



Transitions of determinism and stochasticity in time-delayed complex systems with modulation

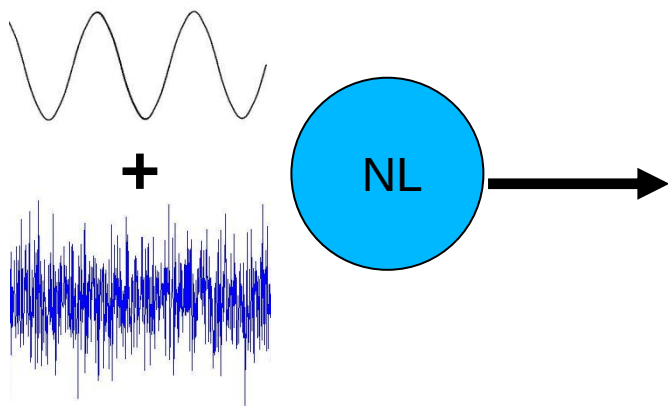
Andrés Aragoneses, Sandro Perrone, Taciano Sorrentino,
Maria Carme Torrent & Cristina Masoller

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Stochastic resonance (periodic, aperiodic)

Ghost resonance (missing fundamental illusion)

Entrainment, synchronization, ...



Laser dynamics

Detection of complex signals

Sensors improvement

Neuronal networks dynamics

Human brain

Animal senses

[A. Neiman, X. Pei, W. Wojtenek, L. Wilkens, F. Moss, H. A. Braun, M. T. Huber and K. Voigt, Phys Rev. Lett., **82**, (1999)]

[L. Gammaitoni, P. Hänggi, P. Jung and F. Marchesoni, Rev. Mod. Phys., **70**, 223 (1998)]

[D. R. Chialvo, Chaos, **13**, (2003)]

[J. Cartwright, D.L. González, O. Piro, Phys. Rev. Lett., **82**, 5389 (1999)]

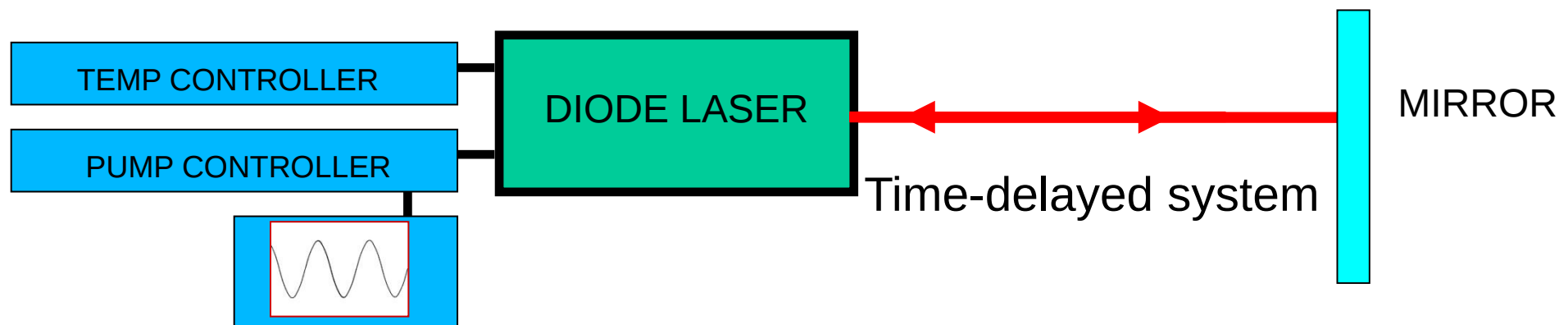
GOAL:

- Characterize the delayed-feedback induced complex dynamics under the influence of an external periodic forcing, at an event-level description.
- Verify an existing model for this type of systems at this level of description.

USING:

- Symbolic time series analysis.

- Applications: fiber optics telecommunications, optical data storage, life science, material processing, sensing, ...
- Non-linear devices easily perturbed by optical feedback.
- Feedback & noise (spontaneous emission) induce complex stochastic dynamics.
- Stochastic resonance, ghost resonance, entrainment, synchronization, ...



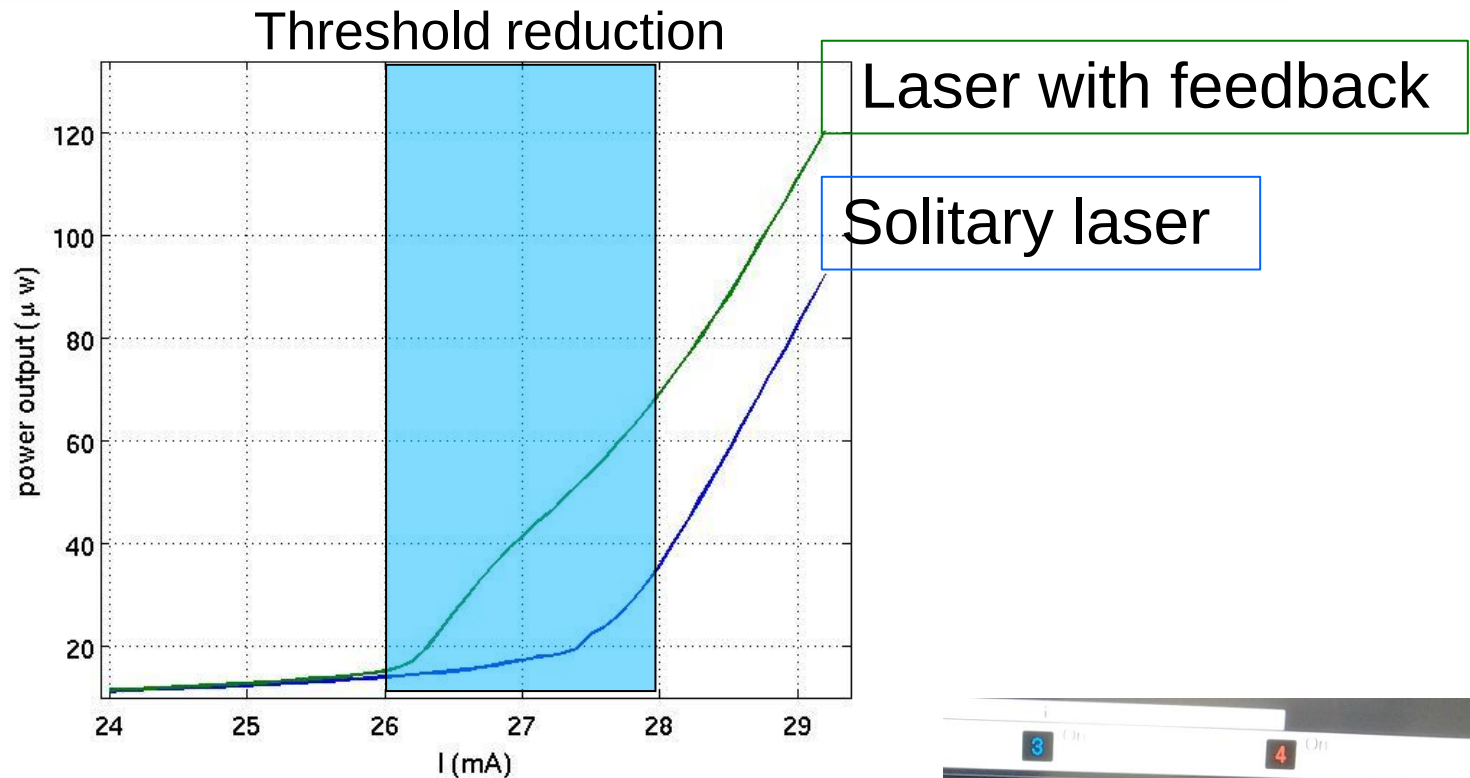
[D. W. Sukow & D. J. Gauthier, IEEE Jour. Quan. Elec., **36**, 175 (2000)]

[J. M. Buldú, J. García-Ojalvo, C.R. Mirasso and M. C. Torrent, Phys. Rev. E, **66**, 021106 (2002)]

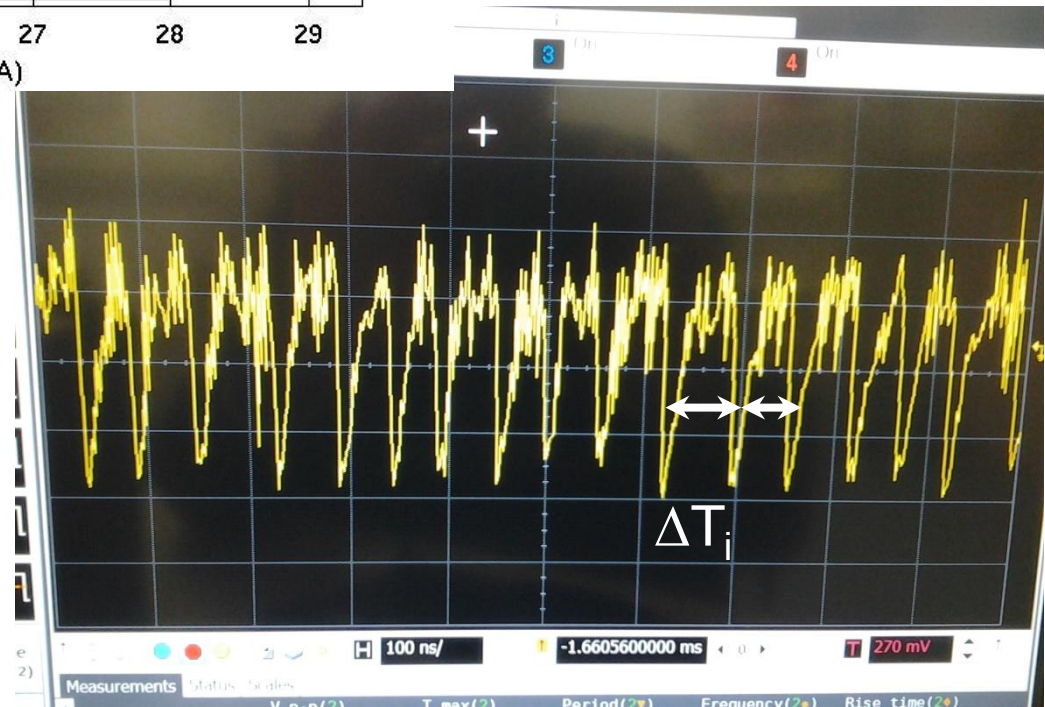
[C. González, J. M. Buldú, M. C. Torrent and J. García-Ojalvo, Phys. Rev. A, **76**, 05824 (2007)]

