Identifying dynamical transitions from data

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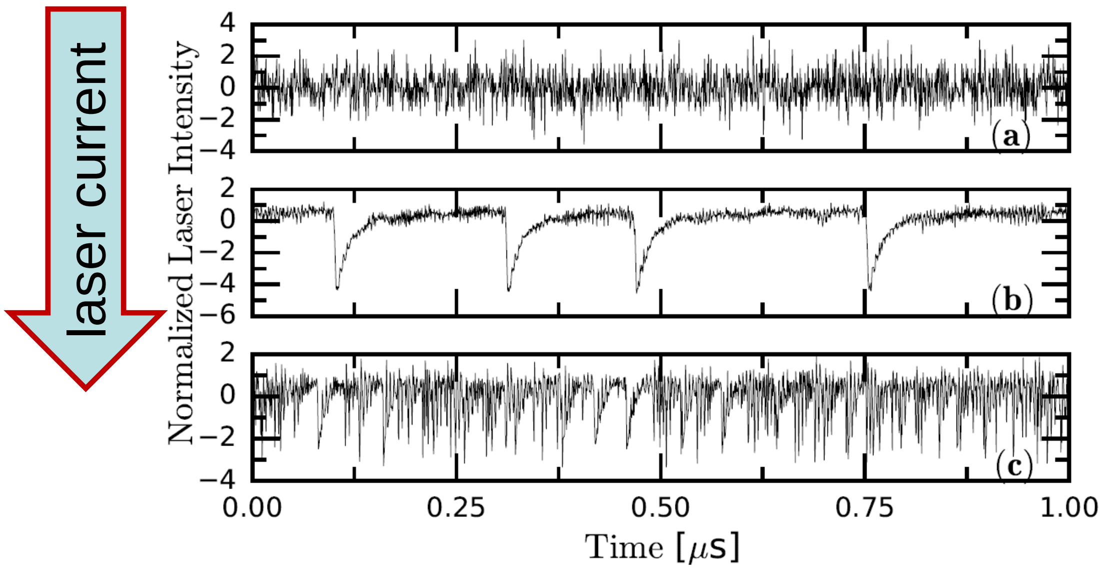
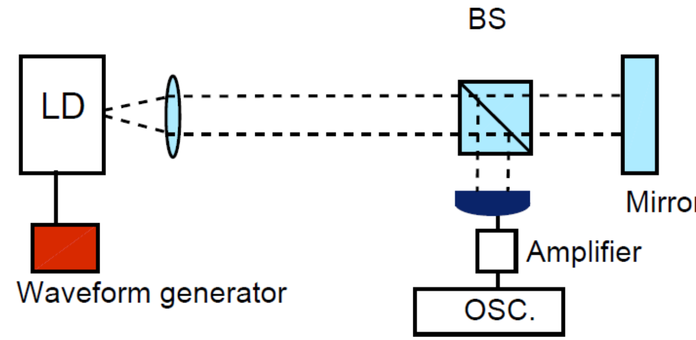
Regime transitions in dynamical systems





Video: how complex optical signals emerge from noisy fluctuations

Similarity with neuronal dynamics?

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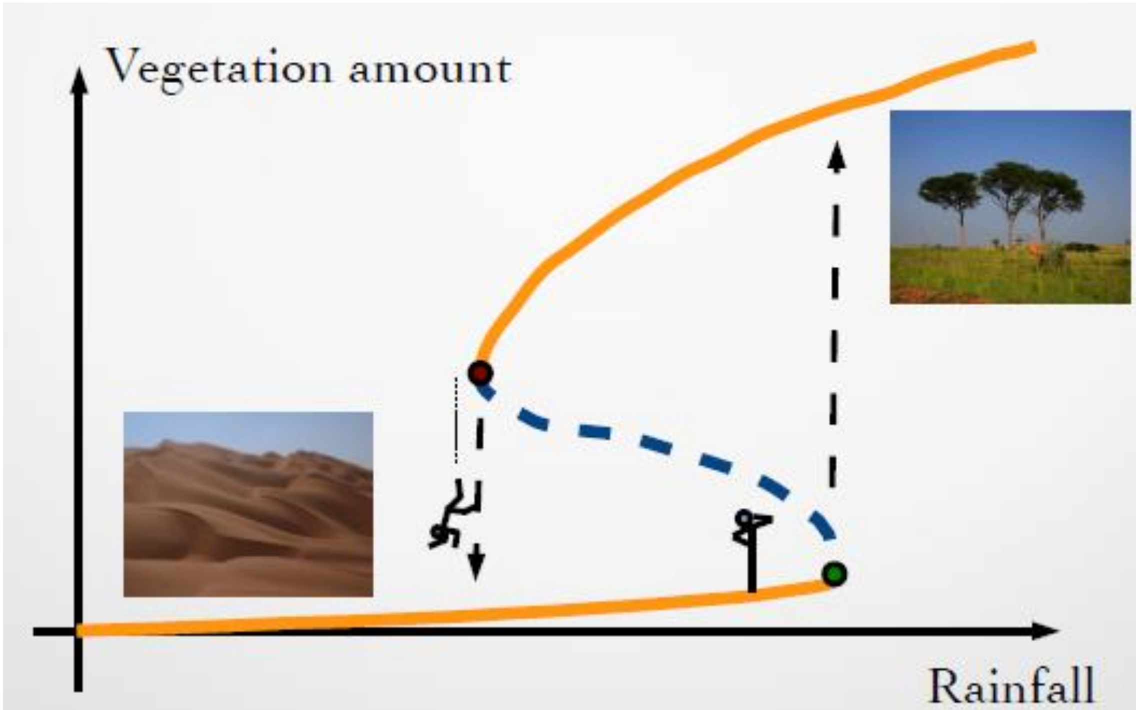
Tipping points in ecosystems



Bangladesh,
Nature 2014

Is there a way to quantify how close we are to the transition point?

Goal: to develop reliable early warning indicators

How can we compare different networks?

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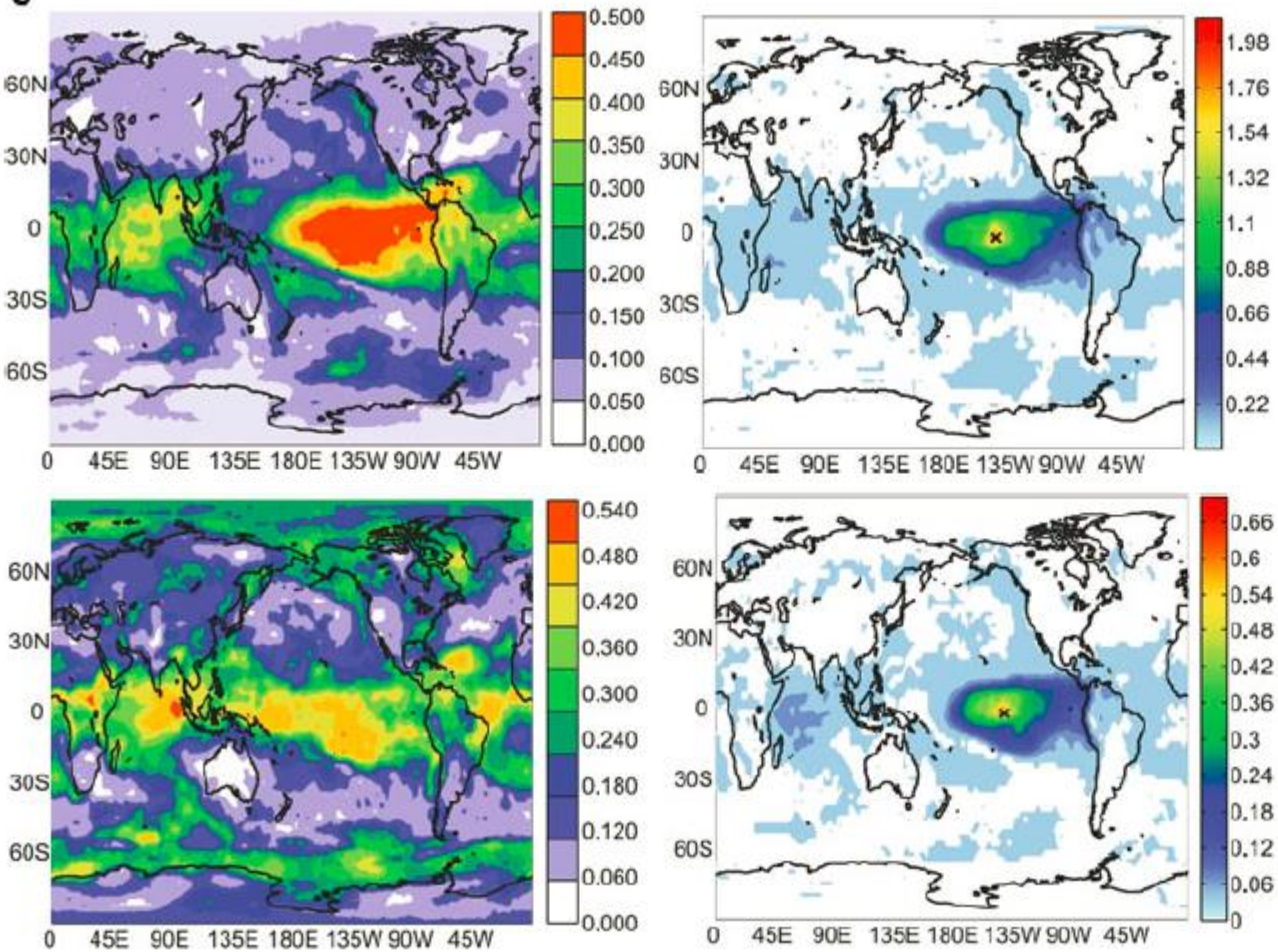
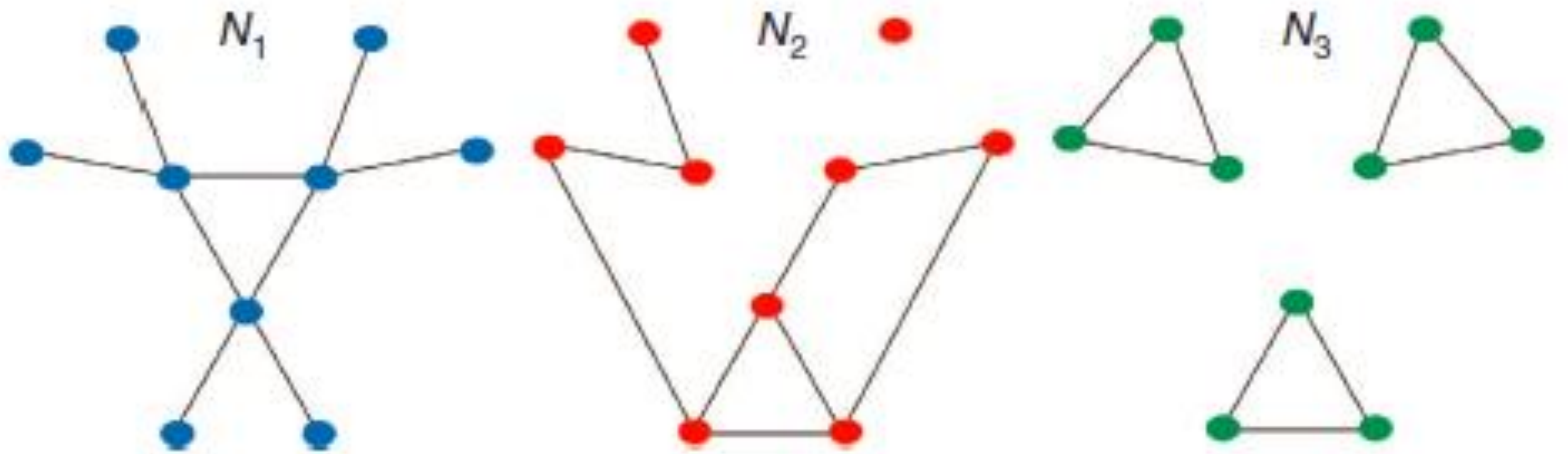


Main Goal: to develop a measure that allows a precise comparison of complex networks (including different sizes)

A.A. Tsonis (ed.), *Advances in Nonlinear Geosciences*,
DOI 10.1007/978-3-319-58895-7_4

Same number of nodes and links



How to measure distances between networks?

Outline

- How optical chaos emerges from noise?
  - Comparison with neuronal dynamics: emergence of temporal correlations in neuronal spikes
- Early-warning indicators of desertification transition
- Quantifying network dissimilarities
- Predicting extreme optical pulses

How optical chaos emerges from noise?

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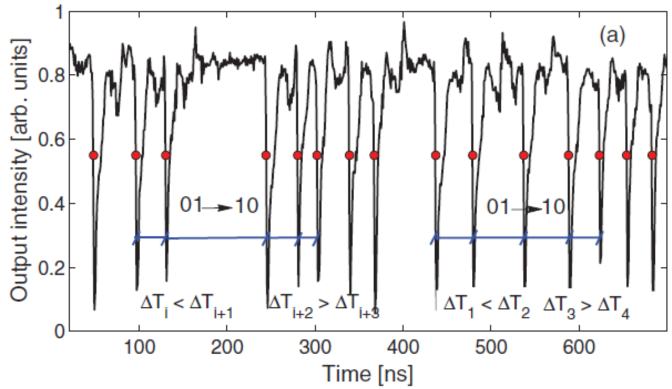
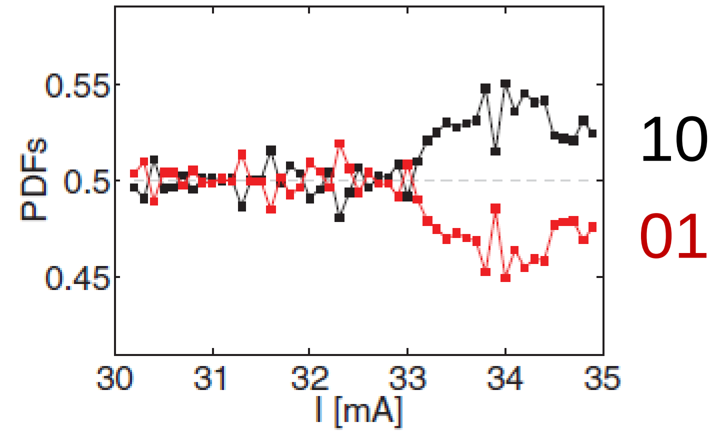
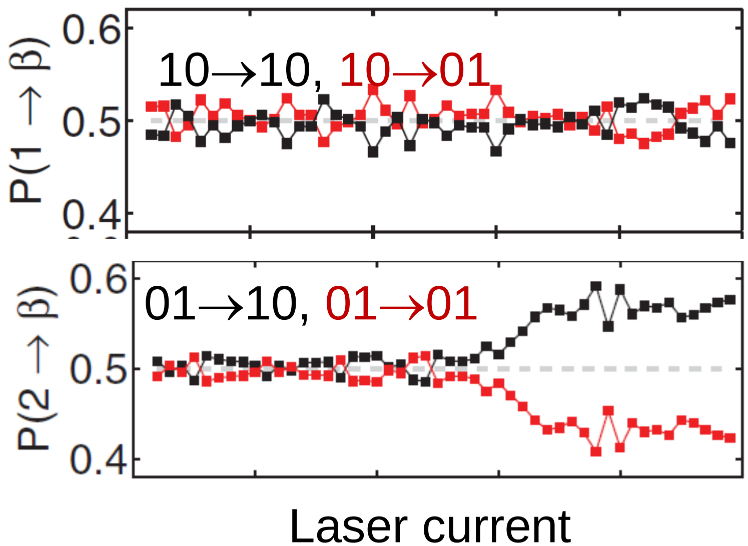
Emergence of temporal correlations in the spiking activity of the laser



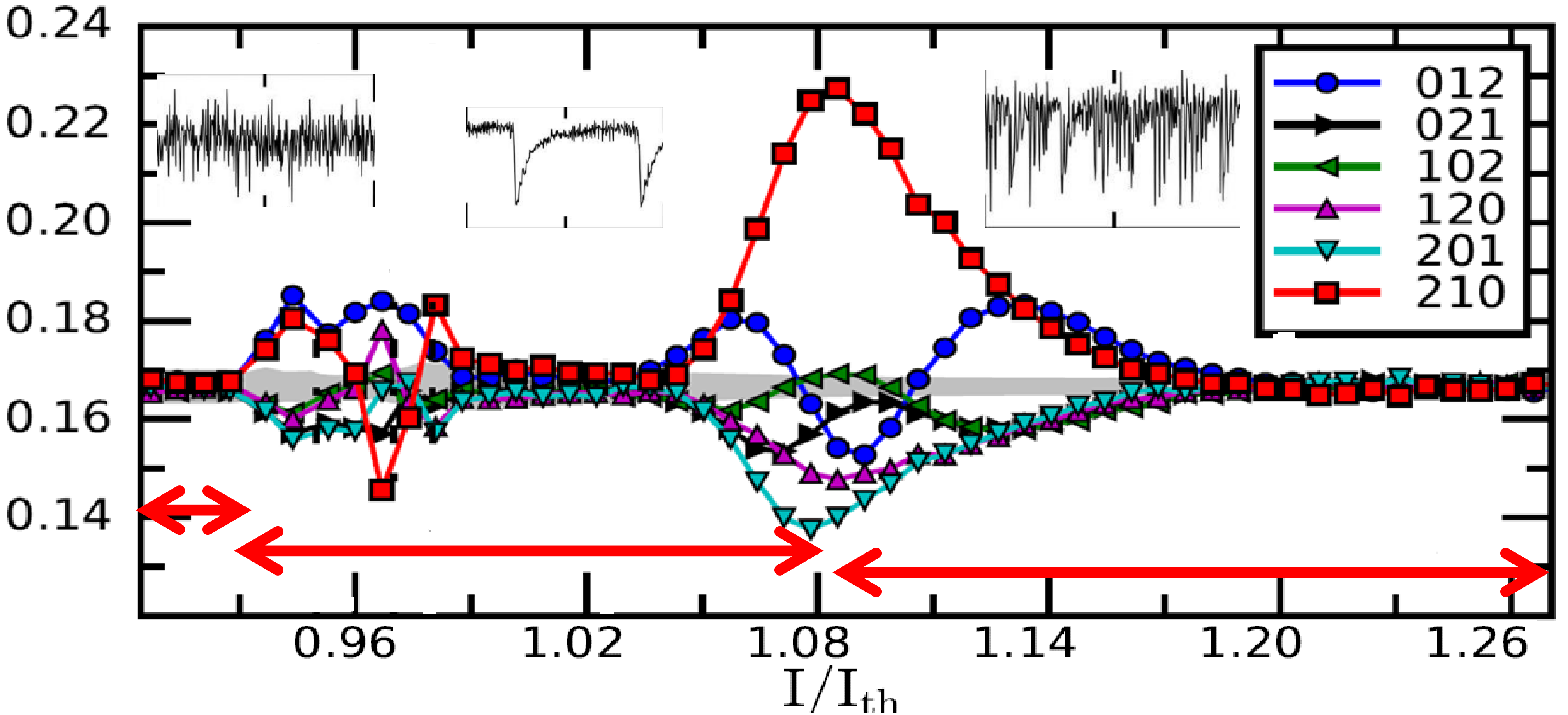




Consistent with stochastic dynamics at low pump current, signatures of "determinism" at higher pump currents.

