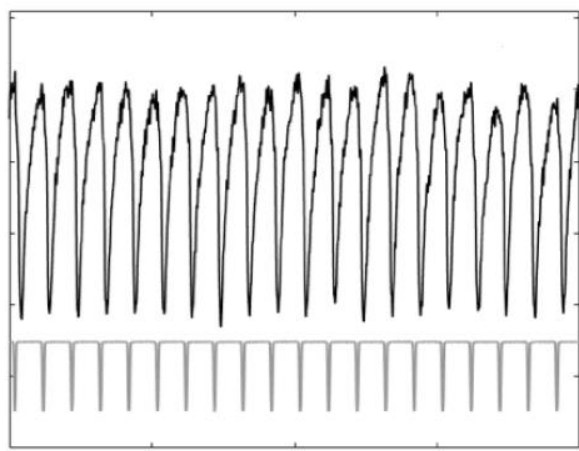
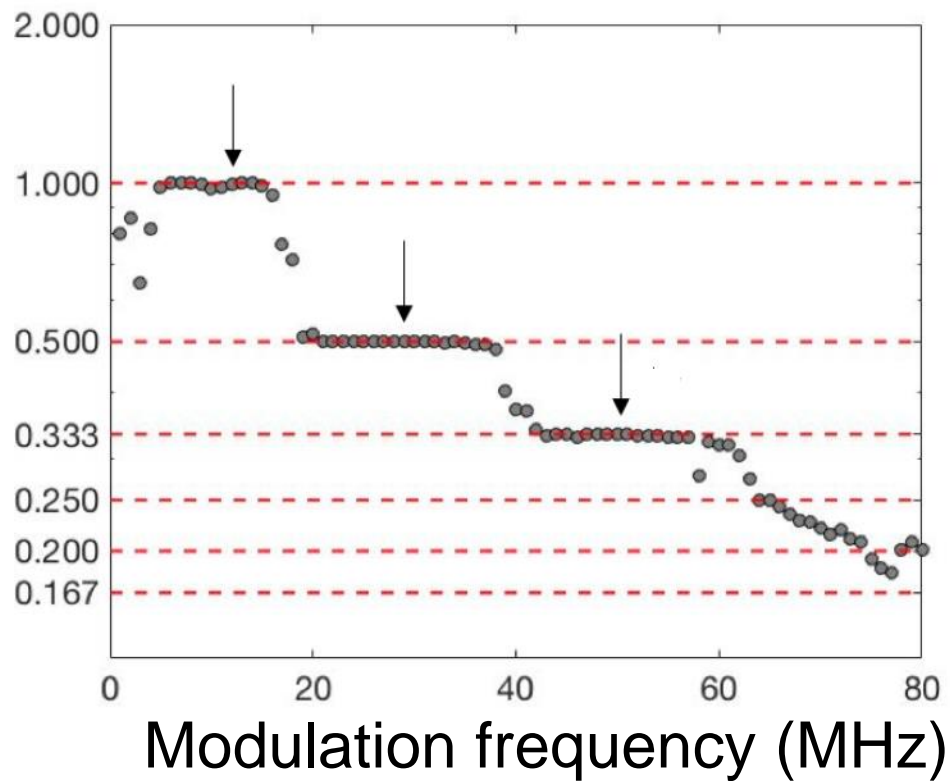


$$SR(\tau) = \frac{\# \text{ of spikes emitted in the interval } \tau}{\# \text{ of modulation cycles}}$$