[M. Ciszak, O. Calvo, C. Masoller, C.R. Mirasso, R. Toral, Phys. Rev. Lett., **90**, 204102 (2003)]

Introduction: Semiconductor lasers

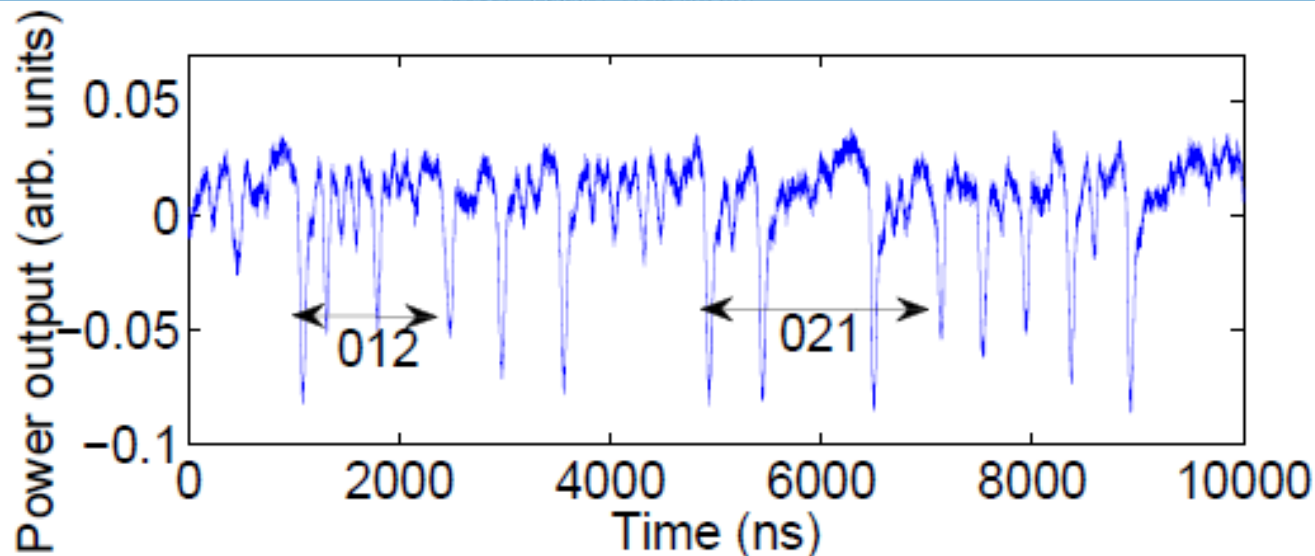


Laser output with feedback



Inter Dropout Interval = ΔT_i

Time delay ~ 5 ns
 $\langle IDI \rangle \sim 200$ ns
1GHz oscilloscope



$$D = 3$$

$$D! = 6$$

Experimental time trace showing the sudden dropouts.

IDI = Inter Dropout Interval (ΔT_i).

Transformation of IDIs into ordinal patterns.

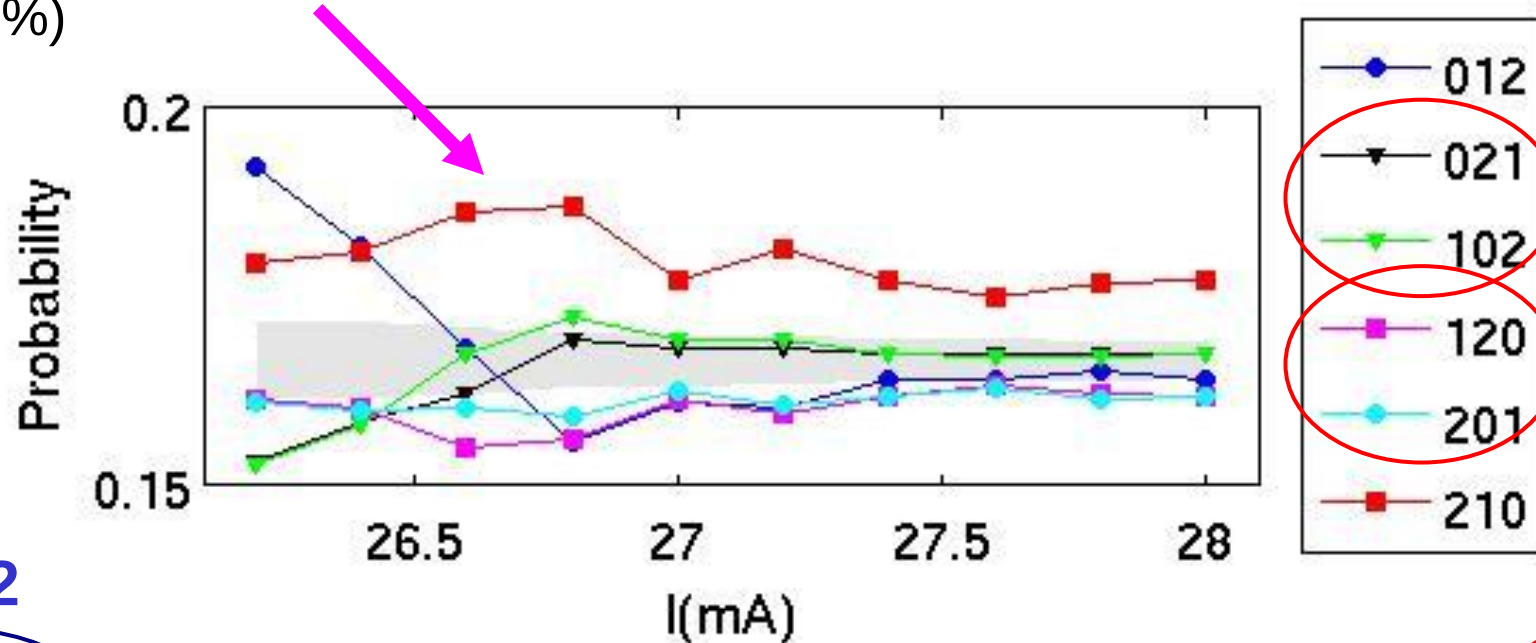
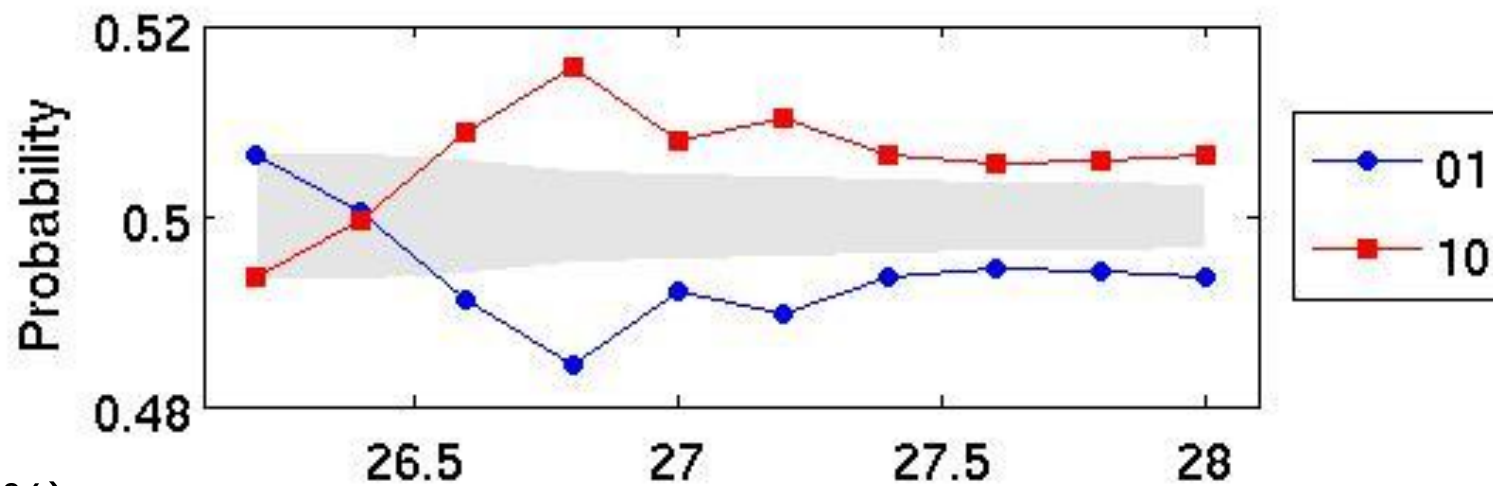
Heart beats, epilepsy, economy, climatology, ...