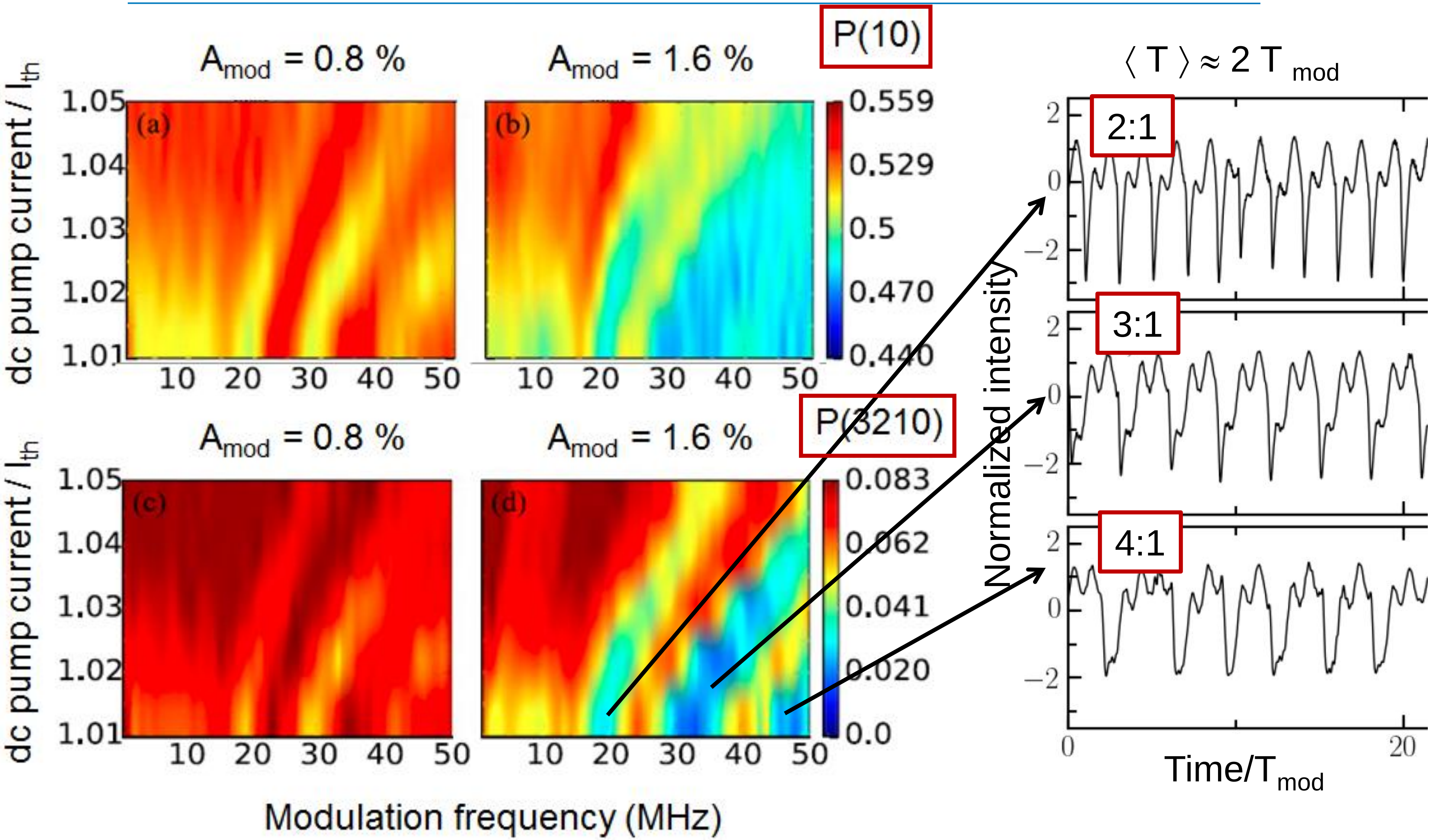
[N. Rubido, J. Tiana-Alsina, et al, Phys. Rev. E 84, 026202 (2011)]

Ordinal analysis allows to quantify the onset of different dynamical regimes



C. Quintero-Quiroz et al, Sci. Rep. 6, 37510 (2016)

Ordinal probabilities identify regions of noisy locking



T. Sorrentino et al, JSTQE 21, 1801107 (2015)

Contrasting empirical optical spikes with synthetic neuronal spikes

- do they have similar ordinal statistics?
- are there more/less frequent patterns?

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Ordinal analysis of ISI correlations in the region of low-frequency fluctuations

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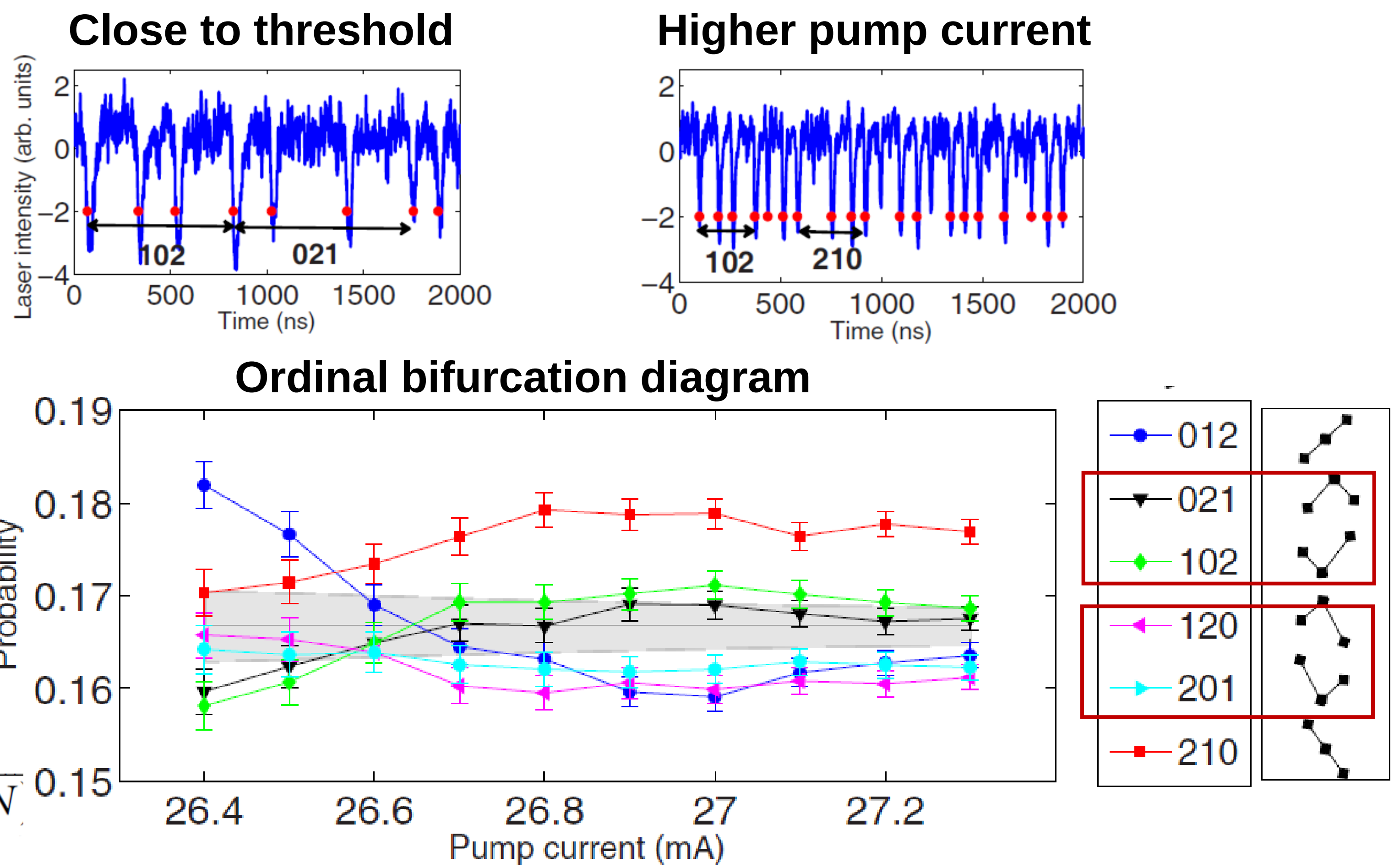


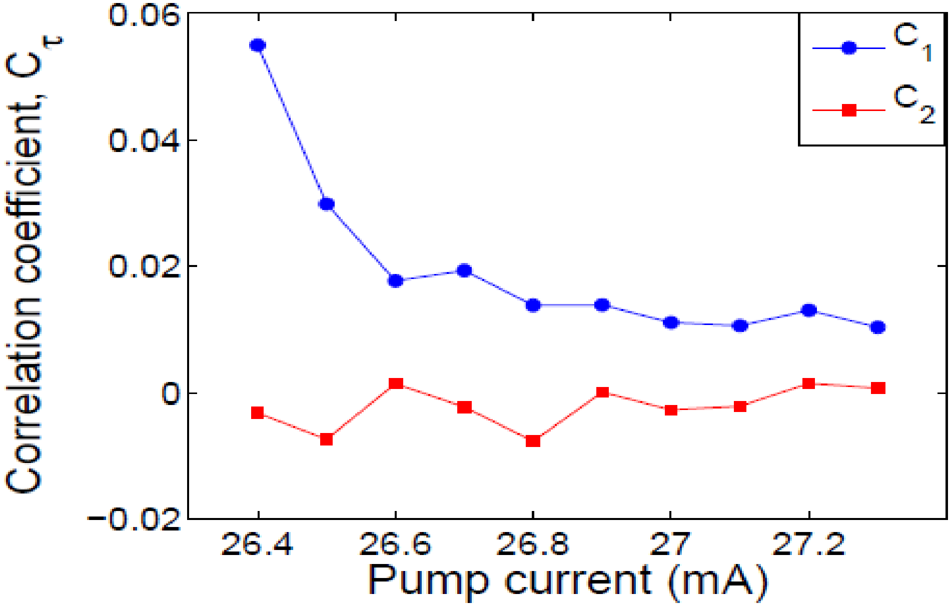
Grey region 99.7% confidence level.

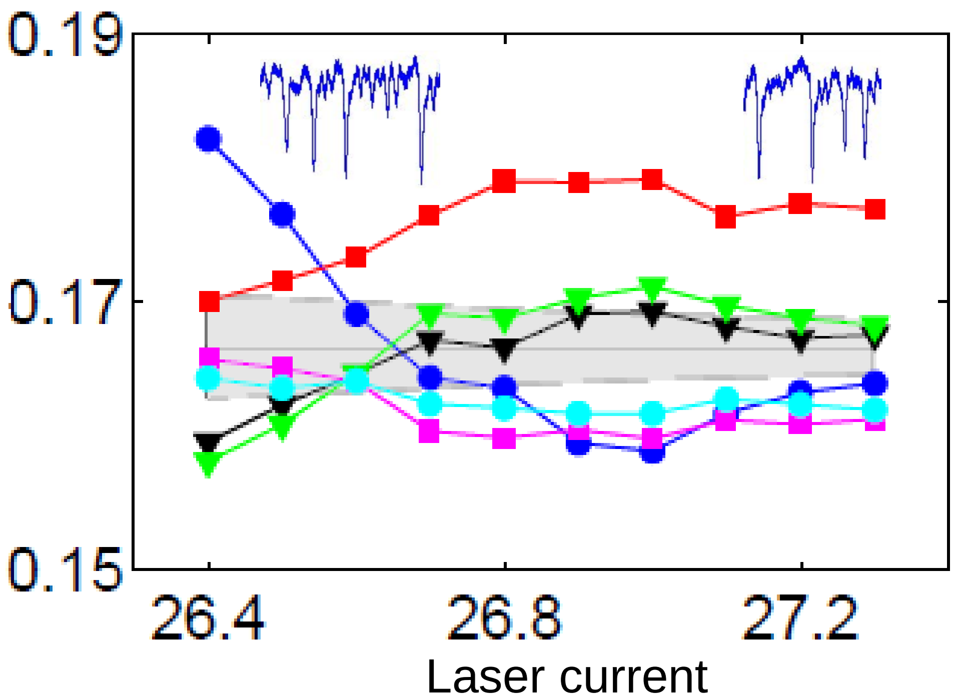
$$p \pm 3\sigma_p$$

$$\sigma_p = \sqrt{(p(1-p)/N}$$

P =1 /6; ***N* > 10,000 ISIs**

A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)





Is the "transition" detected by correlation analysis?

$$C_j = \frac{\langle (I_i - \langle I \rangle)(I_{i-j} - \langle I \rangle) \rangle}{\sigma^2}$$

- not detected.
- C_2 is very small ⇒ no significant linear correlation between I_i and I_{i+2}
- But ordinal probabilities are not consistent with equally probable patterns.

A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)

Minimal model of ISI nonlinear correlations: modified circle map

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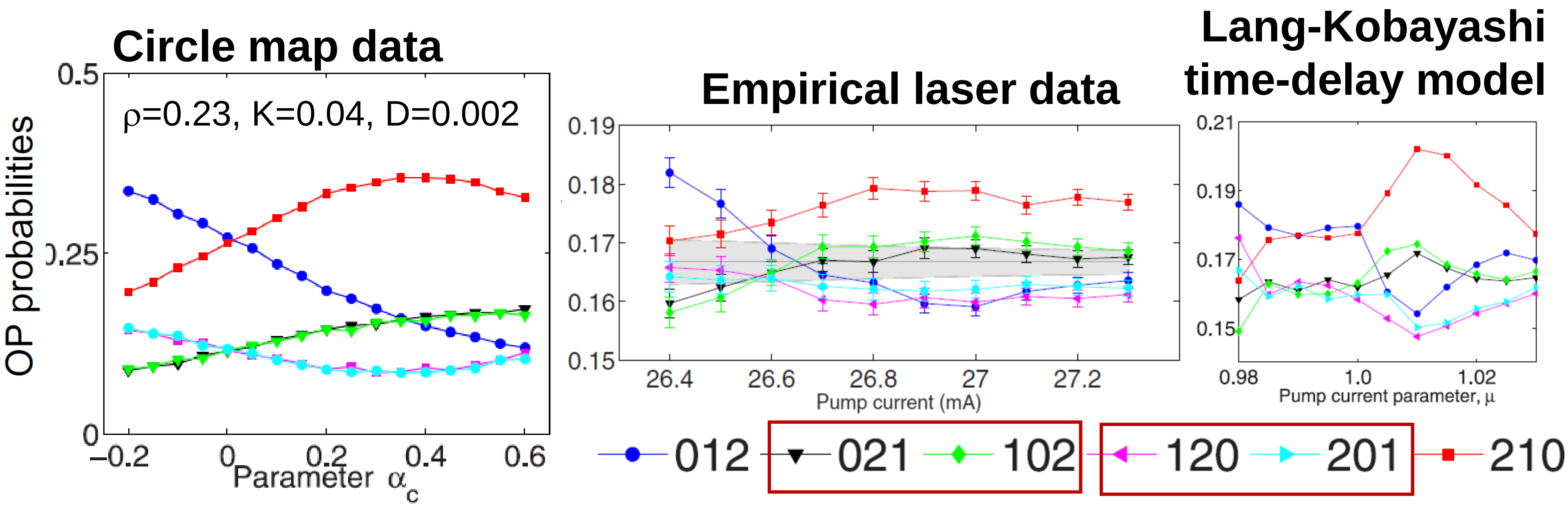
$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha_c \sin(4\pi\varphi_i)\right] + D\zeta$$

$$X_i = \varphi_{i+1} - \varphi_i$$

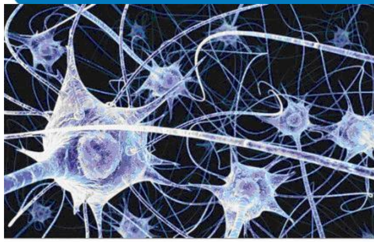
ρ = natural frequency / forcing frequency
K = forcing amplitude
D = noise strength



- Same "clusters" & same hierarchical structure.
- Modified circle map: minimal model for ordinal correlations.
- Same qualitative behavior found with other lasers & feedback conditions.

A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)

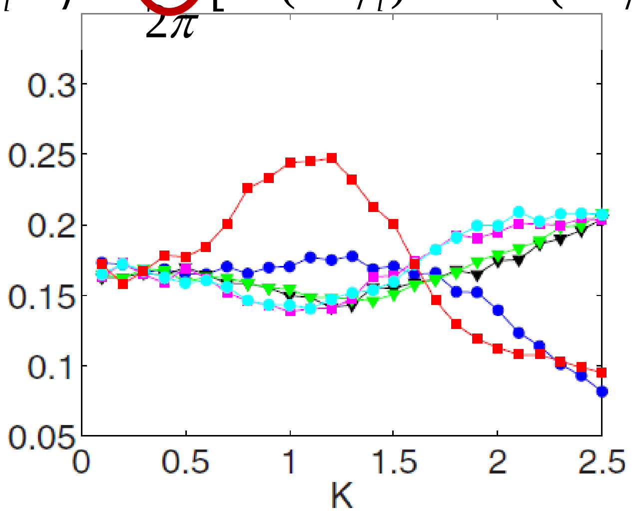
Connection with neurons



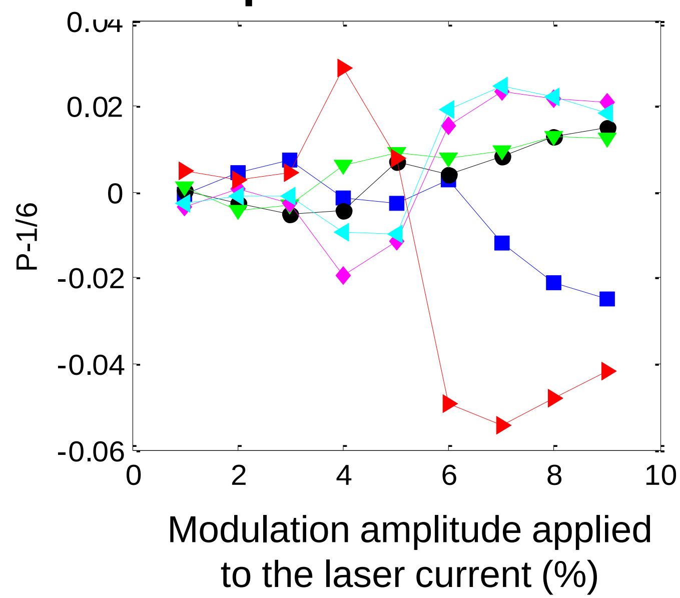
- The circle map describes many excitable systems.
- The modified circle map has been used to describe spike correlations in biological neurons.

