Characterizing and detecting critical transitions using nonlinear data analysis tools

Cristina Masoller

Departament de Física,
Universitat Politecnica de Catalunya

Dynamics Days LAC
Buenos Aires, December 11, 2024

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Campus d'Excel·lència Internacional

Co-authors

Maria Duque-Gijon

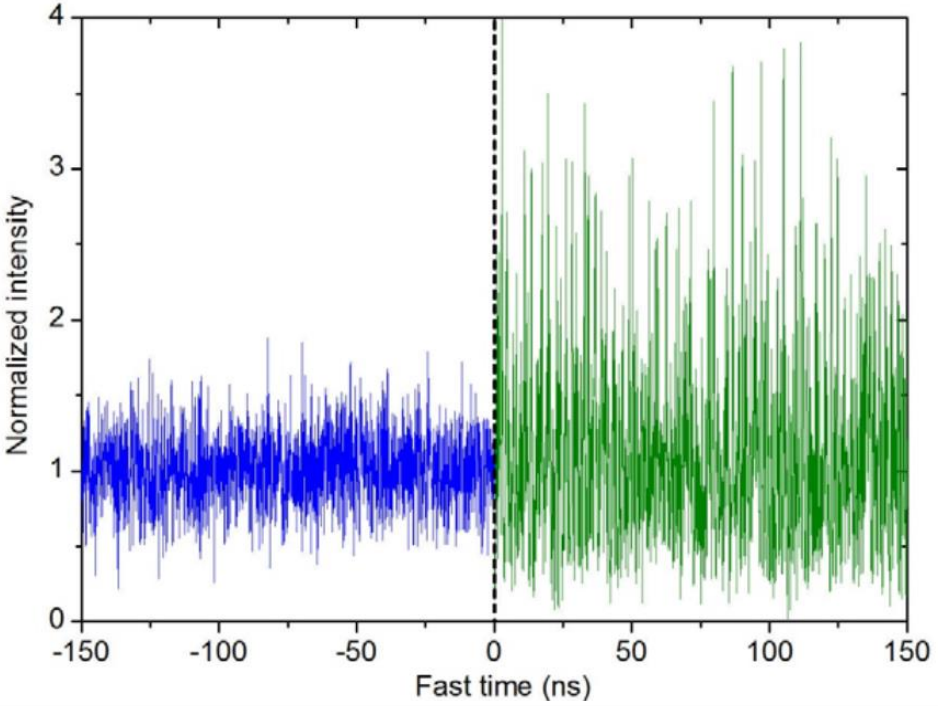
Juan Gancio

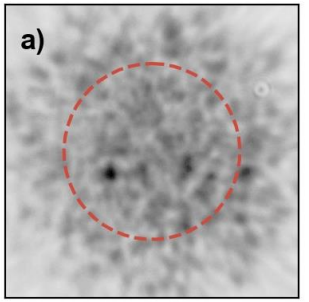
Jordi Tiana-Alsina

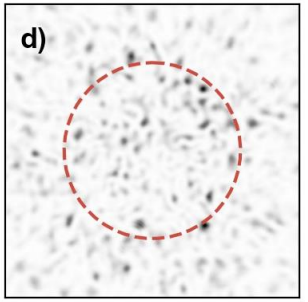
Giulio Tirabassi

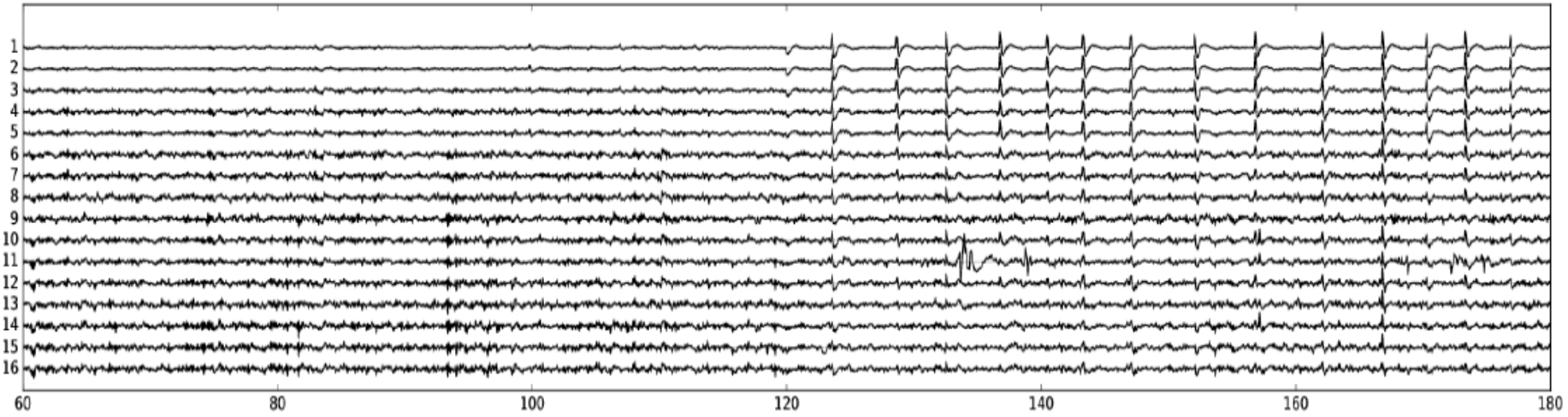
cristina.masoller@upc.edu @cristinamasoll1

Regime transitions in complex systems



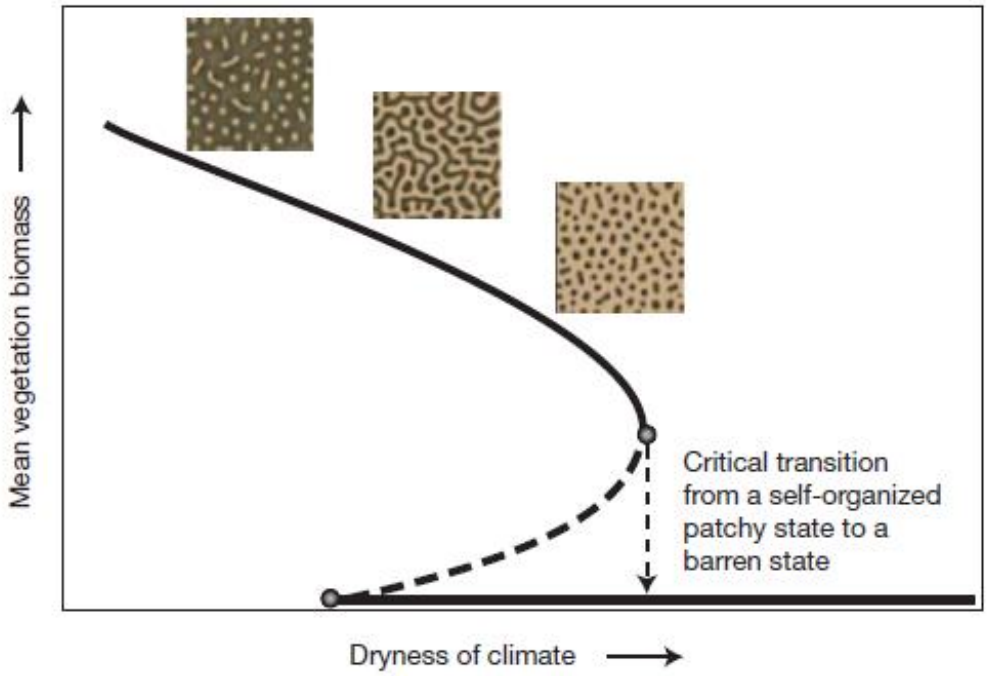






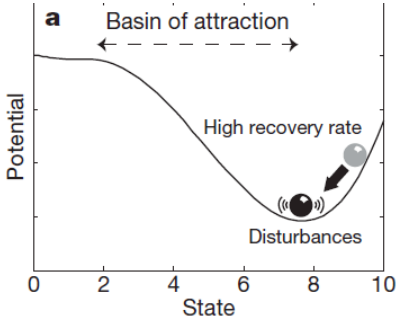
cristina.masoller@upc.edu @cristinamasoll1

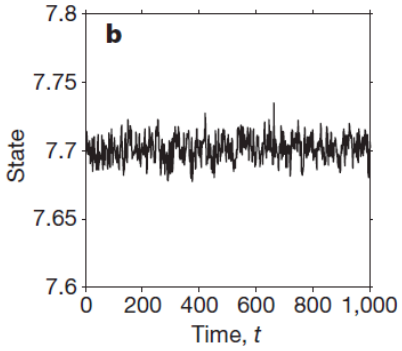
Classical indications of approaching critical transitions

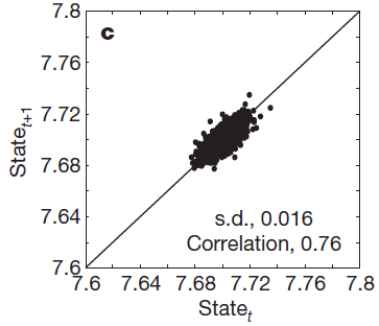


M. Scheffer et al., Nature 461, 53 (2009)

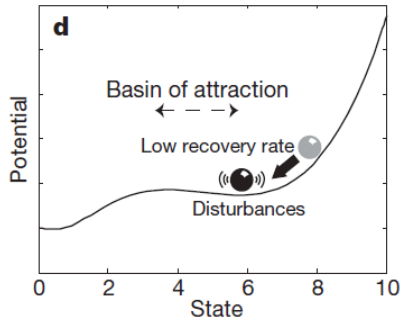
Far from bifurcation

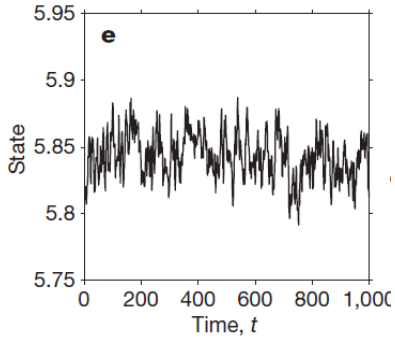


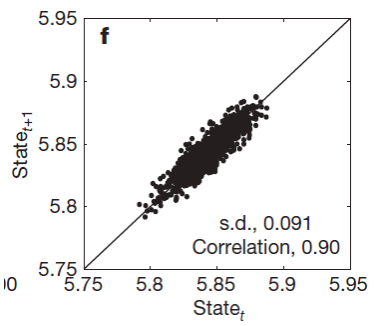




Close to bifurcation







⇒ increase of variance and autocorrelation: *critical slowing down*

cristina.masoller@upc.edu @cristinamasoll1

Critical slowing down can occur after the bifurcation

PHYSICAL REVIEW LETTERS **125,** 134102 (2020)

Testing Critical Slowing Down as a Bifurcation Indicator in a Low-Dissipation Dynamical System

M. Marconi,[1] C. Métayer,[2] A. Acquaviva,[1] J. M. Boyer,[2] A. Gomel,[3]
T. Quiniou,[2] C. Masoller,[4,*] M. Giudici,[1] and J. R. Tredicce[2,3]
[1]*Université Côte d' Azur, Institut de Physique de Nice, CNRS-UMR 7010, Sophia Antipolis, France*
[2]*Université de la Nouvelle Calédonie, ISEA, BP R4-98851 Nouméa Cedex, Nouvelle Calédonie*
[3]*Universidad de Buenos Aires, Departamento de Física, Intendente Guiraldes 2160, CABA, Buenos Aires, Argentina*
[4]*Departamento de Física, Universitat Politecnica de Catalunya, St Nebridi 22, Barcelona 08222, Spain*

at a parameter value well above the bifurcation point. We test experimentally the occurrence of critical slowing down by applying a perturbation to the accessible control parameter and we find that this perturbation leaves the system behavior unaltered, thus providing no useful information on the occurrence of critical slowing down. The theoretical analysis reveals the reasons why these tests fail in predicting an

cristina.masoller@upc.edu @cristinamasoll1

Outline

- Analysis method: ordinal analysis

- Application to regime transitions
  - Desertification: High $\rightarrow$ low biomass (vegetation 2D data – observational and simulated)

