ICTP SAIFR International Centre for Theoretical Physics South American Institute for Fundamental Research

Introduction to nonlinear time series analysis

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Campus d'Excel·lència Internacional

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ICREA

Agència de Gestió d'Ajuts Universitaris i de Recerca

GOBIERNO DE ESPAÑA MINISTERIO DE CIENCIA E INNOVACIÓN

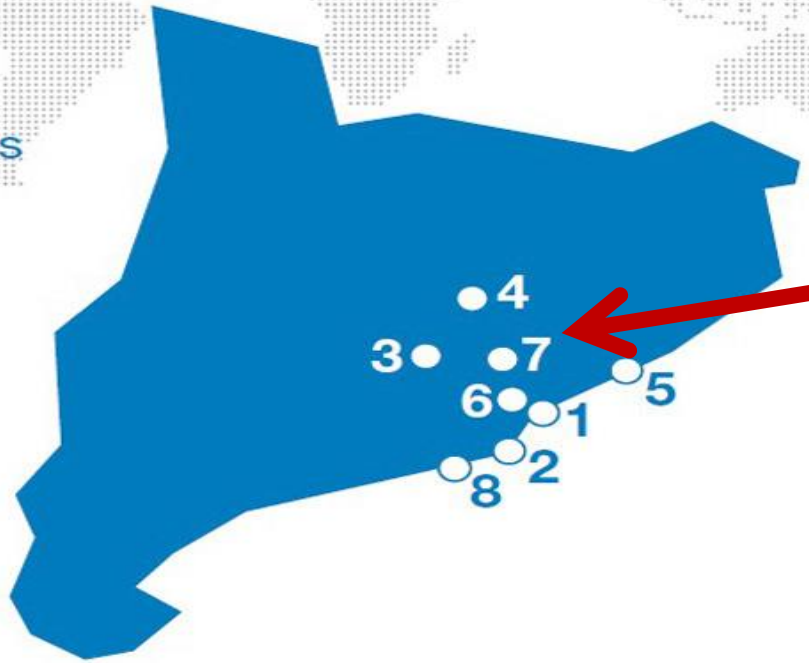
Who are we?

Dynamics, Nonlinear Optics and Lasers

https://donll.upc.edu/en

Where are we?

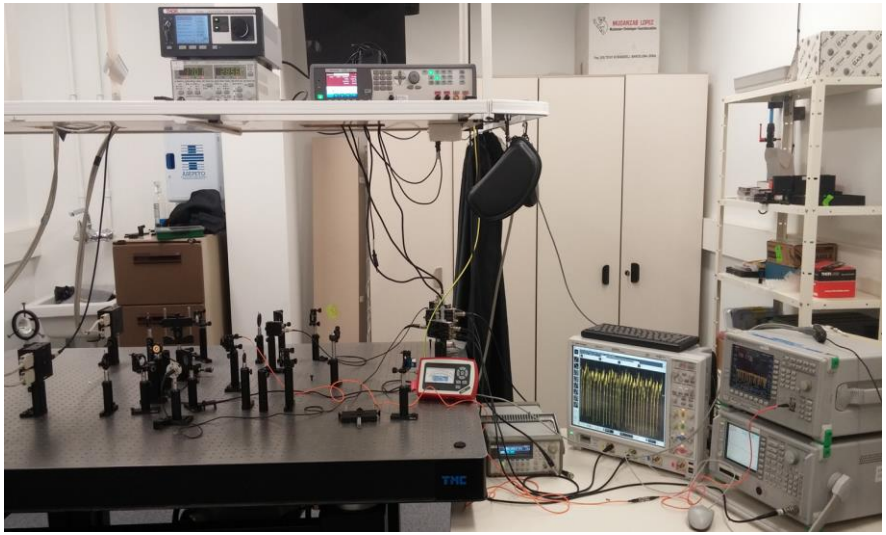
1. Barcelona
2. Castelldefels
3. Igualada
4. Manresa
5. Mataró
6. Sant Cugat del Vallès
7. Terrassa
8. Vilanova i la Geltrú



Viernes, 25 de septiembre de 2009 **Diari de Terrassa**

El edificio Gaia centraliza grupos científicos consolidados y emergentes.

What do we study?



- Nonlinear phenomena
  - Photonics (two labs: dynamics of lasers & nonlinear optics)
  - Biophysics, neuroscience
  - Data analysis and complex systems
- Funding: Agència de Gestió d'Ajuts Universitaris i de Recerca, ICREA, Gobierno de España, Ministerio de Ciencia e Innovación

Outline

- **First lecture**
  - General introduction and historical development
  - Univariate time-series analysis
  - Bivariate and multivariate time-series analysis

- **Second lecture**
  - Identifying dynamical transitions from data

- **Third lecture**
  - Climate data analysis

A. General introduction and historical development

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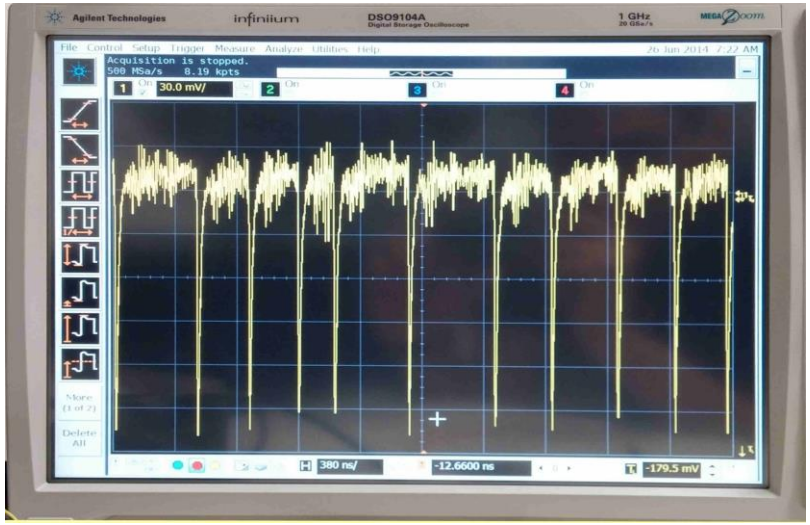
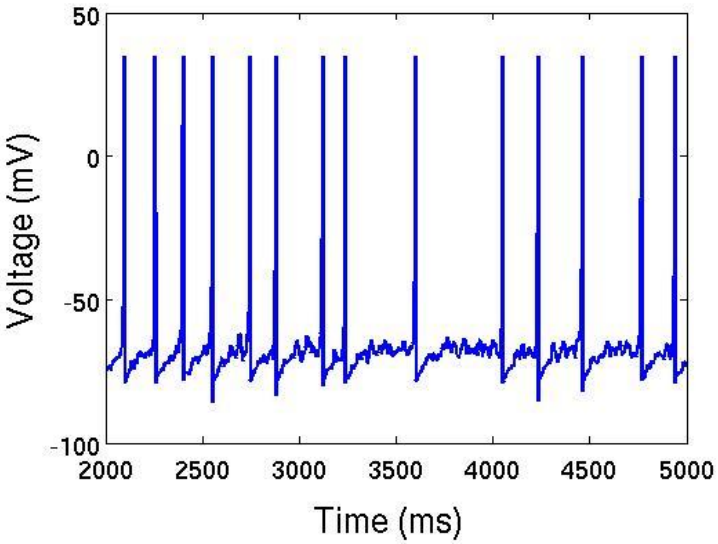
TSA: what is this about?

Optical spikes

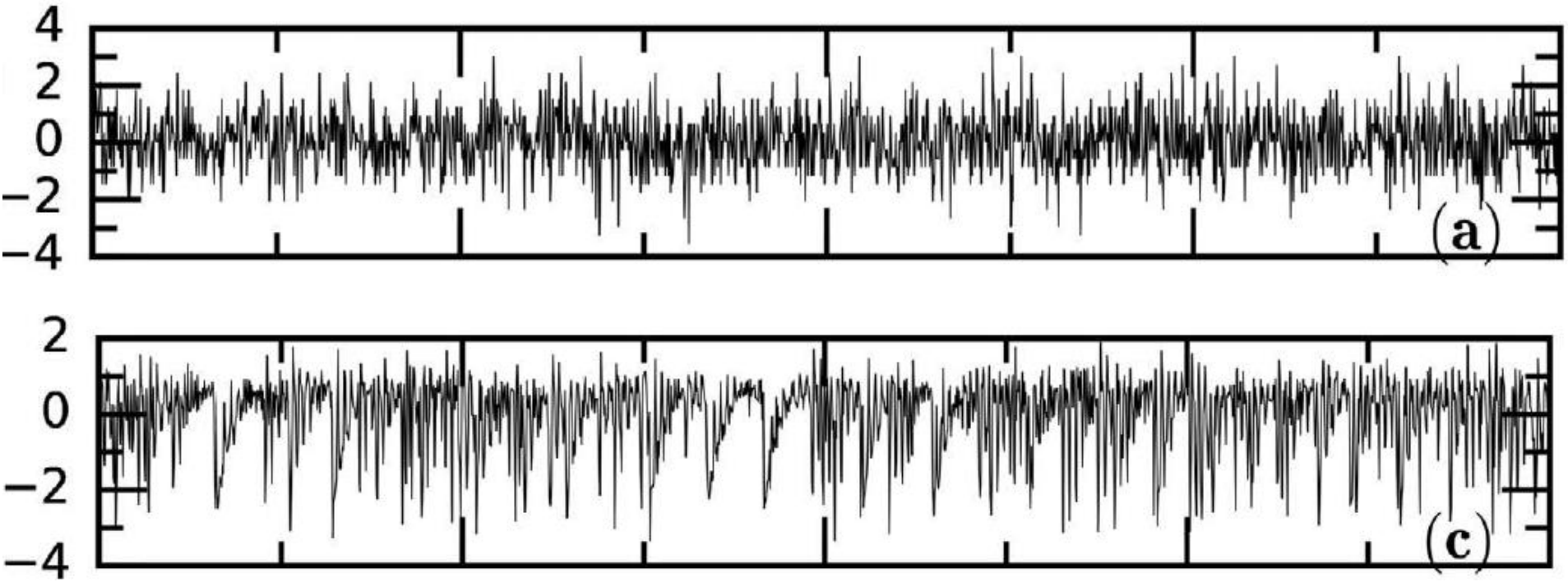


Time (μs)

Neuronal spikes



- Similar dynamical systems generate these signals?
- Ok, very different dynamical systems, but their output signals have similar statistical properties?



Time

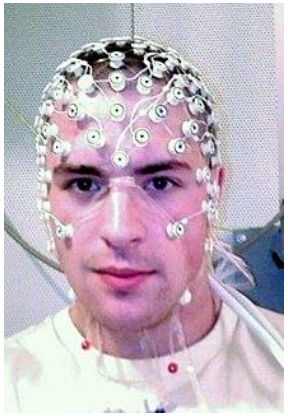
How can differences be quantified? With what reliability?

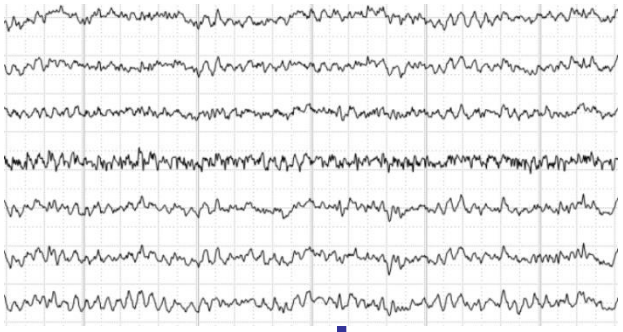
TSA: main goal

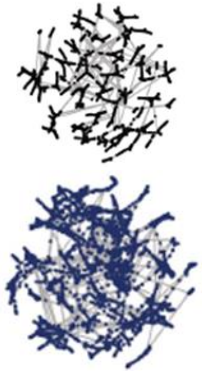
- Extract information from data.

- What for?
  - Classification
  - Prediction
  - Inference
  - Model verification & identification
  - Parameter estimation (assuming we have a good model).

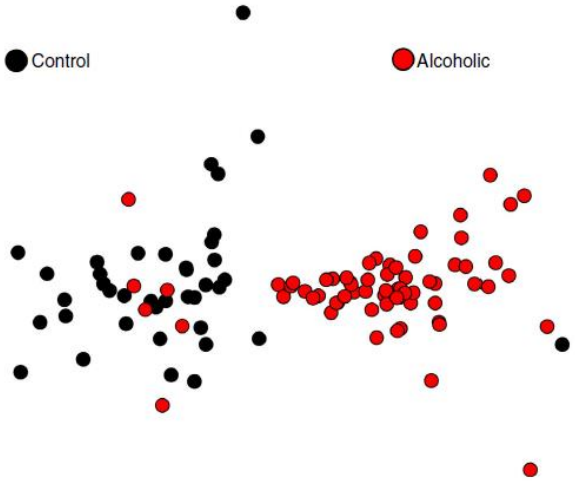
Classification: control vs alcoholic subjects



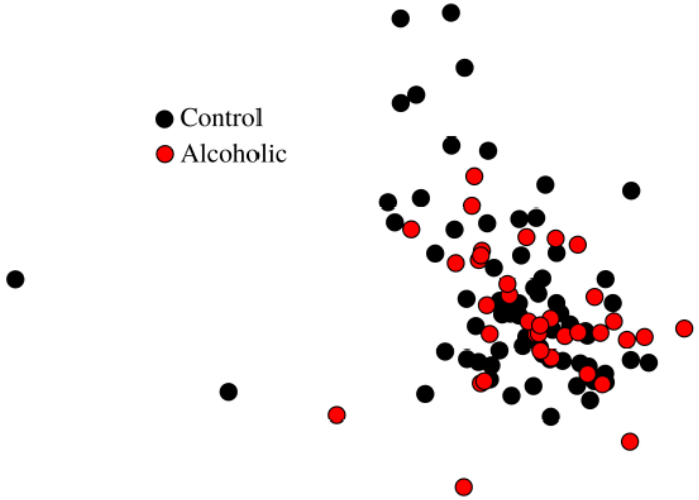




Dissimilarity measure

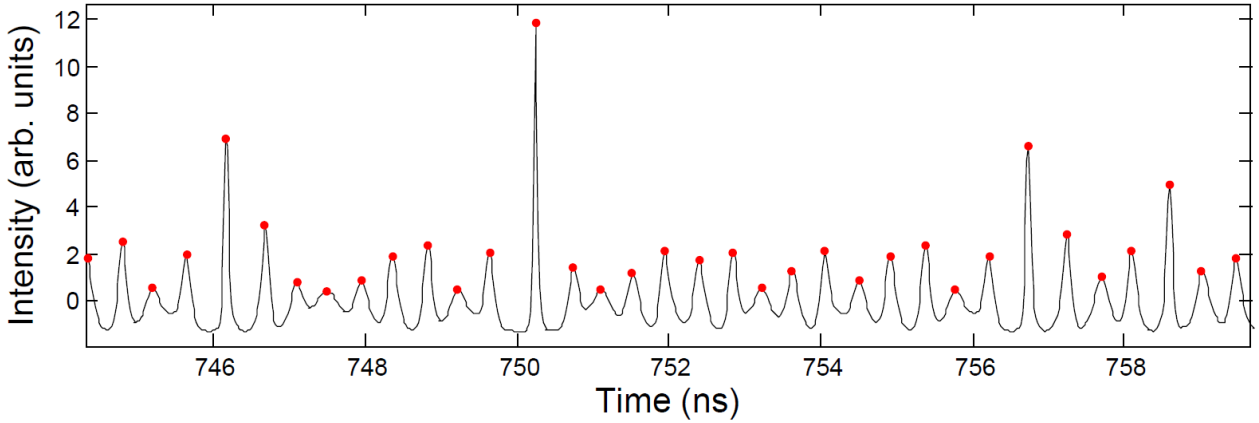


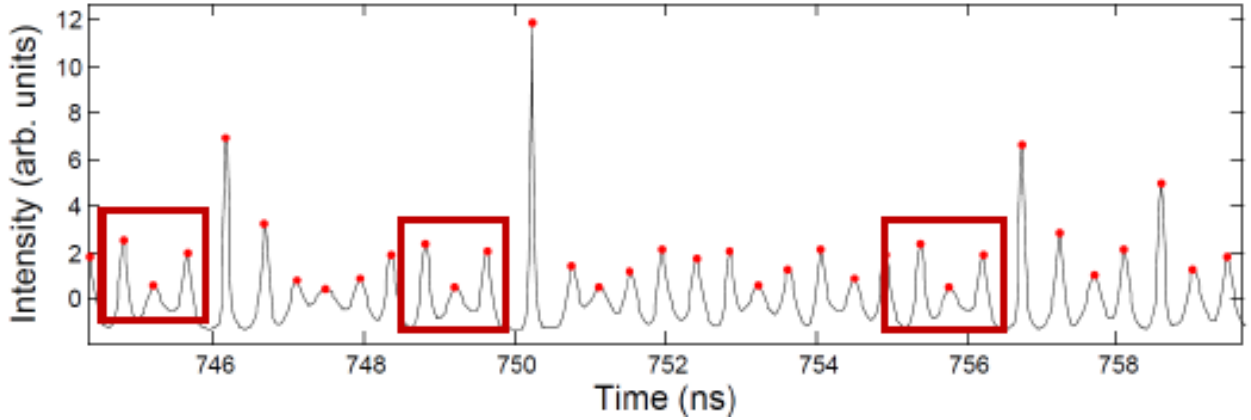
Hamming distance



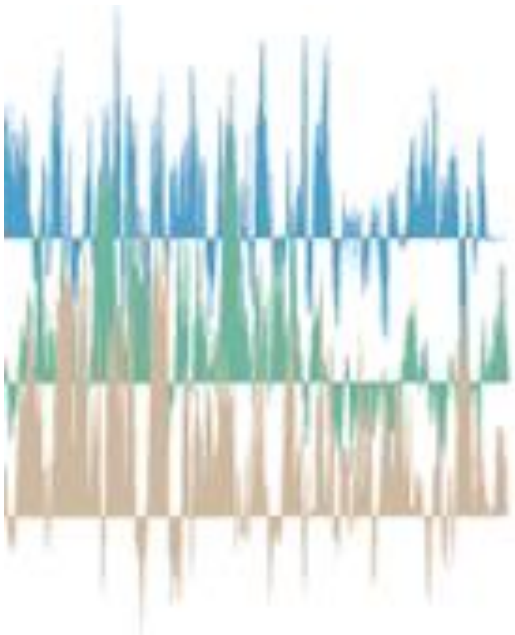
T. A. Schieber et al, Nat. Comm. 8:13928 (2017).

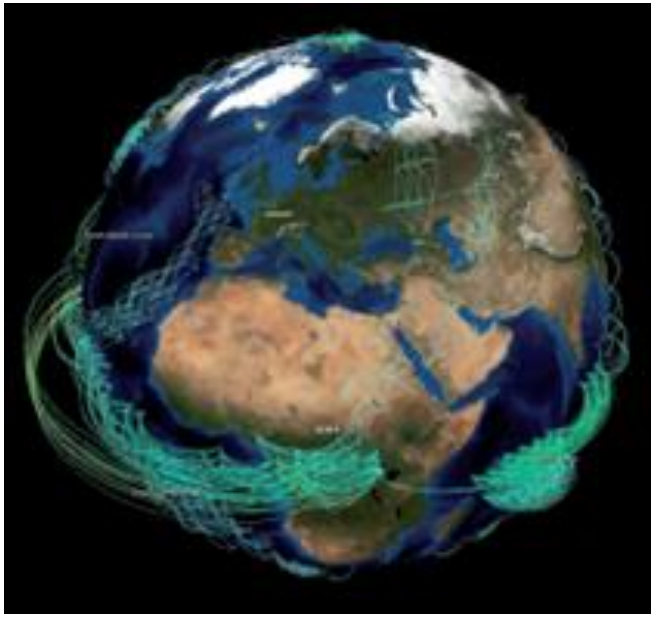
Prediction of extreme events: Ultra-intense light pulses





Inference of underlying interactions



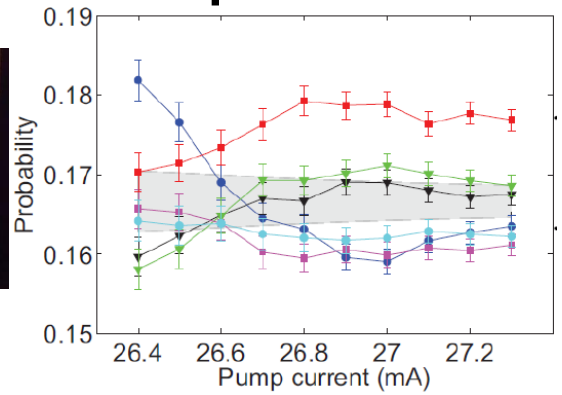


Surface Air Temperature <u>Anomalies</u> in different geographical regions

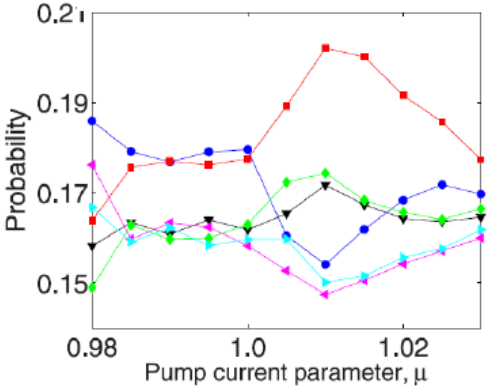
Donges et al, Chaos 2015

Model identification, parameter estimation

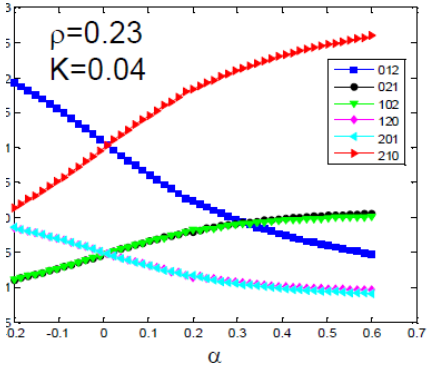
Empirical data

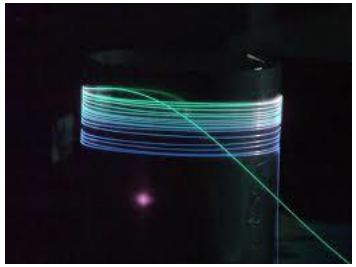


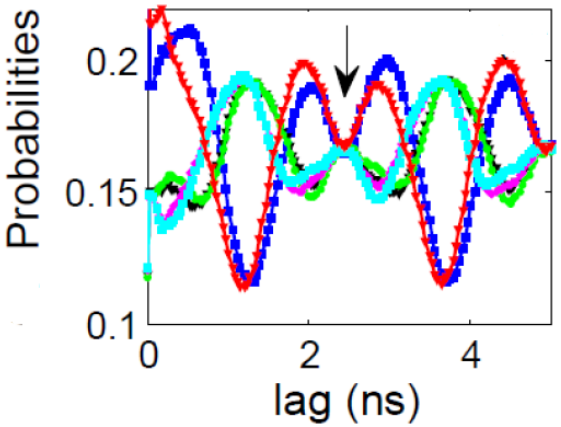
Known model



Minimal model







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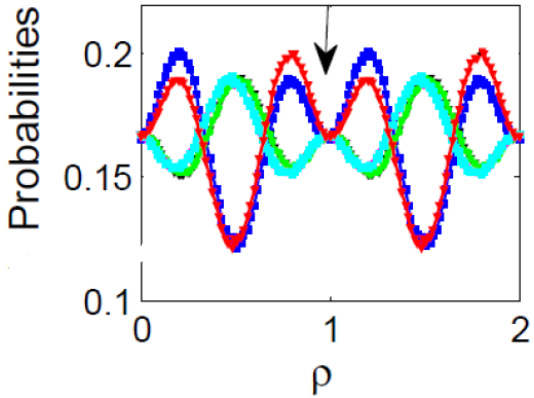

A. Aragoneses et al, Sci. Rep. 4, 4696 (2014)

And much more, so let’s begin!

Methods

- Many methods have been developed to extract information from data.
- The method to be used depends on the characteristics of the data
  - Length of the dataset;
  - Stationary;
  - Level of noise;
  - Temporal resolution;
  - Single-or-Multi variate (single or multi channel measures)
  - etc.
- **Different methods provide complementary information.**

Where the data comes from?

- Modeling assumptions about the **type of dynamical system** that generates the data:
  - Stochastic or deterministic?
  - Regular or chaotic or "complex"?
  - Time-varying parameters?
  - Low or high dimensional?
  - Spatials variable? Hidden variables?
  - Time delays?
  - Etc.
- And because this is a **School**, we start with a brief tour of **dynamical systems.**

Dynamical Systems

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In the beginning…

- Mid-1600s: Ordinary differential equations (ODEs)

- **Isaac Newton**: studied planetary orbits and solved analytically the "two-body" problem (earth around the sun).

- Since then: a lot of effort for solving the "three-body" problem (earth-sun-moon) – Impossible.

Late 1800s

- **Henri Poincare** (French mathematician).

  Instead of asking "*which are the exact positions of planets (trajectories)*?"

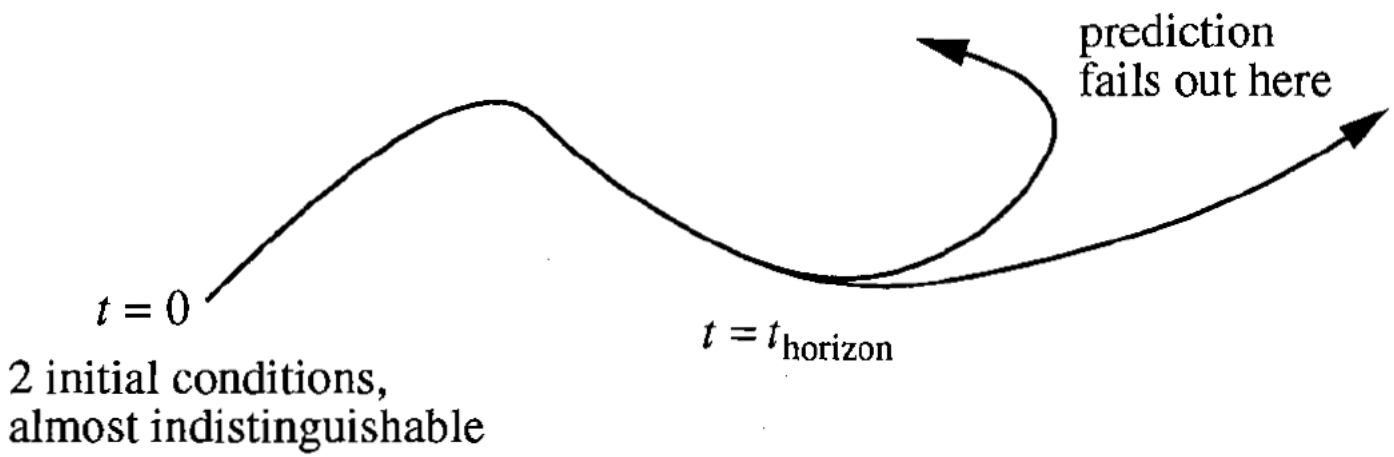
  he asked: "*is the solar system* ***stable*** *for ever, or will planets eventually run away*?"

- He developed a **geometrical** approach to solve the problem.

- Introduced the concept of "phase space".

- He also had an <u>intuition</u> of the possibility of **chaos**

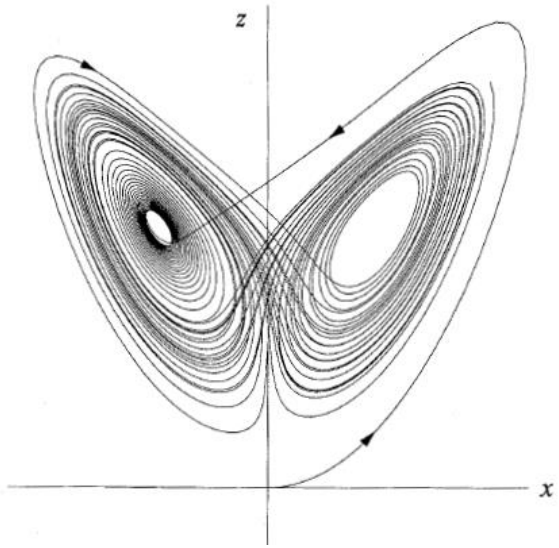
*Poincare: "The evolution of a **deterministic** system can be aperiodic, unpredictable, and strongly depends on the initial conditions"*



Deterministic system: the initial conditions fully determine the future state. **There is no randomness but the system can be unpredictable**.

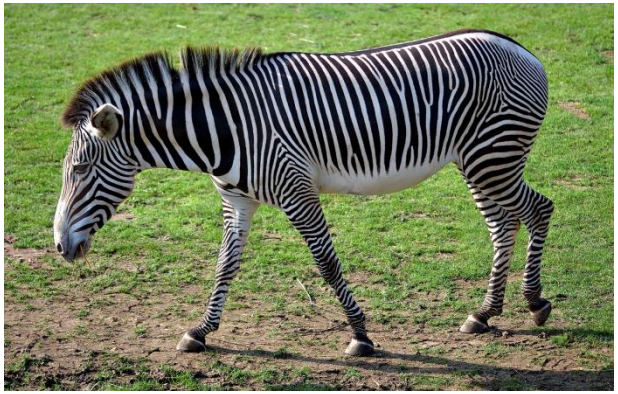
1950s: First simulations

- Computes allowed to experiment with equations.
- Huge advance of the field of "*Dynamical Systems*".
- 1960s: **Eduard Lorentz** (American mathematician and meteorologist at MIT): simple model of convection rolls in the atmosphere.
- **Chaotic** motion.



Order within chaos and self-organization

- **Ilya Prigogine** (Belgium, born in Moscow, Nobel Prize in Chemistry 1977)
- Thermodynamic systems far from equilibrium.
- Discovered that, in chemical systems, the interplay of (external) **input of energy** and **dissipation** can lead to "self-organized" patterns.