[C. Bandt & B. Pompe PRL, **88**, 174101 (2002)]

[N. Rubido, J. Tana-Alsina, M. C. Torrent J. García-Ojalvo & C. Masoller, Phys. Rev. E, **84**, 026202 (2011)]

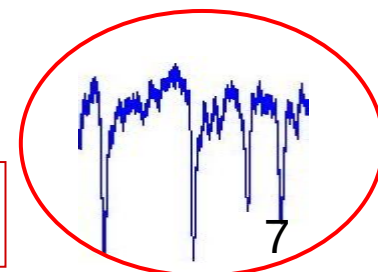
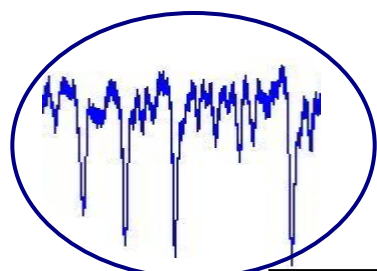
Time series analysis: Ordinal patterns

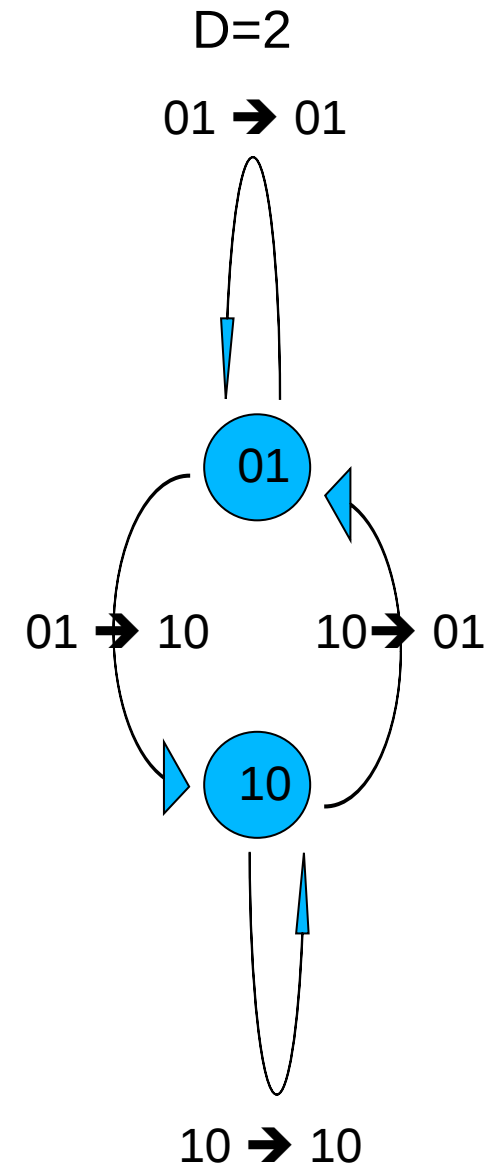
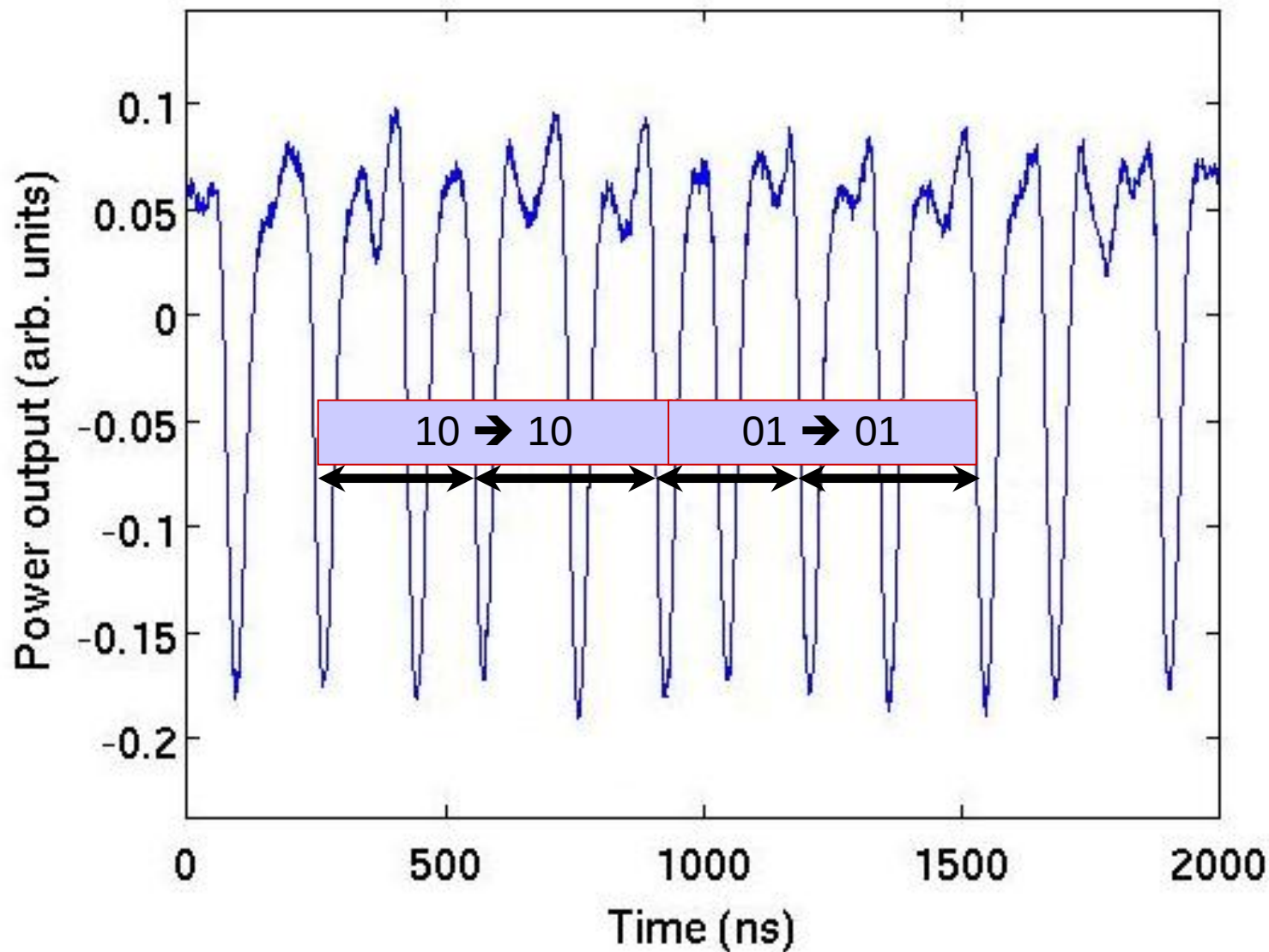


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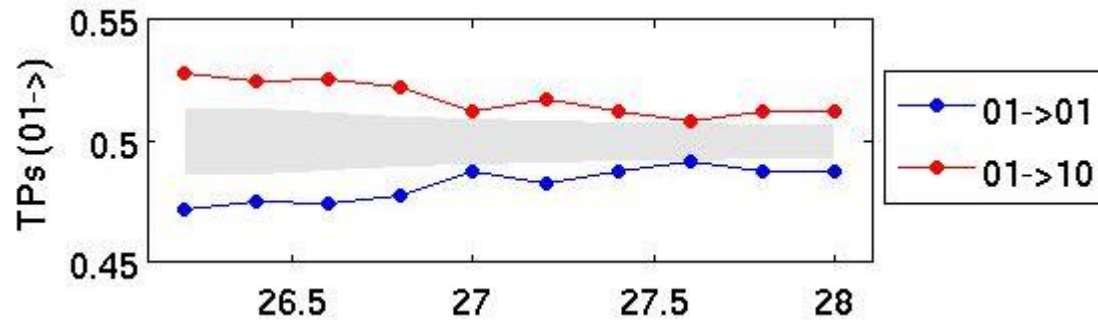
210

[A. Aragonese, N. Rubido, J. Tiana-Alsina, M. C. Torrent & C. Masoller, *Sci. Rep.*, **3**, 1778 (2013)]

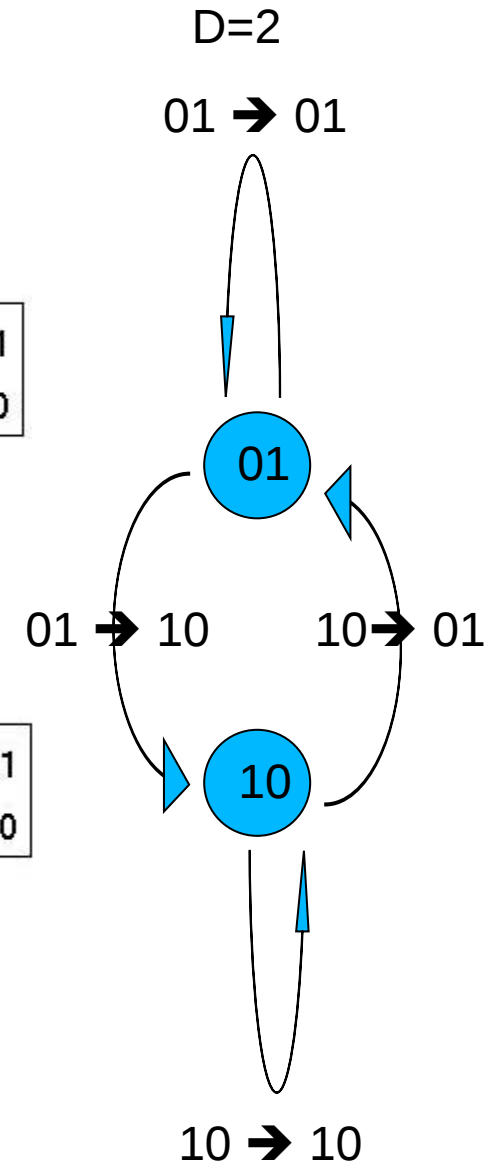
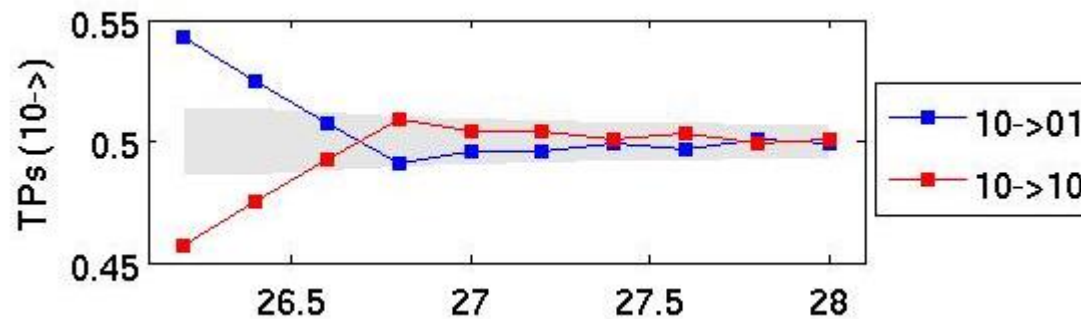