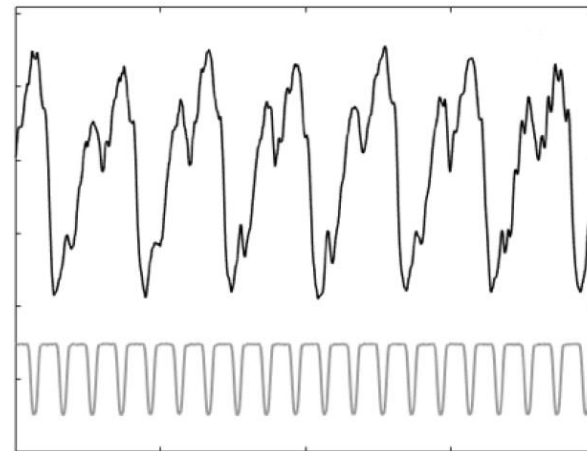
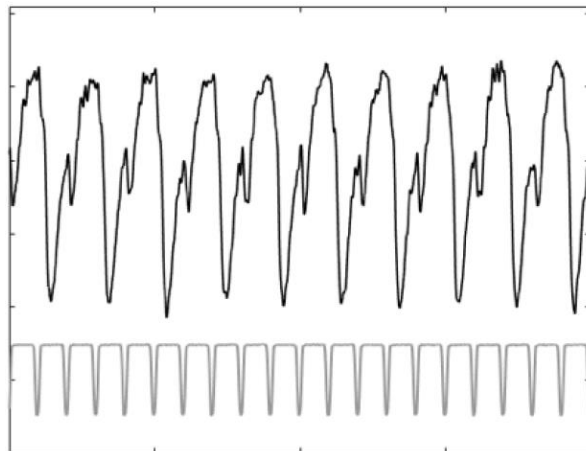
$$FPR(\tau) = \frac{\# \text{ spikes that are not emitted in the time interval } \tau}{\text{Total } \# \text{ of spikes}}$$