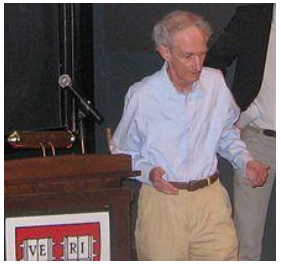
In the 1960s: biological nonlinear oscillators

- **Arthur Winfee** (American theoretical biologist – born in St. Petersburg): Large communities of biological oscillators show a tendency to self-organize in time –**collective synchronization**.

In the 1960's he did experiments trying to understand the effects of perturbations in biological clocks (circadian rhythms).

What is the effect of an external perturbation on subsequent oscillations?

The 1970s



- **Robert May** (Australian, 1936): population biology
- "Simple mathematical models with very complicated dynamics", *Nature* (1976).

$$x_{t+1} = f(x_t)$$

Example: $f(x) = r\,x(1-x)$

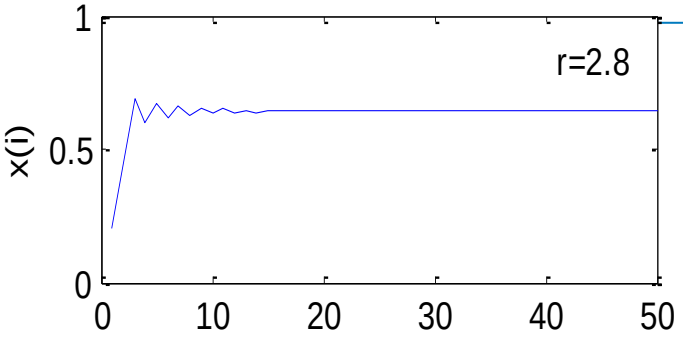
- Difference equations ("iterated maps"), even though simple and deterministic, can exhibit different types of dynamical behaviors, from **stable points**, to a bifurcating hierarchy of **stable cycles**, to **apparently random fluctuations**.

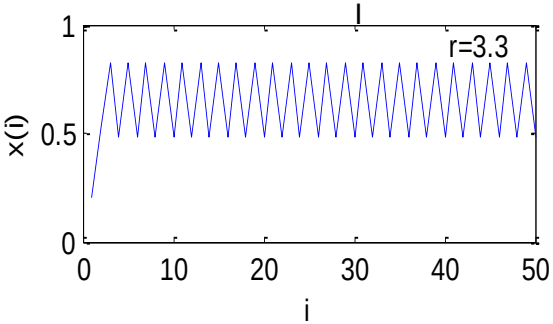
The logistic map

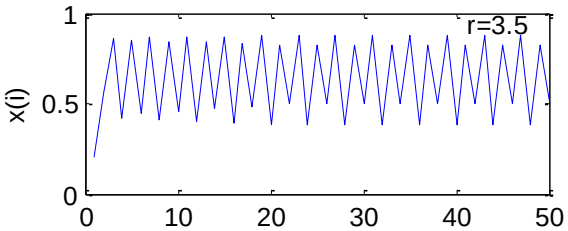
$$x(i+1) = r\,x(i)[1-x(i)]$$

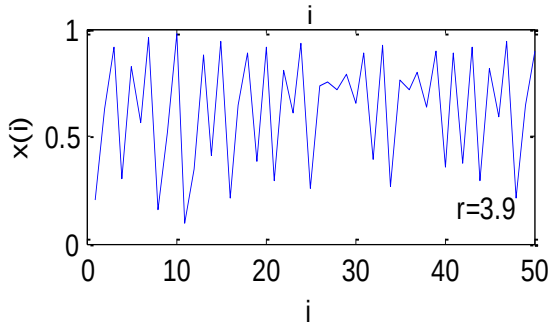
r=2.8, Initial condition: x(1) = 0.2

Transient relaxation → long-term stability



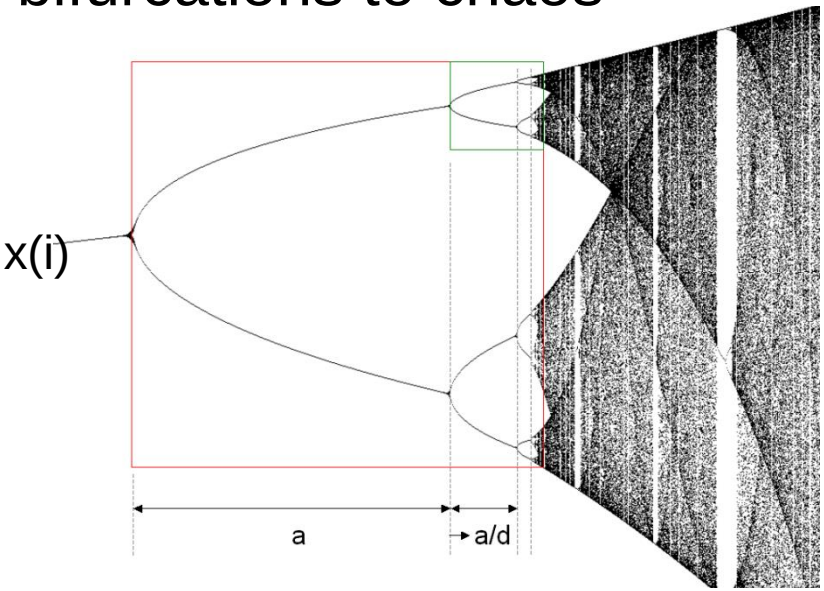






Transient dynamics → stationary oscillations (regular or irregular)

"period-doubling" bifurcations to chaos



Parameter r

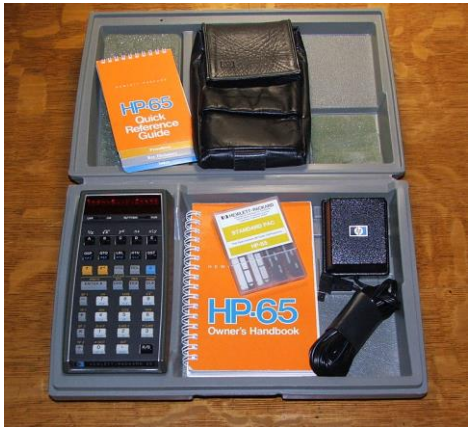
Universal route to chaos

- In 1975, **Mitchell Feigenbaum** (American mathematical physicist), using a small HP-65 calculator, discovered the scaling law of the bifurcation points

$$\lim_{n\to\infty} \frac{r_{n-1} - r_{n-2}}{r_n - r_{n-1}} = 4.6692...$$

- Then, he showed that the same behavior, with the same mathematical constant, occurs within a wide class of functions, prior to the onset of chaos (**universality**).

Very different systems (in chemistry, biology, physics, etc.) go to chaos in the same way, quantitatively.



HP-65 calculator: the first magnetic card-programmable handheld calculator

The late 1970s

- **Benoit Mandelbrot** (Polish-born, French and American mathematician 1924-2010): "self-similarity" and **fractal objects:**

  each part of the object is like the whole object but smaller.

- Because of his access to IBM's computers, Mandelbrot was one of the first to use **computer graphics** to create and display fractal geometric images.

Fractal objects

- Are characterized by a "fractal" dimension that measures roughness.

Broccoli
D=2.66

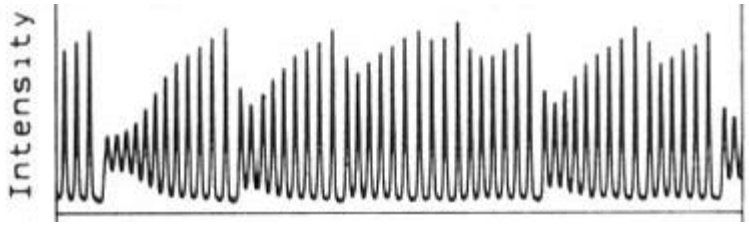
Human lung
D=2.97

Coastline of Ireland
D=1.22

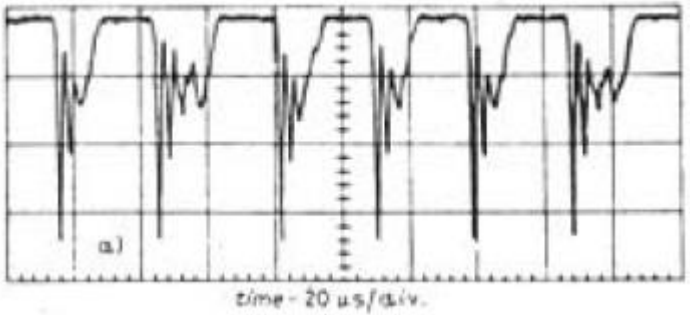
Video: http://www.ted.com/talks/benoit_mandelbrot_fractals_the_art_of_roughness#t-149180

In the 80's: can we observe chaos experimentally?

- **Optical chaos**: first observed in laser systems.



Time



In the 90': can we control chaotic dynamics?

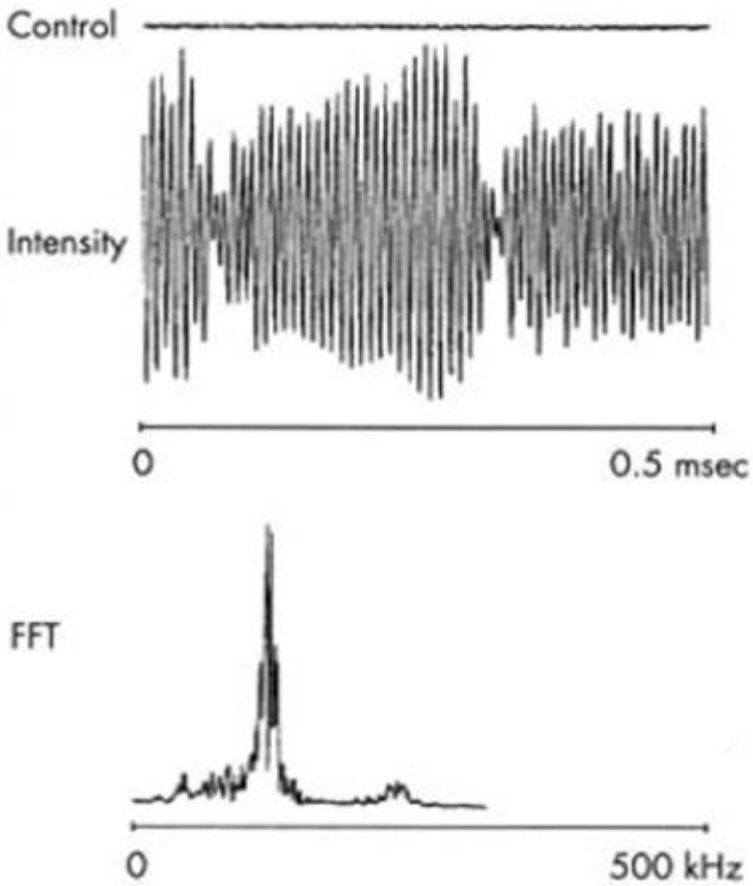
- **Ott**, **Grebogi** and **Yorke** (1990)

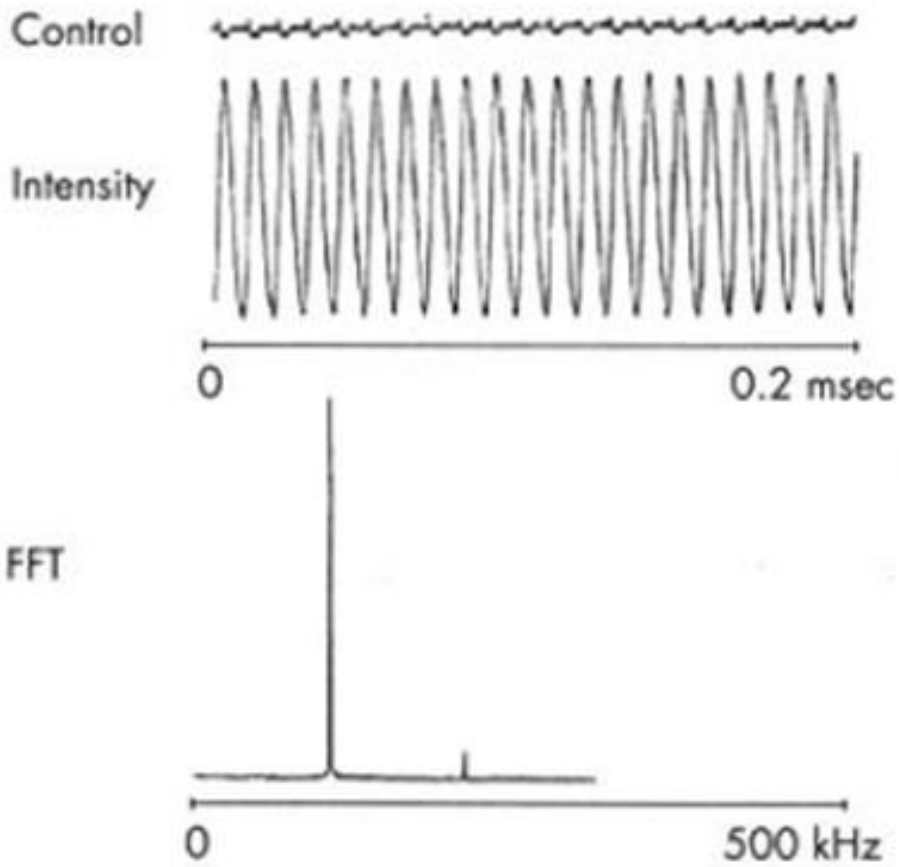
  Unstable periodic orbits can be used for control: wisely chosen **periodic kicks** can maintain the system near the desired orbit.

- **Pyragas** (1992)

  Control by using a **continuous** self-controlling **feedback** signal, whose intensity is practically zero when the system evolves close to the desired periodic orbit but increases when it drifts away.

Experimental demonstration of control of optical chaos

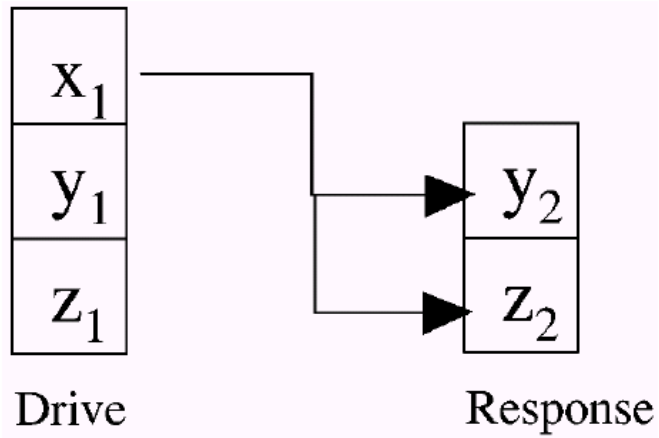




The 1990s: synchronization of two chaotic systems

Pecora and Carroll, PRL 1990

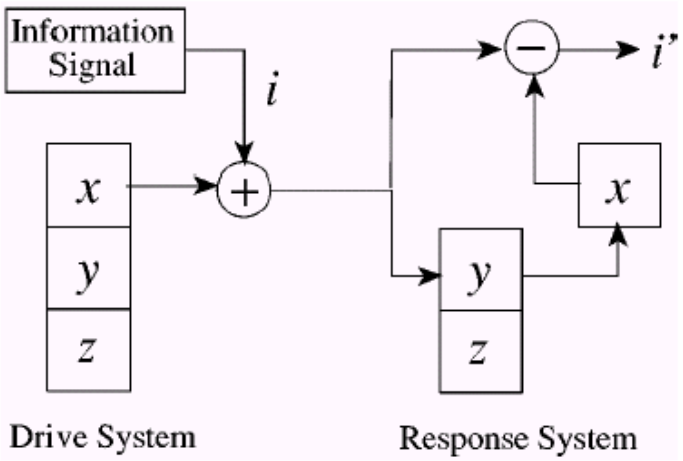
Unidirectionaly coupled Lorenz systems: the '*x*' variable of the response system is **replaced** by the '*x*' variable of the drive system.



$$t \to \infty \quad |y_2 - y_1| \to 0, \; |z_2 - z_1| \to 0$$

Interesting but … useful?

- Transmission of secure information (Cuomo–Oppenheim, PRL 1993)



Is the system secure?

- It is possible to break the system and extract the message (G. Perez y H. Cerdeira, PRL 1995)

Different types of synchronization

$$dx_1 / dt = F(x_1)$$

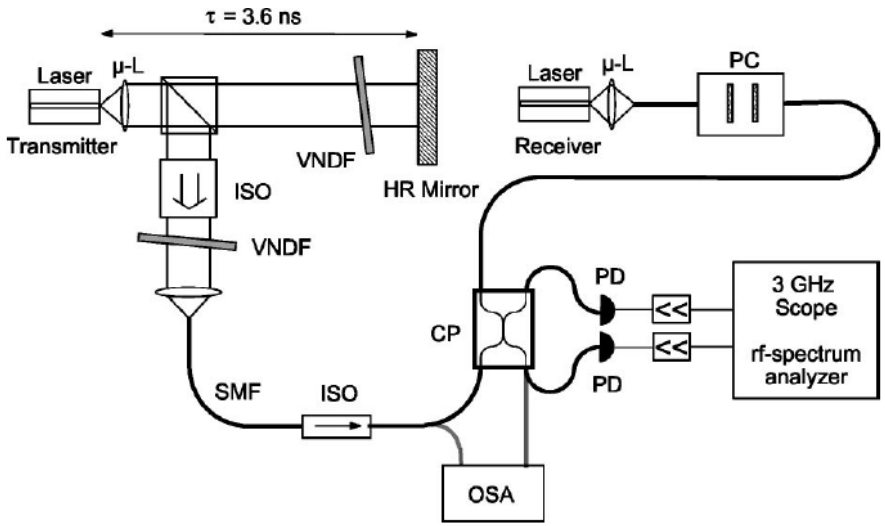
$$dx_2 / dt = F(x_2) + \alpha E(x_1 - x_2)$$

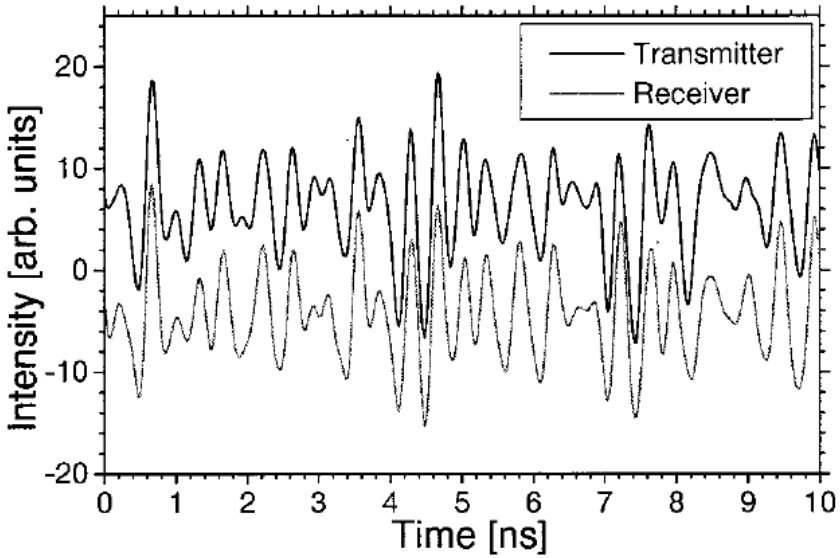
- Complete (CS): $x_1(t) = x_2(t)$ (identical systems)
- Phase (PS): the phases of the oscillations synchronize, but the amplitudes are not.
- Lag (LS): $x_1(t+\tau) = x_2(t)$
- Generalized (GS): $x_2(t) = f(x_1(t))$ (*f* depends on the strength of the coupling)

A lot of work is being devoted to detect synchronization in real-world data.

Experimental observation of synchronization in coupled lasers

Fischer et al Phys. Rev. A 2000





Synchronization of a large number of coupled oscillators

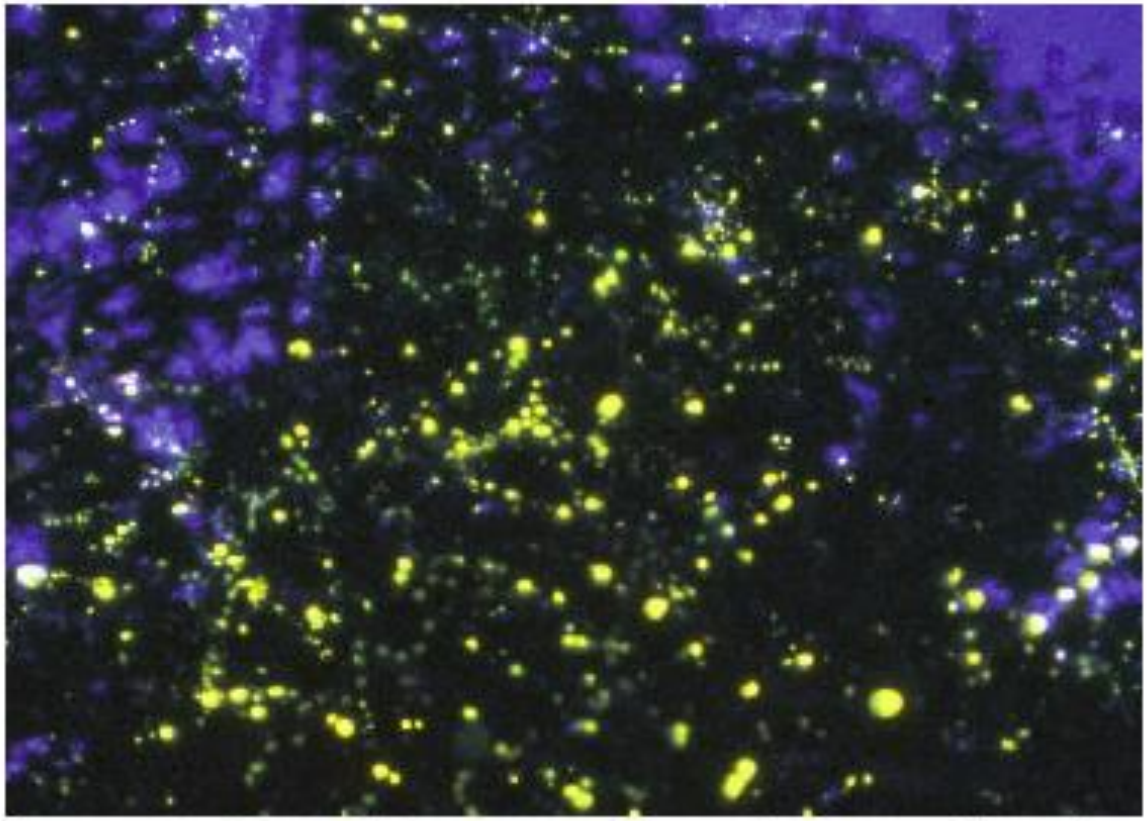
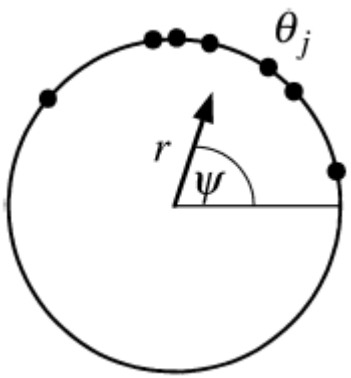


Figure 1 | Fireflies, fireflies burning bright. In the forests of the night, certain species of firefly flash in perfect synchrony — here *Pteroptyx malaccae* in a mangrove apple tree in Malaysia. Kaka *et al.*[2] and Mancoff *et al.*[3] show that the same principle can be applied to oscillators at the nanoscale.

Kuramoto model

(Japanese physicist, 1975)

Model of **all-to-all** coupled **phase oscillators**.



$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) + \xi_i, \qquad i = 1 \ldots N$$

K = coupling strength, ξ_i = stochastic term (noise)

Describes the emergence of collective behavior

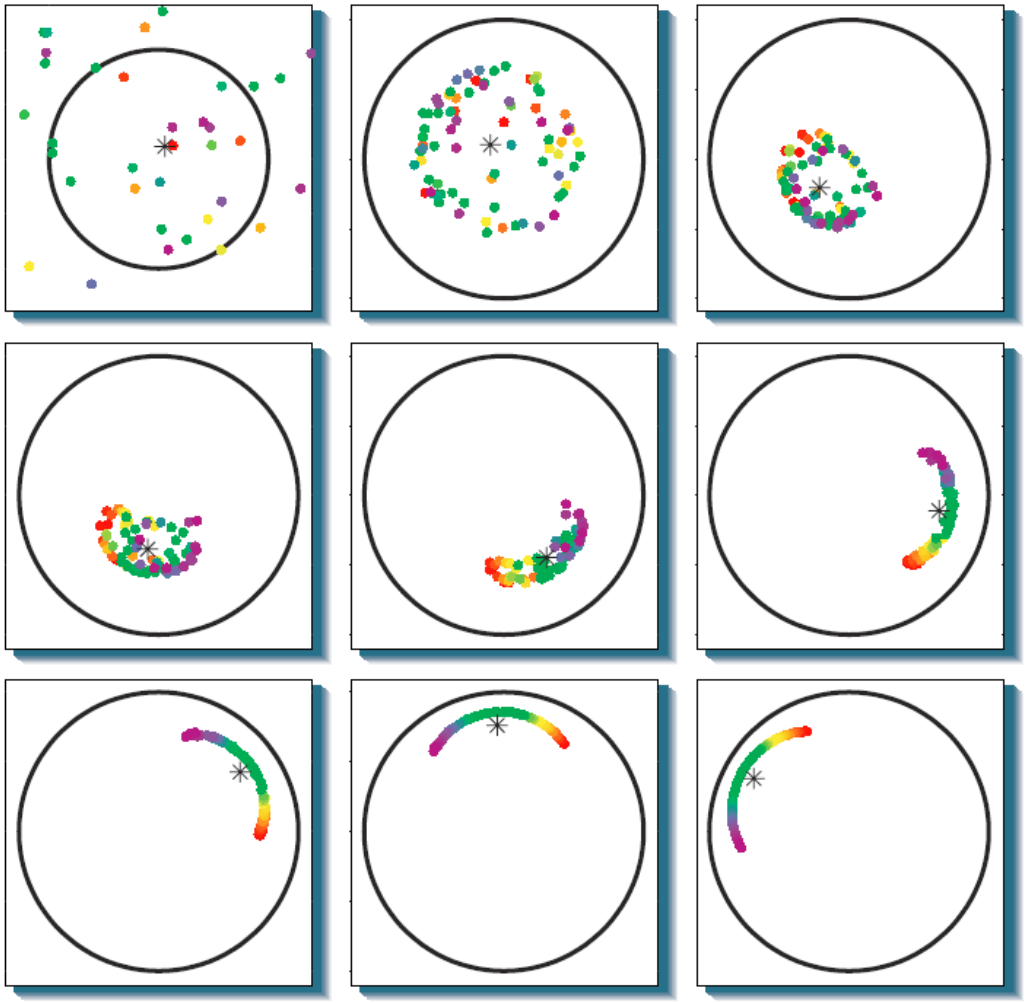
How to quantify?

With the **order parameter**: $$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

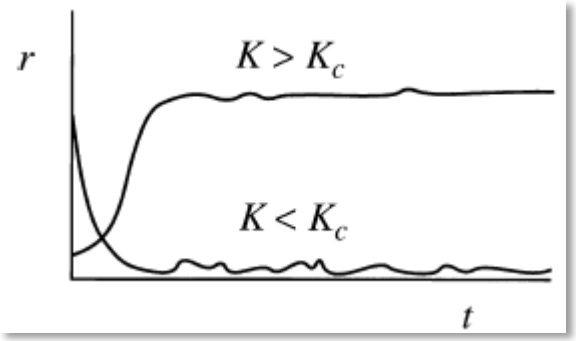
r =0 incoherent state (oscillators scattered in the unit circle)

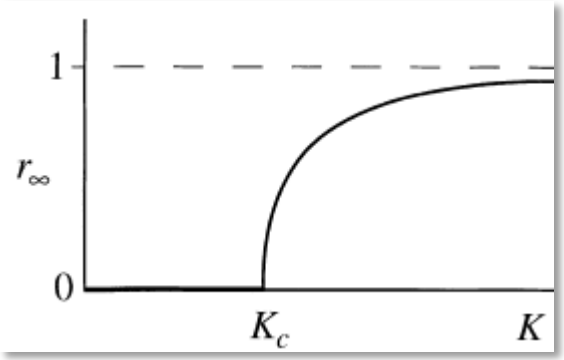
r =1 all oscillators are in phase ($\theta_i = \theta_j \ \forall$ i,j)

Synchronization transition as the coupling strength increases



Strogatz and others, late 90'





Strogatz, Nature 2001

Video: https://www.ted.com/talks/steven_strogatz_on_sync

End of 90's - present

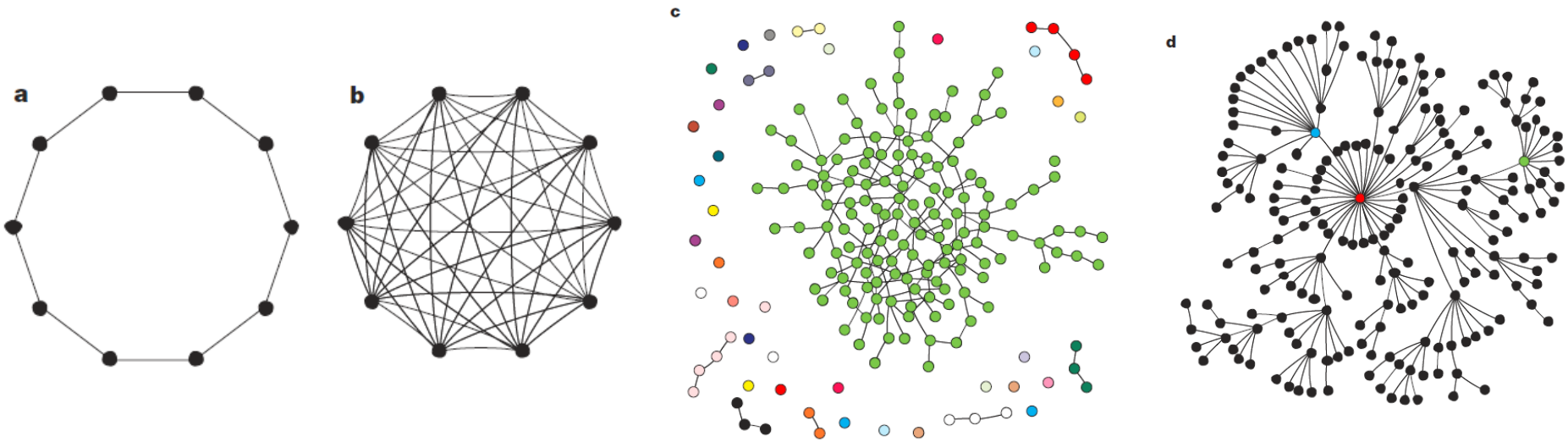
- Interest moves from chaotic systems to complex systems (small vs. very large number of variables).