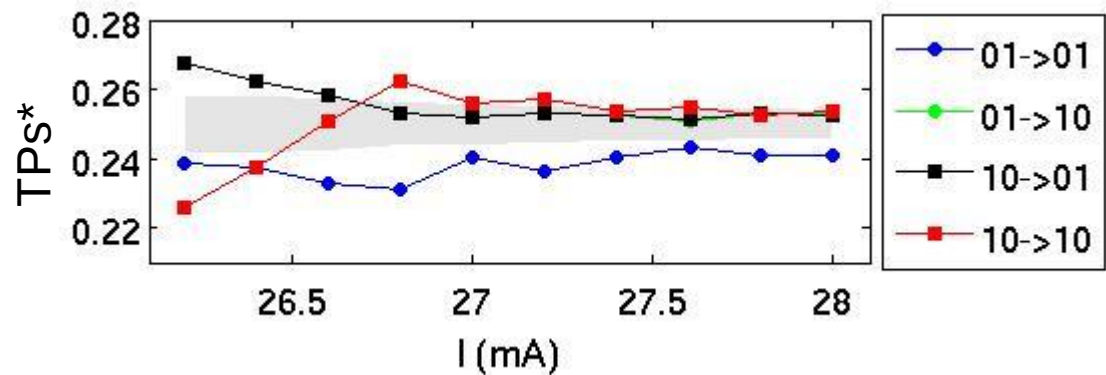


$$T_{01 \rightarrow 01} + T_{01 \rightarrow 10} = 1$$

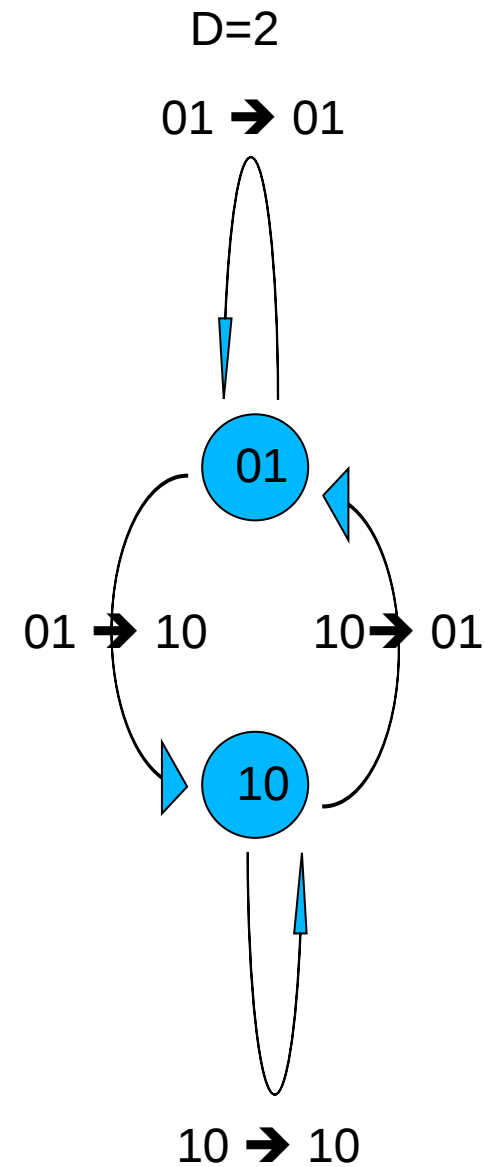


$$T_{10 \rightarrow 01} + T_{10 \rightarrow 10} = 1$$





$$T^*_{01 \rightarrow 01} + T^*_{01 \rightarrow 10} + T^*_{10 \rightarrow 01} + T^*_{10 \rightarrow 10} = 1$$



- Ordinal patterns analysis is a useful tool to distinguish signatures of determinism and stochasticity in spiking complex systems.
- Transition probabilities of $D=2$ show that the system has memory of up to 4 consecutive events.

The Lang-Kobayashi model

$$\dot{E}(t) = k(1+i\alpha)[g(N, |E|^2) - 1]E(t) + \eta e^{-i\omega_0\tau} E(t-\tau) + \sqrt{\beta_{sp}} \xi(t)$$

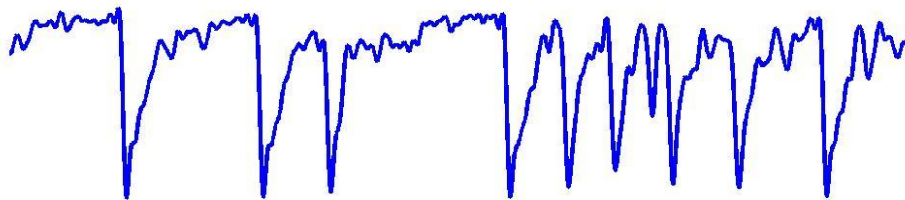
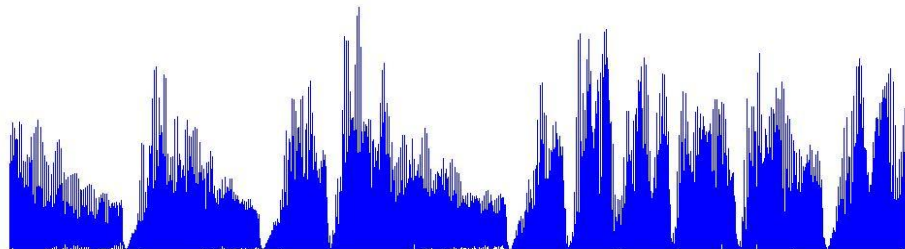
$$\dot{N}(t) = \gamma_N (\mu - N - g|E|^2)$$

Feedback term

Noise term

$$\mu = a_0 + b_0 \sin(\omega t)$$

External periodic forcing



$$\alpha = 4$$

$$\varepsilon = 0.01$$

$$\eta = [10:10:60] ns^{-1}$$

$$k = 300 ns^{-1}$$

$$\tau = 5 ns$$

$$\omega_0\tau = 1.11\pi$$

$$a_0 = 0.98:0.01:1.1$$

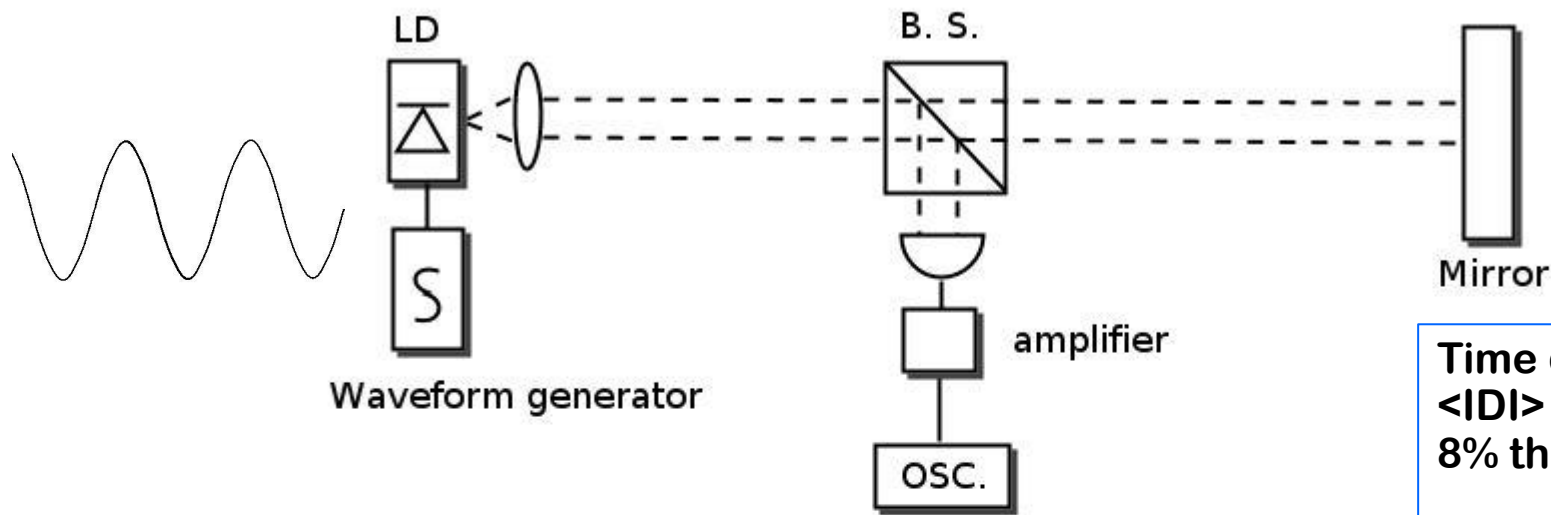
$$b_0 = a_0 \cdot (0.01:0.01:0.1)$$

$$\gamma_N = 1 ns^{-1}$$

$$\beta_{sp} = 10^{-4} ns^{-1}$$

[R. Lang & K. Kobayashi, IEEE Jour. Quan. Elec., **3**, 347 (1980)]