Quantification

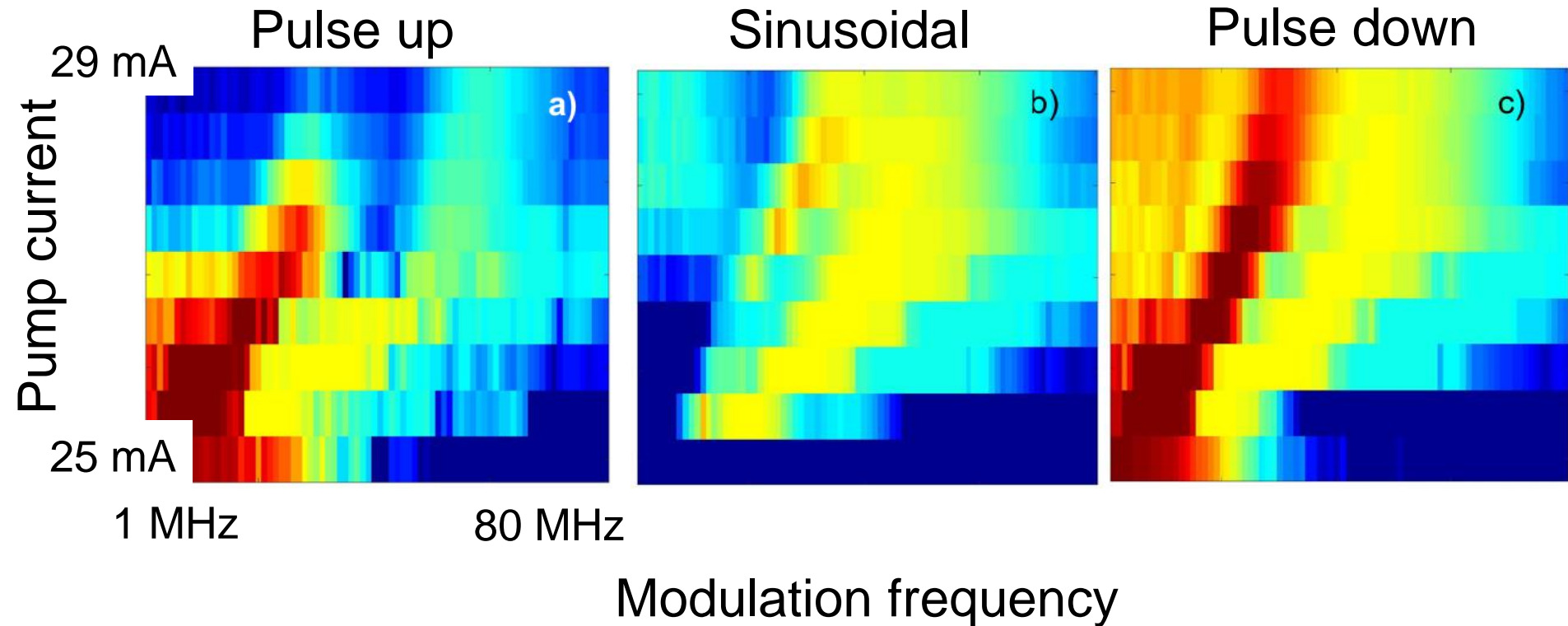
Success
rate



Time/ T_{mod}



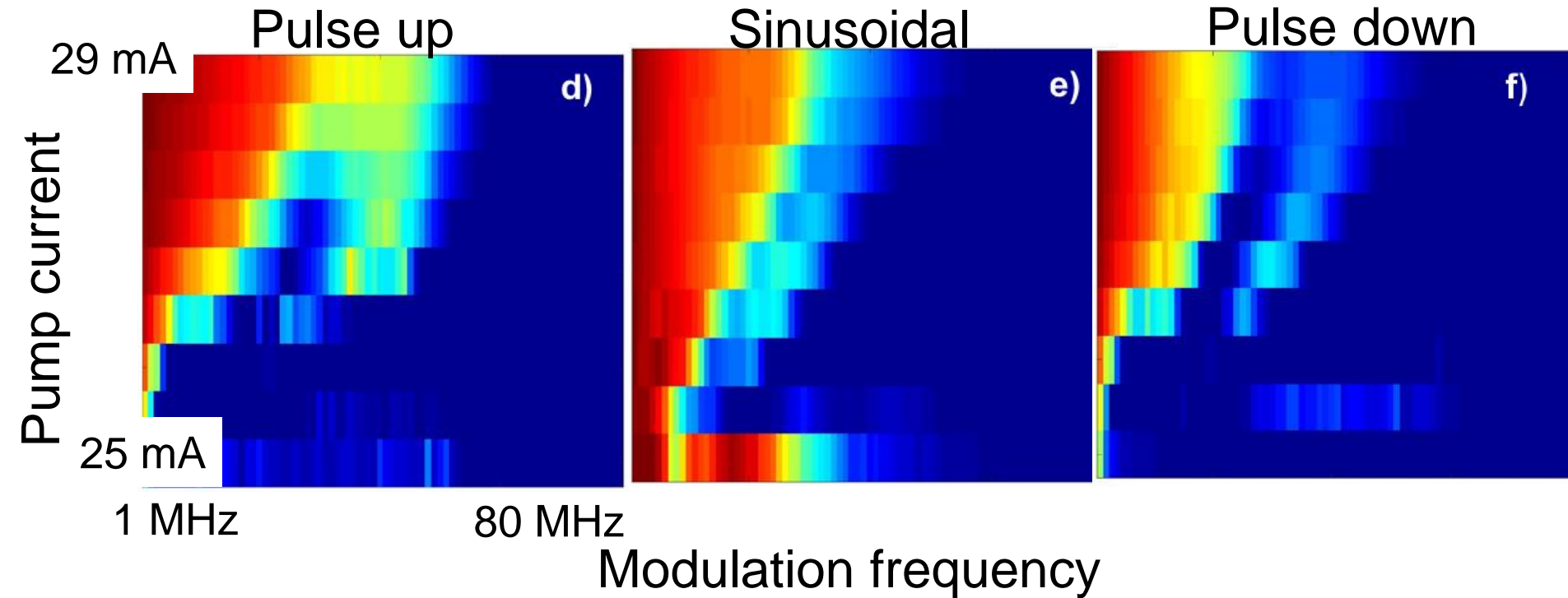
Waveform comparison: in color code the success rate (red SR=1)