  A. B. Neiman and D. F. Russell, *Models of stochastic biperiodic oscillations and extended serial correlations in electroreceptors of paddlefish*, PRE 71, 061915 (2005)

$$\varphi_{i+1} = \varphi_i + \rho + \frac{K}{2\pi}\left[\sin(2\pi\varphi_i) + \alpha\sin(4\pi\varphi_i)\right]$$

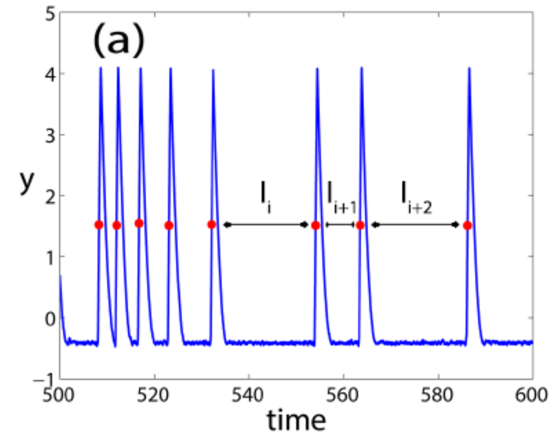


Empirical laser data



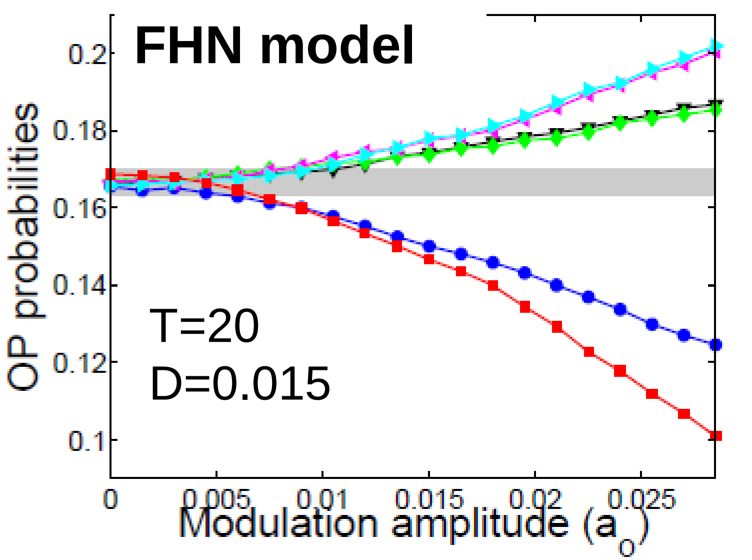
FHN neuron model

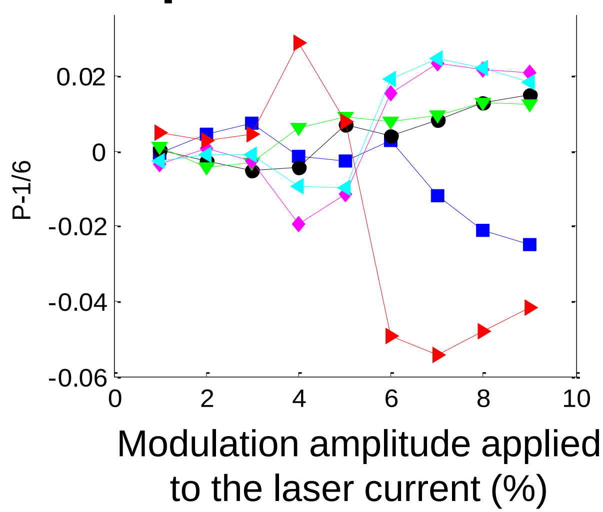
$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o \cos(2\pi t/T)} + D\xi(t),$$



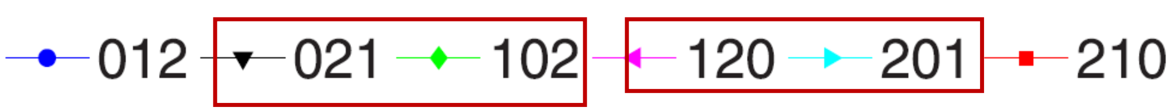
Gaussian white noise and **subthreshold** (weak) modulation: **a_0** and **T** such that spikes are only noise-induced.

Time series with 100,000 ISIs simulated.





$\Rightarrow$ Good qualitative agreement.



Analysis of ISI sequences generated by FHN model

- how a single neuron encodes information about a weak external signal?

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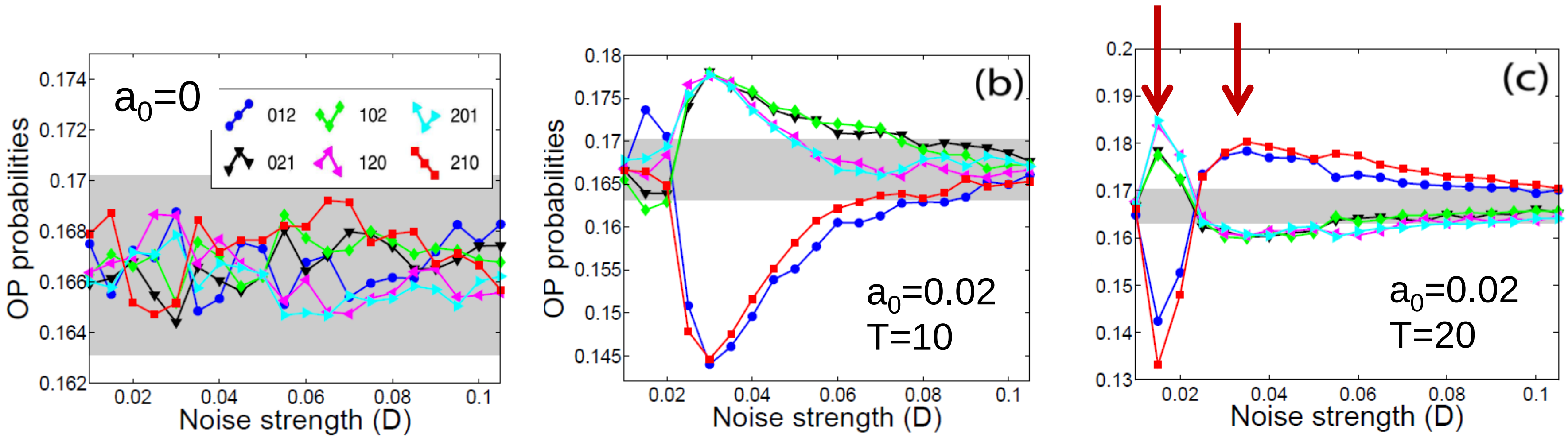
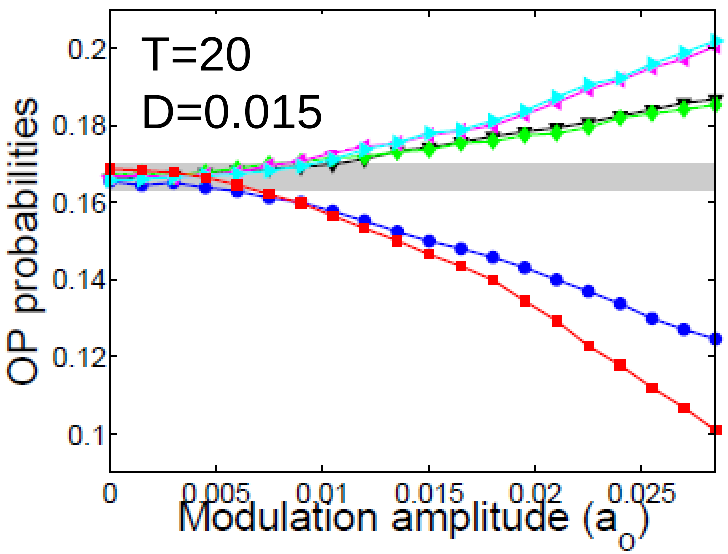
FHN model: role of (white) noise

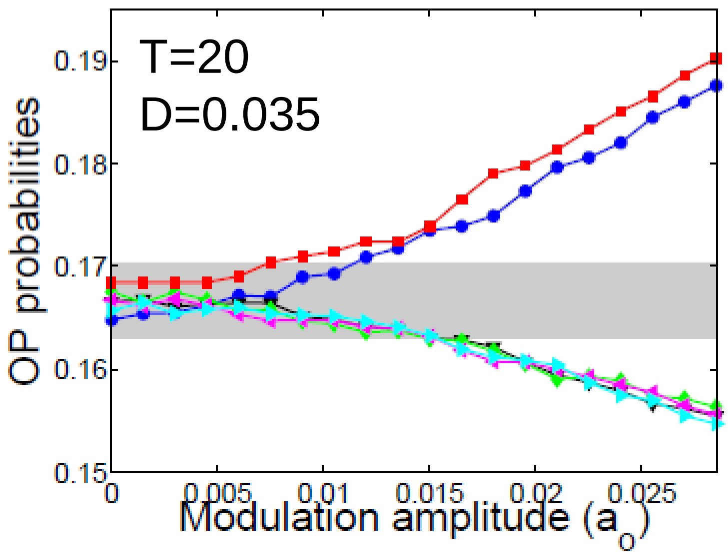


- No signal ⇒ no noise-induced temporal ordering.
- Subthreshold periodic input induces temporal ordering.
- Preferred ordinal patterns depend on the period and on the noise strength.
- Resonant-like behavior.

Aparicio-Reinoso, Torrent and Masoller, PRE 94, 032218 (2016)

Role of the (subthreshold) modulation amplitude





- The amplitude of the (weak) modulation does not modify the preferred and the infrequent patterns.
- The ordinal probabilities encode information about the signal's amplitude.

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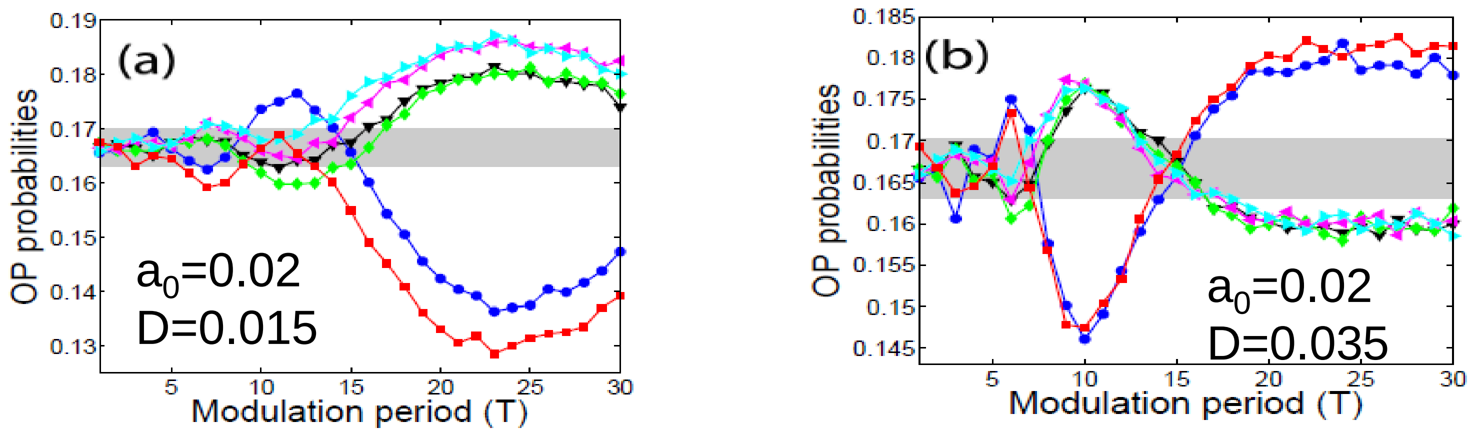
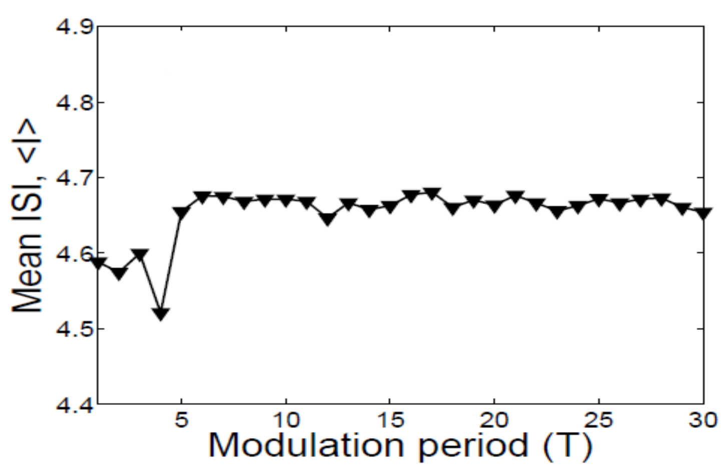
Role of the modulation period



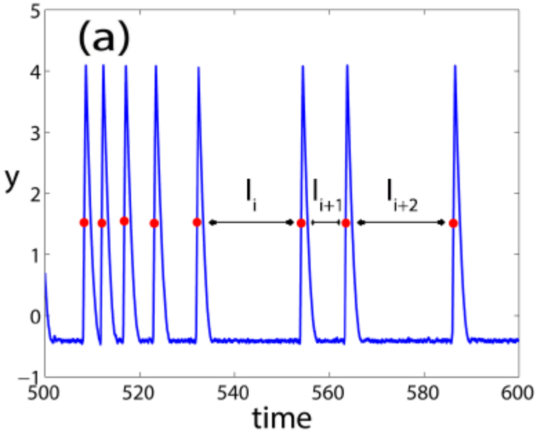
- More probable patterns depend on the period of the external input and on the noise strength.

Which is the underlying mechanism? A change of the spike rate?

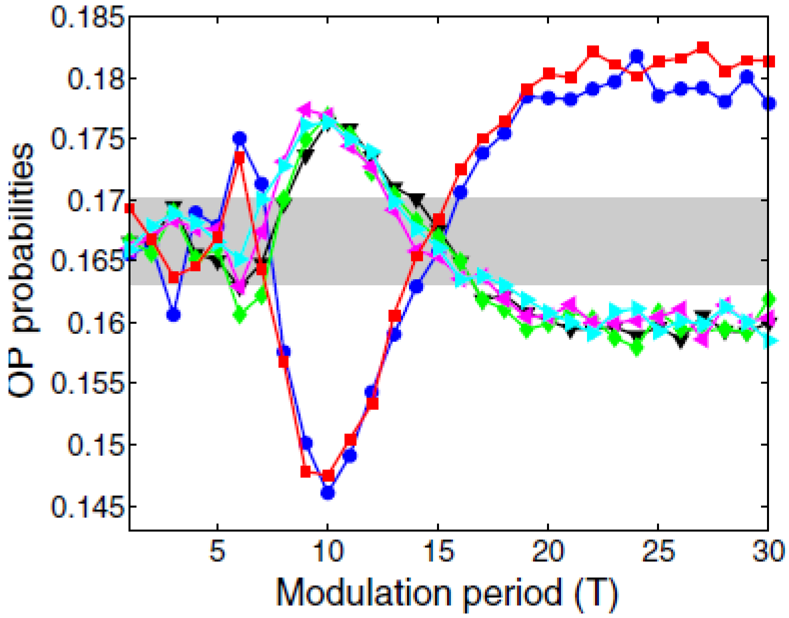


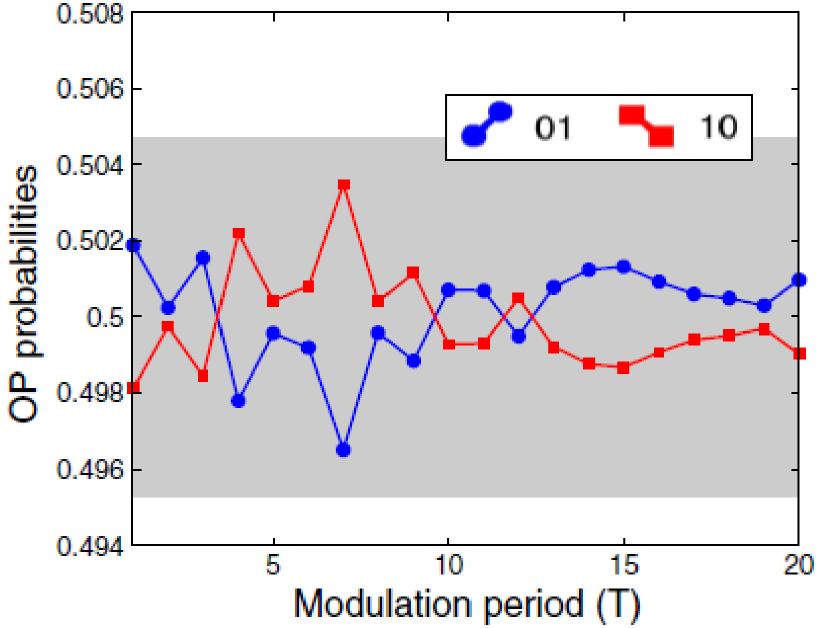
$\Rightarrow$ the spike rate does not encode information of period of the weak signal.