- Networks (or graphs) of interconnected systems

- **Complexity science**: dynamics of emergent properties
  - Epidemics
  - Rumor spreading
  - Transport networks
  - Financial crises
  - Brain diseases
  - Etc.

Network science

The challenge: to understand how the network **structure** and the **dynamics** (of individual units) determine the collective behavior.



Strogatz
Nature 2001,

Summary

- Dynamical systems allow to
  - understand low-dimensional systems,
  - uncover "order within chaos",
  - uncover universal features
  - control chaotic behavior.

- Complexity science: understanding emerging phenomena in large sets of interacting units.

- Dynamical systems and complexity science are interdisciplinary research fields with many applications.

B. Univariate time-series analysis

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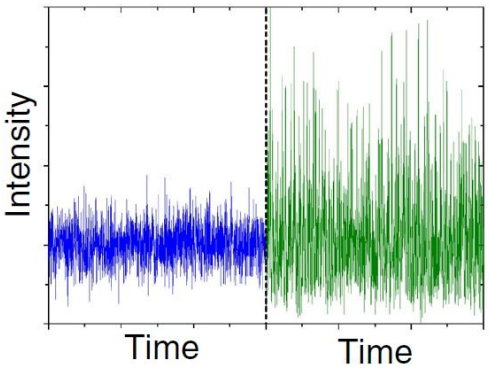
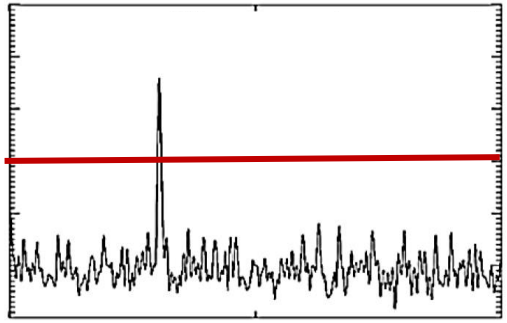
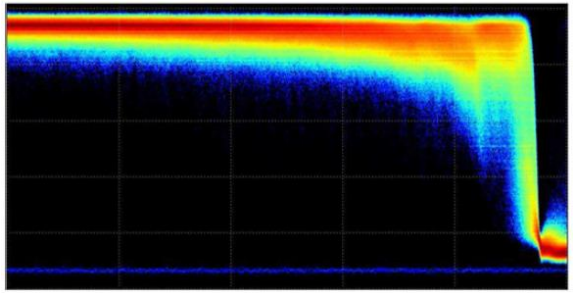
Examples of empirical time series to be discussed

Laser intensity

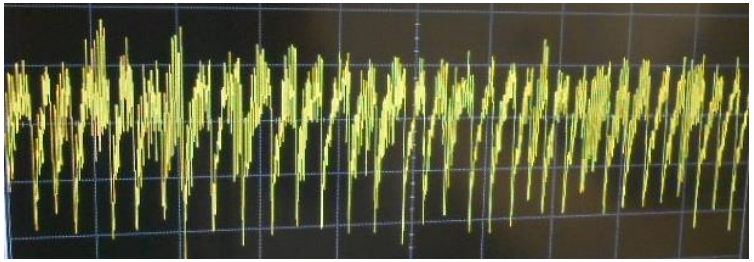




Time



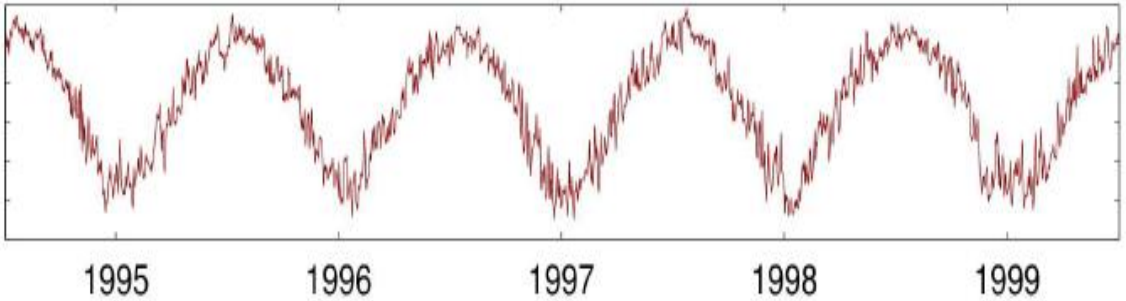
Time



Time

Surface air temperature (SAT)

x



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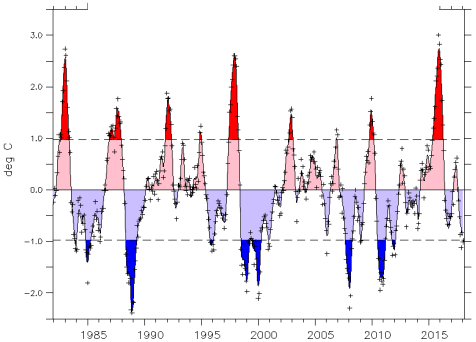
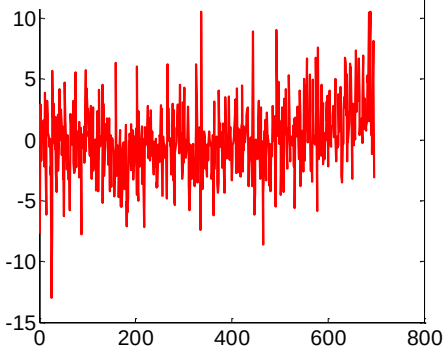
SAT anomalies: solar cycle removed

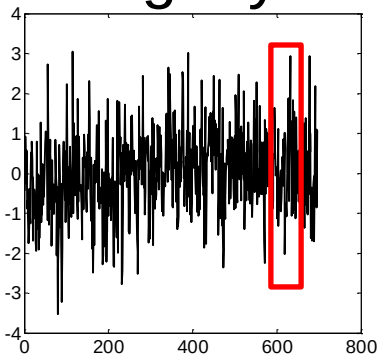
El Niño3.4

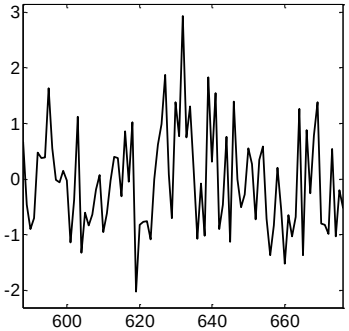


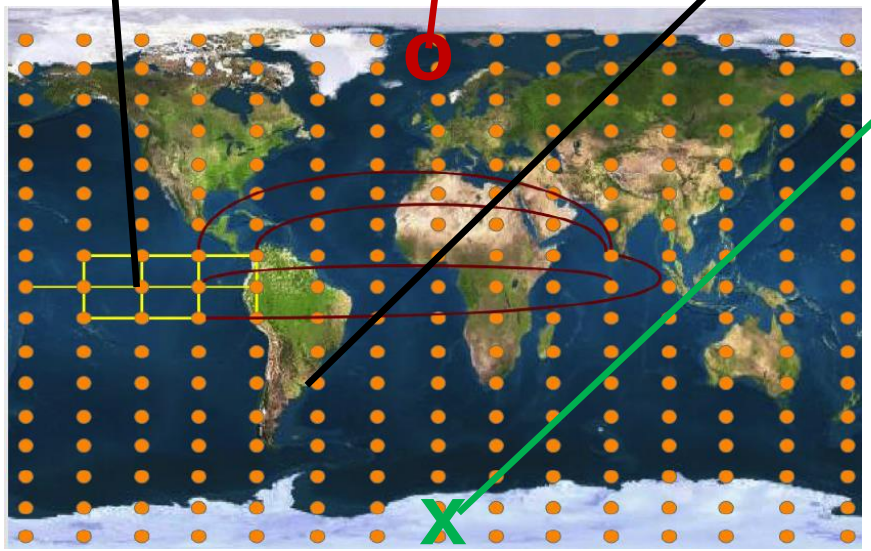
North pole



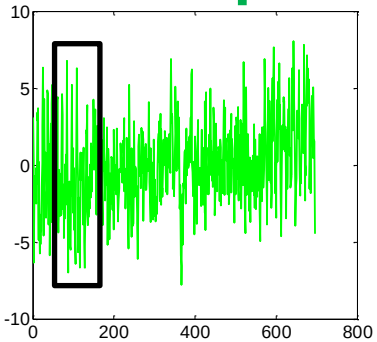
Uruguay

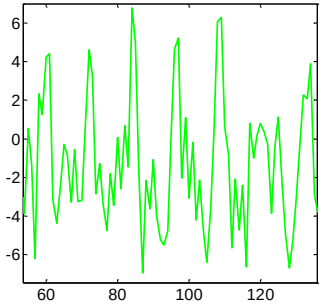






South pole





To begin with

- First step: Look at the data.

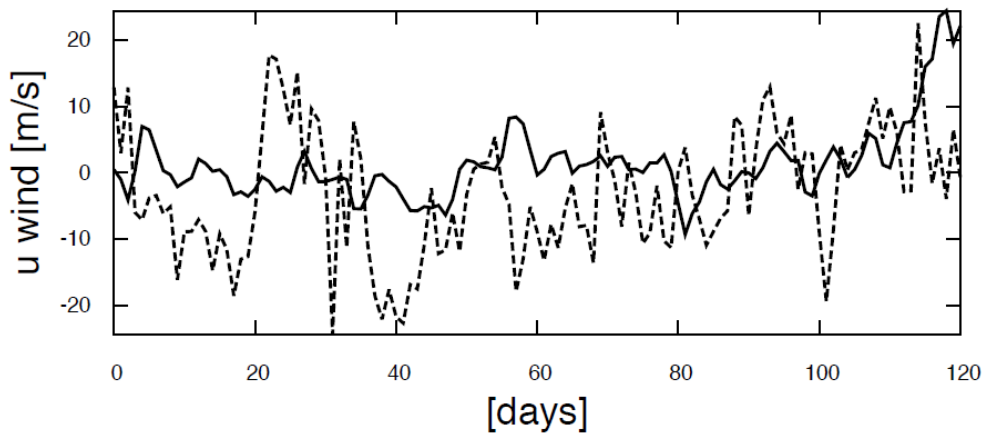
- Examine simple properties:
  - auto correlation,
  - Fourier spectrum,
  - return map (x_i vs $x_{i+\tau}$),
  - histogram

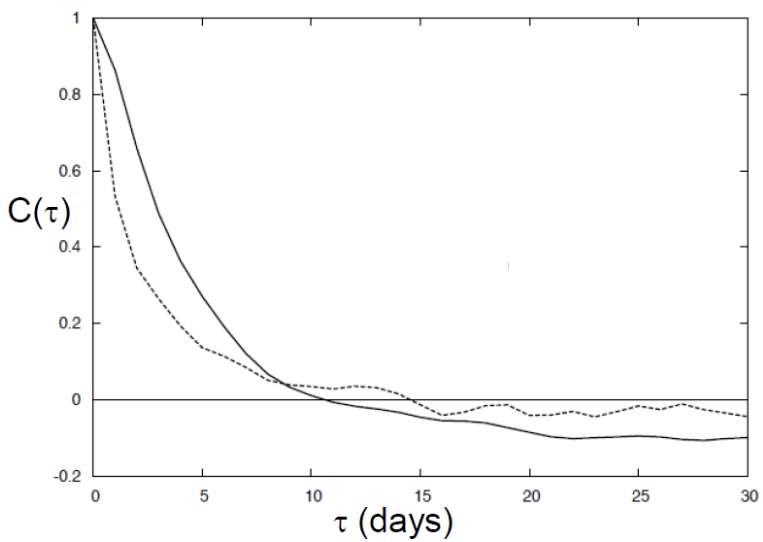
Autocorrelation: measures memory

$$c(\tau) = \frac{\mathrm{cov}(X(t), X(t+\tau))}{\sigma_X^2}$$

$$c(\tau) \sim \exp\{-\tau/\tau_d\}$$

zonal wind (anomalies) at the equator (solid) and at mid-latitudes (dashed)





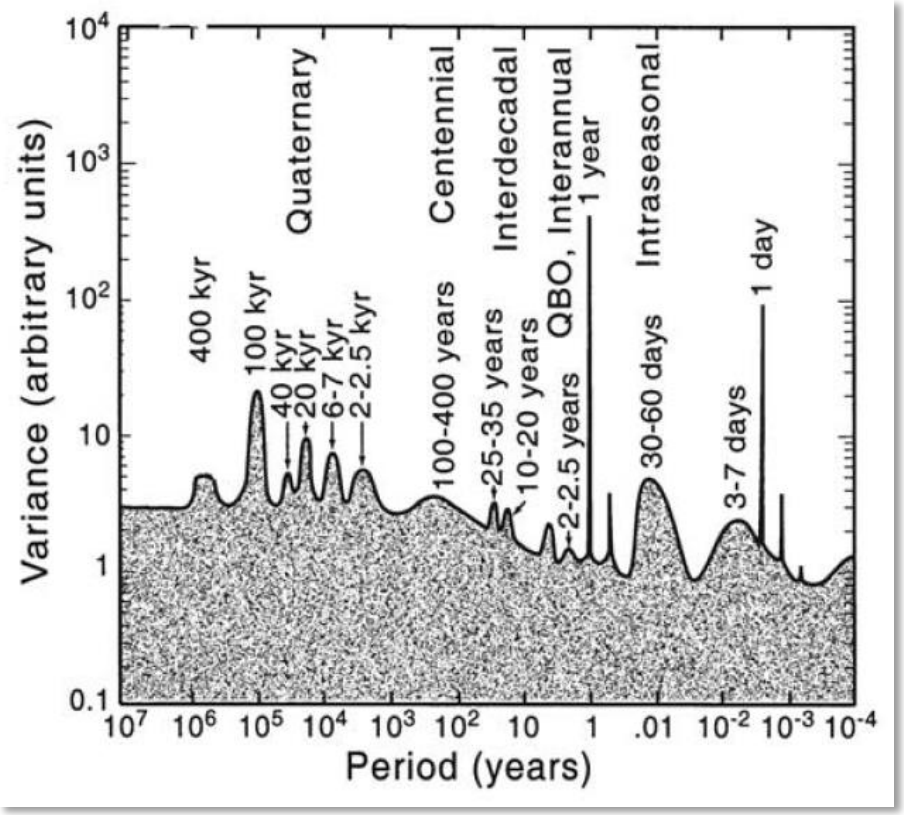
In mid-latitudes the atmosphere is more turbulent and the auto-correlation decay to zero faster than in the equator.

Source: G. Tirabassi PhD thesis

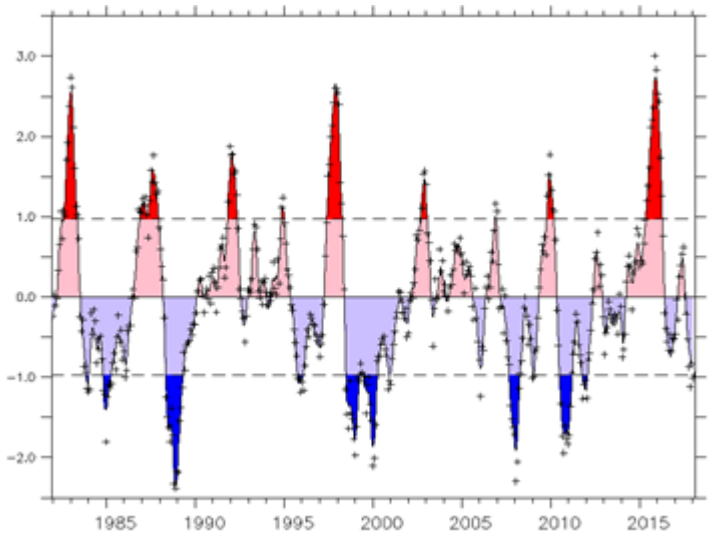
Fourier analysis: characteristic time-scales

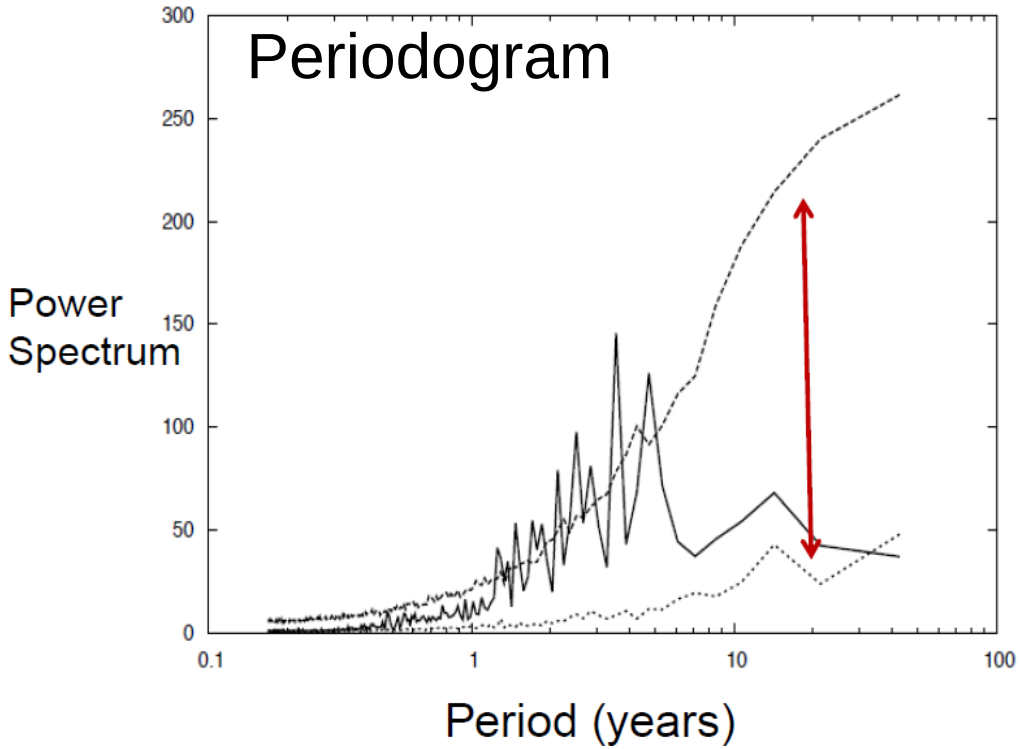
- Our climate: a very complex system with a wide range of time-scales
  - hours to days,
  - months to seasons,
  - decades to centuries,
  - and even longer...

An "artist's representation" of the power spectrum of climate variability (Ghil 2002).



Analysis of NINO3.4 (monthly resolution)





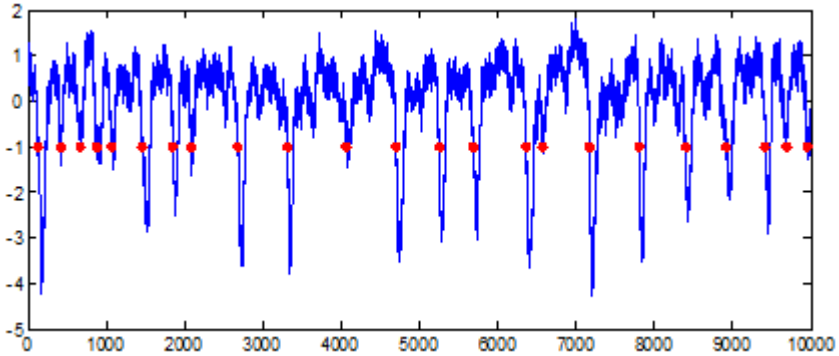
Noise test: 100 noise processes with same autocorrelation at lag 1 and variance as NINO3.4

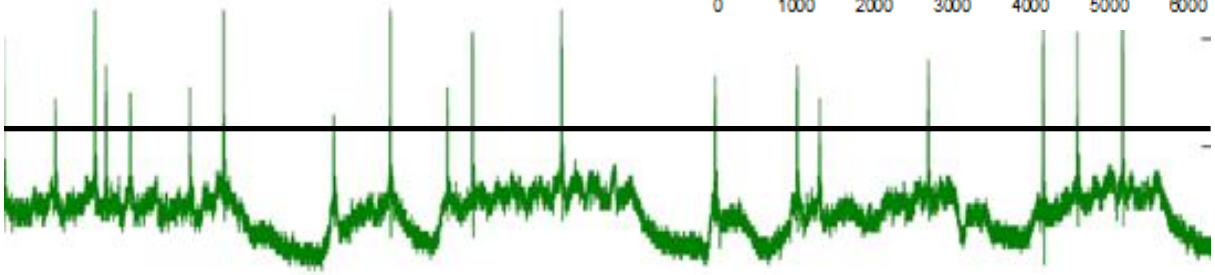
→ significant peaks at 3-4 years

Source: G. Tirabassi PhD thesis

Next: from time-series to sequence of events

- Example: Threshold crossings





inter-spike-intervals (ISIs): $\boxed{\Delta T_i = t_{i+1} - t_i}$

- Problems:
  - How to select the threshold?
  - Threshold dependent results?

ISI distribution (response to sinusoidal forcing)

Empirical neuronal data

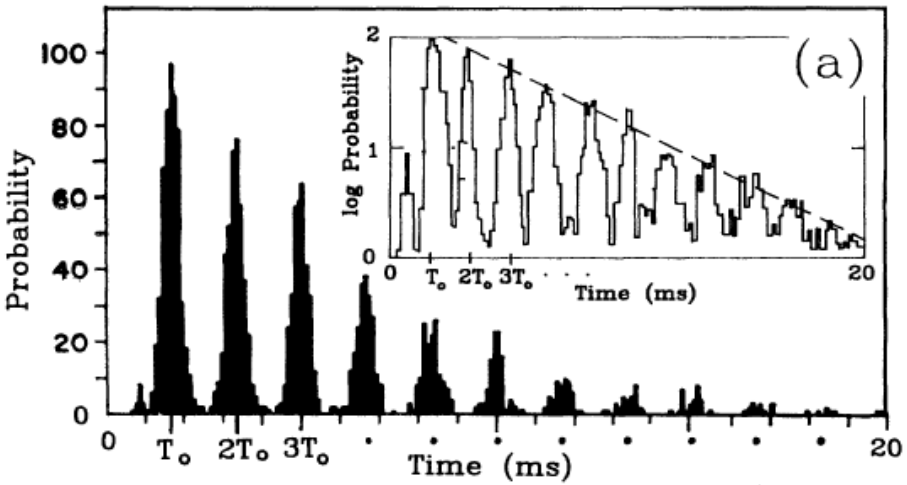
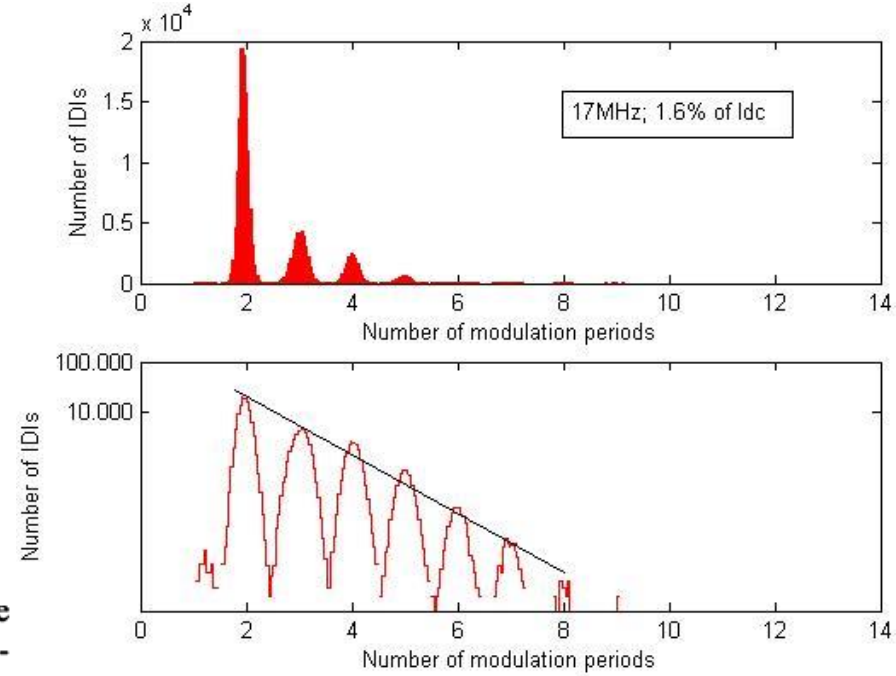


FIG. 1. (a) An experimental ISIH obtained from a single auditory nerve fiber of a squirrel monkey with a sinusoidal 80-dB sound-pressure-level stimulus of period $T_0 = 1.66$ ms applied at the ear. Note the modes at integer multiples of T_0. Inset:

A. Longtin et al, PRL 67 (1991) 656

Laser data from our lab



when a sinusoidal signal is applied to the laser current

Return maps: ΔT_i vs. ΔT_{i+1}

Neuronal ISIs

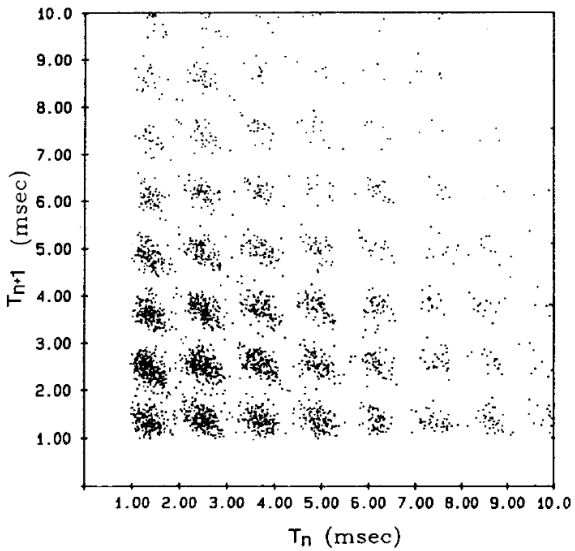
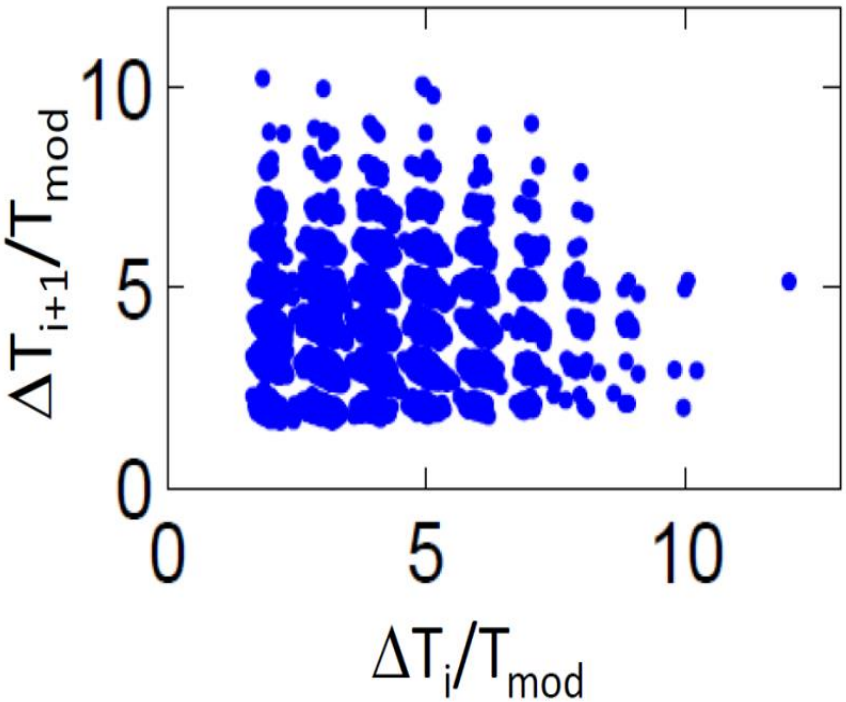


Fig. 4. Scatter plot of spike train data obtained from extracellular measurements of cat auditory fiber activity in response to an 800 Hz 60 dB sound pressure level pure tone presented to the outer ear. The stimulus is discontinuous (see

A. Longtin IJBC 3 (1993) 651

Laser ISIs

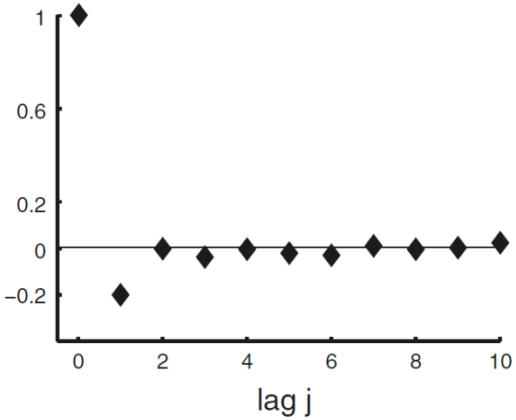


A. Aragoneses et al, Opt. Exp. (2014)

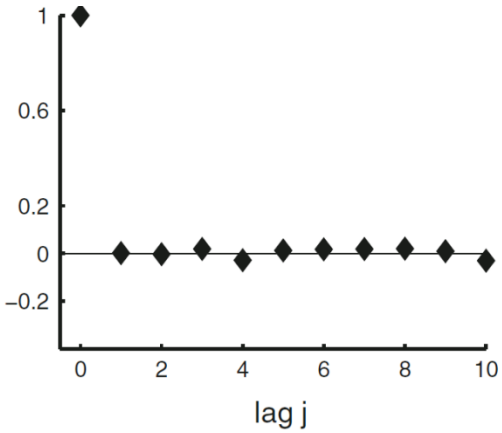
ISI serial correlation coefficients

$$\{\ldots I_{i-1}, I_i, I_{i+1} \ldots\} \qquad C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

Neuron 1

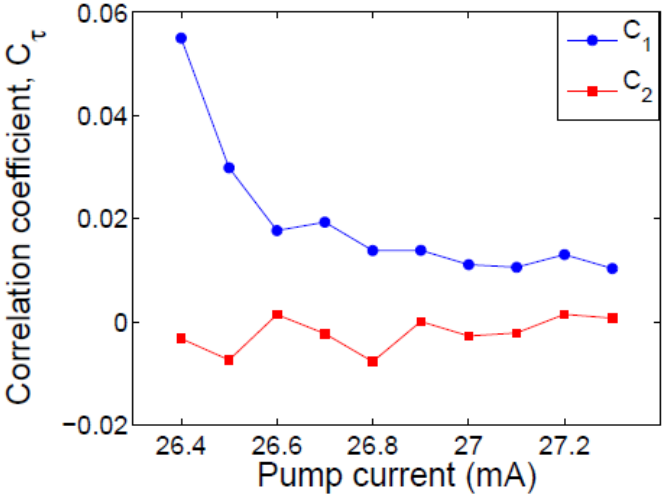


Neuron 2



Exp Brain Res (2011) 210:353–371

Laser ISIs



HOW TO INDENTIFY TEMPORAL ORDER?
MORE/LESS EXPRESSED PATTERNS?