⇒ pulse-down waveform produces a wider locking region

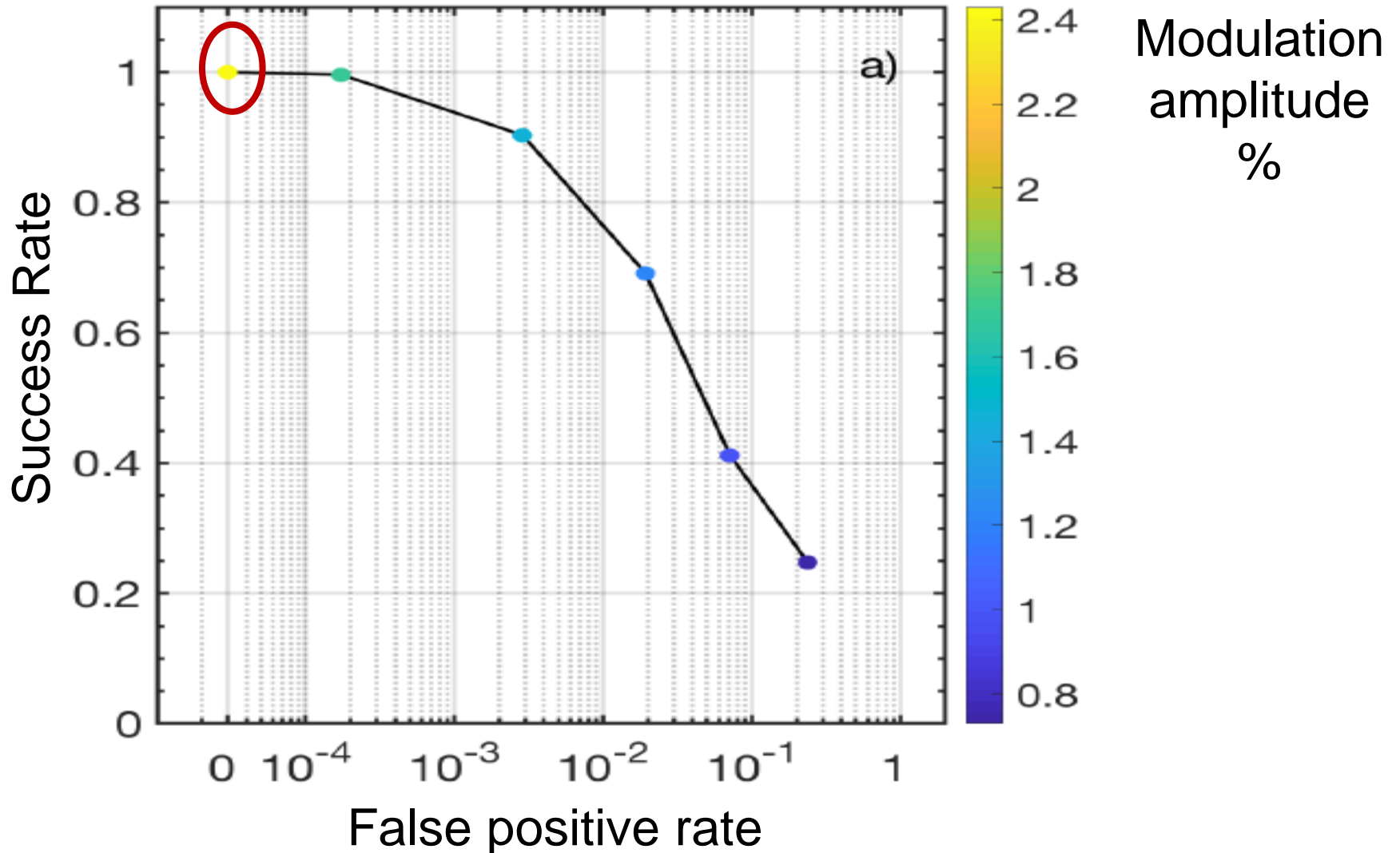
J. Tiana et al., Opt. Express 26 9298 (2018)

And the false positives? (the natural, uncontrolled spikes)