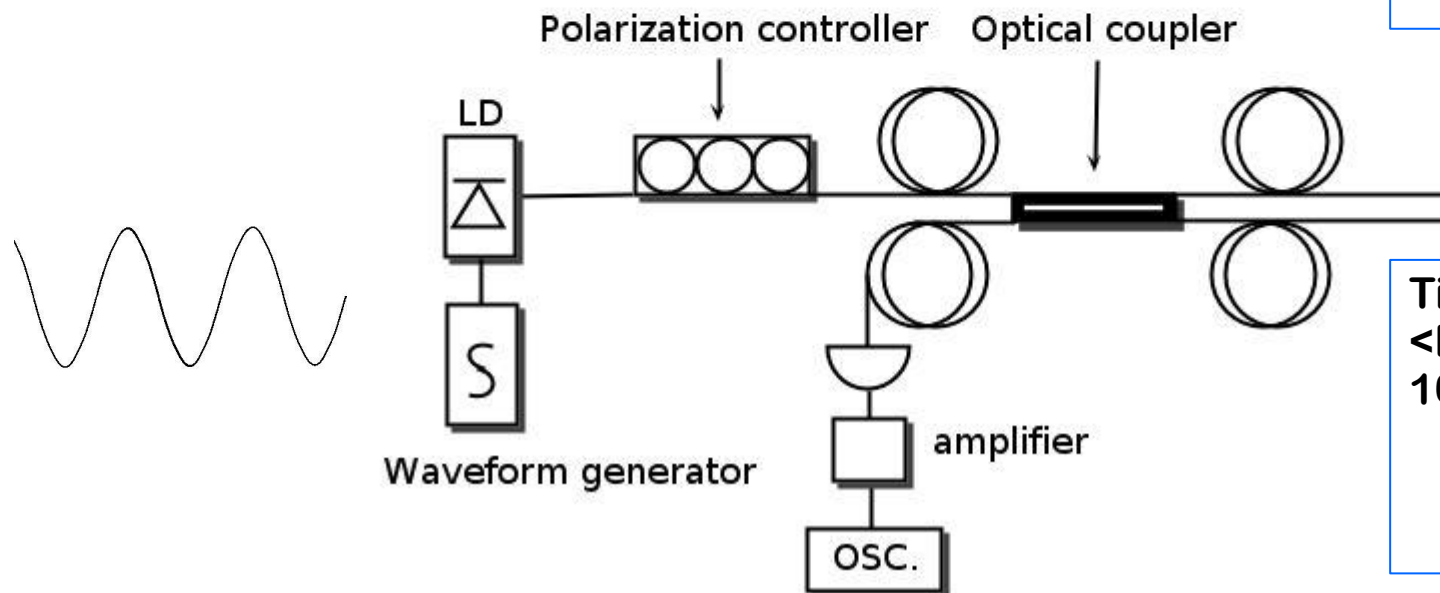
Experimental setup



Time delay $\sim 5\text{ns}$
 $\langle |D| \rangle \sim 200\text{ ns}$
 8% threshold reduction

$$I = 1.03 \cdot I_{th}$$

$$\lambda = 660\text{nm}$$

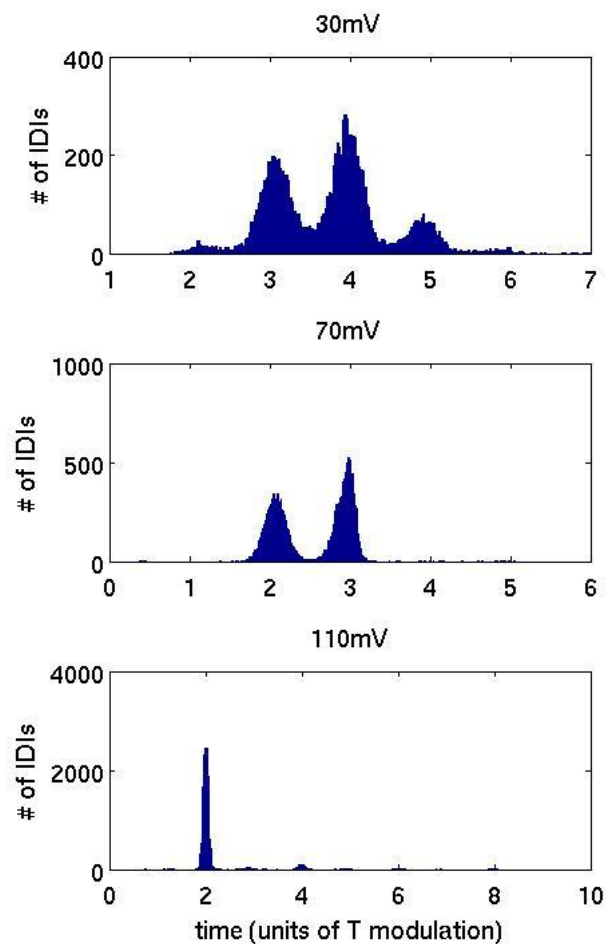
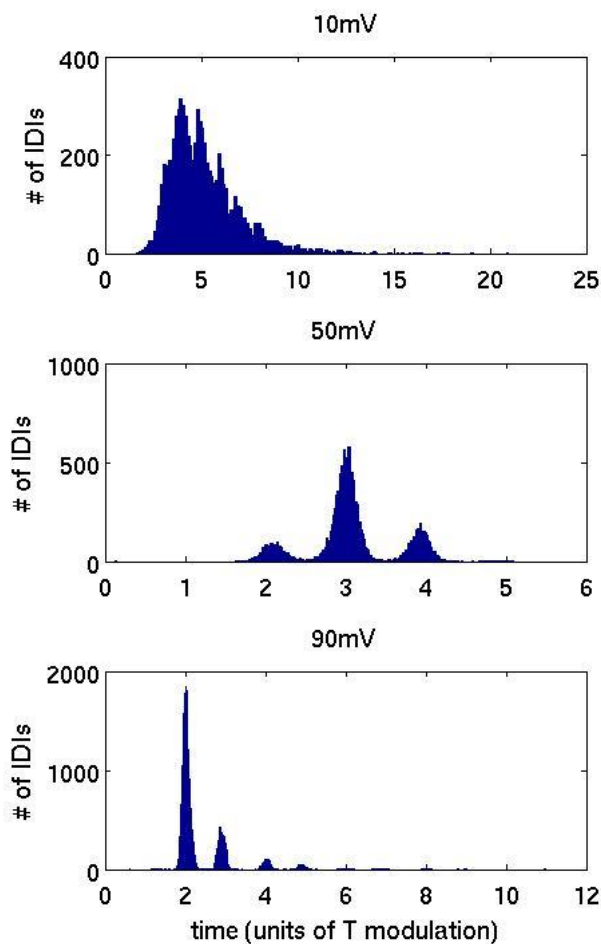
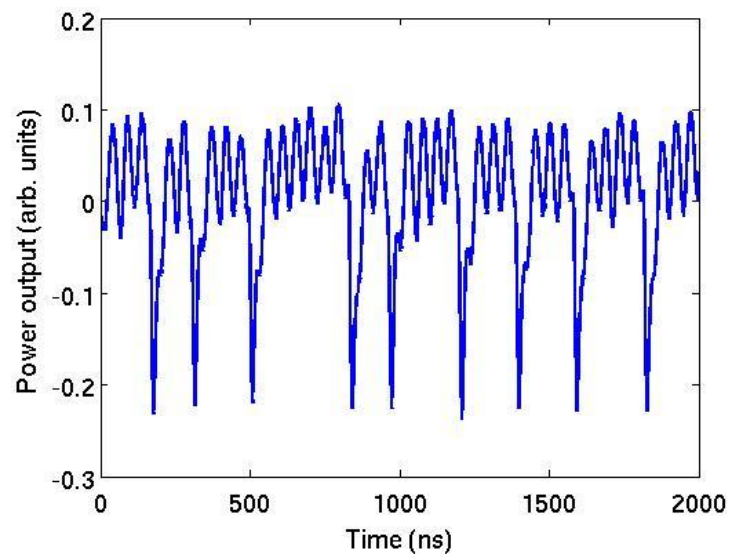


Time delay $\sim 25\text{ns}$
 $\langle |D| \rangle \sim 2000\text{ ns}$
 10% threshold reduction