  - Laser turn-on: Low $\rightarrow$ high coherence (speckle images)

  - Eyes closed $\rightarrow$ eyes open (multichannel EEG data)

cristina.masoller@upc.edu @cristinamasoll1

Data analysis method: ordinal analysis

$$\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$$

Possible order relations among three numbers (e.g., 2, 5, 7)

{…2, 5, 7…}

A

B

{…2, 7, 5…}

{…5, 2, 7…}

C

D

{…5, 7, 2…}

{…7, 2, 5 …}

E

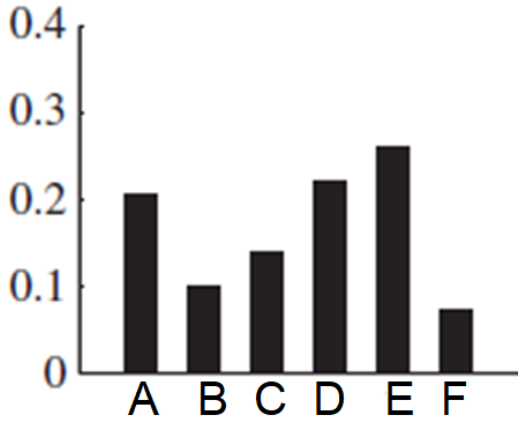
F

{…7, 5, 2…}

Bandt and Pompe: Phys. Rev. Lett. 2002

cristina.masoller@upc.edu @cristinamasoll1

From the frequency of occurrence of the patterns, we calculate the "ordinal probabilities"



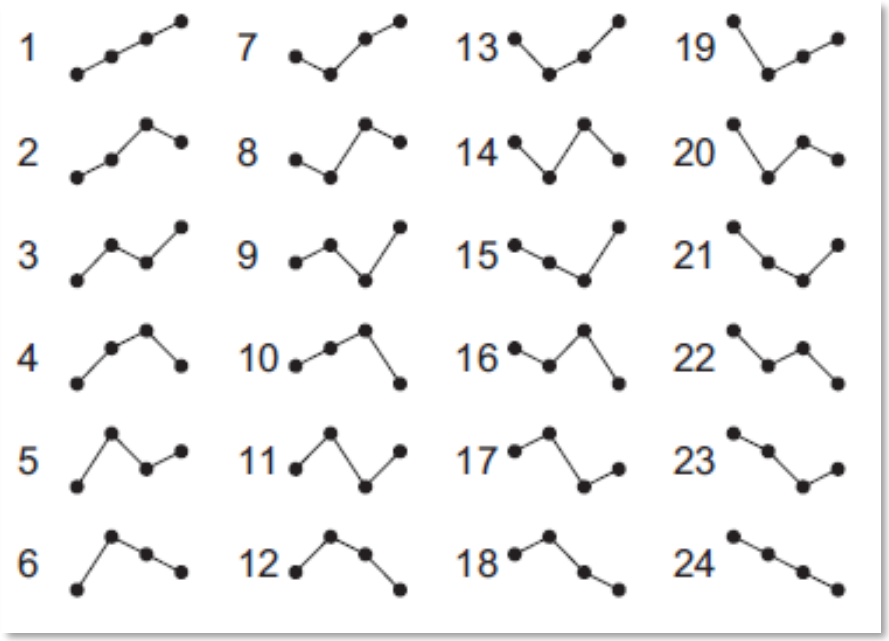
?

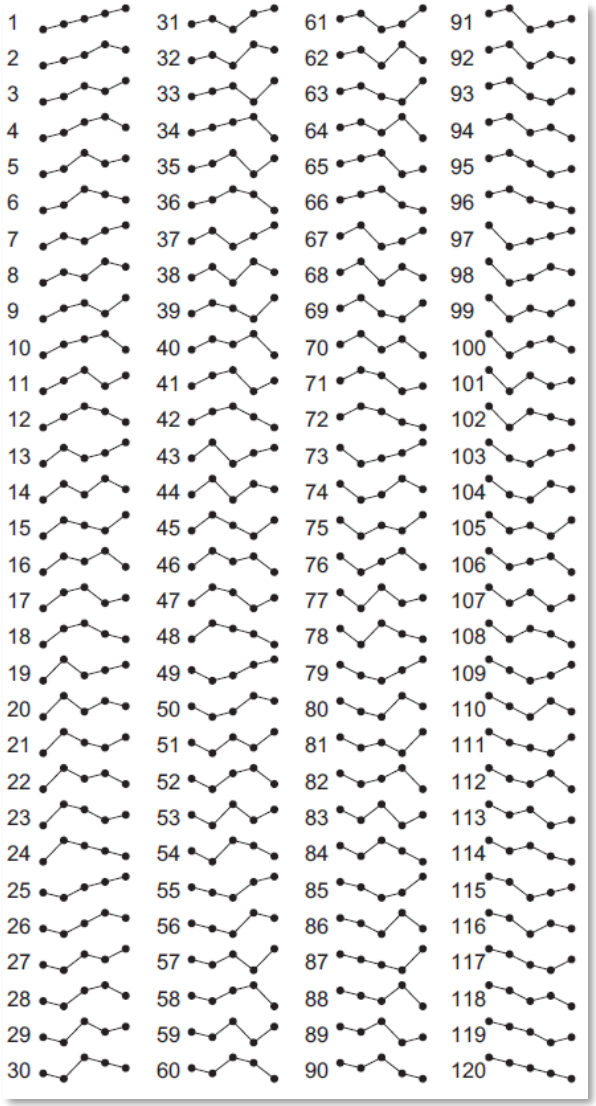
A. Analyze the probability values; use them as features for ML algorithms
B. Analyze "information theory measures" (e.g. entropy)— a form of nonlinear dimensionality reduction.

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

cristina.masoller@upc.edu @cristinamasoll1

The number of ordinal patterns increases as D!





A problem for short datasets.

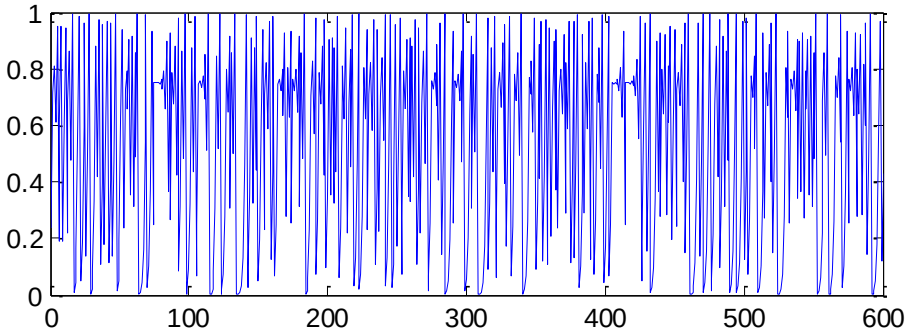
U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

cristina.masoller@upc.edu @cristinamasoll1

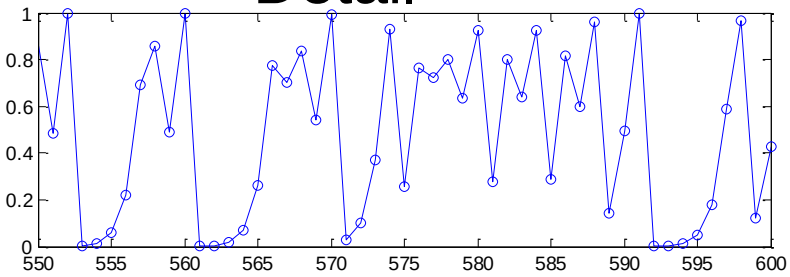
Example: chaotic time series generated with the Logistic map

$$x(i+1) = r\ x(i)[1-x(i)] \quad \text{r=3.99}$$

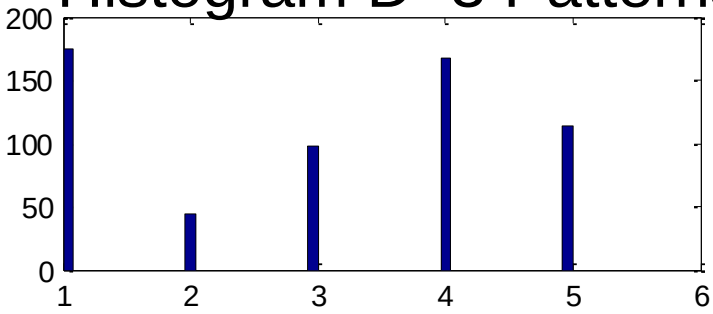
Time series



Detail

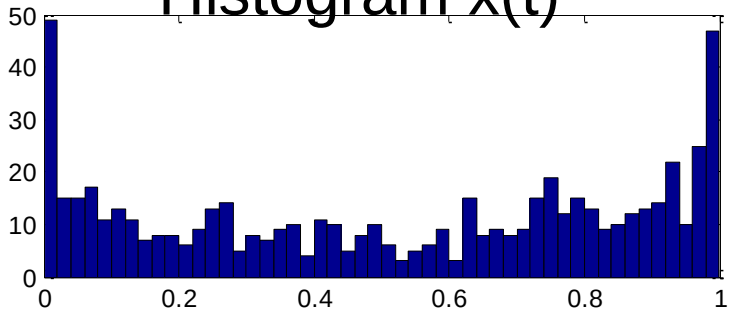


Histogram D=3 Patterns



forbidden

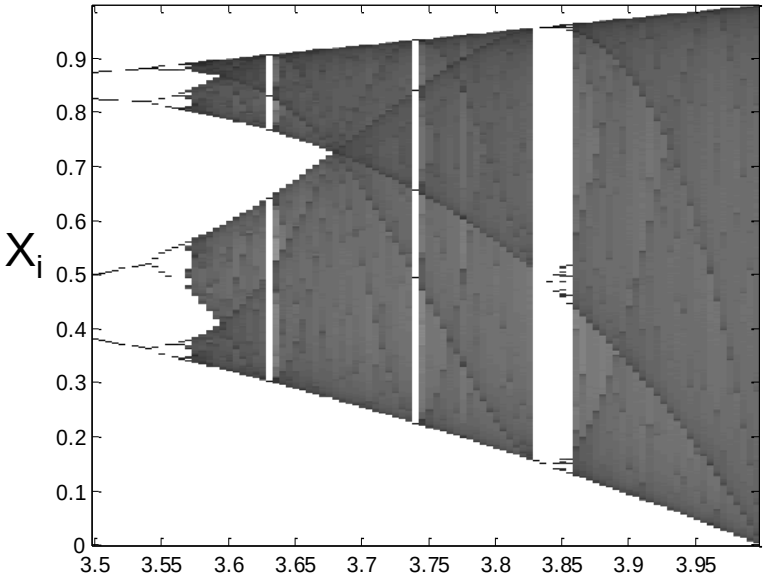
Histogram x(t)