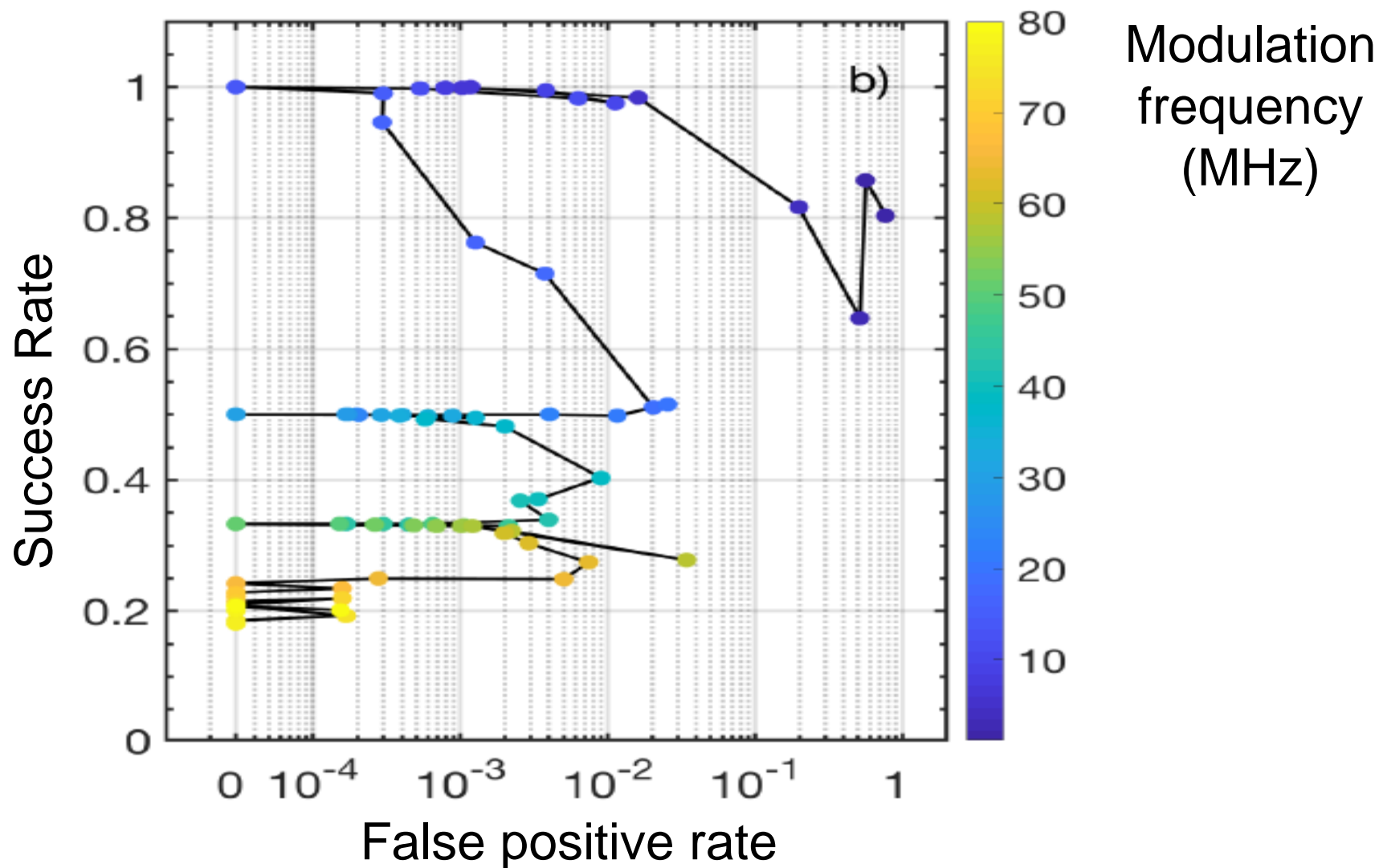


J. Tiana et al., Opt. Express 26, 9298 (2018)

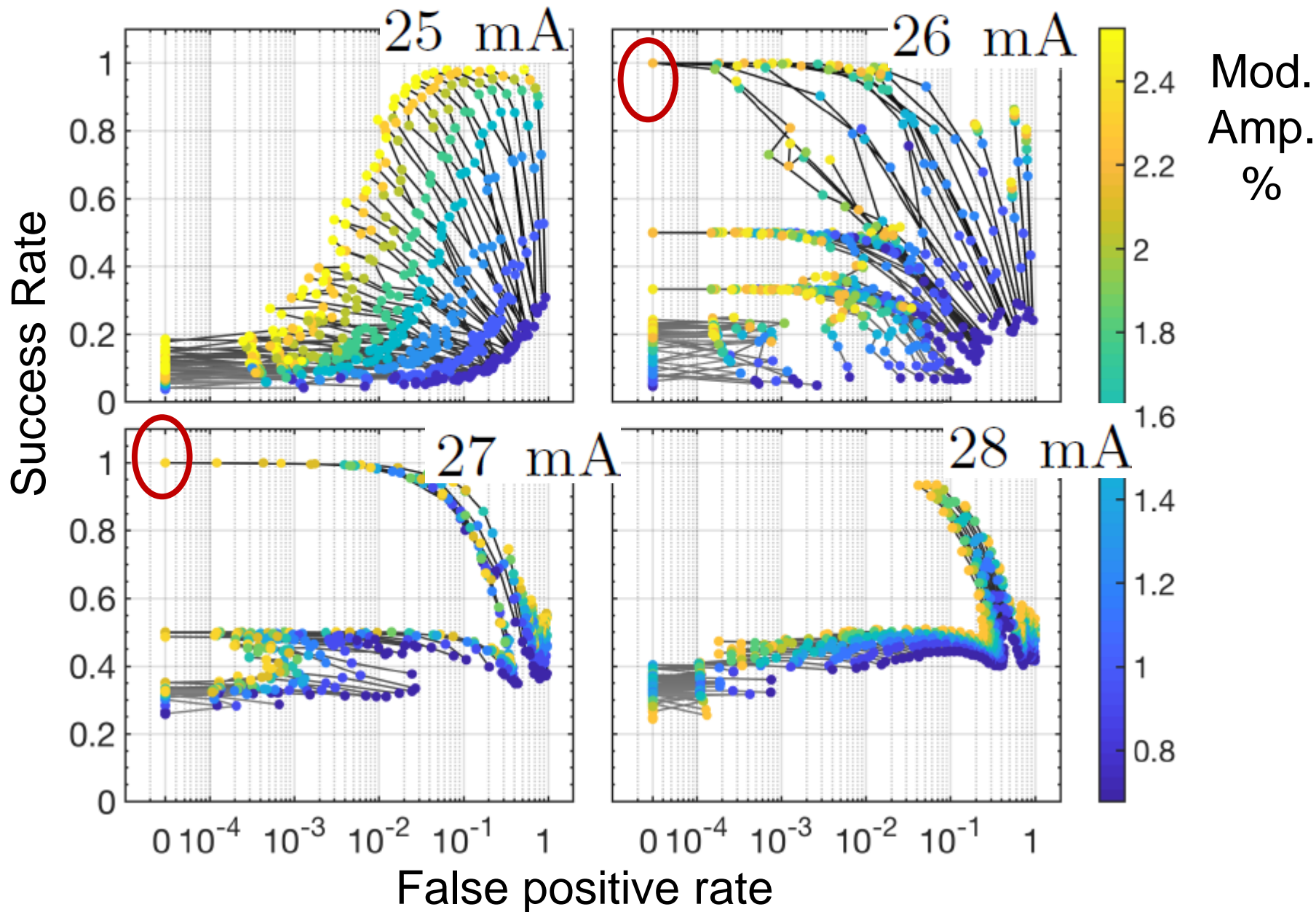
Receiver operating characteristic (ROC) curves



