Synchronization transitions in networks of Hodgkin–Huxley neurons

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Nonlinear Network Dynamics: Complexity and Control
Conference on the occasion of the 75th birthday of Prof. Eckehard Schöll, Berlin, May 4 – 6, 2026

Research question

How the interplay of

- Intrinsic neuronal behavior
- Coupling strength and topology
- Neural noise

generate extreme events and neuronal avalanches?

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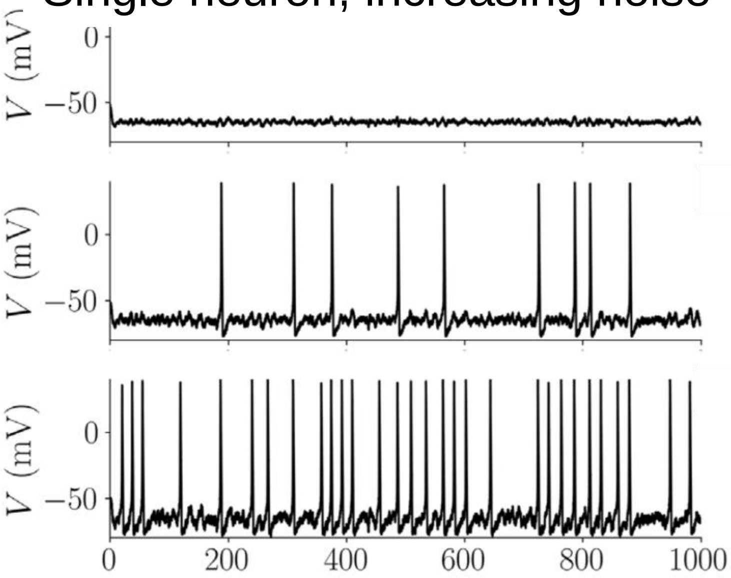
Dynamics of globally coupled Hodgkin–Huxley neurons

$$C_M \frac{dV_i}{dt} = -g_K n_i^4 (V_i - E_K) - g_{Na} m_i^3 h_i (V_i - E_{Na})$$
$$- g_\ell (V_i - E_\ell) + D\xi_i(t) + I_{i,coup}.$$

Single neuron, increasing noise



Time (ms)

$$I_{i,coup} = \frac{\varepsilon}{N} \sum_{j=1}^{N} (V_j - V_i) = \varepsilon(\overline{V} - V_i)$$

Mean field: $\overline{V} = \frac{1}{N} \sum_{i=1}^{N} V_i$

Boaretto, Macao and Masoller, Chaos, Solitons and Fractals 194 (2025) 116133

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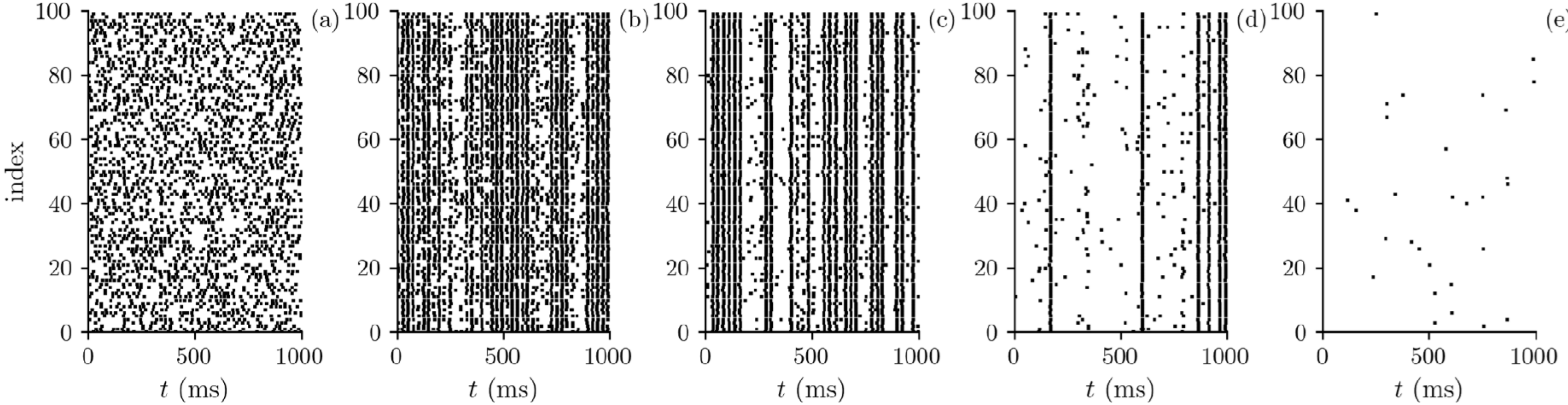
Dynamics of N=100 identical neurons, when the coupling strength increases, keeping constant the noise strength

Weak coupling: incoherent spiking activity

Strong coupling: spiking activity suppressed



Intermediate coupling: synchronized spiking and extreme events

Influence of noise?

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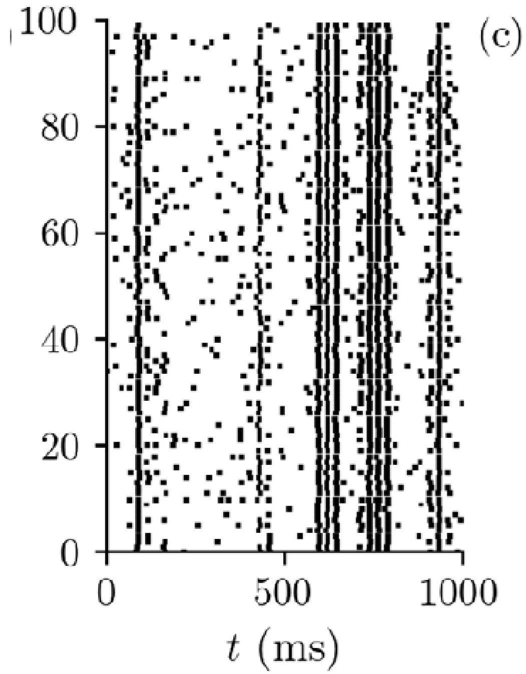
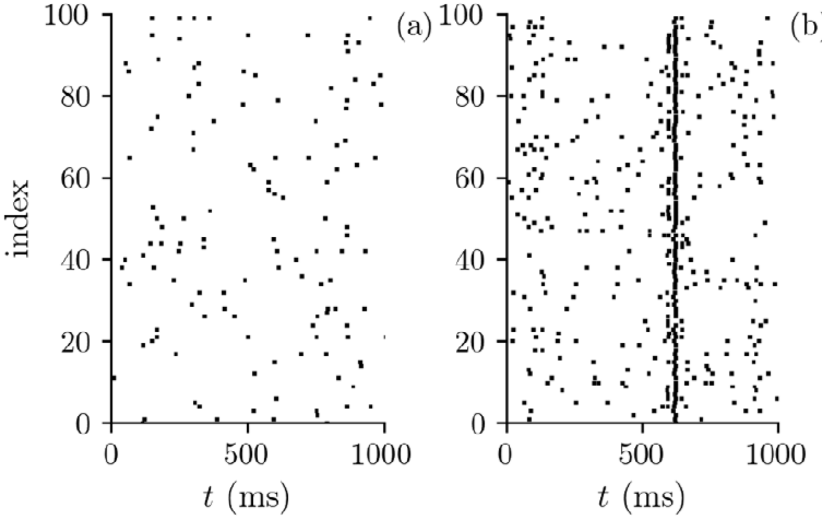
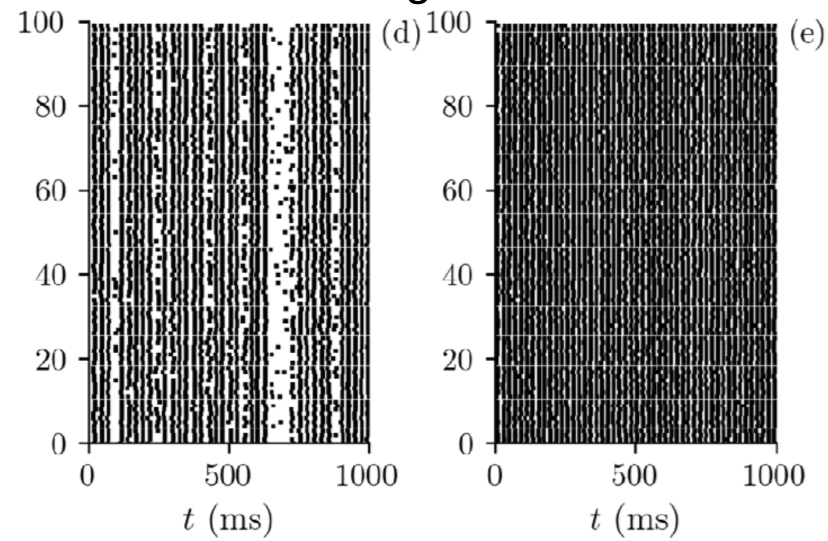