Next step: Methods to identify patterns and structure in time series

1. Phase-space reconstruction
2. Symbolic analysis
3. Mapping a time series into a network
4. Space-time representation of a time series
5. Extracting phase and amplitude information
6. And many more

1. Phase-space reconstruction

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Time-delay coordinates

$$y(t) = \left(s(t),\ s(t+\tau),\ \ldots,\ s(t+(d-1)\tau)\right)$$

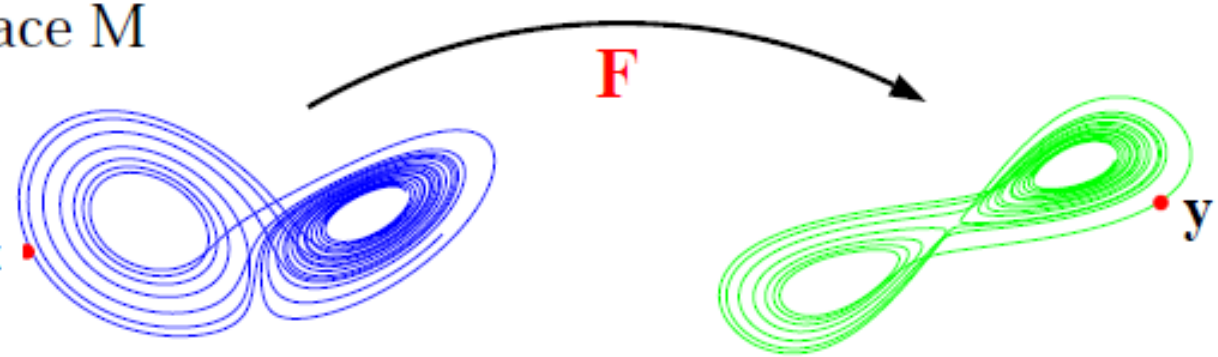
d reconstruction dimension

τ delay time

desirable: F diffeomorphism

state space M

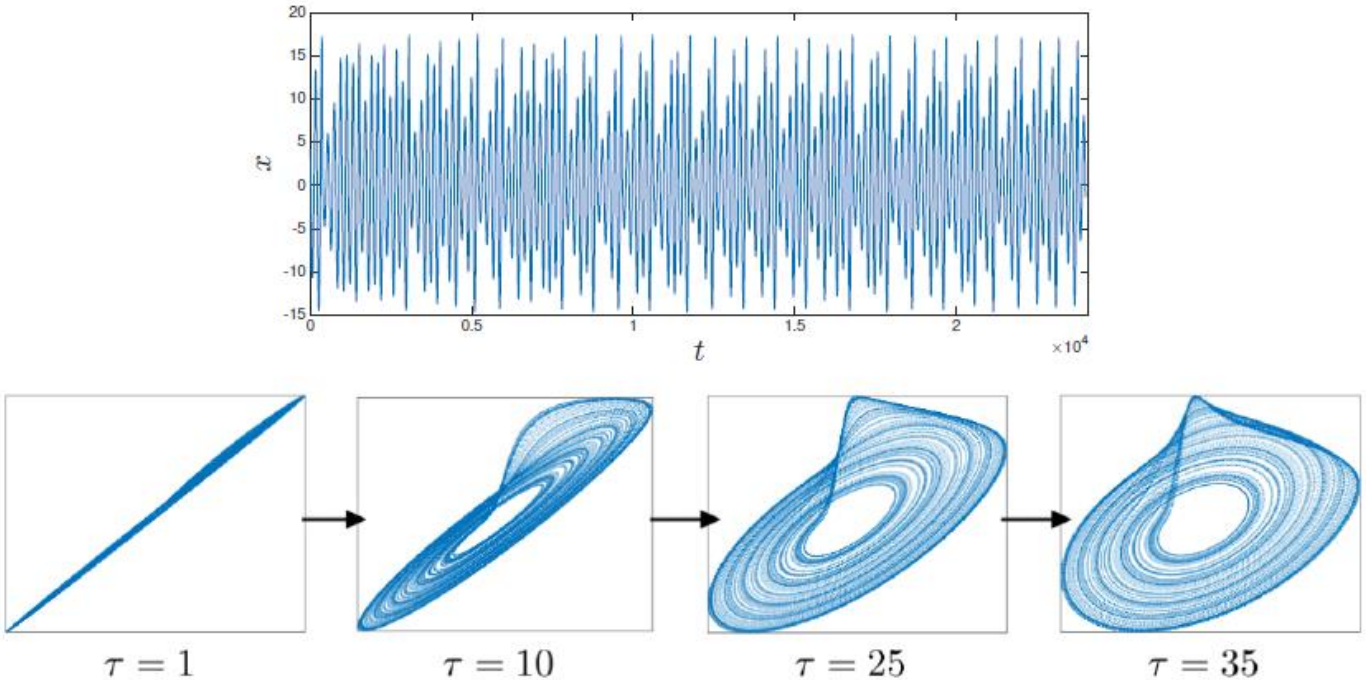
flow ϕ^t



F.Takens, Lecture Notes in Mathematics 366 (1981)
Sauer et al., J. Stat. Phys. 65 (1991) 579

Adapted from U. Parlitz

Reconstruction using delay coordinates



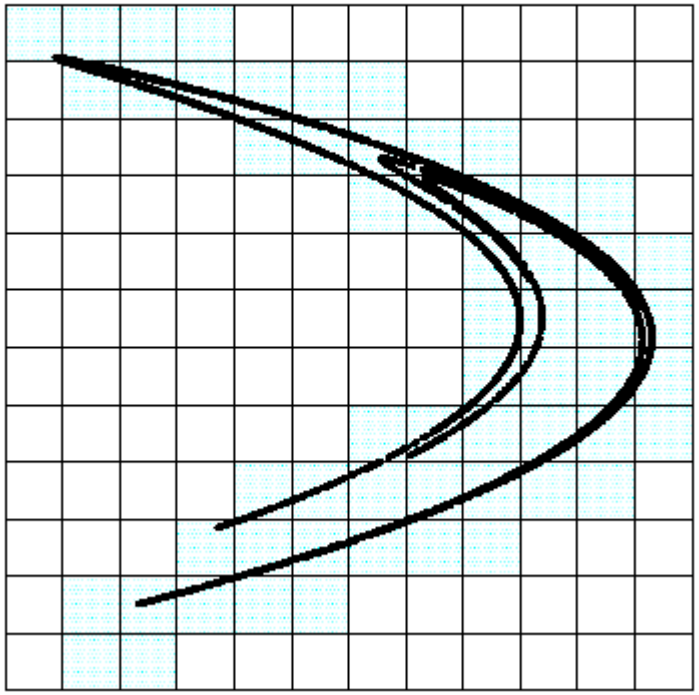
A problem: finding good embedding (lag τ, dimension d)

After reconstructing the attractor, we can characterize the TS by the **fractal dimension** and the **Lyapunov exponent.**

Bradley and Kantz, CHAOS 25, 097610 (2015)

Fractal dimension

covering with cubes of size ε



number of cubes $N(\varepsilon) = \frac{\text{volume} V}{\varepsilon^d}$

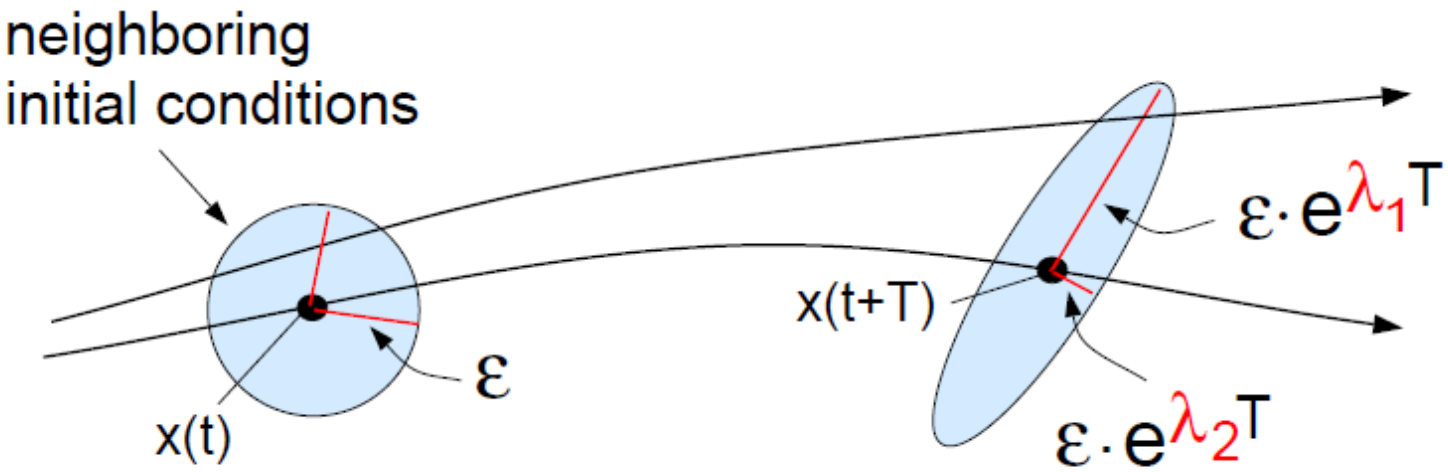
$$d = \frac{\ln N(\varepsilon) - \ln V}{\ln \varepsilon^{-1}}$$

$$d_0 = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln \varepsilon^{-1}}$$

- In general chaotic attractors have fractal dimensions.

Adapted from U. Parlitz

Lyapunov exponents: measure how fast neighboring trajectories diverge



- A stable fixed point has negative λs (since perturbations in any direction die out)
- An attracting limit cycle has one zero λ and negative λs
- A chaotic attractor as at least one positive λ.

Adapted from U. Parlitz

Computing the LEs

- Jacobian methods: compute the complete set of LE from the Jacobian of the underlying flow. Since the dimension of the phase space is often unknown, the number of LEs is unknown.

- Direct methods: compute the largest LE, directly from the time series.

Steps to compute the maximum LE

- Initial distance $\delta_I = |s_i - s_j|$
- Final distance $\delta_F = |s_{i+T} - s_{j+T}|$
- Local *exponential* grow $\lambda^*_{local} = \frac{1}{T}\log(\delta_F/\delta_I)$
- The rate of grow is averaged over the attractor, which gives λ_{max}

A. Wolf, J.B. Swift, H.L. Swinney and J.A. Vastano, Physica D 16 (1985)

a word of warning!

- The rate of grow depends on the direction in the phase space.
- The algorithm returns the value in the direction with fastest expansion.
- Therefore, the algorithm always returns a positive number!
- This is a problem when computing the LE from noise-contaminated data.

Read more: Chaos vs. noise in experimental data.
F. Mitschke and M. Damming, IJBC 3, 693 (1993)

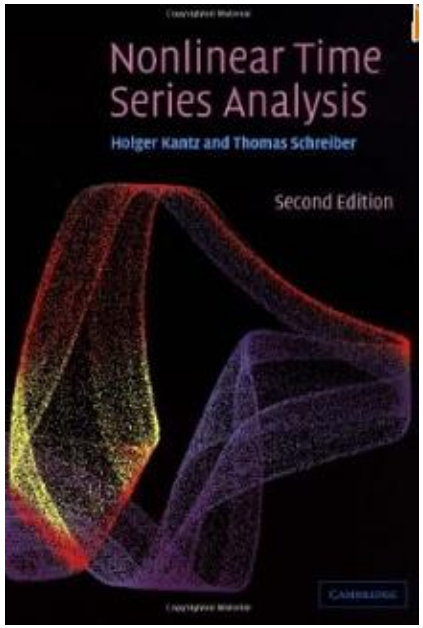
Read more

CHAOS VOLUME 9, 1999

Practical implementation of nonlinear time series methods: The TISEAN package

Rainer Hegger and Holger Kantz[a)]
Max Planck Institute for Physics of Complex Systems, Nöthnitzer Str. 38, D-01187 Dresden, Germany

Thomas Schreiber
Department of Physics, University of Wuppertal, D-42097 Wuppertal, Germany



CHAOS **25**, 097610 (2015)

Nonlinear time-series analysis revisited

Elizabeth Bradley[1,a)] and Holger Kantz[2,b)]
[1]*Department of Computer Science, University of Colorado, Boulder, Colorado 80309-0430, USA and Santa Fe Institute, Santa Fe, New Mexico 87501, USA*
[2]*Max Planck Institute for the Physics of Complex Systems, Noethnitzer Str. 38 D, 01187 Dresden, Germany*

2. Symbolic analysis

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Symbolic analysis

- The time series $\{x_1, x_2, x_3, \ldots\}$ is transformed (using an appropriated **rule**) into a sequence of symbols $\{s_1, s_2, \ldots\}$
- taken from an "**alphabet**" of possible symbols $\{a_1, a_2, \ldots\}$.
- Then consider "blocks" of D symbols ("**patterns**" or "**words**").
- All the possible words form the "**dictionary**".

- Then analyze the "**language**" of the sequence of words
  - the probabilities of the words,
  - missing/forbidden words,
  - transition probabilities,
  - information measures (entropy, etc).

Threshold transformation (phase space partition)

- if $x_i > x_{th} \Rightarrow s_i = 0$; else $s_i = 1$

  transforms a time series into a sequence of 0s and 1s, e.g., {011100001011111…}
- Considering "blocks" of D letters gives the sequence of words. Example, with D=3:

  {011 100 001 011 111 …}
- The number of words (patterns) grows as 2^D
- More thresholds allow for more letters in the "alphabet" (and more words in the dictionary). Example:

  if $x_i > x_{th1} \Rightarrow s_i = 0$;

  else if $x_i < x_{th2} \Rightarrow s_i = 2$;

  else $(x_{th2} < x_i < x_{th1}) \Rightarrow s_i = 1$.

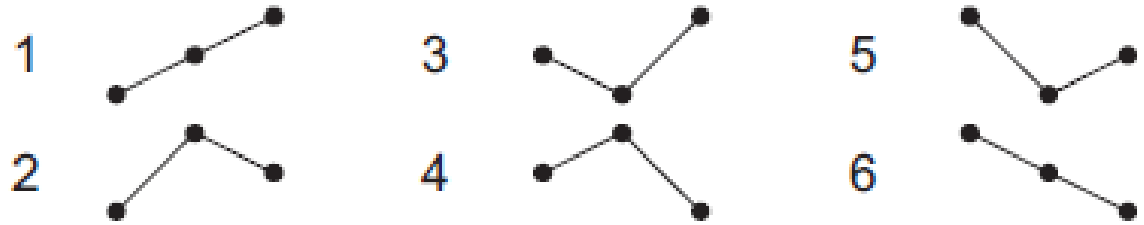
Alternative rule

- **Ordinal** rule: if $x_i > x_{i-1} \Rightarrow s_i = 0$; else $s_i = 1$

  also transforms a time-series into a sequence of 0s and 1s **<u>without</u> using a threshold**

- "words" of D letters are formed by considering the **order relation** between sets of D values $\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$.

D=3

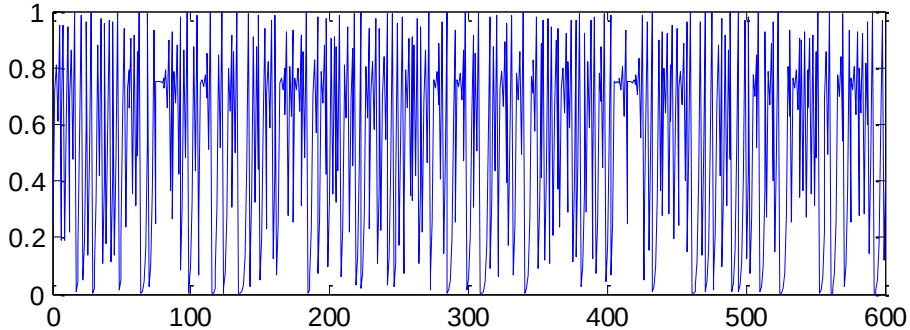


Bandt and Pompe PRL 88, 174102 (2002)

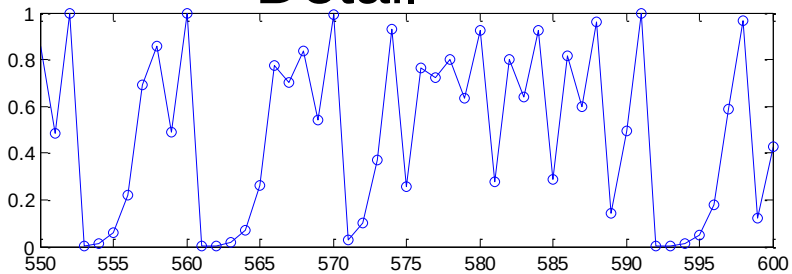
Logistic map

$$x(i+1) = r\ x(i)[1 - x(i)]$$

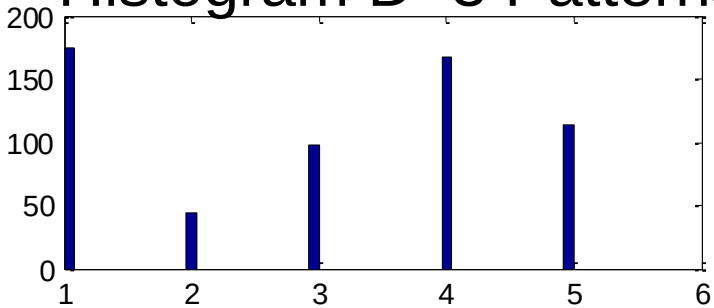
Time series



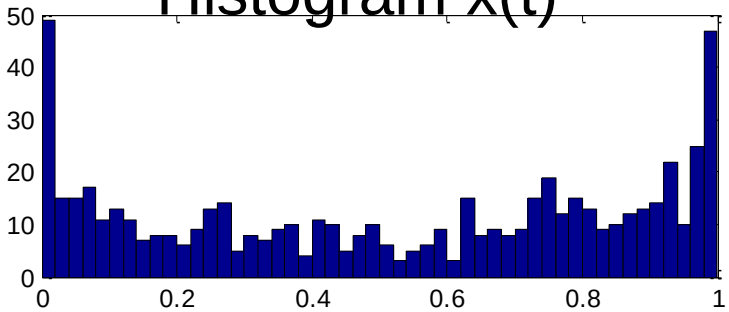
Detail

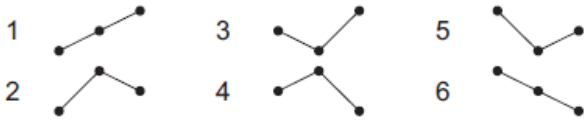


Histogram D=3 Patterns



Histogram x(t)

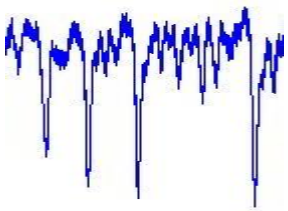




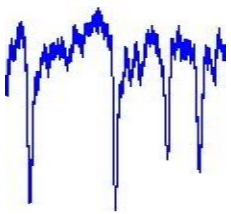
↑ **forbidden**

Ordinal analysis yields information about more and less expressed patterns in the data.

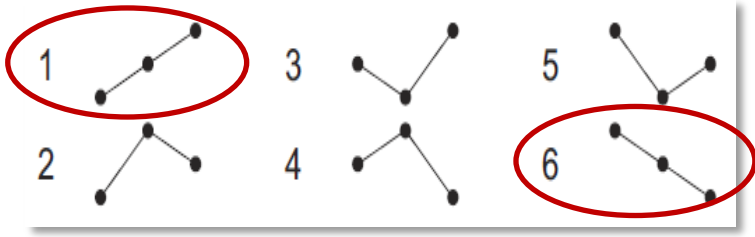
- **D=3**: correlations among 3 ***inter-spike-intervals*** (ISIs).



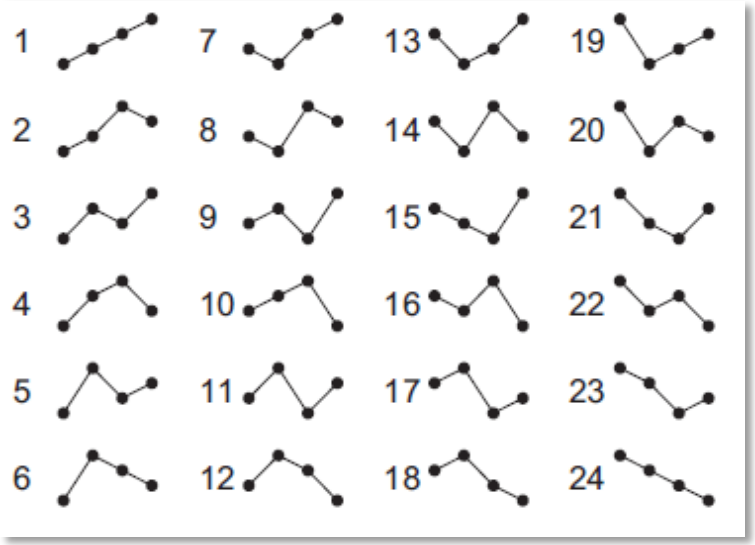
⇒**012**



⇒**210**



- The number of patterns grows as **D!**



- How to quantify the information?
  - Permutation entropy (more latter)

$$s_p = -\sum p_i \log p_i$$

- How to select optimal D? depends on:
  - The length of the data.
  - The length of the correlations

U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Comparison

Threshold transformation:

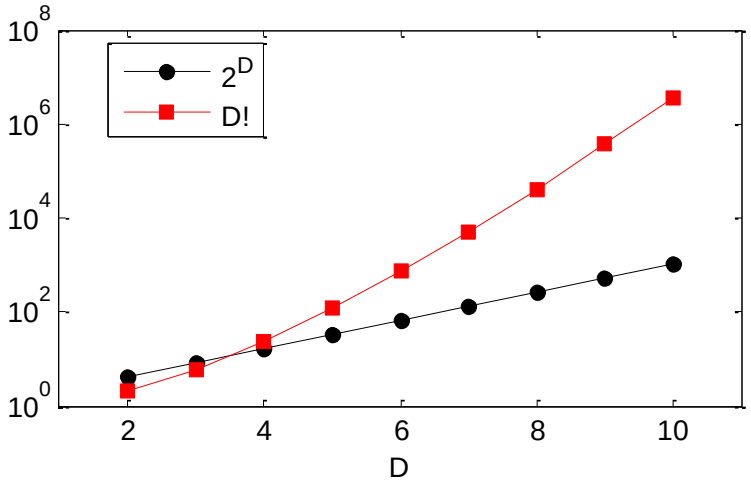
if $x_i > x_{th} \Rightarrow s_i = 0$; else $s_i = 1$

- Advantage: keeps information about the magnitude of the values.
- Drawback: how to select an adequate threshold ("partition" of the phase space).
- 2^D

Ordinal transformation:

if $x_i > x_{i-1} \Rightarrow s_i = 0$; else $s_i = 1$

- Advantage: no need of threshold; keeps information about the temporal order in the sequence of values
- Drawback: no information about the actual data values
- D!



Constructing longer words

$$[\ldots\ x(t), x(t+1), x(t+2), x(t+3), x(t+4), x(t+5)\ldots]$$

- But long time series will be required to estimate the probabilities of the fast growing number of words in the dictionary (D!).
- Solution: a lag τ allows considering long time-scales without having to use words of many letters

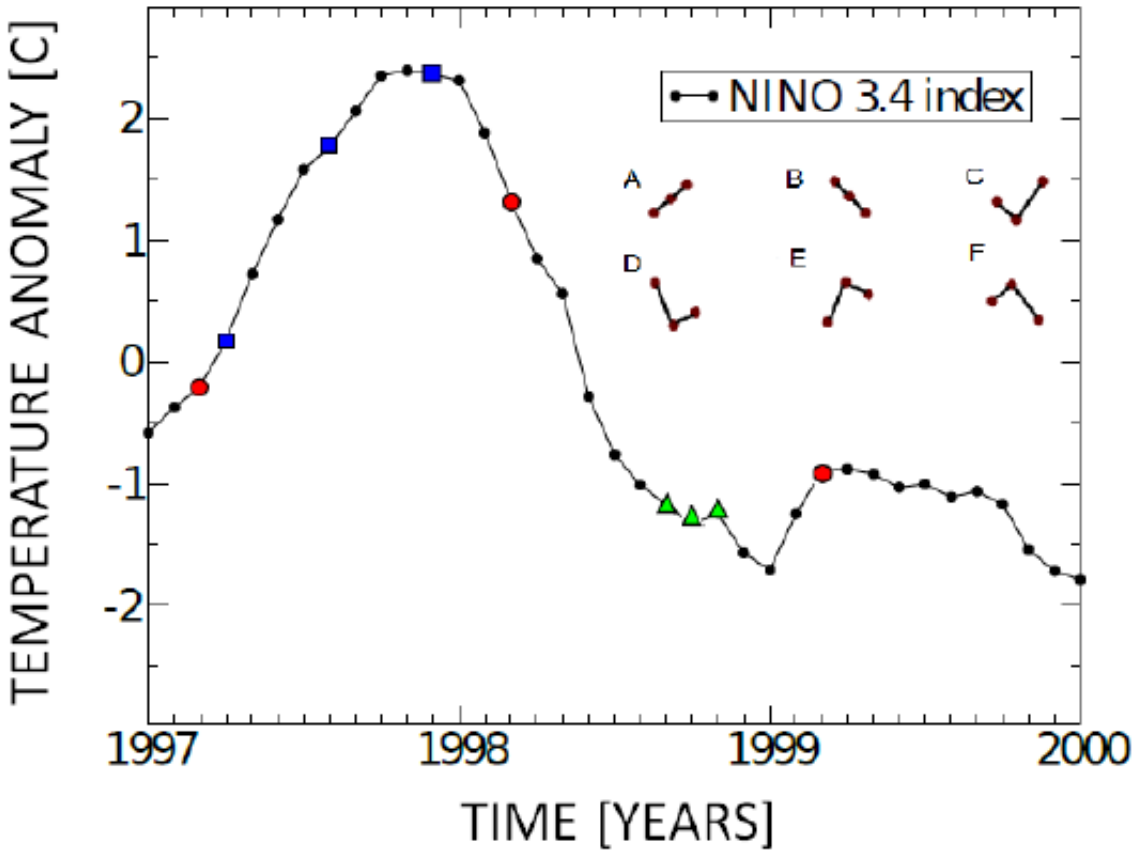
$$[\ldots\ x(t), x(t+2), x(t+4), \ldots]$$

- Example: climatological data (monthly sampled)
  - Consecutive months: $[\ldots x_i(t), x_i(t+1), x_i(t+2)\ldots]$
  - Consecutive years: $[\ldots x_i(t), \ldots\ x_i(t+12), \ldots\ x_i(t+24)\ldots]$
- **Varying τ = varying temporal resolution (sampling time)**

Lag-time is very useful for climate data: allows selecting time-scale of the analysis

- **Green** triangles: intra-seasonal pattern,
- **blue** squares: intra-annual pattern
- **red** circles: inter-annual pattern