Length of ISI correlations



But using patterns with two letters (comparing only two consecutive time intervals)

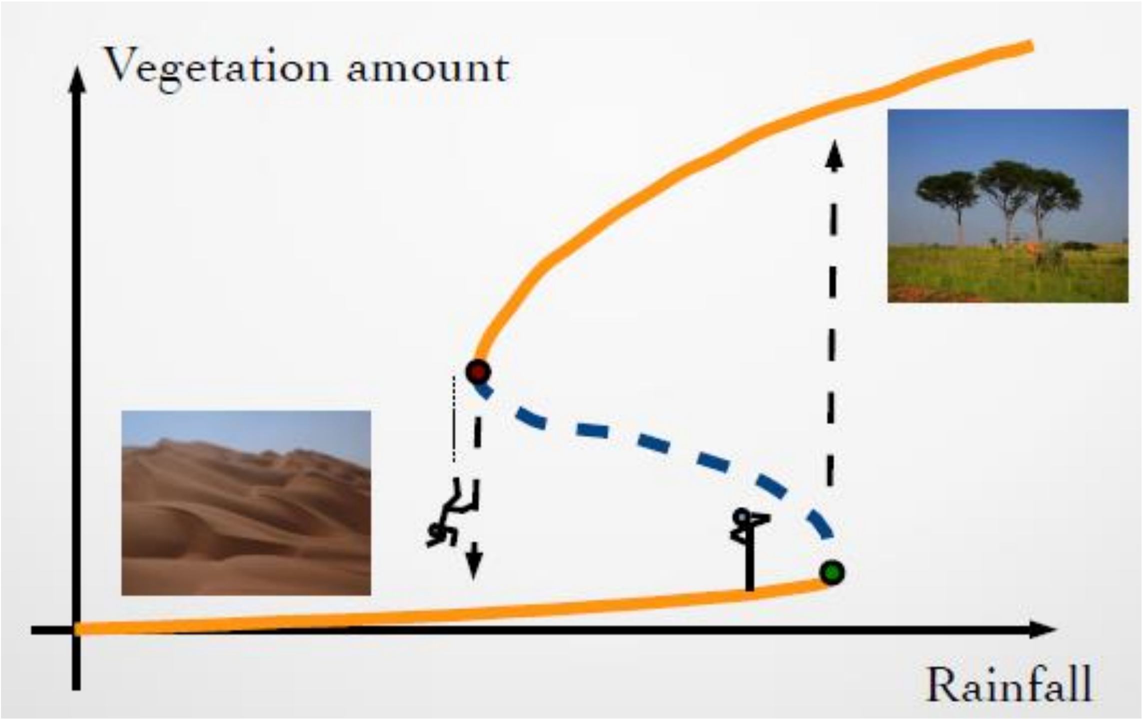




[Aparicio-Reinoso, Torrent and Masoller, PRE 94, 032218 (2016)](#)

What did we learn?

- Transition to optical chaos: different regimes can be *quantitatively* distinguished.
- Minimal model for optical spikes identified (modified circle map)
- Optical & neuronal spikes compared: good qualitative agreement.
- FHN neuron model with weak forcing and white noise:
  - Preferred ordinal patterns depend on the noise strength and on the period of the input signal.
  - resonance-like behavior: certain periods and noise levels maximize the probabilities of the preferred patterns.
- Open questions: why the ordinal probabilities are "clustered"?
- Robust mechanism for neuronal encoding of weak periodic inputs?



Early-warning indicators of desertification transition

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Early warning indicators

- Bifurcation → eigenvalue with 0 real part
- → long recovery time of perturbations
- Critical Slowing Down (CSD)
- CSD → High autocorrelation, variance, spatial correlation, etc.

- Can we use "correlation networks" to detect tipping points?

- "correlation networks"?

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Desertification transition: model

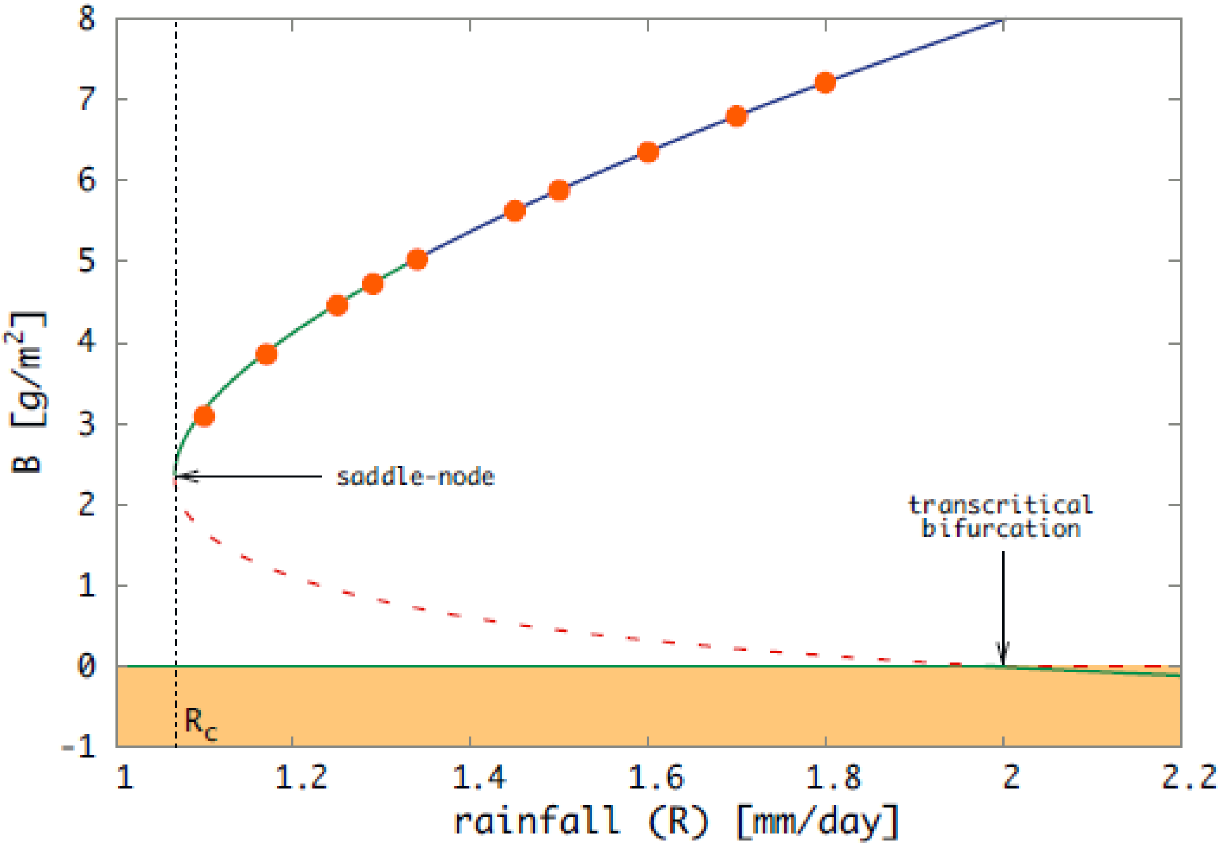
$$dw_t = \left(R - \frac{w}{\tau_w} - \Lambda w B + D \Delta w \right) dt + \sigma_w dW_t$$

$$dB_t = \left(\rho B \left(\frac{w}{w_0} - \frac{B}{B_0} \right) - \mu \frac{B}{B + B_o} + D \Delta B \right) dt + \sigma_B dW_t$$

- *w* (in mm) is the soil water amount
- *B* (in g/m^2) is the vegetation biomass
- Uncorrelated Gaussian white noise
- *R* (rainfall) is the bifurcation parameter

Shnerb et al. (2003), Guttal & Jayaprakash (2007), Dakos et al. (2011)

Saddle-node bifurcation



$R<R_c$: only desert-like solution (B=0)

R_c = 1.067 mm/day

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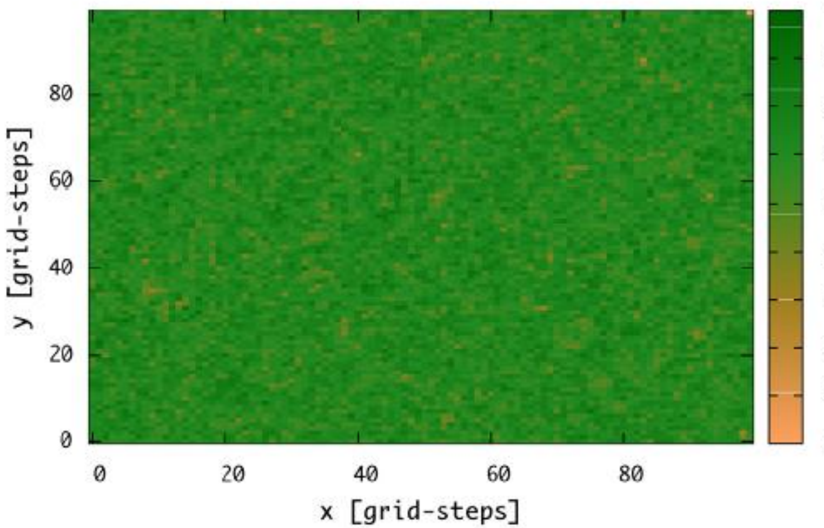
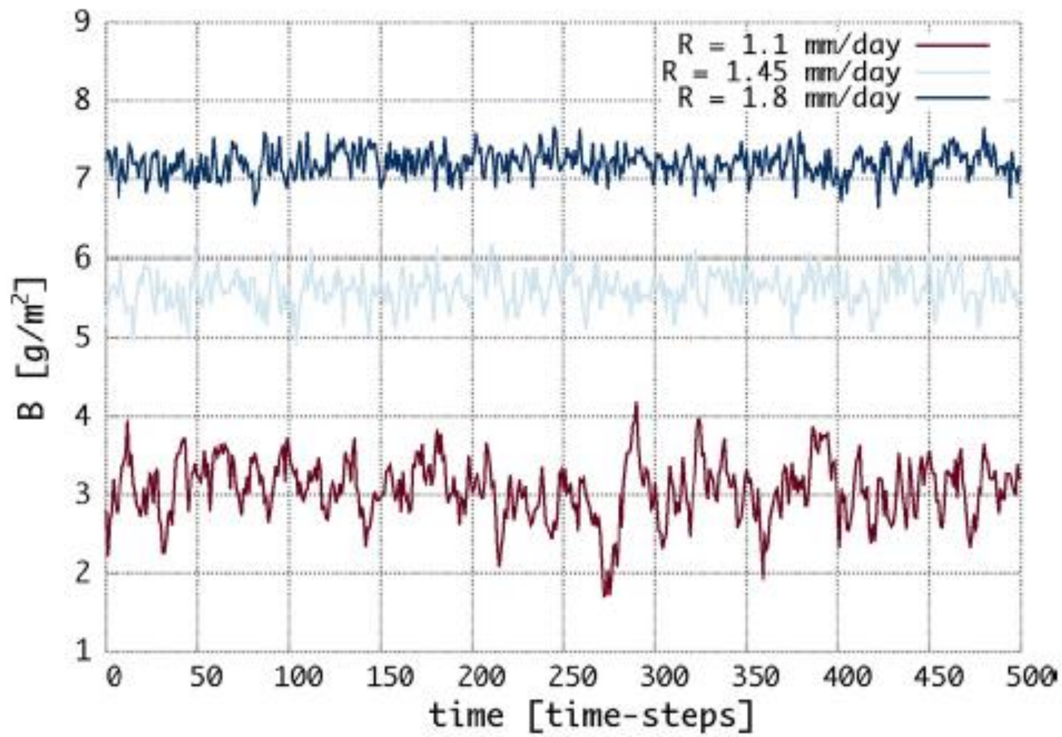
Biomass *B* when *R*=1.1 mm/day





100 m x 100 m = 10^4 grid cells
Simulation time 5 days in 500 time steps
Periodic boundary conditions

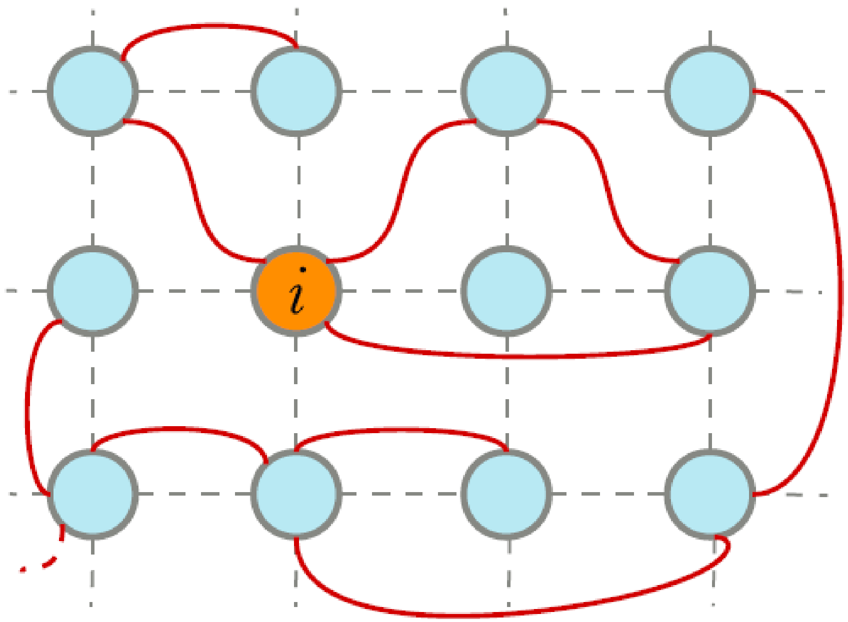
Correlation Network

$$A_{ij} = \mathrm{H}(|\mathcal{C}(B_i, B_j)| - \theta)$$

Adjacency matrix

Zero-lagged cross-correlation

Threshold $\theta=0.2$ gives $p<0.05$



G. Tirabassi et al., Ecological Complexity 19, 148 (2014)

Network analysis

- **Degree** (number of links of a node)

$$k_i \equiv \sum_{j=1}^{N} A_{ij}$$

- **Assortativity** (average degree of the neighbors of a node)

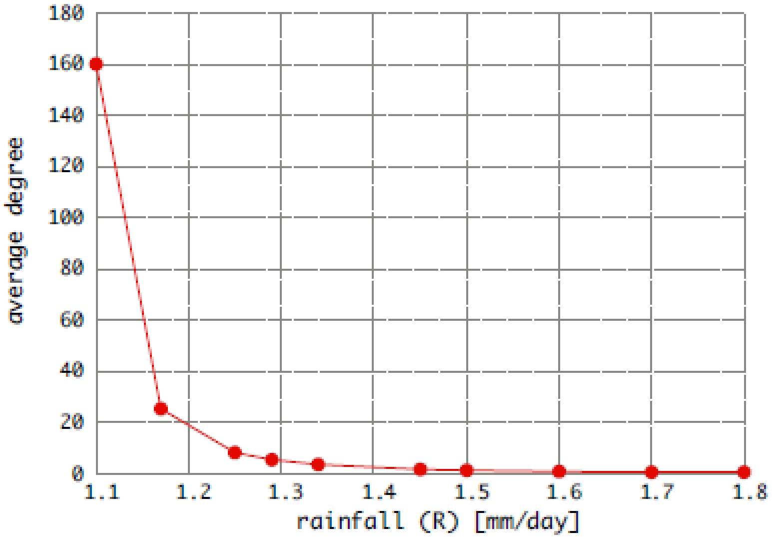
$$a_i \equiv \frac{1}{k_i} \sum_{j=1}^{N} A_{ij} k_j$$

- **Clustering** (fraction of neighbors of a node that are also neighbors among them)

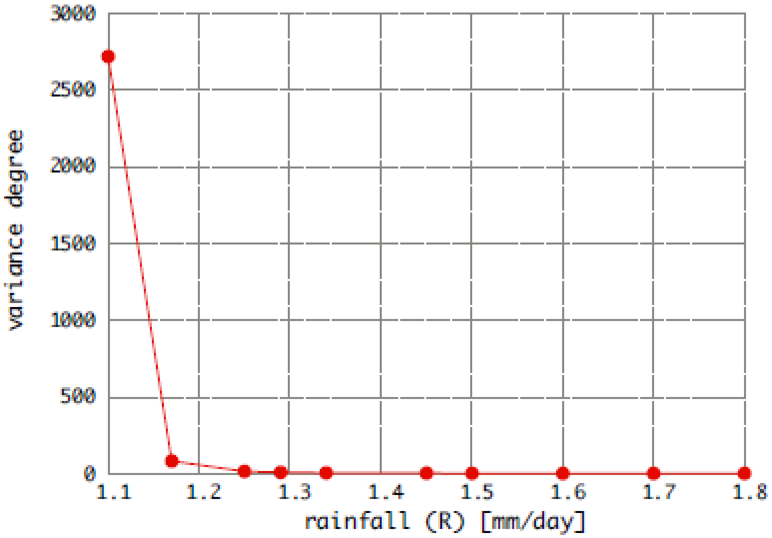
$$c_i \equiv \frac{1}{k_i(k_i - 1)} \sum_{j=1}^{N} \sum_{l=1}^{N} A_{ij} A_{jl} A_{li}$$

Results

Mean degree

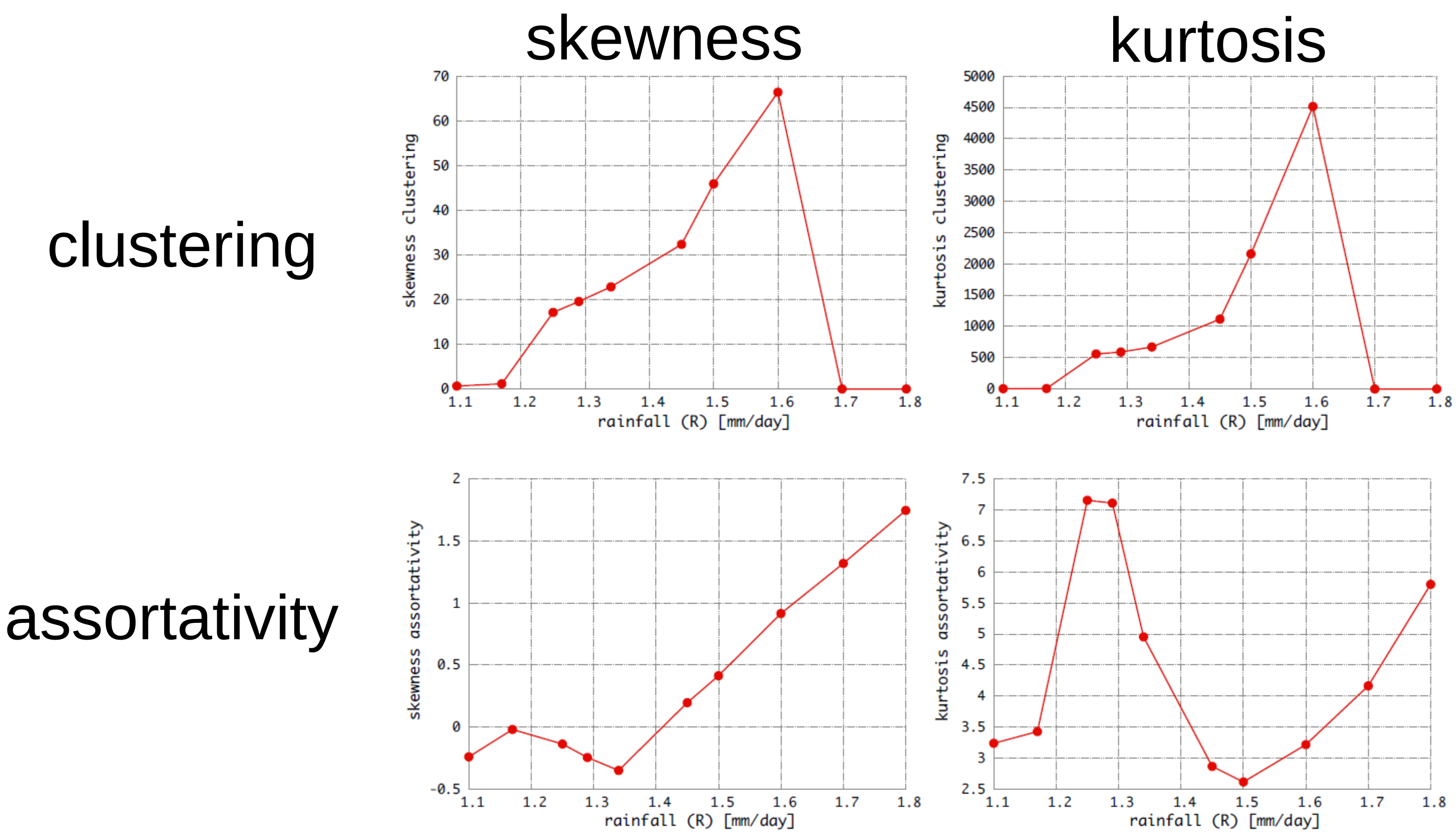


Standard deviation of the degree distribution



Sharp increase close to the transition captures the emergence of spatial correlations