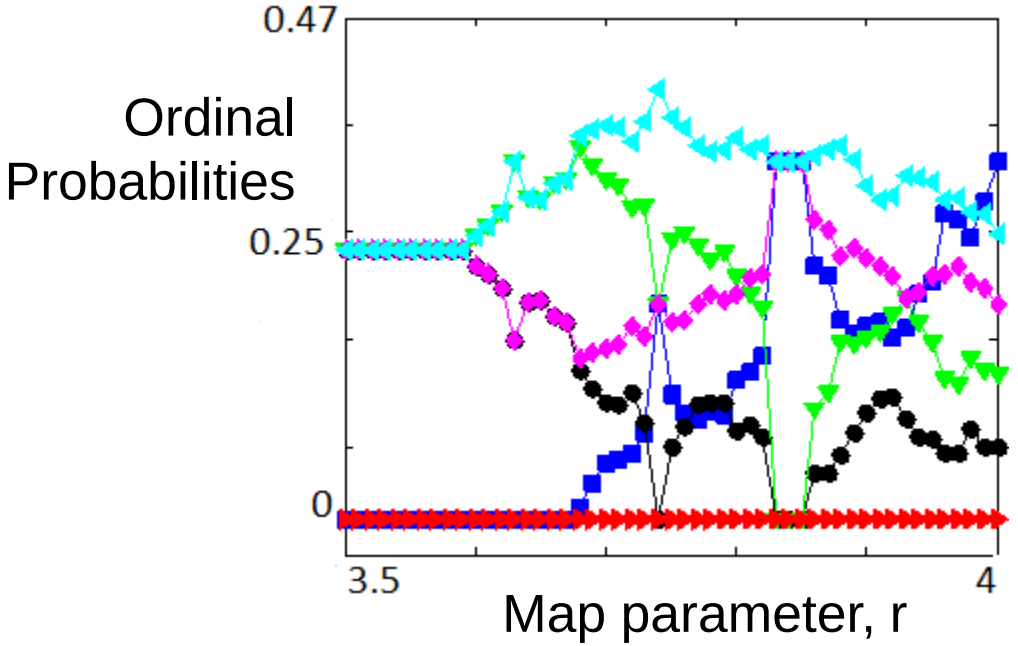
cristina.masoller@upc.edu @cristinamasoll1

“Normal” and “Ordinal” bifurcation diagrams of the Logistic map

Normal bifurcation diagram



Ordinal diagram with D=3

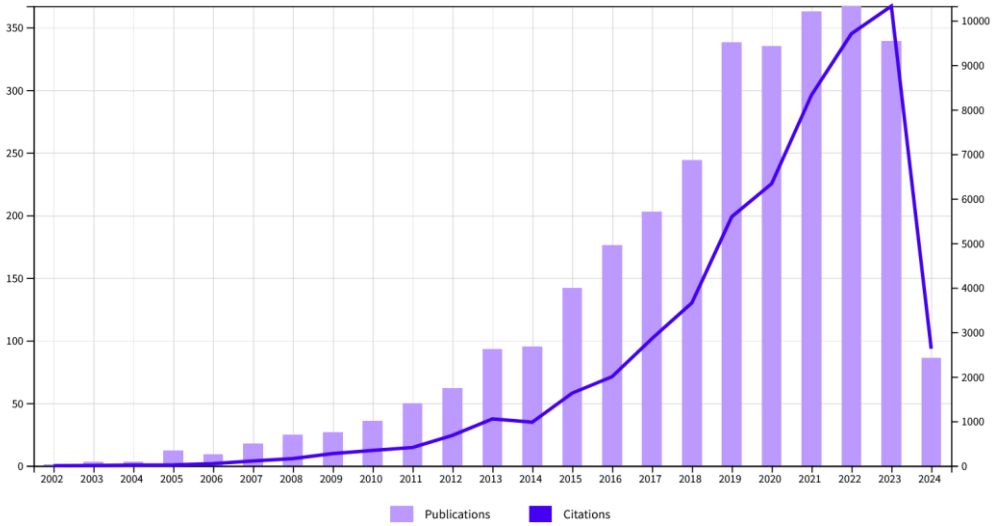


012 021 102 120 201 210

Pattern **210** is always forbidden; pattern **012** is more probable as r increases

cristina.masoller@upc.edu @cristinamasoll1

Ordinal analysis has been extensively used



Citations of the original work by Bandt and Pompe (PRL 2002)

- to test if a model is good for the data,
- to fit the model's parameters,
- to classify different types of data based on similarities of probabilities of ordinal patterns.

Important advantage: robust to noise & outliers.

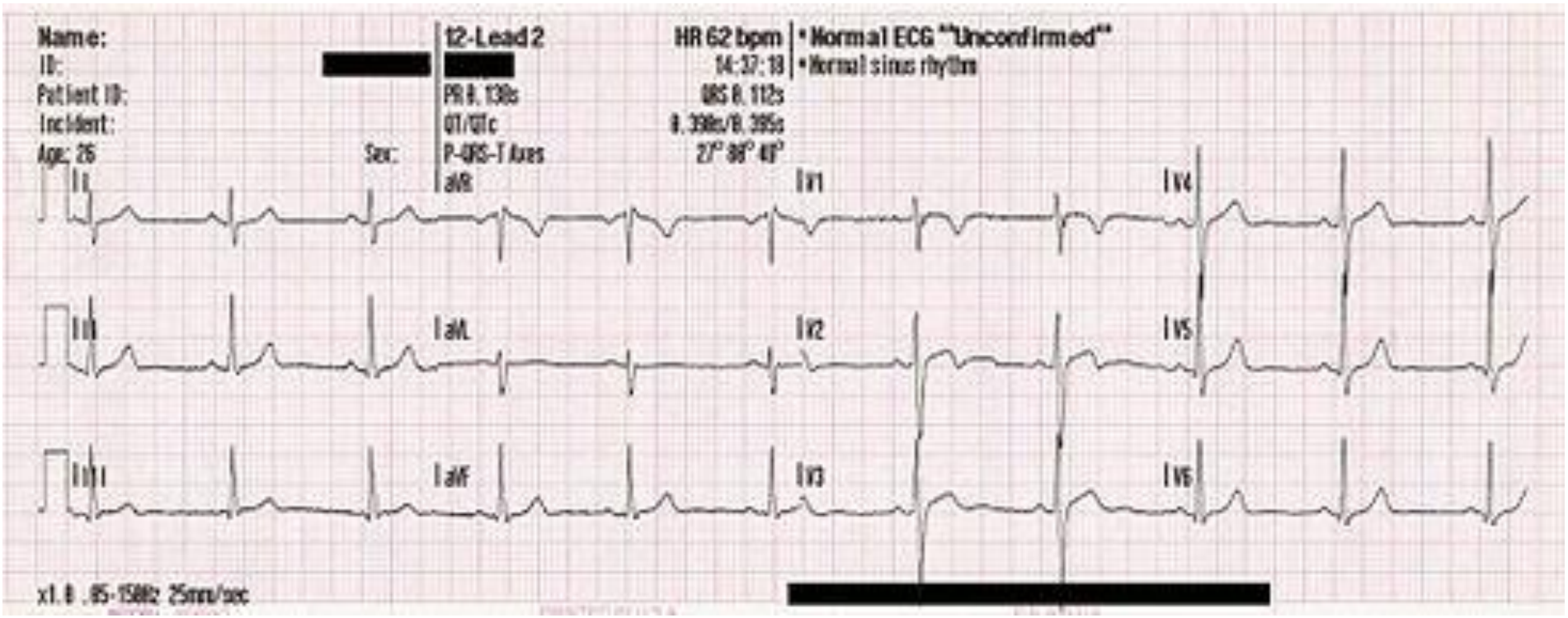
I. Leyva, J. M. Martinez, C. Masoller, O. A. Rosso, M. Zanin, "20 Years of Ordinal Patterns: Perspectives and Challenges", EPL 138, 31001 (2022).

cristina.masoller@upc.edu @cristinamasoll1

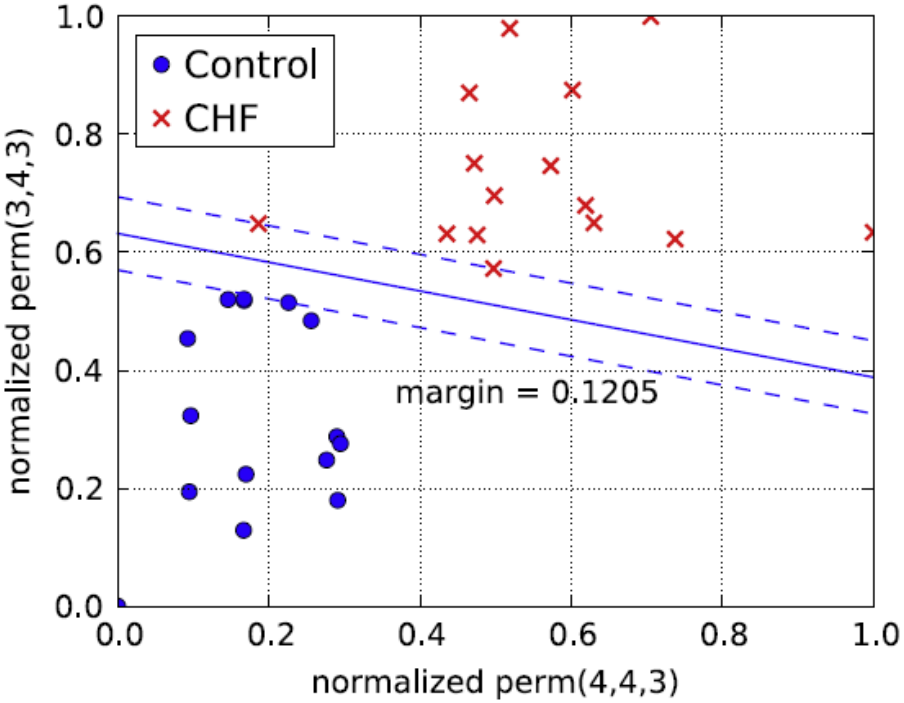
Example of application: analysis of ECG signals

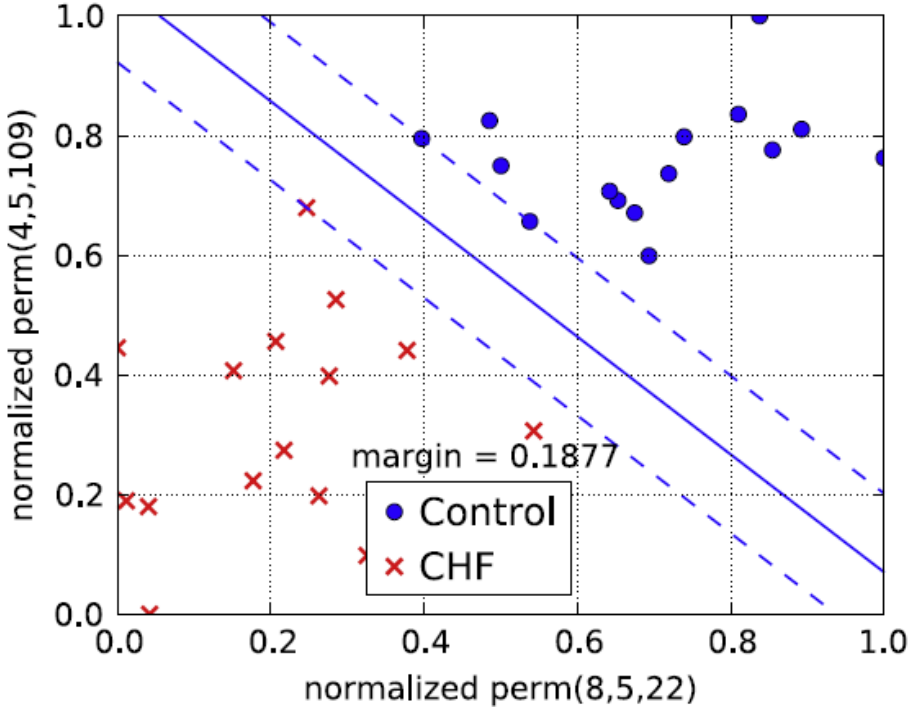
cristina.masoller@upc.edu @cristinamasoll1

Analysis of sequences of inter-beat intervals



cristina.masoller@upc.edu @cristinamasoll1





- Analysis of raw data (statistics of ordinal patterns is almost unaffected by anomalies - outliers)
- The probabilities are normalized with respect to the smallest and largest values occurring in the data set.