Weaker noise



Stronger noise



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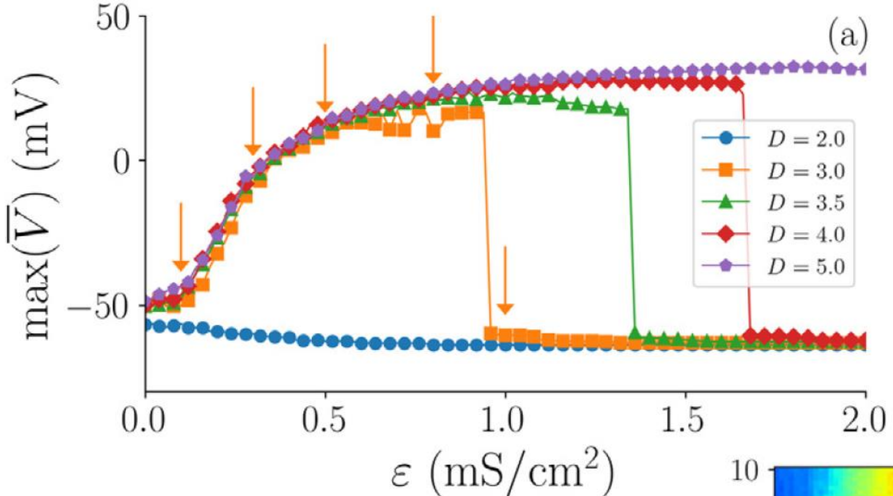
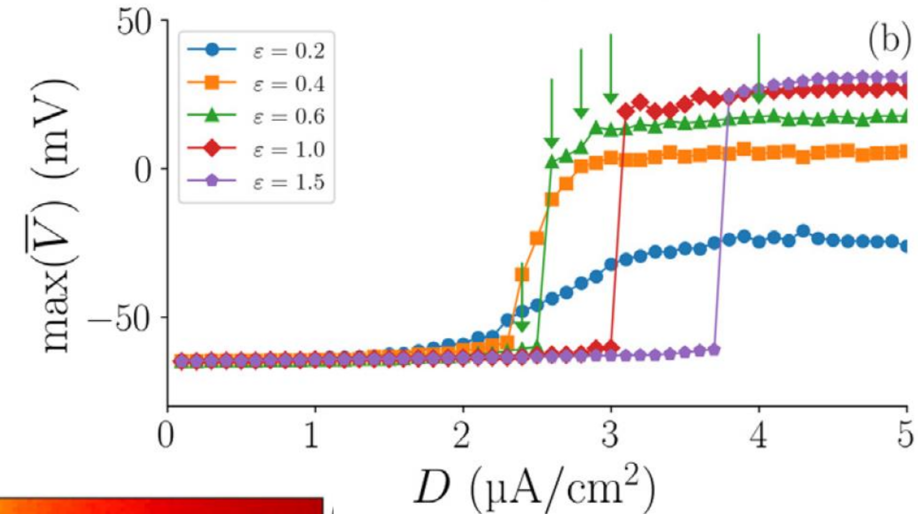
Order parameter: max mean field, max($\overline{V}$)

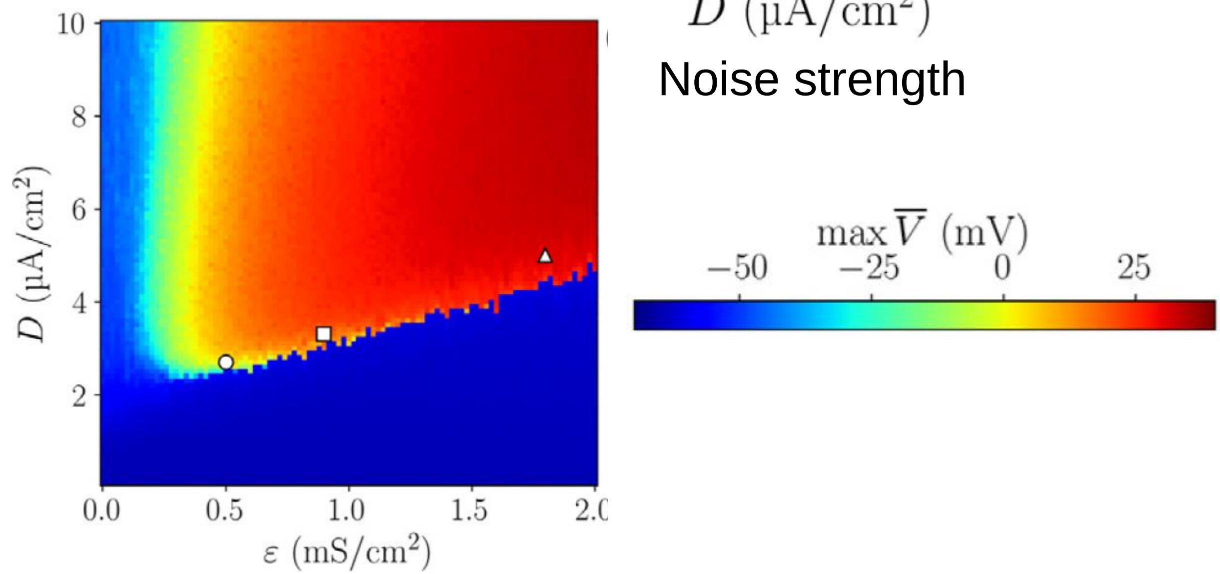
$$\overline{V} = \frac{1}{N}\sum_{i=1}^{N} V_i$$