Network-based indicators

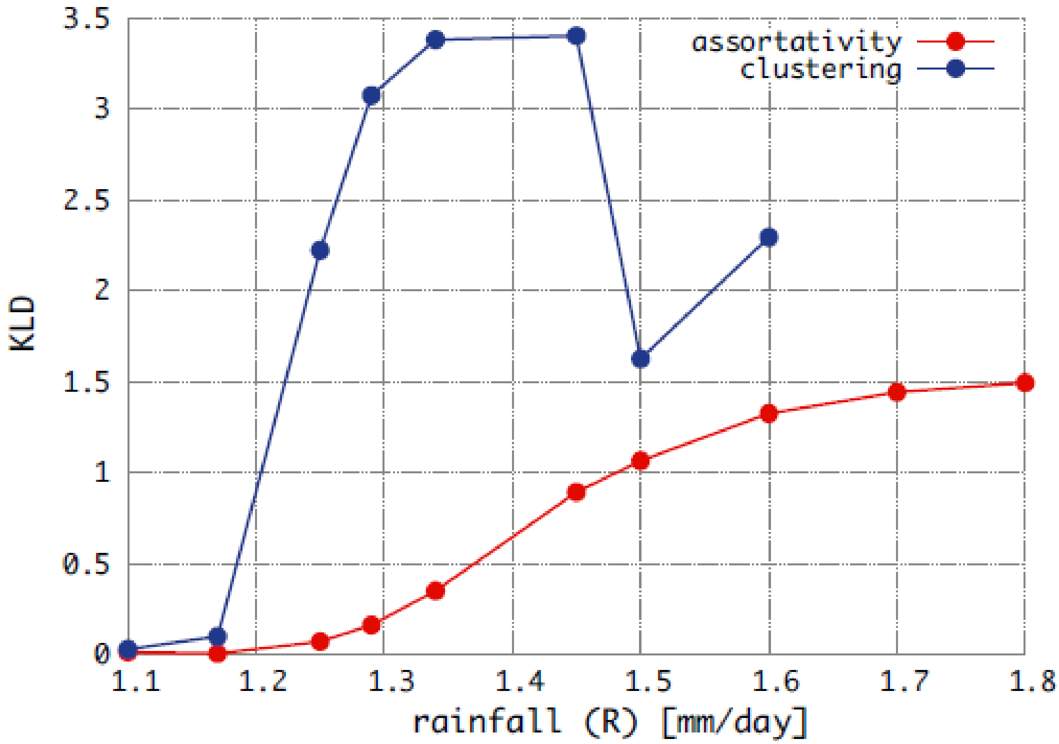


"Gaussianisation" of the clustering and of the assortativity distributions when approaching the tipping point

How to quantify "Gaussianisation"?

Kullback–Leibler Distance (KLD) between 2 PDFs

$$\mathrm{KLD} \equiv \int_{-\infty}^{\infty} \ln\left(\frac{P(x)}{Z(x)}\right) P(x)\, dx.$$



- Open issue: the "Gaussianisation" might be a model-specific feature.

G. Tirabassi et al., Ecological Complexity 19, 148 (2014)

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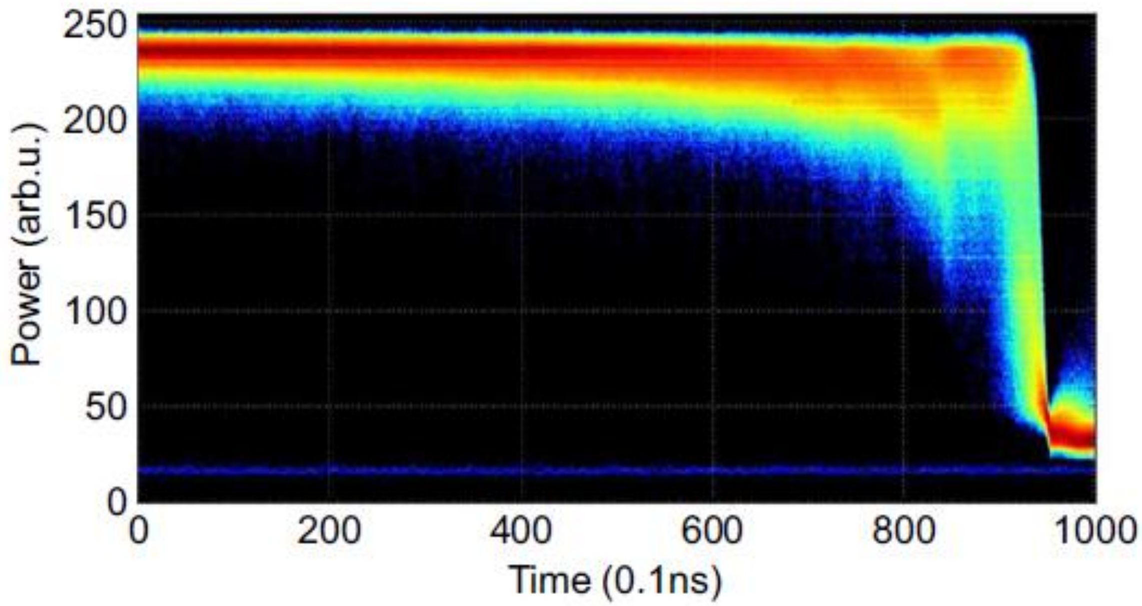
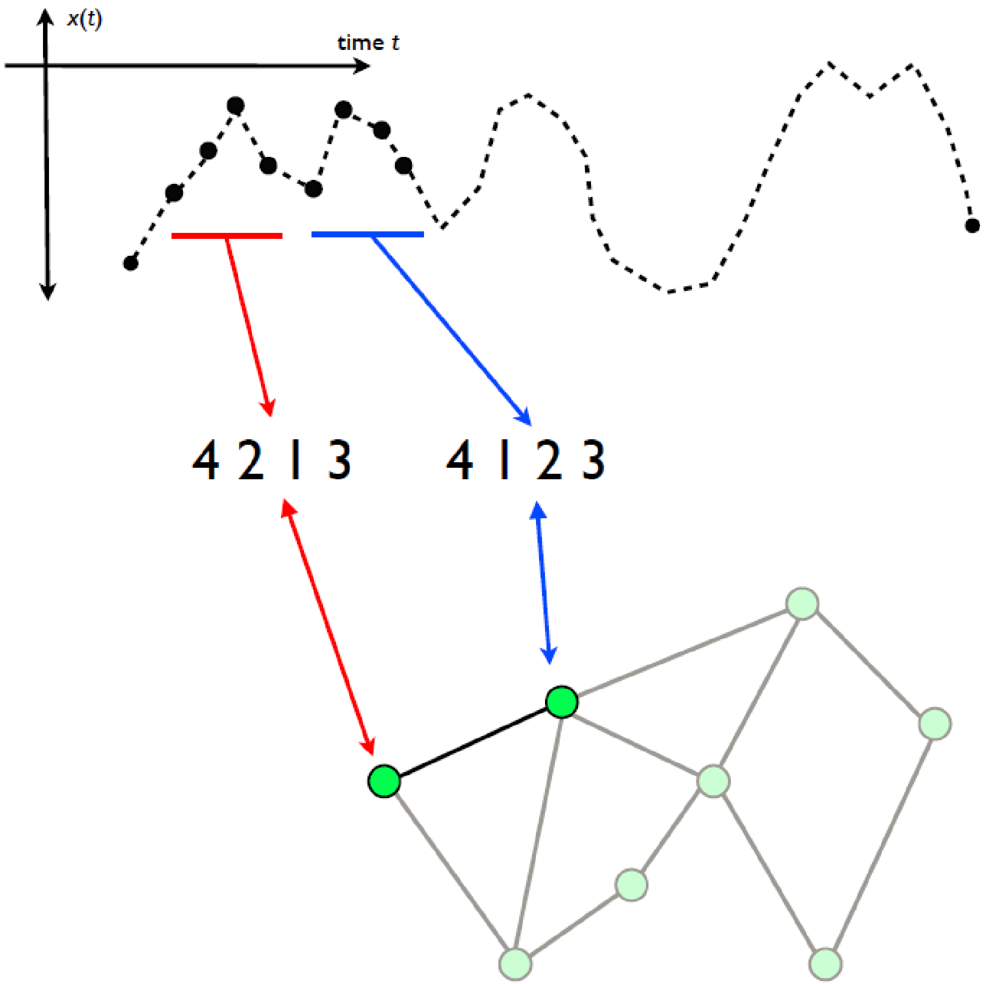
Laser empirical data



- We analyze time series recorded at constant laser current.
- Record the polarization mode that turns on.

Is it possible to anticipate the PS?
No if the mechanisms that trigger the PS are fully stochastic.

The network nodes are the "ordinal patterns", and the links?



- The links are defined in terms of the probability of pattern "β" occurring after pattern "α".
- Weighs of nodes: the probabilities of the patterns (**$\sum_i p_i=1$**).
- Weights of links: the probabilities of the transitions (**$\sum_j w_{ij}=1 \ \forall i$**).

⇒ **Weighted and directed network**

Adapted from M. Small (The University of Western Australia)

Three network-based diagnostic tools

- Entropy computed from the weights of the nodes (**permutation entropy**)

$$s_p = -\sum p_i \log p_i$$

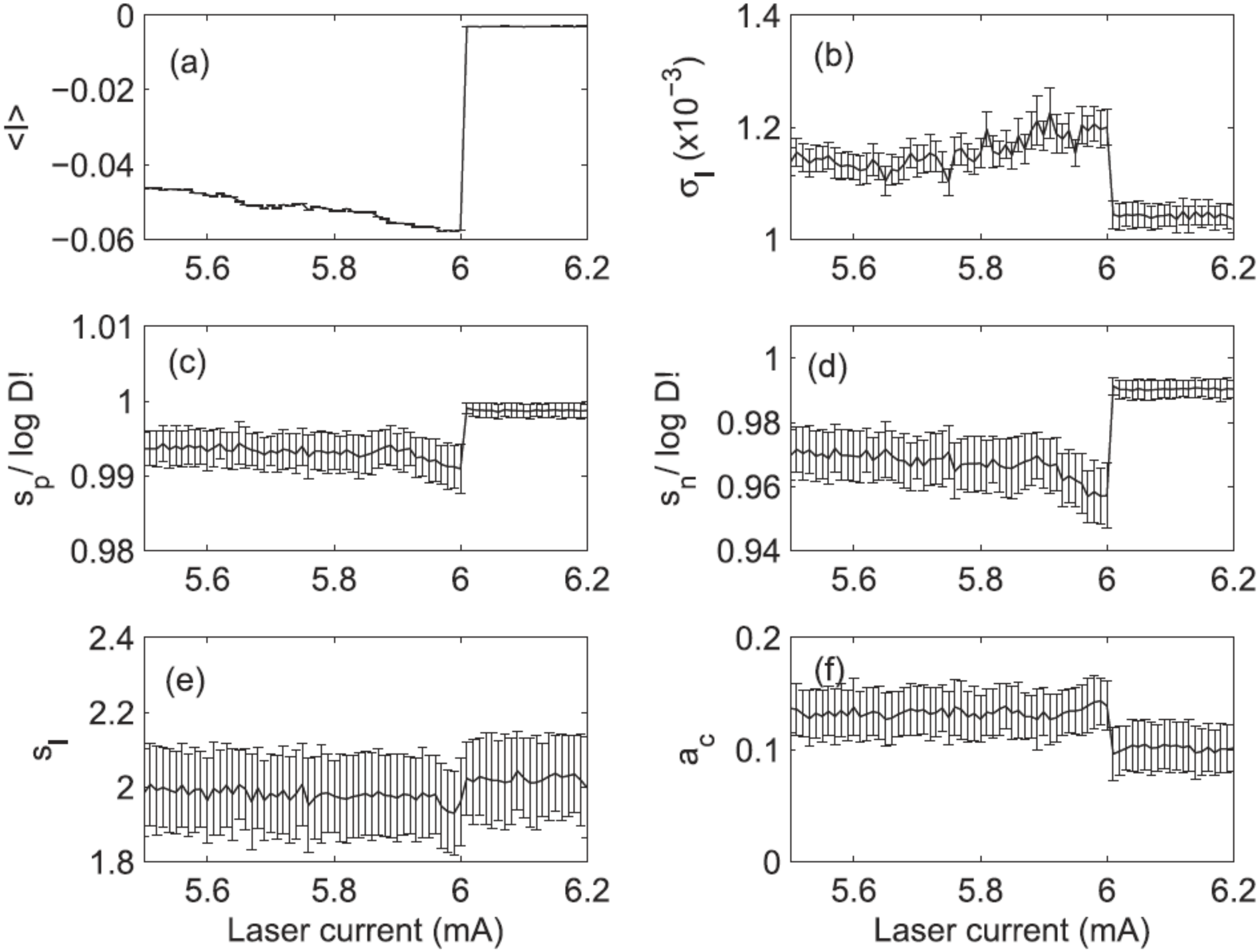
- Entropy computed from weights of the links (**transition probabilities**, '01'→ '01', '01'→ '10', etc.)

$$w_{ij} = \frac{\sum_{t=1}^{L-1} n\left[s(t) = i,\, s(t+1) = j\right]}{\sum_{t=1}^{L-1} n\left[s(t) = i\right]}$$

- Asymmetry coefficient: normalized difference of transition probabilities, P('01'→ '10') - P('10'→ '01'), etc.

$$a_c = \frac{\sum_i \sum_{j \neq i} \left| w_{ij} - w_{ji} \right|}{\sum_i \sum_{j \neq i} \left(w_{ij} + w_{ji} \right)}$$

(0 in a fully symmetric network; 1 in a fully directed network)

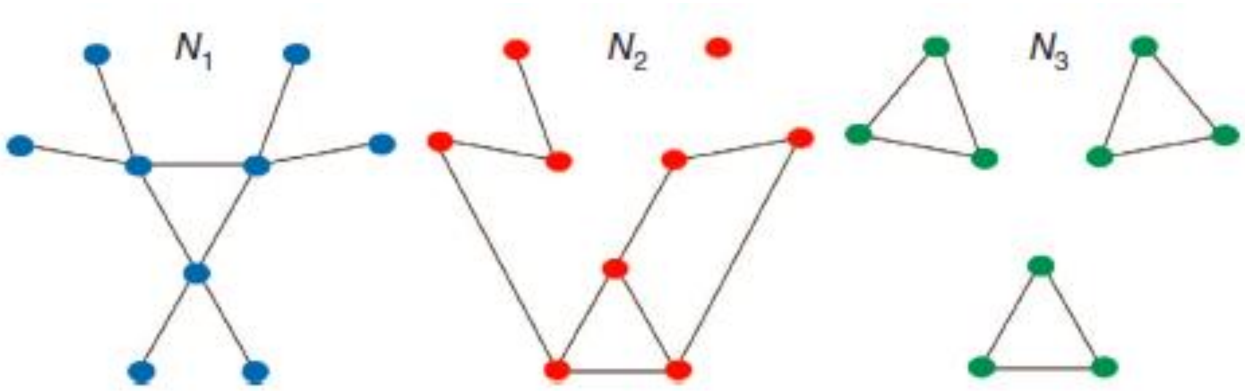


⇒ Despite of the stochasticity of the time-series, the measures "anticipate" the PS.

⇒ Deterministic mechanisms involved.

Error bars computed from 100 non-overlapping windows with L=1000 data points each. Length of the pattern D=3.

C. Masoller et al, NJP 17, 023068 (2015)



Quantifying network dissimilarities

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Coauthors: T. A. Schieber, L. Carpi, M. G. Ravetti (Bello Horizonte, Brazil), A. Diaz-Guilera (UB), P. M. Pardalos (Florida, US)

Complex network measures

- Degree distribution, closeness centrality, betweenness centrality, average path length, etc.
- Provide *partial* information.
- How to define a measure that contains detailed information about the *global topology* of a network, in a *compact* way?

$\Rightarrow$ Node Distance Distributions (NDDs)

- $p_i(j)$ of node "i" is the fraction of nodes that are connected to node i at distance j
- If a network has N nodes:

$$\text{NDDs} = \text{vector of N pdfs } \{p_1, p_2, \ldots, p_N\}$$

- If two networks have the same set of NDDs $\Rightarrow$ they have the same diameter, average path length, etc.

How to condense the information contained in the node-distance distributions?