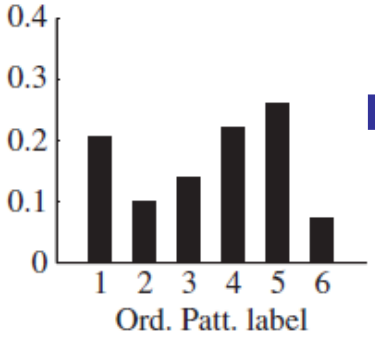
Ulrich Parlitz & coworkers, Computers in Biology and Medicine 42, 319 (2012).

cristina.masoller@upc.edu @cristinamasoll1

Permutation entropy: Shannon's entropy computed from ordinal probabilities

Time series

Ordinal probabilities

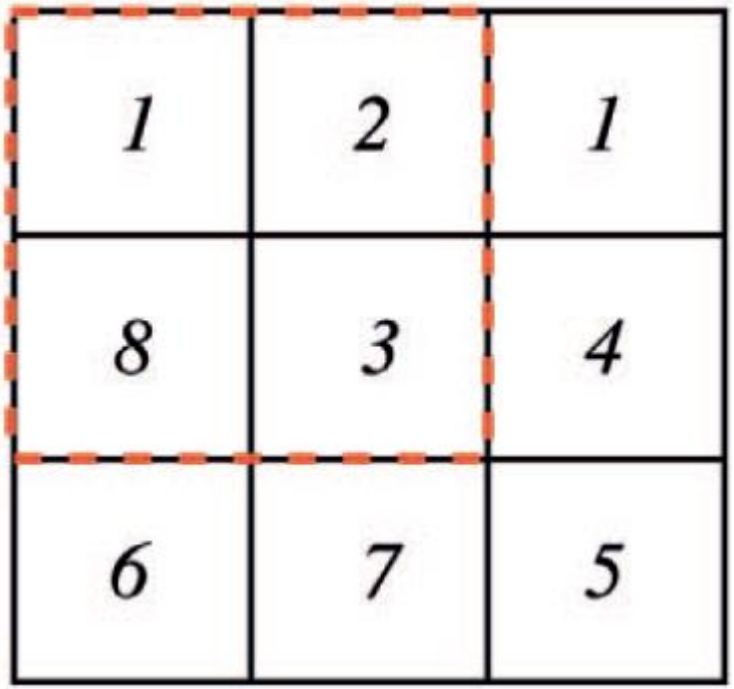


Permutation entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

cristina.masoller@upc.edu @cristinamasoll1

Ordinal analysis of two-dimensional data



1	2
8	3

1283 ⇒0132

2	1
3	4

2134 ⇒1023

8	3
6	7

8367 ⇒3012

3	4
7	5

3475 ⇒0132

Spatial permutation entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

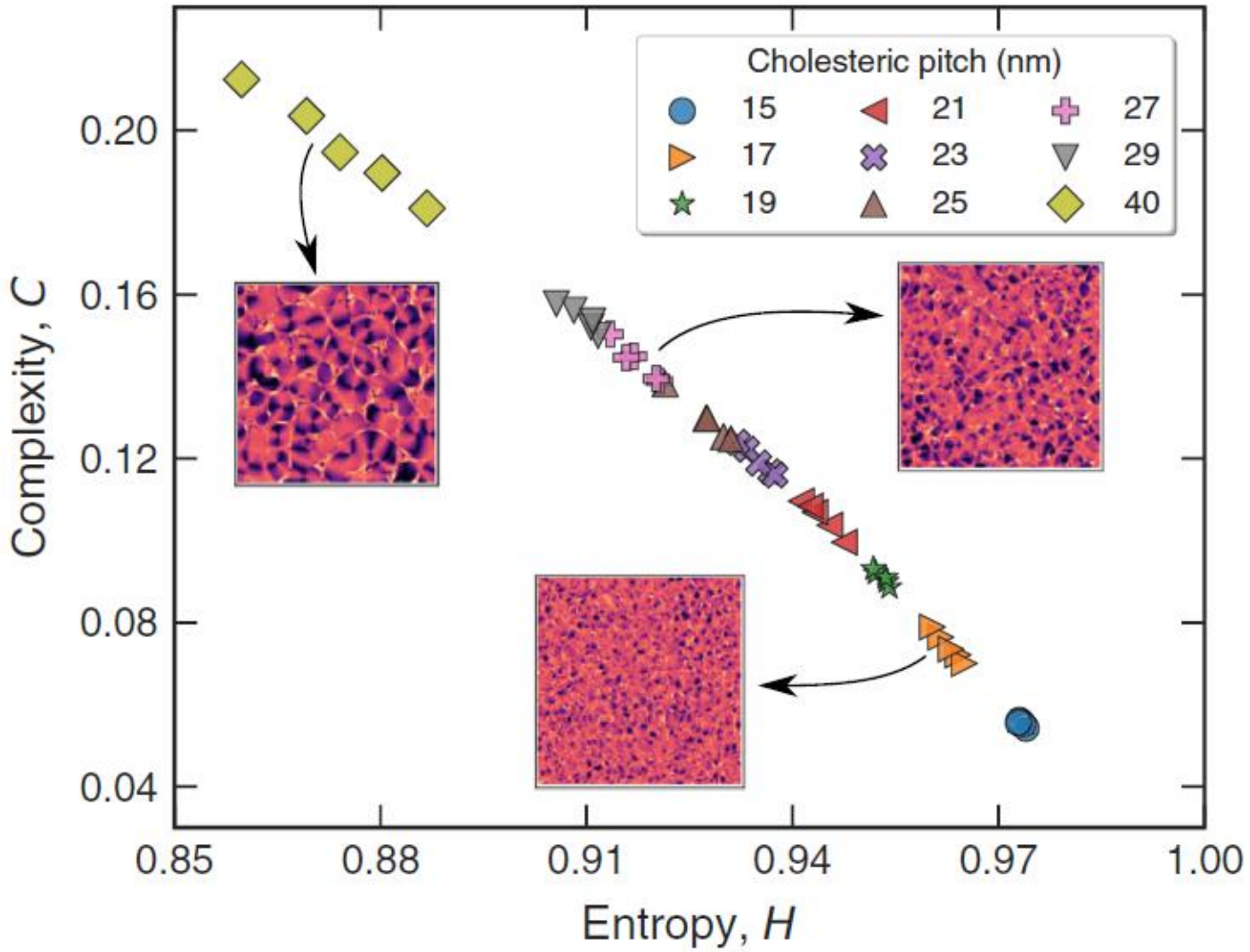
2x2 pixels: 24 possible patterns

Haroldo V. Ribeiro and coworkers, PLoS ONE 7, e40689 (2012)

cristina.masoller@upc.edu @cristinamasoll1

Example of application: analysis of liquid crystal textures

cristina.masoller@upc.edu @cristinamasoll1



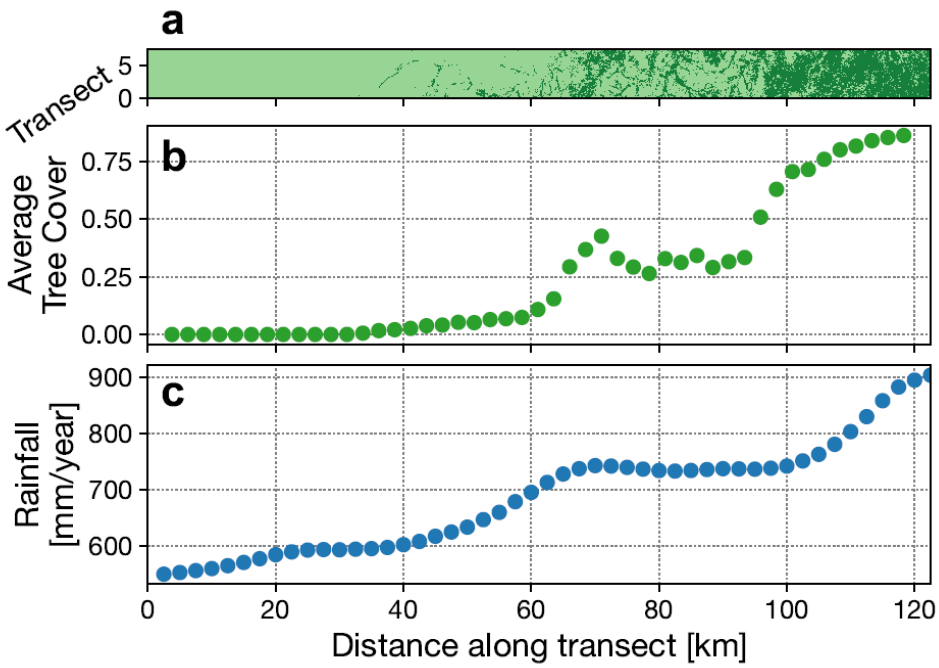
Haroldo V. Ribeiro and coworkers, PRE 99, 013311 (2019)

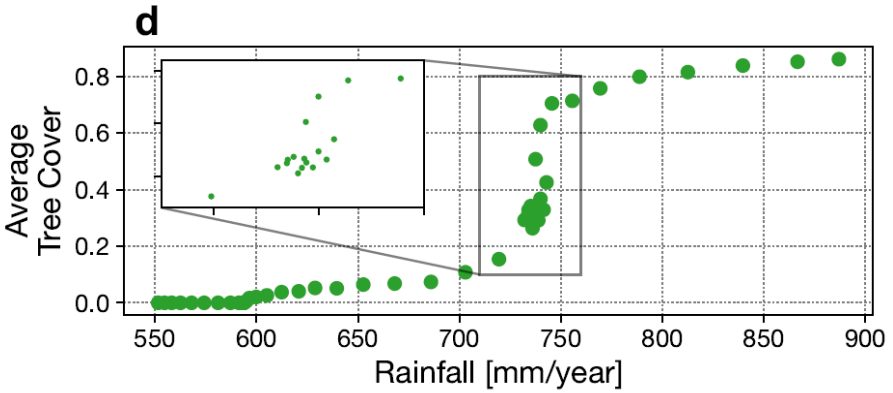
cristina.masoller@upc.edu @cristinamasoll1

Can the variation of the permutation entropy give an early indication of an approaching transition?