Coupling strengt



Noise strength



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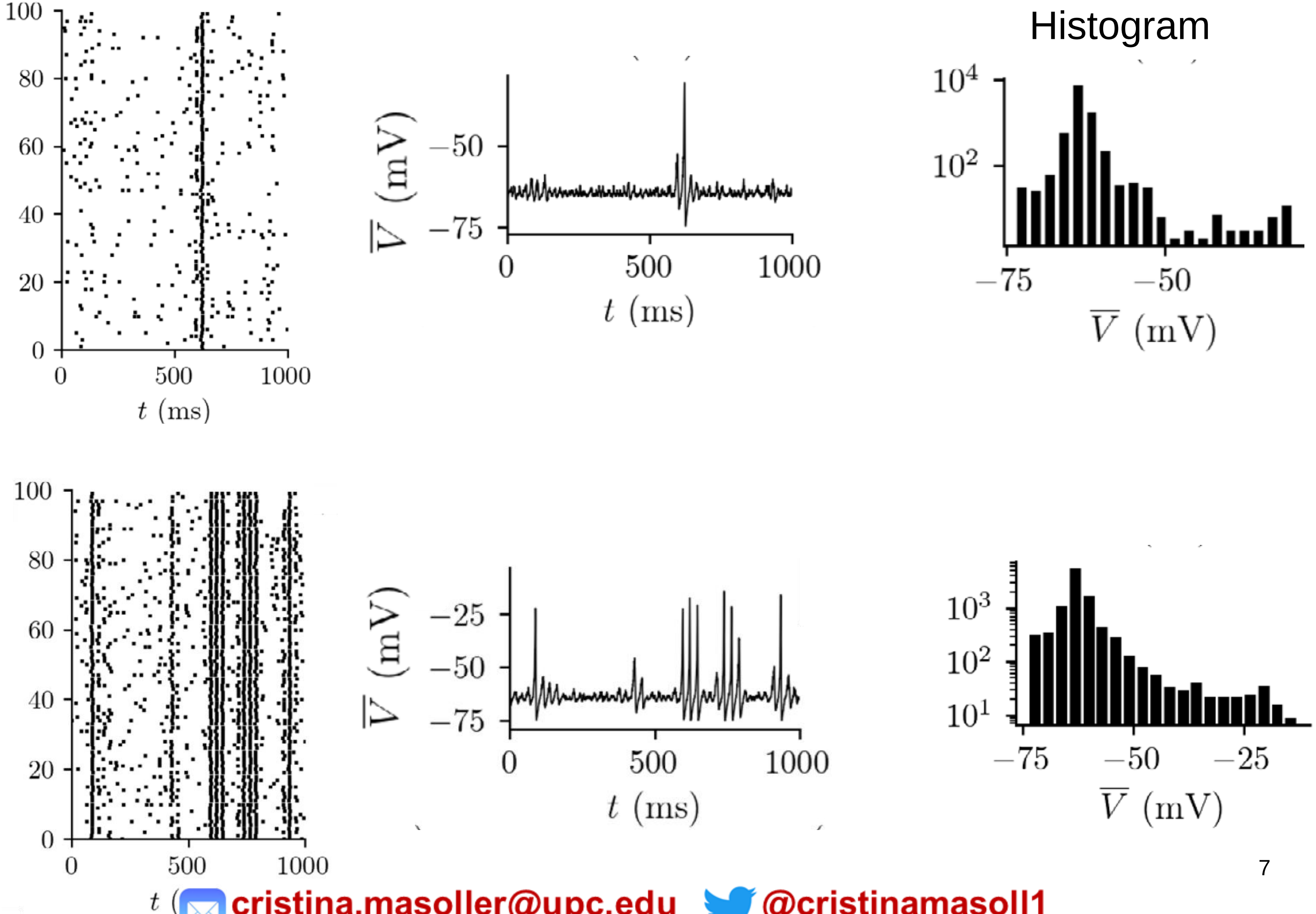
How to identify the parameter region where extreme events occur?

$$\overline{V} = \frac{1}{N}\sum_{i=1}^{N} V_i$$



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Shannon entropy

$$\sum_{i=1}^{N} p_i = 1 \qquad H = -\sum_{i=1}^{N} p_i \ln p_i$$

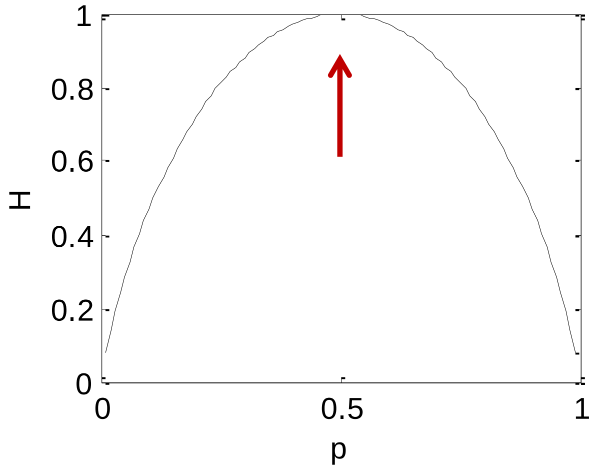
- Interpretation: “*quantity of **surprise** one should feel upon reading the result of a measurement* ”.

- Example: a random variable takes values 0 or 1 with probabilities:

  p(0) = p, p(1) = 1 − p.

  H = −p ln(p) − (1 − p) ln(1 − p).

  ⇒ p=0.5: Maximum **unpredictability.**



C. Shannon, "A Mathematical Theory of Communication", Bell System Technical Journal. 27 (3): 379–423 (1948). Bell System Technical Journal. 27 (4): 623–656 (1948).

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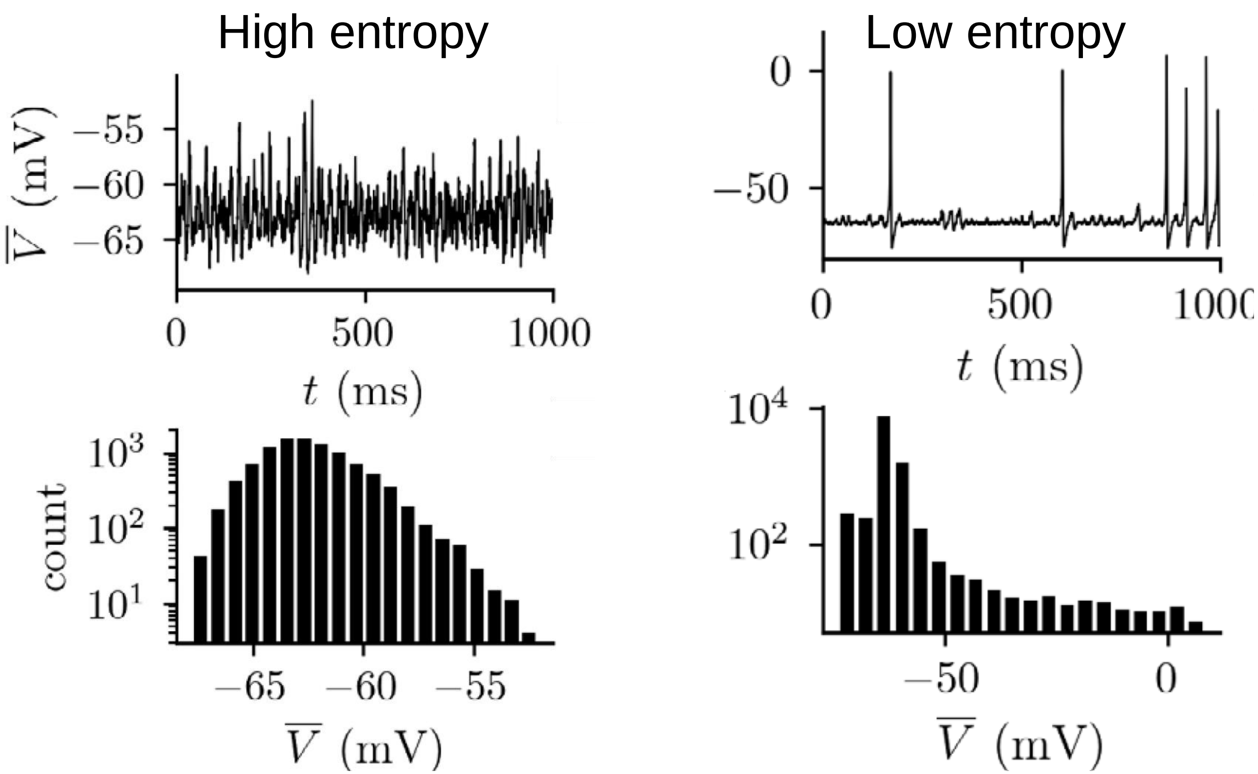
Identifying the extreme event region by using Shannon Entropy of the mean-field distribution

$$H = -\sum_{i=1}^{N} p_i \log p_i$$

p_i (probability that $\overline{V}(t)$ is in the ith bin) is estimated from the histogram of values, using N=100 bins and adjusting their range to the min and max values in each time series.