- The *Network Node Dispersion (NND*) measures the heterogeneity of the N pdfs $\{p_1, p_2, \ldots, p_N\}$
- Quantifies the heterogeneity of connectivity distances.

$$\mathrm{NND}(G) = \frac{\mathcal{J}(\mathbf{P}_1, \ldots, \mathbf{P}_\mathrm{N})}{\log(d+1)}$$

d = diameter

$$\mathcal{J}(\mathbf{P}_1, \ldots, \mathbf{P}_\mathrm{N}) = \frac{1}{N}\sum_{\mathrm{i,j}} p_\mathrm{i}(j)\log\left(\frac{p_\mathrm{i}(j)}{\mu_\mathrm{j}}\right)$$

$$\mu_\mathrm{j} = \left(\sum_{\mathrm{i}=1}^{N} p_\mathrm{i}(j)\right)/N$$

Reminder:
distance between
P and Z

$$\mathrm{KLD} \equiv \int_{-\infty}^{\infty} \ln\left(\frac{P(x)}{Z(x)}\right) P(x)\,\mathrm{d}x.$$

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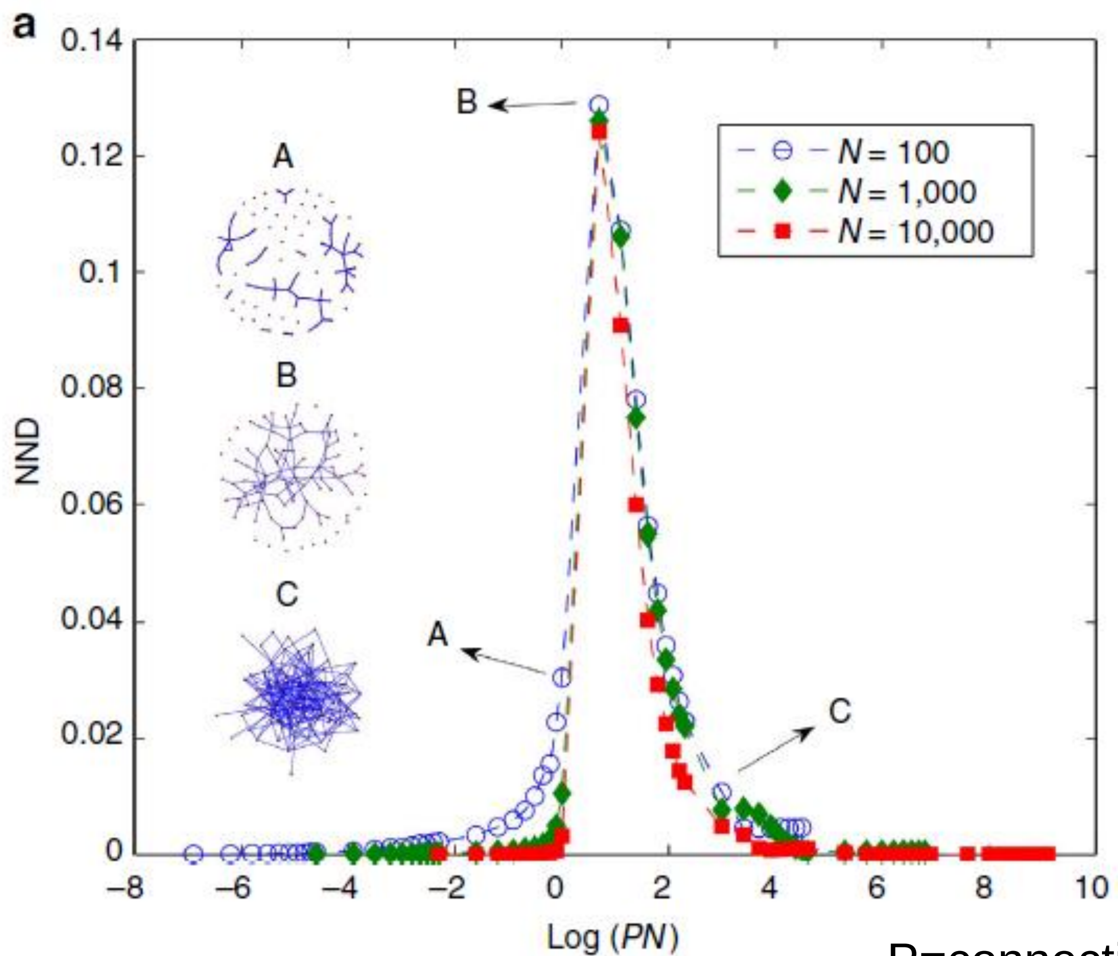
Example of application: percolation transition



⇒ in a random network the Network Node Dispersion detects the percolation transition

P=connection probability

T. A. Schieber et al, Nat. Comm. 8, 13928 (2017)

Dissimilarity between two networks

$$D(G,G') = w_1 \sqrt{\frac{\mathcal{J}(\mu_G, \mu_{G'})}{\log 2}} + w_2 \left| \sqrt{\mathrm{NND}(G)} - \sqrt{\mathrm{NND}(G')} \right|$$

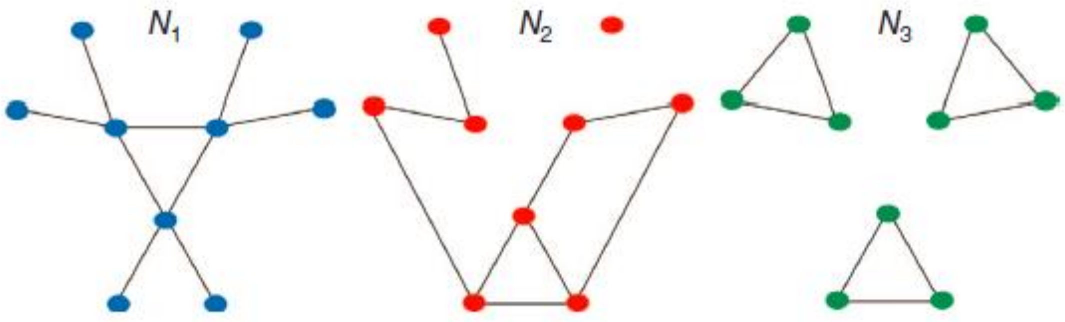
$w_1 = w_2 = 0.5$

compares the averaged connectivity

compares the heterogeneity of the connectivity distances

- Extensive numerical experiments demonstrate that isomorphic graphs return ***D=0***

Comparing three networks with the same number of nodes and links



	D	Hamming	Graph Edit Distance
N_1,N_2	0.25	12	6
N_1,N_3	0.56	12	6
N_2,N_3	0.47	12	6

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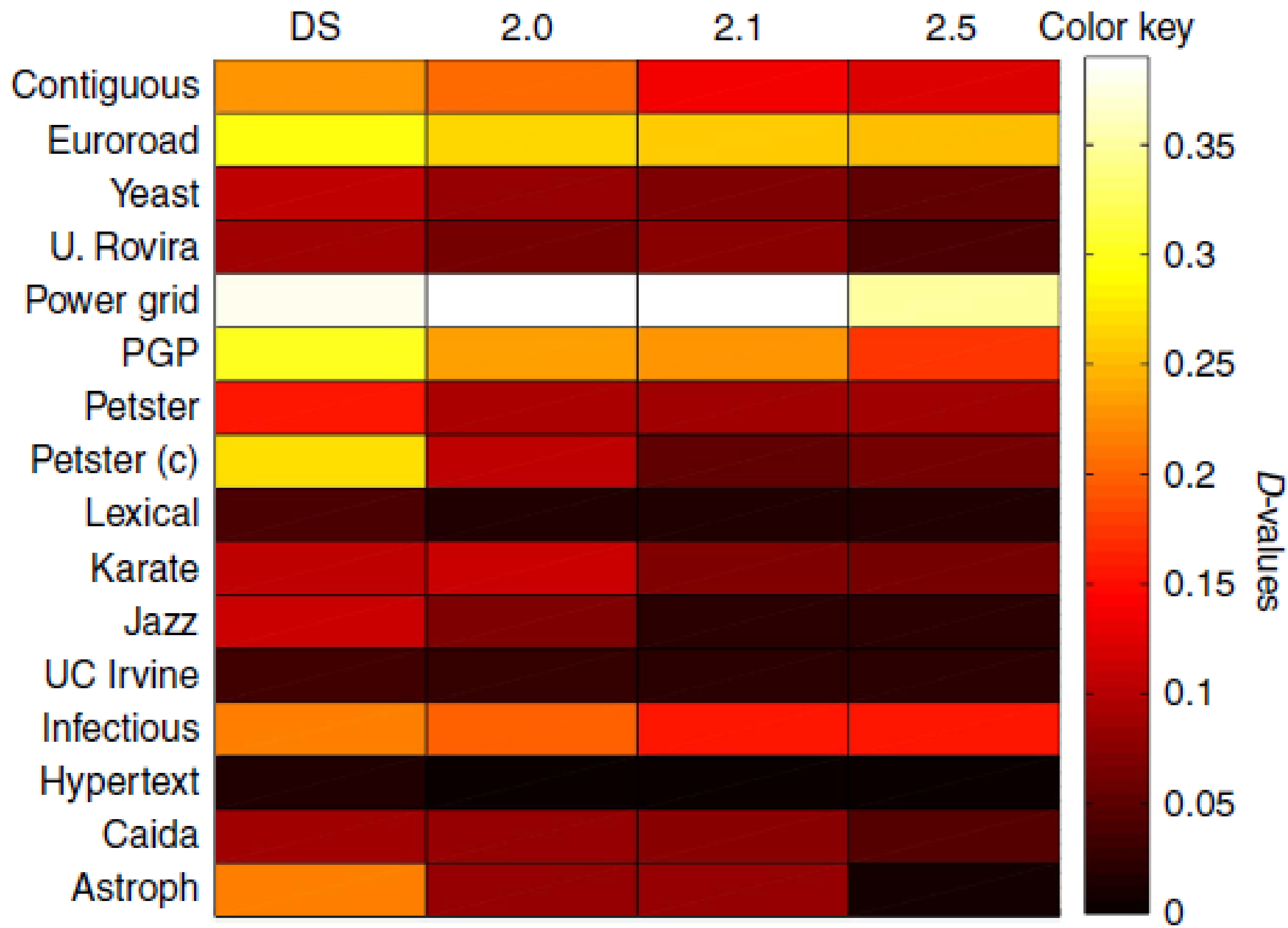
Comparing real networks to null models

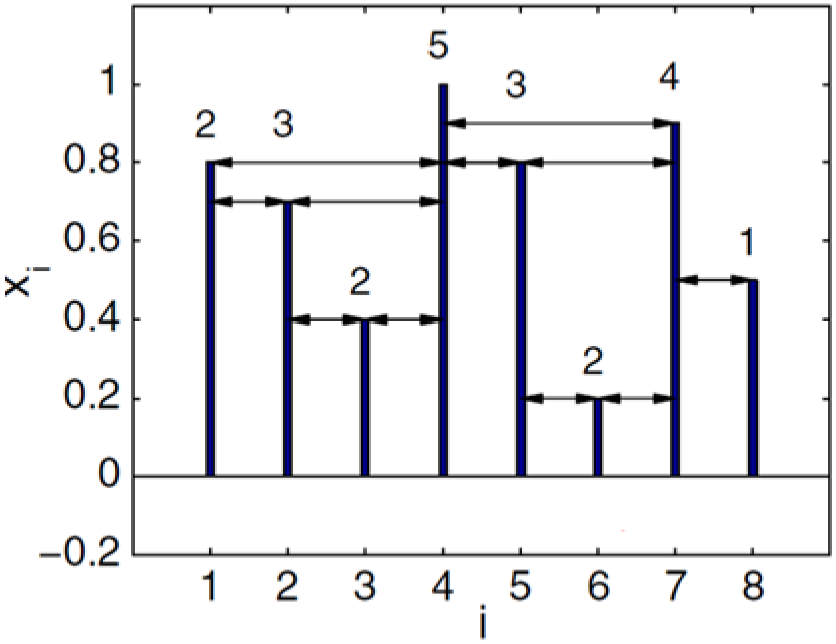
DS preserves the degree sequence; 2.0 also preserves the degree correlation; 2.1 also preserves the clustering coefficient; 2.5 includes the clustering spectrum



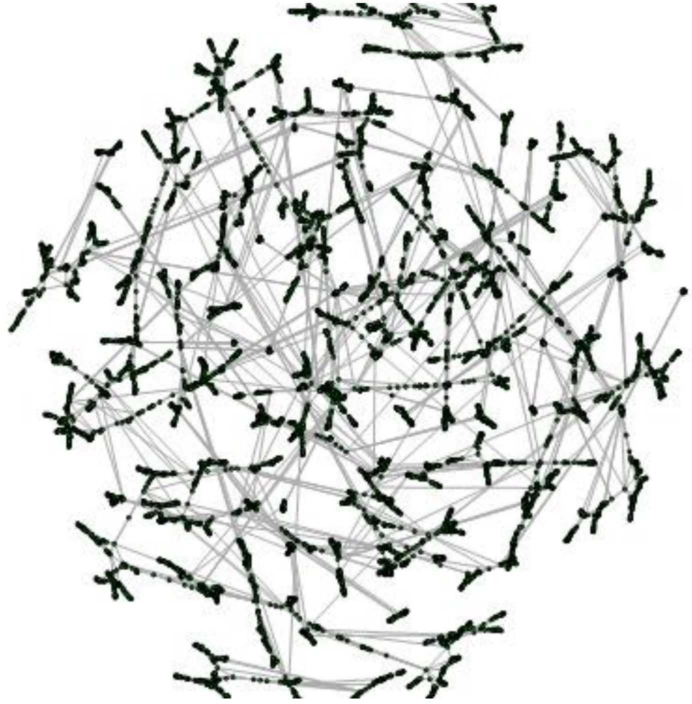
- Synthetic model for Power Grid Network?

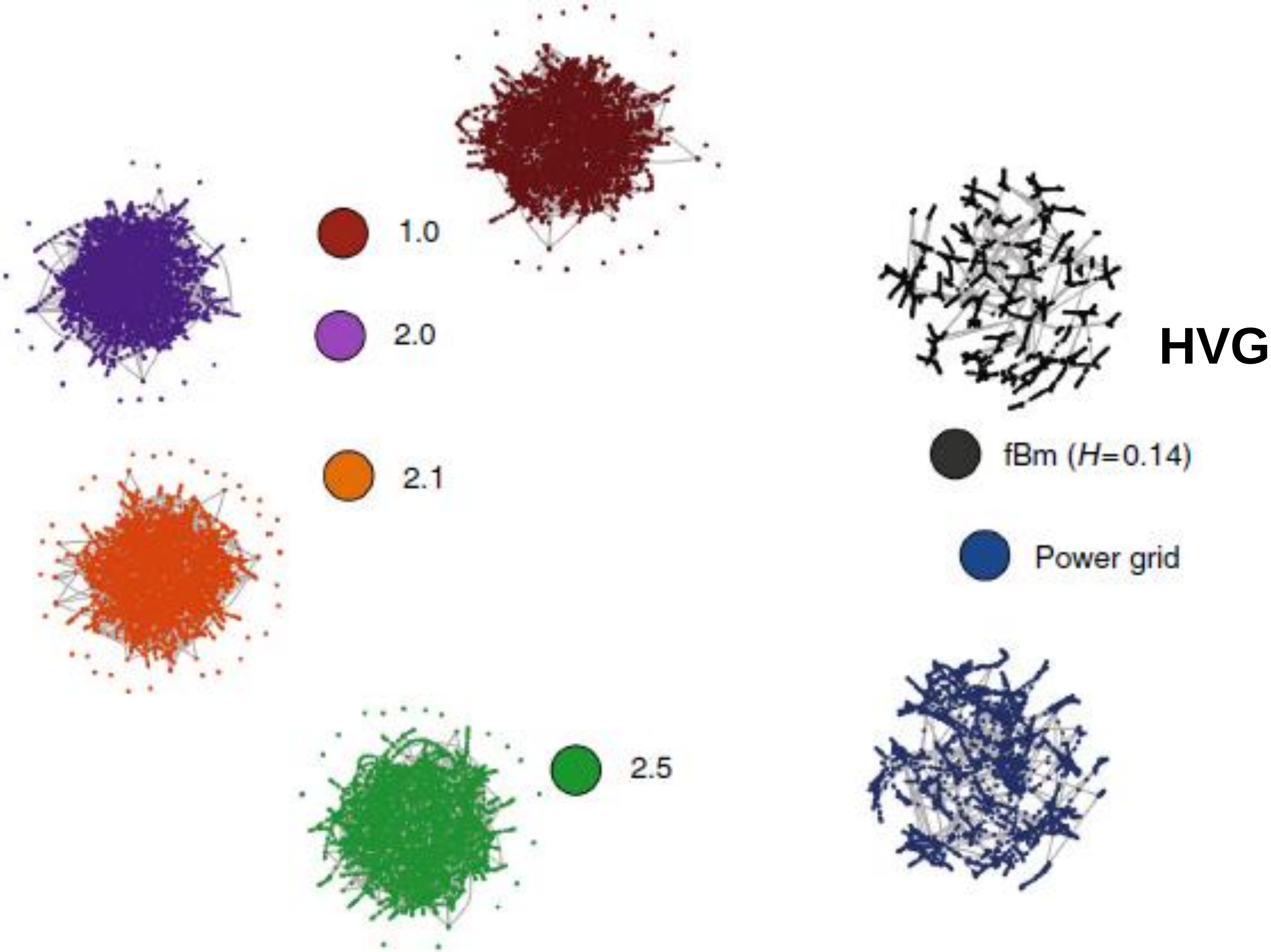
Horizontal Visibility Graph applied to synthetic data

fractional Brownian Motion (fBm) with controllable Hurst exponent



⇒





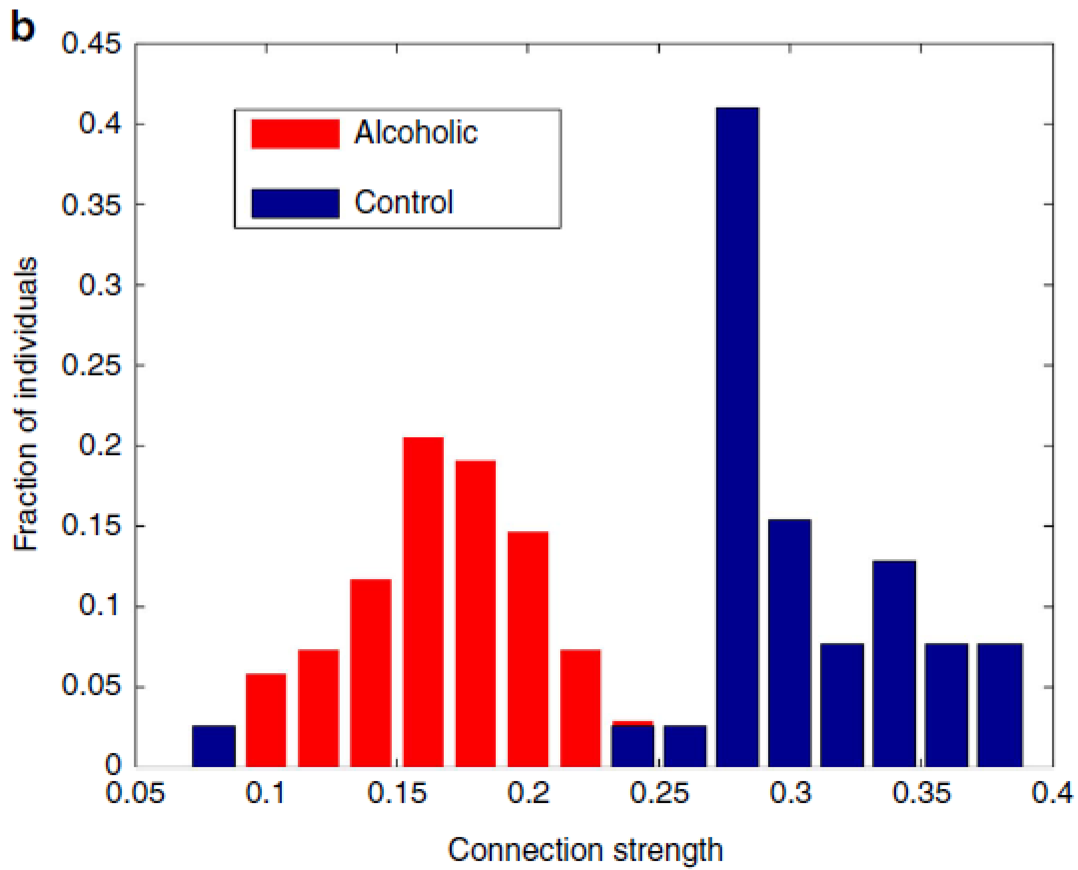
T. A. Schieber et al, Nat. Comm. 8, 13928 (2017)

Comparing brain networks

- EEG data
  - https://archive.ics.uci.edu/ml/datasets/eeg+database
  - 64 electrodes placed on the subject's scalp sampled at 256 Hz during 1s
  - 107 subjects: 39 control and 68 alcoholic
- Use HVG to transform each EEG TS into a network G.
- Weight between two brain regions: **1-*D*(G,G')**
- The resulting network represents the weighted similarity between the brain regions of an individual.