Locked-unlocked transitions when the modulation frequency increases

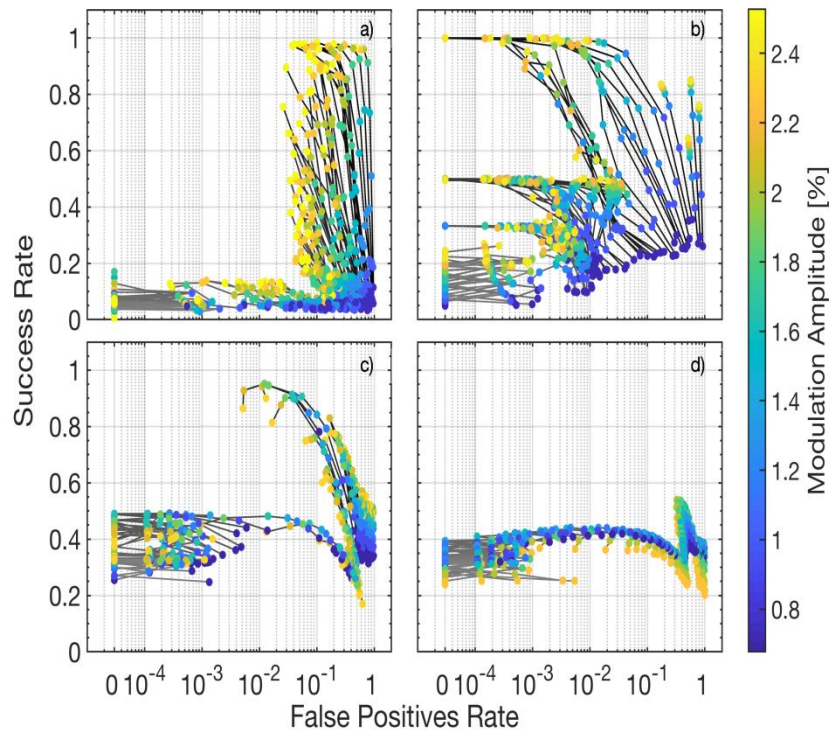


Role of the laser current (controls the natural spike rate)