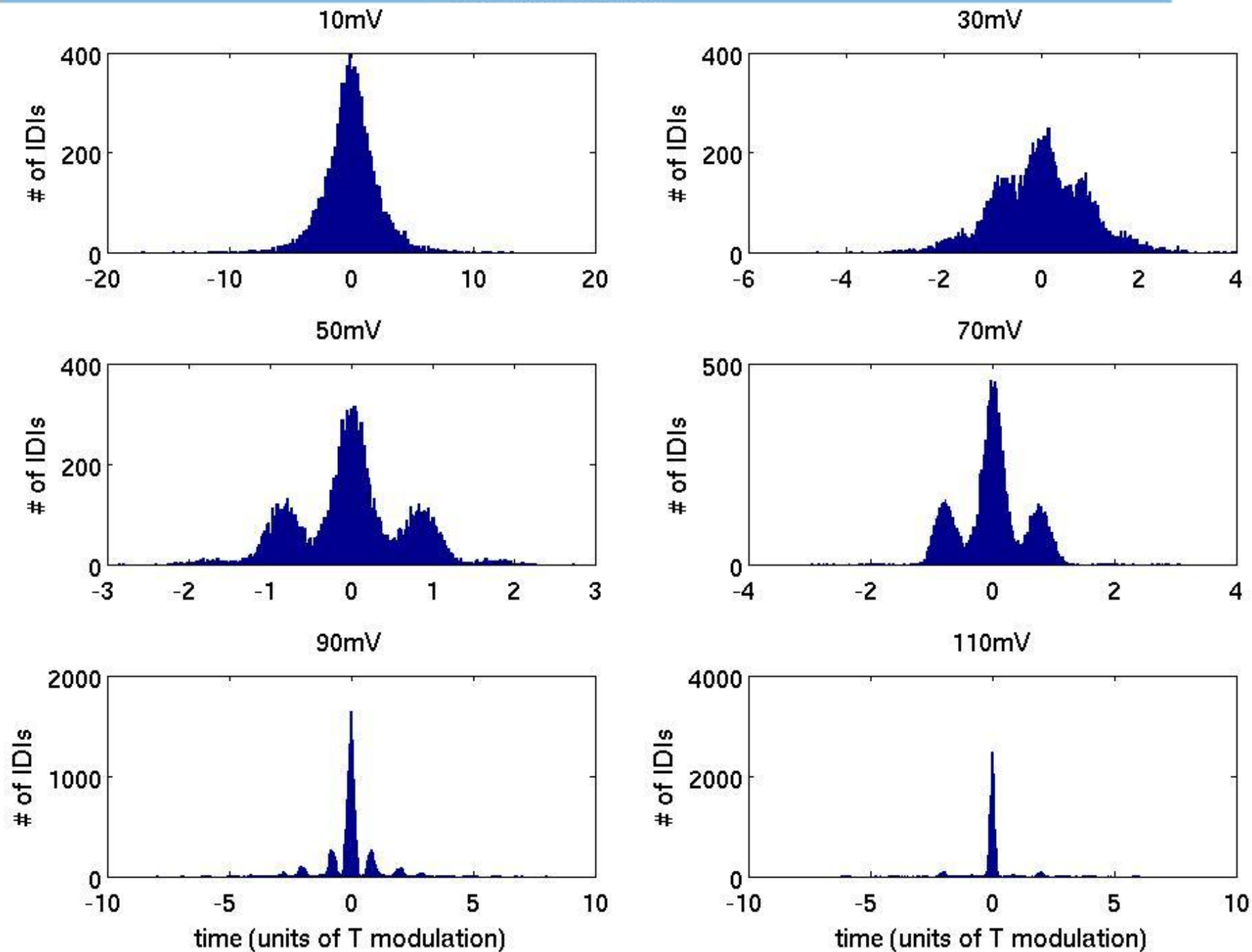
$$I = 1.12 \cdot I_{th}$$

$$\lambda = 1550\text{nm}$$

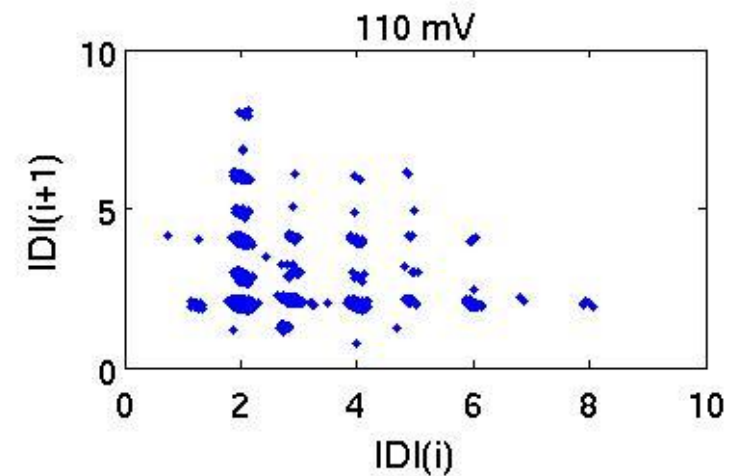
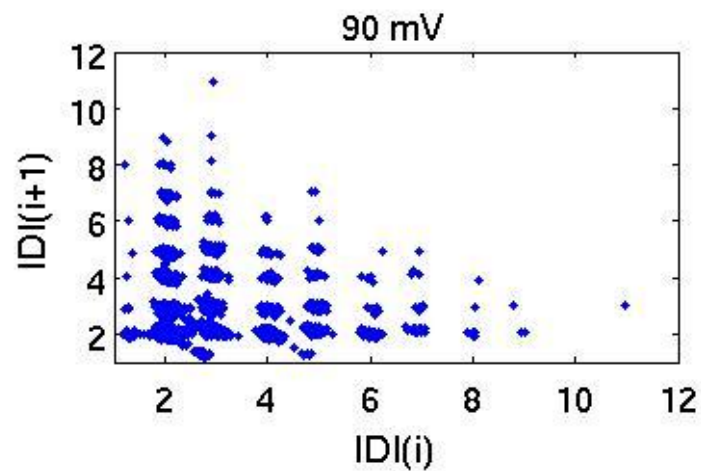
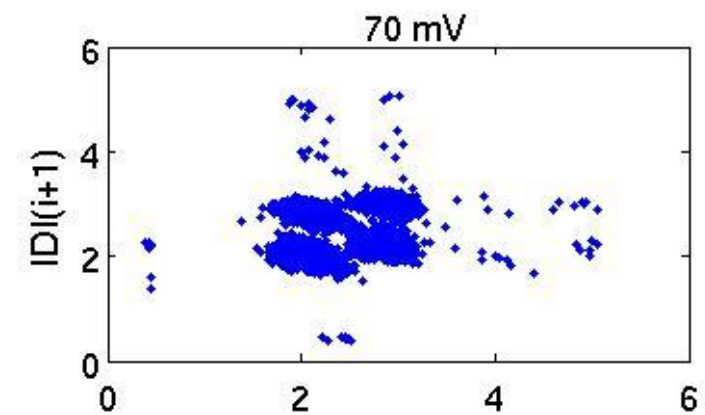
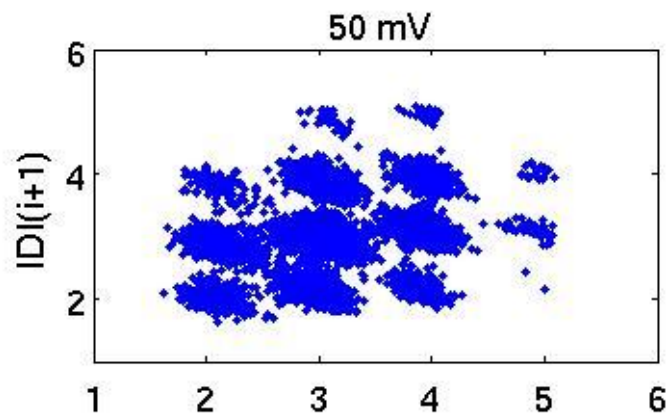
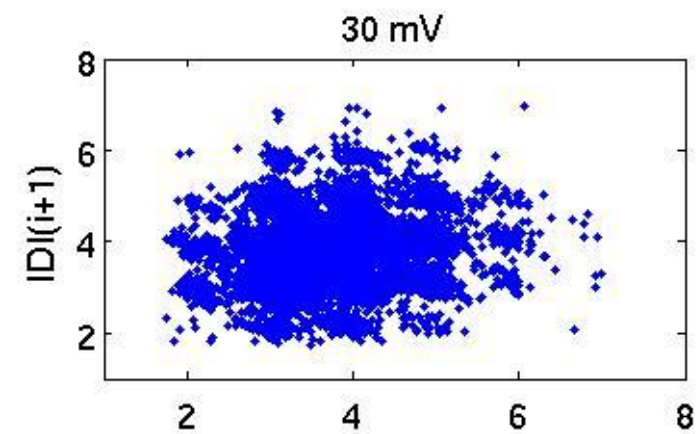
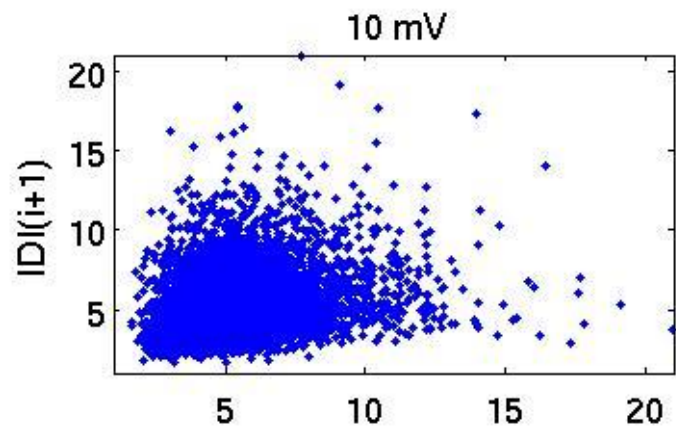
Events histograms