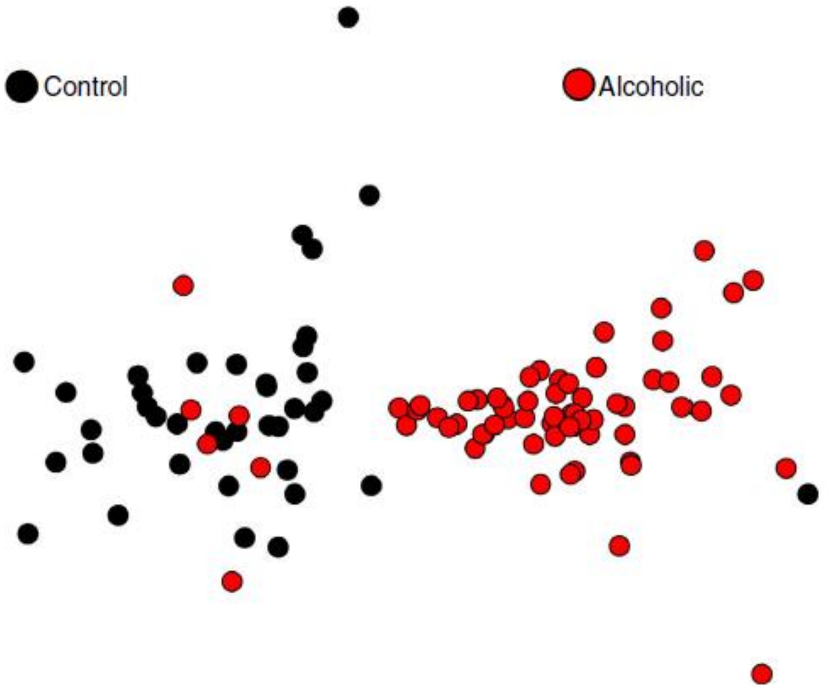
$\Rightarrow$ We can compare the different individuals.

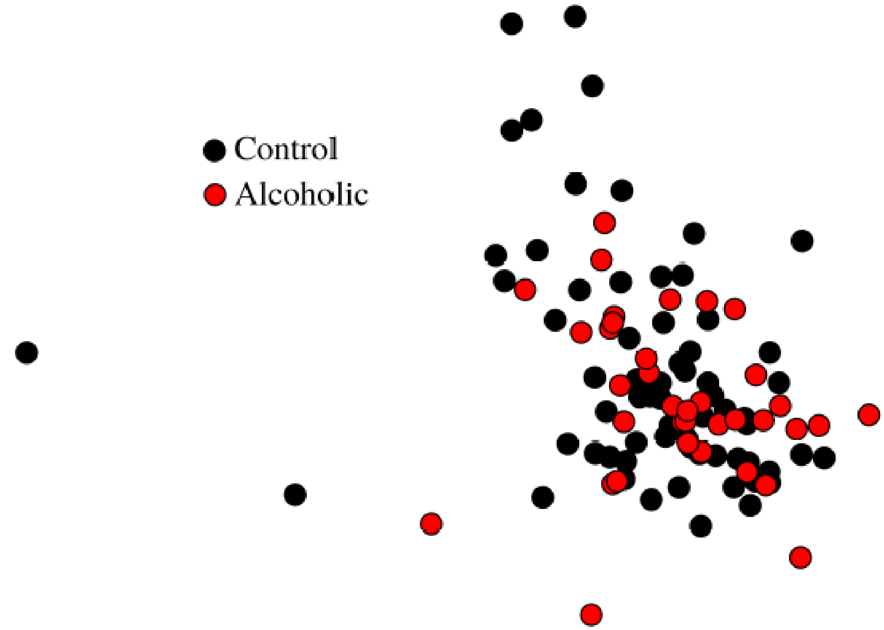
We identified two regions of the brain (called 'nd' and 'y'), where the weight of the connections between these regions is higher in control than in alcoholic networks.



Dissimilarity measure

Hamming distance





T. A. Schieber et al, Nat. Comm. 8, 13928 (2017)

Summary

- New measure to quantify the heterogeneity of the connectivity paths of a single network.
  - detects the percolation transition in a random network.
- New measure to calculate the distance between two networks
  - Can be applied to networks of different sizes.
  - Returns $D=0$ only if the two networks are isomorphic.
- Many possible applications: characterizing time-evolving climate networks, classification of networks generated from biomedical data, etc.

Predicting extreme optical pulses

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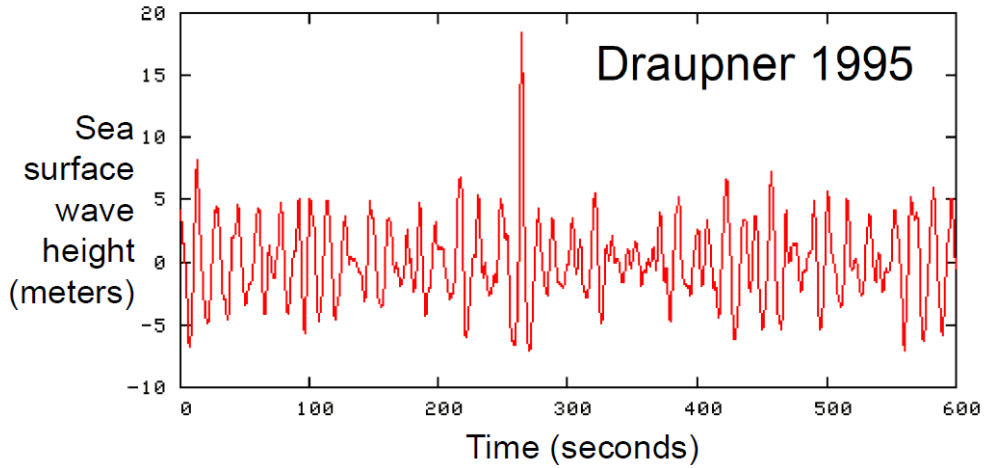
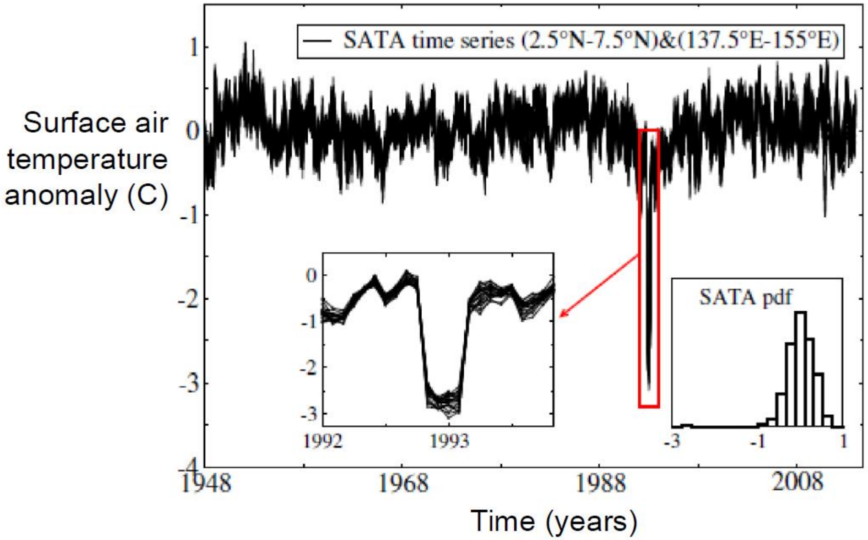
Extreme events in nature



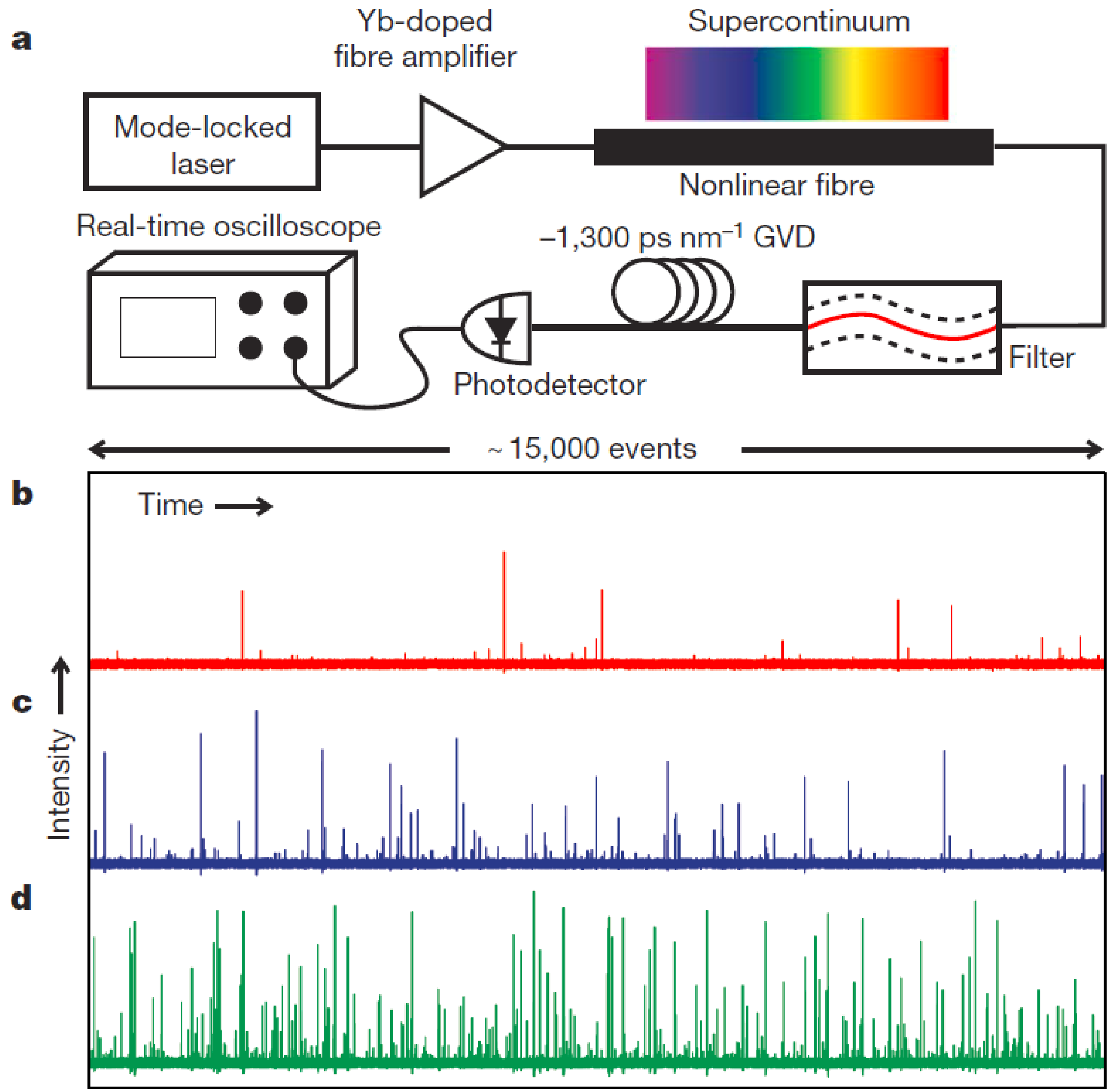


Optical chaos: provides an opportunity to advance predictability.

Optical rogue waves

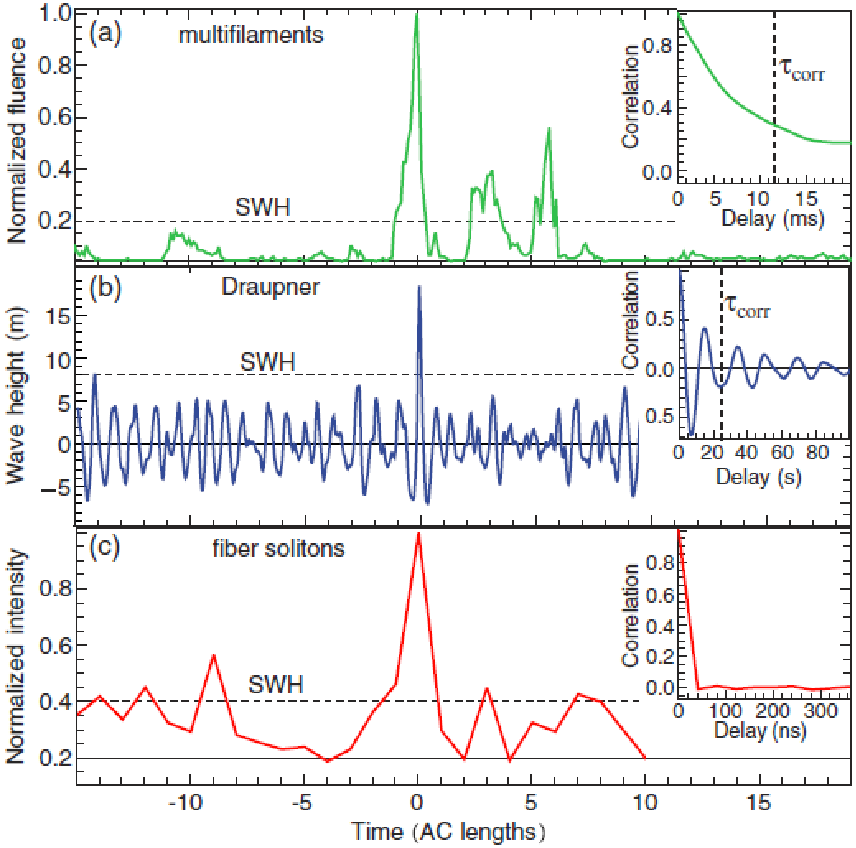
Solli et al, Nature 2007

- Optical systems can contribute to understand the mechanisms capable of triggering / suppressing extreme events.
- Optical systems generate "big data", valuable for testing diagnostic tools for "early warnings" of extreme events.
- The study of extreme pulses can yield new light into nonlinear & stochastic phenomena in optical systems.



Predictability?

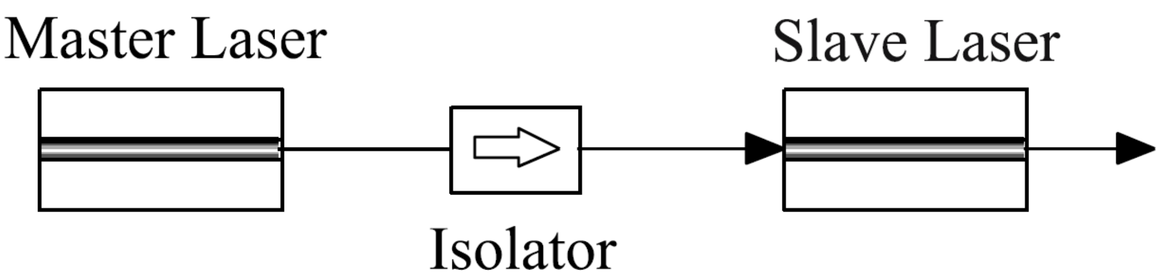
Birkholz et al, *Predictability of Rogue Events*, PRL 114, 213901 (2015)

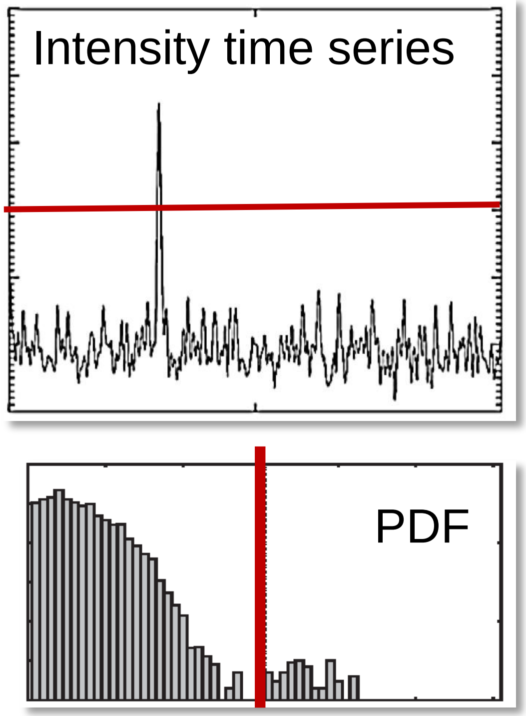


*"Transferring these findings to ocean rogue waves, one may at best expect to predict an ocean rogue wave **a few tens of seconds** before impact, and it would require many future sightings **to isolate characteristic patterns** preceding an ocean rogue wave.*

Therefore any practical rogue wave prediction appears not overly realistic, despite the determinism in the system."