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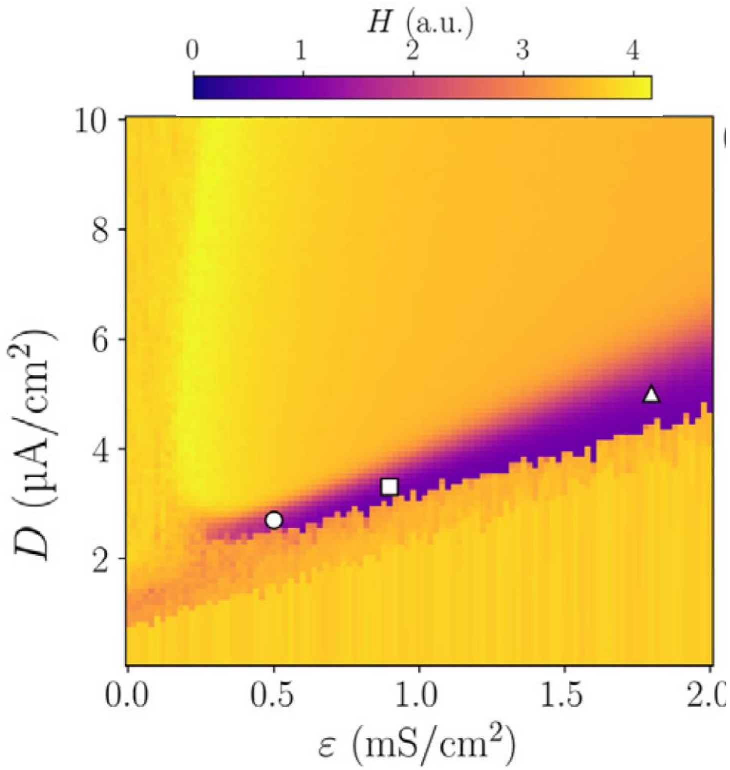
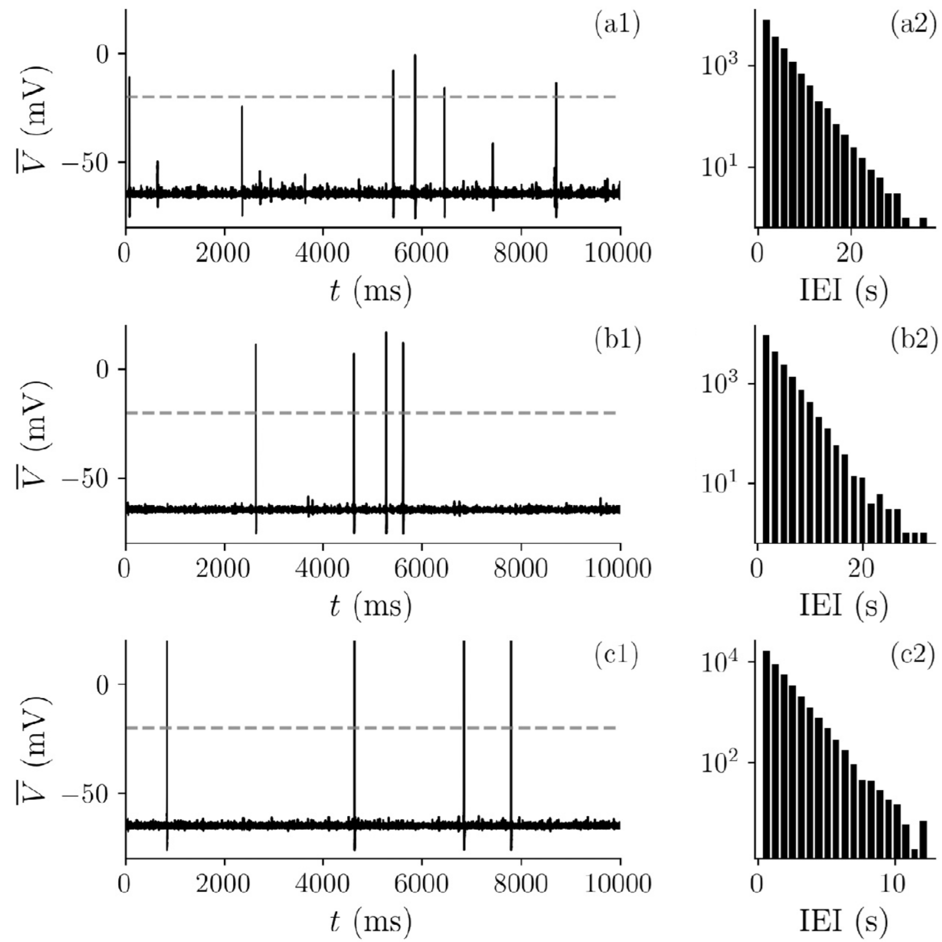
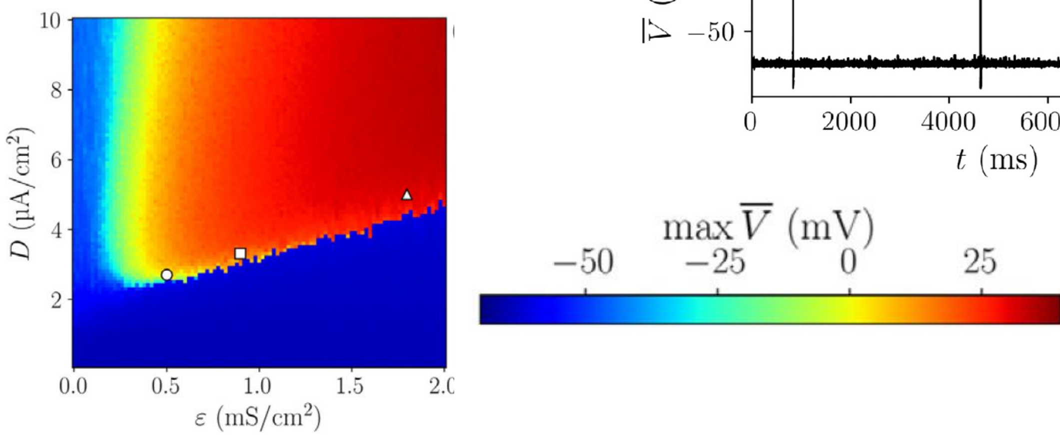
Extreme event region: low entropy