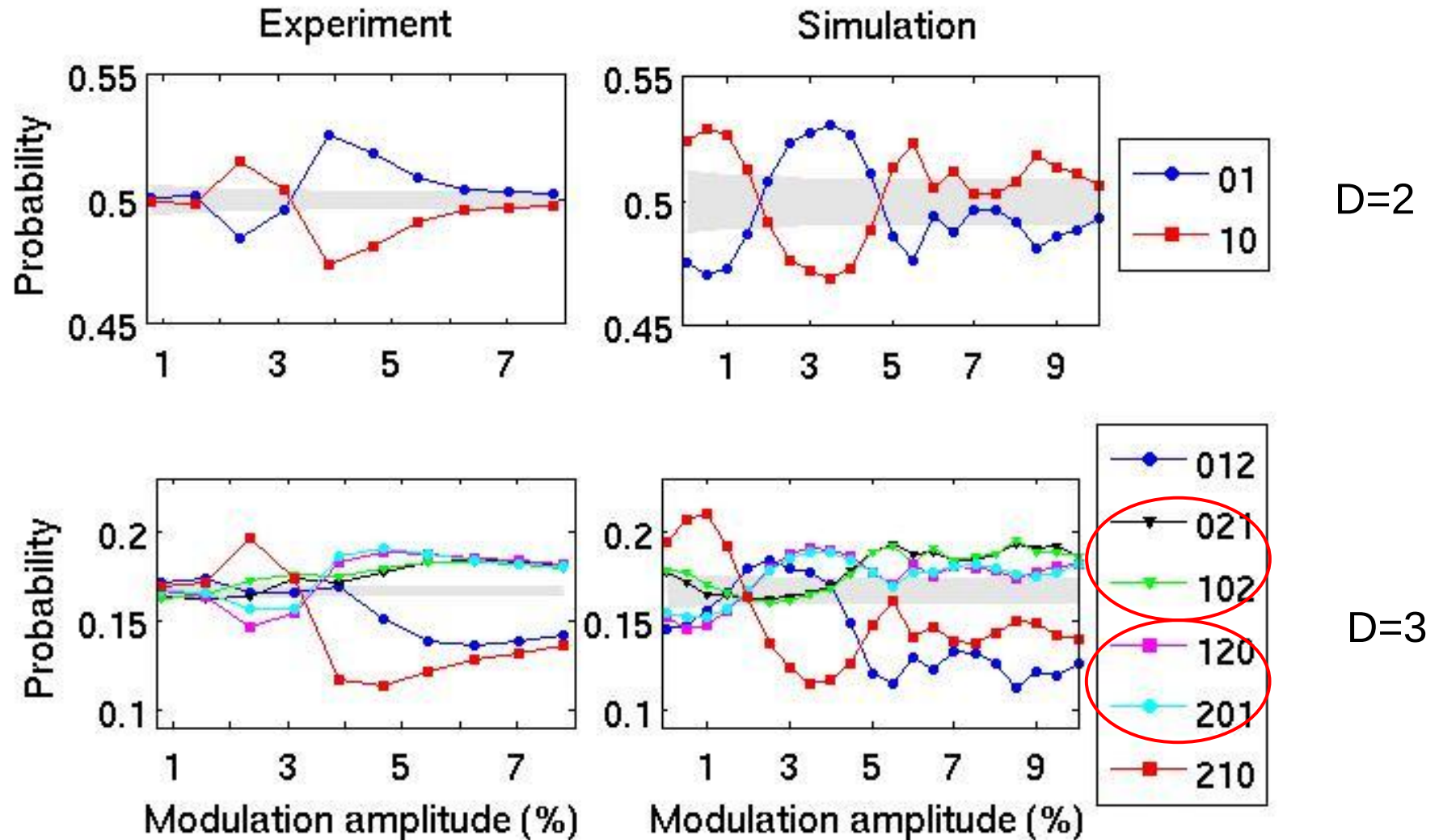
Histograms of $[event(i+1)-event(i)]$



Return maps



Results: Ordinal patterns (experiments and simulations)



$$f = 17\text{MHz}$$

$$I = 1.03 \cdot I_{th}$$

$$I_{th} = 38\text{mA}$$

8% threshold reduction

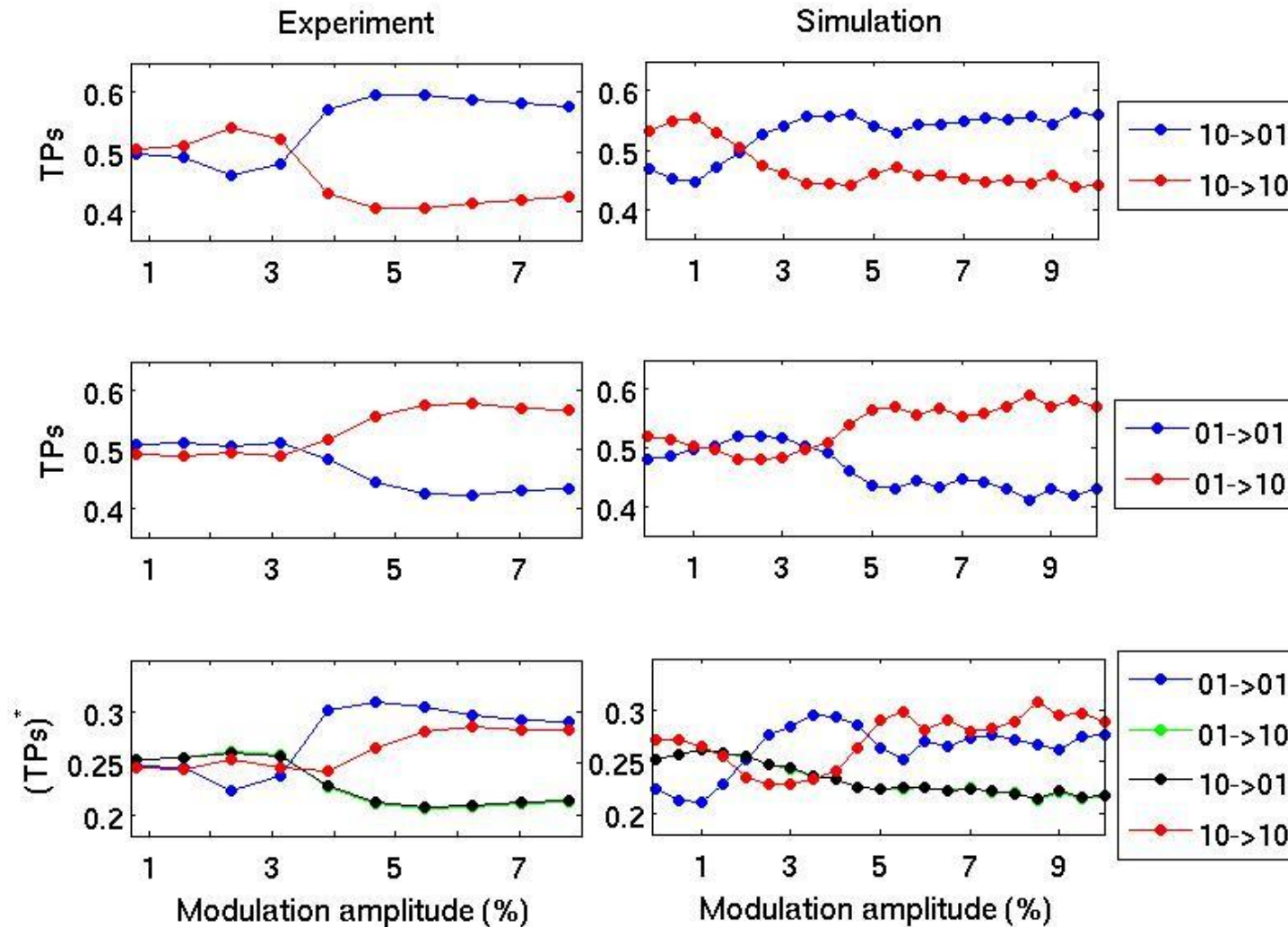
$$f = 15\text{MHz}$$

$$\mu = 1.01$$

$$\eta = 10\text{ns}^{-1}$$

Results: Transition probabilities (experiments & simulations)

D=2



$$f = 17 \text{ MHz}$$

$$I = 1.03 \cdot I_{th}$$

$$I_{th} = 38 \text{ mA}$$

8% threshold reduction

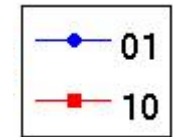
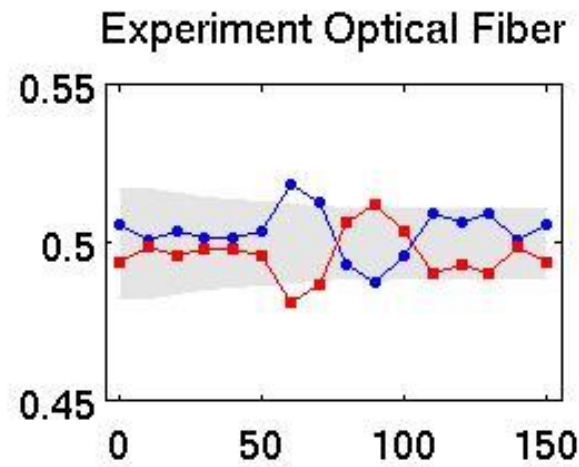
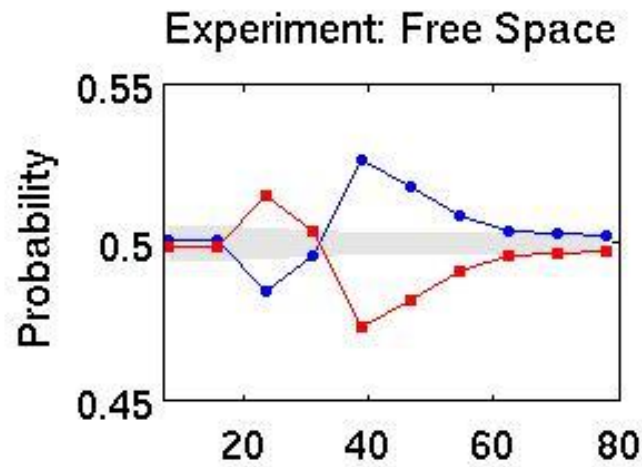
$$f = 15 \text{ MHz}$$