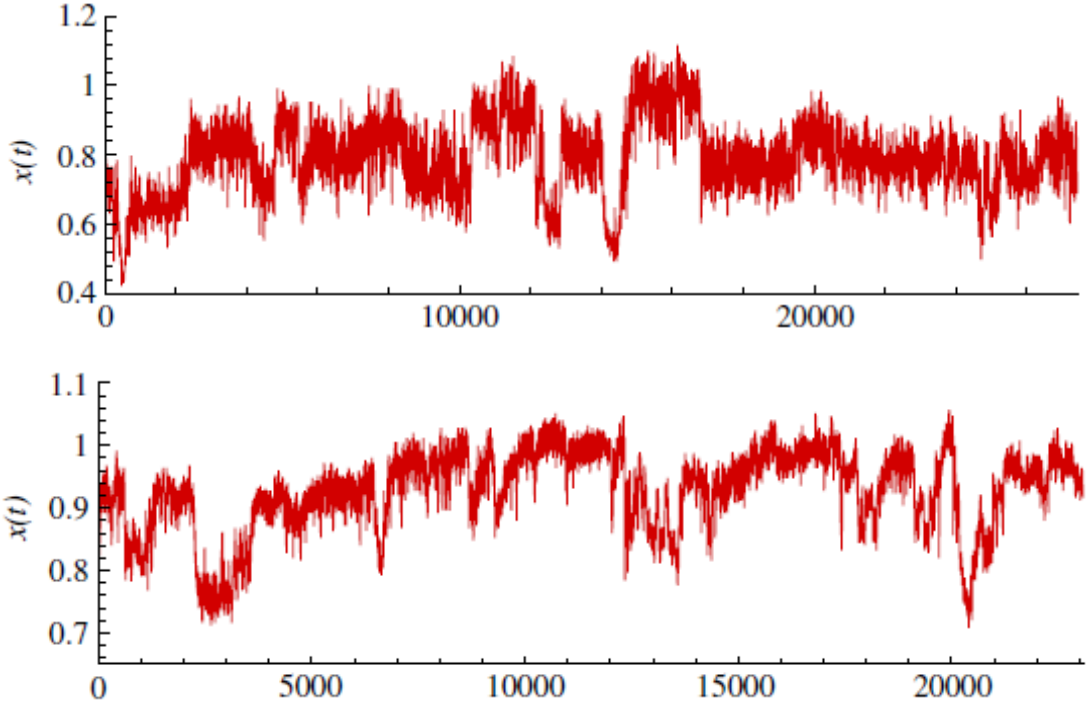
Monthly resolution



J. I. Deza, Eur. Phys. J. ST 222, 511 (2013)

Example of application of ordinal analysis to ECG-signals

Time series of **inter-beat intervals**

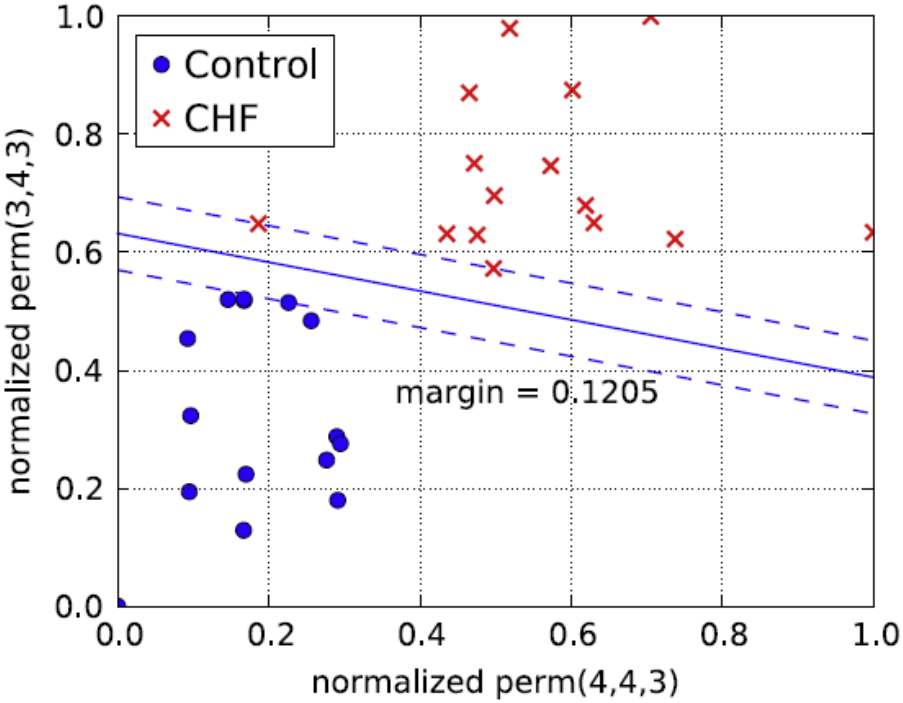


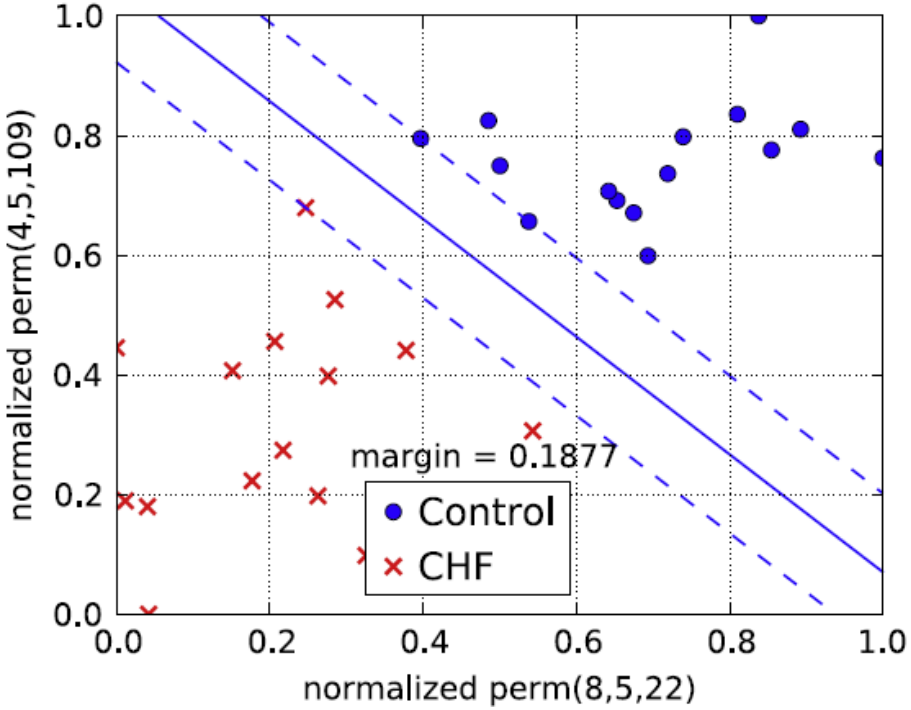
healthy subject

congestive heart failure

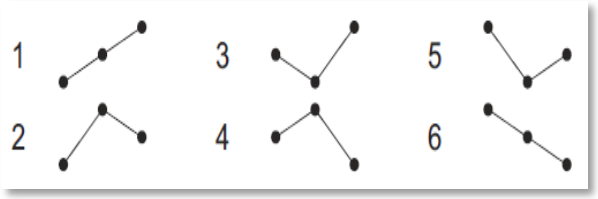
R. Friedrich et al. / Physics Reports 506 (2011) 87–162

Classifying ECG-signals according to the frequency of words





Perm (i,D,lag)



(the probabilities are normalized with respect to the smallest and the largest value occurring in the data set)

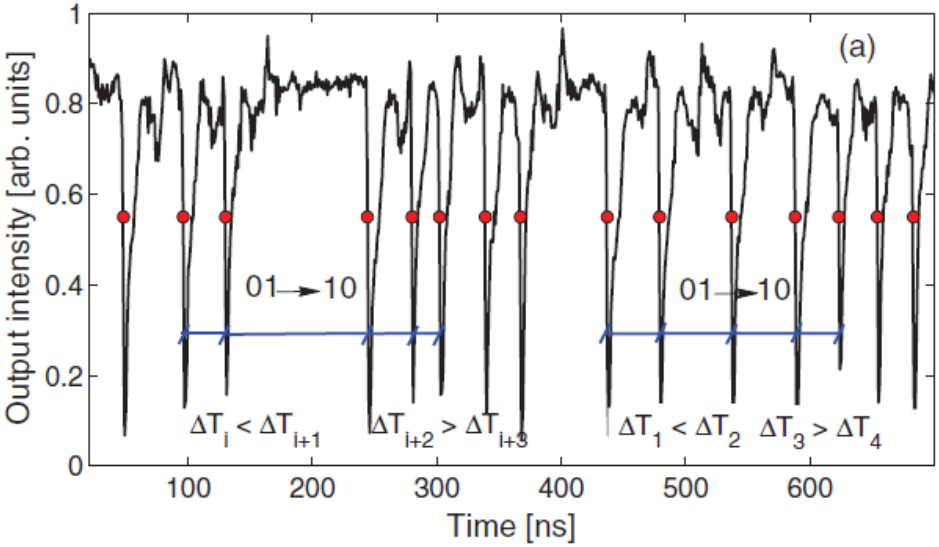
Algorithms

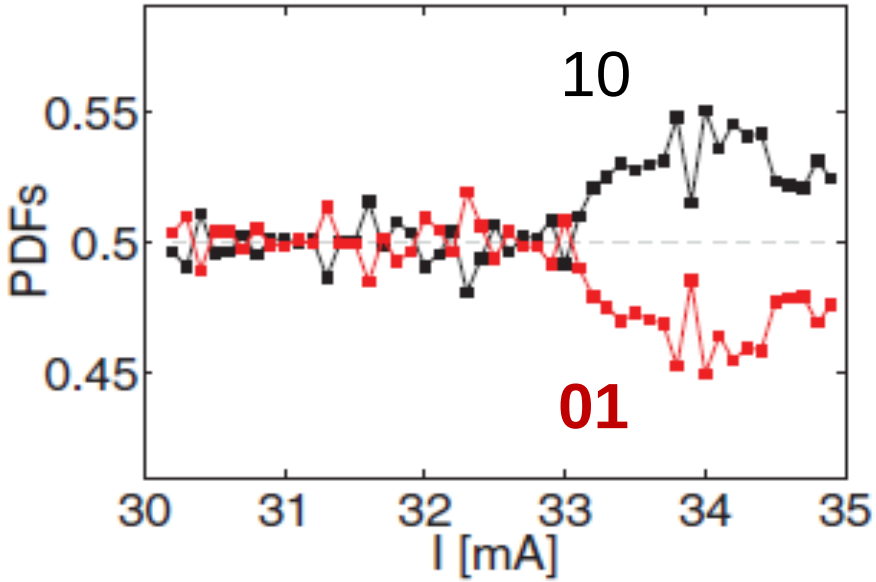
Python and Matlab codes for computing the OP probabilities from a time-series are available here:

U. Parlitz et al. Computers in Biology and Medicine 42, 319 (2012)

Another example of application: the spiking laser output

Video: how complex optical signals emerge from noisy fluctuations





N. Rubido, J. Tiana et al, PRE 84, 026202 (2011)
C. Quintero-Quiroz et al, Sci. Rep. 6, 37510 (2016)

Next?

- Assuming that we have a suitable symbolic description of the time series.
- What **information** can we extract from the sequence of "words"?
- How much information is in the time-series?
- Analogy with deciphering a foreign text.

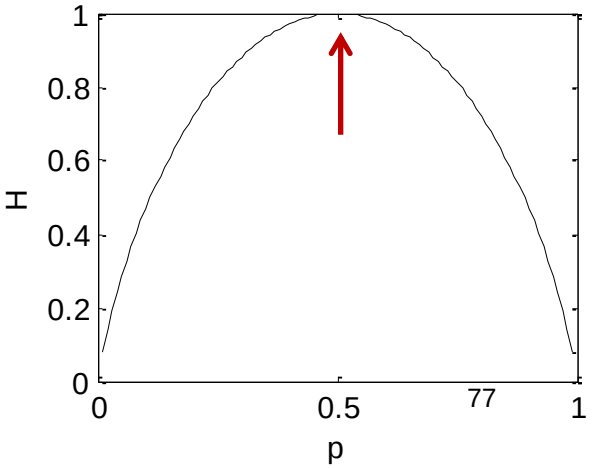
(Information theory quantifiers will be described by Prof. Osvaldo Rosso)

Information theory measure: Shannon entropy

- The time-series is described by a set of probabilities $\sum_{i=1}^{N} p_i = 1$
- Shannon entropy: $H = -\sum_i p_i \log_2 p_i$
- Interpretation: "*quantity of* ***surprise*** *one should feel upon reading the result of a measurement*"

  K. Hlavackova-Schindler et al, Physics Reports 441 (2007)
- Simple example: a random variable takes values 0 or 1 with probabilities: p(0) = p, p(1) = 1 − p.
- H = −p $\log_2$(p) − (1 − p) $\log_2$(1 − p).

  ⇒ p=0.5: Maximum **unpredictability.**



Permutation Entropy: computed from ordinal probabilities

3. Mapping a time series into a network

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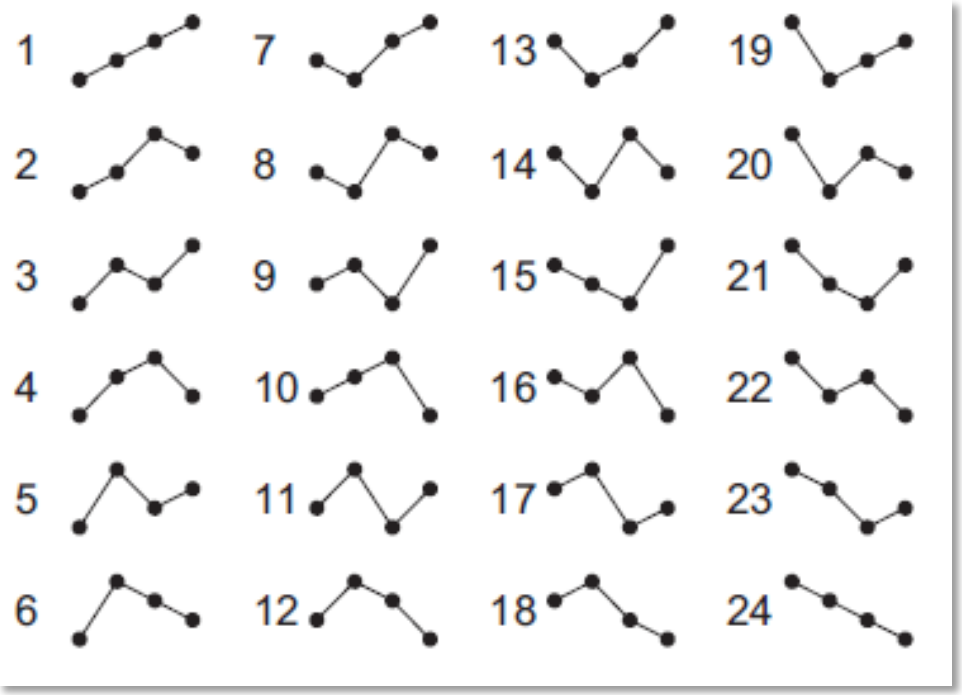
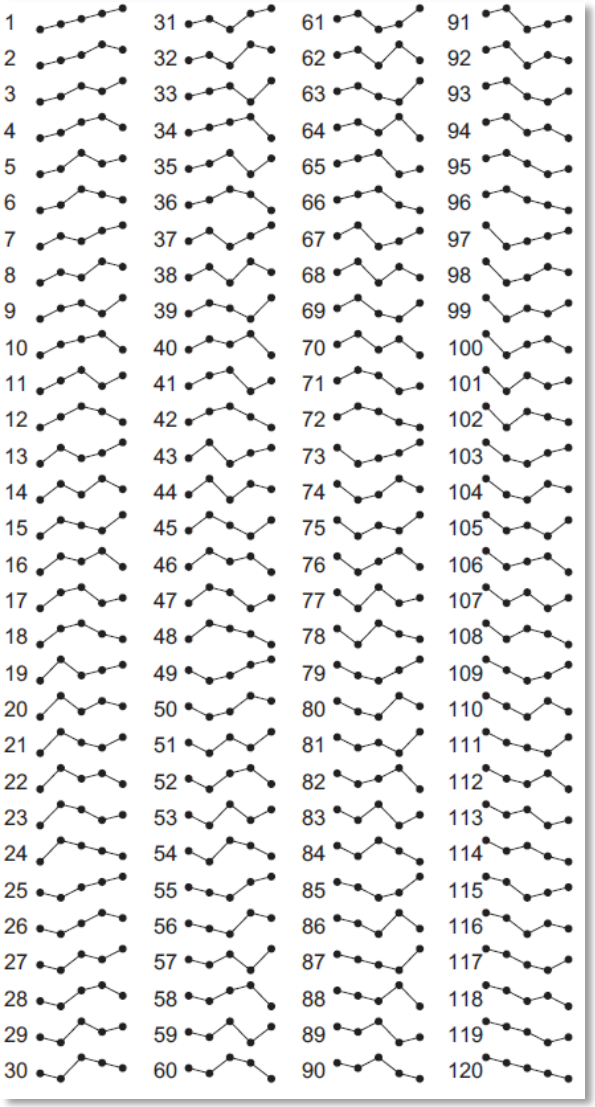
The number of patterns increases as D!

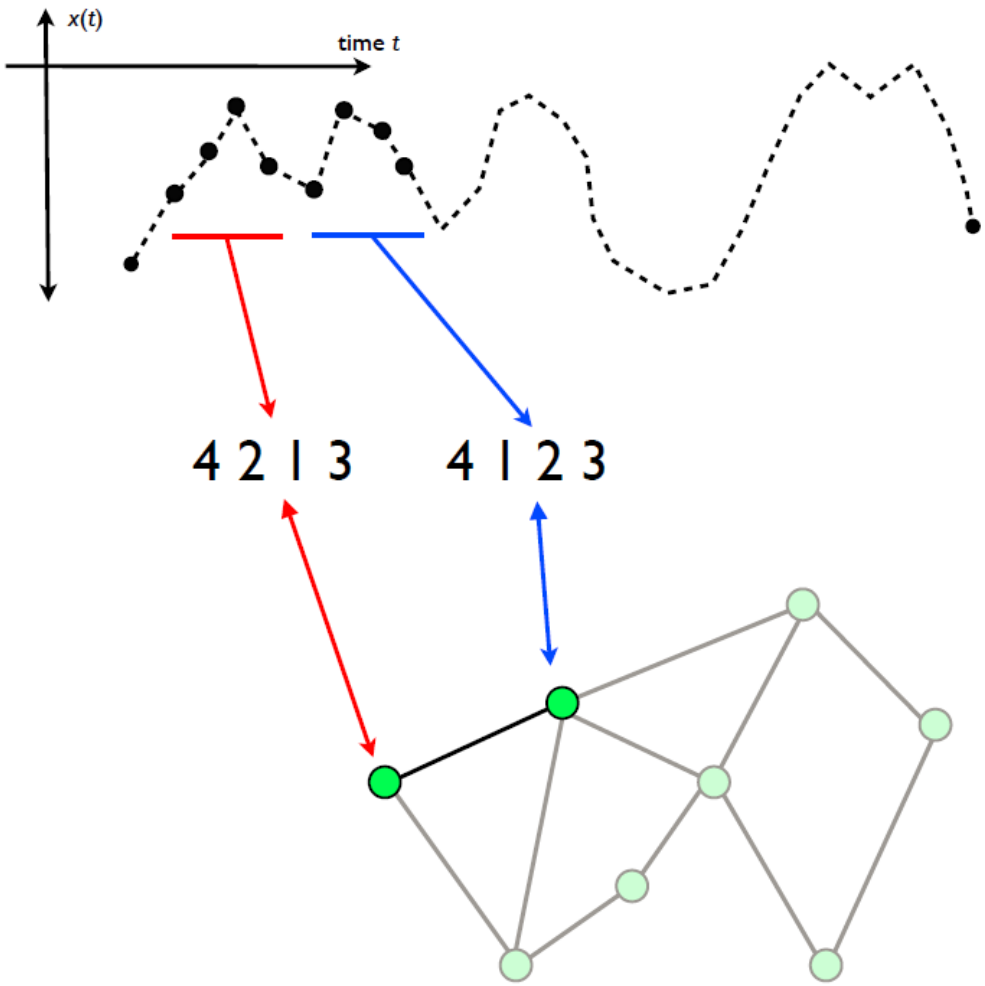




Opportunity: turn a time-series into a network by using the patterns as the "nodes" of the network.

U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

The network nodes are the "ordinal patterns", and the links?



- The links are defined in terms of the probability of pattern "β" occurring after pattern "α".
- <u>Weighs of nodes</u>: the probabilities of the patterns ($\sum_i p_i=1$).
- <u>Weights of links</u>: the probabilities of the transitions ($\sum_j w_{ij}=1\ \forall i$).

$\Rightarrow$ **Weighted and directed network**

Adapted from M. Small (The University of Western Australia)

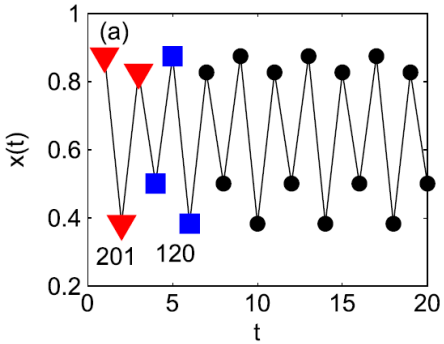
Three network-based diagnostic tools

- Entropy computed from the weights of the nodes (**permutation entropy**)

$$s_p = -\sum p_i \log p_i$$

- Entropy computed from weights of the links (**transition probabilities**, '01'→ '01', '01'→ '10', etc.)

$$w_{ij} = \frac{\sum_{t=1}^{L-1} n\left[s(t) = i,\, s(t+1) = j\right]}{\sum_{t=1}^{L-1} n\left[s(t) = i\right]}$$



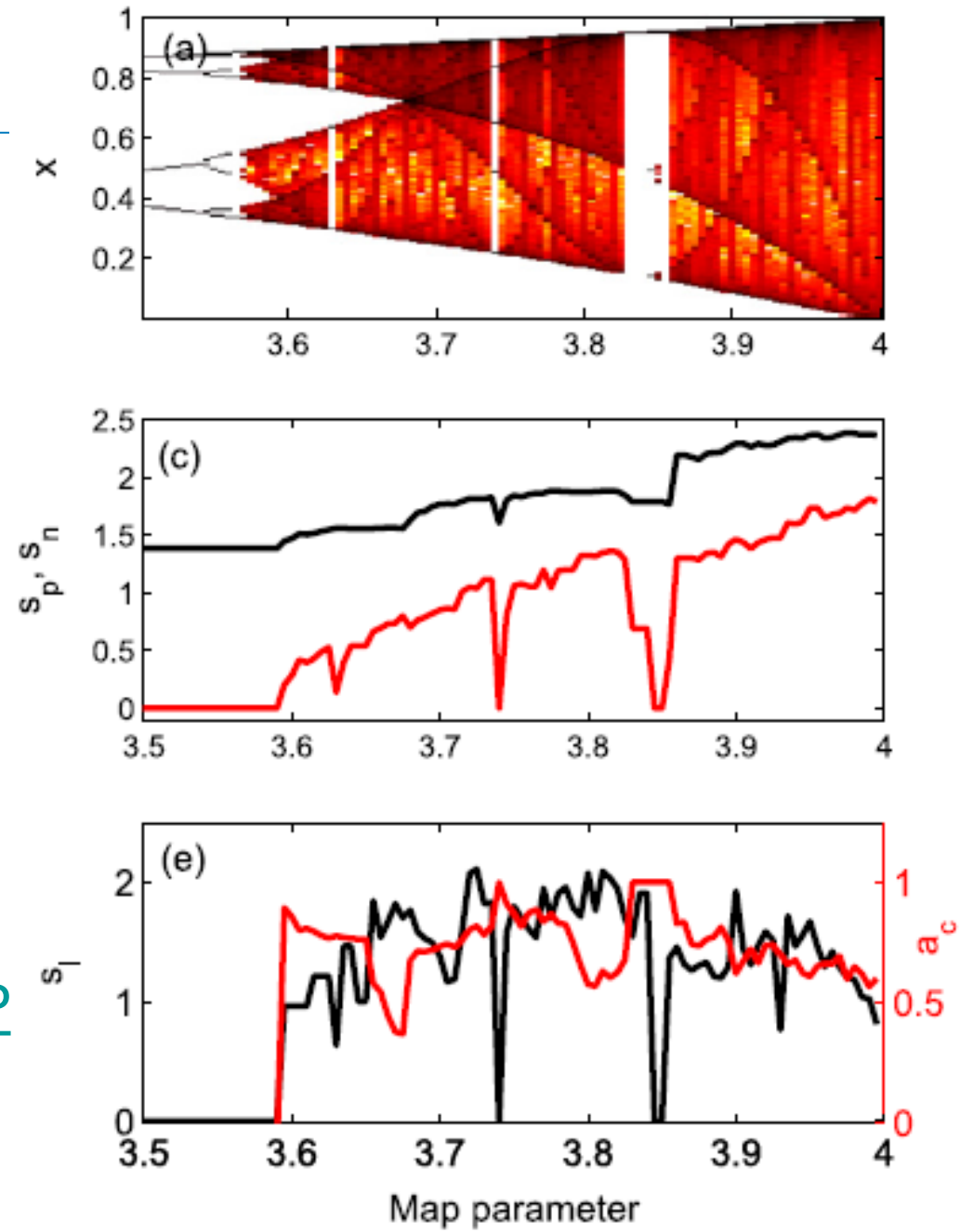
- Asymmetry coefficient: normalized difference of transition probabilities, P('01'→ '10') - P('10'→ '01'), etc.

$$a_c = \frac{\sum_i \sum_{j \neq i} \left| w_{ij} - w_{ji} \right|}{\sum_i \sum_{j \neq i} \left(w_{ij} + w_{ji} \right)}$$

(0 in a fully symmetric network; 1 in a fully directed network)

- Again the Logistic map

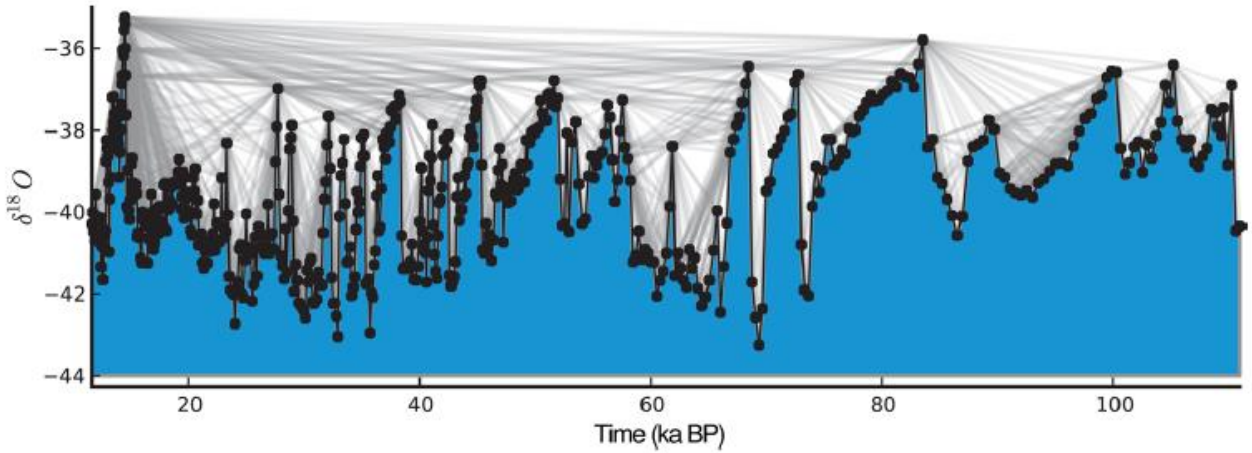
$$x(i+1) = r\, x(i)[1 - x(i)]$$



C. Masoller et al, NJP 17, 023068 (2015)

Another way to turn a time-series into a network: visibility graph

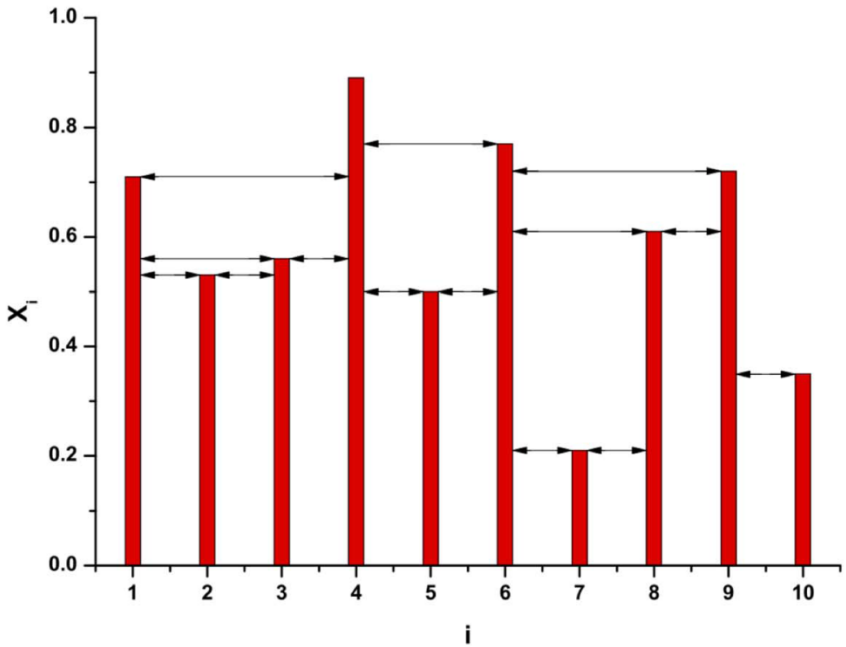
A time-series is represented as a graph, where **each data point is a node.**



- Black dots indicate observations (nodes). Links (gray lines) are added between mutually visible observations.
- Ice core $\delta^{18}O$ isotope record from Greenland during the last glacial allows to study regime shifts in paleoclimate dynamics.