Distribution of Inter-Event-Intervals







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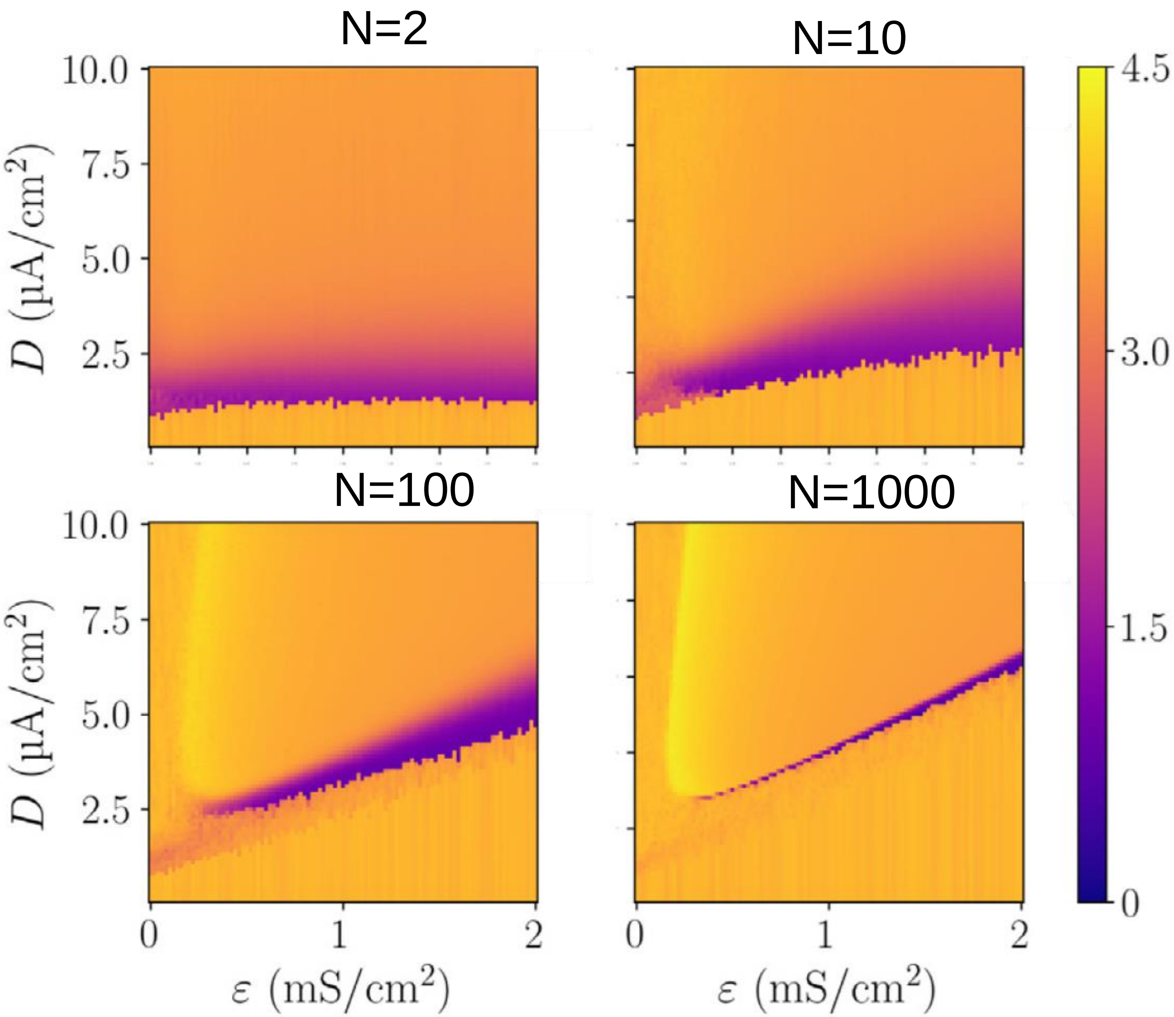
Role of the network size



The “extreme event” region shrinks with N

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Role of the connectivity of the network

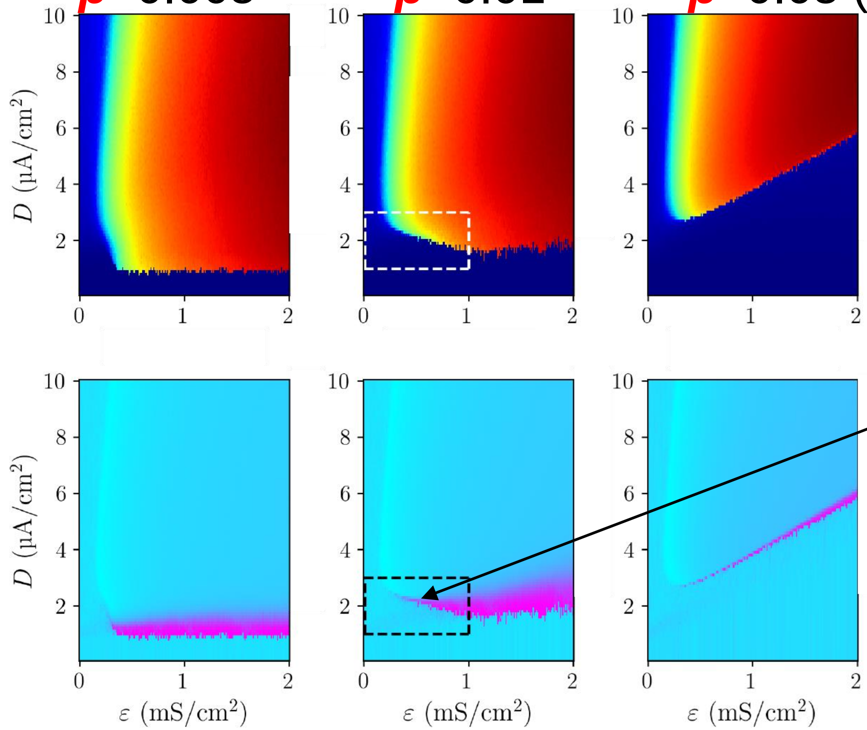
$$I_{i,\mathrm{coup}} = \frac{\varepsilon}{\bar{n}} \sum_{j=1}^{N} e_{ij}(V_j - V_i)$$

N=1000 neurons

Average number of links per neuron

$e_{ij} = e_{ji}$ = 1 with probability ***p***
0 with probability ***1-p***

p=0.005 ***p***=0.01 ***p***=0.05 (same as with p=1)



Color shows:
$A = \max(\bar{V}) - \min(\bar{V})$

Neuronal dynamics in this region?

Boaretto, Macao and Masoller, submitted (2026)

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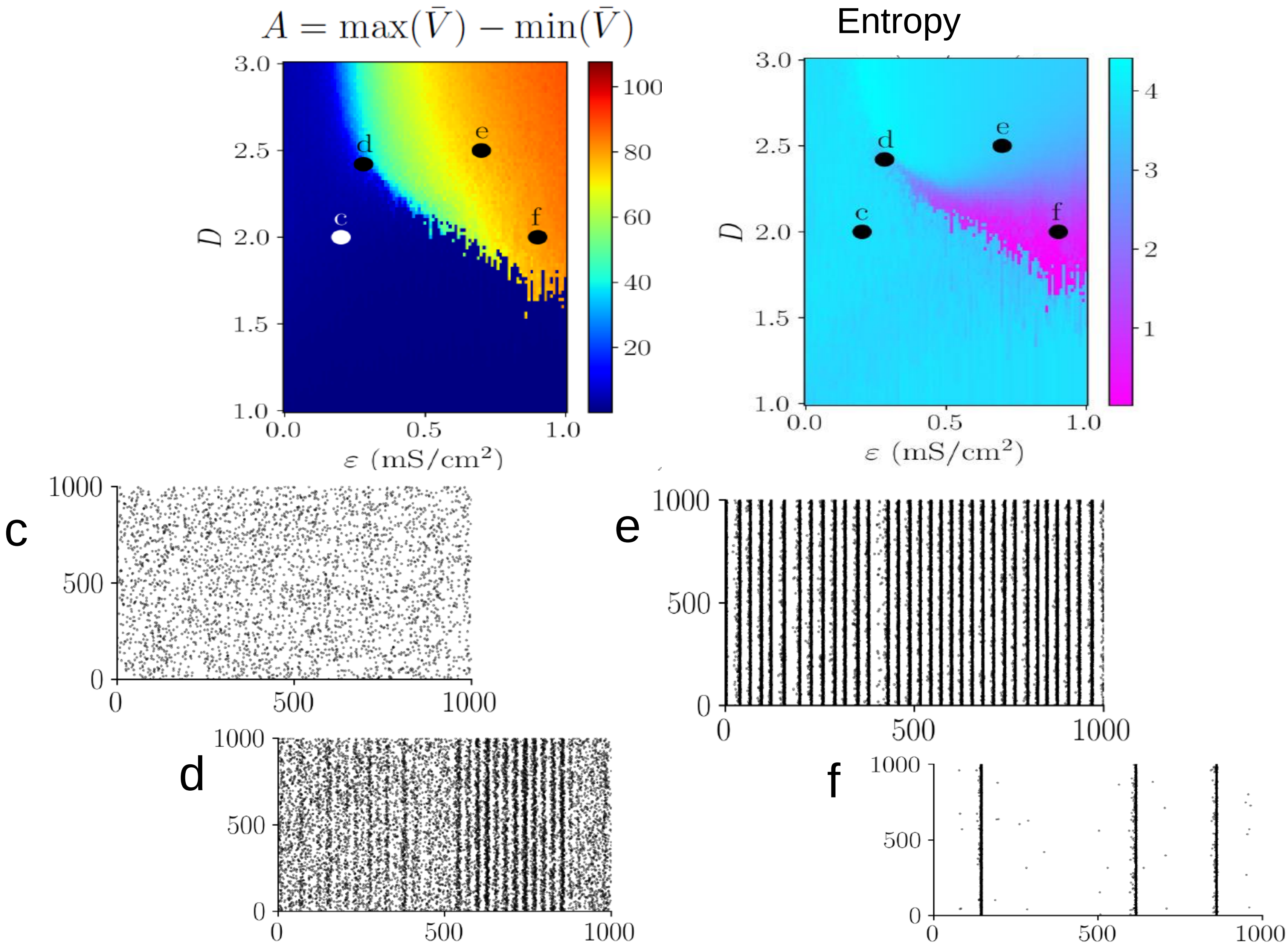