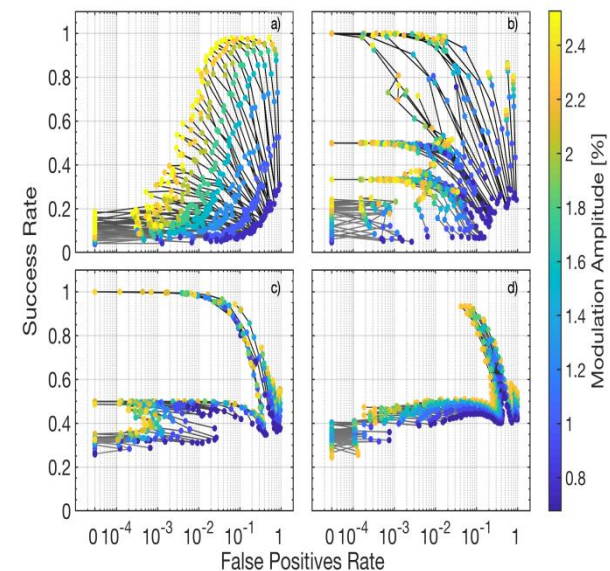


Influence of the modulation waveform

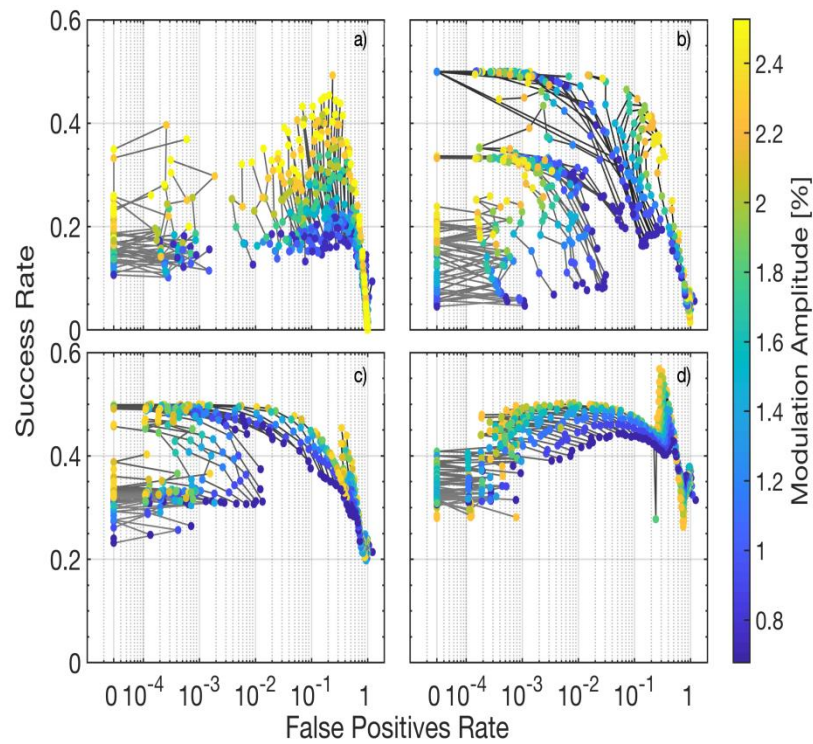
Pulsed-up



Pulsed-down



Sinusoidal



What did we learn?

- Transition to optical chaos: ordinal analysis distinguishes different regimes.
- Spike patterns that are more/less expressed are not always detected by correlation analysis.
- Minimal model identified (a modified circle map).
- Good agreement between optical & neuron (synthetic) spikes.
- ROC curves allow to quantify the entrainment quality.
- Regions of perfect 1:1 locking identified.

Ongoing work: potential for sensing applications?

To do in the future: The connection with the circle map needs to be explored.



Thank you for your attention

<http://www.fisica.edu.uy/~cris>

- A. Aragoneses et al., Opt. Express 22, 4705 (2014)
- A. Aragoneses et al., Sci. Rep. 4, 4696 (2014)
- T. Sorrentino et al., JSTQE 21, 1801107 (2015)
- C. Quintero-Quiroz et al., Sci. Rep. 6, 37510 (2016)
- J.M. Aparicio-Reinoso et al., PRE 94, 032218 (2016)
- M. Panozzo et al., Chaos 27, 114315 (2017)
- J. Tiana et al., Opt. Express 26, 9298 (2018)
- J. Tiana et al., arXiv:1806.08950v1 (2018)