cristina.masoller@upc.edu @cristinamasoll1

Analysis of high-resolution vegetation data



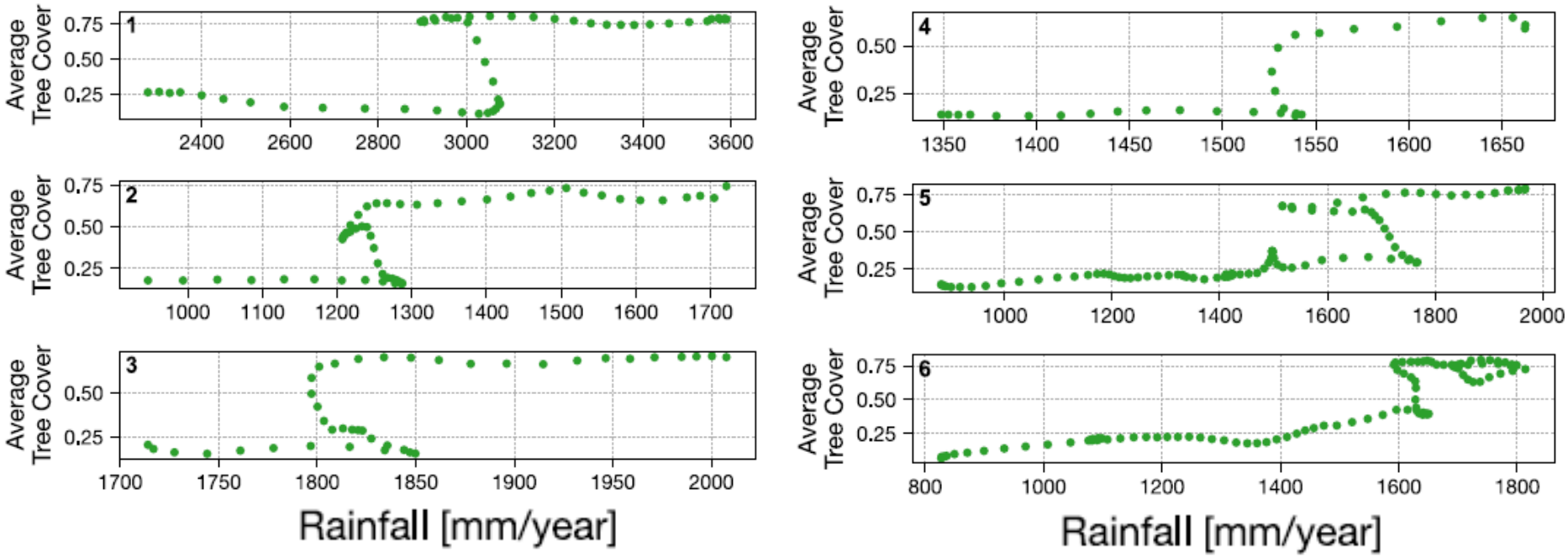


Data from the Serengeti–Mara ecosystem in northern Tanzania and southern Kenya.

G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2022).

cristina.masoller@upc.edu @cristinamasoll1

Analysis of low-resolution satellite vegetation data



cristina.masoller@upc.edu @cristinamasoll1

Two possible indicators of the desertification transition

Spatial Permutation Entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

Ordinal patterns defined by the values of 2x2 pixels

Spatial correlation

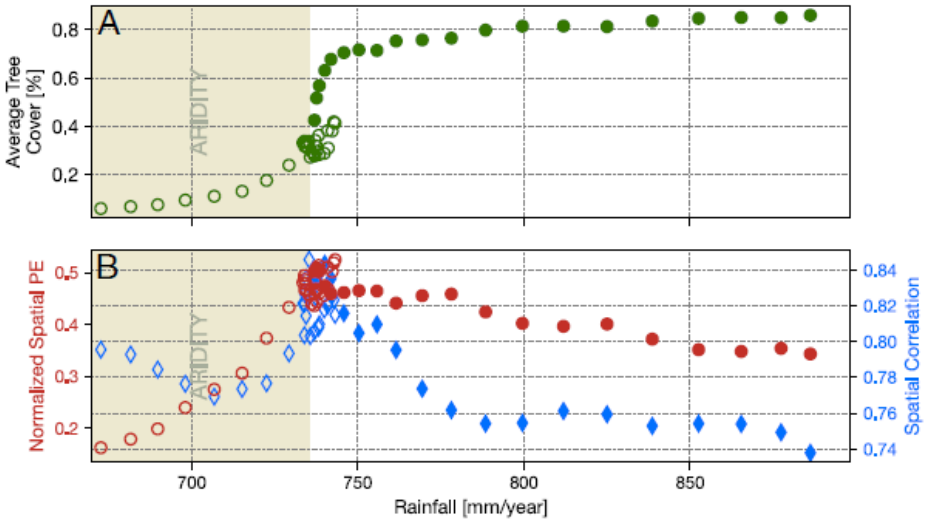
$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

w_{ij}=1 if pixels I and j first neighbors, else w_{ij}=0

cristina.masoller@upc.edu @cristinamasoll1

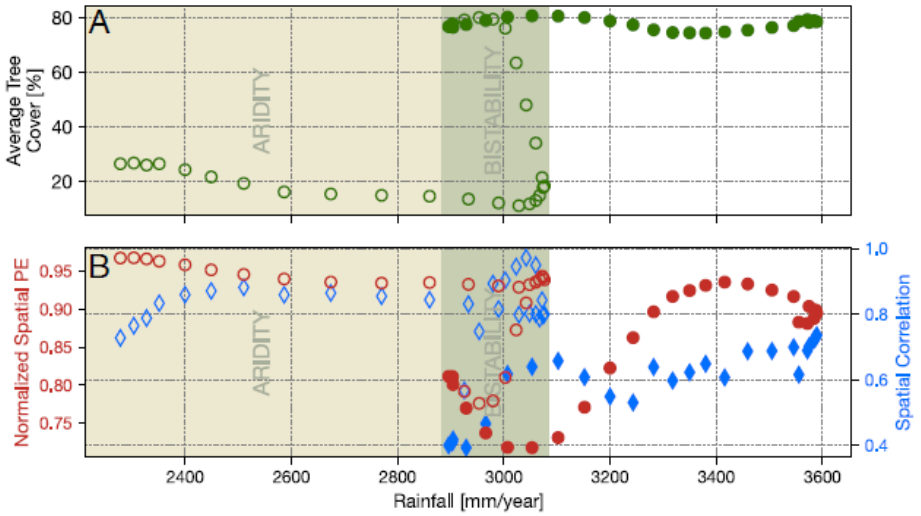
Results

High-resolution data



Low-resolution data

(transect 1; large variability across transects)

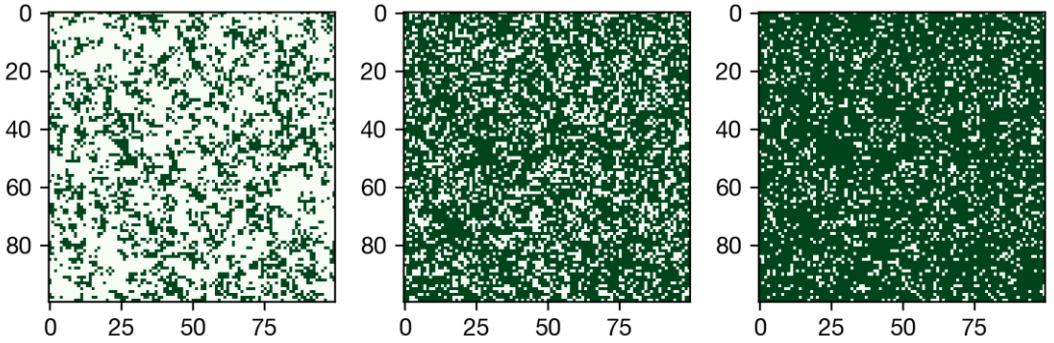


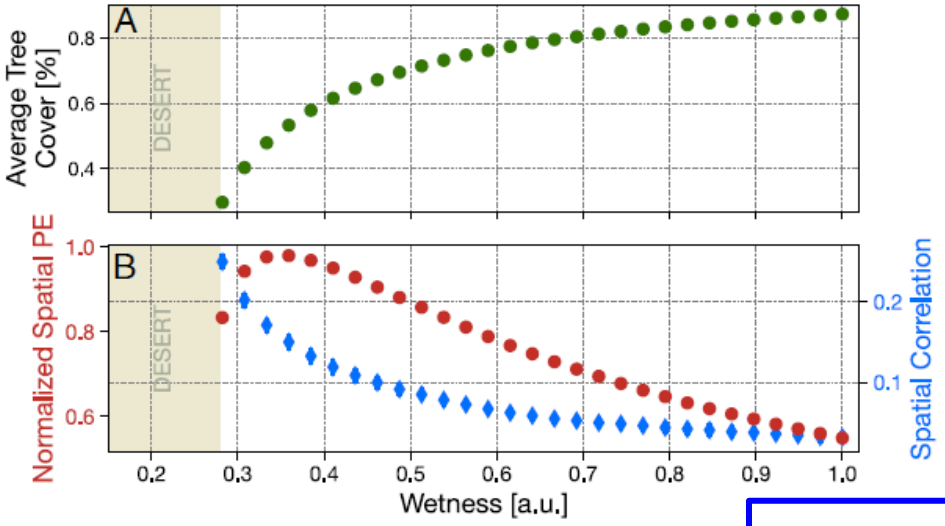
G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2022).

cristina.masoller@upc.edu @cristinamasoll1

To gain insight: simulations of vegetation models

A) Cellular automata model





$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

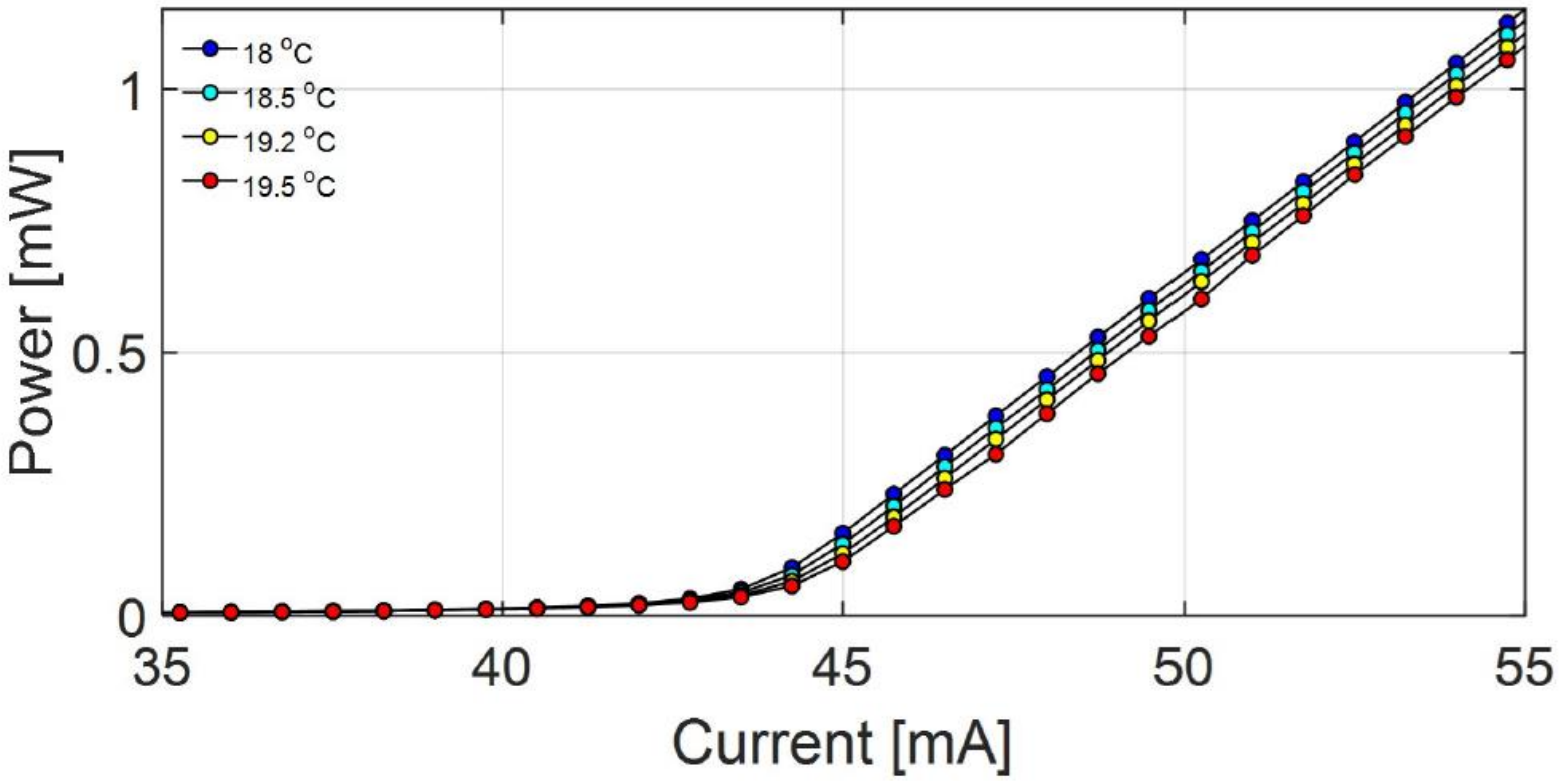
$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

cristina.masoller@upc.edu @cristinamasoll1

Can we test the indicator in controlled experimental data?