Boaretto, Macao and Masoller, submitted (2026)

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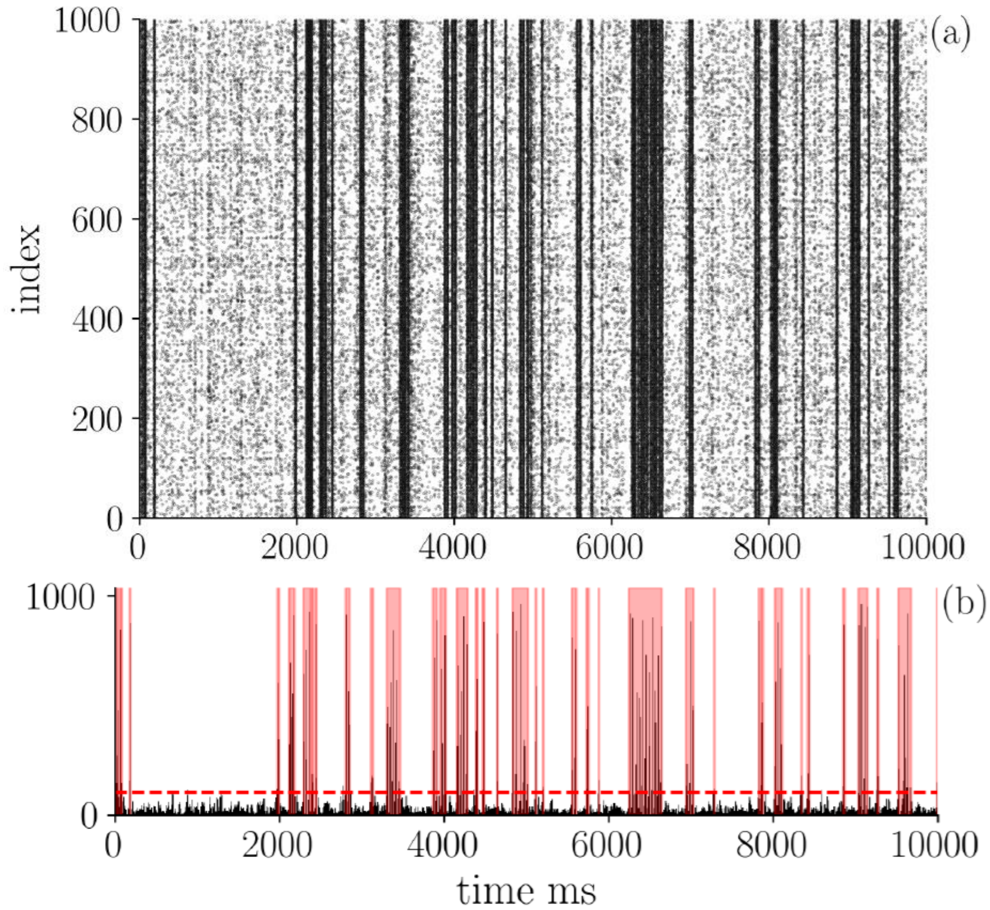
Identifying and characterizing neuronal avalanches



Spike count in each Δt

red dashed line: threshold used to define active bins

- Avalanche duration τ: number of consecutive active bins.
- Avalanche size ***s***: total number of spikes during the avalanche.

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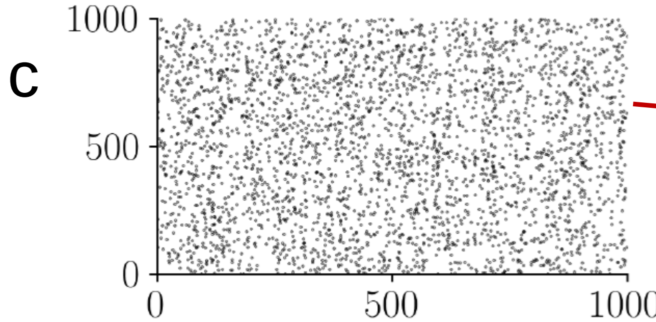
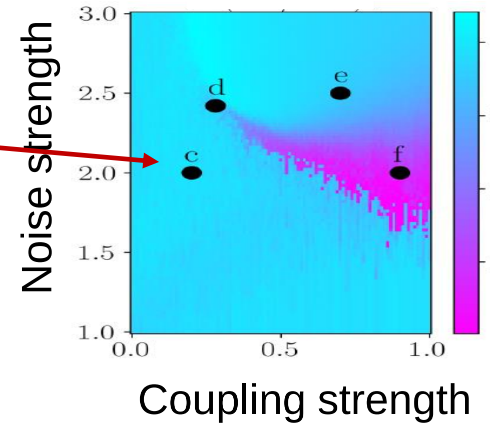
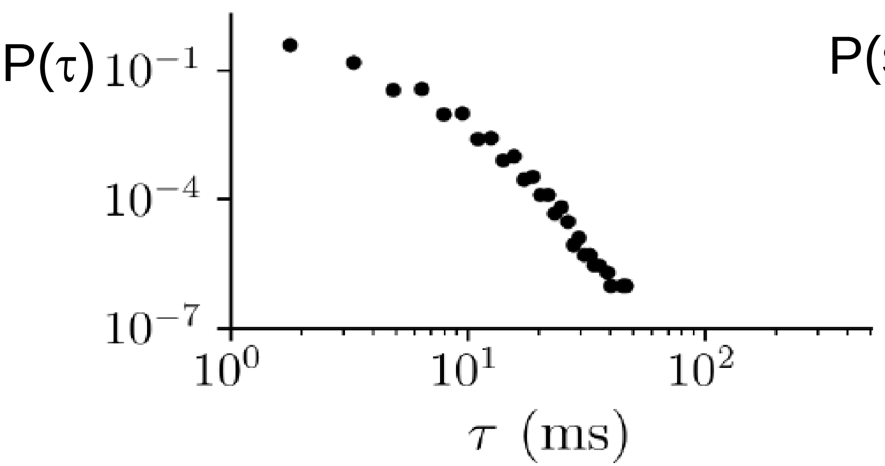
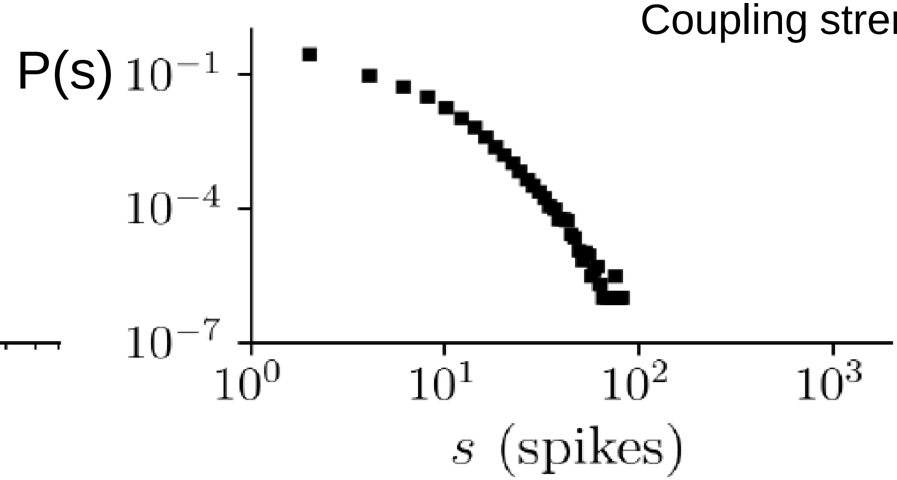
The distributions of avalanches sizes and durations unveil three dynamical regimes