Donges et al, Chaos 2015

A simpler version: the horizontal visibility graph (HVG)



- Rule: data points *i* and *j* are connected if there is "visibility" between them: $I_{max,i}$ and $I_{max,j} > I_{max,n}$ for all n, $i<n<j$

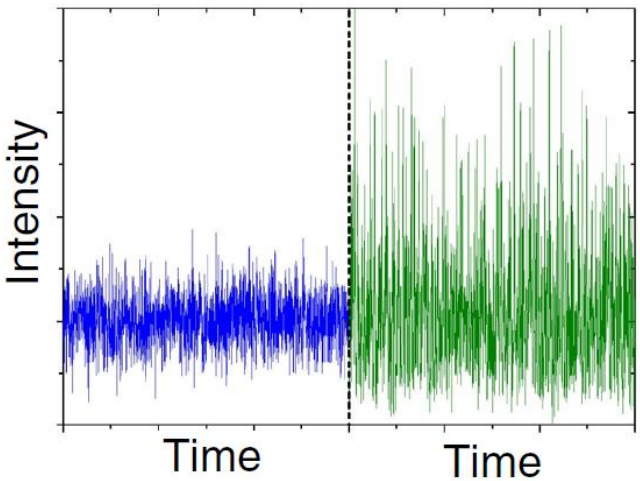
$\Rightarrow$ **Unweighted and undirected graph**

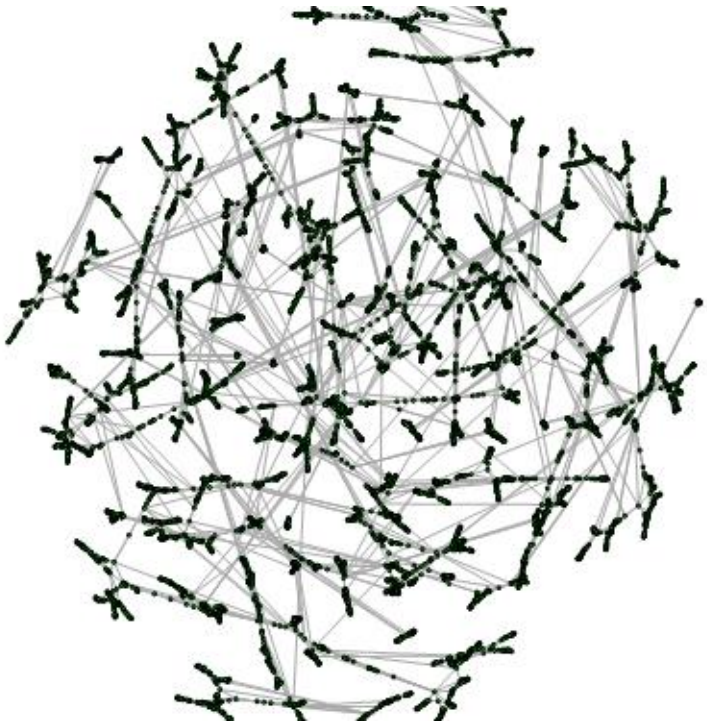
HVG method: B. Luque et al, PRE 80, 046103 (2009)
M. Gomez Ravetti et al, PLOS one 2014

Example

Intensity of a fiber laser

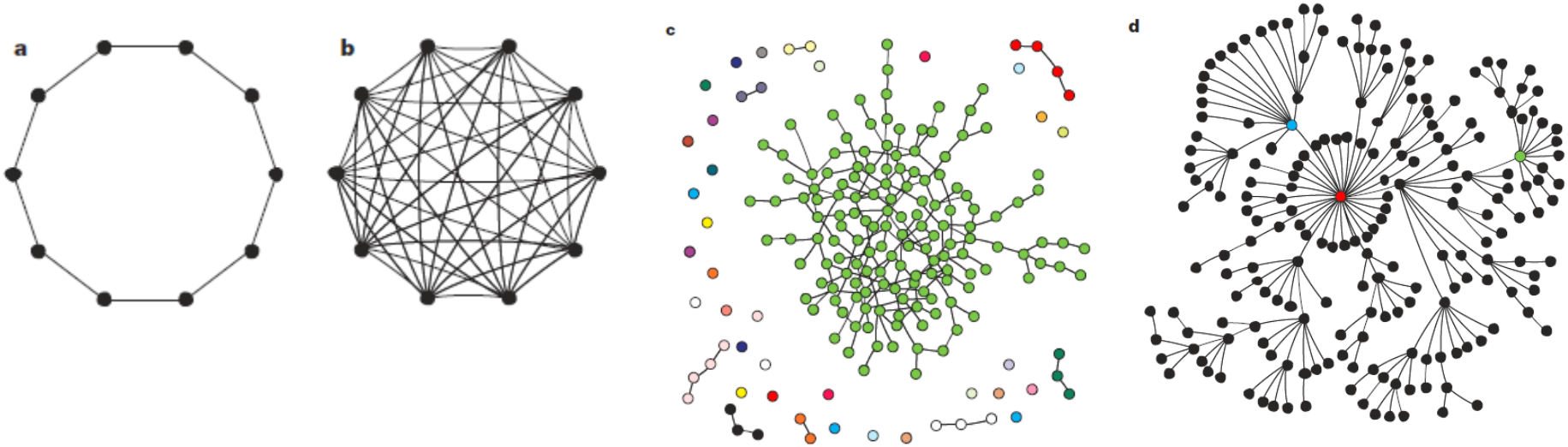
Low -- High pump power



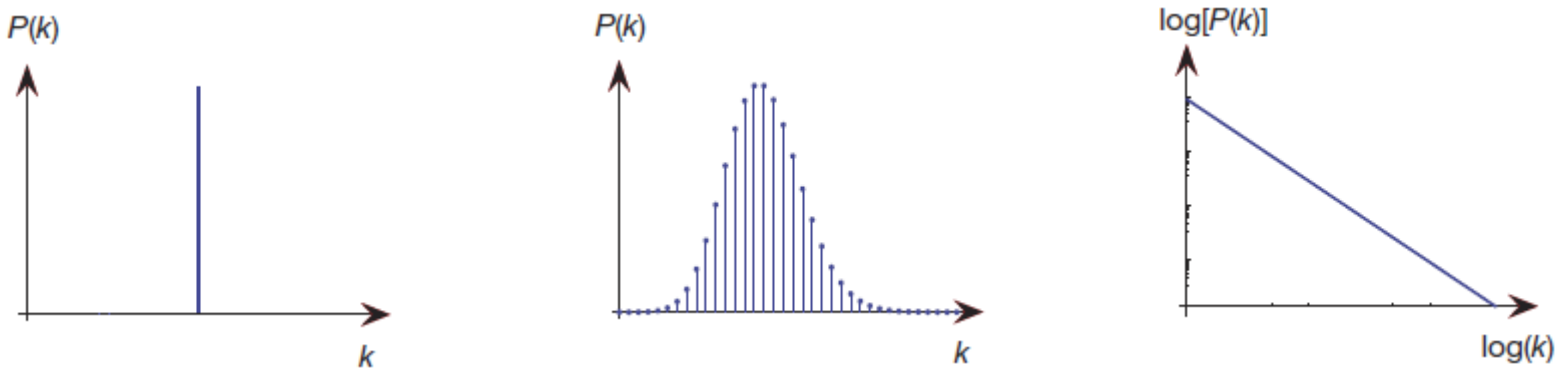


How to characterize the graph?

Degree Distribution (distribution of the number of links)



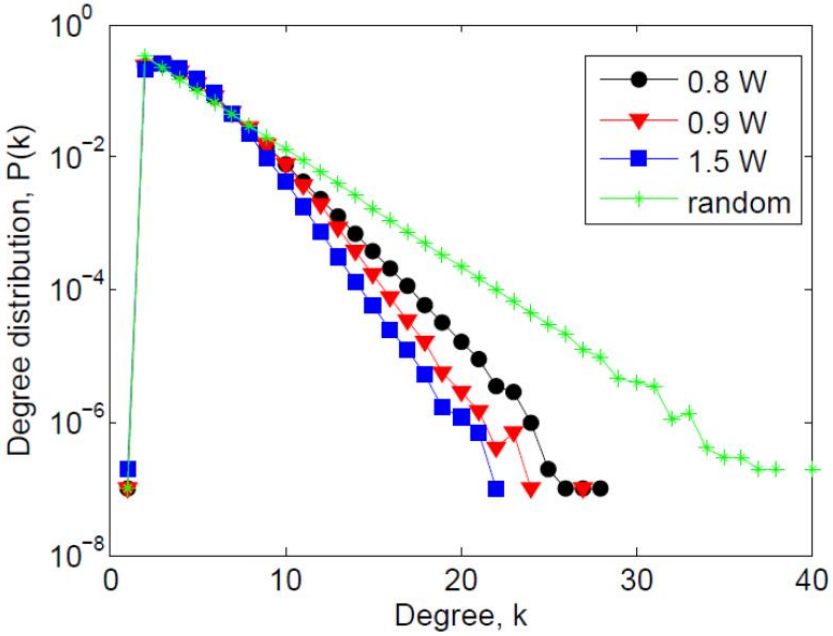
Degree (number of links of a node) distributions:

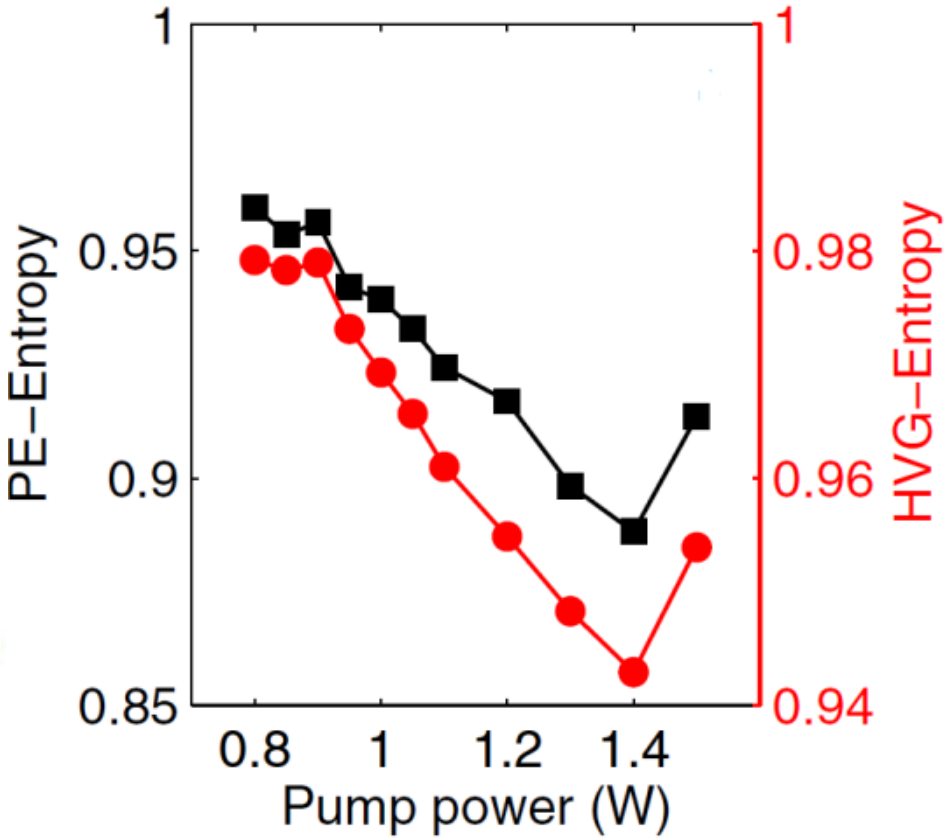


Strogatz, Nature 2001

HVG Entropy: entropy of the degree distribution

Degree distribution





Aragoneses et al, PRL 116, 033902 (2016)

4. Space-time representation of a time series

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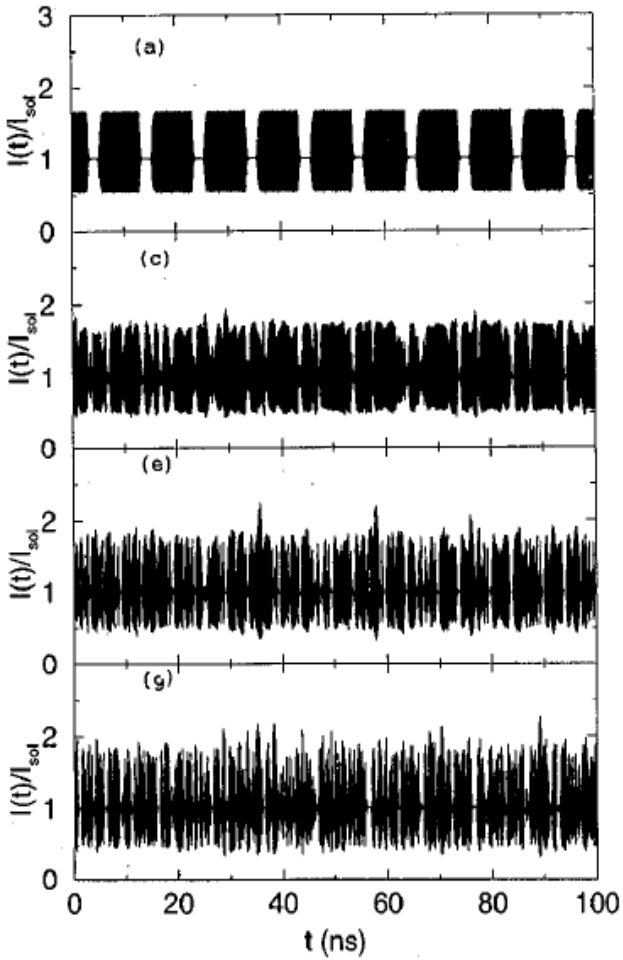
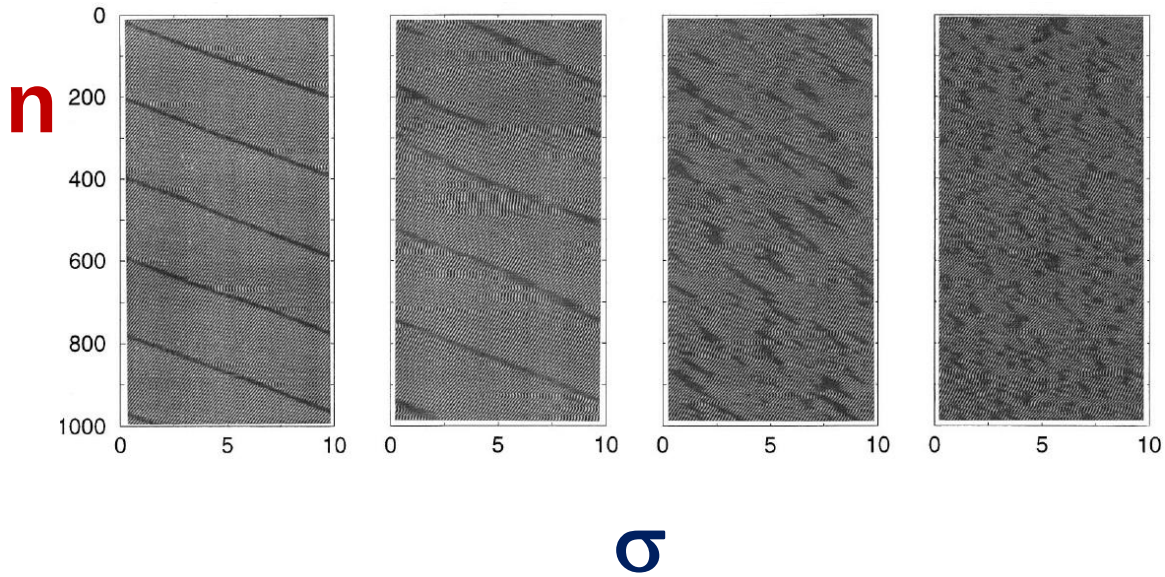
Let's assume that the TS has a characteristic time-scale τ

Synthetic data: laser model, **time-delay τ**

$$t = \sigma + n\,\tau$$

σ= "space-like" variable
n= "time-like" variable



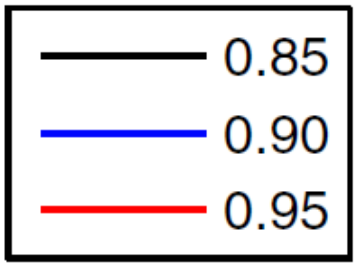


F. T. Arecchi, et al. PRA 45, R4225 (1992)
C. Masoller, Chaos 7, 455 (1997)

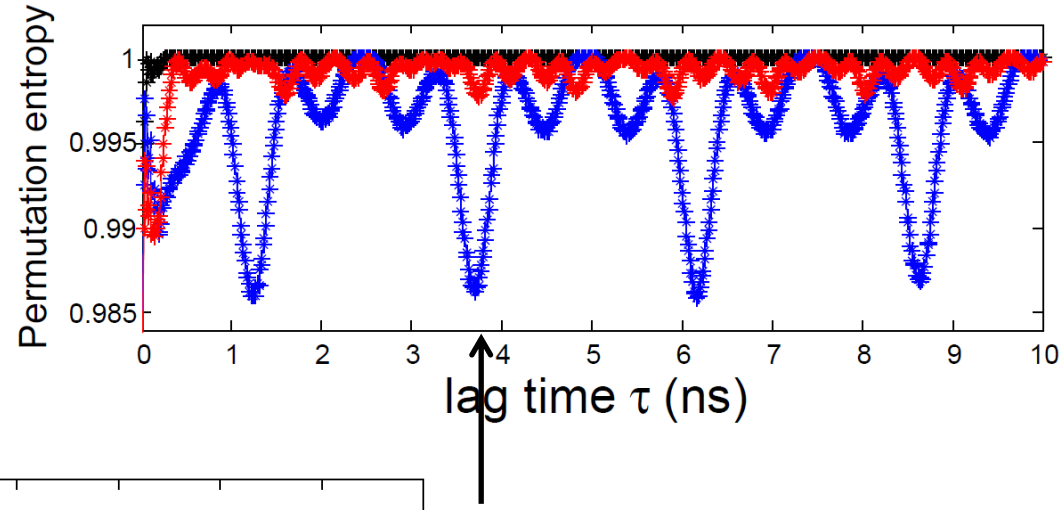
How to identify the characteristic time scale?

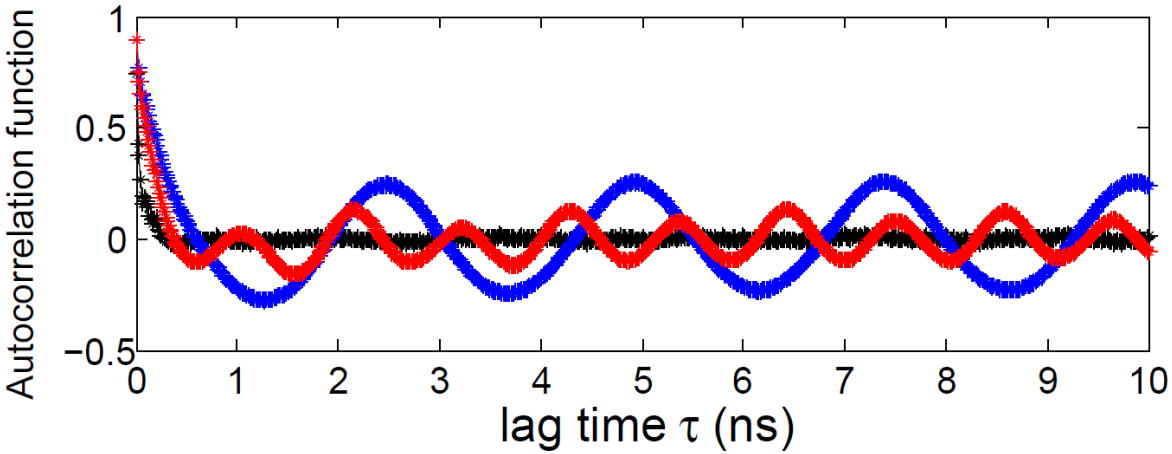
$$\{I_i, I_{i+\tau}, I_{i+2\tau}, \ldots\}.$$

Empirical fiber laser raw data



Pump power below, at, and above the transition.





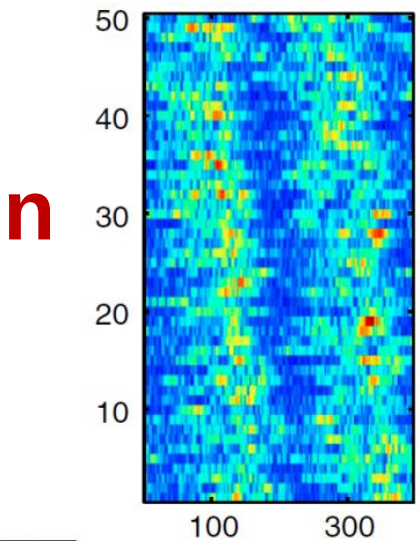
⇒ Sharp variations not captured.

Aragoneses et al, PRL 116, 033902 (2016)

Space-time representation of a time-series

$$t = \sigma + n\,\tau$$

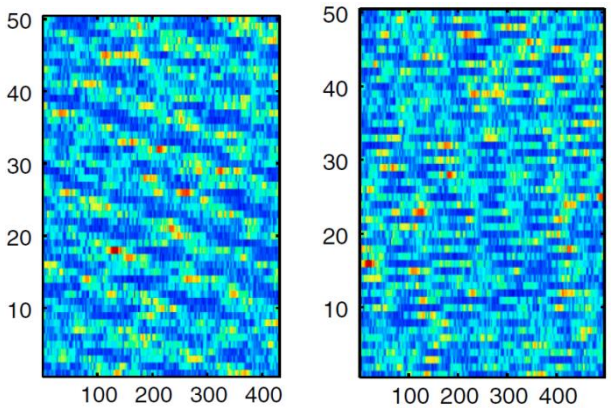
τ=396 dt



Color code: I_i

$$\begin{bmatrix} \dots & \dots & \dots & \dots \\ I_{2\tau+1} & I_{2\tau+2} & \dots & I_{3\tau} \\ I_{\tau+1} & I_{\tau+2} & \dots & I_{2\tau} \\ I_1 & I_2 & \dots & I_\tau \end{bmatrix}$$

τ = 431dt, 496dt



$\Rightarrow$ Different "spatial " structures uncovered with different sampling times.

Aragoneses et al, PRL 116, 033902 (2016)

5. Extracting phase and amplitude information

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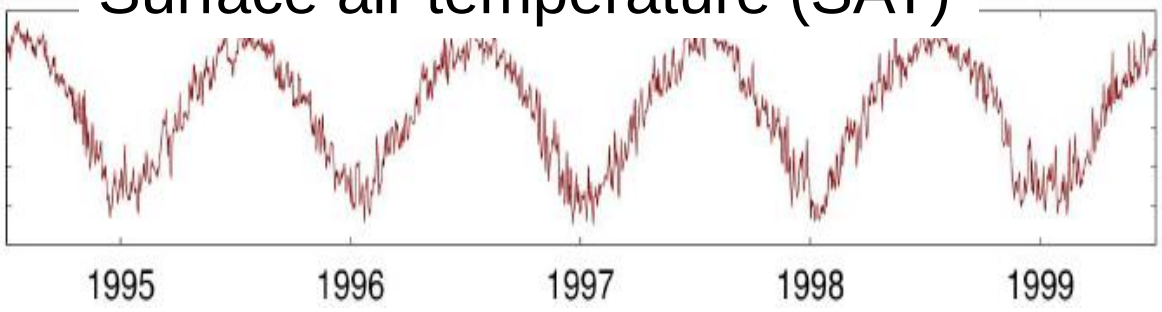
Defining instantaneous amplitude, and phases from data

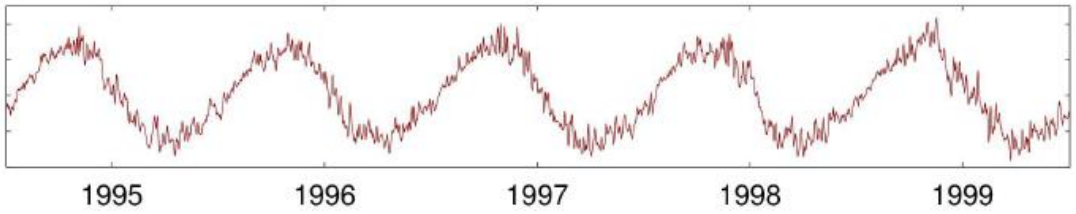
Surface air temperature (SAT)

x

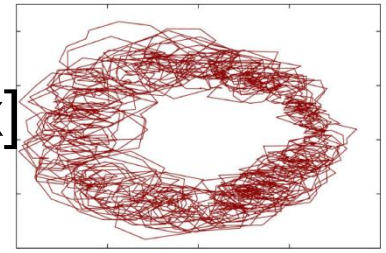


- HT[sin(ωt)]=cos(ωt)

HT[x]



y=HT[x]



x

$$a(t) = \sqrt{[x(t)]^2 + [y(t)]^2}$$

$$\varphi(t) = \arctan[y(t)/x(t)]$$

Hilbert analysis

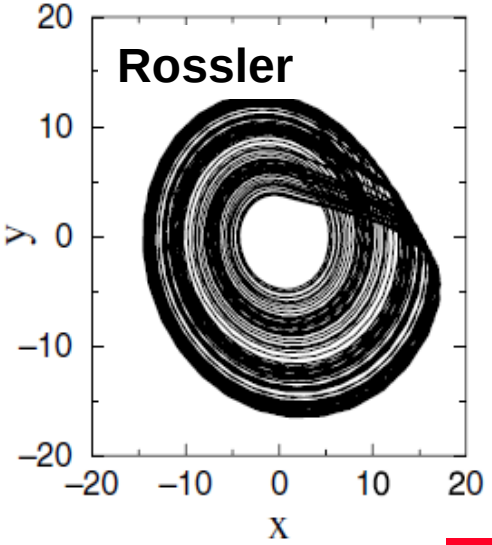
- For a real time series x(t) defines an *analytic signal*

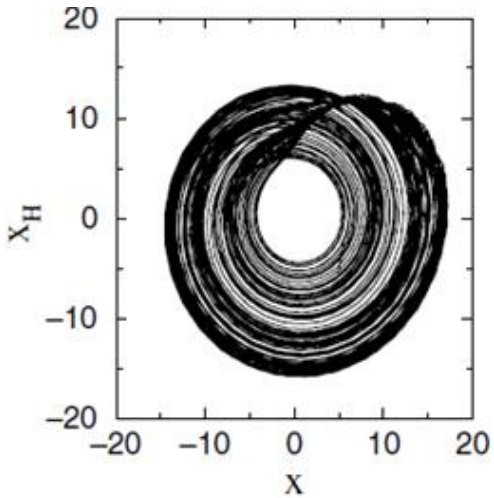
$$\zeta(t) = x(t) + iy(t) = a(t)e^{i\varphi(t)}$$

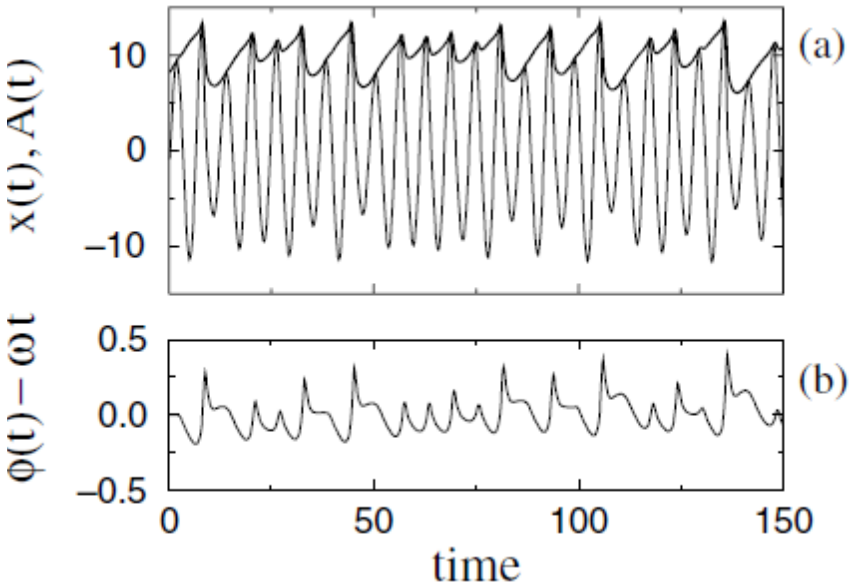
$$y(t) = H[x(t)] = \pi^{-1}\text{P.V.}\int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau}d\tau$$

<u>A word of warning</u>:
Although formally a(t) and φ(t) can be defined for any x(t), **they have a clear physical meaning only if x(t) is a narrow-band oscillatory signal**: in that case, the **a(t)** coincides with the envelope of x(t) and the **instantaneous frequency, ω(t)=dφ/dt**, coincides with the main frequency in the power spectrum.

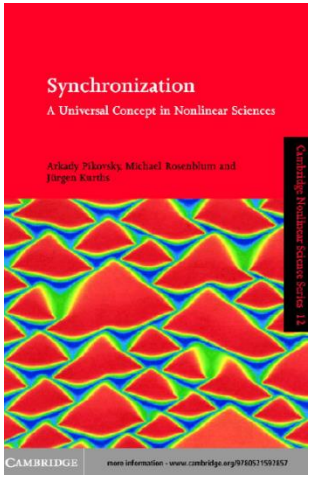
Example







Read more:

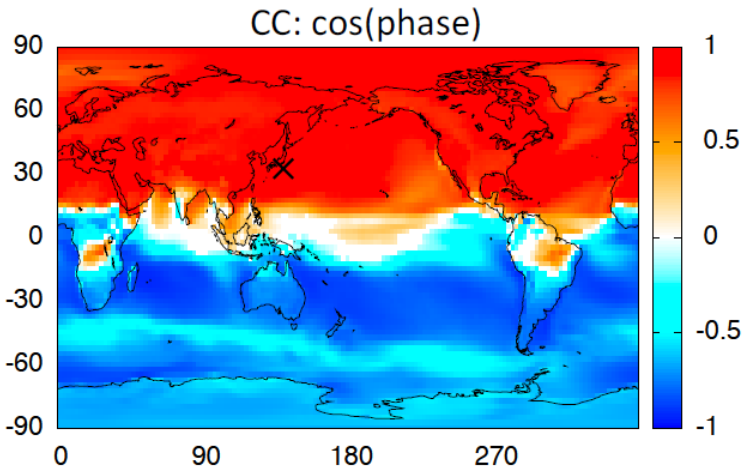


A. Pikovsky, M. Rosenblum and J. Kurths 2001

Application to climate data

- Can we use the Hilbert amplitude, phase, frequency, to investigate:
  - Synchronization in climate data?
  - To identify and quantify regional climate change?
- Problem: climate time series are not narrow-band.
- Usual solution (e.g. brain signals): isolate a narrow frequency band.