$$\mu = 1.01$$

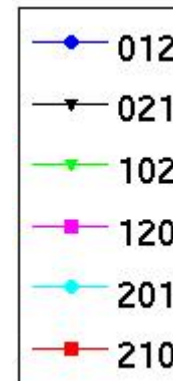
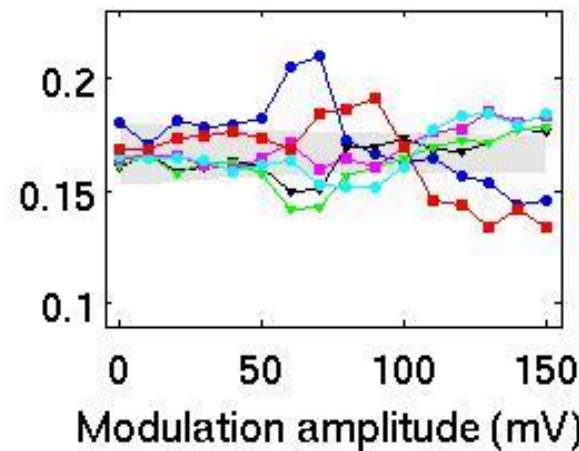
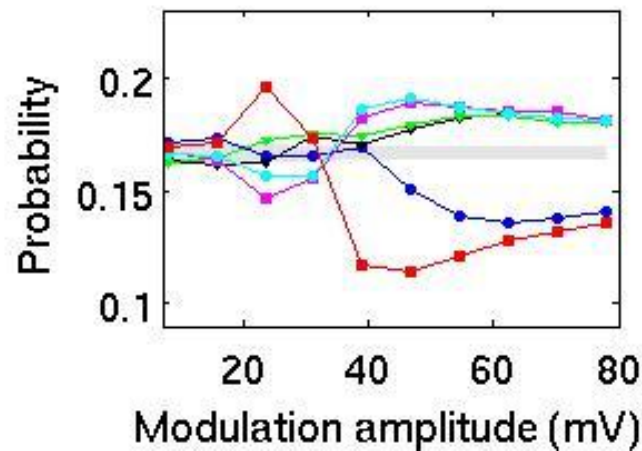
$$\eta = 10 \text{ ns}^{-1}$$

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Results: Free space laser vesus optical fiber laser



D=2



D=3

$$f = 17\text{MHz}$$

$$I = 1.03 \cdot I_{th}$$

$$I_{th} = 38\text{mA}$$

8% threshold reduction

74,000-207,000 IDIs

$$f = 2\text{MHz}$$

$$I = 1.12 \cdot I_{th}$$

$$I_{th} = 12.2\text{mA}$$

10% threshold reduction

8,000-19,000 IDIs

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- Lang & Kobayashi model describes successfully the dynamics found in experiments at the event level of description of the LFFs.
- A structure in the OPs and TPs is found for up to 4 consecutive events.
- A clustering of the ordinal patterns is found for a wide range of conditions and both, for experiments and simulations.



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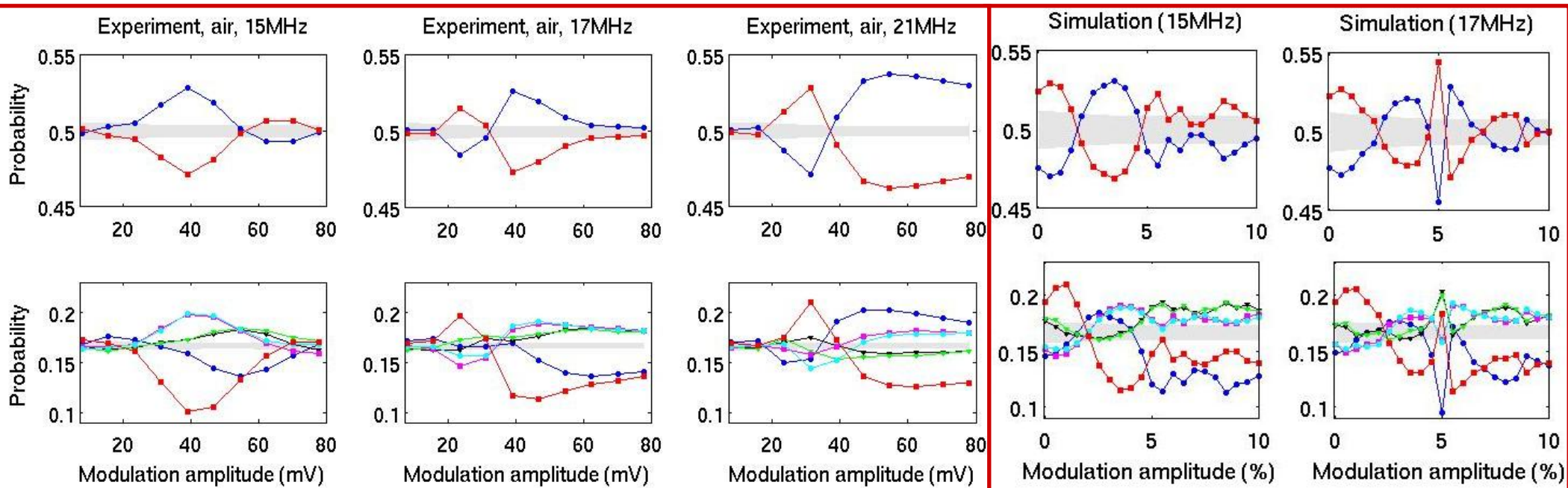
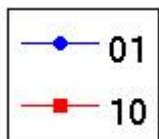
Thank you for your attention

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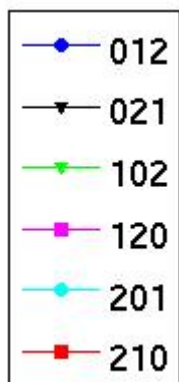
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Results: Effect of frequency (Ordinal patterns)

D=2



D=3



$$f = 17MHz$$

$$I = 1.03 \cdot I_{th}$$

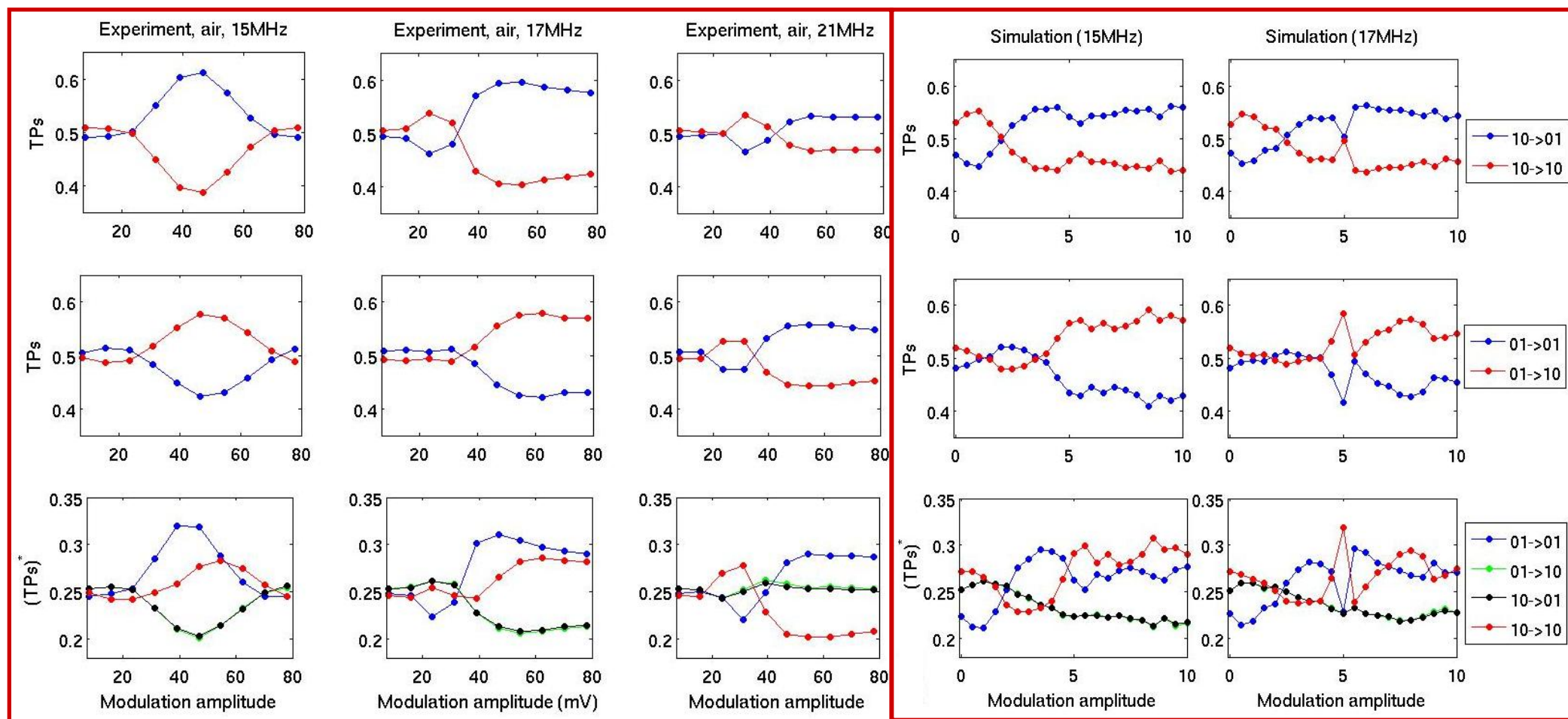
$$I_{th} = 38mA$$

8% threshold reduction

$$f = 15MHz$$

$$\mu = 1.01$$

$$\eta = 10ns^{-1}$$



$$I = 1.03I_{th}$$

8% threshold reduction

$$\lambda = 660nm$$

$$\mu = 1.01$$

$$\eta = 10ns^{-1}$$

Introduction: Semiconductor lasers