- Subcritical: exponential distributions
- Critical: power laws distributions
- Supercritical: heavy-tailed distributions









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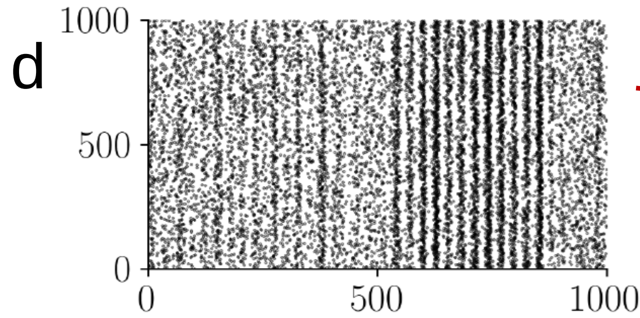
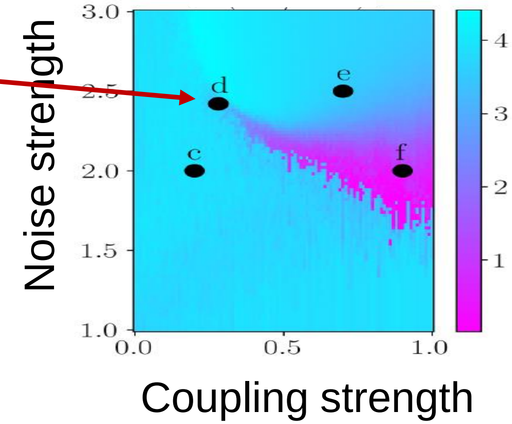
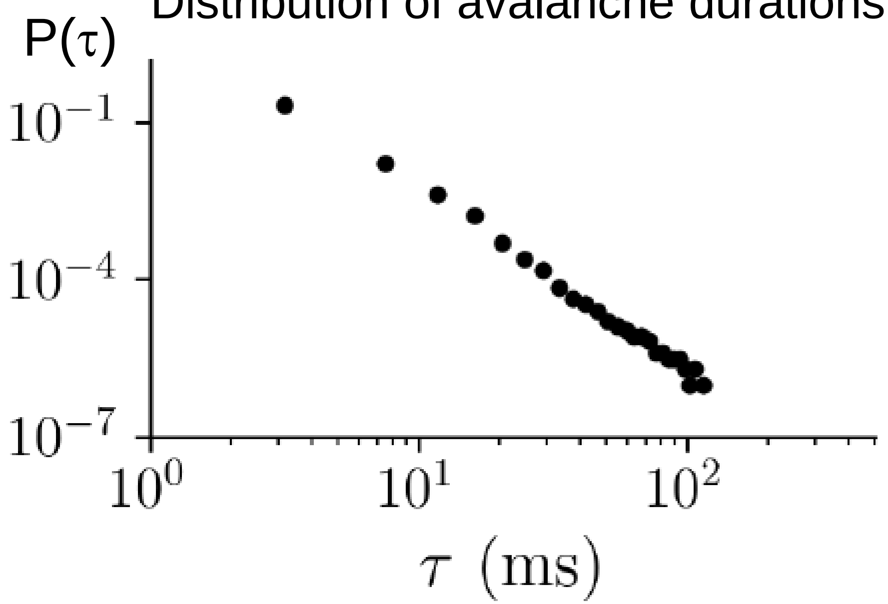
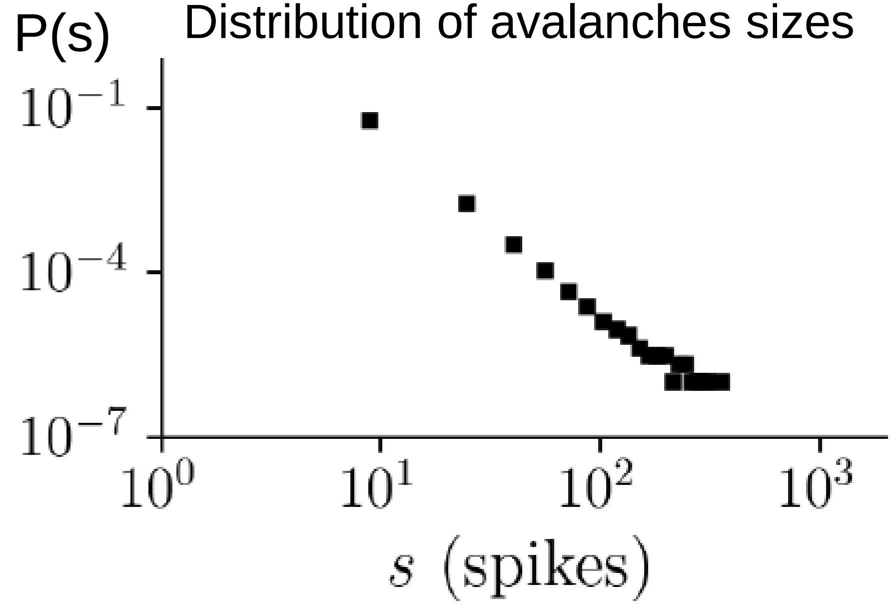




Distribution of avalanche durations



Distribution of avalanches sizes



Critical: power laws distributions

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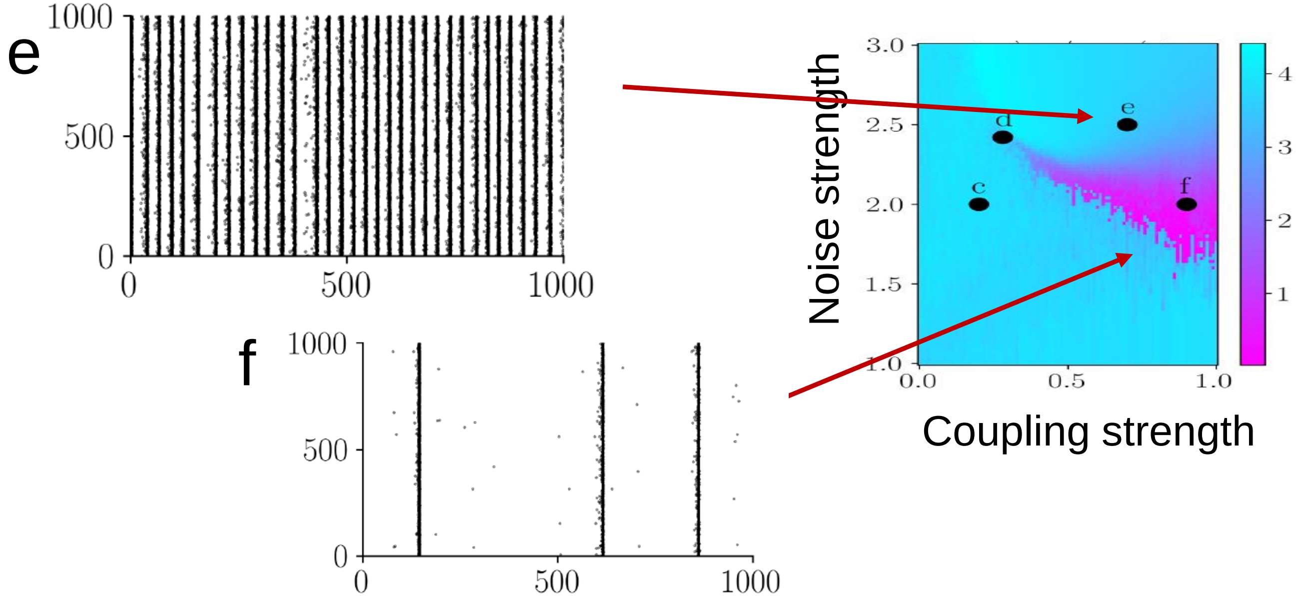