- However, HT applied to Surface Air Temperature time series yields meaningful insights.

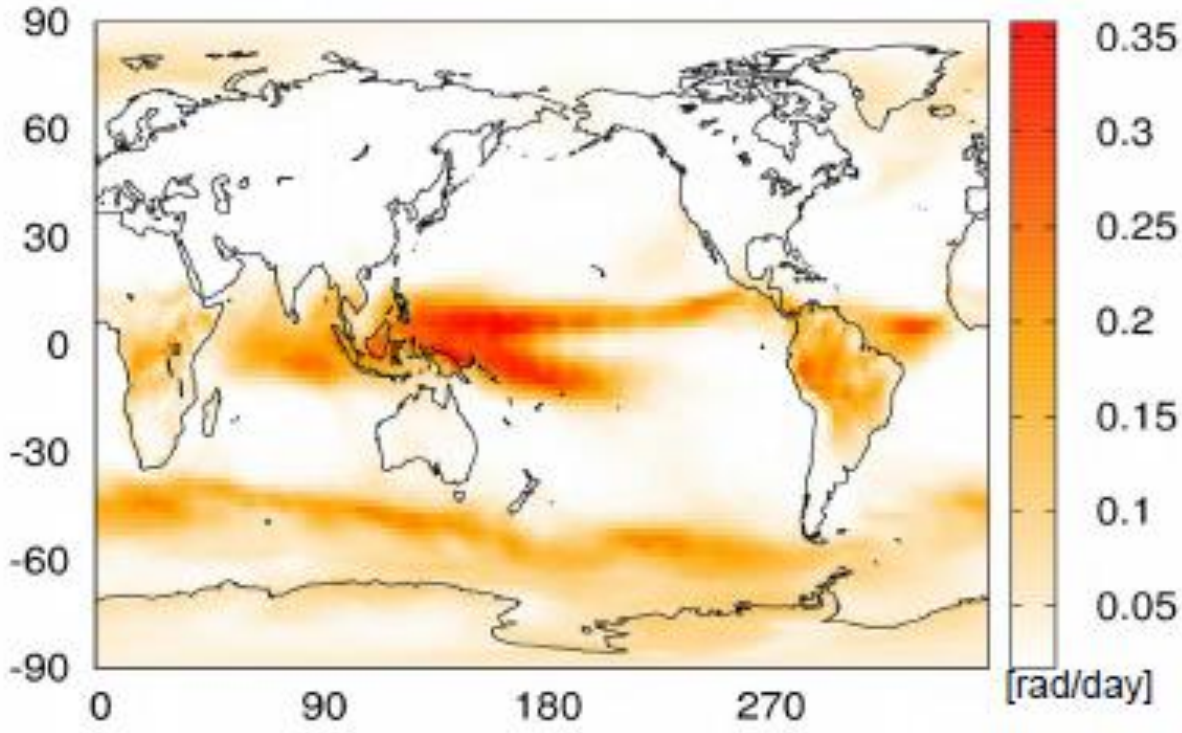


Features extracted

- From each SAT time series we compute
  - Time averaged amplitude, $\langle a \rangle$
  - Time averaged frequency, $\langle \omega \rangle$
  - Standard deviations, σ_a, σ_ω
  - Phase diffusion coefficient, *D*
  - Etc.

Time-averaged instantaneous frequency, ⟨ω⟩

- The expected value is 0.017 rad/day (one year cycle)



Zappala, Barreiro and Masoller, Entropy 18, 408 (2016)

6. And many more TS analysis methods

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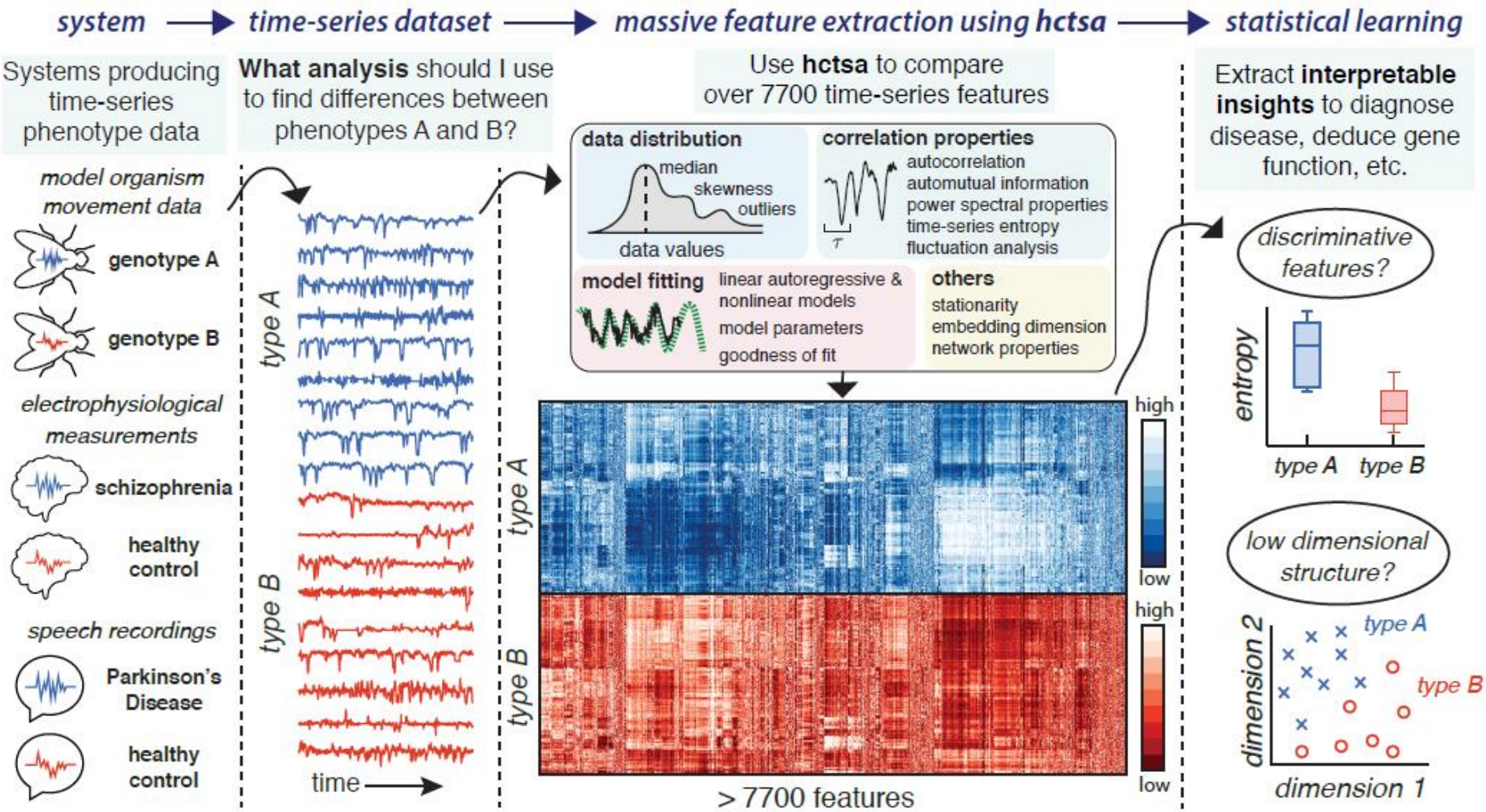
And many MANY other methods to characterize and extract "features" from TS

- How to compare different methods?
- How to identify similar time-series?
- How to identify similar methods?
  - HCTSA: highly comparative time-series analysis
  - From each TS extracts over **7700 features**

B. D. Fulcher, N. S. Jones: *Automatic time-series phenotyping using massive feature extraction.* arXiv:1612.05296v1



Matlab code: www.github.com/benfulcher/hctsa

C. Bivariate and multivariate time-series analysis

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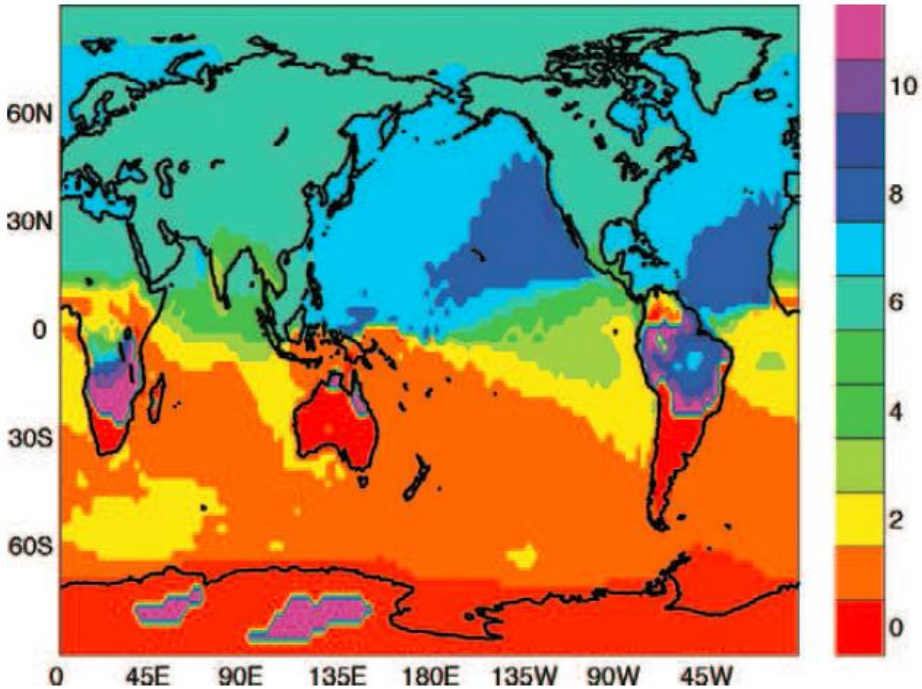
Two time series X, Y: Cross-correlation analysis

$$\rho_{X,Y}(\tau) = \frac{\mathrm{cov}(X(t), Y(t+\tau))}{\sqrt{\sigma_X^2 \sigma_Y^2}} = \frac{E[(X(t)-\mu_X)(Y(t+\tau)-\mu_Y)]}{\sqrt{\sigma_X^2 \sigma_Y^2}}$$

- $-1 \leq \rho_{X,Y} \leq 1$
- $\rho_{X,Y} = \rho_{Y,X}$
- The maximum of $\rho_{X,Y}(\tau)$ indicates the **lag** that renders the time-series X and Y best aligned.

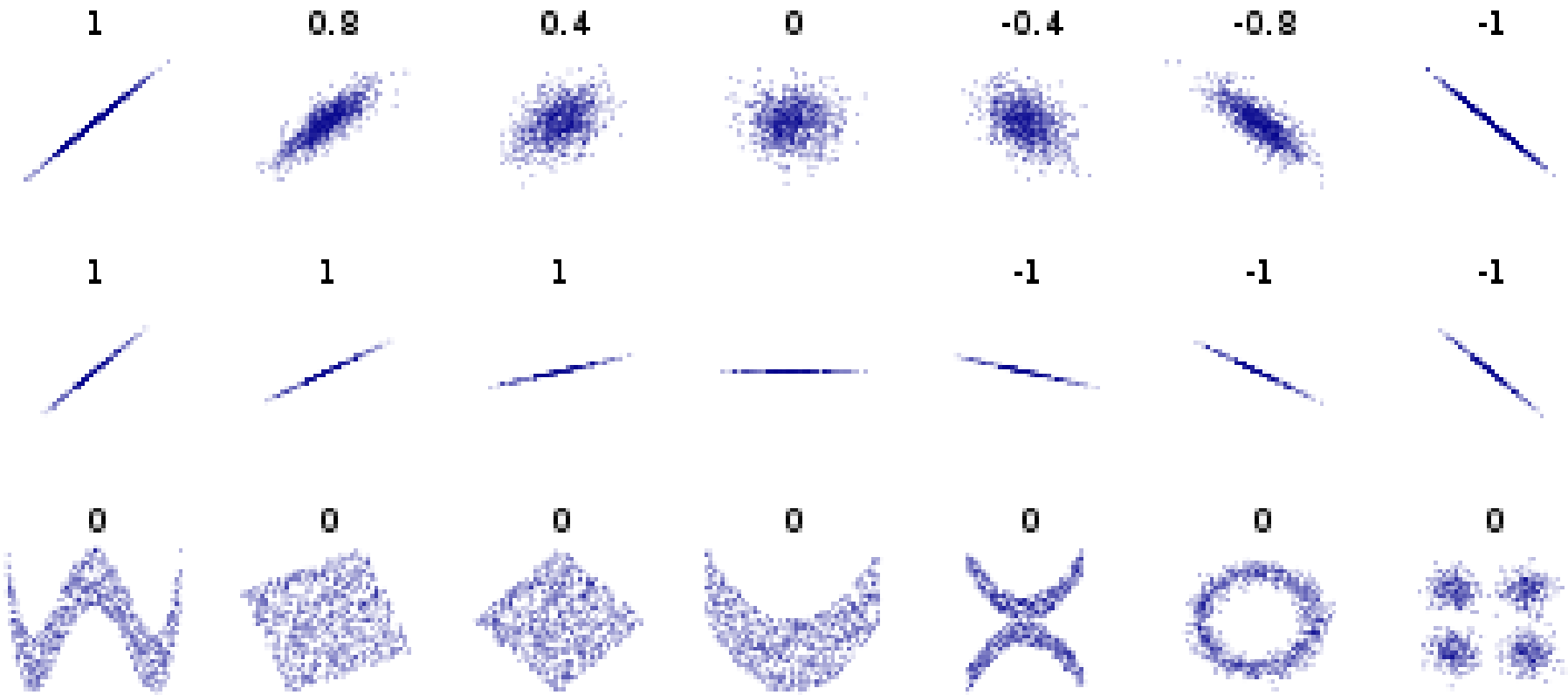
Map of lags of surface air temperature

- Lags between the SAT time-series at a reference point located in Australia, and all the other time-series.



Tirabassi and Masoller EPL 102, 59003 (2013)

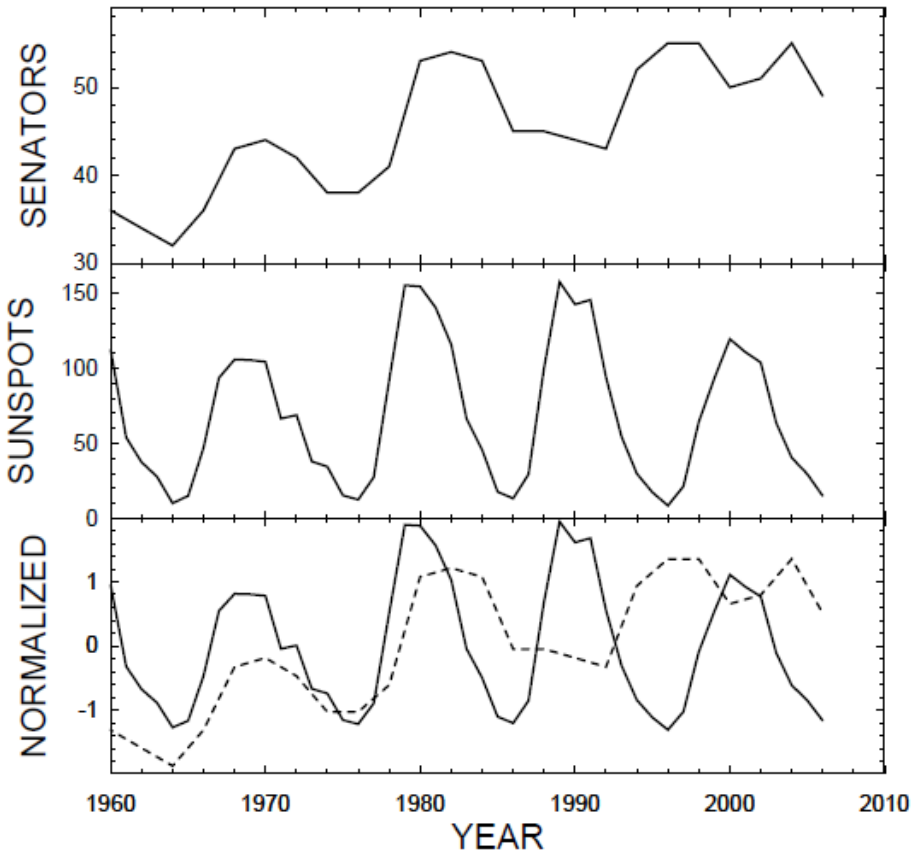
Cross-correlation analysis detects linear relationships only



Source: wikipedia

Correlation is NOT causality

An illustrative example: the story of the number of sunspots and the number of the Republicans in the U.S. Senate in the years 1960-2006



C=0.52

Is this significant???

- For data sets sampled, from independent, identically distributed, normally distributed populations (IID Gaussian distributions) and significance level of 95%, the threshold value is 0.458.
- Therefore, the null hypothesis (independent processes) should be rejected.
- Something is clearly wrong!
- Incorrect interpretation of an incorrectly applied statistical test.

Read more: M. Palus, *From Nonlinearity to Causality: Statistical testing and inference of physical mechanisms underlying complex dynamics*. Contemporary Physics 48(6) (2007) 307-348.

Mutual Information

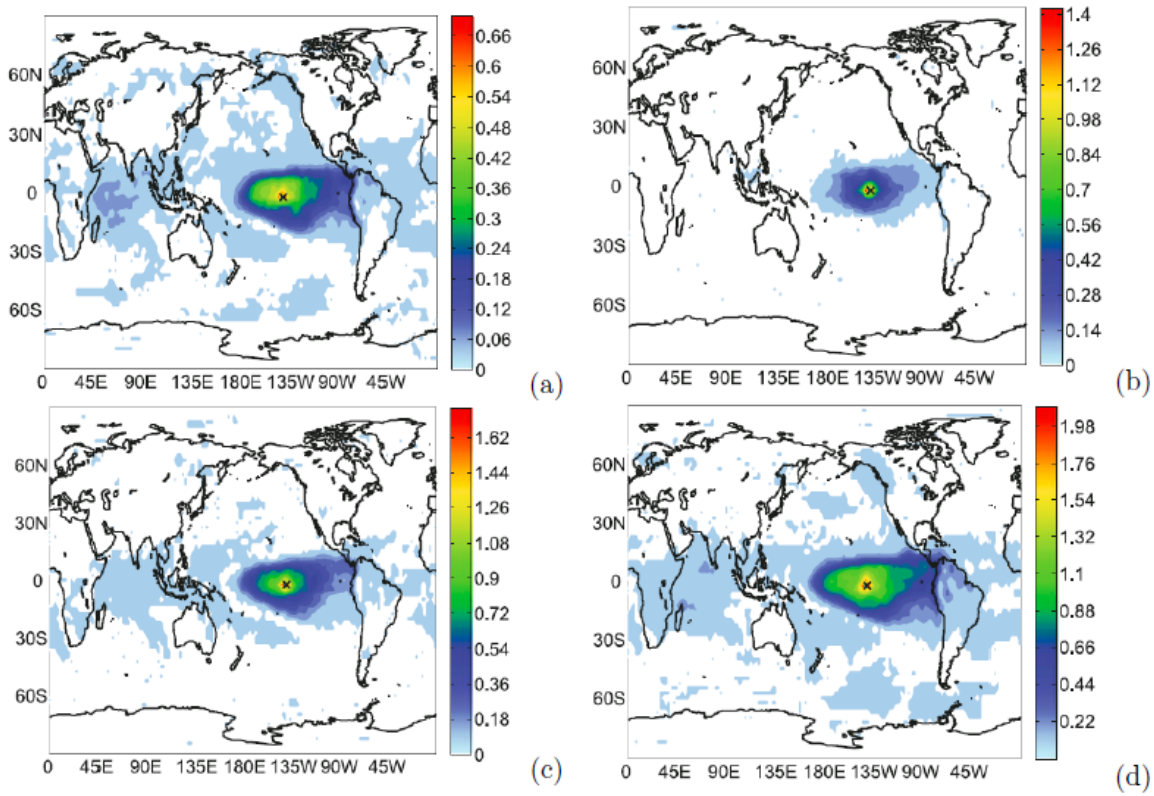
$$MI = \sum_{i \in x} \sum_{j \in y} p(x,y) \ \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

- MI (x,y) = MI (y,x)
- p(x,y) = p(x) p(y) $\Rightarrow$ MI = 0, else **MI >0**
- MI can also be computed with a lag-time.

Mutual information maps

- MI between SAT anomalies time-series at a reference point located in El Niño, and all the other time-series.

Histograms



3 months ordinal patterns

Inter-annual ordinal patterns

3 years ordinal patterns

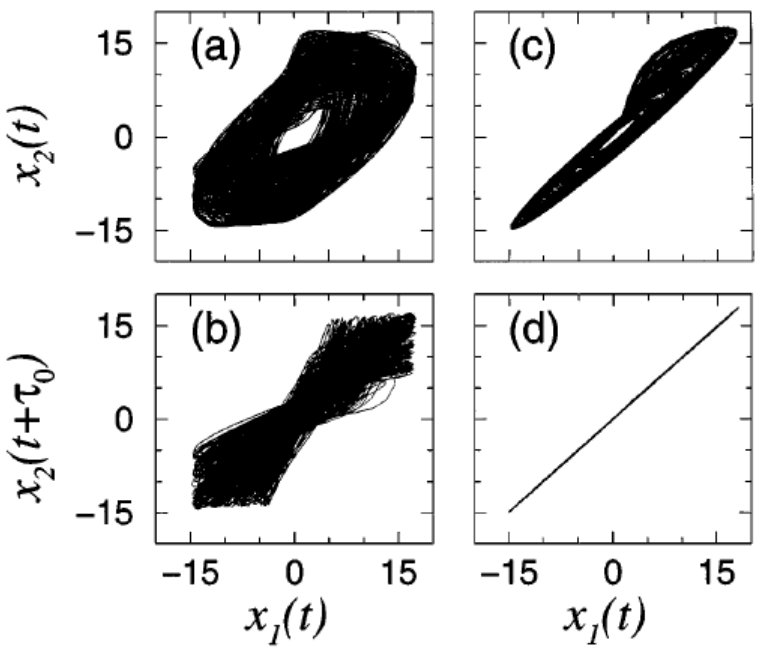
Ordinal analysis separates the times-scales of the interactions

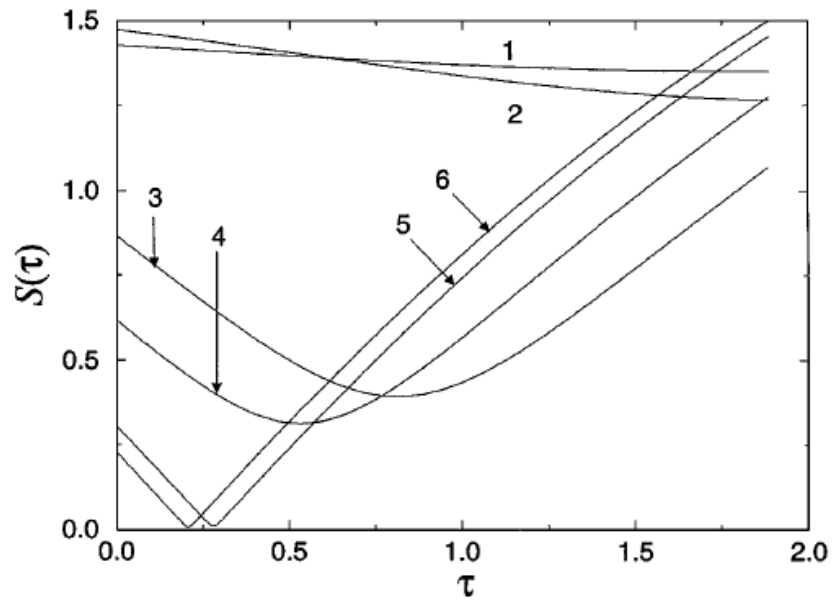
Deza, Barreiro and Masoller, Eur. Phys. J. ST 222, 511 (2013)

Another similarity measure

- Similarity function: $$S^2(\tau) = \frac{\langle [x_2(t+\tau) - x_1(t)]^2 \rangle}{[\langle x_1^2(t) \rangle \langle x_2^2(t) \rangle]^{1/2}}$$

Two mutually coupled (instantaneous coupling) Rossler oscillators: lag synchronization as the coupling increases

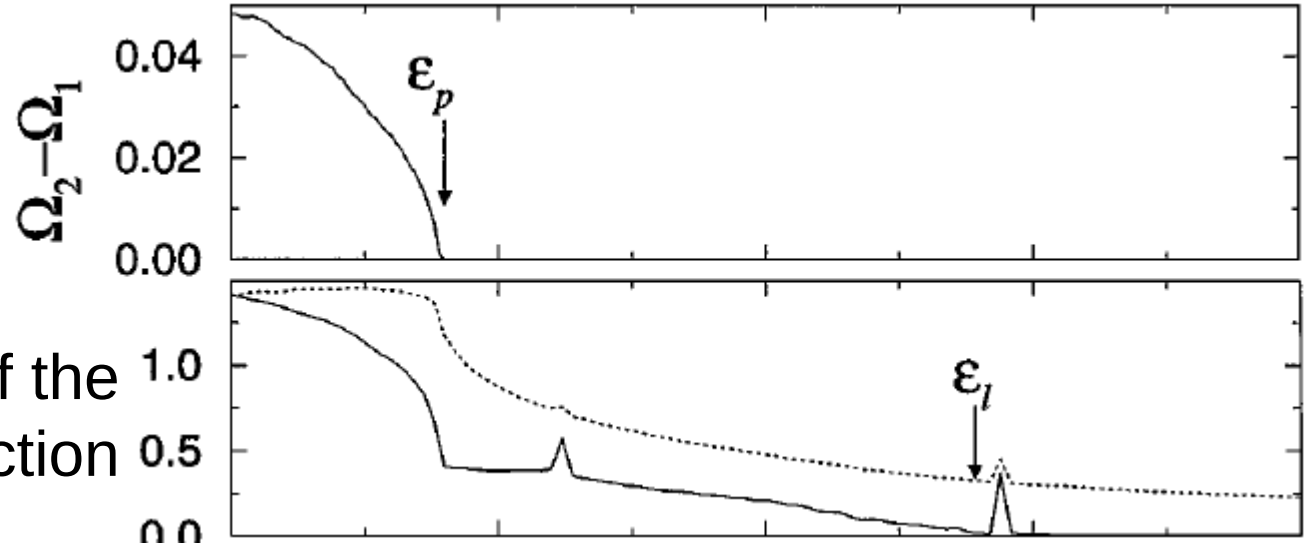




From phase to lag synchronization

Frequency difference

Minimum of the similarity function



Coupling strength ⟶

Read more: M. G. Rosenblum et al, *From Phase to Lag Synchronization in Coupled Chaotic Oscillators*, PRL 78, 4193 (1997)

Using similarity measures to infer interactions: “functional networks”

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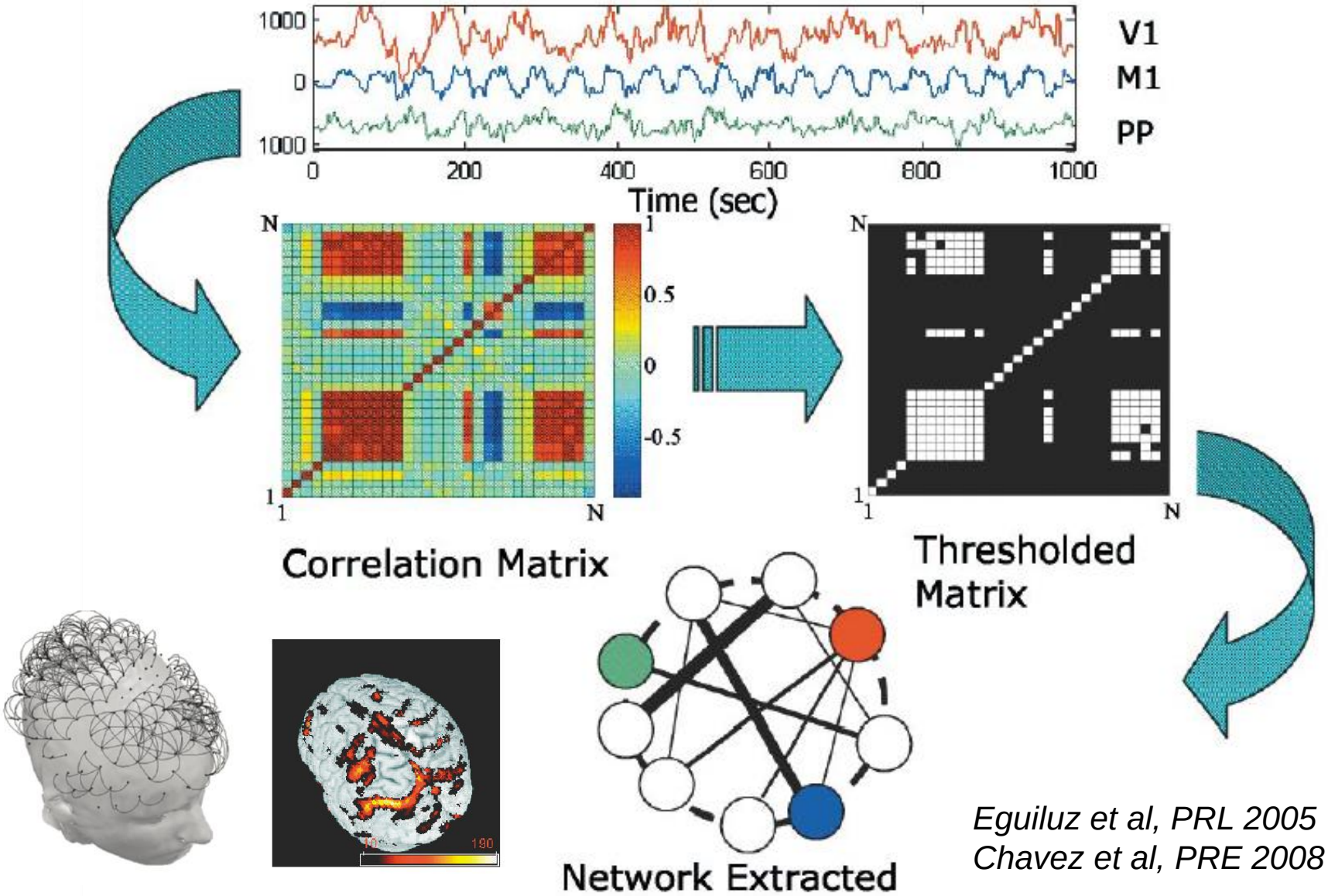
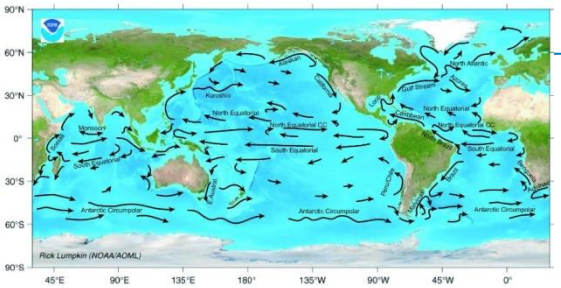
Brain functional network

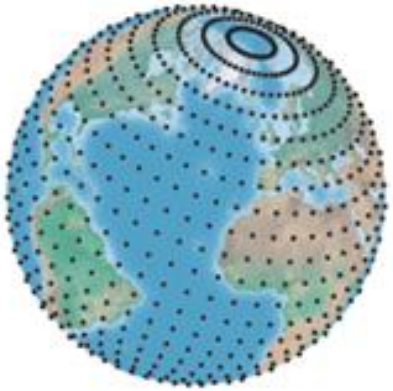


Eguiluz et al, PRL 2005
Chavez et al, PRE 2008

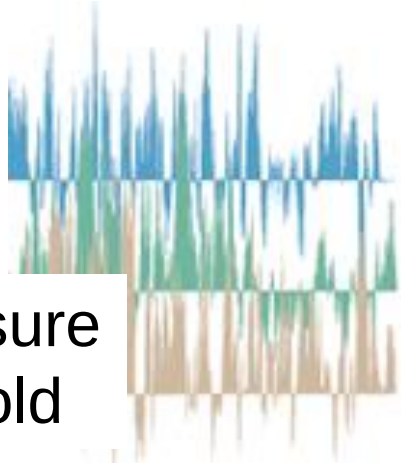
Complex network representation of the climate system



Back to the climate system: interpretation (currents, winds, etc.)