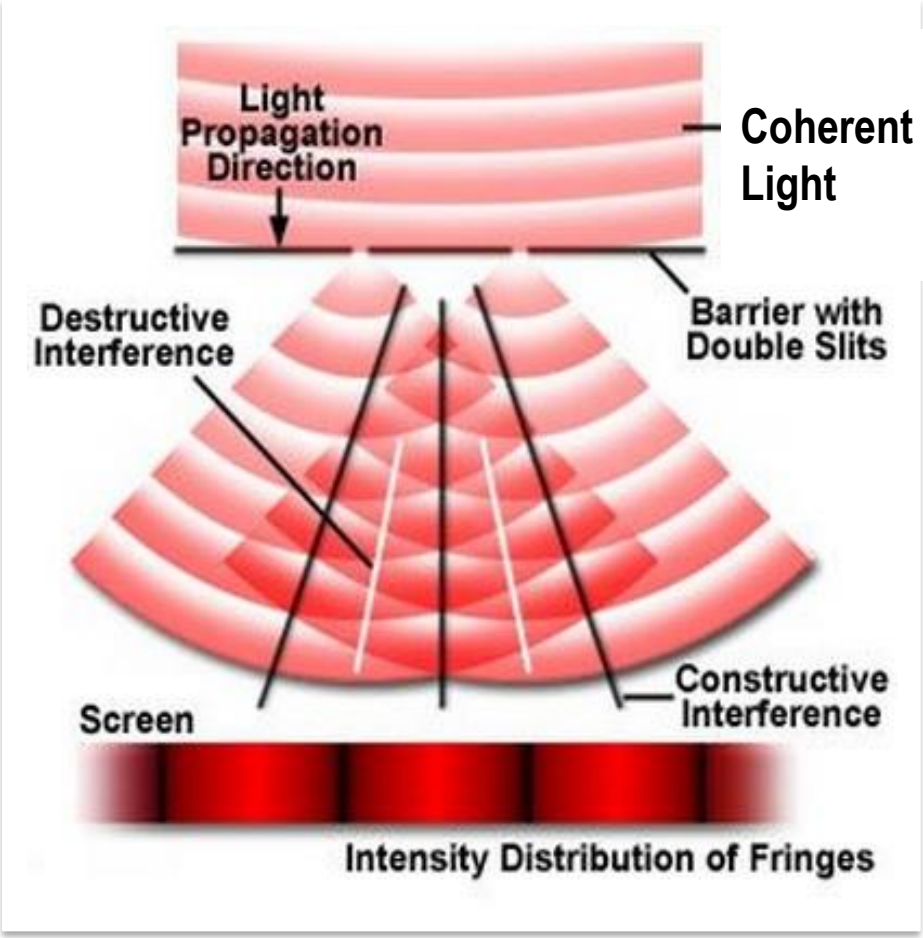
cristina.masoller@upc.edu @cristinamasoll1

Analysis of the turn-on transition of a diode laser



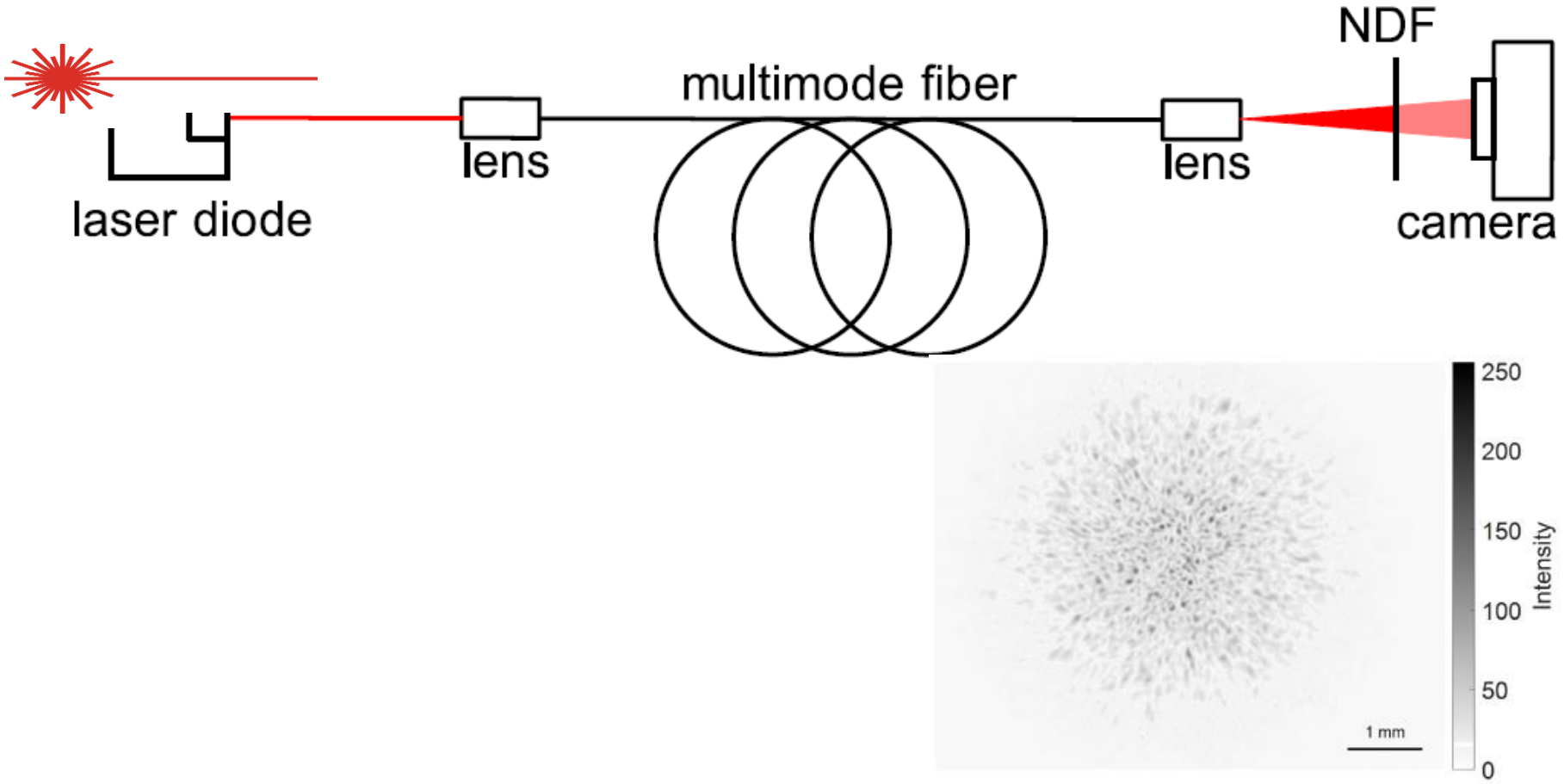
cristina.masoller@upc.edu @cristinamasoll1

Quick reminder of the interference of coherent waves



cristina.masoller@upc.edu @cristinamasoll1

Speckle pattern: generated by the interference / scattering of coherent waves

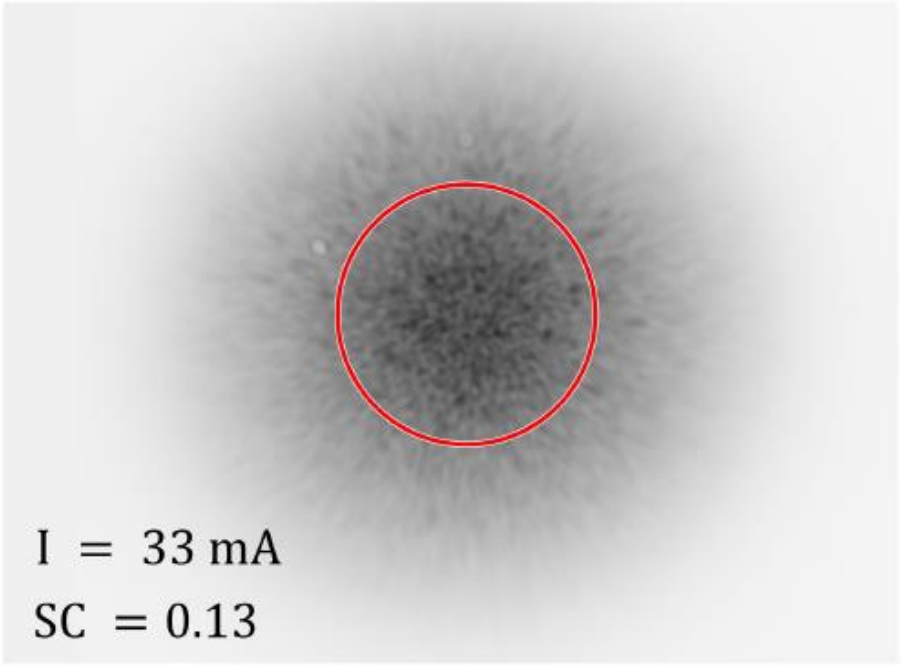


cristina.masoller@upc.edu @cristinamasoll1

The speckle pattern reveals the level of coherence of the laser light

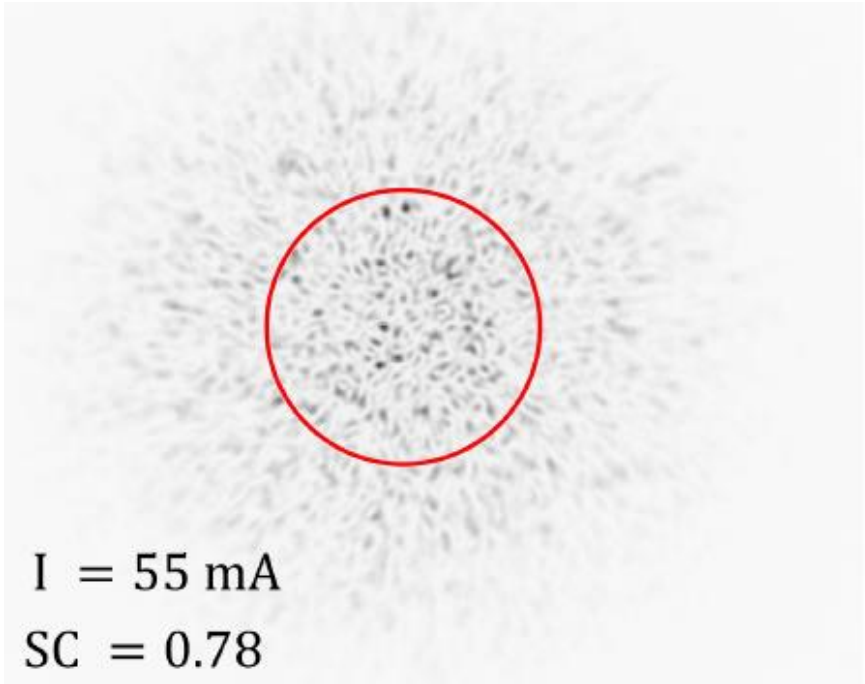
Below threshold

Low light coherence → low speckle contrast



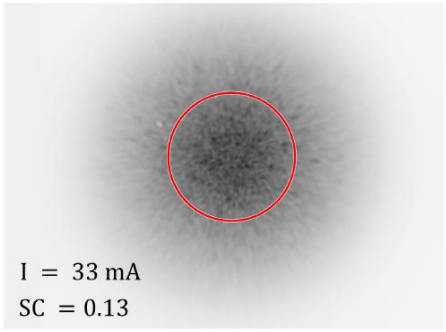
Above threshold

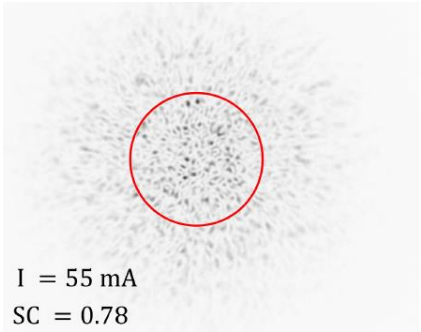
High light coherence→ high speckle contrast