"Deterministic" optical rogue waves





- Parameters:
  - Injection ratio
  - Frequency detuning (controlled via the pump current)

ORW: pulse above

<A> + **6-8** σ

C. Bonatto et al, Phys. Rev. Lett. 107, 053901 (2011)

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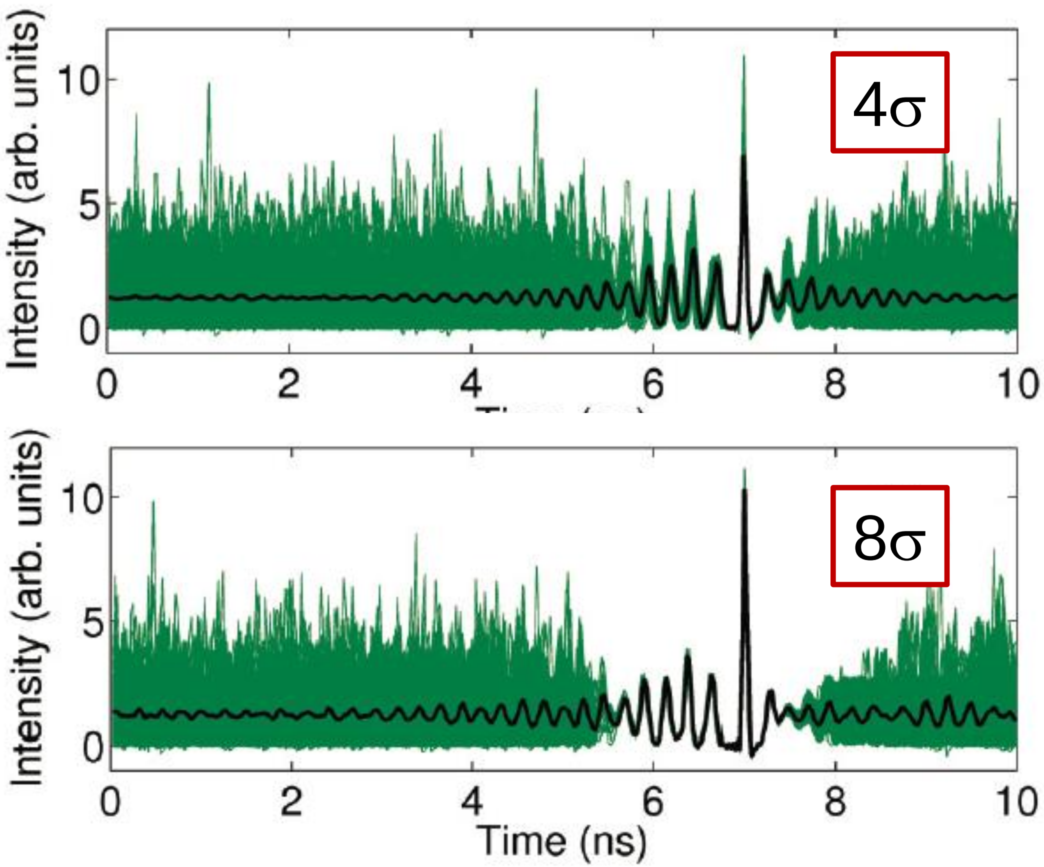
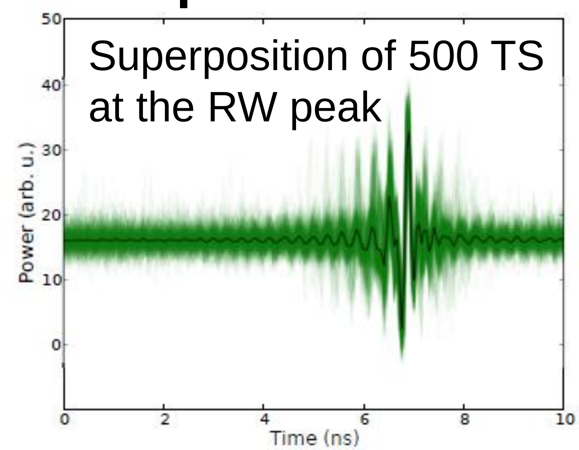
RW predictability

Deterministic simulations



Superposition of 50 time-series at the RW peak

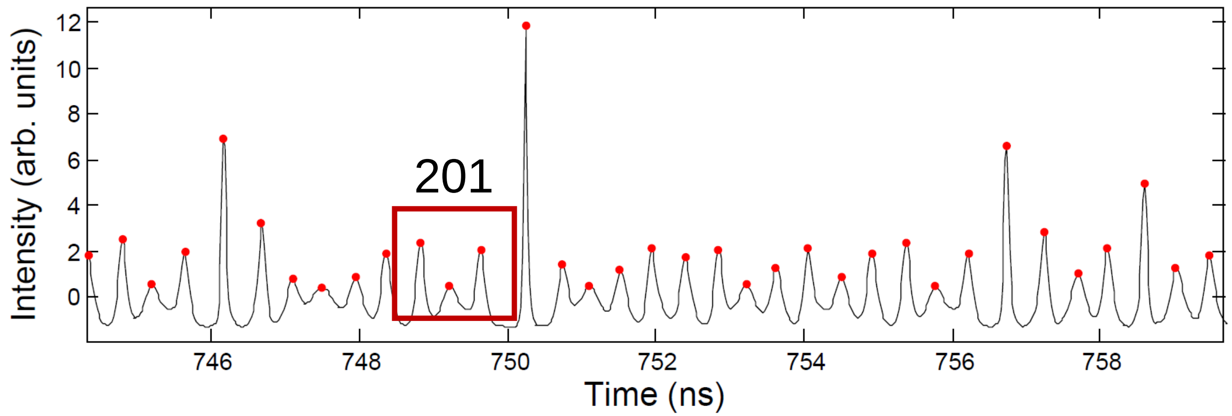
Experiments



⇒ Well-defined oscillation pattern anticipates extreme pulses.

J. Zamora-Munt et al, PRA 87, 035802 (2013)

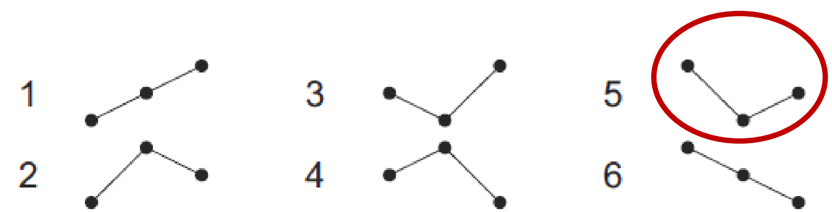
Ordinal analysis



- Consider the sequence of intensity peak heights (red dots):

  $\{\ldots I_i, I_{i+1}, I_{i+2}, \ldots\}$

- Possible order relations of three consecutive values:

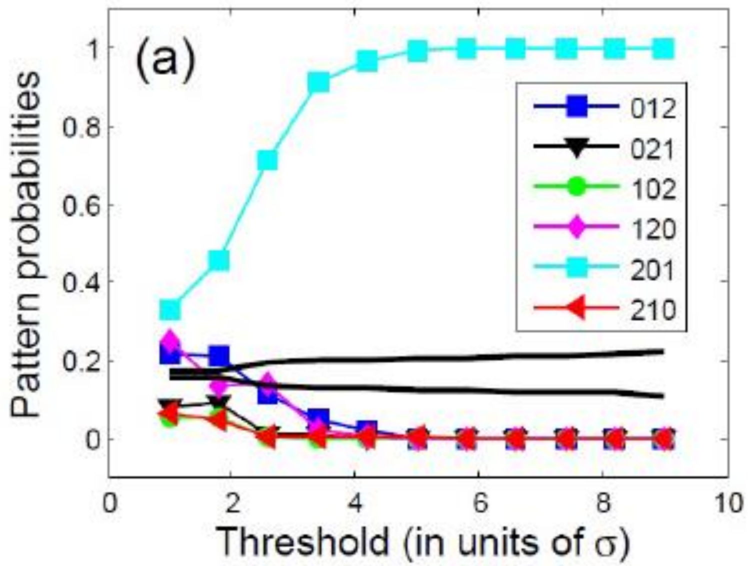


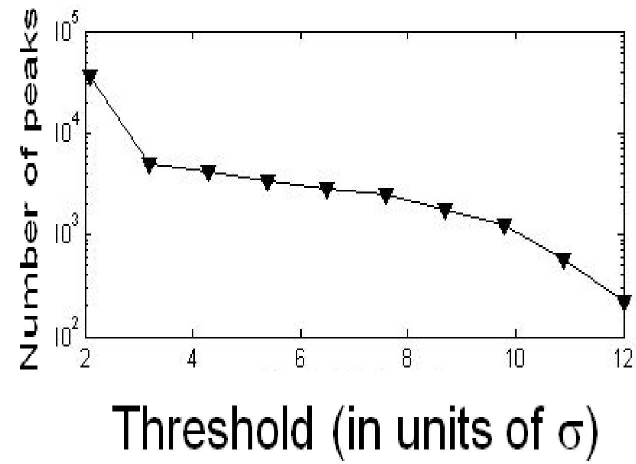
We calculate the probability of the pattern that occurs before each high pulse:

If I_i > **TH**, we analyze the pattern defined by $(I_{i-3}, I_{i-2}, I_{i-1})$

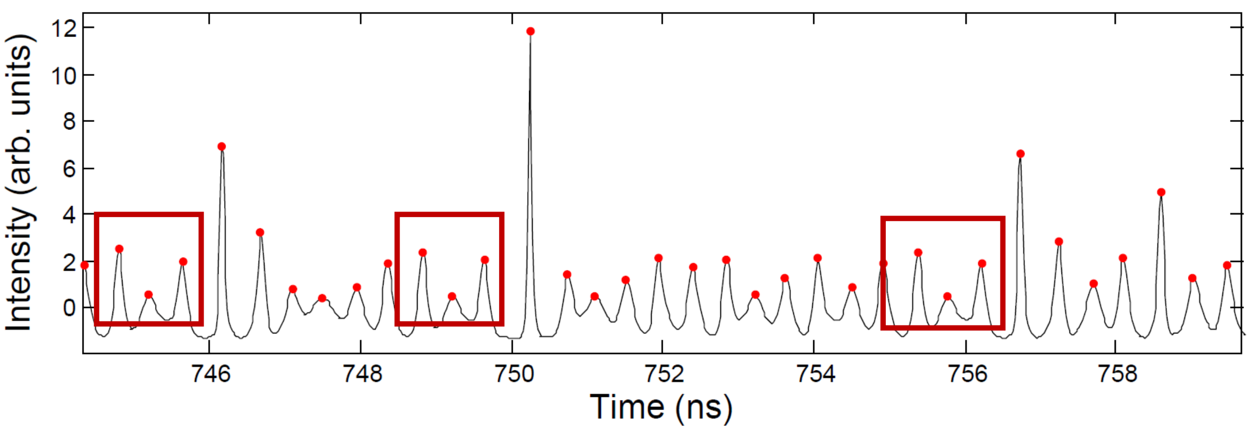
Results: deterministic simulations

Black lines:
99% confidence
$p_i=1/6 \; \forall \; i$





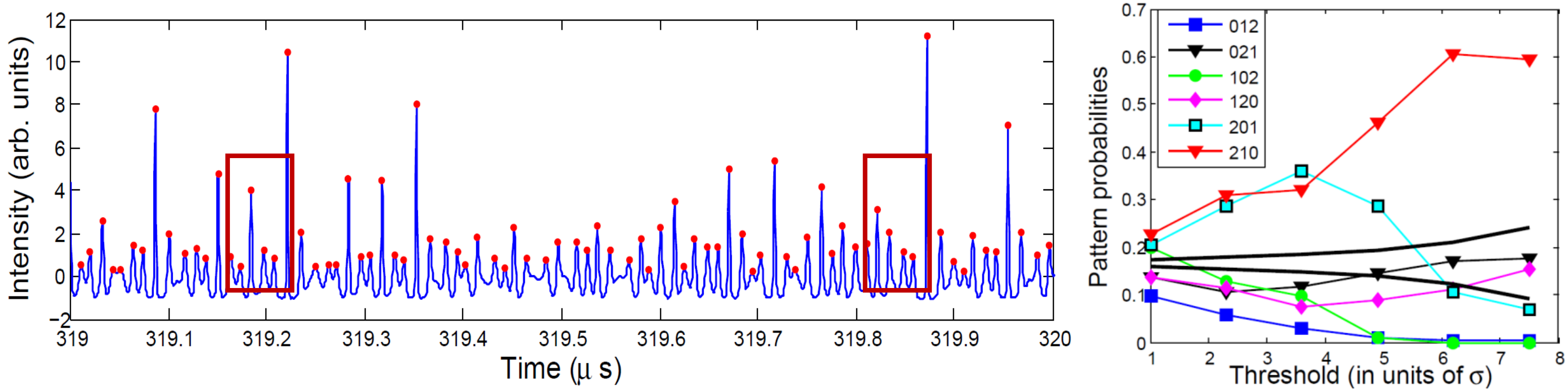
- P(201)=1 if TH >6
- Problem: P(201)≠0 if TH <6 (pattern 201 also anticipates some small pulses) ⇒ false alarms (false positives)

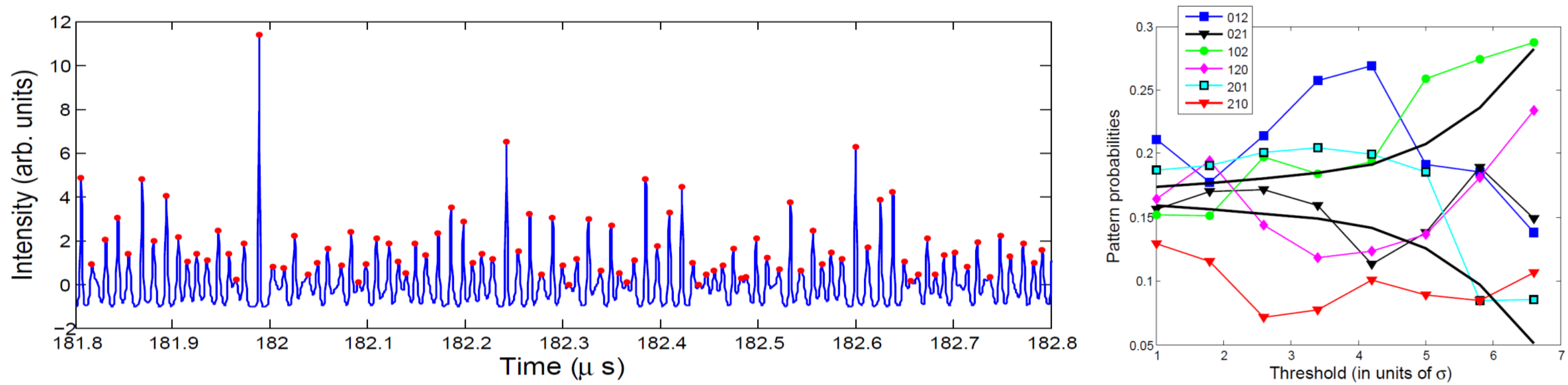


Model and parameters as in J. Ahuja et al, Optics Express 22, 28377 (2014).

Including noise and current modulation

- Two different modulation frequencies

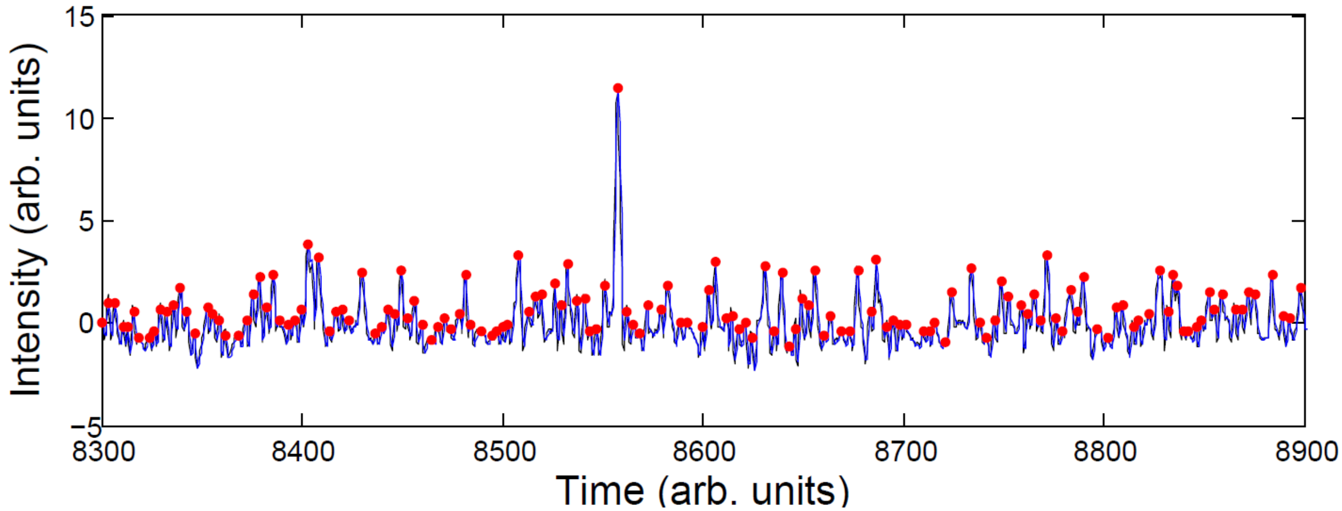


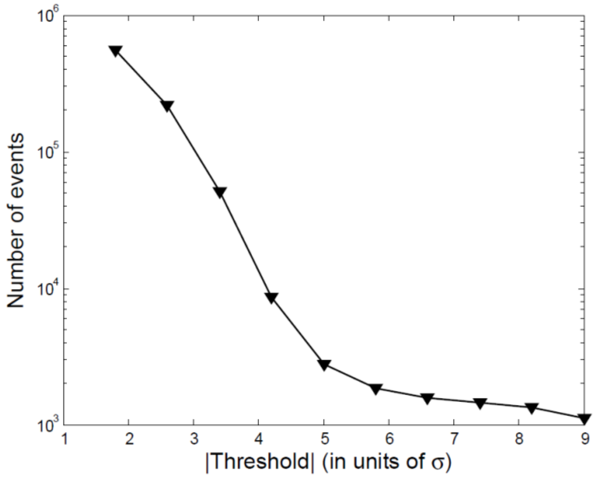


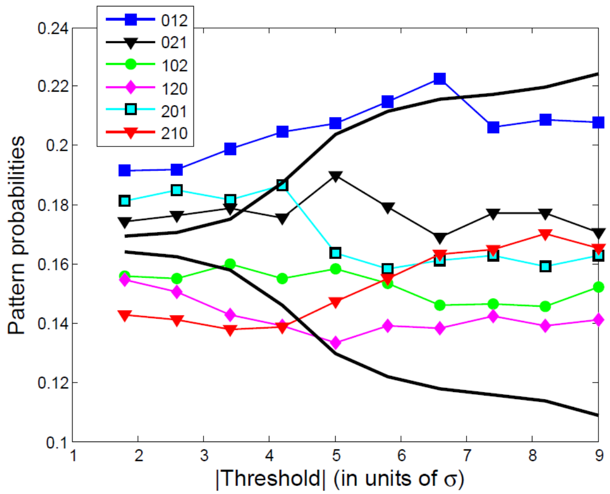
In the first case: 210 is a "good" warning.

$\Rightarrow$"early warning pattern" varies with parameters and might not exist.

Analysis of experimental data







Way to improve the "early warning":

- Filter noise
- Longer patterns

$\{\ldots I_i, I_{i+1}, I_{i+2}, I_{i+3}, \ldots\}$

Summary

- In synthetic data: certain patterns of oscillations can be more (or less) likely to occur before the extreme pulses.
- In experimental data (work in progress): to identify patterns that anticipate the extreme pulses, noise needs to be filtered.
- The analysis of the pattern probabilities can provide complementary information to advance RW predictability.
- Open issue: applicability to real-word time-series?

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