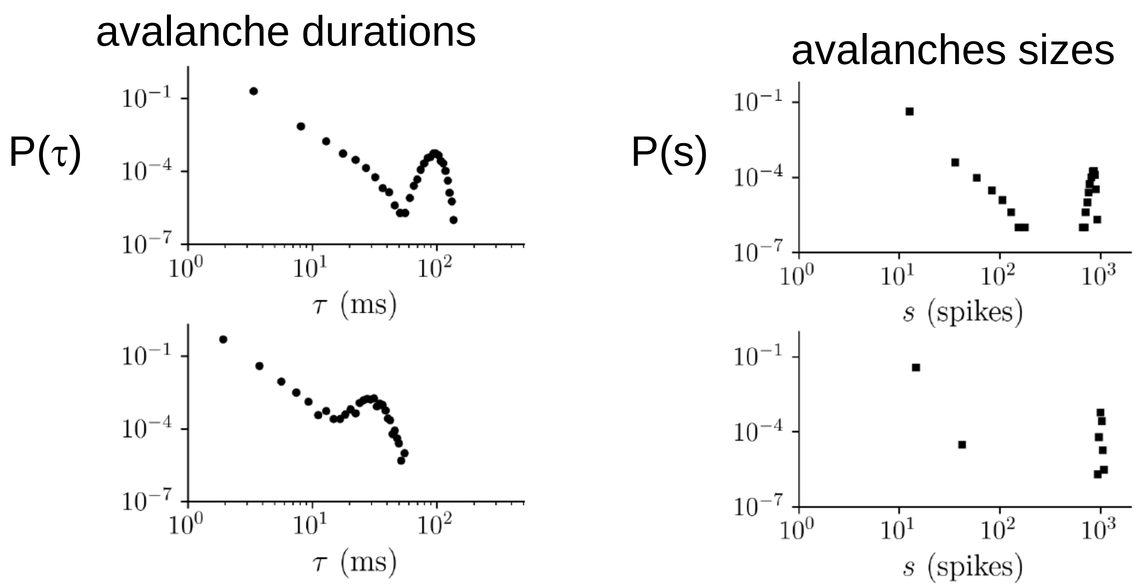

Supercritical: heavy-tailed distributions (extreme events)

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Many studies of avalanches in neural networks with different coupling configurations

Chaos ARTICLE pubs.aip.org/aip/cha

Scale-free avalanches in arrays of FitzHugh–Nagumo oscillators

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II. ARRAY OF NON-LOCALLY COUPLED FITZHUGH–NAGUMO UNITS

Our model is an array of N identical FHN units[39] with a simple non-local interaction scheme where each unit is coupled to P of its neighbors to its left and to its right on a one-dimensional ring,

$$\epsilon \dot{u}_i = u_i - \frac{u_i^3}{3} - v_i + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}(u_j - u_i),$$
$$\dot{v}_i = u_i + \alpha + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}(v_j - v_i). \quad (1)$$

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BRIEF REPORT | NEUROSCIENCE

OP

Signatures of criticality in efficient coding networks

Shervin Safavi[a,b,1], Matthew Chalk[c,1], Nikos K. Logothetis[b,d], and Anna Levina[b,e,f,1]

Edited by Terrence Sejnowski, Salk Institute for Biological Studies, La Jolla, CA; received March 8, 2023; accepted July 24, 2024

The critical brain hypothesis states that the brain can benefit from operating close to a second-order phase transition. While it has been shown that several computational aspects of sensory processing (e.g., sensitivity to input) can be optimal in this regime, it is still unclear whether these computational benefits of criticality can be leveraged by neural systems performing behaviorally relevant computations. To address this question, we investigate signatures of criticality in networks optimized to perform efficient coding. We consider a spike-coding network of leaky integrate-and-fire neurons with synaptic transmission delays. Previously, it was shown that the performance of such networks varies nonmonotonically with the noise amplitude. Interestingly, we find that in the vicinity of the optimal noise level for efficient coding, the network dynamics exhibit some signatures of criticality, namely, scale-free dynamics of the spiking and the presence of crackling noise relation. Our work suggests that two influential, and previously disparate theories of neural processing optimization (efficient coding and criticality) may be intimately related.

criticality | efficient coding | neural dynamics | neural computation

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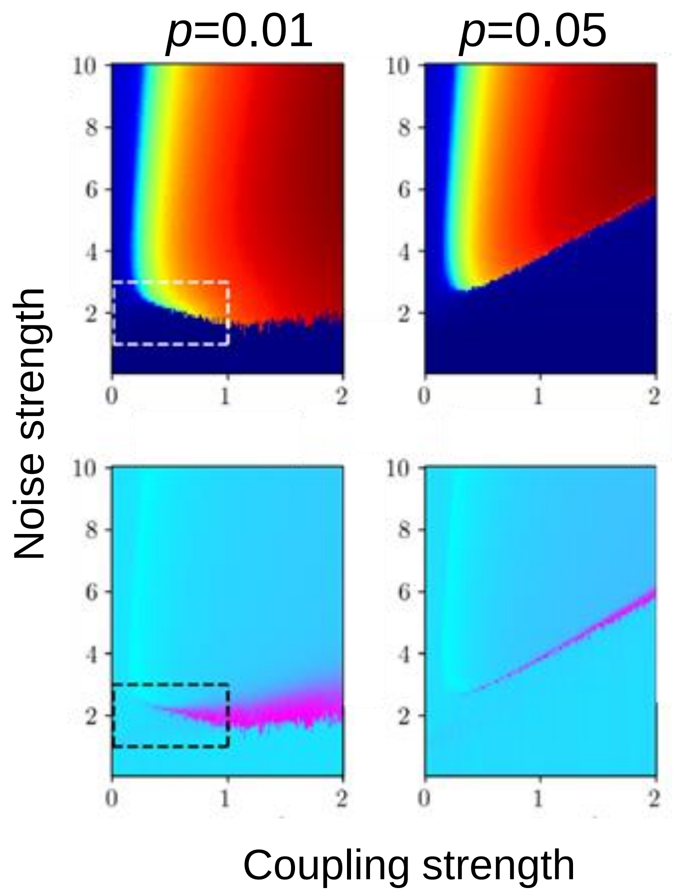
Take home messages, ongoing and future work

- Three regime transitions identified:
  - A smooth transition when increasing the coupling strength that gradually increases neural activity (and the mean field).
  - Abrupt suppression of neural activity when the coupling strength grows above a critical value.
  - Abrupt transition to synchronized spiking when the noise strength grows above a critical value.



- Entropy analysis of the mean field dynamics unveils a critical point at the intersection of coupling-driven and noise-driven transitions.

- Sparce connectivity enlarge the avalanches and extreme events regions.

- Ongoing: more biologically realistic synaptic interactions. Future: network topologies inspired by axonal growth in cultured neuronal networks.

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References and funding

1. B. R.R. Boaretto, E. E.N. Macau, C. Masoller, "*Noise-induced extreme events in Hodgkin–Huxley neural networks*", Chaos, Solitons & Fractals 194, 116133 (2025).

2. B. R. R. Boaretto, E. E.N. Macau, C. Masoller, "*Identifying and characterizing noise-induced avalanches and extreme events on spiking networks*". Submitted (2026).

Thank you for your attention!

Information Processing and Sensing with Photonic Neurons.
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