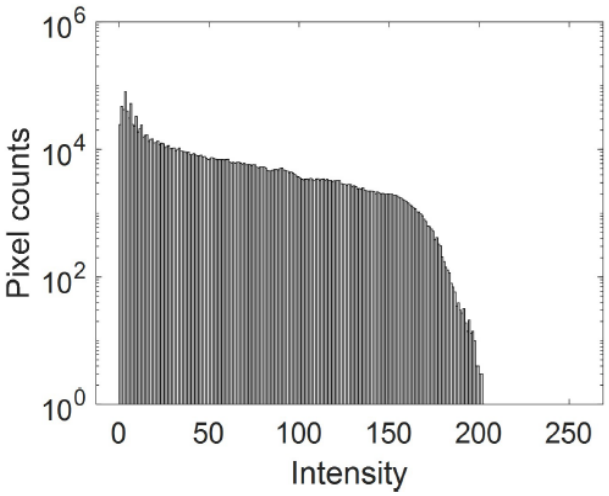


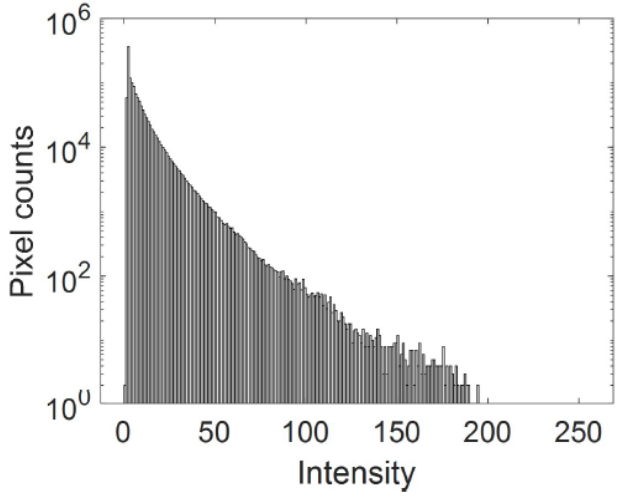
cristina.masoller@upc.edu @cristinamasoll1

Quantification of speckle contrast









$$SC = \sigma/\langle I \rangle$$

cristina.masoller@upc.edu @cristinamasoll1

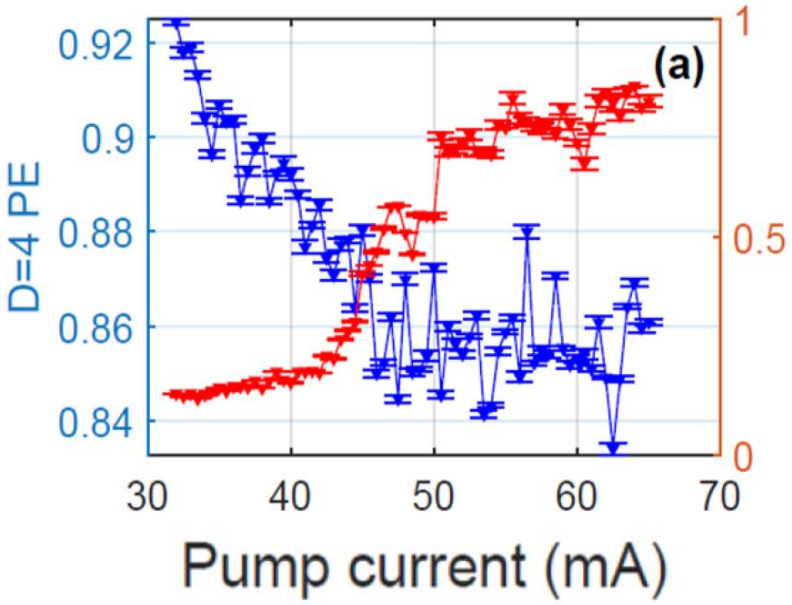
Results

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

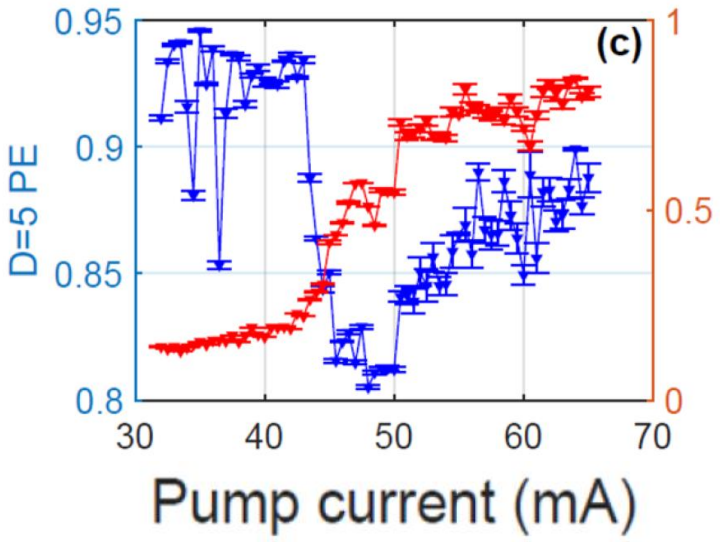
Speckle Contrast

SC = σ/⟨I⟩

Pattern: x x
x x



Pattern: x
x x x
x



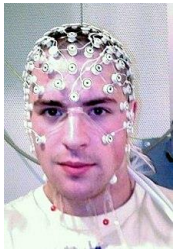
G. Tirabassi et al., "Permutation entropy-based characterization of speckle patterns generated by semiconductor laser light", APL Photonics 8, 126112 (2023).

cristina.masoller@upc.edu @cristinamasoll1

Useful for the analysis of more complex, temporal data?

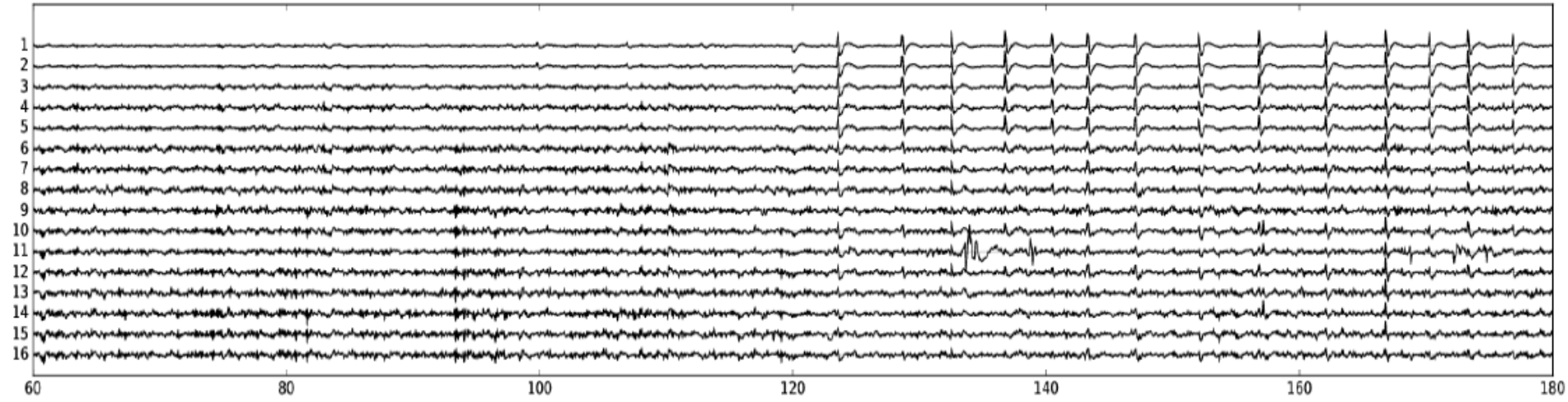
cristina.masoller@upc.edu @cristinamasoll1

Analysis of EEG signals recorded from healthy subjects.



Eyes closed

Eyes open



Time (seconds)

TABLE I. Description of the datasets used.

	DTS1	DTS2
Sampling rate (Hz)	256	160
Time task (seg)	120	60
Total points	30 720	9600
Number of electrodes	16	64
Number of subjects	71	109

DTS1: Britbrain (Zaragoza)
DTS2: Physionet

cristina.masoller@upc.edu @cristinamasoll1

The Permutation Entropy increases in the “eyes open” state

$$\langle \mathrm{PE} \rangle = \frac{1}{N[\text{electrodes}]} \sum_i \mathrm{PE}^i$$

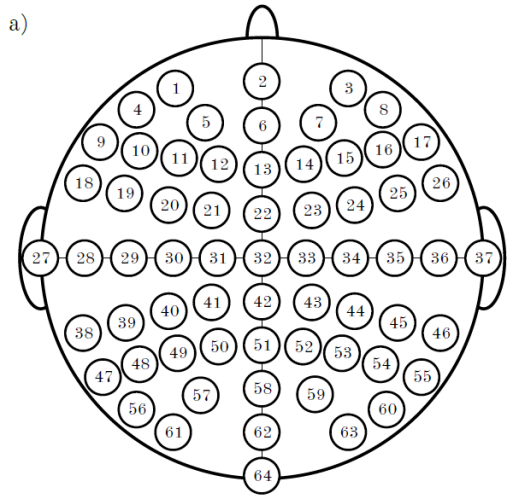
PE was calculated with patterns of length 4 (# of possible patterns 24) in time windows containing >4000 patterns

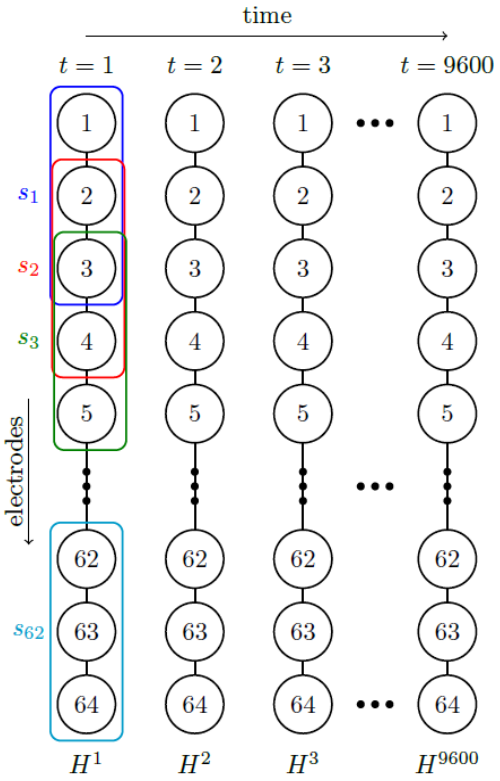
Gray region: σ of $\langle \mathrm{PE} \rangle$ values across subjects