More than 10000 nodes.



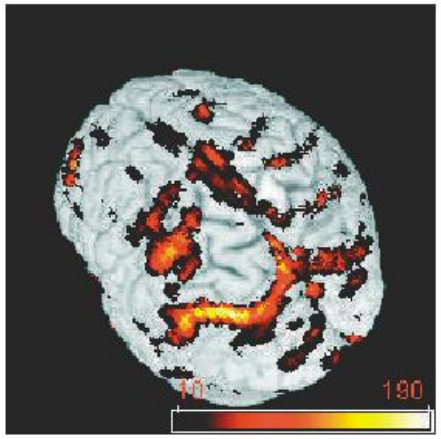
Daily resolution: more than 13000 data points in each TS

Surface Air Temperature <u>Anomalies</u> (solar cycle removed)

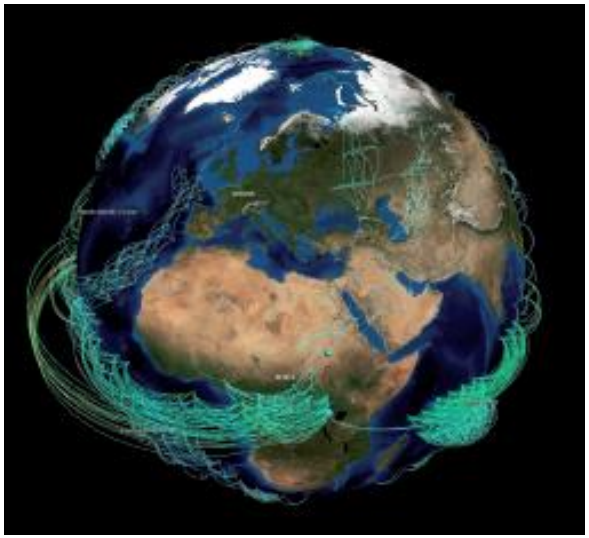
Sim. measure + threshold

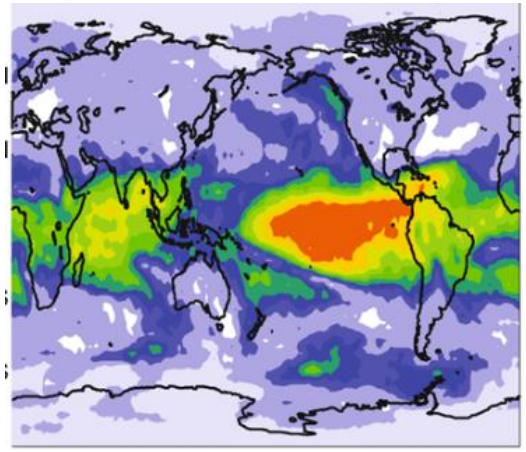
Donges et al, Chaos 2015

Brain network



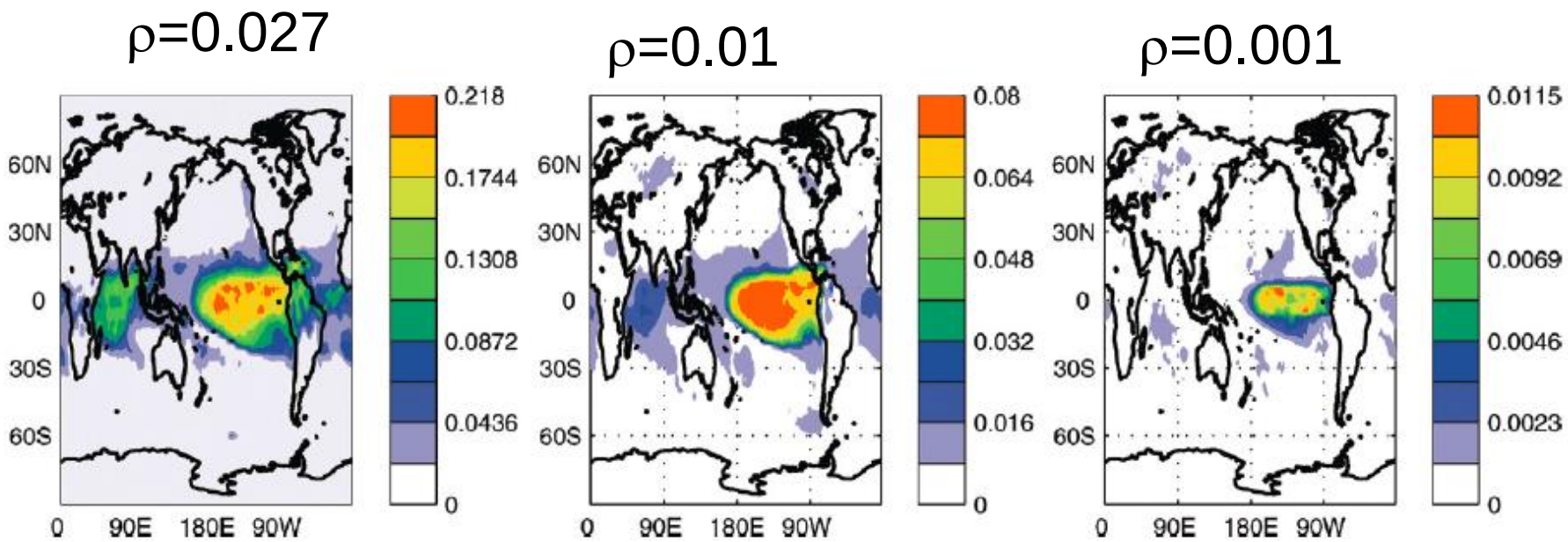
Climate network





Weighted degree

Influence of the threshold



M. Barreiro, et. al, Chaos 21, 013101 (2011)

Software

Unified functional network and nonlinear time series analysis for complex systems science: The pyunicorn package

Jonathan F. Donges' , Jobst Heitzig, Boyan Beronov, Marc Wiedermann, Jakob Runge, Qing Yi Feng, Liubov Tupikina, Veronika Stolbova, Reik V. Donner, Norbert Marwan, Henk A. Dijkstra, and Jürgen Kurths

Citation: Chaos **25**, 113101 (2015); doi: 10.1063/1.4934554

View online: http://dx.doi.org/10.1063/1.4934554

pyunicorn is available at https://github.com/pik-copan/
pyunicorn as a part of PIK's TOCSY toolbox (http://tocsy.pik-potsdam.de/).

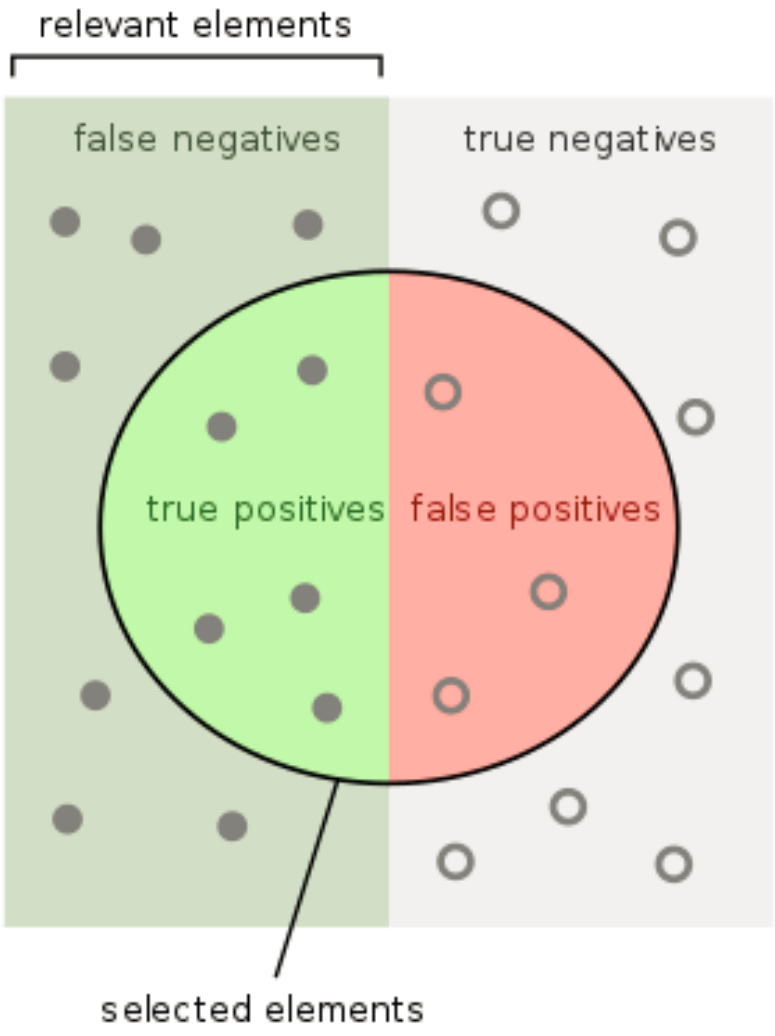
Main problem: how to infer the network?

- How to select the threshold?
- In "spatially embedded networks", nearby nodes have the strongest links.
- How to select **weak-but-significant** links?
- There are many methods to infer bi-variate interactions from observations.

- Can we test them?

- Yes, if we use a "toy model" where **we know** the real connectivity ("the ground truth").

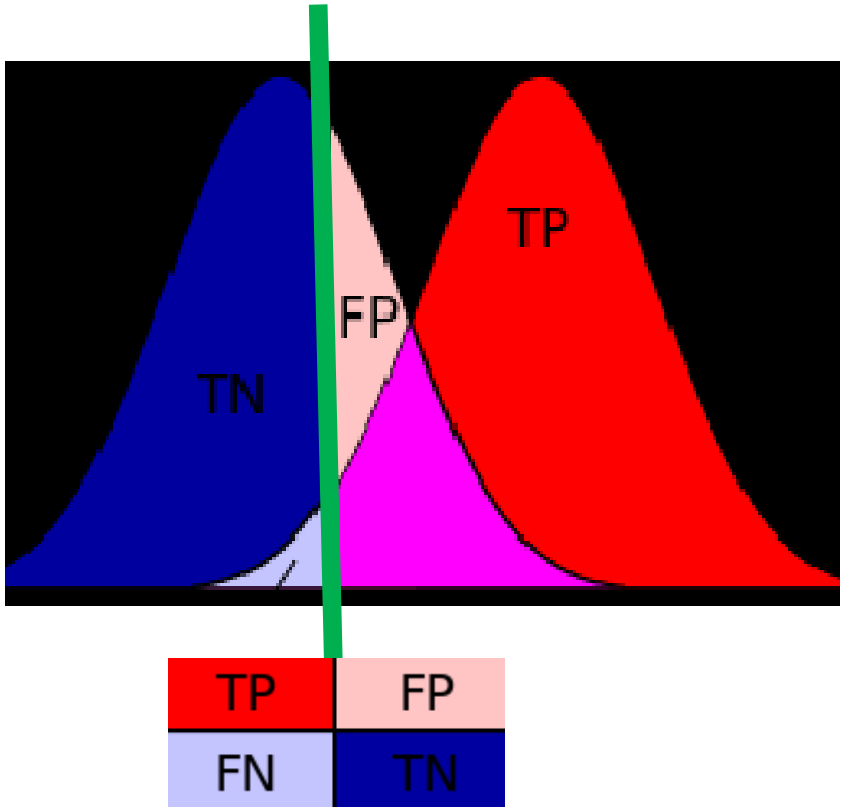
Definitions

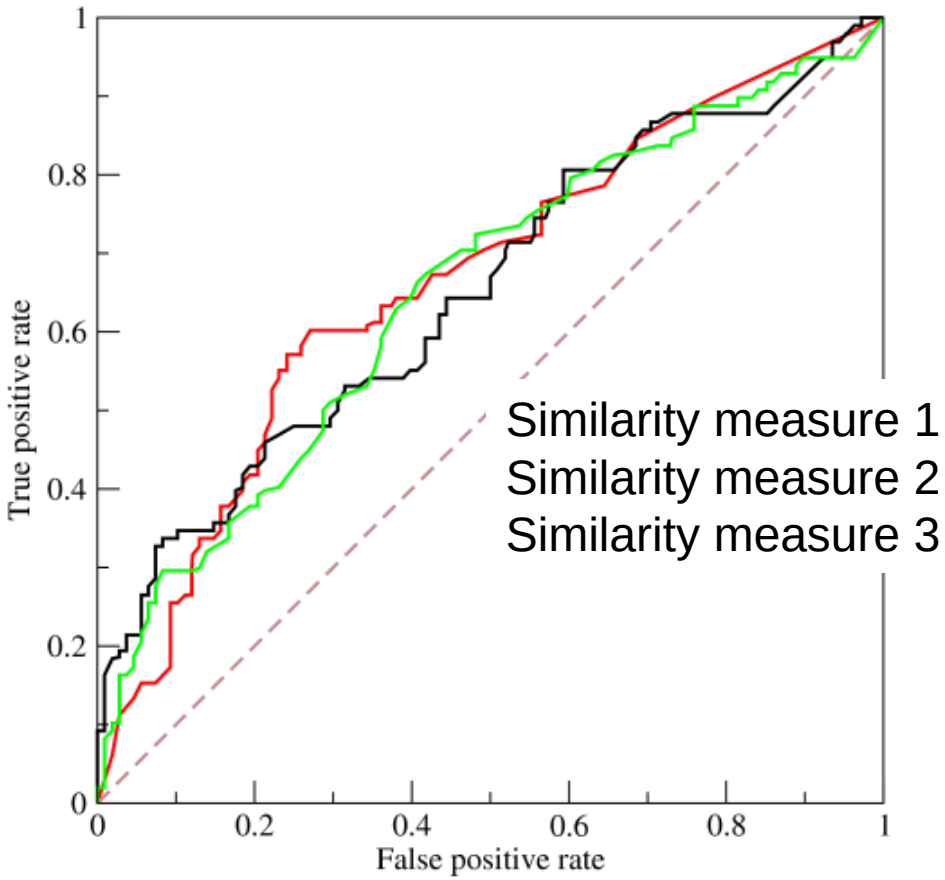


- **Sensitivity** (also called the **true positive rate**): proportion of existing links that are correctly identified.
- **Specificity S** (also called the **true negative rate**): proportion of non-existing links that are correctly identified.
- **False positive rate** ("false alarms") proportion of non-existing links that are incorrectly identified.

Source: wikipedia

Receiver operating characteristic (ROC curve)





Source: wikipedia

Kuramoto oscillators in a random network

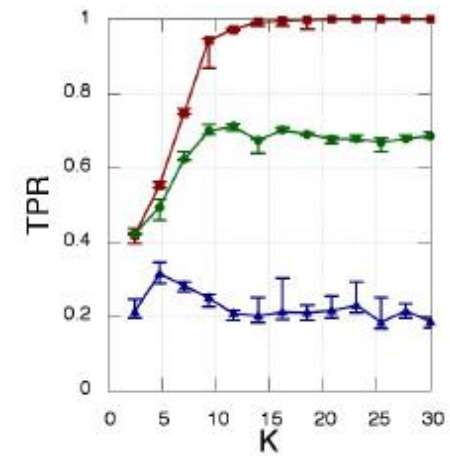
$$\mathrm{d}\theta_i = \omega_i \mathrm{d}t + \frac{K}{N}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i)\mathrm{d}t + D\ \mathrm{d}W_t^i$$

A_{ij} is a symmetric random matrix; N=12 time-series, each with 10^4 data points.

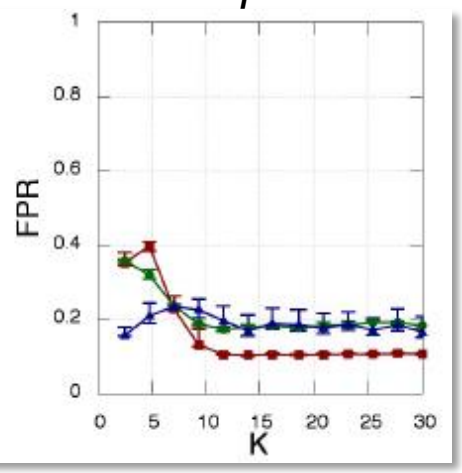
CC MI MIOP

Phases (θ)

True positives

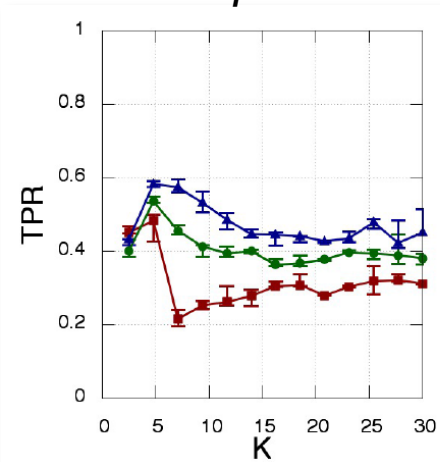


False positives

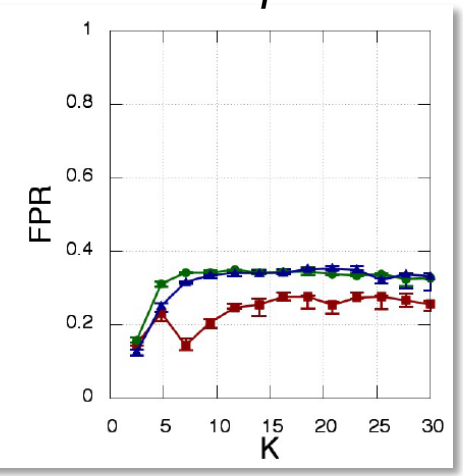


"Observable" Y=sin(θ)

True positives

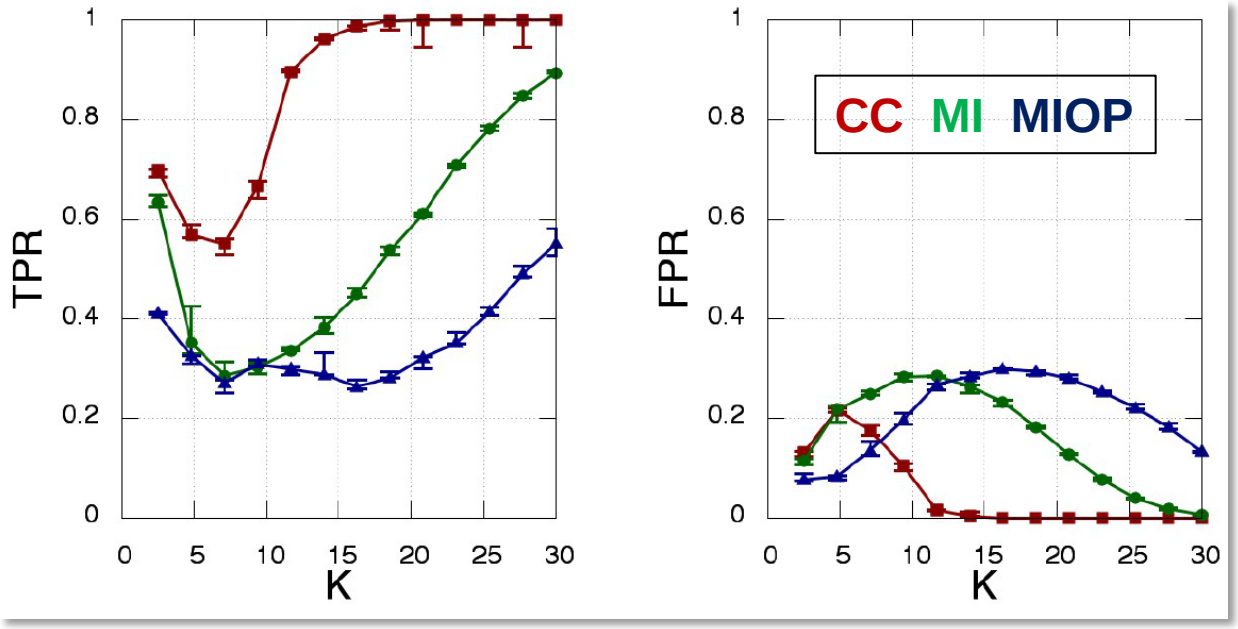


False positives



Results of a 100 simulations with different oscillators' frequencies, random matrices, noise realizations and initial conditions.
For each K, the threshold was varied to obtain optimal reconstruction.

Instantaneous frequencies (dθ/dt)



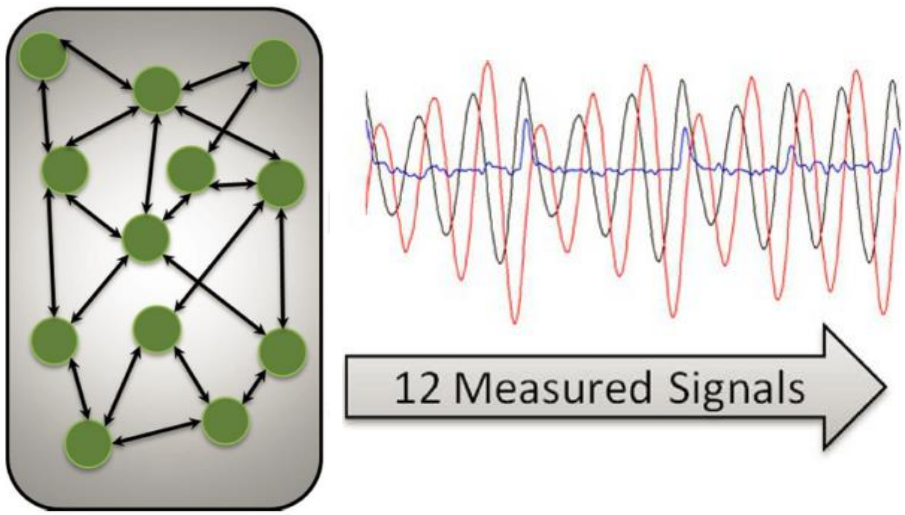
Perfect network inference is possible!

BUT

- the number of oscillators is small (12),
- the coupling is symmetric ($\Rightarrow$ only 66 possible links) and
- the data sets are long (10^4 points)

G. Tirabassi et al, Sci. Rep. **5** 10829 (2015)

We also analyzed experimental data recorded from 12 chaotic Rössler electronic oscillators (symmetric and random coupling)



The Hilbert Transform was used to obtain phases from experimental data

G. Tirabassi et al, Sci. Rep. **5** 10829 (2015)

- Kuramoto Oscillators' Network

$$\theta_i$$

$$f_i = \dot{\theta}_i$$

$$Y_i = \sin(\theta_i)$$

- Rössler Oscillators' Network

$$\varphi_i = HT(x_i)$$

$$f_i = \dot{\varphi}_i$$

$$x_i$$

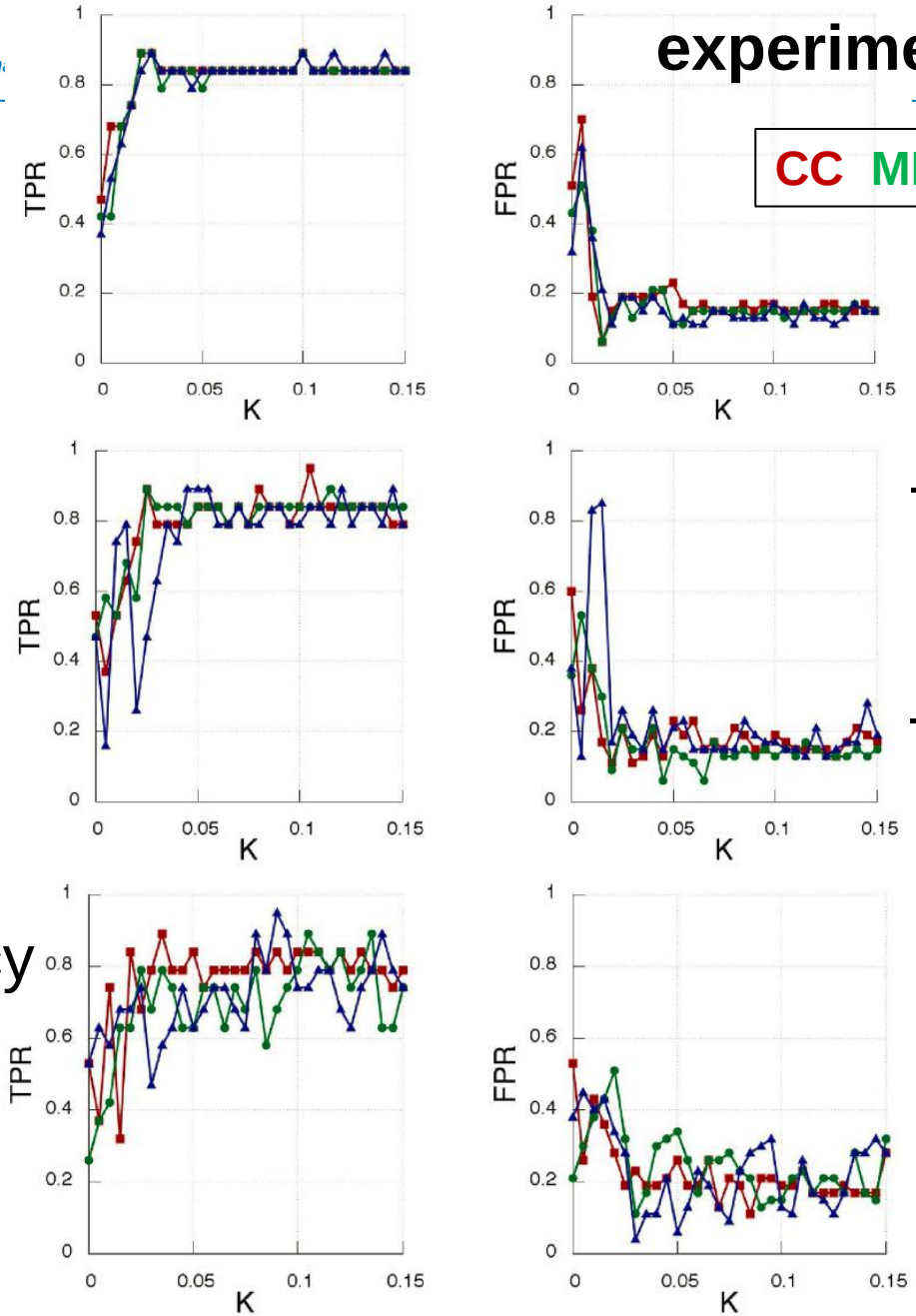
Results obtained with experimental data

CC MI MIOP

Observed variable (x)

Hilbert phase

Hilbert frequency



- No perfect reconstruction
- No important difference among the 3 methods & 3 variables

Concluding

Take home messages

- Symbolic analysis, network representation, spatiotemporal representation, etc., are useful tools for investigating complex signals.
- Different techniques provide *complementary* information.

"…nonlinear time-series analysis has been used to great advantage on thousands of real and synthetic data sets from a wide variety of systems ranging from roulette wheels to lasers to the human heart. Even in cases where the data do not meet the mathematical or algorithmic requirements, the results of nonlinear time-series analysis can be helpful in understanding, characterizing, and predicting dynamical systems…"

Bradley and Kantz, CHAOS 25, 097610 (2015)

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- Tirabassi and Masoller, EPL 102, 59003 (2013)
- C. Masoller et al, NJP 17, 023068 (2015)
- J. Donges et al, Chaos (2015)
- Tirabassi et al, Sci. Rep. **5** 10829 (2015)
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