C. Quintero-Quiroz et al., “Differentiating resting brain states using ordinal symbolic analysis”, Chaos 28, 106307 (2018).

cristina.masoller@upc.edu @cristinamasoll1

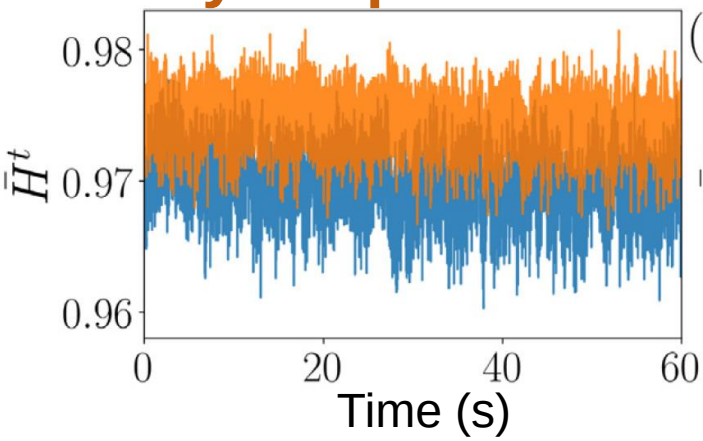
Spatial approach to compute the Permutation Entropy





Eyes closed

Eyes open



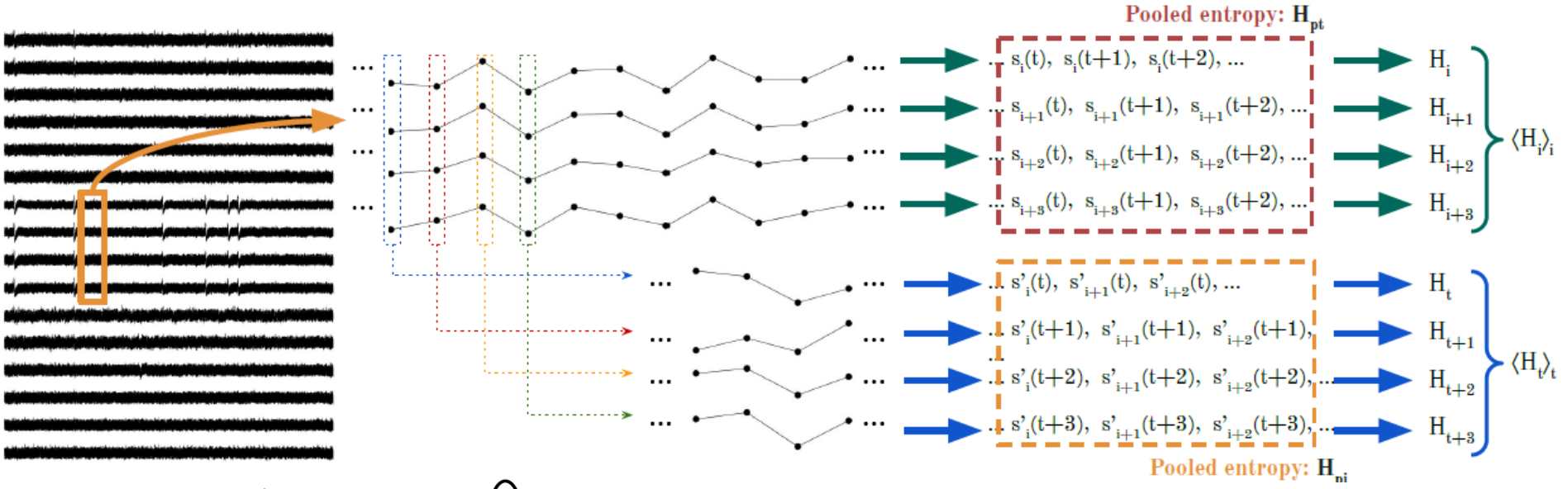
(H_t is averaged over subjects)

At each time: data values of 64 channels ⇒ 62 ordinal patterns to calculate 6 probabilities.

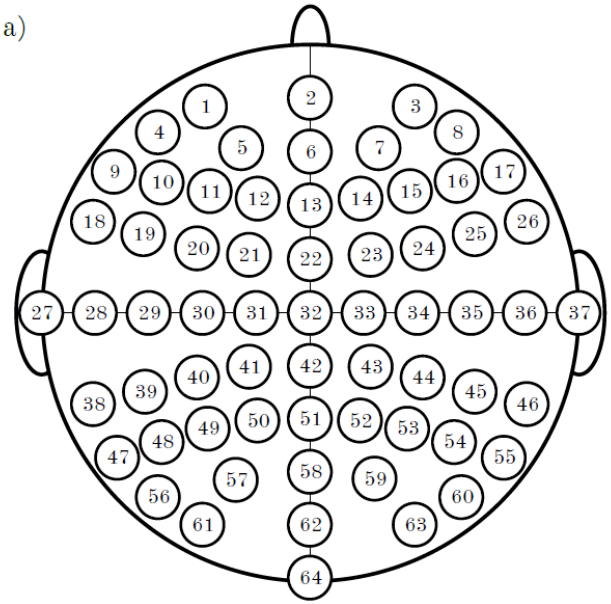
Bruno Boaretto and coworkers, "Spatial permutation entropy distinguishes resting brain states", Chaos, Solitons & Fractals 171, 113453 (2023).

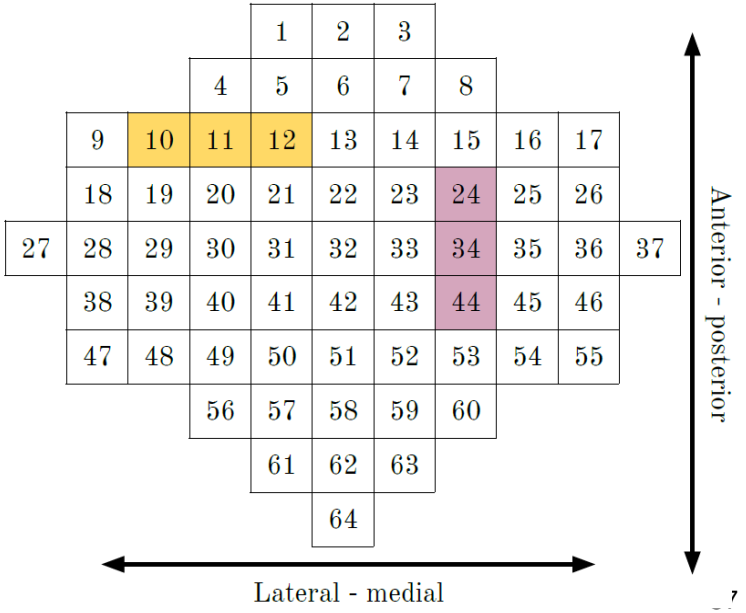
cristina.masoller@upc.edu @cristinamasoll1

Four approaches to calculate the permutation entropy



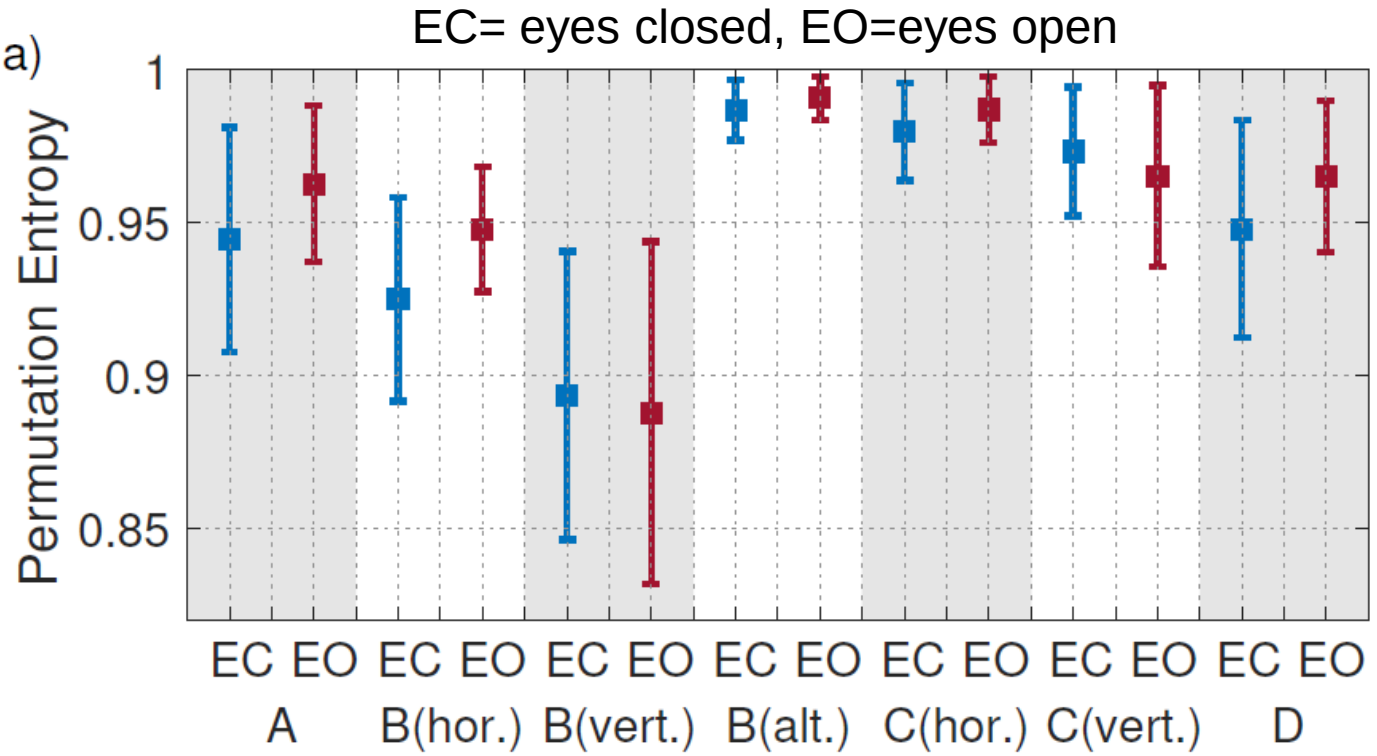
a)





cristina.masoller@upc.edu @cristinamasoll1

Results



error bars: σ over 107 subjects

A: temporal coding
B: spatial coding (horizontal, vertical, and alt. symbols)
C: spatial pooling (horizontal and vertical symbols)
D: temporal pooling

J. Gancio, C. Masoller, G. Tirabassi, "Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches", Chaos 34, 043130 (2024).

cristina.masoller@upc.edu @cristinamasoll1

Take home messages and outlook

Ordinal analysis methods allow us to uncover patterns and relationships in data, which can characterize (and sometimes predict) the behavior of complex systems.

1. We have shown, in empirical, experimental and synthetic data, that the permutation entropy may give an early indication of an approaching transition.

2. It can be used to find differences in high-dimensional datasets.

3. Ongoing work: synchronization transition, climate data

cristina.masoller@upc.edu @cristinamasoll1

- G. Tirabassi and C. Masoller, "*Entropy-based early detection of critical transitions in spatial vegetation fields*", PNAS 120, e2215667120 (2022).

- G. Tirabassi, M. Duque et. al,*"Permutation entropy-based characterization of speckle patterns generated by semiconductor laser light*", APL Photonics 8, 126112 (2023).

- J. Gancio et. al, "*Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches*", Chaos 34, 043130 (2024).

Thank you for your attention!

cristina.masoller@upc.edu @cristinamasoll1